POWER SYSTEM ANALYSIS

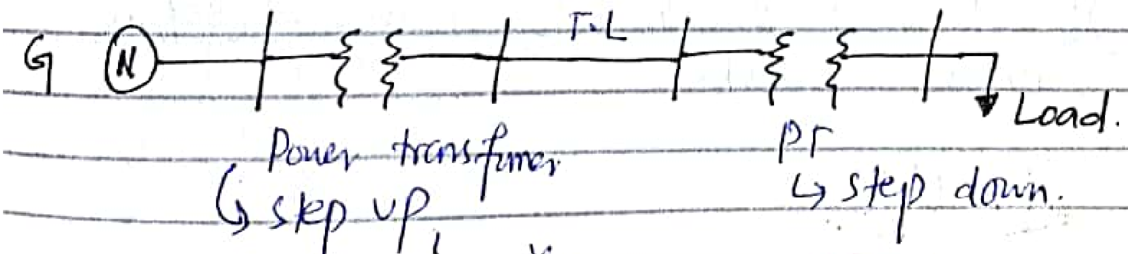
Power System 1

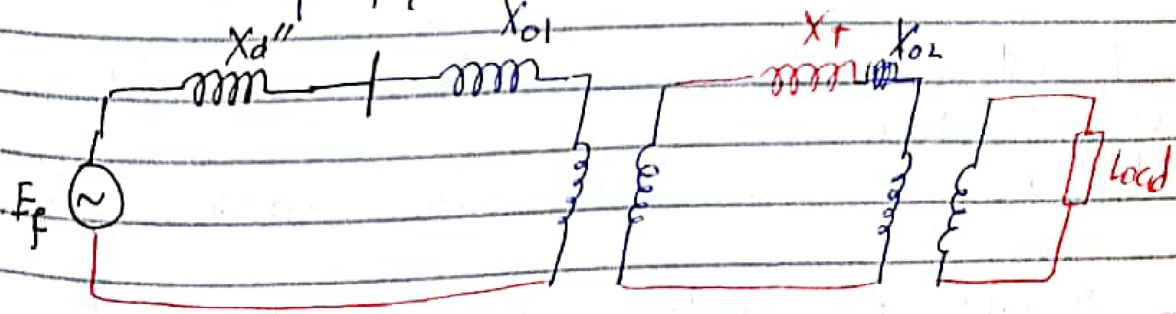
- Transmission line parameter.
- Performance of transmission line.
- Static power equation.
- Voltage control.
- Power factor correction.
- Distribution system.

Power System 2

- Fault.
- Stability.
- Load flow.

Single Line Diagram





We do not have a series network b/c of transformer; so it is very difficult to find the solution. (current and voltages).

The solution to the problem is to represent the system in per unit.

$$\text{Per unit} = \frac{\text{Actual value}}{\text{Base value}}$$

Advantages of per unit?

- The ratings of devices are in KVs, MVAs so the calculations may be difficult.
  In per unit, the range would be near to 1 (somewhat less or greater).

- The reactance / resistance of transformer in pu is same in primary and secondary.
  which means that if we do not draw the magnetiz part of circuit ( ), still it would be fine, b/c the reactance on both sides (X_{01} and X_{02}) is same; so we can draw a series circuit.

Per Unit

Actual value will have a unit, ie say current eg 2A, 5A etc.
Base value will be the rated current.

Similarly for impedance, the actual value will be in ohms whereas the base value will depend on its MVA and voltage rating.

Say impedance; $Z_{pu} = \frac{Z(\Omega)}{Z_b}$

where

$$Z_b = \frac{V_b}{I_b} = \frac{V_{rated}}{I_{rated}} \times \frac{V_{rated}}{V_{rated}} = \frac{V_{rated}^2}{VA}$$

$$\Rightarrow \boxed{Z_{pu} = \frac{Z(\Omega)}{V_{rated}^2} \times (VA)}$$

Normally the ratings are in MVA, not in VA.

$$\Rightarrow \boxed{Z_{pu} = \frac{Z(\Omega) \times MVA}{(KV)^2}}$$

pu values are unitless.

* pu quantities (values) are defined in per phase only. * Its not a 3ϕ value.

Transformers may be connected in star (Y) or delta (Δ).

Y Connection

As $Z_{pu} = Z(\Omega) \times \frac{MVA}{(KV)^2}$ $\rightarrow 3\phi$ MVA $\rightarrow$ ~~per phase~~ $\rightarrow$ phase voltage

$$= Z(\Omega) \times \frac{MVA/3}{\left(\frac{KV_L}{\sqrt{3}}\right)^2}$$

$$\boxed{Z_{pu} = Z(\Omega) \times \frac{3\phi\ MVA}{(KV_L)^2}}$$

Δ Connection

$$Z_{pu} = Z_{(\Omega)} \times \frac{MVA}{(kV)^2} = Z_{(\Omega)} \times \frac{MVA/3}{(kV_L)^2}$$

b/c ϕ and L V are equal.

$$\Rightarrow \boxed{Z_{pu} = \frac{1}{3} Z_{(\Omega)} \times \frac{MVA}{(kV_L)^2}}$$

Y Connection

As $Z_{pu} = Z_{(\Omega)} \times \frac{MVA}{(kV)^2}$

ohmic value does not depend on any thing. (MVA or kV ratings)

$$Z_{pu} \propto 3\phi \text{ MVA}$$

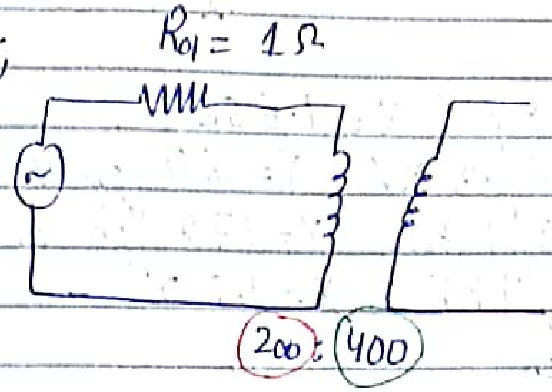
$$Z_{pu} \propto \frac{1}{(kV)^2}$$

$$\frac{Z_{pu(new)}}{Z_{pu(old)}} = \frac{(MVA)_{new}}{(MVA)_{old}} \times \left(\frac{(kV_b)_{old}}{(kV_b)_{new}}\right)^2$$

$$\Rightarrow \boxed{Z_{pu(new)} = Z_{pu(old)} \times \frac{(MVA)_{new}}{(MVA)_{old}} \times \left(\frac{V_{b(old)}}{V_{b(new)}}\right)^2}$$

The pu impedance in primary and secondary is same?

Consider;



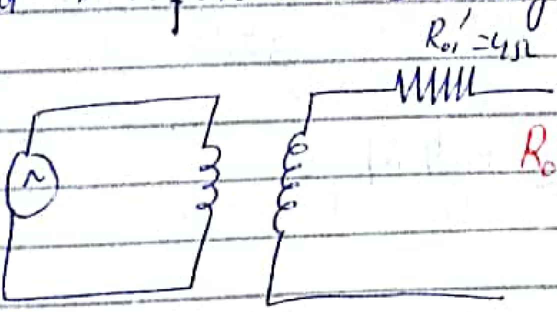
Rating = 1 KVA.

$$R_{01}(pu) = R_{(\Omega)} \times \frac{VA}{V_b^2} = 1 \times \frac{1000}{(200)^2}$$

$$= \frac{1}{40}\ pu.$$

b/c V_b of primary is 200

→ If R_{01} is refereed to secondary;



$$R_{01}' = R_{01} \times \left(\frac{N_2}{N_1}\right)^2$$

$$1 \times \left(\frac{400}{200}\right)^2 = 4\Omega$$

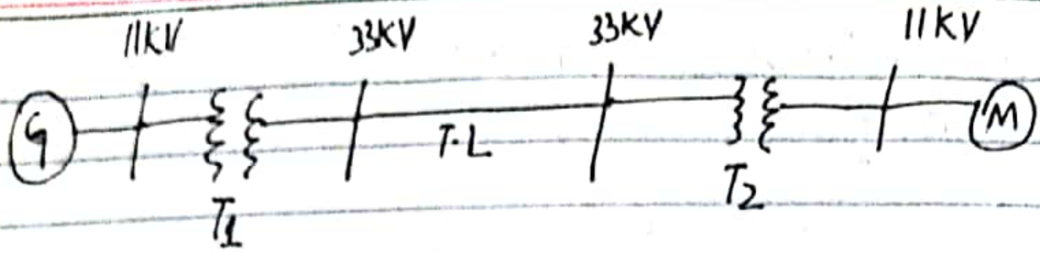
$$R_{01}'(pu) = R_{01}'(\Omega) \times \frac{VA}{V_b^2} = 4 \times \frac{1000}{(400)^2}$$

$$\Rightarrow R_{01}'(pu) = \frac{1}{40}\ pu.$$

Initially we were getting 3 loops in the diagram (eg); so now with the help of pu, we draw a single diagram known as Reactance diagram.
↓(1 loop)

REACTANCE DIAGRAM

Case 1 When reactance are given in ohm.



G : 30 MVA, 11 KV, $X = 1.6\,\Omega$
T_1 : 15 MVA, 11/33 KV, $X = 16\,\Omega$/ph on HV side.
T_2 : 15 MVA, 33/11 KV, $X = 17\,\Omega$/ph on LV side.
M : 25 MVA, 11 KV, $X = 1.2\,\Omega$
TL : 20.5 Ω/ph

Draw reactance diagram by taking any base value.

Considering 30 MVA, 11 KV as base at generating station.

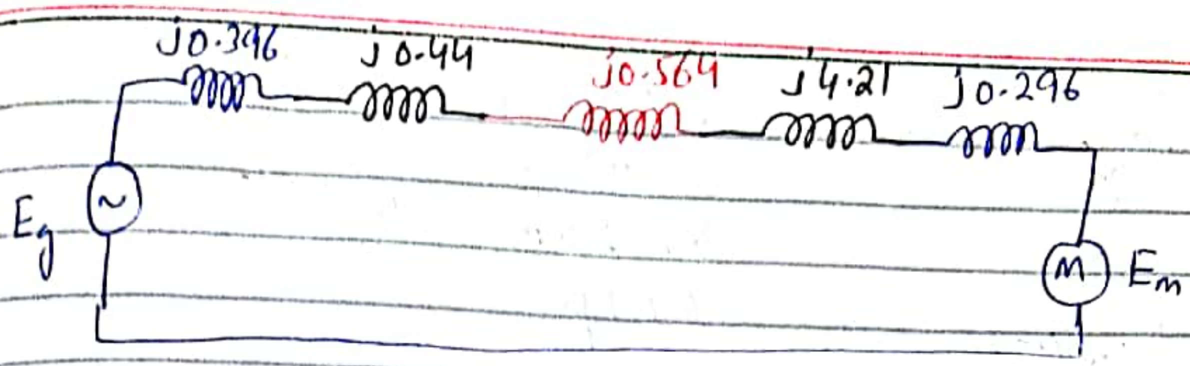
$$X_{G\,pu} = X_{\Omega} \times \frac{MVA_{base}}{(KV_b)^2} = 1.6 \times \frac{30}{11^2} = 0.396 \text{ pu}$$

$$X_{T1\,(pu)} = \cancel{16 \times \frac{30}{11^2}} = 16 \times \frac{30}{33^2} = 0.44 \text{ pu}$$

$$X_{T2\,(pu)} = 17 \times \frac{30}{11^2} = 4.21 \text{ pu}$$

$$X_{TL\,(pu)} = 20.5 \times \frac{30}{33^2} = 0.564 \text{ pu}$$

$$X_{M\,(pu)} = 1.2 \times \frac{30}{11^2} = 0.297 \text{ pu.}$$

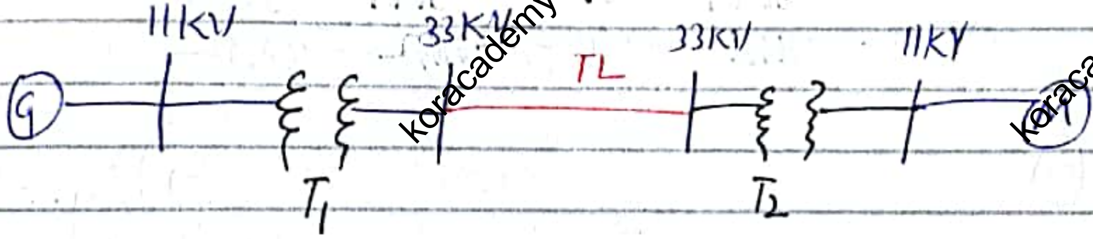


considering 30MVA at 11 KV.

The pu values can be different but the actual value of current and voltage across each element will remain the same.

Case 2.

When reactances are given in per unit.



G: 30 MVA, 11 KV , X = 0.2 pu
T_1 : 15 MVA, 11/33 KV, X = 0.1 pu
T_2 : 15MVA , 33/11 KV, X = 0.1 pu
M : 25 MVA, 11 KV , X = 0.2 pu
T.L : 26.5 Ω/ph

Draw reactance diagram taking 100MVA, 11 KV as base at G.

We know that;

$$Z_{pu(new)} = Z_{pu(old)} \times \frac{(MVA)_{new}}{(MVA)_{old}} \times \left(\frac{V_{b(old)}}{V_{b(new)}}\right)^2$$

$$X_{G(new)pu} = 0.2 \times \frac{100}{30} \times \left(\frac{11\,kV}{11\,kV}\right)^2 = 0.67\,pu$$

$$X_{T1\,(pu)\,new} = 0.1 \times \frac{100}{15} \times \left(\frac{11KV}{11KV}\right)^2 = 0.67\,pu$$

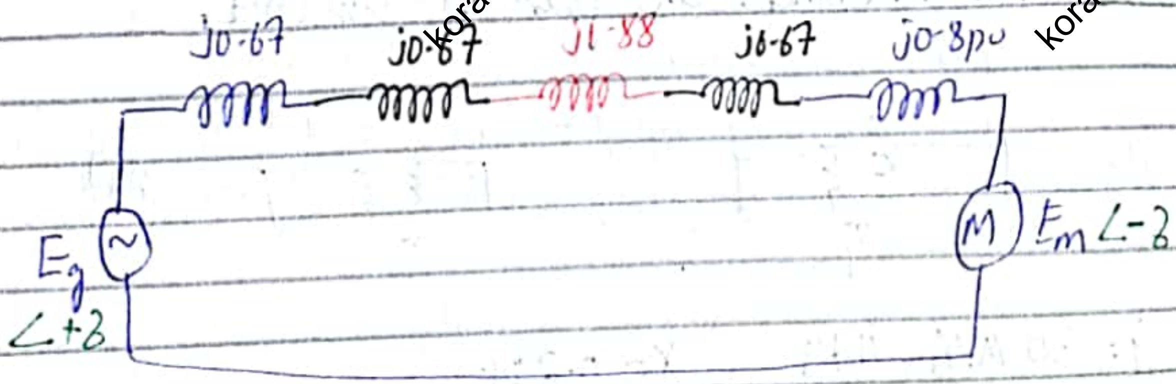
$\hookrightarrow$ or if secondary side $\left(\frac{33KV}{33KV}\right)^2$

$$X_{T2\,pu\,(new)} = X_{T1\,(new)} = 0.67\,pu$$

$\hookrightarrow$ As all ratings are same.

$$X_{M\,(new)} = 0.2 \times \frac{100}{25} \times \left(\frac{11KV}{11KV}\right)^2 = 0.8\,pu$$

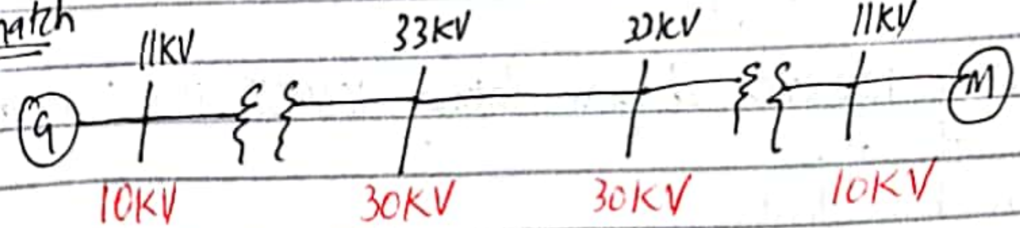
$$X_{TL\,(new)} = X_{(\Omega)} \times \frac{MVA}{(KV_b)^2} = 20.5 \times \frac{100}{33^2} = 1.88\,pu$$



* Case 3 Draw reactance diagram by taking 100MVA, 10KV as base. for data in Q2.

$\downarrow$

Mismatch



$\downarrow$

When $V_{b\,old} \neq V_{b\,new}$

$$X_{G\,(new)} = 0.2 \times \frac{100}{30} \times \left(\frac{11}{10}\right)^2 =$$

$$X_{T_{1\,new}} = 0.1 \times \frac{100}{15} \times \left(\frac{11}{10}\right)^2 =$$

$$\left(\frac{33}{30}\right)^2$$

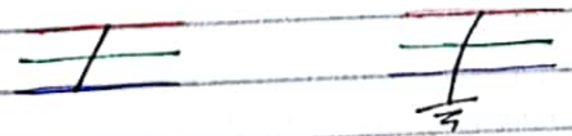
$$X_{T_{2\,new}} = X_{T_{1\,new}}$$

$$X_{M(new)} = 0.2 \times \frac{100}{25} \times \left(\frac{11}{10}\right)^2 =$$

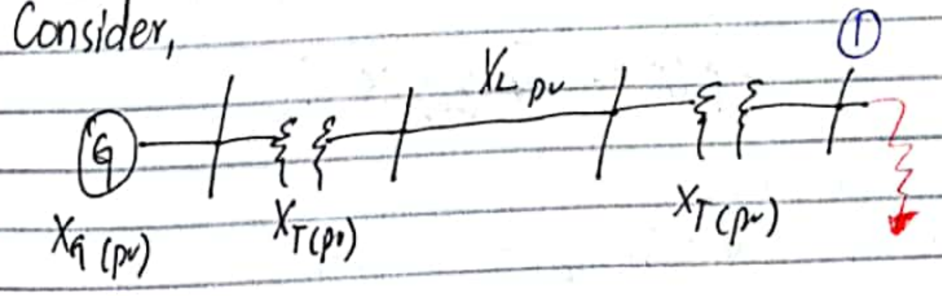
$$X_{TL(pu)} = 20.5 \times \frac{100}{(30)^2} =$$

3φ Symmetrical Fault

A fault is known as symmetrical fault if after the fault the magnitude as well as the phase difference b/w the voltages will remain same as before the fault.



Consider,

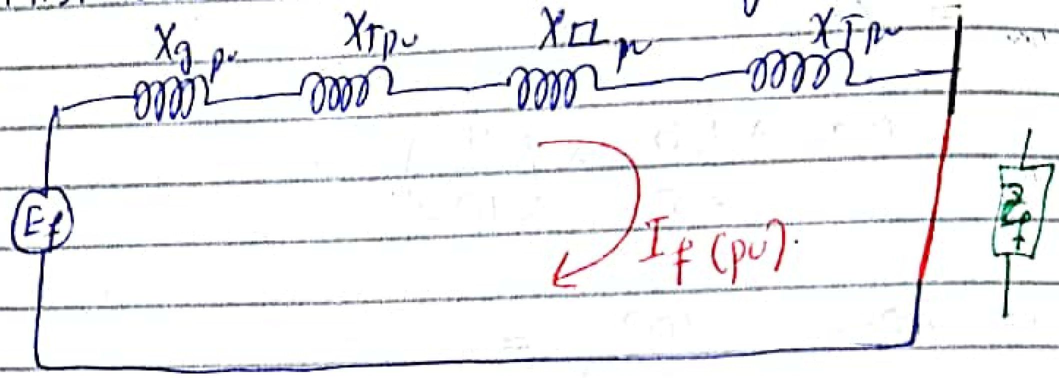


<u>Assumption</u> pu value is given on common base.

Consider a fault occurs at bus ①.
We see that no load is connected ⇒ the current flowing in the TL prior to fault is zero.

So after the fault, a current (fault current) will flow.

First draw the reactance diagram;



We will also have some fault impedance (means the fault/line has not directly touched the ground).
→ If not given, consider it zero.

$$\boxed{I_{f(pu)} = \frac{E_{f(pu)}}{X_{eq(pu)} + Z_{f(pu)}}}$$

As $pu = \text{Actual}/\text{Base}$

$$\Rightarrow I_{f(A)} = I_{f(pu)} \times I_b$$

If three phase power;

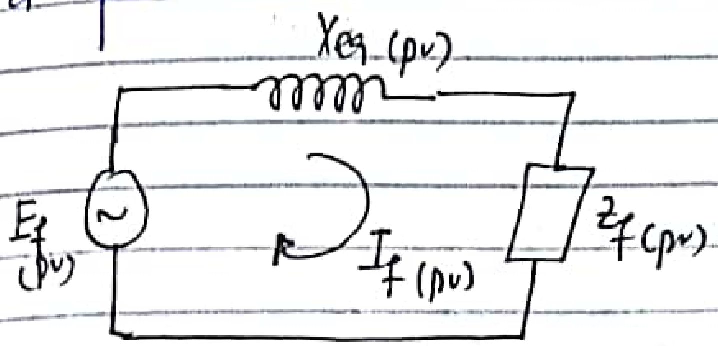
$$\sqrt{3}\, V_L I_L = 3\phi\, MVA$$

$$\Rightarrow I_b = \frac{MVA}{\sqrt{3}\, V_L}$$

$$\rightarrow \boxed{I_{f(A)} = I_{f(pu)} \times \frac{MVA}{\sqrt{3}\, V_L}}$$

Short Circuit MVA is used to define the rating of a circuit breaker.

↳ The amount of power that can flow when a fault occurs.



<u>1st method</u>

$$\boxed{SCMVA = \sqrt{3}\, E_f\, I_f}$$

→ Line quantities

Actual.

<u>2nd method</u>

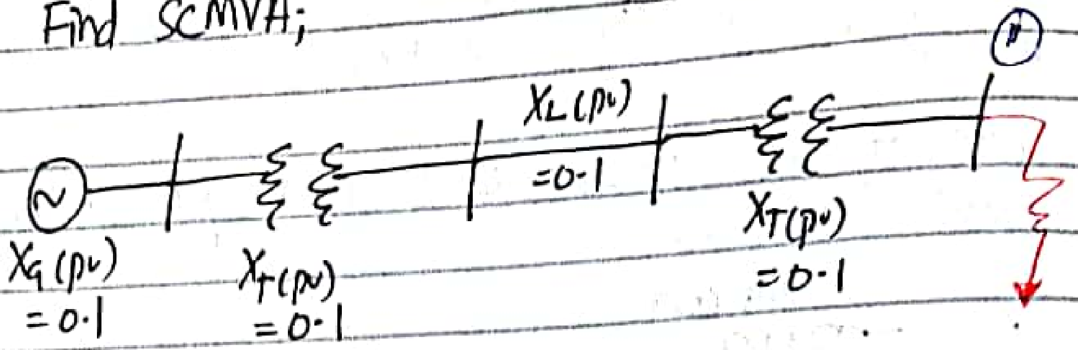
$$\boxed{(SCMVA)_{pu} = E_{f\,(pu)} \times I_{f\,(pu)}}$$

For 3ϕ ; $(SCMVA)_{pu} = 3\, E_{f(pu)} \times I_{f\,(pu)}$ X X

$$\boxed{(SCMVA)_{actual} = (SCMVA)_{pu} \times (3\phi\, MVA)_{base}}$$

pu values are defined per phase only.

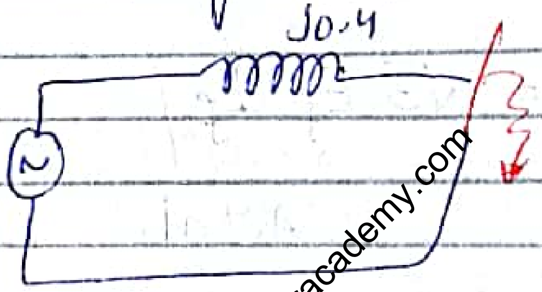
Q. Find SCMVA;



50MVA, $E_f = 11\,KV$

Method 1 Fault current = ?

Reactance diagram;



$$I_{f(pu)} = \frac{E_{f(pu)}}{X_{total}}$$

$$= \frac{1}{j0.4}$$

$$\Rightarrow I_{f(pu)} = -j\,2.5\,pu = 2.5_{pu}\angle -90^\circ.$$

$$\Rightarrow I_{f(act)} = I_{f(pu)} \times \frac{MVA}{\sqrt{3}\,V_L} = 2.5 \times \frac{50\times 10^6}{\sqrt{3}\times 11\times 10^3}$$

$$\Rightarrow I_{f(actual)} = 6.56\,KA$$

$$SCMVA = \sqrt{3} \times V_L \times I_f$$
$$= \sqrt{3} \times 11 \times 10^3 \times 6.56 \times 10^3$$

$$SCMVA = 125\,MVA$$

Method 2

$$I_{f(pu)} = \frac{1}{j0.4} = -j2.5pu$$

$$(SCMVA)_{pu} = E_{f(pu)} \times I_{f(pu)}$$
$$= 1 \times 2.5 = 2.5pu$$

$$(SCMVA)_{actual} = 2.5pu \times (base) = 50$$
$$= 125MVA$$

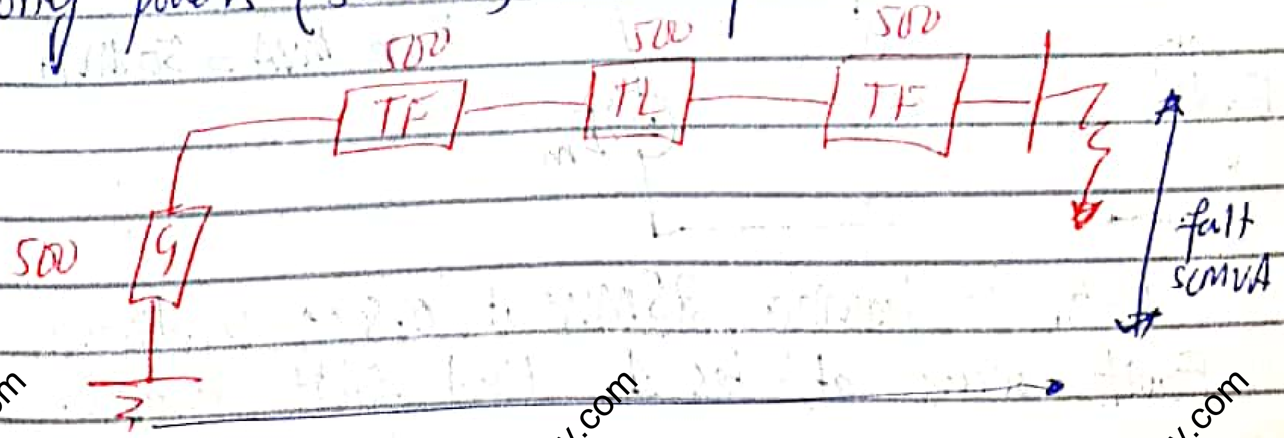
Method 3 (MVA method)

In this method, we find the SCMVA of all the equipment individually

$$\boxed{SCMVA = \frac{MVA}{pu}}$$

	MVA	X_{pu}	SCMVA
generator	50	j0.1	500
Xformer	50	j0.1	500
T.L	50	j0.1	500

Now we draw the equivalent MVA circuit, in which only powers (SCMVA) are represented.



As for n number of same capacitors connected in series;

$$C_{eq} = \frac{C}{n}$$

$$\Rightarrow \quad SCMVA = \frac{500}{4} = 125\,MVA$$

→ If the reactances are given in ohms?

$$SCMVA = \frac{MVA}{X_{pu}} = \frac{\cancel{MVA} \times (kV)^2}{X_{(\Omega)} \times \cancel{MVA}}$$

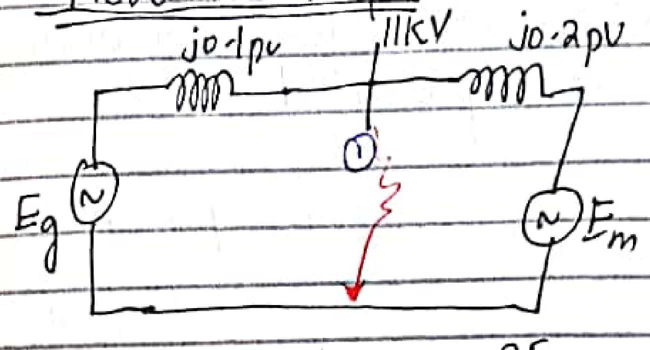
$$\Rightarrow \quad \boxed{SCMVA = \frac{(kV)^2}{X_{(\Omega)}}}$$

We can also find the fault current easily;

ie $\quad SCMVA = E_{pu} \times I_{f\,(pu)}$

$$= \sqrt{3}\,E_L \times I_{f\,(A)}$$

Thevenin Concept

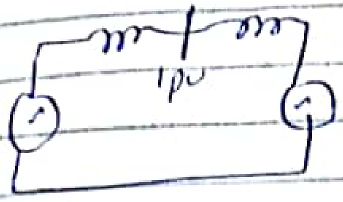


Base MVA = 50 MVA

Motor is drawing 25 MW at 0.8 pf leading. Fault occurs at bus 1. Find fault current.

nected

First we need to find current before fault.



$$V_{pu} = \frac{11KV}{11KV} = 1\ pu$$

$$P_m = \frac{25}{50} = 0.5\ pu$$

$$\frac{MW}{pf} = MVA$$

Now $V_{pu}\, I_{pu} \cos\phi = P = 0.5$

$$I_{pu} = \frac{0.5}{1 \times 0.8} = 0.625 \angle 36.86^\circ$$

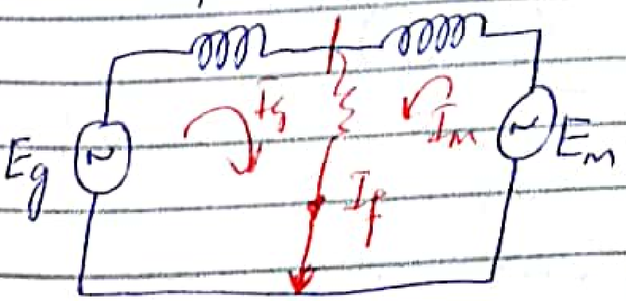
$$E_g = (V_{pu} + I_{pu}) \times j0.1$$

$$= 1 + 0.625 \angle 36.8^\circ \times j0.1$$

$$E_g = 0.9638 \angle 2.97$$

$$E_m = 1 - j0.2 \times 0.625 \angle 36.86$$
$$= 1.0796 \angle -5.315$$

The after fault circuit will be as;



The current delivered by the ~~motor~~ gen will not reach the load (motor) and motor will also behave as generator and supply power to the short cnt path.

$$I_f = I_g + I_m$$

$$I_f = \frac{E_g}{j0.1} + \frac{E_m}{j0.2}$$

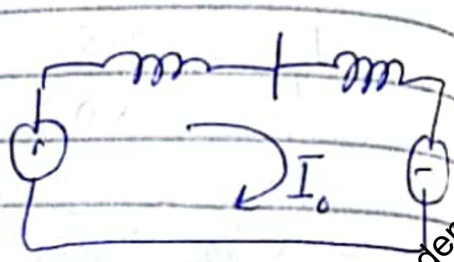
$$= 9.638\angle -87.03 + 5.398\angle -95.315^\circ$$

$$\rightarrow I_f = 14.999\angle -90^\circ$$

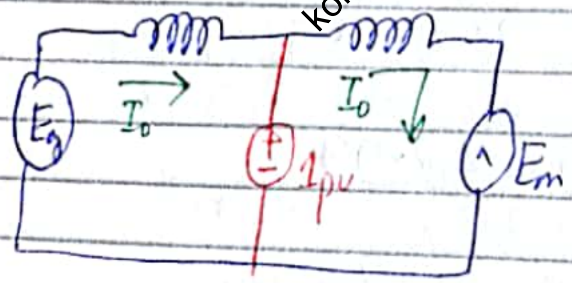
(If system was not loaded, $E_g = 1pu$ no E_m)

<u>Using Thevenin concept;</u>

The pre fault circuit



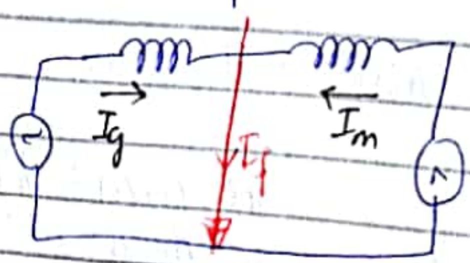
<u>OR</u>



(A)

No current will flow through the 1pu voltage source.

The circuit after fault will be;



(B)

Say we short E_g and E_m,

(C)

The superposition of (A) and (C) will give us circuit after the fault in (B).

Now if add together (algebraic sum) the currents of (A) and (C), it will give us the current in (B).

ie $I_g = I_0 + \Delta I_g$

and $I_m = -I_0 + \Delta I_m$

$$\Rightarrow I_f = I_g + I_m = \underbrace{\boxed{\Delta I_g + \Delta I_m}}_{= I_f}$$

The polarity is opposite so that when it is superimposed, it becomes a shorted path.

From fig; $\Delta I_g = \frac{1}{j0.1} = -j10\,pu$

$\Delta I_m = \frac{1}{j0.2} = -j5\,pu.$

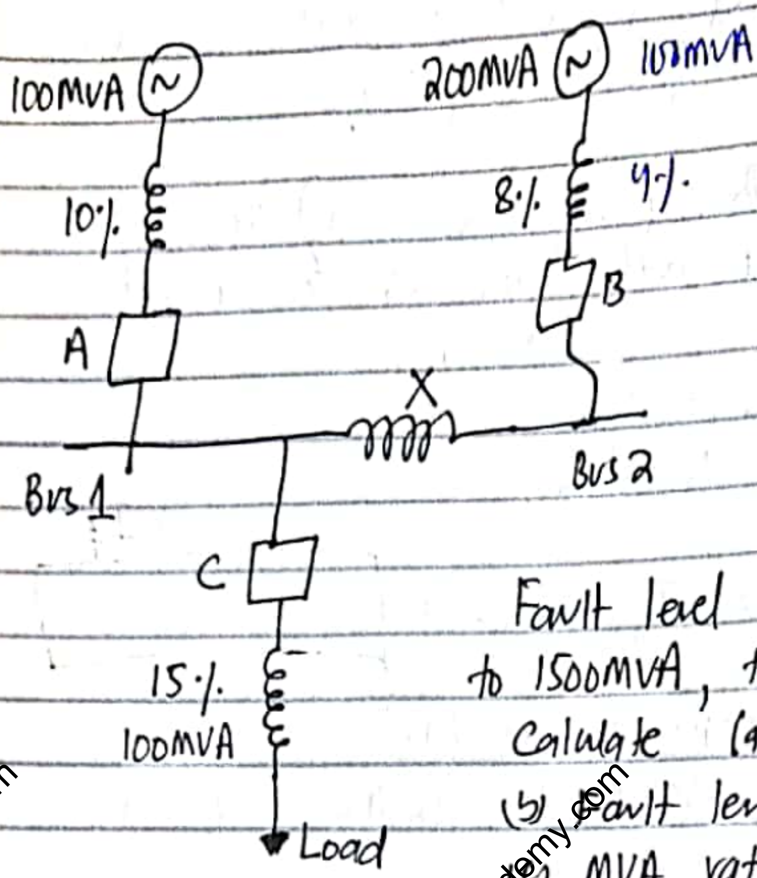
$\Rightarrow I_f = -j15\,pu = 15\,pu \angle -90°$

Similarly, $(SCMVA)_{pu} = E_{(pu)} \times I_{f\,(pu)}$

$= (0.9638 \times 15)$

of the same.

Questions.



Fault level on Bus 1 is restricted to 1500MVA, take 100MVA base.
Calculate (a) Reactor X
(b) Fault level on bus 2.
(c) MVA rating of CB 'C'.

On 100MVA

$$SCMVA = 1500\ MVA$$

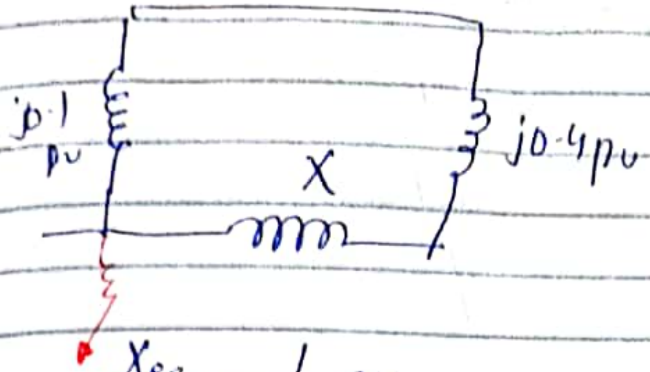
$$(SCMVA)_{pu} = \frac{1500}{100} = 15pu = E_{f(pu)} \times I_{f(pu)}$$

$$E_{pu} = 1pu$$

$$\Rightarrow I_{f(pu)} = 15pu \text{ at bus 1.}$$

As $I_{f(pu)} = \frac{E_{(pu)}}{X_{(pu)}} \Rightarrow X_{(pu)} = \frac{1}{15}$

If fault occurs at bus 1, the net reactance diagram is shown as:



$X_{eq} = \frac{1}{15}$ pu

$$(0.04 + X) \parallel 0.1 = 1/15$$

$$\frac{(0.04 + X)(0.1)}{0.04 + X + 0.1} = \frac{1}{15}$$

$$\Rightarrow \boxed{X = j0.16 \text{ pu}}$$

(b) SCMVA on bus 2 = ?

$$X_{eq} = (0.1 + 0.16) \parallel 0.04$$

$$X_{eq} = \frac{0.26 \times 0.04}{0.26 + 0.04} = j\,0.3467 \text{ pu}$$

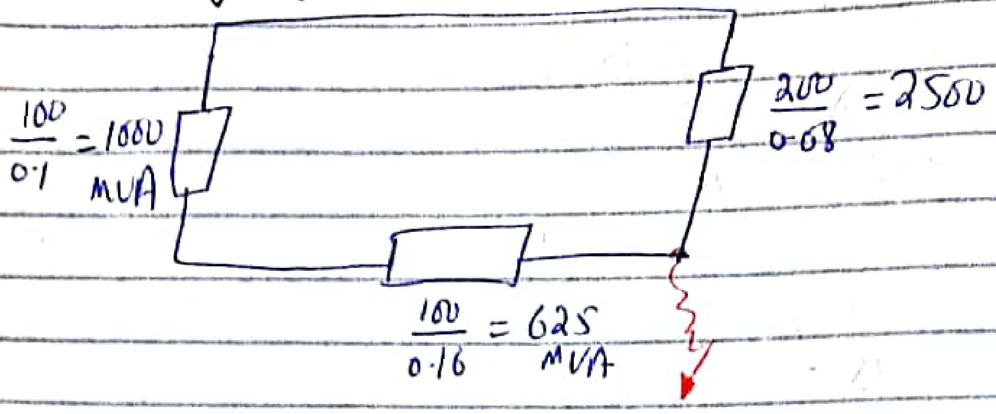
$$I_{f(pu)} = \frac{E_{(pu)}}{X_{pu}} = \frac{1}{j0.3467} = 28.84 \text{ pu} \angle -90^\circ$$

Now,

$$(SCMVA)_{pu} = E_{f(pu)} \times I_{f(pu)} = 28.84 \text{ pu}$$

$$\boxed{(SCMVA)_{act} = 2884 \text{ MVA}}$$

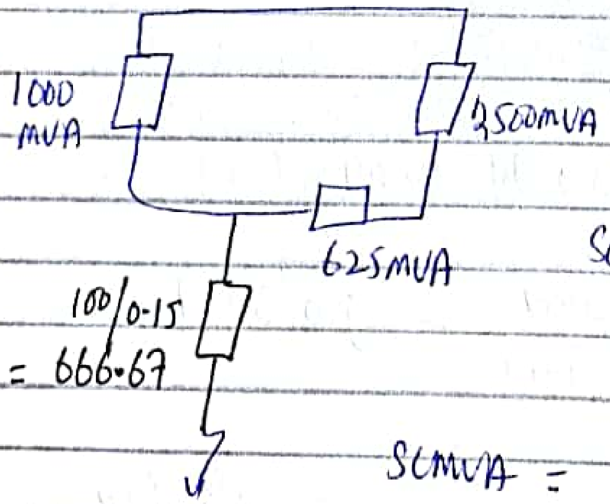
Repeating by MVA method;



$$SCMVA = \left(\frac{1000 \times 625}{1000 + 625}\right) + 2500$$

$$\Rightarrow \boxed{SCMVA = 2884 \text{ MVA}}$$

(c)



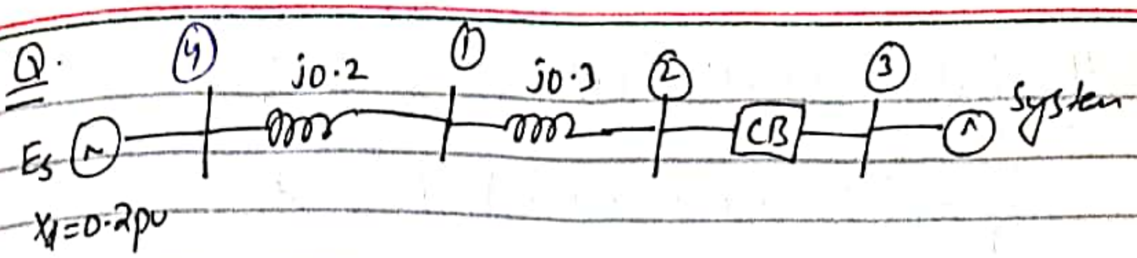
$$SCMVA = \left(\frac{2500 \times 625}{3125}\right) + 1000$$

= 1500 hdf

$$SCMVA = \frac{1500 \times 666.67}{1500 + 666.7}$$

$$\Rightarrow \boxed{SCMVA = 461.53 \text{ MVA}}$$

Q.



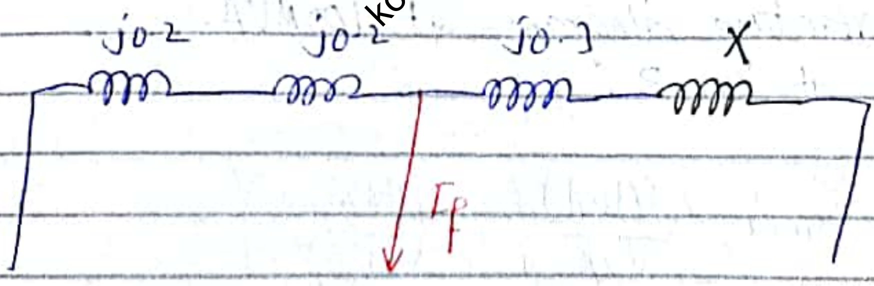
3φ fault current at bus 1 is 5pu.
Find fault current at bus 3.

$$I_{f(pu)} = 5pu = \frac{E_{(pu)}}{X_{eq(pu)}} = \frac{1}{X_{eq(pu)}}$$

$$\Rightarrow X_{eq(pu)} = \frac{1}{5} = 0.2 \, pu$$

↳ across bus 1

Reactance diagram:



$$0.4 \parallel 0.3 + x = \frac{(0.4)(0.3+x)}{0.7+x} = 0.2$$

$$\Rightarrow X = \frac{1}{10} = 0.1 \, pu$$

If fault occurs at bus 3;

$$X_{eq} = 0.7 \parallel 0.1 = \frac{7}{80} \, pu$$

$$I_{f(pu)} = \frac{E_{f(pu)}}{X_{pu}} = \frac{1}{70/80} = \underline{1.14 \, pu.}$$

If fault occurs at bus (4),

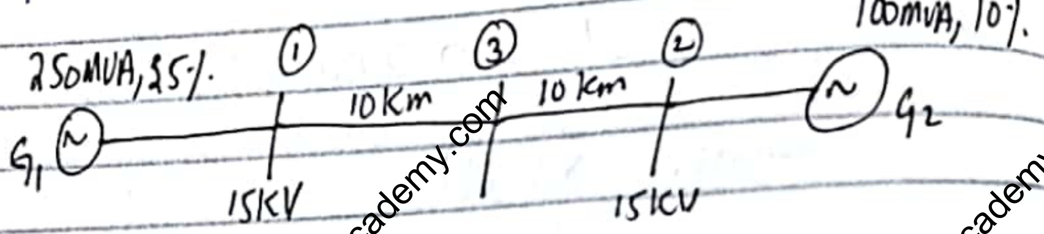
$$X_{eq} = (0.1 + 0.2 + 0.3) \parallel 0.2$$

$$= \_\_\_\_$$

$$I_f = \frac{1}{X_{eq}}$$

If base MVA is given, we can find SCMVA.

Q.



$$X = 0.225\ \Omega/km$$

(a) Draw reactance diagram at 100MVA.
(b) SCMVA at Bus 3.

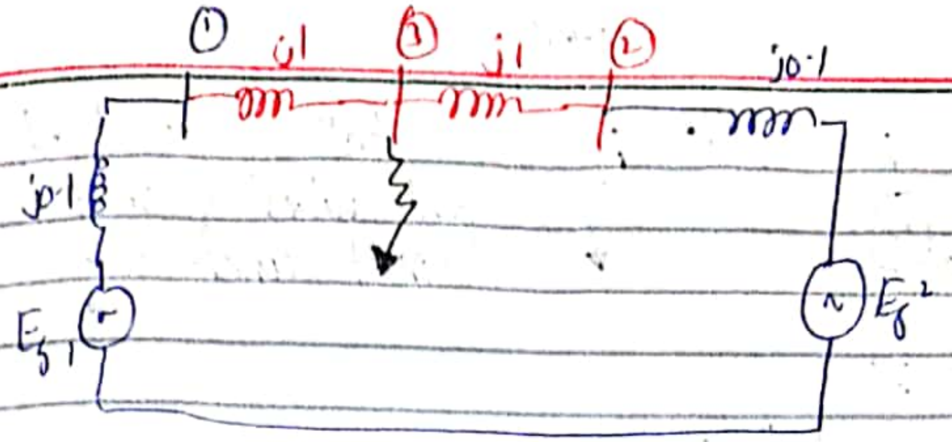
$$X_{G1_{new}} = X_{G1_{old}} \times \frac{(MVA)_{new}}{MVA_{old}} \times \left(\frac{(KV_b)_{old}}{(KV_b)_{new}}\right)^2$$

$$= 0.25 \times \frac{100}{250} \times \left(\frac{15}{15}\right)^2 = j0.1$$

$$X_{G2} = j0.1\,pu$$

$$X_{TL} = 0.225 \times 10 = 2.25\ ohm$$

$$X_{TL(pu)} = \frac{\underset{X(\Omega)}{2.25} \times \overset{MVA}{100}}{\underset{kV^2}{15^2}} = j\,1\,pu$$



(3) $X_{eq} = (j0.1 + j1) \,\|\, (j0.1 + j1) = \dfrac{1.1 \times 1.1}{2.2}$

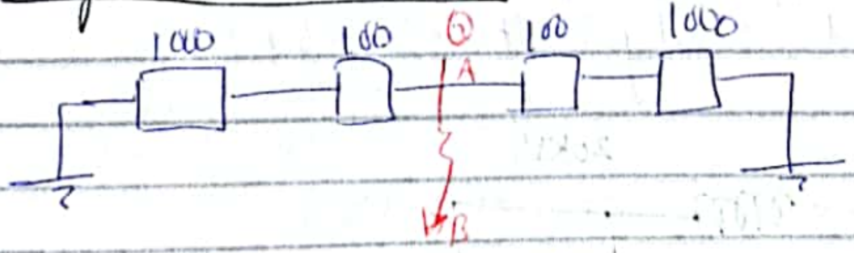
$\Rightarrow X_{eq} = \cancel{1.8182\ pu}\ 0.55\,pu$

$I_{f(pu)} = \dfrac{1}{X_{pu}} = \dfrac{1}{0.55} = 1.8181\,pu$

$SCMVA_{pu} = E_{fpu} \times I_{fpu} = 1.8181\,pu$

$SCMVA = 1.8181 \times 100 = \boxed{181.81\ MVA}$

Or by shortcut MVA method.



$SCMVA = \dfrac{1000 \times 100}{1100} + \dfrac{100 \times 100}{100 + 100} = 181.81\ MVA$

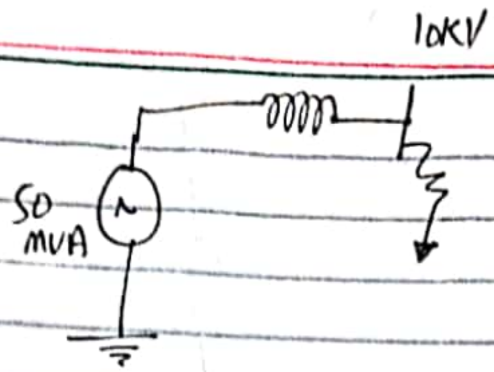
$X_d \rightarrow$ Reactance at steady state.

$X_d'' \rightarrow$ sub transient reactance.

$X_d' \rightarrow$ transient reactance.

$$\boxed{X_d'' < X_d' < X_d}$$

Q.



Ratio of initial and final values of short circuit current = ?

$X_d = j1\,pu$, $X_d'' = j0.2\,pu$.

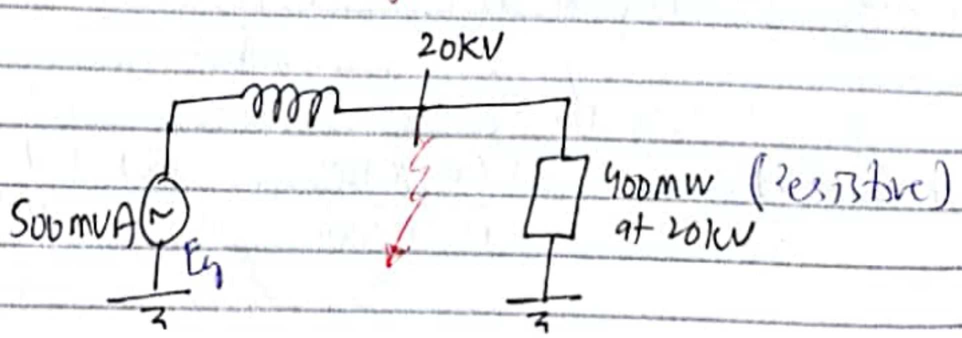
$$\text{Initial } I_f = \frac{1}{X_d''} = \frac{1}{j0.2} = -j5\,pu$$

$$\text{Final } I_f = \frac{1}{X_d} = \frac{1}{j1} = -j1$$

$$\boxed{\frac{\text{Initial}}{\text{Final}} = 5}$$

At fault we take X_d'', after 3 to 4 cycles we take X_d' and after 6 to 7 cycles we talk about steady state ie X_d.

Q.



$X_d'' = 0.2\,pu$, $X_d = 1\,pu$

Find initial rms current of generator when symmetrical fault occurs at load terminal.

→ When E_g is not given, we cannot find the fault current in loading condition.

→ Under no load condition, $E_g = 20\,kV$ and no current will flow

But here loaded → $E_g > 20\,kV$

Pre Fault Case

$$P_{(pu)} = \frac{400}{500} = 0.8\,pu = V_{pu} I_{pu} \cos\phi$$

Resistive load ⇒ $\cos\phi = 1$

Also $V_{pu} = 20/20 = 1\,pu$

$\Rightarrow I_{pu} = 0.8$

$$E_g = 1\angle 0 + 0.8\angle 0 \times 1\,pu$$

→ X

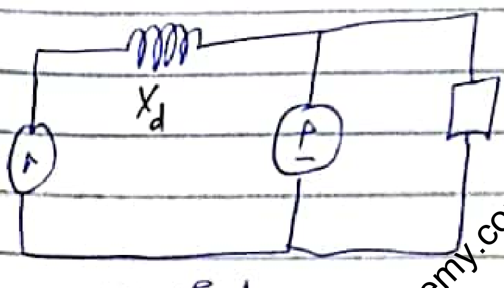
$$E_g = 1.28\angle 38.65^\circ$$

Now fault

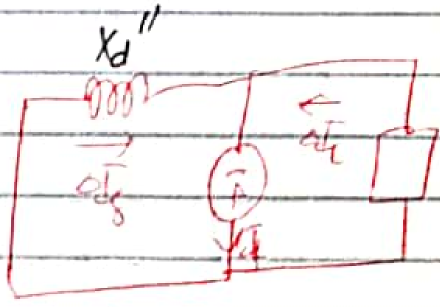
$$I_{F(pu)} = \frac{E_g(pu)}{X_d''} = \frac{1.28}{0.2}$$

$$\Rightarrow I_{F(pu)} = 6.4\,pu$$

Doing using Thevenin Concept



pre fault



Thevenin:

$$I_f = \Delta I_f + \Delta I_L$$

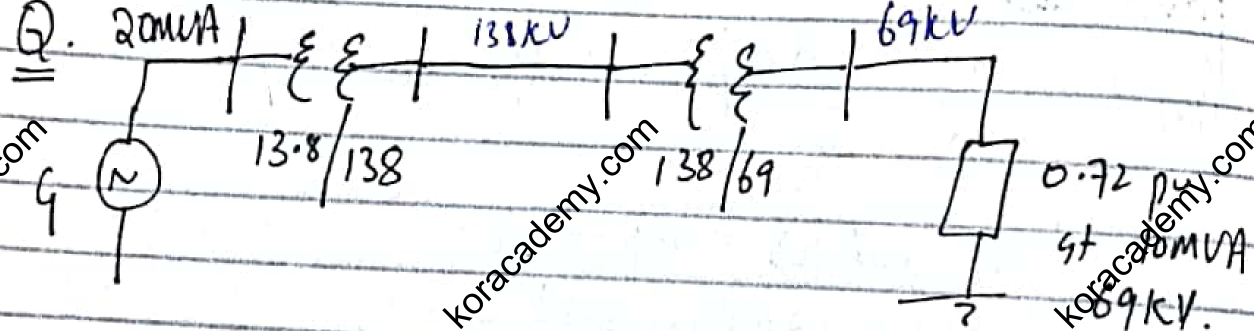
$I_f = -j5 + 1\angle 0 = 5.09\ pu$ previously 6.44 ?

This is b/c in one circuit we have X_d and in the other we have X_d''.

So for superposition we need them to be same.

So Thevenin concept cannot be applied here.

Q.



The eq value of load at gen end will be ?

$$X_{pu(new)} = X_{pu(old)} \frac{(MVA)_{new}}{(MVA)_{old}} \left(\frac{(kV_b)_{old}}{(kV_{b,new})}\right)^2$$

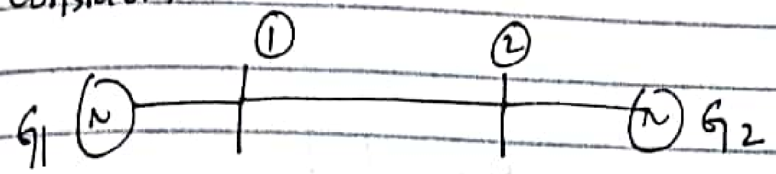
$$= (0.72)\left(\frac{20}{10}\right)\left(\frac{69}{13.8}\right)^2$$

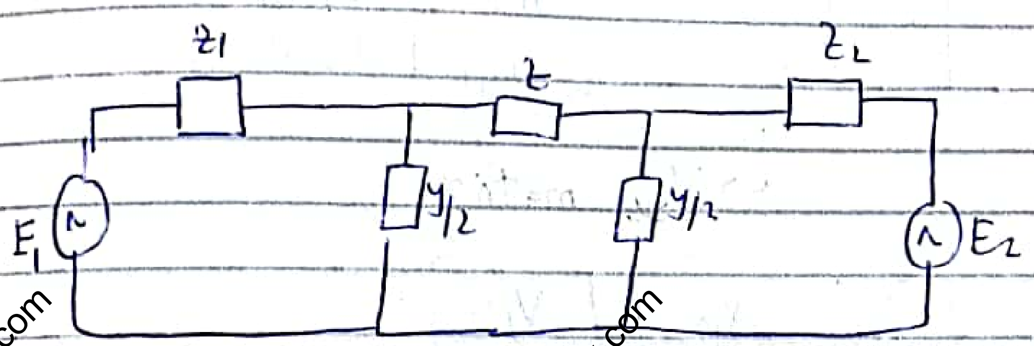
$$\Rightarrow \boxed{X_{pu\,(new)} = 36\ pu.}$$

LOAD FLOW STUDIES

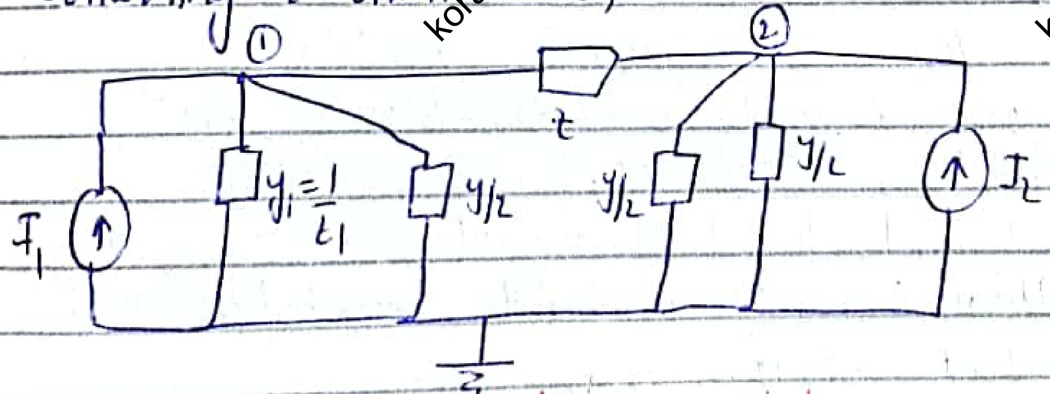
Y_{bus} matrix → stability analysis
Z_{bus} matrix → fault analysis

Considers;



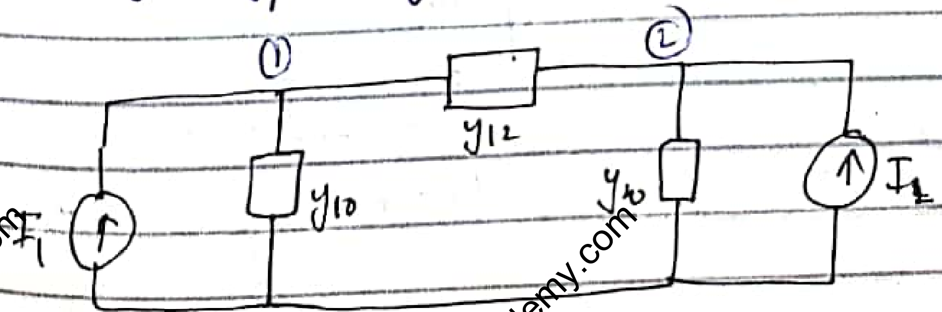


Converting to current source;



Admittance → added in parallel

$y_1 + y/2 = y_{10}$ → Adm of bus 1 wrt gnd.
$y_2 + y/2 = y_{20}$ → Adm of bus 2 wrt gnd.



$$I_1 = (V_1 - V_2)y_{12} + (V_1 - 0)y_{10}$$

$$\Rightarrow I_1 = (y_{10} + y_{12})V_1 - y_{12}V_2$$

$$I_2 = -y_{12}V_1 + (y_{20} + y_{12})V_2$$

In matrix form;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{10} + y_{12} & -y_{12} \\ -y_{12} & y_{20} + y_{12} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

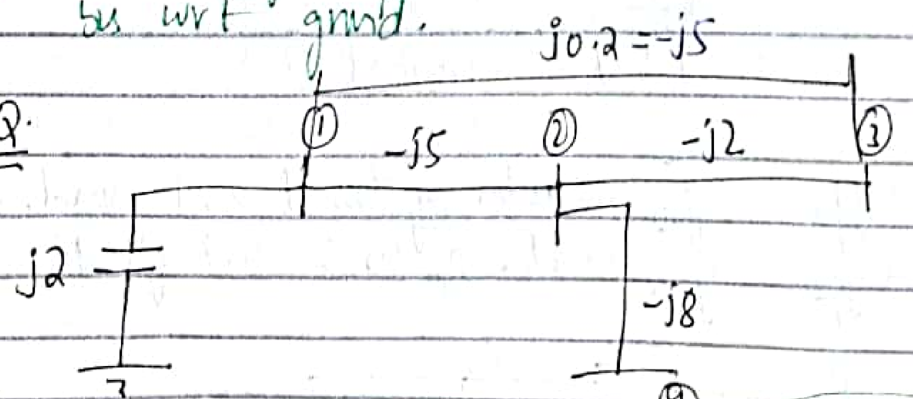
↳ Y_{bus} matrix

(or)

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

1. The Y_{bus} is a symmetric matrix
2. Diagonal element is summation of admittance connected at that bus.
3. Off diagonal element = −(adm b/w two buses)
4. Addition of row/column = Adm connected at that bus wrt ground.

Q.



$Z = R + j\omega L = j\omega L$

$Z = j(\ )$ +ve

$Y = -j(\ )$ −ve

$X_C = Z = 1/j\omega C = -j$

$Y = j(\ )$ +ve

$Y = \frac{1}{R} + jB$

↳ susceptance

$y_{12} = -j5$, $y_{23} = -j2$, $j_{24} = -j8$

$y_{10} = j2$

$$\begin{bmatrix} y_{12}+y_{10} & -y_{12} & -y_{13} & -y_{14} \\ -y_{21} & y_{23}+y_{12}+y_{24} & -y_{23} & -j_{24} \\ -y_{31} & -y_{32} & y_{23} & -y_{34} \\ -y_{41} & -y_{42} & -y_{43} & y_{24} \end{bmatrix}$$

$$\begin{bmatrix} -j3 & +j5 & 0 & 0 \\ +j5 & -j15 & +j2 & +j8 \\ 0 & j2 & -j2 & 0 \\ 0 & j8 & 0 & -j8 \end{bmatrix}$$

Y_{bus} → Sparse matrix → many elements = zero
less storage

Now if another Xmission line is added b/w ① and ③ with $X = j0.2$

The new Y_{bus} matrix will be;

$$\begin{bmatrix} -j8 & +j5 & +j5 & 0 \\ +j5 & -j15 & j2 & j8 \\ +j5 & j2 & -j7 & 0 \\ 0 & j8 & 0 & -j8 \end{bmatrix}$$

Classification of Bus

1. Slack bus / Reference bus / Swing bus

known = V, δ
unknown = P, Q

Slack bus is a generator bus only.

$$P_{slack} + P_{gen} = P_{loss} + P_{demand}$$

$$\boxed{P_{slack} = P_{demand} + P_{loss} - P_{gen}}$$

2. Generator bus / PV bus

known = P, V
unknown = Q, δ

$$\boxed{\text{No. of PV bus} = \text{Total no. of buses} - \underbrace{1}_{\text{slack bus}}}$$

For i^{th} bus;

$$\boxed{Q_{gi\,min} < Q_{gi} < Q_{gi\,max}}$$

If this condition is violated
ie $Q_{gi} < Q_{gi\,min} \rightarrow Q_{gi} = Q_{gi\,min}$

$Q_{gi} > Q_{gi\,max} \rightarrow Q_{gi} = Q_{gi\,max}$

PV bus is converted to PQ bus.

3. Load Bus / PQ bus

P, Q = known V, δ = unknown

$$\boxed{\text{No. of PQ bus} = \text{Total buses} - \text{slack bus} - \text{PV bus}}$$

4. Voltage controlled bus.

The bus where compensation is applied for voltage control.

Type of PV bus -

Compensation technique
- → SVC (Static VAR compensator) → PV
- → shunt capacitor / inductor → PQ

Q. 100 buses, 10 gen buses; size of Jacobian.

Jacobian = (2PQ + PV) x (2PQ + PV)

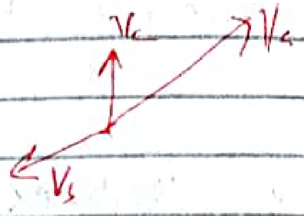
Gen bus = 10
- 1 slack
- PV bus = 9

PQ bus = total - gen buses
= 100 - 10 = 90

⇒ Jacobian matrix size = (2 x 90 + 9) = 189 x 189

Symmetrical Fault: The voltages and currents are displaced at 120° before and after the fault.

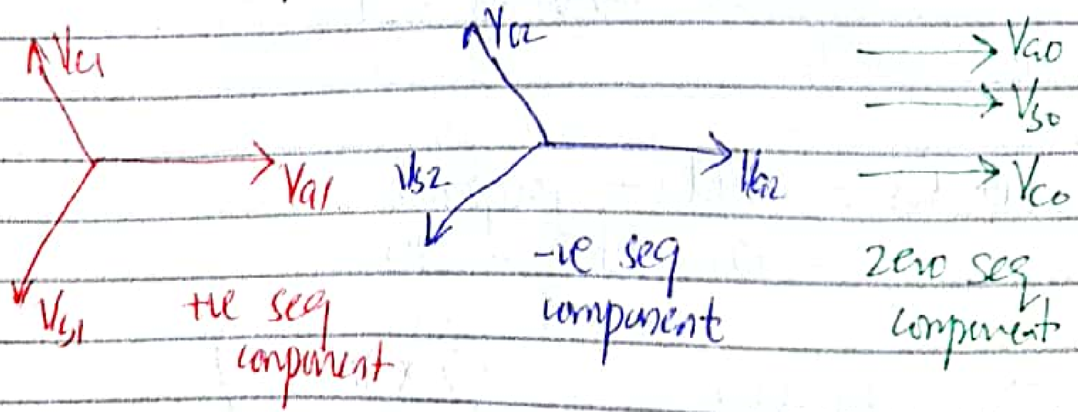
Unsymmetrical Fault: The voltages and currents before the fault are 120° displaced whereas after the fault their magnitude is also different and they are not at 120°.



- Line to ground fault
- Double line to ground fault
- Line to line fault.

Fortescue Theorem

We can resolve n unsymmetrical phasors into n-1 symmetrical phasors and one cophasor.



$$V_a = V_{a1} + V_{a2} + V_{a0}$$
$$V_b = V_{b1} + V_{b2} + V_{b0}$$
$$V_c = V_{c1} + V_{c2} + V_{c0}$$

Shift operator; $a = 1\angle 120°$, $a^2 = 1\angle 240°$

$$\boxed{1 + a + a^2 = 0}$$

$V_{b0} = V_{a0}$, $V_{b1} = |V_{a1}| \angle 240^\circ = a^2 V_{a1}$

$V_{b2} = a V_{a2}$, $V_{c0} = V_{a0}$, $V_{c1} = a V_{a1}$

$V_{c2} = a^2 V_{a2}$

$\Rightarrow$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$$
$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2}$$

In matrix form;

$$\boxed{\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}}$$

$$[V]^{abc} = [A][V]^{012}$$

$$\Rightarrow [V]^{012} = [A]^{-1} [V]^{abc}$$

$$\Rightarrow \boxed{\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}}$$

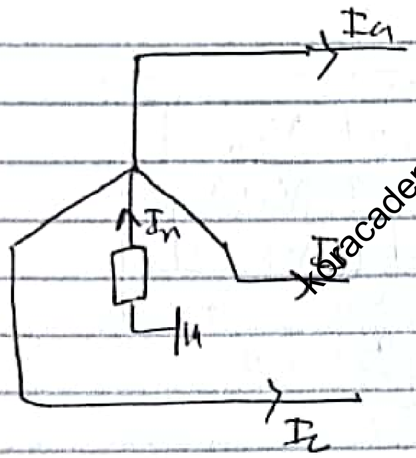
Similarly for currents;

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

and
$$\boxed{\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}}$$

→ Star = 3 phase 4 wire system.
→ Delta = 3 phase 3 wire system.

consider a star connection;



$$\boxed{I_n = I_a + I_b + I_c}$$

① $V_{ab0} = \frac{1}{3}\left[V_{ab} + V_{bc} + V_{ca}\right]$

$= \frac{1}{3}\left[V_a - V_b + V_b - V_c + V_c - V_a\right]$

$= 0$

No zero sequence component of line voltage in star connection!

① $I_n = I_a + I_b + I_c$

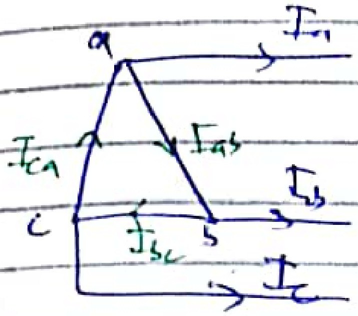
$I_n = \left[I_{a0} + I_{a1} + I_{a2}\right] + \left[I_{a0} + a^2 I_{a1} + a I_{a2}\right] + \left[I_{a0} + a I_{a1} + a^2 I_{a2}\right]$

$= 3 I_{a0} + I_{a1}(1 + a + a^2) + I_{a2}(1 + a + a^2)$

$\Rightarrow \boxed{I_n = 3 I_{a0}}$

In neutral ~~component~~ current, the +ve and -ve seq components are absent.

Consider a delta connection;



(1) $\boxed{I_n = 0}$

(2) $I_{ab} = \left[I_{ab0} + I_{ab1} + I_{ab2} \right]$

Zero seq component of current is present in phase.

(3) $I_a = \left[I_{ca} - I_{ab} \right]$, $I_b = \left[I_{ab} - I_{bc} \right]$

$I_c = \left[I_{bc} - I_{ca} \right]$

$I_{a0} = \frac{1}{3} \left[I_a + I_b + I_c \right] = 0$

No zero seq component in current in line.

In symmetrical fault:

Complex 3ϕ power $= \sqrt{3} V_L I_L^* = 3 V_p I_p^*$

In unsymmetrical fault,

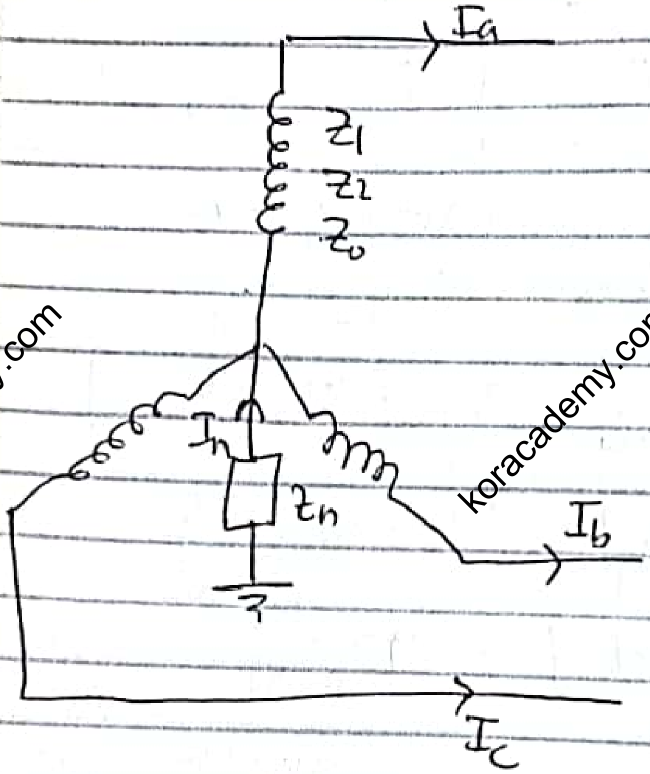
$$\boxed{3\phi \text{ complex } P = \underline{3} \left[V_{a0} I_{a0}^* + V_{a1} I_{a1}^* + V_{a2} I_{a2}^* \right]}$$

(actual value)

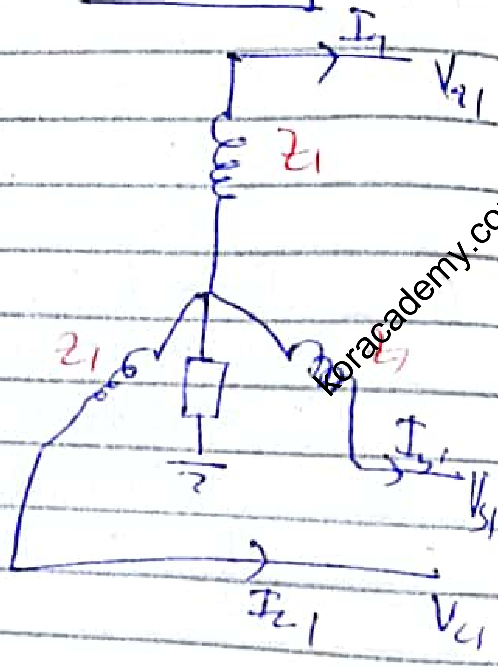
$$\boxed{3\phi \text{ complex power}_{(pu)} = \left[\underset{(pu)}{V_{a0}} \underset{(pu)}{I_{a0}^*} + \underset{(pu)}{V_{a1}} \underset{(pu)}{I_{a1}^*} + \underset{(pu)}{V_{a2}} \underset{(pu)}{I_{a2}^*} \right]}$$

(pu)

Sequence network of star connected load.



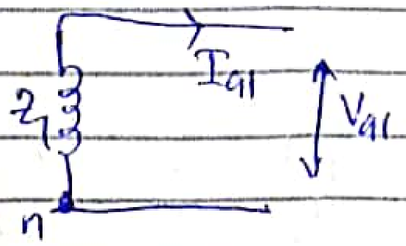
Positive seq



$$I_{a1} = \frac{V_{a1}}{Z_1}, \quad I_{b1} = \frac{V_{b1}}{Z_1}, \quad I_{c1} = \frac{V_{c1}}{Z_1}$$

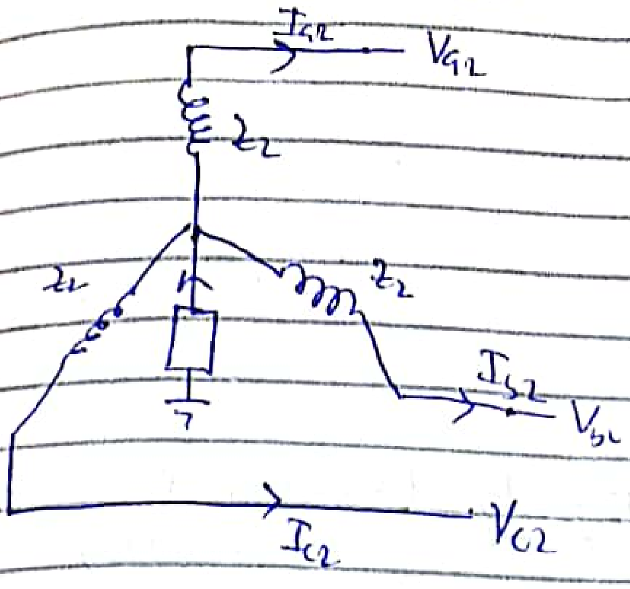
$$I_n = 0$$

Per phase equivalent circuit;



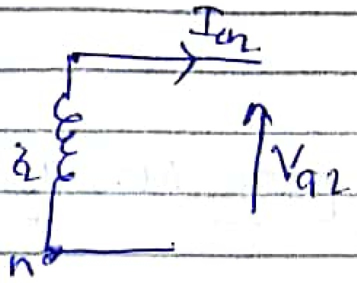
$$\boxed{V_{a1} = -I_{a1} Z_1}$$

Negative sequence network



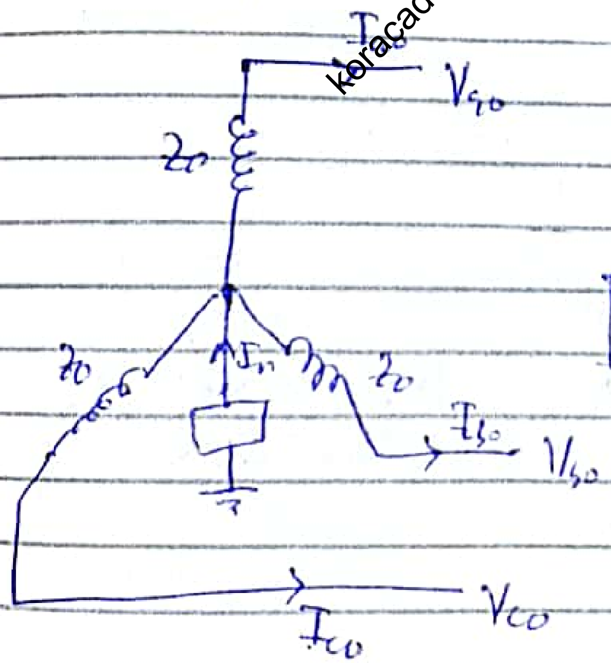
$\boxed{I_n = 0}$

Per phase equivalent ckt.



$\boxed{V_{a2} = -I_{a2} Z_2}$

Zero sequence network

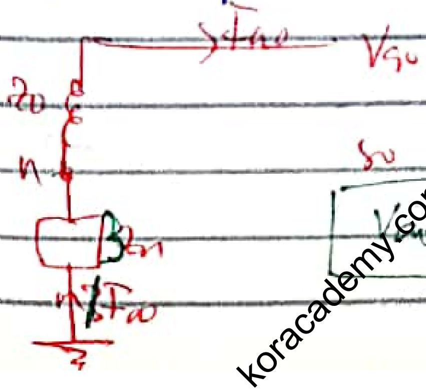


$\boxed{I_n = 3 I_{a0}}$

$\boxed{V_n = -I_n Z_n = -3 I_{a0} Z_n}$

$= -I_{a0} 3 Z_n$

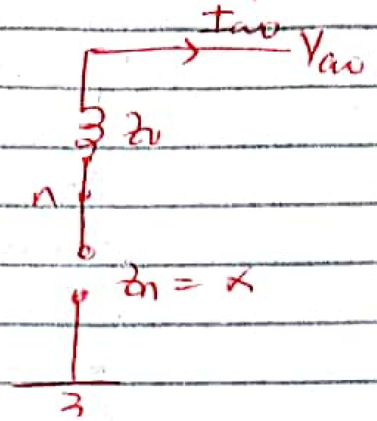
Per phase eq ckt.



different currents cannot flow in series (I_{a0} and $3I_{a0}$) so we make Z_n as $3Z_n$.

$\boxed{V_{a0} = -I_{a0}(Z_0 + 3Z_n)}$

→ If star with ungrounded; ⇒ $Z_n = \infty$



Sequence Network of Alternator (Synchronous Generator)

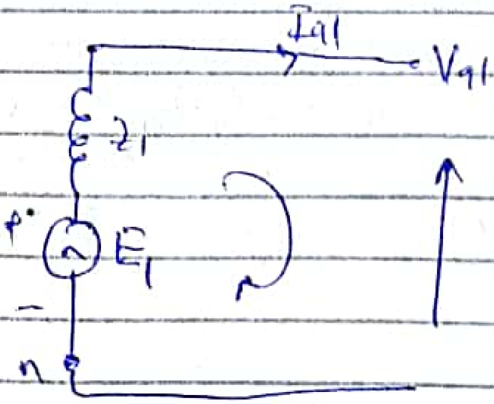
Generally we take star connection

Positive seq component

rotation of rotor = fixed = anticlockwise

+ve seq emf is present.

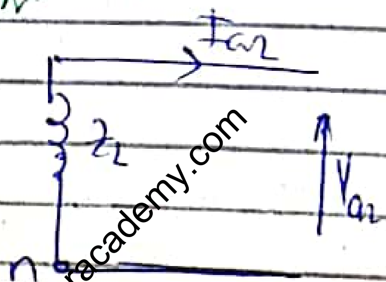
-ve seq emf = 0 seq emf = 0



$$\boxed{V_{a1} = E_1 - I_{a1} Z_1}$$

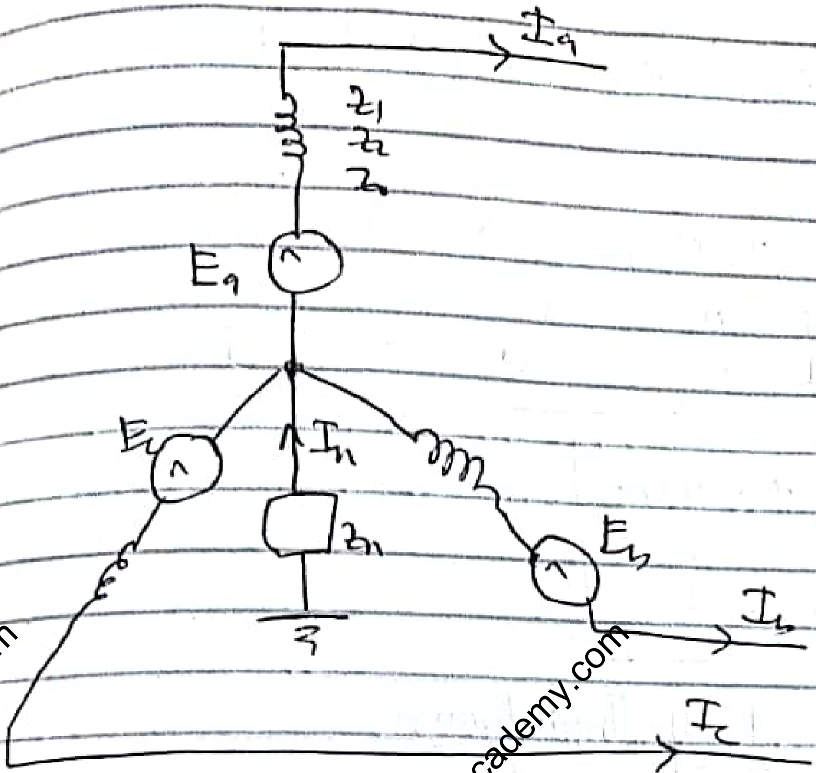
we will get -ve seq emf if we change direction of rotation of rotor

-ve seq

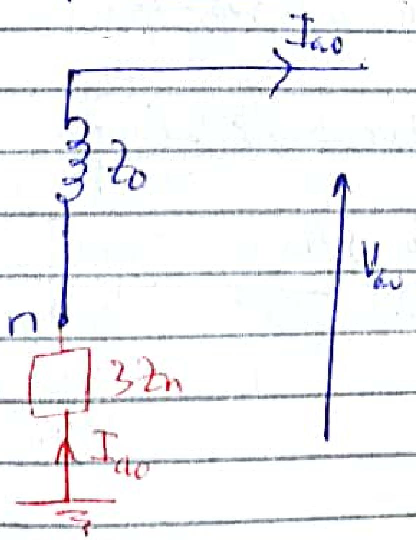


$$\boxed{V_{a2} = -I_{a2} Z_2}$$

Alternator circuit:



Zero seq network

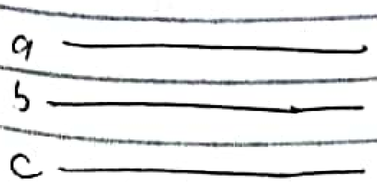


$$V_n = (3I_{a0})Z_n = I_{a0}(3Z_n)$$

$$\boxed{V_{a0} = -I_{a0}(Z_0 + 3Z_n)}$$

$$\boxed{Z_{0eq} = Z_0 + 3Z_n}$$

Transmission Line



Here the +ve, -ve and 0 seq impedance comes from two components
- Self impedance
- Mutual impedance.

$Z_{aa} = Z_{bb} = Z_{cc} = Z_s$ = self impedance

$Z_{ab} = Z_{bc} = Z_{ca} = Z_m$ = mutual inductance

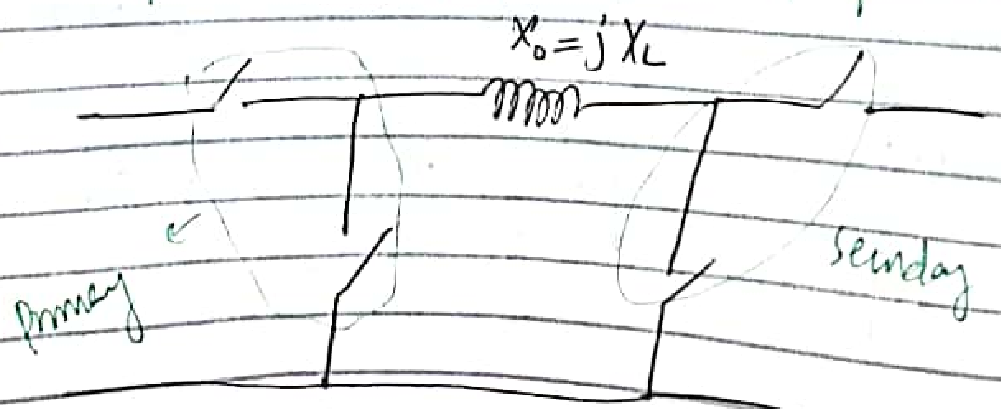
$$\boxed{\begin{bmatrix} Z_0 \\ Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}}$$

$$\Rightarrow \boxed{\begin{aligned} Z_0 &= Z_s + 2Z_{sm} \\ Z_1 &= Z_s - Z_m \\ Z_2 &= Z_s - Z_m \end{aligned}}$$

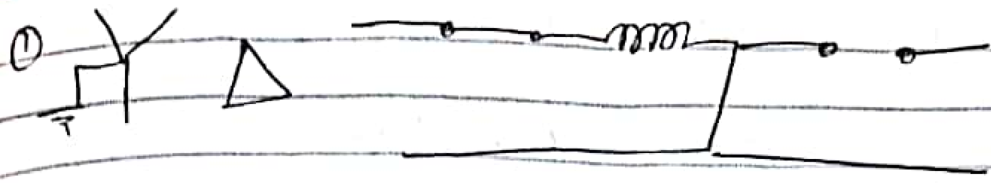
Sequence Network of Transformer

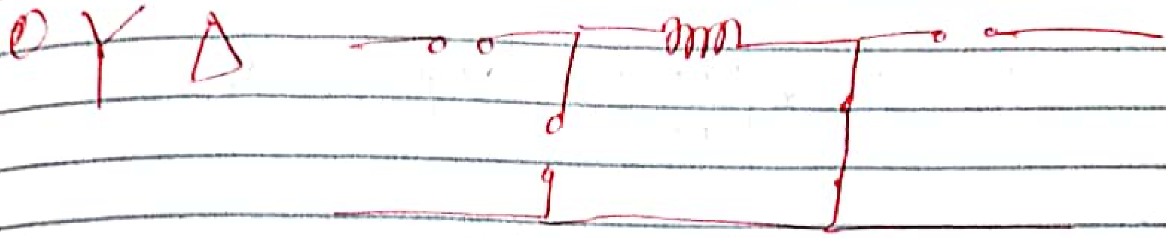
Transformer = static device = if phase sequence is changed, so no change in impedance.

In transformer, +ve seq impedance = -ve seq imp = 0 seq imp



series switch = active → Star connection (grounded)
parallel switch = active → delta

1.


2.


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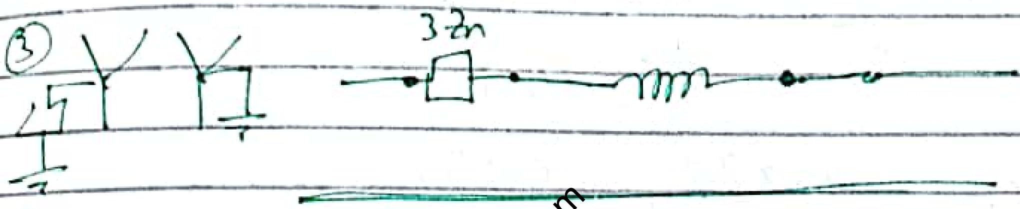
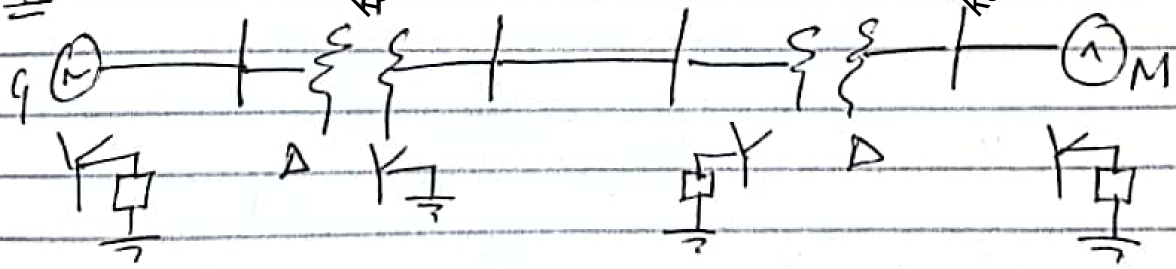


$X_{0eq} = X_L + 3Z_n$

4.


G : 25 MVA, 11 kV, 0.2 pu
T_1 : 30 MVA, 10 kV/60 kV, 0.2 pu
T_2 : 30 MVA, 60 kV/10 kV, 0.2 pu.
M : 15 MVA, 10 kV, $X = 0.25$ pu.
TL : $X_s = 100\Omega$, $X_n = 20\Omega$

Assume +ve, -ve and 0 seq networks of G, M and Transformer are equal.
Draw +ve, -ve and 0 seq reactance diagram.

say $S_{base} = 30MVA, 10KV$

$$X_g = 0.2 \times \frac{30}{25} \times \left(\frac{11}{10}\right)^2 = 0.2904\,pu$$

$$X_{T1} = 0.2 \times \frac{30}{30} \times \left(\frac{10}{10}\right)^2 = 0.2\,pu = X_{T2}$$

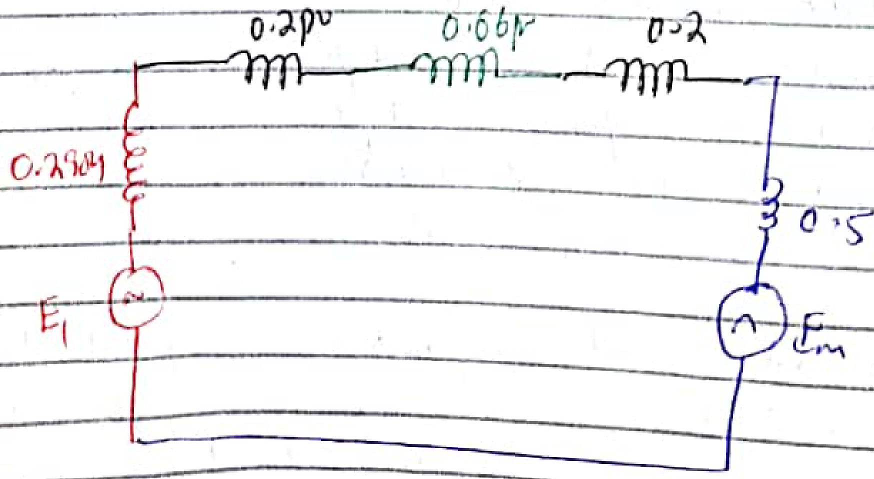
$$X_M = 0.25 \times \frac{30}{15} \times \left(\frac{10}{10}\right)^2 = 0.5\,pu$$

TL $X_{s_{pu}} = 100 \times \frac{30}{60^2} = 0.833\,pu$

$$X_{m_{pu}} = 20 \times \frac{30}{60^2} = 0.167\,pu$$

$$X_1 = X_2 = X_s - X_m$$

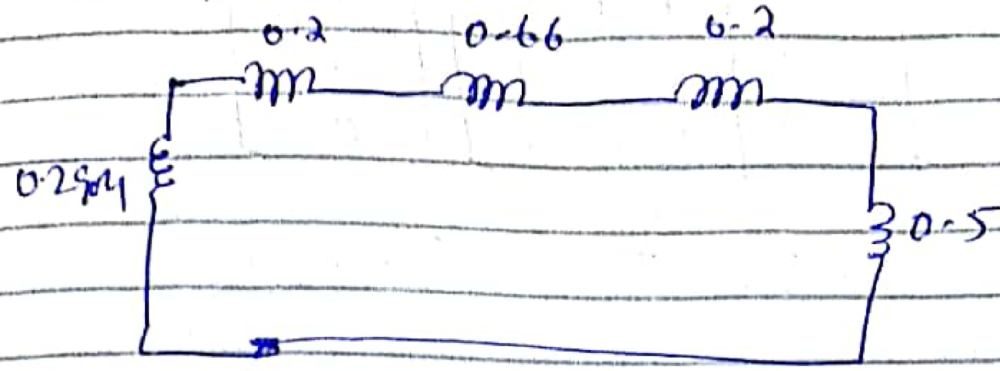
$$X_0 = X_s + 2X_m$$



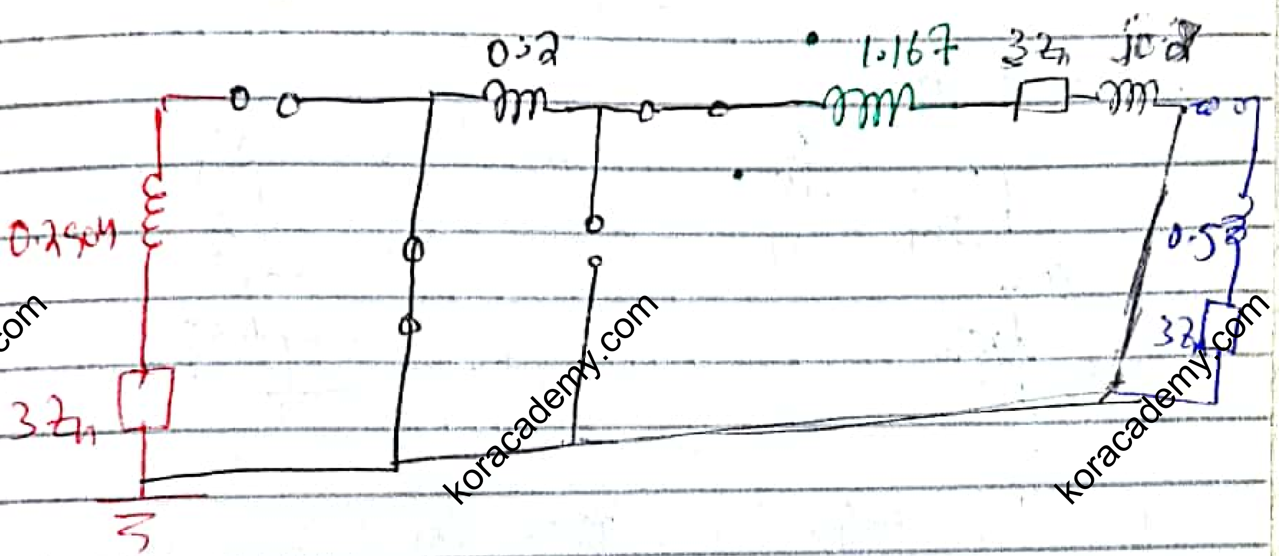
+ve seq network.

G, T2, T, M

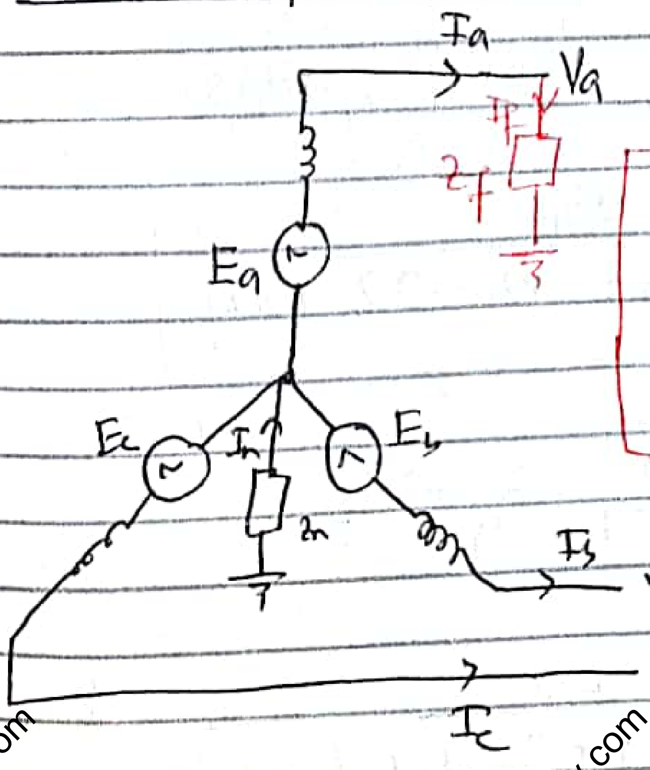
−ve seq network



Zero seq network



Line to Ground Fault



System is at no load.

ie before fault

$I_a = I_b = I_c = 0$

$V_a, V_b, V_c \rightarrow$ balanced.

If fault occurs at A,

$I_a \neq 0$

$I_b = I_c = 0$

$V_a, V_b, V_c \rightarrow$ unbalanced

$V_a = I_f Z_f = I_a Z_f$

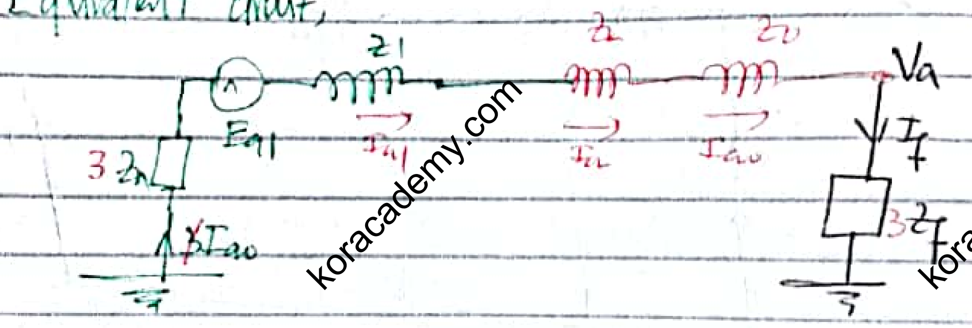
$I_a = I_f$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{I_{a0} = I_{a1} = I_{a2} = \frac{I_a}{3} = \frac{I_f}{3}}$$

$$\boxed{I_f = 3 I_{a0}}$$

Equivalent circuit;



As $\quad I_{a0} = \dfrac{V_{a0}}{Z_0 + 3Z_n}, \quad I_{a1} = \dfrac{V_{a1}}{Z_1}, \quad I_{a2} = \dfrac{V_{a2}}{Z_2}$

$$V_a = I_f Z_f = 3 I_{a0} Z_f = I_{a0}(3Z_f)$$

$$\boxed{I_{a0} = I_{a1} = I_{a2} = \frac{E_{a1}}{Z_1 + Z_2 + Z_0 + 3Z_n + 3Z_f}}$$

or $\quad I_{a0} = \dfrac{E_{a1}}{Z_{1eq} + Z_{2eq} + Z_{0eq} + 3Z_f} \quad$ (pu)

where $Z_{0eq} = Z_0 + 3Z_n$

$$\Rightarrow \boxed{I_f = 3 I_{a0} = \frac{3E_{a1}}{Z_1 + Z_2 + Z_0 + 3(Z_n + Z_f)} \text{ (pu)}}$$

to find actual current;

$$\boxed{I_f(A) = I_f(pu) \times \frac{MVA}{\sqrt{3} \times V_L}}$$

Short Circuit MVA

The power delivered by the source during fault is known as SCMVA

Method 1

$$\boxed{SCMVA = E_{a(V)} \times I_{f(A)}}$$ actual value

$E_{a(V)}$ ↓ phase voltage

↓ from first figure

Method 2

$$P_{3\phi} = 3\left[E_{a1} I_{a1}^* + E_{a2} I_{a2}^* + E_{a0} I_{a0}^*\right]$$

↓ actual value.

$$\boxed{SCMVA = 3 \; \underset{(V)}{E_{a1}} \cdot \underset{(A)}{I_{a1}^*}}$$

↳ phase value

Method 3

$$(SCMVA)_{pu} = \underset{(pu)}{E_{a1}} \cdot \underset{pu}{I_{a1}^*}$$

$$\Rightarrow SCMVA = (SCMVA)_{pu} \times 3\phi \text{ base SCMVA.}$$

Q. 100 MVA, 11 kV → line voltage

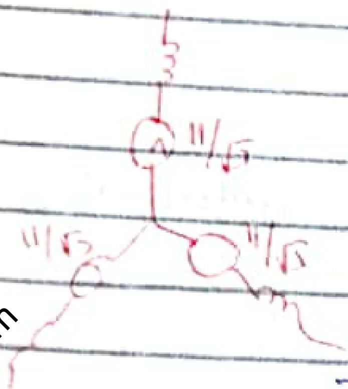
$Z_1 = Z_2 = 0.1\,pu \quad Z_0 = 0.1\,pu \quad Z_n = 0.05\,pu = Z_f$

L-G fault occurs at a phase.

Find;

$I_{f(pu)}, \; I_{f(A)}, \; SCMVA, \; SCMVA_{pu}$

→ $I_{f(pu)} = \dfrac{3E_{a1}}{Z_1 + Z_2 + Z_0 + 3Z_n + 3Z_f}$



$I_f = \dfrac{3}{0.1 + 0.1 + 0.1 + 0.15 + 0.15}$

$\Rightarrow I_{f(pu)} = 5\,pu$

$I_{a0} = I_{a1} = I_{a2} = \dfrac{I_f}{3} = \dfrac{5}{3}\,pu$

→ $I_{f(A)} = I_{f(pu)} \times \dfrac{MVA}{\sqrt{3}\,V_L} = 5 \times \dfrac{110\,M}{\sqrt{3} \times 11k}$

$\Rightarrow I_{f(A)} = 26.24\,kA$

→ $SCMVA = E_{a1(ph)} \times I_{f(A)} = \dfrac{11000}{\sqrt{3}} \times 26.4 \times 10^3$

$= 166.64\,MVA$

M2 $SCMVA = 3\,E_{a1(ph)}\,I_{a1}^*$

$= 3 \times \dfrac{11000}{\sqrt{3}} \times \dfrac{5}{3} \times \dfrac{100}{\sqrt{3} \times 11k}$

$= 166.67\,MVA$

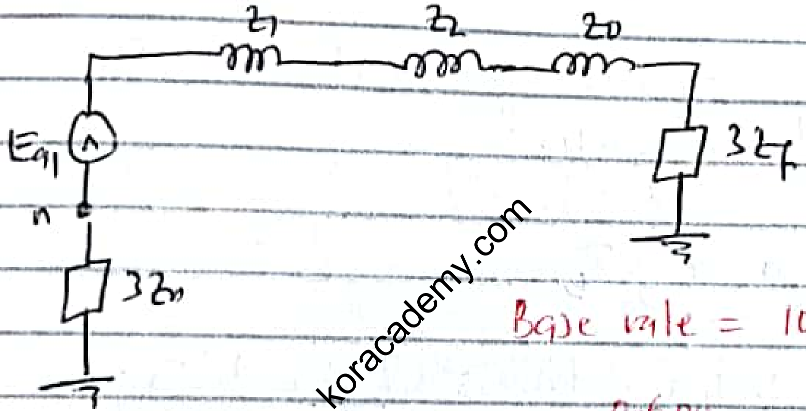
M III $(SCMVA)_{pu} = E_{a1(pu)} \times I_{a1(pu)}$

$= 1 \times \frac{5}{3} = \frac{5}{3}$ pu.

$(SCMVA) = \frac{5}{3} \times 100 = \frac{500}{3} = 166.67$ MVA.

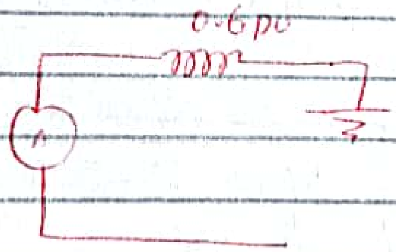
Method 4

The pu equivalent circuit;



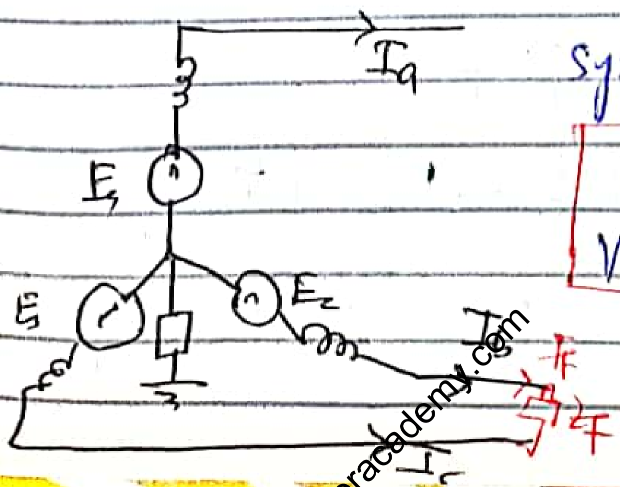
Base rate = 100MVA

eq circuit



$SCMVA = \frac{100}{0.6} = \frac{500}{3} = 166.67$ MVA

Line to Line Fault



Before fault; system is at no load.

$\boxed{I_a = I_b = I_c \quad V_a, V_b, V_c \rightarrow \text{balanced}}$

After fault;

$$\boxed{\begin{aligned} I_b &= I_f = -I_c \\ V_b - V_c &= I_f Z_f \end{aligned}}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$I_{a0} = \frac{1}{3} [I_b - I_b] = 0$$

No zero seq component of current will flow in line to line fault.

$$I_{a1} = \frac{1}{3} [a - a^2] I_b$$

$$= \frac{1}{3} [1\angle 120 - 1\angle 240] I_f = \frac{1}{3} \sqrt{3} j I_f$$

$$\Rightarrow \boxed{I_f = -j \sqrt{3} I_{a1} = |\sqrt{3} I_{a1}| \angle -90^\circ}$$

$$I_{a2} = \frac{1}{3} [a^2 - a] I_b = -I_{a1}$$

$$\boxed{I_{a2} = -I_{a1}}$$

Now As

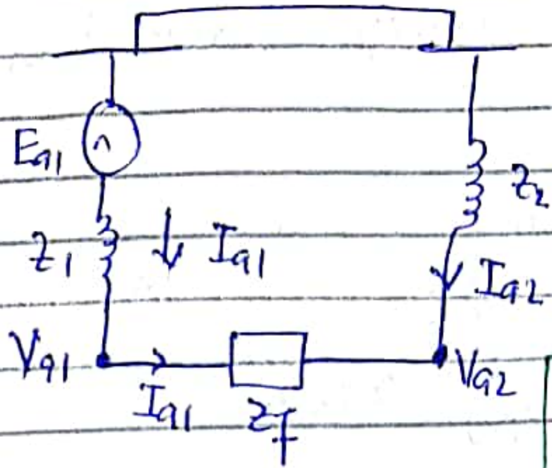
$$V_b - V_c = I_f Z_f$$

$$[V_{a0} + a^2 V_{a1} + a V_{a2}] - [V_{a0} + a V_{a1} + a^2 V_{a2}] = I_f Z_f$$

$$\Rightarrow (-a + a^2) V_{a1} + (-a^2 + a) V_{a2} = -j\sqrt{3} I_{a1} Z_f$$

$$j\sqrt{3}\,V_{a1} - j\sqrt{3}\,V_{a2} = -j\sqrt{3}\,I_{a1} Z_f$$

$$\boxed{V_{a1} - V_{a2} = I_{a1} Z_f}$$



$$\boxed{I_{a1} = \frac{E_{a1}}{Z_1 + Z_2 + Z_f}}\ pu$$

$$\boxed{I_f = \frac{\sqrt{3}\,E_{a1}}{Z_1 + Z_2 + Z_f}}\ pu$$

M1

$$\boxed{SCMVA_{(A)} = (\vec{E_b} - \vec{E_c}) \times I_{f(A)}}$$

$$= (|E| \angle -120 - |E| \angle -240^\circ)\, I_{f(A)}$$

$$\boxed{SCMVA = \sqrt{3}\ |E| \times I_{f(A)}}$$

phase value

M2

$$\boxed{3\phi\ pwr = SCMVA = 3\left[E_{a1\,(V)} \times I_{a1\,(V)}\right]_{\text{Actual}}}$$

M3

$$\boxed{SCMVA = E_{a1\,(pu)} \times I_{a1\,(pu)}}$$

Q. 100MVA, 11 KV. $Z_1 = Z_2 = Z_0 = 0.1$ pu
$Z_n = Z_f = 0.05$ pu

Line to line fault occurs b/w b and c phase

Find $I_{f(pu)}$, $I_{f(A)}$, $SCMVA_{(pu)}$, $SCMVA_{(A)}$.

$$I_{f(pu)} = \frac{\sqrt{3}\, E_{a1(pu)}}{Z_1 + Z_2 + Z_f} = \frac{\sqrt{3}}{0.25}$$

$$I_{f(pu)} = 4\sqrt{3}\ pu$$

$$I_{f(A)} = I_{f(pu)} \times \frac{MVA}{\sqrt{3}\, V_L} = \frac{4\sqrt{3} \times 100M}{\sqrt{3} \times 11k}$$

$$I_{f(A)} = 36.3\ kA$$

i. $SCMVA = \sqrt{3}\,|E| \times I_{f(A)}$

$$= \sqrt{3} \times \frac{11k}{\sqrt{3}} \times 36.3k = 400\ MVA$$

ii. $SCMVA = E_{a1\,pu} \times I_{a1\,pu}$

where $I_{a1(A)} = \frac{E_{a1\,pu}}{Z_1 + Z_2 + Z_f}$

$SCMVA_{pu} = 4.$

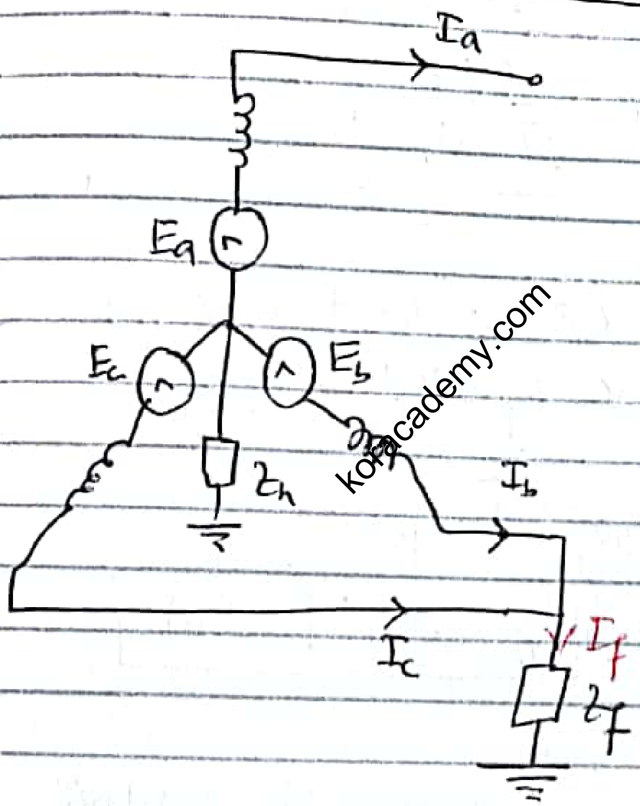
$SCMVA = 4 \times 100 = 400$ MVA
↓
S_{base} MVA

m. $SCMVA = 3 E_{a1(V)} \cdot I_{a1(A)}$

$$= 3 \times \frac{11000}{\sqrt{3}} \times \frac{I_f}{\sqrt{3}} \left(\frac{36.63\,kA}{\sqrt{3}} \right)$$

$$\Rightarrow SCMVA = 400.$$

Double Line to Ground Fault



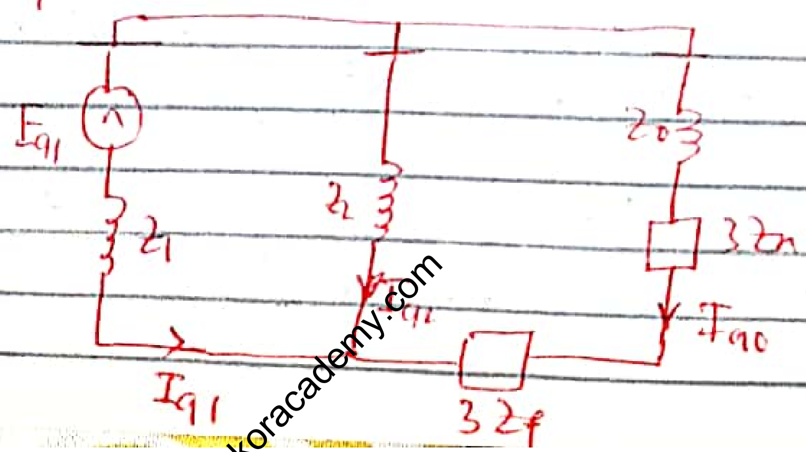
$V_b = V_c = I_f Z_f$

$I_a = 0$

$$\boxed{I_f = 3 I_{a0}}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

The equivalent circuit;



$$\boxed{I_{a1} = \frac{E_{a1}}{Z_1 + (Z_2 \parallel Z_0 + 3Z_n + 3Z_f)}} \text{ (pu)}$$

KCL @ current divider:

$$\boxed{I_{a0} = -I_{a1} \left(\frac{Z_2}{Z_2 + Z_0 + 3Z_n + 3Z_f} \right)} \text{ pu}$$

$$\boxed{I_f = 3 I_{a0}}$$

$$\boxed{I_{f(A)} = I_{f(pu)} \times \frac{MVA}{\sqrt{3}\, kV_L}}$$

For SCMVA:

We will install CB in that arm where we have emf (+ve seq network).

$$\boxed{SCMVA = E_{a1\,pu} \times I_{a1\,pu}}$$

$$\boxed{SCMVA = (SCMVA)_{pu} \times 3\phi \text{ MVA}}$$

Q. 100 MVA, 11 kV. $Z_1 = Z_2 = Z_0 = 0.1$ pu

$Z_n = Z_f = 0.05$

Find $I_{f\,(pu)}$, $I_{f\,(A)}$, SCMVA.

$$I_{a0} = -\frac{I_{a1} Z_2}{Z_2 + (Z_0 + 3Z_n + 3Z_f)} = 1.11 \text{ pu}$$

$$I_{a1} = \frac{E_{a1}}{Z_1 + [Z_2 \parallel (Z_0 + 3Z_n + 3Z_f)]} \doteq 5.55 \text{ pu}$$

$$I_f = 3 I_{a0} = 3.33 \text{ pu}$$

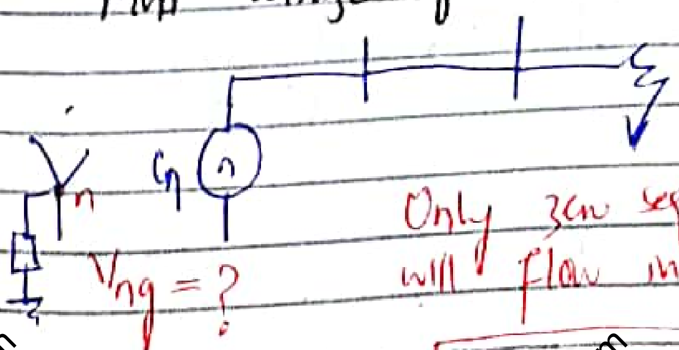
$$I_{f(A)} = 3.33 \times \frac{100}{\sqrt{3} \times 11} = 17.47 \text{ kA}$$

$$I_{G1(A)} = 5.55 \times \frac{100}{\sqrt{3} \times 11} = 29.12 \text{ kA}$$

$$SCMVA = E_{G1\,pu} \cdot I_{n\,pu} = 5.55$$

$$SCMVA_{(act)} = 5.55 \times 100 = 555 \text{ MVA}$$

Q. 20 MVA, 6.6 kV star connected alternator.
$X_g \Rightarrow X_{g1} = X_{g2} = j0.1$, $X_{g0} = j0.04$
Neutral of alternator is connected to ground with a reactor of $j0.05$ pu. It is connected with a T.L having $X_{T1} = X_{T2} = j0.1$ and $X_{T0} = j0.3$ pu.
L-G fault occurs at TL.
Find voltage of neutral wrt ground.



Only zero seq component of current will flow in neutral.

$$\boxed{I_n = 3 I_{a0}}$$

$$V_{ng} = I_n X_n = 3 I_{a0} X_n$$

$I_{a0} = ?$

In LG fault $I_{a0} = I_{a1} = I_{a2} = \dfrac{E_{a1\,pu}}{X_{1eq} + X_{2eq} + X_{0eq}}$

where

$X_{1eq} = X_{G1} + X_{T1} = j0.2$

$X_{2eq} = X_{G2} + X_{T2} = j0.2$

$X_{0eq} = (X_{G0} + 3X_n) + X_{T0} = j0.49$

$\Rightarrow I_{a0} = \dfrac{1}{j0.89} =$

$\Rightarrow V_{ng} = 3 I_{a0} X_n = 3 \times \dfrac{1}{0.89} \times 0.05$

$= 0.168$ pu

$V_{ng\,(V)} = V_{ng\,pu} \times V_{base} = 0.168 \times \dfrac{6.615}{\sqrt{3}}$

$= 642.8$ V.

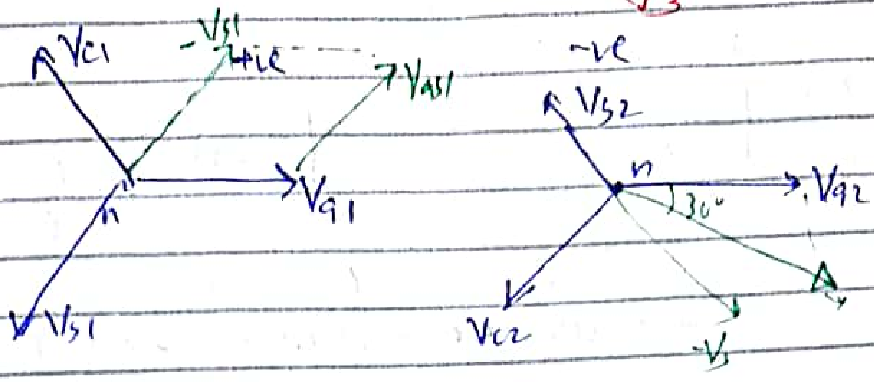
Q. Symmetrical component of line voltage in pu are;

$V_{ab1} = X \angle \theta_1$, $V_{ab2} = Y \angle \theta_2$

The phase to neutral seq voltage are;

$V_{an1} = \dfrac{X}{\sqrt{3}} \angle \theta_1 - 30°$, $V_{an2} = \dfrac{Y}{\sqrt{3}} \angle \theta_2 + 30°$

$V_L = \sqrt{3} V_p$ (or) $V_p = \frac{V_L}{\sqrt{3}}$

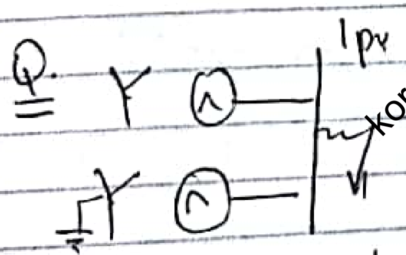


$V_{a1} = \frac{V_{ab1}}{\sqrt{3}} \angle -30^\circ$

lagging by 30°

V_p leading V_L by 30°

Q.



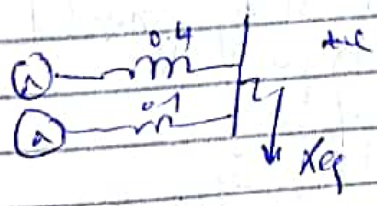
$X_1 = j0.4$ pu $\quad X_2 = j0.3$ pu

$X_0 = j0.15$ pu.

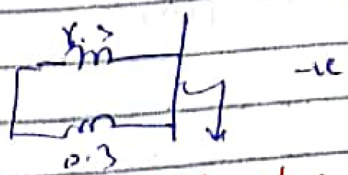
L-G fault occurs at 1 pu bus.

Find I_f (pu).

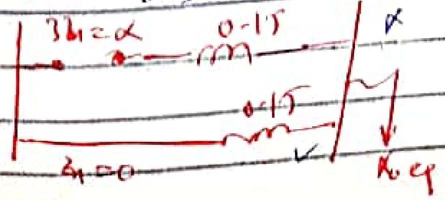
$$I_{f(p)} = 3 I_{a0} = \frac{3 E_{a1}}{X_{1eq} + X_{2eq} + X_{0eq}}$$



$X_{1eq} = 0.4 \| 0.4 = 0.2$ pu.



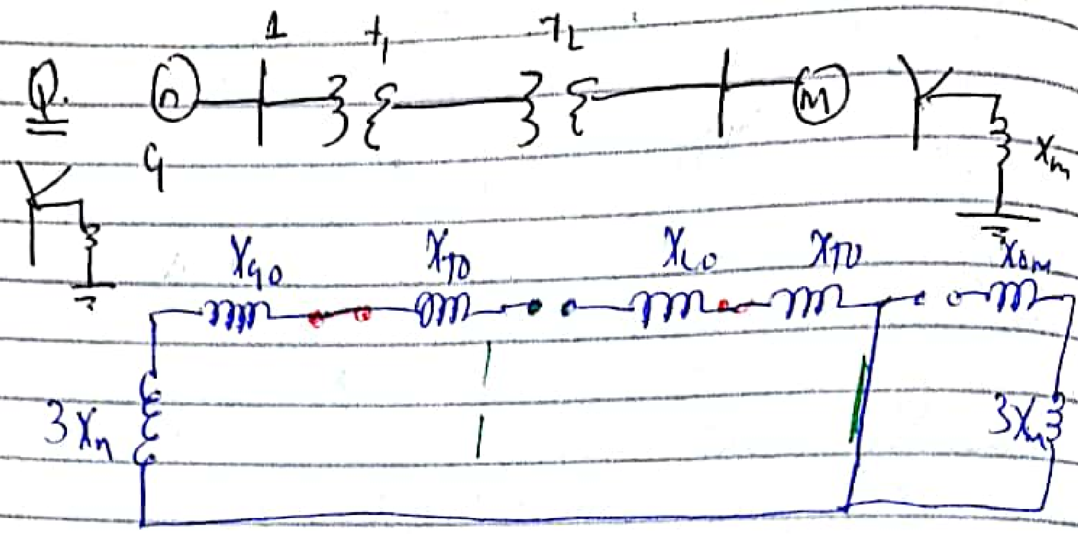
$X_{2eq} = 0.3 \| 0.3 = 0.15$ pu



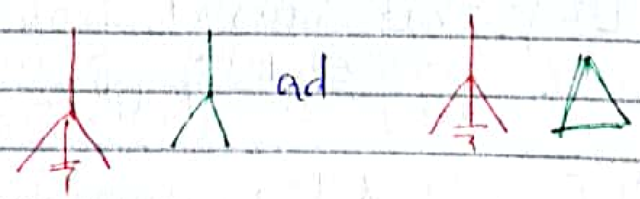
$X_{0eq} = j0.15$

$$\Rightarrow I_{f(N)} = \frac{3\times 1}{0.5} = 6\,pu$$

Q.



T_1: p shorted → p is star and grounded
no parallel so no delta, O.C → s is star connected but ungrounded.
T_2: p series switch shorted → star with grounded.
parallel short → delta



Q. 100 MVA, 25 kV Neutral is solidly grounded. → zero seq comp of current will flow.
$X_1 = X_2 = 0.2\,pu$ $X_0 = 0.05\,pu$

L-G fault occurs at terminal of the generator; $I_{f(A)}$?

$$I_f = 3I_{a0} = \frac{3E_{a1}}{Z_1+Z_2+Z_0+3\cancel{Z_n}+3\cancel{Z_f}}$$

$$I_f = \frac{3}{0.45} = 6.67\,pu$$

$$I_{f(A)} = I_{f(PU)} \times \frac{MVA}{\sqrt{3}\, V_L}$$

$$= 6.67 \times \frac{100}{\sqrt{3} \times 25} = 15.39\ KA.$$

Stability

Speed governer → keep speed of rotor constant.

K.E → $\boxed{\text{Synchronous motor}}$ → Electrical energy

$\frac{1}{2} J \omega_{sm}^2 \downarrow$

$\uparrow \downarrow N_s = \frac{120 f}{P} \downarrow$

① **Voltage stability**

overvoltage or undervoltage

Range 0.95 pu to 1.5 pu

ie ± %5 of V_{rated}

② **Frequency stability**

$K.E \downarrow \uparrow \Rightarrow N_s \downarrow \uparrow \rightarrow f \downarrow \uparrow$

Range 49.5 Hz to 50.5 Hz

ie ± 1 % of f_{rated}

③ **Angular stability** (change in rotor angle when disturbance occurs)

(a) **Steady state stability** (swing equation)

Change in δ when small disturbance occurs in system.

(b) Transient stability (Equal area criteria)
$\delta(t)$ when large disturbance occurs.

(c) Dynamic stability
Small disturbance but periodic.

Mechanical parameters to Electrical

$$KE = \frac{1}{2} J \omega_{sm}^2 \longrightarrow \boxed{\text{Alternator}} \longrightarrow P = \frac{EV}{X} \sin\delta$$

J = moment of inertia
ω_{sm} = mechanical speed

As $\quad \theta_m = \frac{2}{P}\theta_e \Rightarrow \frac{d\theta_m}{dt} = \frac{2}{P}\frac{d\theta_e}{dt}$

$\omega_s = \frac{120}{P}$ $\quad \boxed{\omega_{sm} = \frac{2}{P}\omega_s}$

$$KE = \frac{1}{2}\left[J \left(\frac{2}{P}\right)^2 \omega_s\right]\omega_s \Rightarrow \boxed{KE = \frac{1}{2}[M]\omega_s}$$

MJ-sec/elect radian = unit $\leftarrow M = 2KE/\omega_s \leftarrow$ inertia constant

Also $\quad \boxed{KE = GH}$ $\quad G$ = base MVA
H = inertia constant.

$$\Rightarrow \frac{1}{2} M\omega_s = GH$$

$$\Rightarrow \boxed{M = \frac{2GH}{\omega_s} = \frac{2GH}{2\pi f} = \frac{GH}{\pi f} = \frac{GH}{180 f}}$$

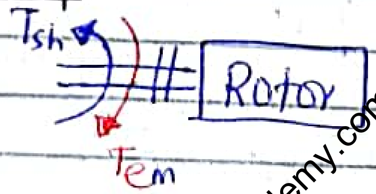
MJ-sec/elec radn $\quad$ MJ-sec/elec degree

$$\boxed{M_{pu} = \frac{M}{G} = \frac{H}{\pi f}}$$

Also $\quad H = \frac{KE}{G} = \frac{\text{Energy}}{\text{power}} = t$

unit of H → sec
→ MJ/MVA

Swing Equation



when no disturbance; $T_{sh} = T_{em}$ ⇒ stable system

Disturbance ⇒ change in T_{em}.

$$T_{sh} - T_{em} = T_J + T_B$$

T_J → inertia torque, T_B → damping torque

For worst case analysis; $T_B = 0$

$$\Rightarrow T_J = T_{sh} - T_{em}$$

$$\Rightarrow \boxed{J \frac{d^2\theta_m}{dt^2} = T_{sh} - T_{em}}$$

$$\omega_{sm}\left(J \frac{d^2\theta_m}{dt^2}\right) = (T_{sh} - T_{em})\,\omega_{sm}$$

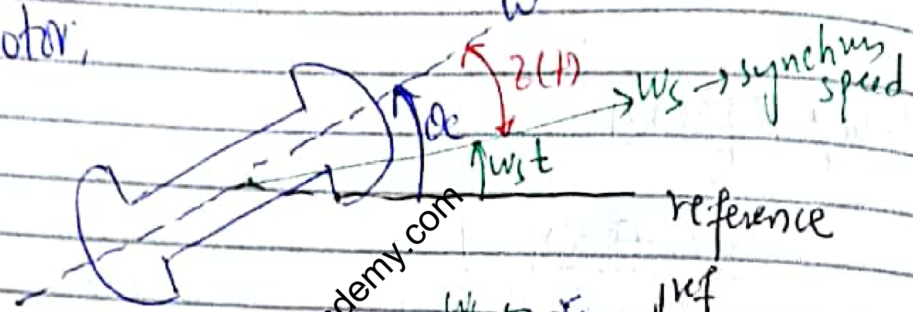
$$J \cdot \frac{2}{P} \cdot \omega_s \cdot \frac{2}{P} \frac{d^2\theta_e}{dt^2} = P_m - P_e$$

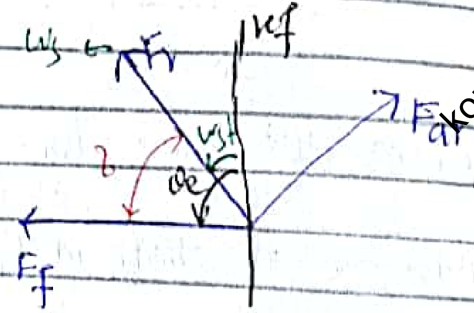
$$\left[J\left(\frac{2}{P}\right)^2 \omega_s\right]\frac{d^2\theta_e}{dt^2} = P_m - P_e$$

$$M\frac{d^2\theta_e}{dt^2} = P_m - P_e$$

For synchronous machine; $\theta_e = \omega_s t + \delta(t)$

Rotor,





ω_s = fixed

$$\frac{d^2\theta_e}{dt^2} = \frac{d^2}{dt^2}\delta(t)$$

$$\Rightarrow \boxed{M\frac{d^2}{dt^2}\delta(t) = P_m - P_e}$$

↳ Swing equation

Relation of δ with power

$\delta \to$ angle, $\frac{d}{dt}\delta \to$ speed, $\frac{d^2}{dt^2}\delta(t) \to$ angular acceleration

$$M\alpha = P_m - P_e$$

under steady state; $\boxed{P_m = P_e}$

$$M\alpha = 0 \Rightarrow \boxed{\alpha = 0}$$

$$\frac{d^2}{dt}\delta = 0 \Rightarrow \boxed{\begin{array}{l}\frac{d\delta}{dt} = \text{constant} \\ \Downarrow \\ \text{speed} = \text{constant}\end{array}}$$

Now when disturbance occurs, δ there are two possibilities;

i: $P_m - P_e > 0$

$\Rightarrow M\alpha > 0$

$\Rightarrow$ Acceleration $\alpha > 0$

$\Rightarrow$ $\boxed{\text{Speed of rotor will increase}}$

ii: $P_M - P_e < 0$

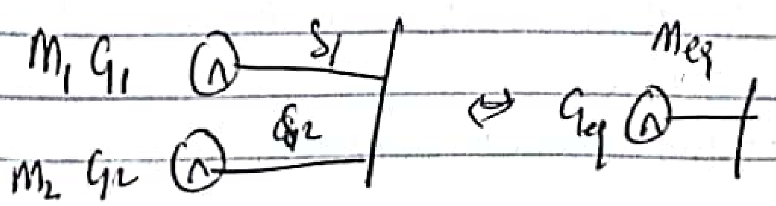
$M\alpha < 0$

$\Rightarrow \alpha < 0$ Decelleration.

$\Rightarrow$ $\boxed{\text{Speed of rotor will decrease.}}$

For 2 machine system:

If two alternators are connected in parallel.



(a) Coherent operation

Frequency is same and $\boxed{\delta_1 = \delta_2} = \delta$

$$M_1 \frac{d^2}{dt^2}\delta_1 = P_{m_1} - P_{e_1} \quad \text{— (1)}$$

$$M_2 \frac{d^2}{dt}\delta_2(t) = P_{m2} - P_{e2} \quad \text{— (1)}$$

(1) + (2)

$$\Rightarrow \boxed{(M_1 + M_2)\frac{d^2}{dt^2}\delta(t) = (P_{m1} + P_{m2}) - (P_{e1} + P_{e2})}$$

$$\Rightarrow \boxed{\begin{aligned} M_{eq} &= M_1 + M_2 \\ P_{meq} &= P_{m1} + P_{m2} \\ P_{eeq} &= P_{e1} + P_{e2} \end{aligned}}$$

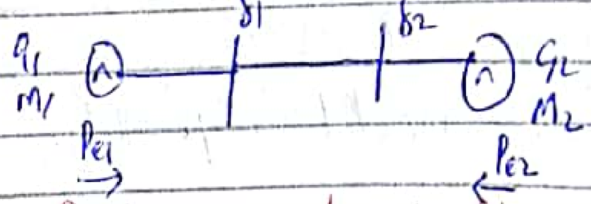
$$\frac{G_{eq} H_{eq}}{\pi f} = \frac{G_1 H_1}{\pi f} + \frac{G_2 H_2}{\pi f}$$

→ individual

$$\Rightarrow \boxed{H_{eq} = \frac{G_1 H_1 + G_2 H_2}{G_{eq}}}$$

→ common base

(b) <u>Incoherent operation</u> $\boxed{\delta_1 \neq \delta_2}$



Real power always flow from high rotor angle to low rotor angle!

Say $\delta_1 > \delta_2 \Rightarrow$ pwr will flow for 1 to 2.

1 will behave like generator and 2 will behave like motor

$$\boxed{P_{e2} = -P_{e1}}$$

Now swing equations;

$$M_1 \frac{d^2}{dt}\delta_1(t) = P_{m_1} - P_{e_1} \quad —① \times M_2$$

and
$$M_2 \frac{d^2}{dt^2}\delta_2 = P_{m_2} - P_{e_2} \quad —② \times M_1$$

①-②

$$\Rightarrow M_1 M_2 \frac{d^2}{dt_2}(\delta_1 - \delta_2) = (P_{m_1}M_2 - P_{m_2}M_1) - (P_{e_1}M_2 - P_{e_2}M_1)$$

As $P_{e_2} = -P_{e_1}$

$$\Rightarrow M_1 M_2 \frac{d^2}{dt_2}(\delta_1 - \delta_2) = (P_{m_1}M_2 - P_{m_2}M_1) - (P_{e_1}(M_1+M_2))$$

Divide by $M_1 + M_2$

$$\Rightarrow \boxed{\frac{M_1 M_2}{M_1+M_2}\frac{d^2}{dt^2}(\delta_1-\delta_2) = \frac{P_{m_1}M_2 - P_{m_2}M_1}{M_1+M_2} - P_{e_1}}$$

M_{eq} = eq inertia constant

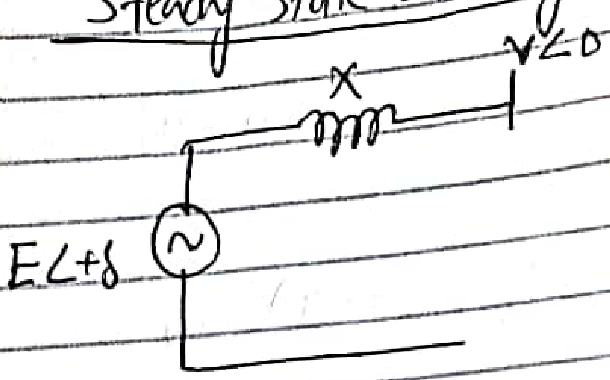
mechanical equivalent

equivalent electrical power

As $\frac{G H_{eq}}{\pi f} = M_{eq}$

$$\Rightarrow \boxed{H_{eq} = \frac{M_{eq} \cdot \pi f}{G}}$$

Steady state stability



Power delivered by machine.

$$P = \frac{EV}{X} \sin\delta$$

Under no fault; (stable)

$$P_{eo} = \frac{EV}{X} \sin\delta_o = P_m$$

$\delta_o \rightarrow$ initial rotor angle $\delta^o = \sin^{-1}\left(\frac{P_{eo}}{P_{max}}\right)$

After disturbance;

$$\delta = \delta_o + \Delta\delta$$

$$\rightarrow P_e = \frac{EV}{X} \sin(\delta_o + \Delta\delta)$$

$$= P_{max} \sin(\delta_o + \Delta\delta)$$

$$P_e = P_{max}\left[\sin\delta_o \cos\Delta\delta + \cos\delta_o \sin\Delta\delta\right]$$

For small disturbance;

$\cos\Delta\delta \approx 1$, $\sin\Delta\delta \approx \Delta\delta$

$$P_e = P_{max}\left[\sin\delta_o + \Delta\delta \cos\delta_o\right]$$

$$P_e = P_{max}\sin\delta_o + \Delta\delta\, P_{max}\cos\delta_o$$

$$\boxed{P_e = P_{eo} + \left.\frac{\partial P_e}{\partial\delta}\right|_{\delta=\delta_o} \cdot \Delta\delta}$$ rad/s

new

$P_e = P_{max}\sin\delta_o$ $\left.\frac{\partial P_e}{\partial\delta}\right|_{\delta=\delta_o} = P_{max}\cos\delta_o$

As swing equation:

$$M\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$= \cancel{P_m} - \cancel{P_{e0}} - \frac{dP_e}{d\delta}\bigg|_{\delta=\delta_0} \cdot \Delta\delta$$

$$\Rightarrow M\frac{d^2\delta}{dt^2} = -\frac{dP_e}{d\delta}\bigg|_{\delta=\delta_0} \cdot \Delta\delta$$

As $\delta = \delta_0 + \Delta\delta$

$$\Rightarrow \frac{d}{dt}\delta = \frac{d}{dt}\Delta\delta$$

$$\Rightarrow \frac{d^2}{dt^2}\delta = \frac{d^2}{dt^2}\Delta\delta$$

$$\Rightarrow M\frac{\partial^2}{\partial t^2}\Delta\delta + \frac{\partial P_e}{\partial t}\bigg|_{\delta=\delta_0} \cdot \Delta\delta = 0$$

$$\Rightarrow \frac{\partial^2}{\partial t^2}\Delta\delta + \frac{K}{M}\Delta\delta = 0$$

where $K = \frac{\partial P_e}{\partial \delta}\bigg|_{\delta=\delta_0}$

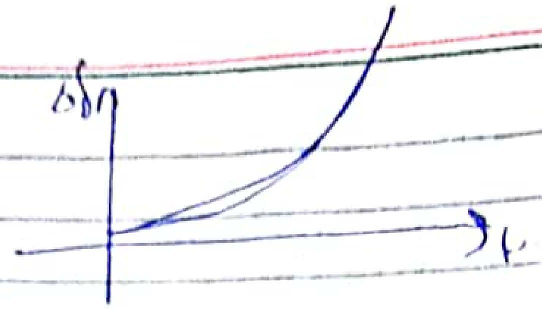
$$\left(D^2 + \frac{K}{M}\right)\Delta\delta = 0$$

$$\boxed{D = \pm\sqrt{\frac{-K}{M}}}$$ roots.

i. $K<0$ $\Rightarrow$ $D =$ equal and real roots.

Response: $\Delta\delta = C_1 e^{\sqrt{\frac{K}{M}}t} + C_2 e^{-\sqrt{\frac{K}{M}}t}$

↳ unstable



ii. K>0

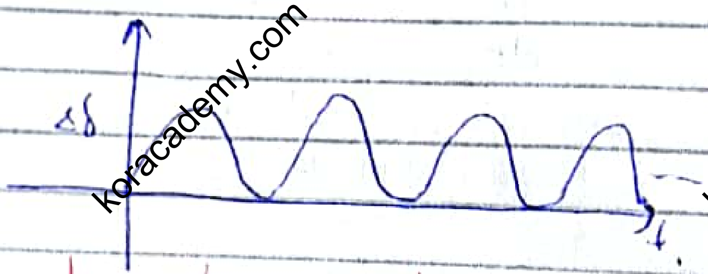
Then → roots are complex.

$$D = \pm j \sqrt{K/M}$$

Response,

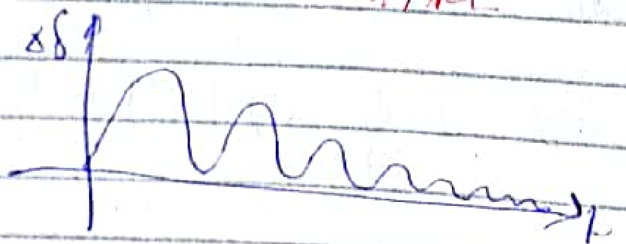
$$\boxed{\Delta\delta = C_1 \cos\left(\sqrt{\tfrac{K}{M}}\, t\right) + C_2 \sin\left(\sqrt{\tfrac{K}{M}}\, t\right)}$$

marginal Stable



The damping torque will try to damp these oscillation

stable



frequency of oscillations,

$$w = j \sqrt{\frac{\frac{\partial P_e}{\partial \delta}\Big|_{\delta=\delta_0}}{M}}$$

or

$$w = \sqrt{\frac{\frac{dP_e}{d\delta}\Big|_{\delta=\delta_0}}{GH} \pi f} \quad \text{rad/s}$$

Stability criteria (conclusion)

① $\frac{\partial P_e}{\partial \delta} > 0$, $P_{max} \cos \delta > 0$ stable, $\delta \in (0, 90)$

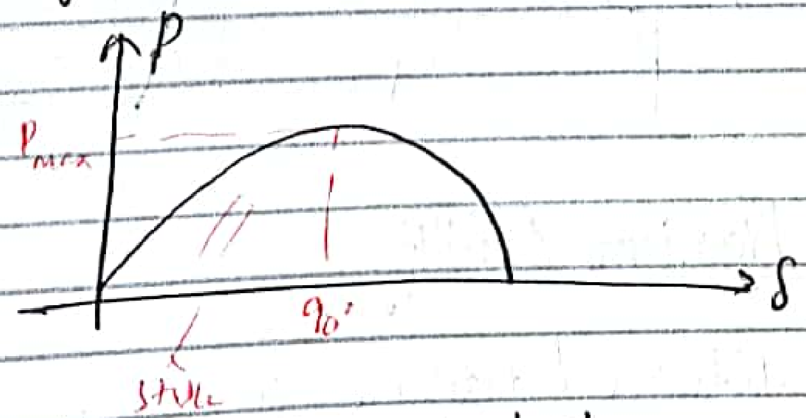
② $\frac{\partial P_e}{\partial \delta} < 0$ unstable, $\delta \in (90, 180)$ $P_{max} \cos \delta < 0$

③ $\frac{\partial P_e}{\partial \delta} = 0$ marginally stable, $\delta = 90^\circ$ $P_{max} \cos \delta = 0$

Maxi steady state limit $P_e = P_{max} \sin \delta = P_{max}$
↳ $\delta = 90^\circ$

$\delta > 90^\circ$ → unstable state

④ Max steady state limit = P_{max}.



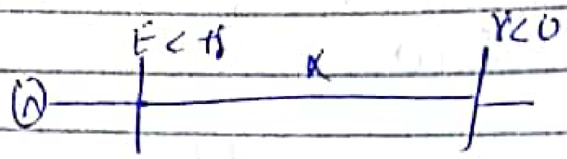
Method to improve steady state limit

$$P_{max} = \frac{EV}{X}$$

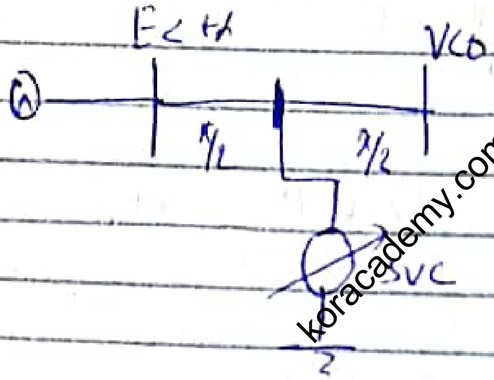
To increase P_{max}
① Increase excitation (emf) overexcited condition
② Decrease reactance ↓ ↳ bundling of conductor
↳ Double circuit line / parallel line

→ Series compensation.

(3) By use of SVC (static VAR compensator)
↓
will inject reactive power and will not absorb any real power.

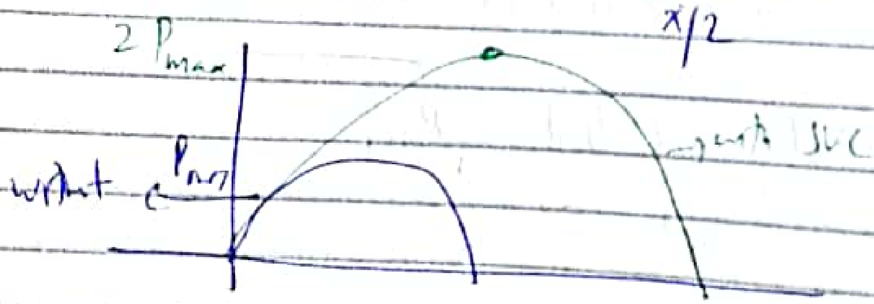


$$P_{max} = \frac{EV}{X}$$



$$P_{max} = \frac{EV}{X/2}$$

$$P = \frac{EV}{X/2}\sin\delta_2$$



Transient Stability

large disturbance / disturbance for a longer duration / continuous disturbance

↳ short circuit fault.

↳ contingency → removal of any instrument from transmission line due to fault / disturbance

Critical clearing time (t_{CR}) maximum time taken by C.B to maintain the system in synchronism.

(the max time to remove fault)

time taken by CB to remove fault $\leq t_{CR}$

Assumptions-

1. Neglect saturation. ($E_f \to E_f$)
2. Neglect saliency (cylindrical rotor)
3. Neglect damping.
4. Neglect winding resistance
5. Frequency = constant
6. Speed governor is deactivated.
7. Automatic voltage regulator (AVR) is deactivated.

Equal Area Criteria

As swing equation;

$$M\frac{d^2\delta}{dt^2} = P_m - P_e = P_a$$

↳ accelerating power

$$\Rightarrow 2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = \left(\frac{P_a}{M}\right)2\frac{d\delta}{dt}$$

$$\Rightarrow \int \frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 dt = \int 2\frac{P_a}{M}\frac{d\delta}{dt}\,dt$$

$$\Rightarrow \left(\frac{d\delta}{dt}\right)^2 = \int 2\frac{P_a}{M}\,d\delta$$

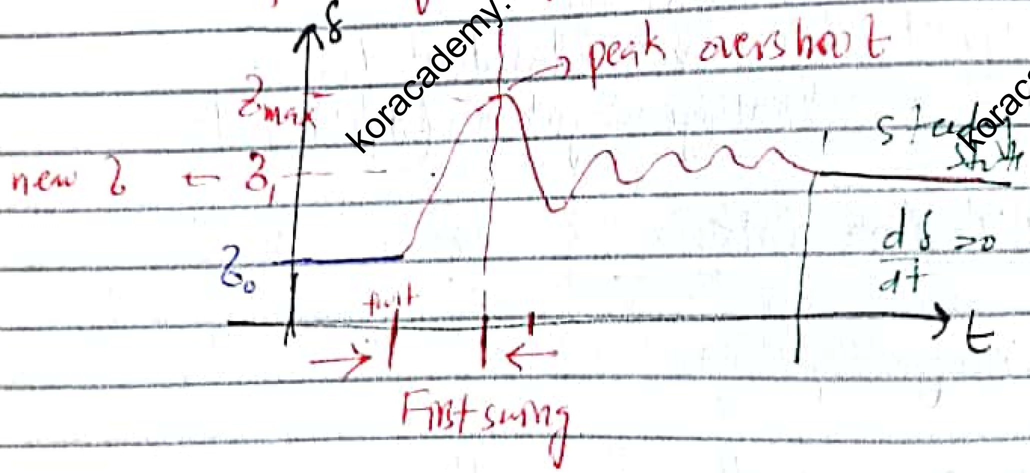
$$\rightarrow \frac{d\delta}{dt} = \sqrt{\frac{2P_a}{M}\, d\delta}$$

or
$$\boxed{\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int P_a\, d\delta}}$$

↳ during the fault.

(or)
$$\boxed{\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int (P_m - P_{max}\sin\delta)\, d\delta}}$$

alternate form of swing equation.



Under Steady state; $\frac{d\delta}{dt} = 0$

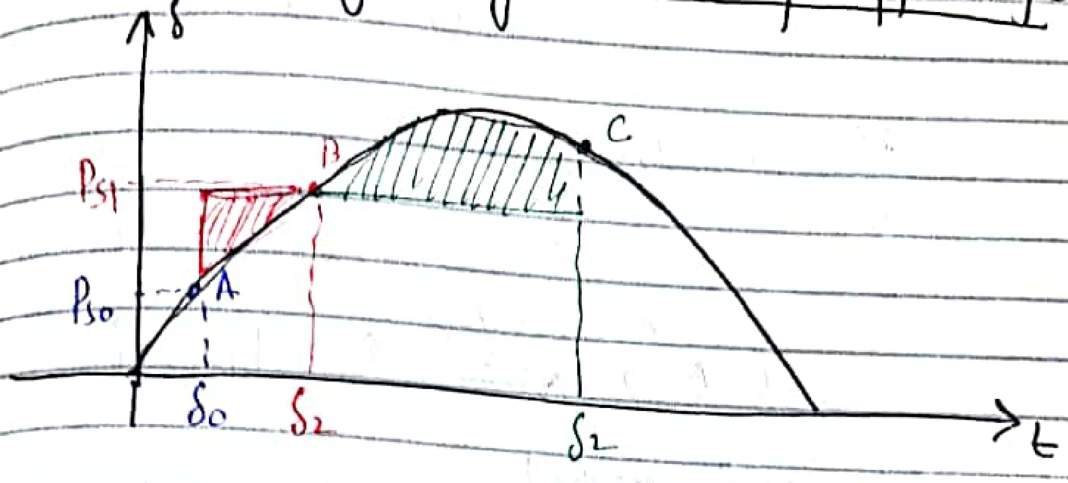
$$\Rightarrow \boxed{\int P_a\, d\delta = 0}$$

↳ Area under accelerating pow = Area under decellerating pow

→ Equal area criterion

↳ (seeing swing equation graphically)

Transient stability analysis when shaft i/p changes.



$P_{s0} = P_e = P_{max} \sin \delta_0$ → steady state.

suddenly increase shaft power from P_{s0} to P_{s1}
(i/p P_s)

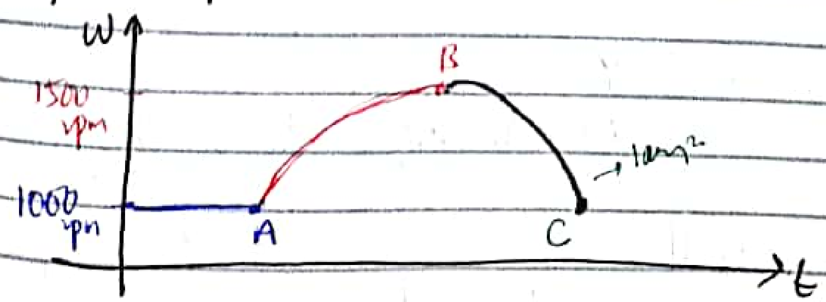
$$M \frac{d^2\delta}{dt^2} = P_{s1} - P_e = P_a > 0$$

rotor → accelerate ⇒ ω ↑

$$\frac{d\delta}{dt} = \uparrow\omega - \omega_s \qquad \frac{d\delta}{dt} > 0$$

↳ rotor angle ↑

Say for example;



At point B;
(electrical power also increases until max in δ and becomes equal to P_{mech} at δ2)

$P_{s1} = P_e \Rightarrow P_a = 0$

↳ no acceleration

$\Rightarrow 1500 - 1000 = 500$

$$\frac{d\delta}{dt} = \omega - \omega_s > 0$$

↳ rotor angle will still increase.

Now as $\delta \uparrow \Rightarrow \cancel{P_{max} \sin\delta}$ $EV/X_s \sin\delta \uparrow \Rightarrow P_{electrical} \uparrow$

(say due to inertia)

<u>From B to C</u>

$P_e > P_{s1} \Rightarrow P_a < 0$

↳ deceleration

$$\frac{d\delta}{dt} = \omega - \omega_s > 0$$

↳ rotor angle still increase

At point C.

$P_e > P_{sh} \Rightarrow P_a < 0$

↳ Decelleration.

$$\frac{d\delta}{dt} = \omega - \omega_s = 0$$

↳ rotor angle will not increase further

In a nutshell;

AB → acceleration BC → deceleration.

CB → deceleration BA → acceleration

and this repeats for a few cycles and

finally it will become stable at a new operating point (B).

Applying equal area criteria to the first swing;

ie $\int P_a \, d\delta = 0$

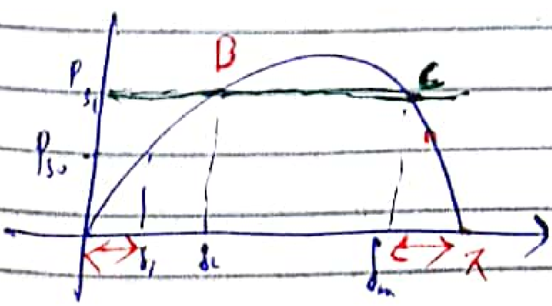
$$\boxed{\int_{\delta_0}^{\delta_1} (P_{s1} - P_e) \, d\delta = \int_{\delta_1}^{\delta_2} (P_e - P_{s1}) \, d\delta}$$

$$\int_{\delta_0}^{\delta_1} (P_s - P_{max} \sin\delta) \, d\delta = \int_{\delta_1}^{\delta_2} (P_{max} \sin\delta - P_s) \, d\delta$$

$$\Rightarrow \boxed{P_s(\delta_1 - \delta_0) - P_{max}(\cos\delta_0 - \cos\delta_1) = P_{max}(\cos\delta_1 - \cos\delta_2) - P_s(\delta_2 - \delta_1)}$$

Limit / Range of c?

↳ Rotor angle $\delta_2 = \delta_{max} = ?$ (range)



$$\boxed{\delta_{max} = \pi - \delta_1}$$

$\delta_1 = ?$

As $P_{s1} = P_{max} \sin\delta_1$

$$\boxed{\delta_1 = \sin^{-1}\left(\frac{P_{s1}}{P_{max}}\right)}$$

What if we take C beyond this point?

B to C — deceleration.

At C, $P_{s1} = P_e$

If δ is further increased; $P_e < P_{s1}$

$\Downarrow$

rotor speed increases (ω) $\Leftarrow$ acceleration

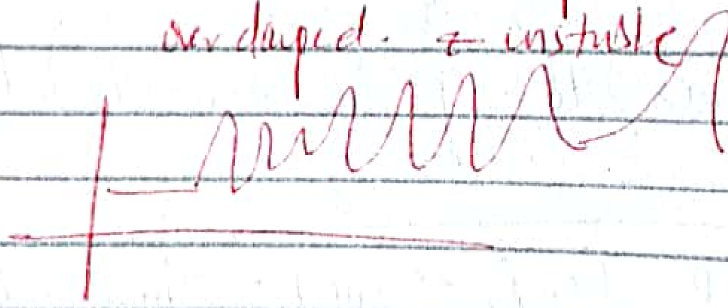
↳ It will never decelerate again.

B/c after C; P_{s1} is always greater than P_e.

↳ acceleration

↳ δ will keep on increasing

overdamped. $\Leftarrow$ unstable



δ_{max} → max rotor angle below which the system will not loose synchronism.

⊙ The angle above which system will loose synchronism

Generally mechanical power does not increase, we have disturbance in electrical side.