Electrostatics

It is that branch of physics which deals with the properties of charges when they are at rest.

What is charge? It is the property of matter which may be defined as the source of electric field. In general this property is acquired either by losing or by gaining electron. A charged object exerts force on a nearby charged/neutral object. The S.I unit of charge is called coulomb (C). To be very specific we say that charge is an intrinsic property of matter.

Millikan's Findings By oil-droplet experiment:

Following two are the great discoveries of Millikan.

(1) The smallest amount of charge which can be developed on a neutral object is 1.6×10^{-19} C. Millikan associated this amount of charge with an electron. Therefore, $e = 1.6 \times 10^{-19}$ C and it is also considered as the basic unit of charge.

(2) Quantisation of charge: It states that charge exists in the form of discrete packets, each of magnitude "e", which represents the smallest amount of charge that can exists freely. The total magnitude of charge which a body may acquireis equal to integral multiple of "e". i.e.

$$Q = 1e, 2e, 3e, 4e \ldots = ne \text{ where } n = 1, 2, 3, 4 \ldots \text{ and } e = 1.6 \times 10^{-19} \text{ C.}$$

Note: The charge of how many electrons will be equal to 1 C?

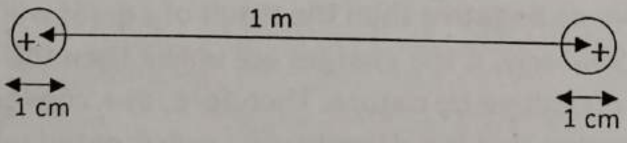
$$\text{As we know, } Q = ne \Rightarrow n = \frac{Q}{e} = \frac{1\,C}{1.6 \times 10^{-19} C} = 6.25 \times 10^{18}$$

Conservation of charge: It states that charge can neither be created nor destroyed but can be transferred from one body to another but the total magnitude of charge remains constant.

Point charges: In general we say that a body whose charge is concentrated at its centre or appear to be concentrated at its centre is called a point charge. e.g, an atom which has lost all of its electrons, a charged conducting sphere, etc.

The most acceptable definition of point charges is:

"Two charged bodies are considered point charges if separation between them is greater than their size, as shown."



Point charges can be positive or negative.

Source Charge: The charge whose field is under consideration is called the source charge. It may be positive or negative.

Field Charge: The charge placed in the field of a source charge for some investigations/study is called the field charge. It may be positive or negative.

Test Charge: The charge which is used to find the direction of an electric field/force is called the test charge. It is always equal to unit positive charge.

Electrostatic force: The force of attraction or repulsion between two charges when the magnitude of displacement between them remains constant. Like charges repel and unlike charges attract each other.

Coulomb's Law: This law states that the force of attraction or repulsion between two point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between their centres.

$$F \propto q_1 q_2 \rightarrow (1)$$

$$F \propto \frac{1}{r^2} \rightarrow (2)$$

Combining the two proportionalities

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\text{Or } \vec{F} = k\,\frac{q_1 q_2}{r^2} \rightarrow (3)$$

Where "k" is constant of proportionality and its value depends upon the system of units used and the medium between the charges. For vacuum, $k = \frac{1}{4\pi\,\epsilon_0} \rightarrow (4)$

Where, $\epsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$(S.I base unit= $kg^{-1}\ m^{-3}\ s^4\ A^2$) , and it represents the permittivity of vacuum (free space).

What is Permittivity of a medium? The resistance of a medium for electric field. A strong electric field cannot be easily established in a medium of large permittivity.

Substituting values in equation (4) , $k = 9 \times 10^9\ N\ m^2\ C^{-2}$

When the charges are placed in vacuum, then $F = \frac{1}{4\pi\,\epsilon_0}\frac{q_1 q_2}{r^2} \rightarrow (5)$

The electrostatic force always acts along the line, joining the two charges. If r is the separation between them and $\hat{r}$ is its direction then in vector form,

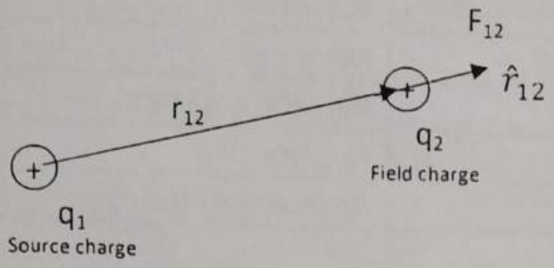
$$\vec{F} = \frac{1}{4\pi\,\epsilon_0}\frac{q_1 q_2}{r^2}\,\hat{r} \rightarrow (6)$$

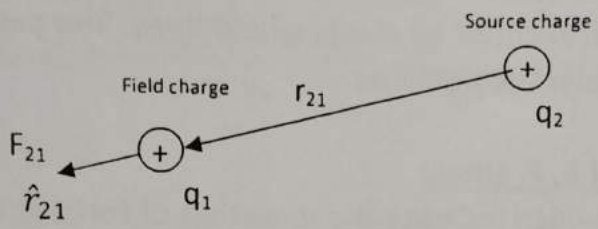
If q_1 and q_2 are both either positive or negative then the result of eq. (6) will be positive and the force will be of repulsive nature. Similarly, if the charges are unlike then the result of eq. (6) will be negative and the force will be attractive by nature. Therefore, as a convention attractive force is represented with negative sign and repulsive force is represented with positive sign.
If we consider q_1 as a source charge and q_2 as a field charge then eq. (6) can be written as,

$$\overrightarrow{F_{12}} = \frac{1}{4\pi\,\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12} \rightarrow (7)$$

Where, F_{12} = force due to q_1 on q_2 , and r_{12} = distance from q_1 to q_2, $\hat{r}_{12}$ represents a direction pointing from q_1 to q_2. Similarly, if we consider q_2 as a source charge and q_1 as a field charge then eq. (6) can be written as, $\overrightarrow{F_{21}} = \frac{1}{4\pi\,\epsilon_0}\frac{q_1 q_2}{r_{21}^2}\,\hat{r}_{21} \rightarrow (8)$

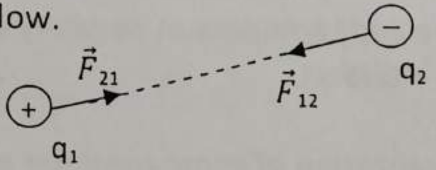
Where, F_{21} = force due to q_2 on q_1 , and r_{21} = distance from q_2 to q_1, $\hat{r}_{21}$ represents a direction pointing from q_2 to q_1. Eq. (7) and eq. (8) are illustrated in the following figure.





From the above diagrams it is clear that $\vec{F}_{12} = - \vec{F}_{21}$

If q_1 and q_2 are opposite charges then they will attract each other and their forces will act along the line joining them , as shown below.



Coulomb's Law for dielectric medium:

Electric force between two point charges is dependent upon the medium in which the charges are placed. A dielectric medium means an insulating medium. The permittivity of a medium (ϵ_m) can be determined with respect to vacuum. This concept is called relative permittivity (ϵ_r).

$$\epsilon_r = \frac{\epsilon_m}{\epsilon_o}$$

Relative permittivityϵ_r is a dimensionless physical quantity. From above equation we can write that the permittivity of a medium can be determined as:

$$\epsilon_m = \epsilon_r \epsilon_o$$

Thus, $F_m = \frac{1}{4\pi\, \epsilon_m} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\, \epsilon_r \epsilon_o} \frac{q_1 q_2}{r^2} = \frac{1}{\epsilon_r} \left\{ \frac{1}{4\pi\, \epsilon_o} \frac{q_1 \;\; q_2}{r^2} \right\}$

$$F_m = \frac{1}{\epsilon_r} F \Rightarrow F = \epsilon_r F_m$$

The relative permittivity of various dielectric media is greater than one as given below. It means that electric force between two point charges when they are placed in vacuum (F) is greater than the force (F_m) when the same charges are placed in a dielectric medium at same distance from each other.

Medium	ϵ_r	Medium	ϵ_r
Vacuum	1	Glass	4.8 to 10
Air	1.0006	Mica	3 to 7.7
Benzene	2.284	Paraffine paper	2
Germinium	16	Rubber	2.94
Water	78.5	Ammonia(liquid)	22 to 25

Electric Field and Electric field intensity:

The region around a charge where it can exert force on another charge is called electric field. Electric field around a charge is represented by electric field lines. The path followed by a test charge in an electric field is called electric field line.

Some Important Characteristics of E. F. Lines:

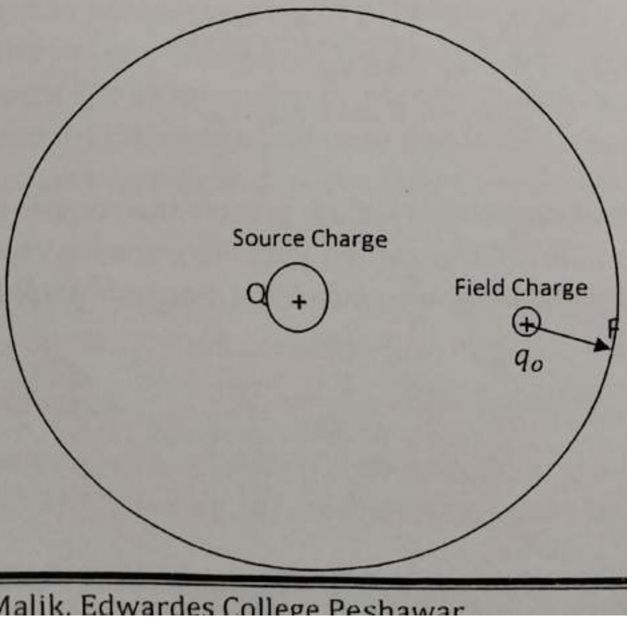
1. These are imaginary lines, which indicate the direction of force acting on a test charge at a point in an electric field.
2. The region in which electric field lines are closely packed represents strong electric field.
3. They are always perpendicular at the surface of the charged object.
4. They originate at positive and terminate at negative charge.
5. Electric field lines never intersect.

Note: The diagrammatic representation of some common electric field patterns are given in book.

Electric Field Intensity:

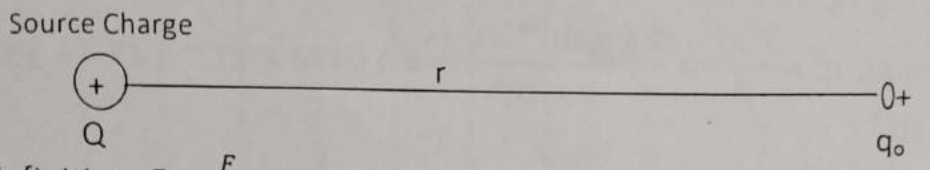
That physical quantity which indicates the strength of electric field at a specific point is called electric field intensity. It can be defined as "Electric force acting per unit charge at a point in an electric field is called electric field intensity".Mathematically, $E = \frac{F}{q_o} \rightarrow (1)$

Its S.I unit is N C^{-1} (S.I. Base unit = kg m s^{-3} A^{-1}). It is a vector quantity and its direction is always the same as that of the force acting on q_o .



Electric field intensity at a distance r due to a point (source) charge:

Consider a source charge Q. Let q_o is a test charge placed in its field at a distance r as shown.



Then by definition, E = $\frac{F}{q_o}$

According to coulomb's law.

F = $\frac{1}{4\pi\,\epsilon_0}\frac{Q\,q_o}{r^2}$, therefore, E = $\frac{1}{q_o}[\frac{1}{4\pi\,\epsilon_0}\frac{Q\,q_o}{r^2}]$

E = $\frac{1}{4\pi\,\epsilon_0}\frac{Q}{r^2}$→ (3)

Example 11.1

Find electric field at a distance of 30 cm from a 3 μC point charge?

Solution:

As given, r = 30 cm = 0.3 m , and the source charge, Q = 3 μC = 3 × 10^{-6} C

Formula, E = $\frac{1}{4\pi\,\epsilon_0}\frac{Q}{r^2}$ = (9 × 10^9) $[\frac{3\times10^{-6}}{0.3}]$ = 3 × 10^5 N C^{-1}

Example 11.2

Two small spheres, each of mass 0.1 g are suspended from a common point with the help of 20 cm long strings. They are equally charged and they repel each other to a distance of 24 cm. what is the charge on each sphere?

Solution:

Let q be the magnitude of charge on each sphere.

According to Coulomb's law

F = $k\frac{q^2}{r^2}$→ (1)

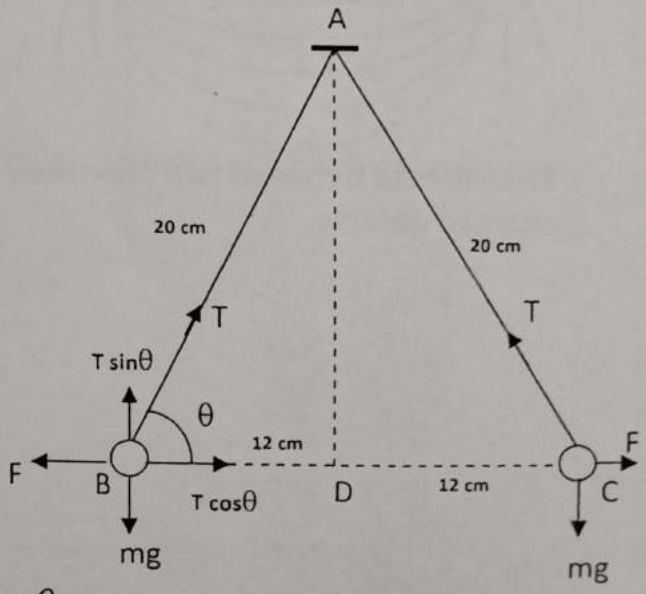
Here k = 9 × 10^9 N m^2 C^{-2} and r = 24 cm = 0.24 m

We need to calculate F to find q.

Consider the sphere at B, which is under the action Of three forces F, mg and T but it is still in equilibrium. To apply first condition of equilibrium, we resolve T Into its components, as shown.

Thus according to Σ F = 0

Or F = T cos θ→ (2) and mg = T sin θ→ (3)



To eliminate T, divide eq. 3 by eq. 2 ⇒$\frac{mg}{F} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

$\frac{mg}{\tan\theta} = F$→ (4)

To find tan θ consider ΔADB, tan θ = $\frac{AD}{BD}$→ (5)

To find AD, apply Pythagoras theorem to ΔADB, $(AB)^2 = (BD)^2 + (AD)^2$

$(AD)^2 = (AB)^2 - (BD)^2 = (0.2)^2 - (0.12)^2 = 0.04 - 0.0144 = 0.0256$

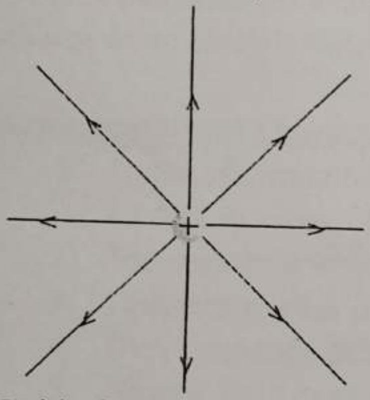
AD = 0.16 m, Thus $F = \frac{0.1 \times 10^{-3} \times 9.8}{0.16} = 7.4 \times 10^{-4} N$

Thus eq. (1) can be written as, $q^2 = \frac{F r^2}{k} = \frac{7.4 \times 10^{-4} \times 0.24}{9 \times 10^9} = 0.0474 \times 10^{-13} = 47.4 \times 10^{-16}$

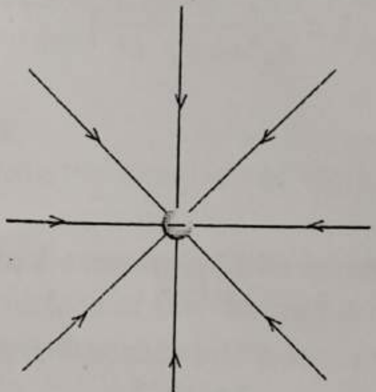
$q = 6.889 \times 10^{-8}$ C $= 6.9 \times 10^{-8}$ C

Representation of electric field lines:

According to convention the electric field lines around a positively and negatively charged bodies can be represented as.

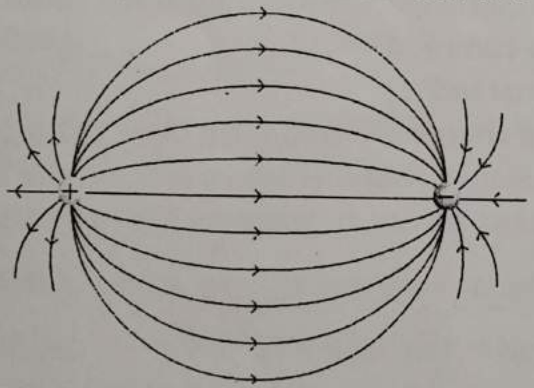


Field of a positive charge

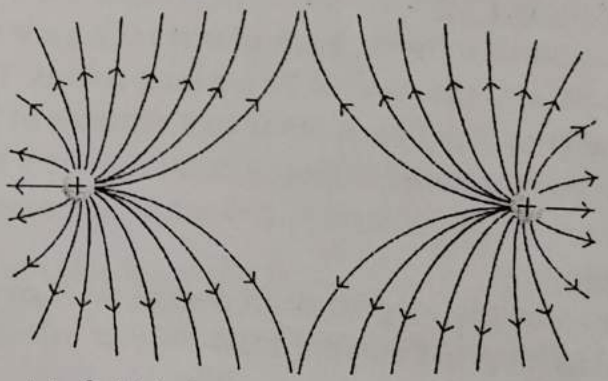


Field of a negative charge

Similarly electric field between two opposite and two similar charges is:

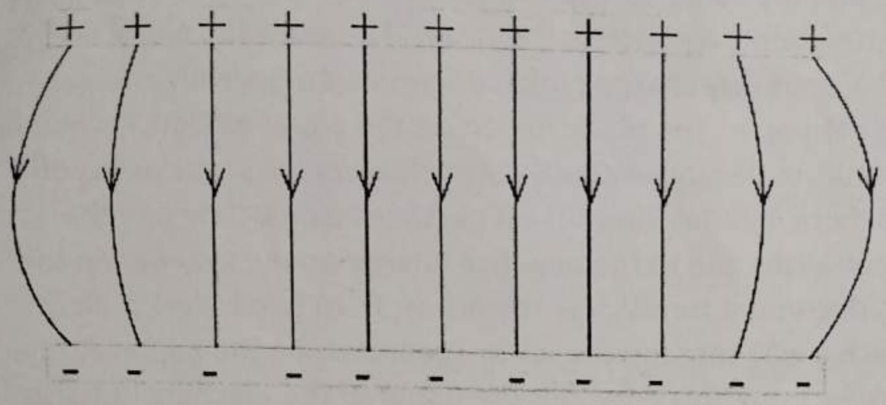


Electric field between two oppositely charged objects.



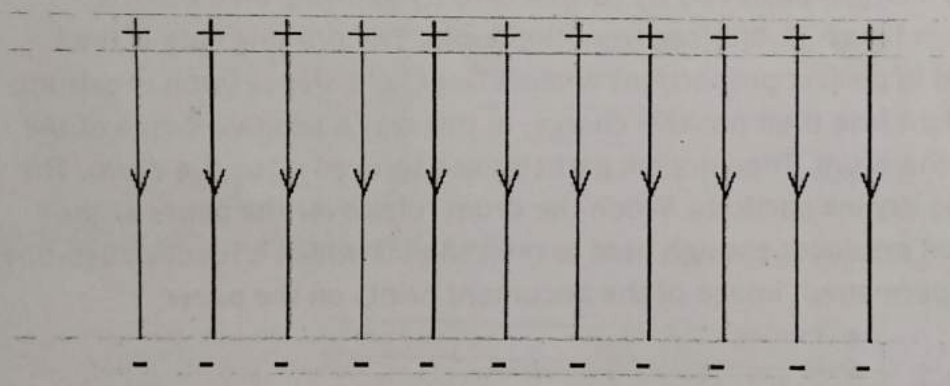
Electric field between two similar charges.

Electric field between two oppositely charged parallel plates:

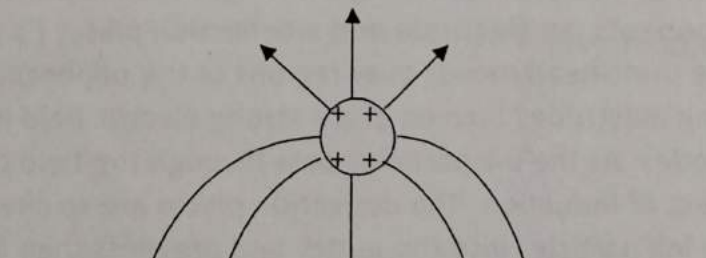


The region in which electric field lines are parallel represents uniform electric field, i.e. the strength and direction of the field remain same within this region. The bending of field at the edges is called fringing effect.

If the plates are of infinite length then the field looks like below:



The filed pattern of a positively charged sphere and a negatively charged plate looks like below:



Applications of Electrostatics:

a) **Photocopiers:**

It consists of a drum which is made of aluminum and is coated with selenium. Selenium is a photo conductor i.e. it becomes conducting when light shines upon it and behaves like a perfect insulator when there is no light. A dry ink called toner is usually positively charged whereas the drum is negatively charged. When the button of the photocopier machine is pressed to make a photocopy of the document which is laid on the glass plate of the copier machine, first the toner sprays charged ink particles on the aluminum drum. Due to electrostatic attraction they stick to the drum. The selenium works like a perfect insulator between the positively charged ink particles and negatively charged drum. Then light illuminates the paper. The blank region on the paper reflects maximum light, which is focused on the drum through a convex lens. The dry ink particles fall off the region on the selenium where light falls because it becomes conducting and the positive charge of the ink neutralizes due to the negative charge on the cylinder. In this way an image of the written document forms over the drum. Then two rollers pulls in the paper and it passes in such a way that the dry ink image leaves on the paper. At the end the paper two heating rollers not only bring out the paper of the machine but also melt the ink, which is readily absorbed by the paper and a permanent impression forms on the paper.

b) Laser Printers:

Laser printer work is largely due to process called xerography. Initially the photoreceptor drum is charged positively by corona wire by applying an electrical current on it as shown in figure 11.8(b)(see from the book). The printing data is read from the computer and in perfect proportions when a laser light shines upon the drum, areas exposed to the light lose their positive charge. In this way a positive image of the written text leaves on the drum. Then dry ink particles are sprayed upon the drum. The charged areas catch the dry ink particles. When the drum roles over the paper at the same time a heating coil produces enough heat to melt the ink which is readily absorbed by the paper. In this a permanent image of the document prints on the paper.

c) Inkjet Printers:

An inkjet printer is another type of printer that uses electrostatic charges in its operation. While shuttling back and forth across the paper, the inkjet printhead ejects a thin stream of ink. The schematic block diagram about its different parts and its internal working is shown in fig. 11.9 (book). During their flight the ink particles pass through two electrical components, an electrode and a deflection plates (a parallel plate capacitor) when the print-head moves over regions of the paper that are not to be printed, the charging electrodes turn on and a strong electric field is developed between the charging electrodes. As the ink particles pass through the field they acquire a net charge by the process of induction. The deflection plates are so charged that they deflect the charged ink particles into the gutter and prevents then from reaching the paper. For those areas on the paper where printing has to be done the charging electrodes remain off and in this way the stream of ink jet reaches the paper. The charging electrodes get instruction from a computer system.

Example 11.3:

A metallic sphere of diameter 40 cm carries a charge of 600 μC. Find the electric field intensity (a) at a distance of 1.5 m from the centre of the sphere and (b) at the surface of the sphere.

Solution:

A charged sphere behaves as a point charge. In real the charge is uniformly distributed upon its surface but the charge works that as it is concentrated at its centre.

Diameter = d = 40 cm = 0.4 m, Radius, r = 0.2 m, Q = 600 μC = 6×10^{-4} C

(a) For a distance of d_1 = 1.5 cm = 1.5 m

$$E = k\frac{Q}{d_1^2} = (9 \times 10^{9})\frac{6 \times 10^{-4}}{(1.5)^2} = 24 \times 10^{5}\ N\ C^{-1}$$

(b) $$E = k\frac{Q}{d_2^2} = (9 \times 10^{9})\frac{6 \times 10^{-4}}{(0.2)^2} = 13.5 \times 10^{7}\ N\ C^{-1}$$

Electric Flux:

The number of electric lines passing through an area is called electric flux.

Electric flux will be maximum if the area is held perpendicular to the field, as shown.

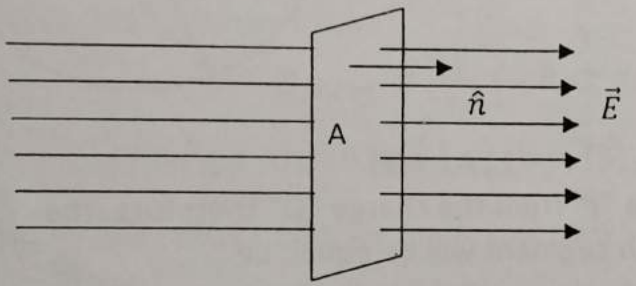


Figure-1

Electric flux will be minimum if the area is held parallel to the field, as shown.

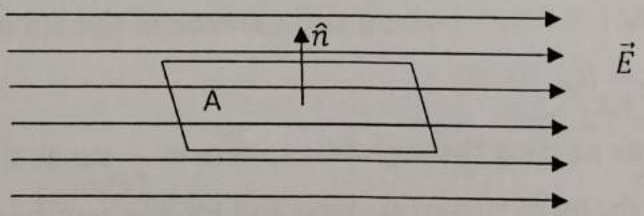


Figure-2

If the plane area is neither perpendicular nor parallel then in general the formula, $\phi = \vec{E} \bullet \vec{A}$ = EA cos θ , is used.

Mathematically,

$\phi = \vec{E} \bullet \vec{A} = EA \cos\theta \rightarrow (1)$

Here θ represents the angle between electric field and normal at the surface of the area. The S.I. unit of electric flux is N m^2 C^{-1}.

For fig.1 $\phi = \vec{E} \bullet \vec{A} = EA \cos 0 = EA \rightarrow (2)$

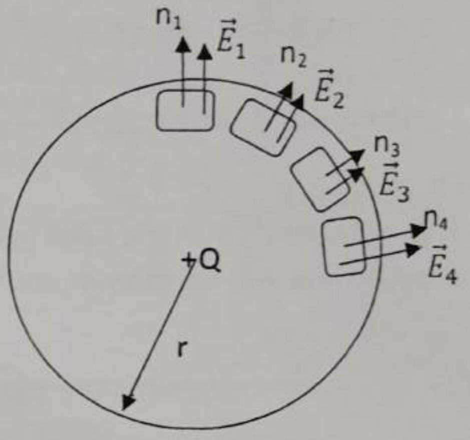
For fig.2 $\phi = \vec{E} \bullet \vec{A} = EA \cos 90 = 0 \rightarrow (3)$

Electric flux through a closed surface:

are the electric field intensities at these area segments respectively as shown. In order to calculate the total flux through this sphere, we need to add all the fluxes passing through each individual area segment. i.e.

$\emptyset_{total} = \emptyset_1 + \emptyset_2 + \emptyset_3 + \ldots + \emptyset_n \rightarrow (2)$

Where $\emptyset_1, \emptyset_2, \emptyset_3, \ldots$ are the fluxes passing through
Area segments $A_1, A_2, A_3, A_4, \ldots$ respectively.
From the diagram it is clear that $\vec{E}_1 // \hat{n}_1, \vec{E}_2 // \hat{n}_2, \vec{E}_3 // \hat{n}_3$
And so on. Therefore,



$\emptyset_1 = \vec{E}_1 \cdot \vec{A}_1 = E_1 A_1 \cos 0^\circ = E_1 A_1$

$\emptyset_2 = \vec{E}_2 \cdot \vec{A}_2 = E_2 A_2 \cos 0^\circ = E_2 A_2$

$\emptyset_3 = \vec{E}_3 \cdot \vec{A}_3 = E_3 A_{13} \cos 0^\circ = E_3 A_3$

$\vdots \qquad \vdots \qquad \vdots$

$\emptyset_n = \vec{E}_n \cdot \vec{A}_n = E_n A_n \cos 0^\circ = E_n A_n$

Thus eq. (2) $\Rightarrow \emptyset_{total} = E_1 A_1 + E_2 A_2 + E_3 A_3 + \ldots\ldots + E_n A_n \rightarrow (3)$

As all the area segments are at same distance "r" from the charge "Q" therefore, the magnitudes of electric field intensities at each segment will be equal. i.e.

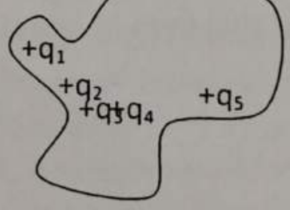
$E_1 = E_2 = E_3 = \ldots\ldots = E_n = E = \frac{1}{4\pi \epsilon_o} \frac{Q}{r^2} \rightarrow (4)$

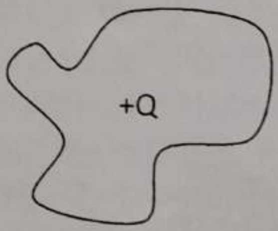
Thus eq. (3) $\Rightarrow \emptyset_{total} = E(A_1 + A_2 + A_3 + \ldots\ldots + A_n) \rightarrow (5)$

Here, $(A_1 + A_2 + A_3 + \ldots\ldots + A_n) = 4\pi r^2$ (whole surface area of the sphere)

$\emptyset_{total} = [\frac{1}{4\pi \epsilon_o} \frac{Q}{r^2}](4\pi r^2) = \frac{1}{\epsilon_o}(Q) \rightarrow (6)$

Eq. (6) shows that the total flux passing through this sphere is $\frac{1}{\epsilon_o}$ times the charge enclosed by it. Gauss's law is true for any shape of closed surface and for any number of charges enclosed by that surface.





Applications of Gauss's Law:

(1) Location of Excess Charge on a Conductor:

Consider a conductor which has some excess charge. In order to prove that whether this excess charge reside on its external surface or inside the conductor, first we need to understand that in the state of electrostatic equilibrium, electric field inside a conductor is zero. (E = 0)

A conductor consists of a large number of free charges (electrons) throughout its volume. Under the electrostatic equilibrium condition, we say that there is no electric force on these free charges otherwise they would exhibit a net flow in a specific direction and a current would establish in the conductor. But no net movement of charge in a specific direction is ever detected. Thus it is very logical to conclude that electric field inside the conductor is zero. i.e. $E = \frac{F}{q} = \frac{0}{q} = 0$.

We will use the above fact as a base or pre-requisite information and apply Gauss's law to find the location of excess charge on a conductor.

For this purpose consider a closed Gaussian surface "S" just below the external surface of a charged solid conducting sphere, as shown in diagram.

The total flux through this closed surface "s"

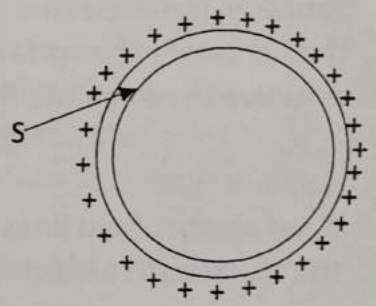
Is, $\emptyset_{total} = \vec{E} \cdot \Sigma\vec{A} \rightarrow (1)$

Where, $\Sigma\vec{A}$ = surface area

Under the electrostatic condition, inside a conductor, $|E| = 0$

Therefore, $\emptyset_{total} = 0$

According to Gauss's law,



$$\emptyset_{total} = \frac{1}{\epsilon_o}\,(\,charge\ enclosed\,) \rightarrow (2)$$

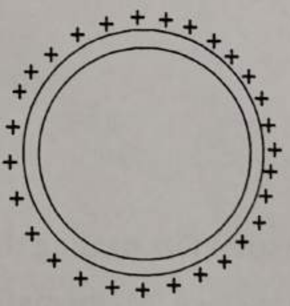
Or $\frac{1}{\epsilon_o}(\,charge\ enclosed\,) = 0 \rightarrow (3)$

As, $\frac{1}{\epsilon_o} \neq 0$, thus $(\,charge\ enclosed\ by\ S) = 0$

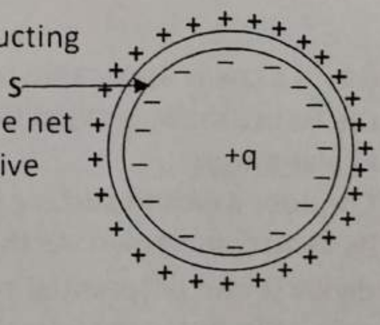
Hence proved that the excess charge resides only on the external surface of the conductor.

Gauss's law is also true for a hollow conductor which has excess charge on its outer surface. The net charge inside it can also be proved zero by using Gauss's law.

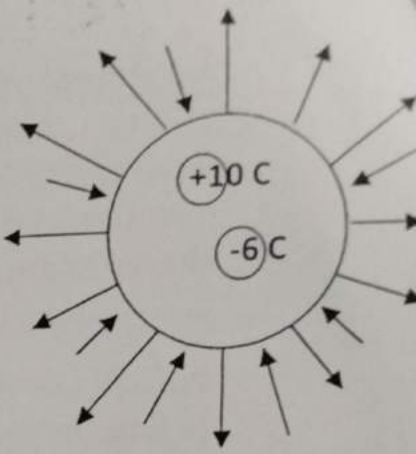
Charge Hollow Sphere



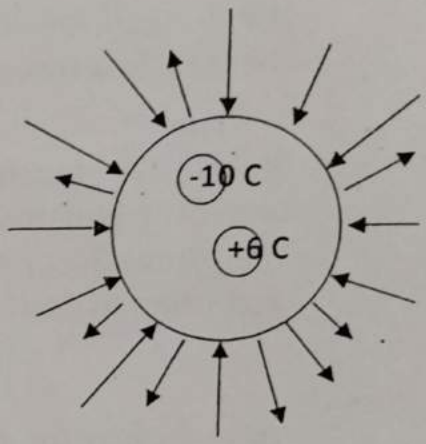
In the third case if a charge q is placed inside a charged conducting hollow sphere, as shown. Even in this case an equal amount of negative charge induces at the inner surface "S" making the net charge inside the conductor equal to zero. And only the positive charge at the outer surface could be detected.



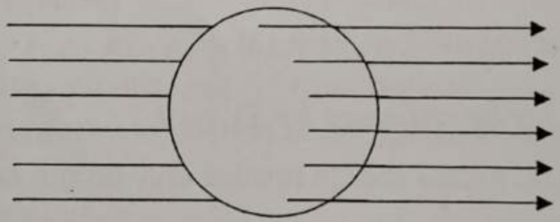
If the electric field is not uniform or the area is a curved surface then in such cases the surface area is divided into such small area segments each of magnitude ΔA, that this area could be assume flat and the electric field within these area could be assumed constant. These areas are called differential areas. These area segments are so small that electric field with each area segment can be assumed uniform.If net electric field lines leaving the surface are greater than entering the surface or if net charge enclosed by the surface is positive then electric flux is considered positive, as shown below.



Similarly, if net electric field lines leaving the surface are lesser than entering the surface or net charge enclosed by the surface is negative then electric flux is considered negative, as shown below.



If net electric field lines leaving the surface are equal to entering the surface then electric flux is considered zero, as shown below.



If the surface area is not flat or the E.field is not uniform then the whole surface area is divided into such small area segments that each segment must appear flat and it must be so small that E. fild can be considered constant within that area. Then total flux will be;

$$\emptyset_{total} = \sum_{i=1}^{i=n} \overrightarrow{E_\iota} \cdot \overrightarrow{A_\iota}$$

Gauss's Law:

This law states that the total flux passing through a closed surface is equal to $\frac{1}{\epsilon_o}$ times the charge enclosed by the surface. Mathematically,

$$\emptyset_{total} = \frac{1}{\epsilon_o} \; (charge\ enclosed) \rightarrow (1)$$

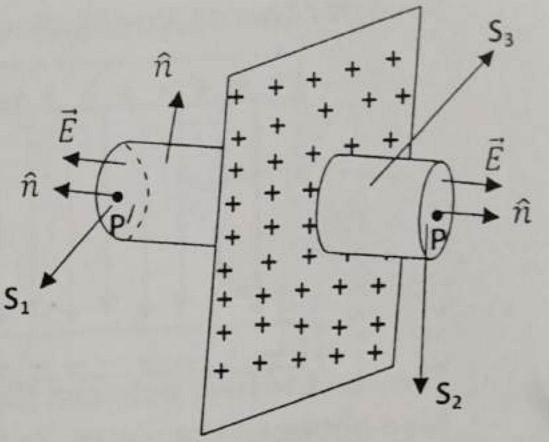
Gauss's law is applicable to any shape of closed surface. The closed surface for which we apply the Gauss's law is called Gaussian surface.

Explanation:

Consider a closed surface in the form of a sphere of radius "r". Let +Q be the charge placed at its centre, as shown. As the surface area of the sphere is a curved surface therefore, we need to divide it into differential areas, A_1, A_2, A_3, A_4, . . . , A_n, each of magnitude ΔA, such that these area segments are so small that within each segment electric field can be considered constant. Let n_1, n_2, n_3, . . . represent the direction of each area segment, as shown. Let $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$, . . .

(2) Electric Field Intensity Due to an infinite sheet of charges:

An infinite sheet of charge we mean a sheet which is either of infinite dimensions or the point at which electric field is to be determined must be very close to the sheet, as shown.



Let P and $P^/$ are points at front and back respectively, at which electric field intensity needs to be determined. Let both points are at a distance "r" from the sheet. In order to apply Gauss's law we consider aclosed surface in the form of a cylinder of length "2r" such that it contains the points P and $P^/$ at its end faces, as shown. To find the total flux through this cylinder, we divide it into three surfaces, S_1, S_2, and S_3. As clear from the diagram that the end faces have equal areas. Let its magnitude be "A". The Area of the curved surface S_3 is A_3. Therefore,

$$\emptyset_{total} = \emptyset_1 + \emptyset_2 + \emptyset_3 \rightarrow (1)$$

Flux Through $S_1 = \emptyset_1 = \vec{E} \bullet \vec{A} = EA$ because electric field and normal are parallel.

Flux Through $S_2 = \emptyset_2 = \vec{E} \bullet \vec{A} = EA$ because electric field and normal are parallel.

And $\emptyset_3 = \vec{E} \bullet \vec{A}_3 = 0$ because electric field and normal are perpendicular.

Thus Eq. (1) $\Rightarrow \emptyset_{total} = 2EA \rightarrow (2)$

According to Gauss's law flux through a closed surface is:

$\emptyset_{total} = \frac{1}{\epsilon_o}\,(charge\ enclosed) \rightarrow (3)$

Let "Q" be the magnitude of charge enclosed by the cylinder on the sheet of charges. Thus eq. 3,

$$\emptyset_{total} = \frac{1}{\epsilon_o}\,(Q) \rightarrow (4)$$

Comparing eq. 2 and eq. 4,

$$2EA = \frac{1}{\epsilon_o}\,(Q) \Rightarrow \quad E = \frac{Q}{2\,A\,\epsilon_o} \rightarrow (5)$$

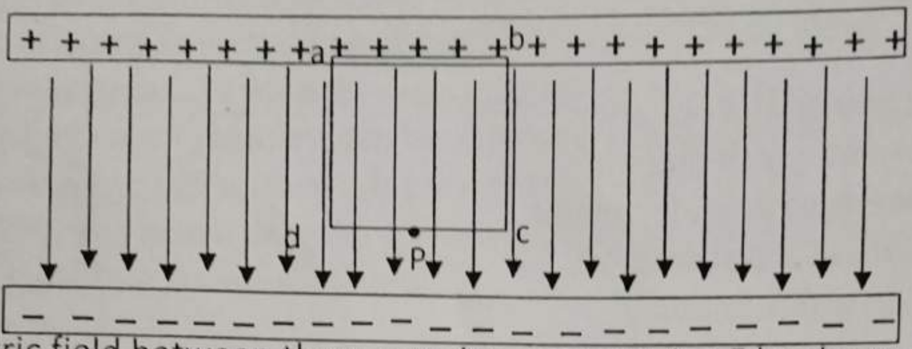
Since surface charge density, $\sigma = \frac{Q}{A}$

Therefore, $E = \frac{\sigma}{2\,\epsilon_o} \rightarrow (6)$

Eq. (5) or (6) can be used to find electric field intensity due to an infinite sheet of charges at point P or $P^/$ close to it.

(3) Electric field between to oppositely charged parallel plate:

Consider two oppositely charged parallel plates, as shown.



The electric field between them remains constant. Let E be the magnitude of electric filed between the plates. To find its magnitude at point P between the plates, we use Gauss's law. To apply Gauss's law consider a closed surface in the form of a box such that the top face of the box is inside the upper plate and the lower face contains that point P. Let $\emptyset_{total}$ be the total flux through this box and is;

$\emptyset_{total} = \emptyset_{ab} + \emptyset_{bc} + \emptyset_{cd} + \emptyset_{da} \rightarrow$ (1)

$\emptyset_{ab} = 0$, because E = 0 inside the conductor.

$\emptyset_{bc} = 0$, because electric field is perpendicular to the normal at this face.

$\emptyset_{cd} = EA$, because electric field is parallel to the normal at this face.

$\emptyset_{da} = 0$, because electric field is perpendicular to the normal at this face.

Thus eq. (1) $\Rightarrow \emptyset_{total} = EA \rightarrow$ (2)

According to Gauss's law, $\emptyset_{total} = \frac{1}{\epsilon_o}$(Charge enclosed) $\rightarrow$ (3)

Let Q be the charge which this box encloses on the plates. Thus eq. (3)

$\emptyset_{total} = \frac{1}{\epsilon_o}(Q) \rightarrow$ (4)

Now comparing eq. 2 and eq. 4

EA = $\frac{1}{\epsilon_o}(Q)$ or E = $\frac{Q}{A\,\epsilon_o} \rightarrow$ (5)

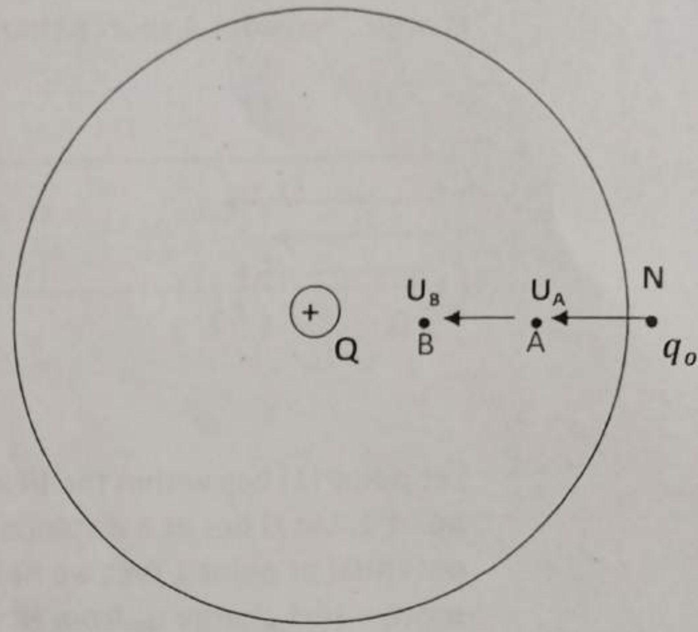
As we know, Surface charge density, $\sigma = \frac{Q}{A}$, therefore,

E = $\frac{\sigma}{\epsilon_o} \rightarrow$ (6)

Eq. 5 or eq. 6 can be used to find the magnitude of electric field intensity between two oppositely charged parallel plates.

Electric Potential Energy:

The work done on a charge against electric field stores in it in the form of electric P.E. Consider a source charge Q, as shown along with its electric field.



In order to understand the concept of electric potential energy consider a test charge q_o at point N just outside the field, as shown. From the diagram it is clear that at point N its electric P.E is zero. In order to move it from N to a field point A we need to do some work $W_{N\rightarrow A}$, which stores in it in the form of electric P.E. Thus electric P.E of q_o at A is:

$U_A = W_{N\rightarrow A} \rightarrow (1)$

Similarly, in order to move the test charge from N to B, we need to do work $W_{N\rightarrow B}$. Thus electric P.E. at B is, $U_B = W_{N\rightarrow B} \rightarrow (2)$

It is clear from the diagram that $U_B > U_A$.

Electric Potential (V):

The electric potential at point can be defined as the electric P.E. per unit charge at that point. Thus electric potential at point A is:

$V_A = \frac{U_A}{q_o} \rightarrow (3)$ Since $U_A = W_{N\rightarrow A}$ therefore, $V_A = \frac{W_{N\rightarrow A}}{q_o} \rightarrow (4)$

On the basis of eq.4, electric potential at point A can also be defined as the work done per unit charge in moving it from zero potential to a field point A. The S.I. unit of electric Potential is volt (V).

$1V = \frac{1J}{1C}$

Similarly, $V_B = \frac{U_B}{q_o} = \frac{W_{N\rightarrow B}}{q_o} \rightarrow (5)$

As $U_B > U_A$ therefore, $V_B > V_A$.

Potential Difference (ΔV):

The difference between electric potentials of two different points is called potential different. It is usually denoted by ΔV.

$\Delta V = V_B - V_A \rightarrow (6)$

Other commonly used multiples and submultiples of volt are:

$1\ kV = 10^3\ V$, $1\ MV = 10^6\ V$, $1\ GV = 10^9\ V$, $1\ mV = 10^{-3}\ V$, $1\ \mu V = 10^{-6}\ V$

Electric Potential due to a point source charge:

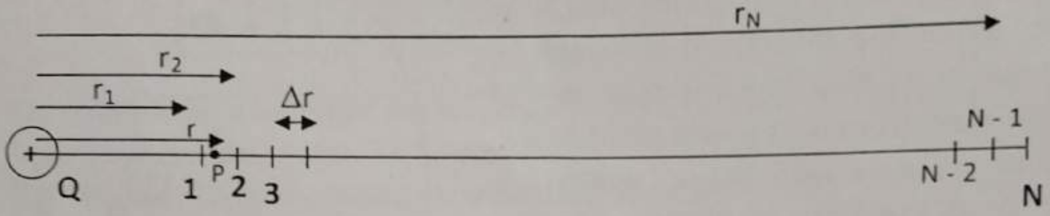
Definition:

Electric potential at a field point (1) is the amount of work done per unit charge in moving it from infity to that field point.

Mathematically, $V_1 = \frac{W_{\infty \to 1}}{q_o} \to (1)$

Explanation:

In order to derive a formula to calculate electric potential at point due to a point source charge , consider a source charge +Q, as shown.



Let point (1) lies within the field at a distance r_1 from Q and point (N) lies very far from point 1. Let N lies at a distance r_N from the source charge. In order to calculate electric potential at point 1 first we need to calculate the magnitude of work done required to move a test charge q_o from N → 1.

As N lies very far from 1 therefore the electric force against which q_o is moved will be a variable force. Therefore, we need to apply the concept of work done by a variable force. For this purpose we need to divide the whole displacement from N to 1 into small displacement segments, each of Δr, such that E.field could be assumed constant within each step. Let the steps are 1, 2, 3, 4, ..., N-2, N-1, N, which lie at distances r_1, r_2, r_3, ..., r_N respectively from the source charge Q, as shown. Thus total work done from N to 1 is:

$W_{N \to 1} = W_{N \to (N-1)} + W_{(N-1) \to (N-2)} + \ldots + W_{3 \to 2} + W_{2 \to 1} \to (2)$

In order to calculate work done from 2 →1 (i.e. $W_{2 \to 1}$), consider a test charge q_o which is supposed to be moved from 2 → 1. From the diagram it is clear that electric force at point 1 will be slightly greater than electric force at point 2. Let F be the force which we are considering constant within this step and of course it will be an average value of the two forces which corresponds to the mid-point P of the step 2 to 1. Let P lies at a distance r from Q. To find the magnitude of force F at P, consider a test charge q_o at P. Now according to coulomb's law:

$F = k \frac{Q q_o}{r^2} \to (3)$

In order to introduce r_1 in eq. 3, from the diagram we can write:

$\Delta r = r_2 - r_1 \to (4)$

$r_2 = r_1 + \Delta r \to (5)$

Similarly, $r = \frac{r_1 + r_2}{2} \to (6)$, Now put eq. 5 in eq. 6

$r = \frac{r_1 + (r_1 + \Delta r)}{2} = (r_1 + \frac{\Delta r}{2}) \to (7)$

Take square of eq. 7 $\Rightarrow r^2 = r_1^2 + \frac{\Delta r^2}{4} + 2 r_1 (\frac{\Delta r}{2}) = r_1^2 + \frac{\Delta r^2}{4} + r_1 \Delta r \to (8)$

As stated earlier, that $\Delta r<<<$, therefore, $\frac{\Delta r^2}{4} \to 0$

Eq. (8) $\Rightarrow r^2 = r_1^2 + r_1 \Delta r \rightarrow (9)$

Put eq. 4 in eq. 9, thus $r^2 = r_1^2 + r_1 (r_2 - r_1) = r_1^2 + r_1 r_2 - r_1^2) = r_1 r_2 \rightarrow (10)$

Put eq. 10 in eq. 3, $F = k\ \frac{Q\,q_o}{r_1 r_2} \rightarrow (11)$

Thus work done from 2→1 is

$W_{2_{\to}1} = \vec{F} \cdot \Delta\vec{r} = F\,\Delta r \rightarrow (12)$ because F // Δr

Now put eq. (11) and eq. (4) in eq. (12)

$W_{2_{\to}1} = [k\ \frac{Q\,q_o}{r_1 r_2}]\,[r_2 - r_1] = k\,Qq_o\,(\frac{1}{r_1} - \frac{1}{r_2})$

Similarly, $W_{3_{\to}2} = k\,Qq_o\,(\frac{1}{r_2} - \frac{1}{r_3})$

$W_{4_{\to}3} = k\,Qq_o\,(\frac{1}{r_3} - \frac{1}{r_4})$

And so on, $W_{(N-1)_{\to}(N-2)} = k\,Qq_o\,(\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}})$

$W_{N_{\to}(N-1)} = k\,Qq_o\,(\frac{1}{r_{N-1}} - \frac{1}{r_N})$

Therefore, eq. 3 $\Rightarrow$

$W_{N_{\to}1} = kQq_o\,[(\frac{1}{r_{N-1}} - \frac{1}{r_N}) + (\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}) + \ldots + (\frac{1}{r_3} - \frac{1}{r_4}) + (\frac{1}{r_2} - \frac{1}{r_3}) + (\frac{1}{r_1} - \frac{1}{r_2})$

Or $W_{N_{\to}1} = kQq_o\,(\frac{1}{r_1} - \frac{1}{r_N}) \rightarrow (13)$

If N lies at infinity then $W_{\infty_{\to}1} = kQq_o\,(\frac{1}{r_1} - \frac{1}{\infty}) = \frac{kQ\,q_o}{r_1}$

Or $W_{\infty_{\to}1} = \frac{1}{4\pi\epsilon_0}\frac{Q\,q_o}{r_1} \rightarrow (14)$

This work stores in q_o as an electric P.E. therefore, electric P.E of q_o at point 1, in the electric field of source charge Q is:

$U_1 = \frac{1}{4\pi\epsilon_0}\frac{Q\,q_o}{r_1} \rightarrow (15)$

Put eq. 14 in eq. 1 $\Rightarrow V_1 = \frac{1}{q_o}[\frac{1}{4\pi\epsilon_0}\frac{Q\,q_o}{r_1}]$

Or $V_1 = \frac{1}{4\pi\epsilon_0}\frac{Q}{r_1} \rightarrow (16)$

Eq. 16 gives a formula to find electric potential at point (1) due to the source charge Q in general for a distance r, eq. 15 and 16 can be expressed as:

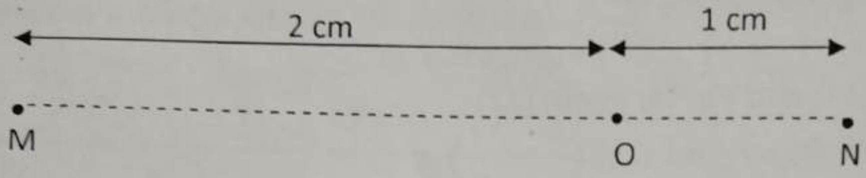
$U = \frac{1}{4\pi\epsilon_0}\frac{Q\,q_o}{r} \rightarrow (17)$

$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \rightarrow (18)$

Example 11.6

A point charge of 3 μC is placed at point O between M and N which are 3 cm apart. Point M is 2 cm from the point charge and N is 1 cm away from it. What is the potential difference $V_M - V_N$?

Solution:



To find electric potential at a distance r due to a point charge, we use:

$$V = \frac{1}{4\pi\epsilon_o}\frac{Q}{r}$$

Thus, $V_M = \frac{1}{4\pi\epsilon_o}\frac{Q}{r} = 9 \times 10^9\,[\frac{3 \times 10^{-6}}{2 \times 10^{-2}}] = 13.5 \times 10^5$ V

And $V_N = \frac{1}{4\pi\epsilon_o}\frac{Q}{r} = 9 \times 10^9\,[\frac{3 \times 10^{-6}}{1 \times 10^{-2}}] = 27 \times 10^5$ V

$V_M - V_N = (13.5 \times 10^5) - (27 \times 10^5) = -\,13.5 \times 10^5$ V

Example 11.7

Equipotential line or equipotential surface:

In most of the practical cases it is usually difficult to represent electric field pattern because both strength and direction should exclusively be represented at each point. As an alternate to field lines diagram, "contour maps" of electric field can be drawn using equipotential lines or surfaces. An equipotential line or surface connects points in space where the potential of every point is same. **As shown in figure 11.26(a) book.**

For a point charge electric potential at a distance r is:

$V = \frac{q_o}{4\pi\varepsilon_o r}$. As all points on a specific line are at same distance from the charge, therefore, electric potential of each point on a specific line will have same value.

If there are two or more charges placed closely in a region, as shown in figure 11.26(b) Book, then at every point electric potential will be equal to the sum of electric potentials due to each charge. Electric potential is a scalar quantity. However, electric potential can be positive or negative, depending upon the sign of the charge. In figure 11.26(b) Book, the electric field lines and the equipotential line exclusively represented. It can be noted that at each point E.F. line and equipotential line make 90° angle.

Potential Gradient:

Change in electric potential with respect to displacement moved along the field is called potential gradient. Mathematically,

Potential gradient = $\frac{\Delta V}{\Delta r} \rightarrow$ (1)

Its S.I unit is volt per metre ($V\ m^{-1}$).

Show that electric field intensity is equal to the negative of potential gradient. OR

$$E = -\frac{\Delta V}{\Delta r}$$

Proof: Consider a uniform electric field between two oppositely charged parallel plates, as shown. Let E be the electric field intensity. According to the convention, positive is taken as high potential point and negative is taken as least potential point. Therefore, let ΔV be the potential difference between these plates. Let Δr be the separation between the plates. Consider a test charge $q_o$which is placed near negative plate and is required to move towards positive with constant speed. Let F be the applied force and w is the work done needed for this task. By definition,

$W = F \bullet \Delta r = F\,\Delta r \cot 0^\circ = F\,\Delta r \rightarrow (1)$

$\theta = 0^\circ$ because displacement is moved in the direction of applied force.

From the diagram it is clear that the applied force is acting against the electric force Eq_o on the charge. Therefore, $F = -E\,q_o \rightarrow (2)$

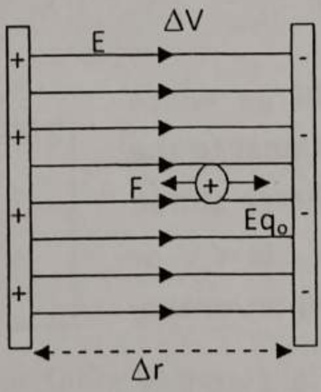
Put eq. 2 in eq. 1 $\Rightarrow$ $w = -E\,q_o\Delta r \rightarrow (3)$

By definition, the potential difference ΔV between the plates is:

$$\Delta V = \frac{w}{q_o} = \frac{-E\,q_o\,\Delta r}{q_o} = -E\,\Delta r$$

$$E = -\frac{\Delta V}{\Delta r} \rightarrow (4)$$

The negative sign shows that the direction of field is opposite to the direction in which electric potential increases.Hence proved that electric field intensity is equal to the negative of electric potential. From eq. 4 it is clear that another valid unit of electric field intensity is $V\,m^{-1}$.



Show that $V\,m^{-1} = N\,C^{-1}$

Proof: L.H. Side = $V\,m^{-1} = \frac{V}{m} = \frac{J}{C\,m} = \frac{N\,m}{C\,m} = \frac{N}{C} = N\,C^{-1}$

The Electron Volt (eV)

It is the unit of energy. One electron volt is the amount of K.E acquired or lost by an electron when it moves through a potential difference of one volt.

As we know, $\Delta V = \frac{\Delta U}{q}$ or $\Delta U = q\,\Delta V \rightarrow (1)$

If $\Delta V = 1$ V then the K.E acquired by an electron , $q = e$ is:

K.E = 1 eV → (2)

In units of joule, $1\ eV = 1 \times 1.6 \times 10^{-19}\ C \times \frac{J}{C} = 1.6 \times 10^{-19}\ J$

Some common multiples of eV are:

1 MeV = 10^6 eV , 1 GeV = 10^9 eV

Example 11.8

A particle carrying a charge of 3e falls through a potential difference of 5V. Calculate the K.E acquired in units of joules.

Solution: As K.E = q V = 3e × 5V = 15 eV = $15 \times 1.6 \times 10^{-19} = 2.4 \times 10^{-18}$ J

Capacitor

A device which can store electric charge is called capacitor.

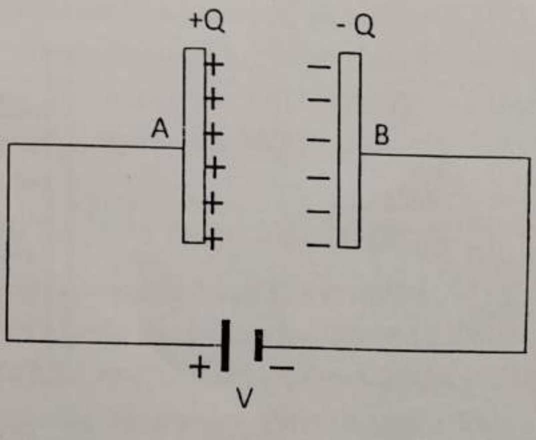
There are many types of capacitor. The one which is of our interest at present is parallel plate capacitor. The medium between the plates may be vacuum, air, paper or any dielectric substance.

How it stores charge:

Consider a parallel plate capacitor, connected to a battery, as shown. As these plates are made of metal, i.e. each plate is initially neutral but has large number of free electrons. Therefore, the positive terminal of the battery will attract free electrons from plate A attached to it. As a result a positive charge +Q appears on plate A. Due to the positive field of plate A, the plate B drags the same number of free electrons from the battery and a –Q charge appears on it, to maintain the equilibrium. Mutual attraction between the plates bound the charges on each plate to the inner surface, as shown. This charge remains stored even when the battery is detached from the capacitor.

Capacitance of a Capacitor:

Experiments have proved that the charge which stores on a capacitor is directly proportional to the voltage of the battery. Let Q be the charge stored and V is the voltage of the battery, then $Q \propto V$ or $Q = CV$ →(1) Where C is constant of proportionality and is called capacitance of a capacitor.



Mathematically, $C = \frac{Q}{V}$ → (2)

From eq (2) capacitance can be defined as "charge stored per unit volt". Its S.I. unit is farad (F). $1\ F = \frac{1C}{1V}$. i.e. if a capacitor stores 1 C charge when it is connected with a battery of 1 V then its capacitance will be 1 F.

Farad is a large unit. In real practice the capacitors which are used usually have capacitance of the order of 1 micro-farad and 1 pico-farad.

$1\,\mu F = 10^{-6}\,F$, $1\,pF = 10^{-12}\,F$

How a capacitor stores charge?

When a capacitor is connected with a battery, the plate connected to positive terminal stores positive charge (by losing/donating electrons to the positive terminal of the battery). As a consequence the plate attached with the negative terminal of the battery will store same amount of negative charge (by dragging electrons from negative terminal of the battery) due to electrostatic induction. in the charging process the p.d between the plates will gradually increase with time. And the flow of electrons (current) decreases with time.

Important Note:

in charging circuit, Q and V are directly proportional, but I is inversely proportional to both Q and V.

Capacitance of a Parallel Plate Capacitor:

Consider a parallel plate capacitor of capacitance C, which can store charge Q when connected with a battery of V volts. Then, $C = \frac{Q}{V} \rightarrow (1)$

Let d be the separation, A be the area of the plate and E is the electric field intensity between the plates.

As $E = -\frac{\Delta V}{\Delta r} = \frac{-(V_2 - V_1)}{d} = \frac{V_1 - V_2}{d} = \frac{V}{d} \Rightarrow V = E\,d \rightarrow (2)$

According to one of the applications of Gauss's law, electric field between two oppositely charged parallel plates is, $E = \frac{\sigma}{\varepsilon_o} \rightarrow (3)$

Where, σ(surface charge density) $= \frac{Q}{A} \rightarrow (4)$

Thus eq. 3 $\Rightarrow E = \frac{Q}{A\,\varepsilon_o} \rightarrow (5)$

Put eq. 5 in eq. (2) $\Rightarrow V = \frac{Q}{A\,\varepsilon_o}\,d \rightarrow (6)$

Put eq. 6 in eq. (1) $\Rightarrow C = \frac{Q}{Qd/A\,\varepsilon_o}$

$\Rightarrow C = \frac{A\,\varepsilon_o}{d} \rightarrow (7)$

Eq. 7 can be used to calculate capacitance of a parallel plate capacitor in terms of area of the plate and separation between the plates when the gap between the plate either has vacuum or air. For air or vacuum filled capacitor eq. 7 is usually written as: $C_{vac} = \frac{A\,\varepsilon_o}{d} \rightarrow (8)$

When the gap between the plates is filled with a dielectric medium of permittivity ε_m then capacitance, $C_{med} = \frac{A\,\varepsilon_m}{d} \rightarrow (9)$

As , $\varepsilon_m = \varepsilon_r \varepsilon_o$, therefore, eq. 9 $\Rightarrow C_{med} = \frac{A\,\varepsilon_r \varepsilon_o}{d} = \varepsilon_r \left(\frac{A\ \varepsilon_o}{d}\right) = \varepsilon_r\, C_{vac} \rightarrow (10)$

For all dielectric media, $\varepsilon_r > 1$. Therefore, $C_{med} > C_{vac}$

Eq. (10) can also be written as $\varepsilon_r = \frac{C_{med}}{C_{vac}} \rightarrow (11)$

Eq. (11) gives the definition of relative permittivity of a medium as, " the ratio of capacitance of medium filled capacitor to the capacitance when the same capacitor is filled with vacuum".

Combinations of capacitors:

In real practice capacitors are either connected in series or parallel.

Series Combination of Capacitors:

Consider three different capacitors of capacitances C_1, C_2 and C_3 which are connected in series with a voltage source V, as shown in figure 11.30 (a) Book. From the diagram it is clear that in real the left plate of C_1 and the right plate of C_3 are connected directly with the battery. Thus +Q and –Q charge will store on them respectively. According to the phenomenon of electrostatic induction the same amount of charge of opposite type will induce on the corresponding plate. Thus same magnitude of charge will store on each capacitor. Let V_1, V_2 and V_3 are the p.ds across capacitors respectively. For series combination,

$V = V_1 + V_2 + V_3 \rightarrow (1)$

By definition, $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$, $V_3 = \frac{Q}{C_3}$

Thus eq. (1) $\Rightarrow V = \frac{Q}{C_1} + \frac{Q}{C_2} .+ \frac{Q}{C_3} \rightarrow (2)$

Let C be the total capacitance of this combination. Then, $V = \frac{Q}{C}$

Thus, $\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \rightarrow (3)$

Eq. (3) can be used to calculate the equivalent capacitance of capacitors when they are connected in series. For series combination, the equivalent capacitance will always be smaller than the smallest capacitance in the combination.

Parallel Combination of Capacitors:

Consider three different capacitors of capacitances C_1, C_2 and C_3 which are connected in parallel with a voltage source V, as shown in figure 11.30 (b) Book.

From the diagram it is clear that every capacitor is directly connected with the voltage source. Thus p.d across each capacitor will be same. In this case each capacitor will store different charge according to their capacitances. Let Q_1, Q_2 and Q_3 be the charges stored by C_1, C_2 and C_3 respectively. Let Q be the total charge stored collectively by the combination, then

$Q = Q_1 + Q_2 + Q_3 \rightarrow (1)$

Let C be the total or equivalent capacitance the eq. 1 can be expressed as:

$CV = C_1 V + C_2 V + C_3 V \Rightarrow \quad C = C_1 + C_2 + C_3 \rightarrow (2)$

Eq. 2 can be used to calculate the equivalent capacitance of parallel combination of capacitors. Eq.2 also shows that equivalent capacitance will be greater even than the highest capacitor connected in the combination.

Electric Polarization:

When a dielectric material of relative permittivity ε_r is placed between the plates of a charged capacitor and then opposite charges appear at the end faces of the material. This phenomenon is called electric polarization.

There are two types of dielectric material, polar and non-polar dielectric. The polar dielectric material can retain permanently positive or negative charge at opposite ends even in the absence of external electric field. But as a whole it remains neutral.e.g. NaCl. Whereas in a non-polar dielectric material no charge exists at any corner of its molecules, in the absence of external electric field. In the absence of external electric field in non-polar dielectric material in each atom of molecule the centres of positive and negative charges coincide. But when it is exposed to an external electric field, it's each atom or molecule acquires oblong shape due to the drift of electrons and protons towards oppositely charged plates, as shown. In this state we say that the material has polarized electrically. Due to electric polarization the capacitance increases because net electric field intensity between the plates decreases. In this state each atom or molecule haves like a dipole. "A small object which has opposite equal amount of opposite charge at opposite ends separated by a small distance".

Electric dipole moment is that physical quantity which shows the behaviour of a material when it is exposed to an external electric field. It is a vector quantity. Mathematically, Electric dipole moment, $P = |qd|$

Its S.I. unit is C m.

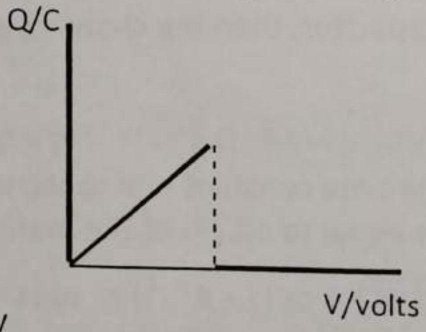
Example 11.9

Example 11.10

Energy stored In a Capacitor:

When the circuit is switched on in which a capacitor is connected with a battery, the charging starts and the battery starts moving electrons from one plate to another. For doing so the battery performs work done which stores in these charges as a potential energy. Thus in charging process capacitor stores electrical energy. The magnitude of this energy can easily be calculated by calculating area under Q-V graph for a capacitor.

As we know, $Q \propto V$, therefore, their graph will be a straight line starting from origin as shown.



From the graph it is clear that area under the
Is of triangular shape. There fore,
Energy Stored (U) = Area under Q-V graph
$U = \frac{1}{2}$ (base × height)
Base gives the value of applied voltage and height gives the value of corresponding charge.
Therefore, $U = \frac{1}{2} QV \rightarrow (1)$
Eq. (1) gives the energy stored in a capacitor. As $Q = CV$
Therefore, eq (1) $\Rightarrow U = \frac{1}{2} C V^2 \rightarrow (2)$
Put $V = Q/C$ in eq. (1) then $U = \frac{Q^2}{2C} \rightarrow (3)$

This energy is stored in the electric field between the plates. To derive a formula for this stored energy in terms of electric filed intensity, we know that,
$V = Ed \rightarrow (4)$
If a dielectric medium of relative permittivity ε_r exists between the plates then

C = $\frac{A\,\varepsilon_r\varepsilon_o}{d} \rightarrow$ (5)

Put eq. (4) and eq. (5) in eq. (2)

U = ½ [$\frac{A\,\varepsilon_r\varepsilon_o}{d}$] $(Ed)^2$

U = ½ [$\varepsilon_r\varepsilon_o E^2\,(Ad)$] → (6)

Energy Density (u):

Energy stored per unit volume is called energy density.

In eq. (6) (Ad) represents the volume of the gap between the plates.

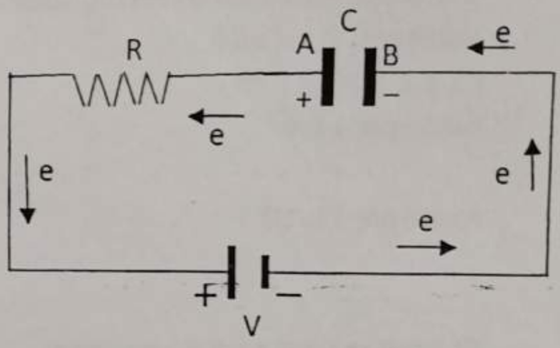
Therefore, u = $\frac{U}{volume} = \frac{½\,[\varepsilon_r\varepsilon_o E^2\,(Ad)]}{Ad}$

u = ½ [$\varepsilon_r\varepsilon_o E^2$] → (7)

eq. (7) gives the energy density of the capacitor.

Charging of a capacitor:

In order to charge a capacitor, we need to connect it with a d.c source through a resistor as shown. Let R be the resistance in the circuit, C is the capacitance of the capacitor and V is the voltage of the source. The arrows represent the direction of flow of electrons which positive terminal of the battery will drag from plate "A" and the battery deposits them on plate "B".



The process will continue unless the capacitor is fully charged. By connecting an ammeter in the circuit the completion of the charging process can be identified. Initially the ammeter will be giving large deflection but slowly and gradually the deflection will vanish. When the ammeter gives zero deflection in a properly connected circuit then it means that the transferring process of electrons has stopped. If q and q_o represent the instantaneous charge and maximum charge stored in a capacitor, then the charging process takes place according to the following equation:

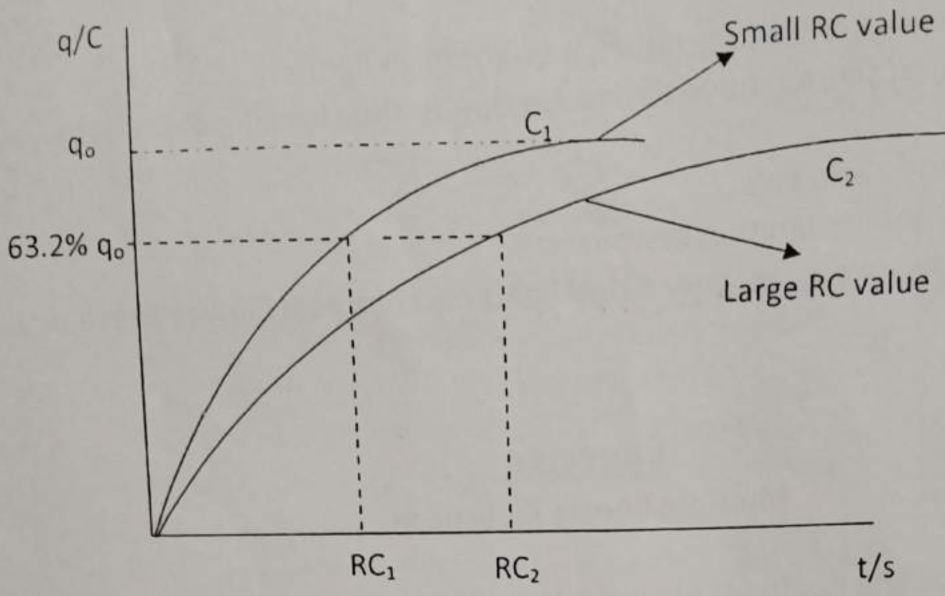
$$q = q_o\,(1 - e^{-\frac{t}{RC}}) \rightarrow (1)$$

Here RC is the time constant of a capacitor. It is the time during which the charge that stores in a capacitor is equal to 63.2% of the maximum charge. To verify this value, put RC = t in eq. (1).

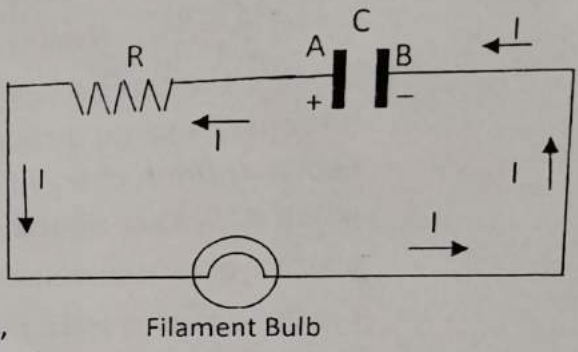
$q = q_o\,(1 - e^{-\frac{t}{RC}}) = q_o\,(1 - e^{-1}) = q_o\,(1 - \frac{1}{e}) = q_o\,(1 - \frac{1}{2.718}) = q_o\,(0.632)$

$q = 63.2\ \%\ q_o$

The variation of charge q with time on a capacitor during charging process is shown in fig. (2)



Discharging of a Capacitor:



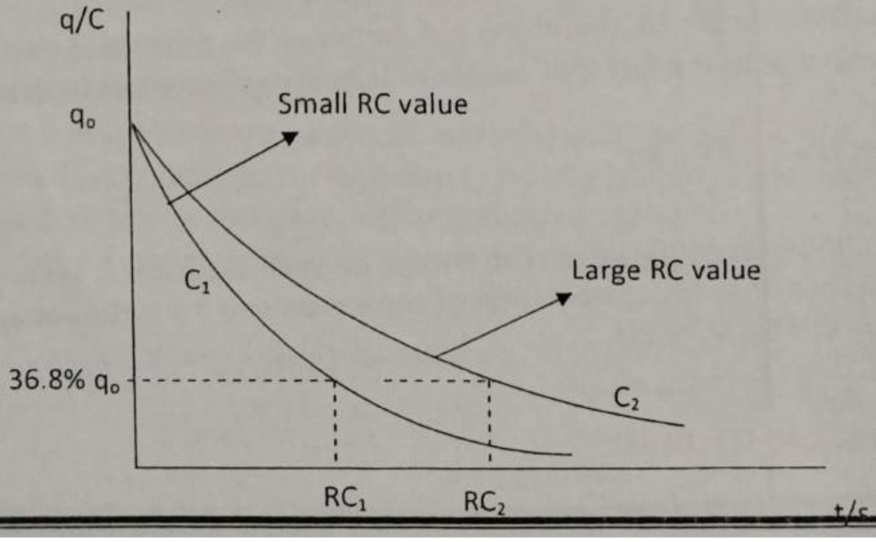
In order to study the discharging of a capacitor, consider the circuit in which a charged capacitor is connected a filament bulb through a resistor R. Electron will start flowing from "B" to "A". As the direction of current is always taken opposite to the flow of electrons, thus the direction of current is shown in the diagram. As initially the capacitor is fully charged therefore, the bulb will shine with maximum brilliance but with the passage of time due to the deficiency of charge in the capacitor the brilliance fades out and ultimately vanishes. When the bulb stops glowing any more, we say that the capacitor has discharged. The discharging of a capacitor Takes place according to the following equation:

$$q = q_o e^{-\frac{t}{RC}} \rightarrow (1)$$

Here RC is the time constant. At the time of discharging time constant can be defined as,"it is the time during which the charge that leaves n a capacitor is 36.8% of maximum charge." To prove this value, put RC = t in eq.(1)

$q = q_o\, e^{-1} = q_o\, (\frac{1}{e}) = q_o\, (\frac{1}{2.718}) = q_o(0.368) = 36.8\%\, q_o$

The variation of charge q with time on a capacitor during discharging process is shown in fig. (2)



Some common Applications:

1. Capacitor is used in camera to produce a bright flash light.
2. Wipers of cars wind screen move to and fro due to the charging and discharging of a capacitor.
3. It is used in T.V and radio sets.
4. It is used in automobiles ignition system.
5. To light up a gas burner a capacitor fitted lighter is commonly in use these days.

Exercise

Multiple Choice Questions

1. (b) $q = Q/2$

   Explanation: According Coulomb's law the force between q and $(Q - q)$ is $F = k\,\frac{q\,(Q-q)}{R^2}$, Now substitute the values given in each option in place of q. Thus for $q = Q/2$ the force F will be maximum.
2. (c) remains same throughout the volume of the conductor.

   Explanation: A charged conducting sphere behaves like a point charge. Therefore, whether we consider a point at its surface or within the conductor the magnitude of electric potential remains same. But if we move away from its surface then $V \propto \frac{1}{r}$, where r is the distance from the centre of the conductor.
3. (b) Conductor

   Explanation: Earth is a large reservoir of electrons. It can absorb or supply as many electrons as required.
4. (c) 1 keV

   Explanation: The K.E. acquired by a charged particle when it moves through a p.d. can be determined as: K.E. = QV → (1)

   If Q = 1e and V = 1 kV then K.E. = 1 keV
5. (c) P.D. between the plates decreases, stored energy decreases and charge remains the same.

   Explanation: In the process of electric polarization of the dielectric material some energy stored in the capacitor is utilized and as a result net electric field intensity between the plates decreases. As $V = Ed$, therefore, p.d. between the plates also decreases. The remains same due to the fact that capacitor is now not in contact with battery.
6. (a) 8 J

   Explanation: $U = \frac{1}{2} C V^2 = 8$ J
7. (b) 10 μC

   Explanation: In series combination the charge on each capacitor is same. If C_{eq} is the total capacitance of series combination of capacitors and if V is the voltage of the battery then, $Q = C_{eq} V$ → (1)

   $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow C_{eq} = 1\ \mu F$$

   Thus from eq. 1 ⇒ $Q = 10\ \mu C$