PROBLEMS

1. Water at 20 °C falls from a height of 854 meters. If the whole energy is used in increasing the temperature, find out the final temperature. Specific heat of water is 4200 J K⁻¹ kg⁻¹.

SOLUTION

$T_1 = 20\ °C = 273 + 20 = 293\ K$ h = 854 m

$C = 4200\ J\ K^{-1}\ kg^{-1}$ $T_2 = ?$

Since $\Delta Q = C\ m\ \Delta T$ (1)

Also $\Delta Q = m\ g\ h$ (2)

Comparing Eq. 1 and Eq. 2, we get;

$$\Delta Q = C\ m\ \Delta T = m\ g\ h$$

$$C\ (T_2 - T_1) = g\ h$$

$$T_2 = (g\ h/C) + T_1 = (9.8 \times 854\ /\ 4200) + 293$$

$$T_2 = (1.99) + 293 = 294.99\ K$$

$$T_2 = (294.99 - 273)\ °C = 22\ °C$$

2. 25200 J of heat is supplied to the system while the system does 6000 J of work. Calculate the change in internal energy of the system.

SOLUTION

$\Delta Q = 25200\ J$ $\Delta W = 6000\ J$ $\Delta U = ?$

Since $\Delta Q = \Delta U + \Delta W$

$\Rightarrow$ $\Delta U = \Delta Q - \Delta W = 25200\ J - 6000\ J = 19200\ J$

3. A sample of ideal gas is uniformly heated at constant pressure. If the amount of 180 J of heat is supplied to the gas (γ = 1.41), Calculate the

- **Change in internal energy of the gas**
- **Work done by the gas.**

SOLUTION

$\Delta Q_p = 180\ J$ $\Delta U = ?$ $\Delta W = ?$

$\gamma = 1.41$

As $\Delta Q_p = n\ C_p\ \Delta T$ (1)

Since $\gamma = \dfrac{C_P}{C_V}$ $\Rightarrow$ $C_p = \gamma C_v$ (2)

Putting Eq. 2 in Eq. 1, we get;

$$\Delta Q_p = n\ \gamma C_v\ \Delta T = \gamma\ (n\ C_v\ \Delta T) = \gamma\ \Delta U$$

$$180\ J = 1.41\ \Delta U$$

$\Rightarrow$ $\Delta U = 180\ J\ /1.41 = 127.66\ J$

Also $\Delta W = \Delta Q_p - \Delta U = 180\ J - 127.66\ J = 52.34\ J$

4. 5 moles of oxygen is heated at constant volume from 10 °C to 20 °C. What will be the change in the internal energy of the gas? The specific heat of oxygen at constant pressure is 8 Cal mol⁻¹ °C⁻¹. R = 8.36 J mol⁻¹ °C⁻¹

SOLUTION

$n = 5\ mol$ $T_1 = 10\ °C$ $T_2 = 20\ °C$

$\Delta T = T_2 - T_1 = 20\ °C - 10\ °C = 10\ °C$

$\Delta U = ?$

$C_p = 8$ Cal mol^{-1} °C^{-1} 1J = $\frac{1}{4.186}$ cal

R = 8.36 J mol^{-1} °C^{-1} = (8.36/4.186) Cal mol^{-1} °C^{-1}

R = 2 Cal mol^{-1} °C^{-1}

$\Delta U = \Delta Q_v = n\, C_v \Delta T$ $C_p - C_v = R$

$\Delta U = n\,(C_p - R)\,\Delta T$ = 5 mol × (8 – 2) Cal mol^{-1} °C^{-1} × 10 °C

ΔU = 5 mol × 6 Cal mol^{-1} °C^{-1} × 10 °C

ΔU = 300 Cal

5. Find the efficiency of a Cornot's heat engine working between the steam and ice points?

SOLUTION

Temperature of HTR, T_1 = 100 °C = (273 + 100) K = 373 K

Temperature of LTR, T_2 = 0 °C = (273 + 0) K = 273 K

$$\eta = 1 - \frac{T_2}{T_1} = \eta = 1 - \frac{273}{373} = 0.268$$

$\eta = 0.268 \times 100\,\% = 26.8\%$

6. A Carnot heat engine absorbs 2000 J of heat from the source of heat engine at 227 °C and rejects 1200 J of heat during each cycle to sink. Calculate.

- **Efficiency of engine**
- **Temperature of sink**
- **Amount of work done during each cycle.**

SOLUTION

Q_1 = 2000 J T_1 = 227c + 273 = 500 K Q_2 = 1200 J

η =? T_2 =? ΔW =?

Since $$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{1200}{2000} = 0.4 = 0.4 \times 100\,\% = 40\,\%$$

Now for temperature of sink; C = K – 273

$$\eta = 1 - \frac{T_2}{T_1}$$

$\Rightarrow$ $$T_2 = (1 - \eta)\, T_1 = \left(1 - \frac{40}{100}\right) 500\text{ K} = 300\text{ K} = 27\text{ °C}$$

The work done during each cycle is;

$\Delta W = Q_1 - Q_2$ = 2000 J – 1200 J = 800 J

7. In a refrigerator, heat from inside at 277 K is transferred to a room at 300 K. How many joules of heat will be delivered to the room for each joule of electric energy consumed ideally?

SOLUTION

T_2 = 277 K T_1 = 300 K

Electrical energy consumed W = 1 J

Heat delivered to the room Q_1 =?

Since the Coefficient of performance is;

$$\eta = \frac{T_2}{T_1 - T_2} = \frac{277}{300 - 277} = 12.04$$

Let Q2 amount of heat is absorbed from inside of refrigurator at the cost of W amount of work done. Now we can calculate the heat rejected ot the room as follow;

$$\eta = \frac{Q_2}{Q_1 - Q_2} = \frac{Q_2}{W} \qquad (W = Q_1 - Q_2)$$

$\Rightarrow$ $Q_2 = \eta\, W$ = 12.04 × 1 J = 12.04 J

Now the heat delivered to the room is;

$$Q_1 = Q_2 + W = 12.04\ J + 1\ J = 13.04\ J$$

8. What is the least amount of work that must be performed to freeze one gram of water at 0 °C by means of a refrigerator? Take the temperature of the surrounding as 37°C. How much heat is passed on to the surrounding during this process?

SOLUTION

$m = 1\ g = 10^{-3}\ kg$

Latent heat of Fusion of water, $H_f = 336000\ J/Kg$

$T_2 = 0\ °C + 273 = 273\ k \quad T_1 = 37°C + 273 = 310\ K$

$W = ? \qquad Q_1 = ?$ (1)

$Q_2 = m\,H_f = 10^{-3} \times 336000 = 336\ J$

$$\eta = \frac{T_2}{T_1 - T_2} = \frac{273}{310 - 273} = 7.37$$

$$W = \frac{Q_2}{\eta} = \frac{336}{7.37} = 45.53\ J$$

The heat passed on to the environment is;

$$Q_1 = Q_2 + W = 336\ J + 45.53\ J = 381.53\ J$$

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9. Calculate the change in entropy when 10 kg of water is heated from 90°C to 100 °C? Specific heat of water = 4180 J mol⁻¹ K⁻¹.

SOLUTION

$m = 10\ Kg = 0.01\ kg$

Molecular mass of H_2O, $M = 2 \times 1 + 16 \times 1 = 18\ g/mol$

$T_1 = 100°\ C = (100 + 273)\ K = 373\ K$

$T_2 = 90°C = (90 + 273)\ K = 363\ K$

$C = 4180\ J\ mol^{-1}\ K^{-1}$

$\Delta S = ?$

Since

$$\Delta Q = n\,C\,\Delta T = \frac{m}{M} C\,\Delta T$$

$$\Delta Q = \frac{0.01}{18} \times 4180 \times (T_2 - T_1) = 2.32 \times (373 - 273) = 23.22\ J$$

Now

$$\Delta S = \frac{\Delta Q}{T_2} - \frac{\Delta Q}{T_1} = \frac{23.22}{363} - \frac{23.22}{373} = 0.00171 = 1.71 \times 10^{-3}\ J/K$$

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10. A system absorbs 1176 J of heat and at the same time does 352.8 cal of external work. Find the change in internal energy of the system? Find the change in internal energy in the system when it absorbs 1176 J of heat while 84 J of work is done on the system? What will be the change in internal energy of the gas from 210 J of heat is removed at constant volume? · 1135.8

SOLUTION

1. $\Delta Q = 1176\ J \qquad \Delta W = -352.8\ cal$

$$\Delta W = -\frac{352.8\ cal}{4.186\ cal/J} = -84\ J$$ (work done by the system is negative)

$\Delta U = ?$