ELECTRICAL MACHINES

Basics

Concept of phasors

Say $V_1(t) = V_m \sin(wt + \phi_1)$

or $V_1(t) = V_m \cos(wt \pm \theta_2)$

V_m → peak value w → frequency θ → phase shift.

⇒ If Sin is the refeence;

$V_m \angle \pm \theta_1$ → in terms of peak value.

$\frac{V_m}{\sqrt{2}} \angle \pm \theta_1$ → in term of rms value.

⇒ If Cos is the refeence,

$V_m \angle \pm \theta_2$, $V_m/\sqrt{2} \angle \pm \theta_2$

If directly given, eg $5 \angle 30^\circ$,

↳ This 5 is rms value.

Comparison b/w two sinosoids.

i. Both signals must be in same form.
ii. Both signals must have same frequency.
iii. Both signals must have positive amplitude.

lead → anticlockwise → +ve ↺

lag → clockwise → -ve ↻

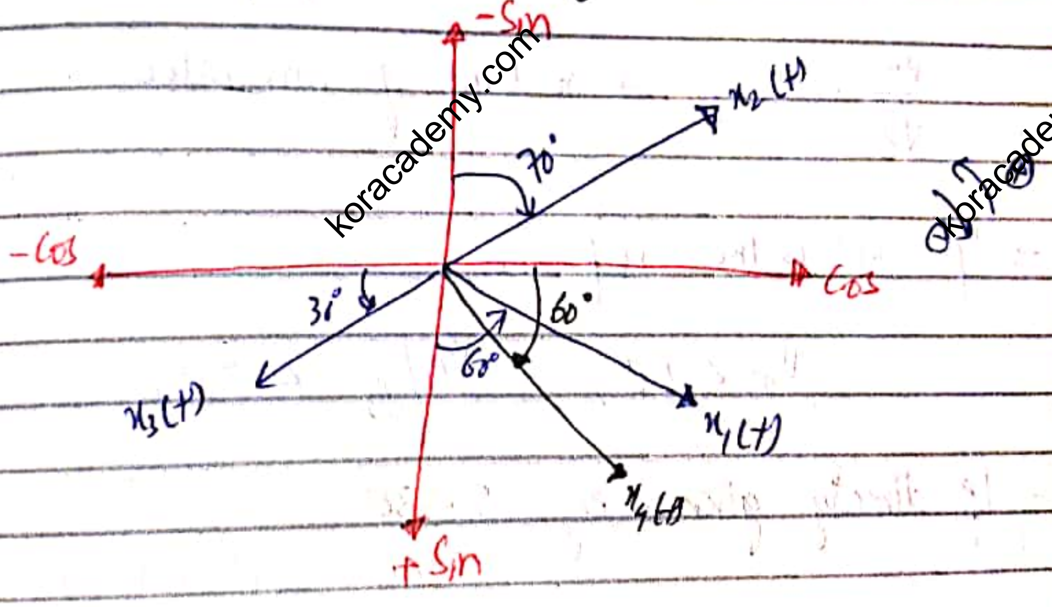
Sin laggs cosine by 90°.
Cosine leads sine by 90°.

Say my reference is sine. So my standard signal will be;

$$x(t) = x_m \sin(\omega t \pm \theta)$$

In phasor form, $x_m \angle \pm\theta$ or $x_m/\sqrt{2} \angle \pm\theta$

Consider;
$x_1(t) = 5\sin(\omega t + 60°) = 5\angle 60° = \frac{5}{\sqrt{2}}\angle 60°$
$x_2(t) = -10\sin(\omega t - 70°) = 10\angle 110°$
$x_3(t) = -5\cos(\omega t + 30°) = 5\angle -60°$
$x_4(t) = 5\cos(\omega t - 60°) = 5\angle 30°$



See x_2 is not in standard form.
To convert into standard form, take the reference and take it to the required signal.

ie here take +Sin to $x_2(t)$

So $x_2(t) = 10\sin(\omega t + 110°)$
$x_3(t) = 5\sin(\omega t - 60°)$
$x_4(t) = 5\sin(\omega t + 30°)$

Talking abut lead ad lagg.

Again we need a reference

Say $x_1(t)$ is my reference.

x_3 laggs x_1 by 120°

x_4 laggs x_1 by 30° etc

x_2 leads x_1 by 50°.

Say $x_2(t)$ is reference.

x_4 laggs x_2 by 80°.

Now talk about power

Instantaneous Power

$$p(t) = v(t) \times i(t) = V_m \sin(wt + \theta_1) \cdot I_m \sin(wt + \theta_2)$$

$$= \frac{V_m I_m}{2} \cos(\theta_1 - \theta_2) + \frac{V_m I_m}{2} \cos(2wt + \theta_1 + \theta_2)$$

$$= \frac{V}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos(\theta_1 - \theta_2) + \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2wt + \theta_1 + \theta_2)$$

$$\boxed{p(t) = V_{rms} I_{rms} \cos(\theta_1 - \theta_2) + V_m I_{rms} \cos(2wt + \theta_1 + \theta_2)}$$

↓

independent of time

↓

average power.

$$P_{avg} = V_{rms} I_{rms} \cos(\theta_1 - \theta_2)$$

where $\cos(\theta_1 - \theta_2) = \cos\phi$ = PF

P_{avg} = Real power = wattage = Active = useful
= AC power → avg. dc quantity → dc quantity of Multiple power (AC power)
↳ power in AC.

ii. Complex power

$$= \left(\vec{V}_{rms} \times \vec{I}_{rms}^{*}\right)$$

= P ± jQ
↳ DC power also w pf included

$$= |V_{rms}| \angle \theta_1 \cdot |I_{rms}| \angle -\theta_2$$

$$= |V_{rms}|\,|I_{rms}| \cdot \angle \theta_1 - \theta_2$$

DC $P = V_{DC} I_{DC}$

$$= \boxed{V_{rms} I_{rms} \cos(\theta_1 - \theta_2) + j V_{rms} I_{rms} \sin(\theta_1 - \theta_2)}$$

↓ P_{avg}

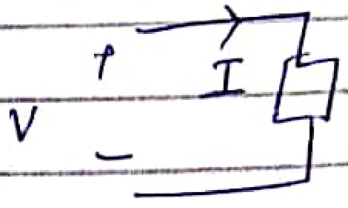
↓ Reactive power (Q)

iii. Apparent power

$$= \boxed{V_{rms} \times I_{rms}}$$

As $P = V_{rms} I_{rms} \cos\phi$.

φ is the angle b/w V and I if they are defined at the same terminals.



↳ φ is the angle b/w phases not b/w line quantities.

absorbing delivering

Short circuit → V = 0V
open circuit → I = 0A

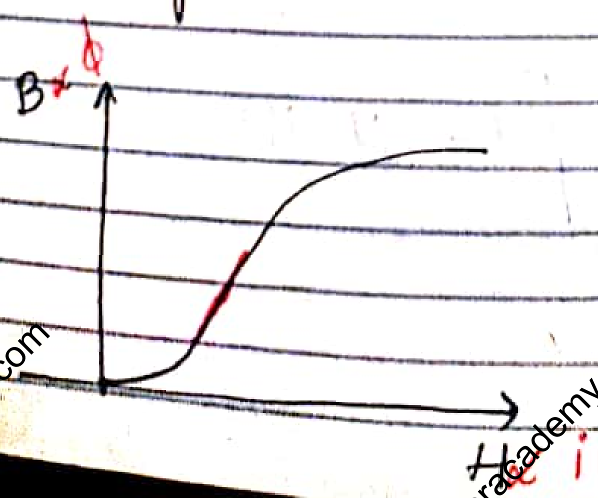
Three types of magnetic materials.

i. Diamagnetic materials do repulsion when placed in an external magnetic field.
ii. Paramagnetic materials are like diamagnetic but they can create a little attraction when subjected to a magnetic field.
iii. Feromagnetic materials strongly attract when an external magnetic field is applied.
Their permeability is very high.

The type of magnetic material is decided by the allignment of dipoles in the atom.

Residual magnetism → When the external magnetic field is removed and the material is still somewhat magnetized.

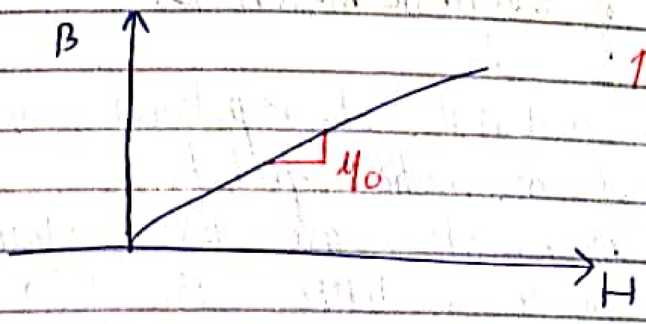
The magnetization curve is as:



The slope in the linear region represent permeability.

$$\boxed{B = \mu H}$$

For air, μ is very low; $= \mu_0$
So the BH curve is assumed to be linear.



The air do not saturate.

Some important relations;

$$B = \mu H$$

B → flux density, H → field intensity

Also $B = \frac{\phi}{A} \Rightarrow \boxed{\phi = BA}$

Also $H = \frac{B}{\mu} = \left(\frac{\phi}{A\mu}\right) = \frac{mmf}{R\mu A} = \frac{N \cdot i(t)}{R \cdot \mu A}$

$\left[\phi = \frac{mmf}{Reluctance},\ mmf = N \cdot i(t),\ R = \frac{l}{\mu A}\right]$

$$H = \frac{N \cdot i(t)}{\frac{l}{\cancel{\mu A}} \cdot \cancel{\mu A}} \qquad \boxed{H = \frac{N \cdot i(t)}{l}}$$

Important Points for Rotating Machine

If a material is being magnetized for the first time, the curve is known as virginal curve.

Conditions for induced voltages

$$\boxed{e = N\frac{d\phi}{dt}} \qquad e \propto \frac{d\phi}{dt}$$

or $$\boxed{e = \frac{d\psi}{dt}}$$

ψ = flux linkage.

$$\boxed{\psi = N.\phi}$$

i. If the magnetic field is fixed, the conductor should be rotating in nature!
ii. If the mag field is changing in nature, the conductor is fixed.
iii. If both mag field and conductor are changing in nature.

i. Dynamically induced emf.
ii. Statically induced emf eg transformer.
iii. Static + dynamic.

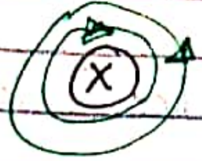
Fleming Right Hand Rule is applicable in generator.
Fleming Left hand rule is applicable in motors.

Thumb → motion or speed or torque
First finger → Flux direction
Middle finger → Direction of current.

⊗ into the surface.
⊙. out of the surface.

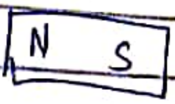
Thumb Rule   Thumb = current

finger curl = magnetic flux.



↳ or the screw rule.

Fixed pole If poles do not change

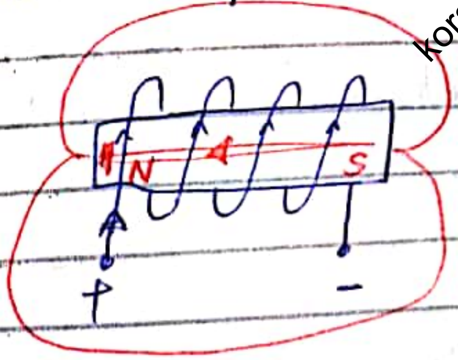


↳ natural magnet.

↳ using DC supply.

For wrapping of coil, thumb rule is opposite.

ie Thumb = flux   curl = current



fixed pole.

$$\phi = \frac{N \cdot i(t)}{R}$$

~~Electromagnet~~

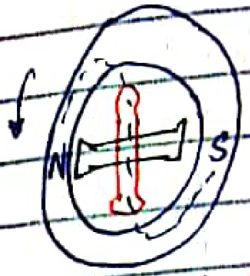
Varying pole from electromagnet.

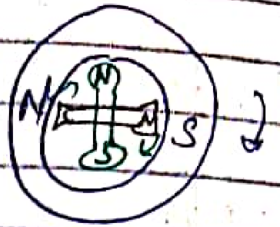
↳ poles change.

↳ using AC supply. ↳ during +ve and −ve cycles.

Reluctance Torque is only possible in salient pole machine. ie it is not possible in cylindrical machine.

Electromagnetic Torque is possible in both salient pole and cylindrical machine.





Rotor initially non magnetized.

↳ Reluctance torque

↳ will align the rotor in direction to get minimum reluctance path.

Initially magnetized.

N-S will lock each other

No further movement.

Electromagnetic torque.

↳ due to interaction of fluxes of stator and rotor.

If cylindrical rotor;

$$\boxed{T_e \propto \phi_r \phi_s \sin \delta}$$

δ is angle b/w axis of stator and rotor.

Each and every point has almost same reluctance

↳ no rotation → no torque.

Some Important terms.

Conductor: The ~~length~~ portion of ~~ct~~ wire in the influence of mag field.

Turn (t) Minimum no. of conductors to form a turn = 2.

if z no of conductors -

total turn possible

$$\boxed{T = \frac{z}{2}}$$

Coil Minimum turn = 1 ⇒ Minimum conductor = 2

slot where you house (put) a conductor.

Coil side The part of the coil in a slot.

Pole pitch The angular displacement from pole to pole.

$$\boxed{1 \text{ pole pitch} = 180^\circ \text{ electrical.}}$$

(magnitude)

$$\boxed{\Phi_{\text{per pole}} = \frac{4B_p l r}{P}}$$

→ The amount of flux useful for induction process.

↳ This is not the flux generated by individual pole

$$\boxed{\theta_e = \frac{P}{2}\theta_m}$$

$$\boxed{N = \frac{120f}{P}}$$

N → mechanical quantity

f → in Hz → electrical quantity

↳ rotation per minute (rpm)

$$\boxed{w_e = \frac{P}{2} w_m}$$

$$\boxed{w_m = \frac{2\pi N}{60}}$$

w_e ✗

w_e → angular speed

DC Machines

Generator

→ i/p variable is a mechanical quantity (T torque, ω angular speed).

→ o/p variable is V voltage or current I.

Torque related to current

Speed related to voltage.

Motor

→ i/p variable is an electrical quantity ie V or I

→ o/p is a mechanical quantity ie T or ω.

$V \to \omega$, $I \to T$

Construction

Stator is responsible for the production of main flux

↳ stationary part.

↳ The supply on dc machine is always dc supply on the stator to get fixed poles.

Stator — field poles - field winding - produces main flux - DC supply.

Rotor

It has armature core (if gen, you take voltage at this point and if motor you apply at this point).

Rotor - armature core - armature winding - produces induced voltage (AC).

$$\boxed{E_{ph} = 4.44 \cdot f \, \phi \cdot N}$$

Commutator.

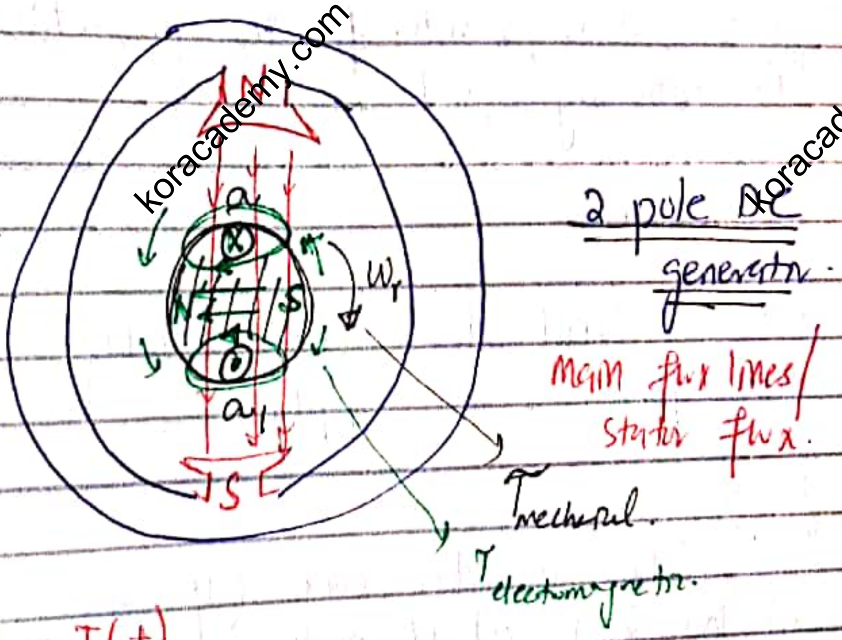
In case of generator, converts AC voltage from rotor to DC.

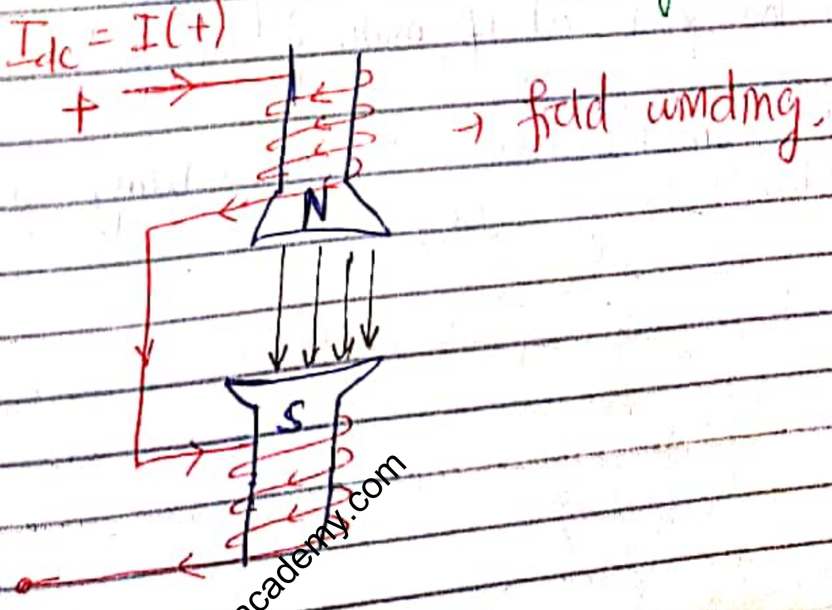
ⓐ converts bidirectional voltage/current into a unidirectional.

↳ Acting as a rectifier.

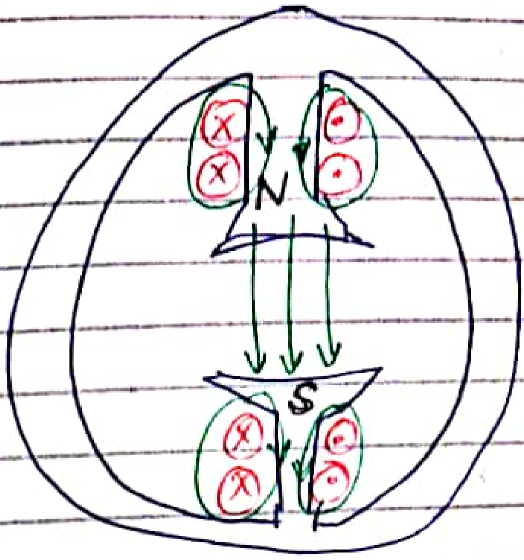
In case of motor, i/p is always a dc voltage. I/p is always applied at the rotor. So the dc will not operate the rotor./machine.

So here commutator converts DC to AC.





It can also be represented as;

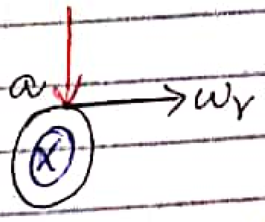


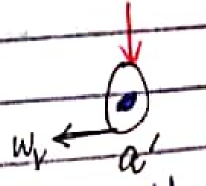
Generator

① Rotate the rotor with a constant speed. (ω_r)
② Switch on the excitation system.

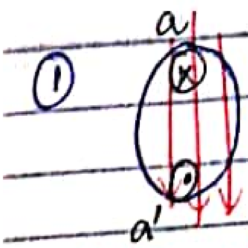
Fleming right hand rule.

current into the coil.

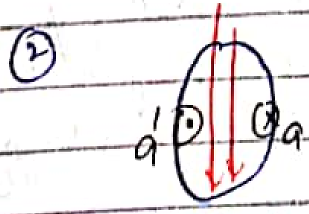




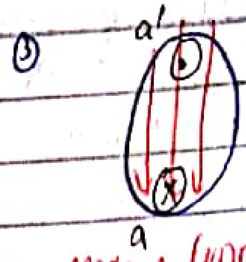
out of the coil.



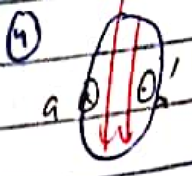
maximum cutting



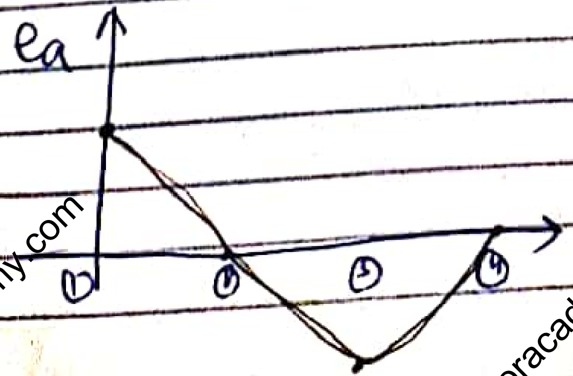
almost 0 cutting

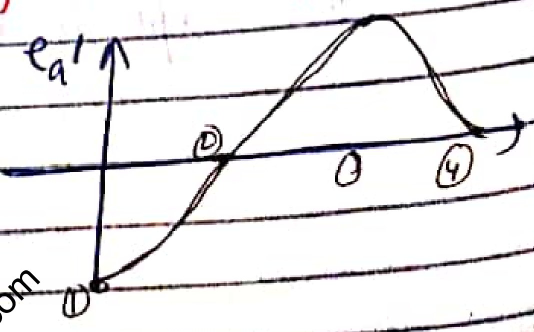


maximum (opposite)



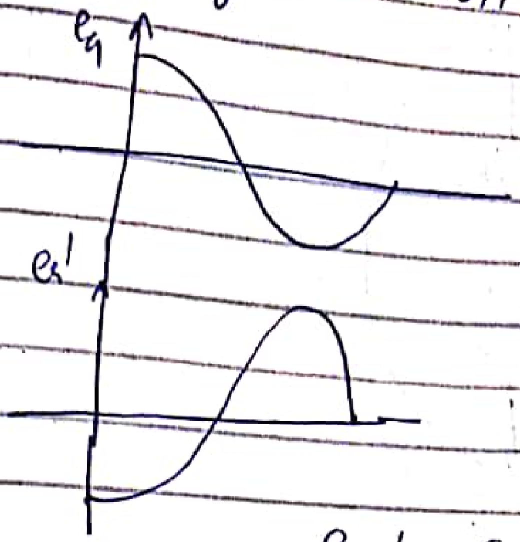
zero



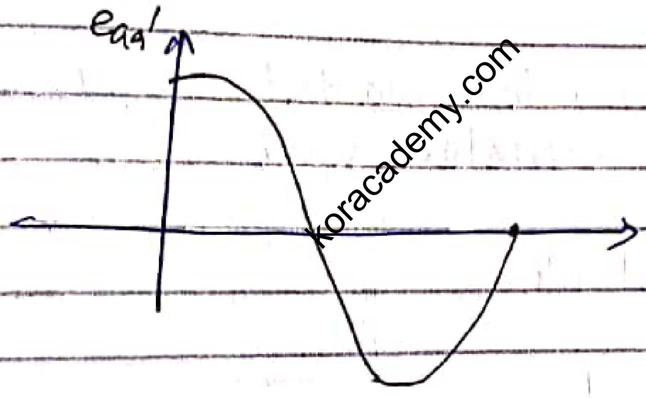


The rotor waveform in rotor is always AC.

The voltage of total coil $e_{aa'}$;



$$e_{aa'} = e_a - e_a' = e_a + (-e_a')$$



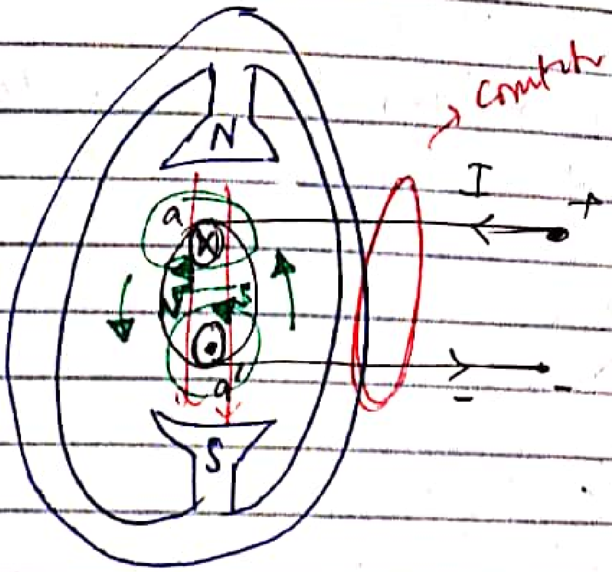
Electromagnetic torque always opposes mechanical torque.

The amount of mechanical energy wasted to compensate for the electromagnetic torque is converted into electrical energy.

DC generator → o/p is electrical and DC.
DC motor → i/p is DC → at rotor

not directly.

DC Motor

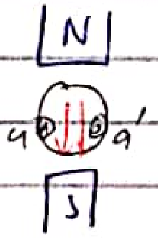


let DC supply is directly applied at the rotor.

main flux

T_e

After some time, N-S will lock each other.



No further rotation.

$$\boxed{T = \Phi_s \Phi_r . \sin \delta}$$

$\delta = 0$

T_e will vanish in a little time if DC supply is directly connected.

→ For electromagnetic torque to be produced;
- Both stator and rotor should have AC supply.
- If one is DC, the other must have AC supply.

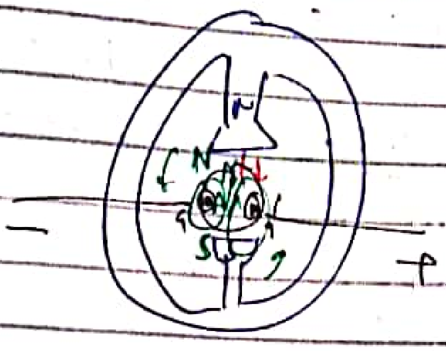
T_e cannot be produced if both stator and rotor have DC supplies.

So a commutator is introduced after the DC supply to get an AC current and hence get steady state torque.

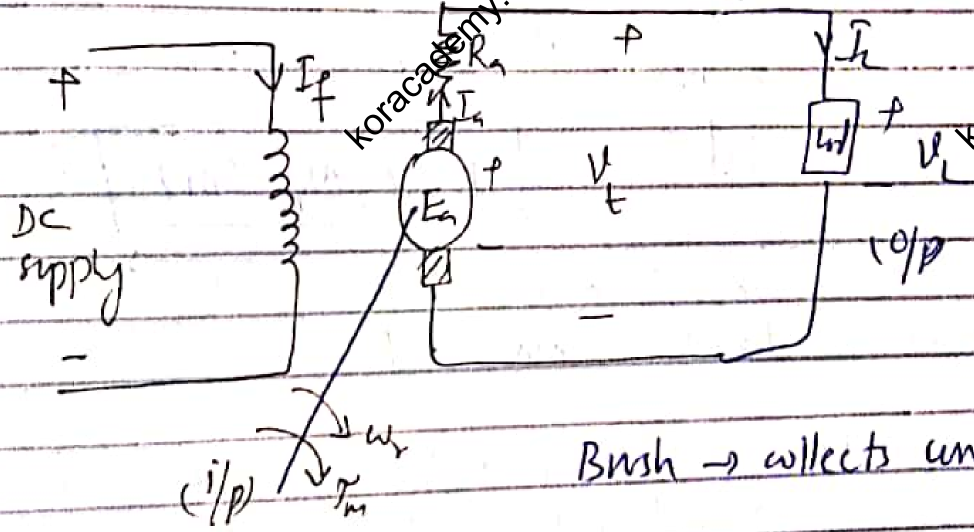
T_e is called & b/c it is produced from the interaction of stator and rotor fluxes. (also called interaction torque).

Load torque and electromagnetic torque always oppose each other.

When supply at coils becomes AC after commutator the T_e is produced.

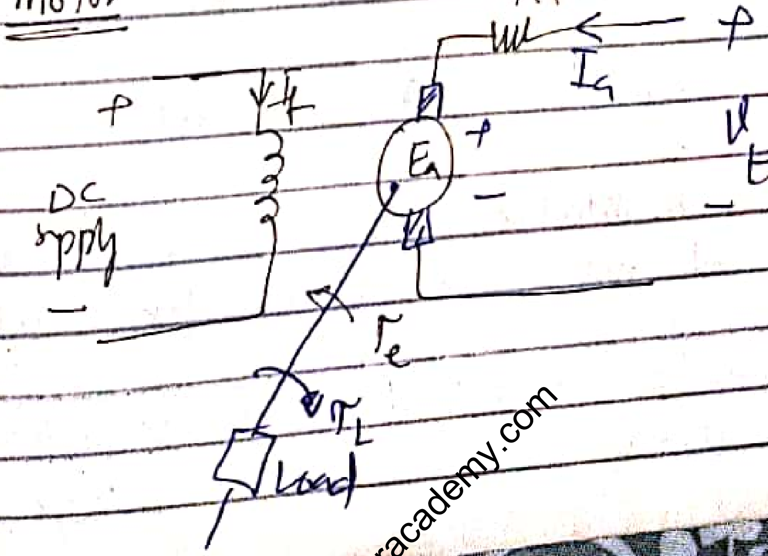


Generator



Brush → collects current.

Motor



$$\boxed{E_{a(avg)} = K\phi\omega_r}$$ $\omega_r \rightarrow$ mechanical.

↳ induced voltage in dc machine.

where $\boxed{K = \frac{PZ}{2\pi A}, \quad \omega_r = \frac{2\pi N_r}{60}}$

P = no. of poles.
Stator and rotor always has the same no. of poles.

→ The electromagnetic torque developed in armature;

$$\boxed{\tau_{a(avg)} = \tau_e = K\phi I_a}$$

As $\tau_a = \frac{P_a}{\omega_r} = \frac{E_a \times I_a}{\omega_r} = \frac{K\phi\omega_r I_a}{\omega_r} = K\phi I_a$

→ No. of Parallel paths (A)

$\boxed{\begin{array}{l} A = P \\ A = 2 \end{array}}$ for lap windings.
for wave windings.

Z → number of total armature conductors.
T → no. of turns = Z/2

$$\boxed{N_r = rpm = \frac{120 f_e}{P}}$$

$f_e \rightarrow$ electrical.
↳ the frequency of induced voltage in rotor.

$$\boxed{\omega_r = \frac{2\pi N_r}{60}}$$

Types of DC machines (Gen or motor)

① Seperately excituted DC machines.

② Self excited DC machine (residual flux).

- DC shunt machine
- DC series machine
- DC compund machine
  - On basis of field flux contribution.
    - Differentially compound.
    - Cummulatively compund
  - Field winding connection.
    - Long shunt
    - Short shunt

→ Excitation refers to the field winding.

① If a separate DC supply is provided to the field winding → separately excited.
↳ little application.
↳ if not mentioned → by default separately excited.

② No seperate supply to field winding. The supply will only be that generated or that given to the rotor.

→ If the field winding is connected in parallel with total armature → shunt machine
→ in series → series machine.

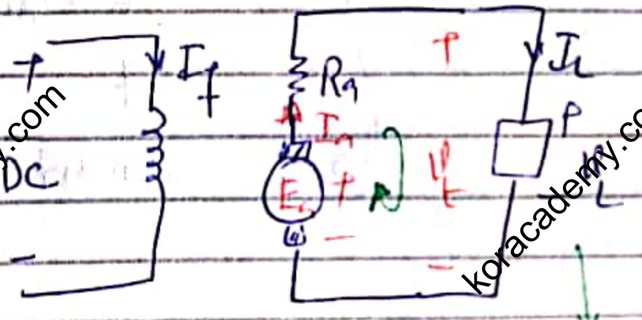
→ compund = series + shunt.

→ If series and shunt winding fluxes support each other → commulative
↳ If oppose → differential.

(1) DC Separately Excited Machine.

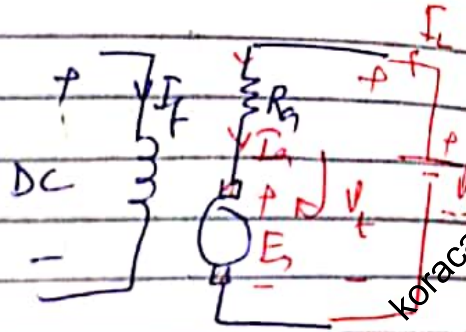
The voltage E_a combined with the voltage of R_a is armature voltage.

i- Gen



$P_o = V_t \times I_L$

ii- Motor



As $\phi_f \propto I_f$

DC ⇒ I_f = constant ⇒ ϕ_f = constant

$\phi_f \propto I_f$

ϕ_f = constant

I_f = constant

KVL ⇒ $\boxed{E_a = V_t + I_a R_a}$ $+V_{BD}$

$\boxed{I_a = I_L}$

KVL ⇒ $\boxed{E_a = V_t - I_a R_a}$ $-V_{BD}$

$I_a = I_L$

Armature power, $\boxed{P_a = E_a I_a}$

$P_a = V_t I_a + I_a^2 R$

$\boxed{P_a = V_t I_a + I_a^2 R}$

i/p o/p copper losses.

$E_a I_a = V_t I_a + I_a^2 R$

$P_a = V_t I_L + I_a^2 R$

$\boxed{P_a = P_m + I_a^2 R}$

o/p

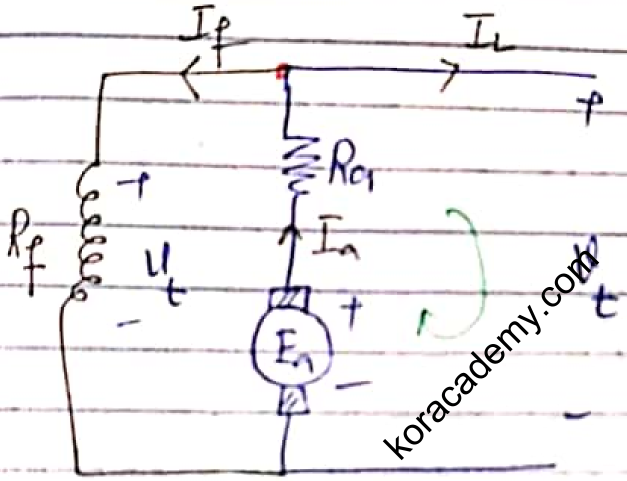
V_{BD} → brush drop

(2) Self Excited DC machine

No need of a separate DC supply.

(i) Shunt Machine

(a) Generator



As $\phi_f \propto I_f$

$I_f = \frac{V_t}{R_f}$ = const

↳ ϕ_f = constant

KCL $\quad I_a = I_L + I_f$

KVL $\quad E_a = V_t + I_a R_a$

$$\boxed{\begin{aligned} I_a &= I_L + I_f \\ E_a &= V_t + I_a R_a \end{aligned}}$$

$$\text{Power} = E_a I_a = V_t I_a + I_a^2 R_a$$

$$P_a = V_t (I_L + I_f) + I_a^2 R$$

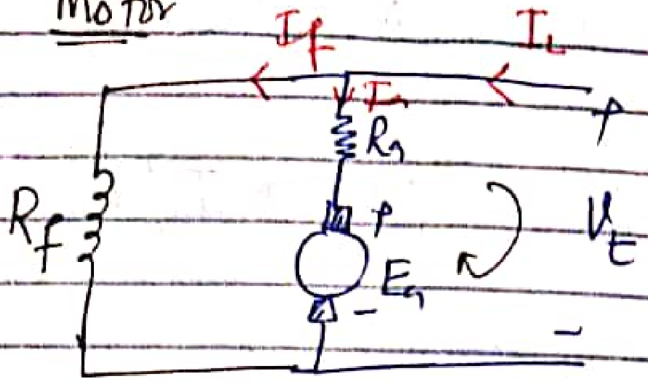
$$= V_t I_L + V_t I_f + I_a^2 R_a$$

$\because V_t = I_f R_f$

$$\boxed{P_a = \underbrace{V_t I_L}_{P_{out}} + \underbrace{I_f^2 R_f}_{\text{field winding loss}} + \underbrace{I_a^2 R_a}_{\text{armature resistive loss.}}}$$

↳ copper loss

(b) Motor



$$I_L = I_a + I_f$$

$$\boxed{\boxed{I_a = I_L - I_f} \quad \boxed{E_a = V_t - I_a R_a}}$$

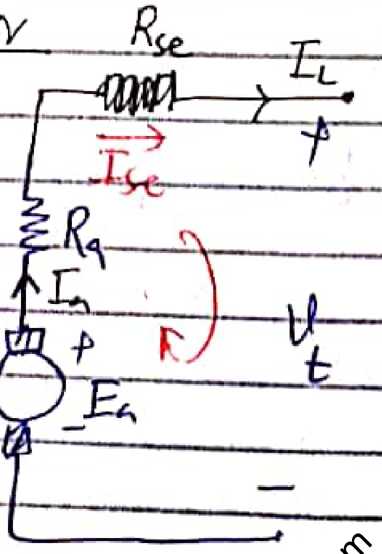
$$P = E_a I_a = V_t I_a - I_a^2 R_a = V_t (I_L - I_f) - I_a^2 R_a$$

$$E_a I_a = V_t I_L - \underset{\downarrow \; V_t = I_f R_f}{V_t I_f} - I_a^2 R_a$$

$$\boxed{\underset{o/p}{P_a} = \underset{i/p}{V_t I_L} - \underbrace{I_f^2 R_f - I_a^2 R_a}_{losses}}$$

ii. Series Machine

(a) Generator



$\phi_{se} \propto I_{se}$

$$\boxed{I_{se} = I_L = I_a}$$

$\phi_{se} \neq$ constant

b/c I_L is not constant

→ depends on the load.

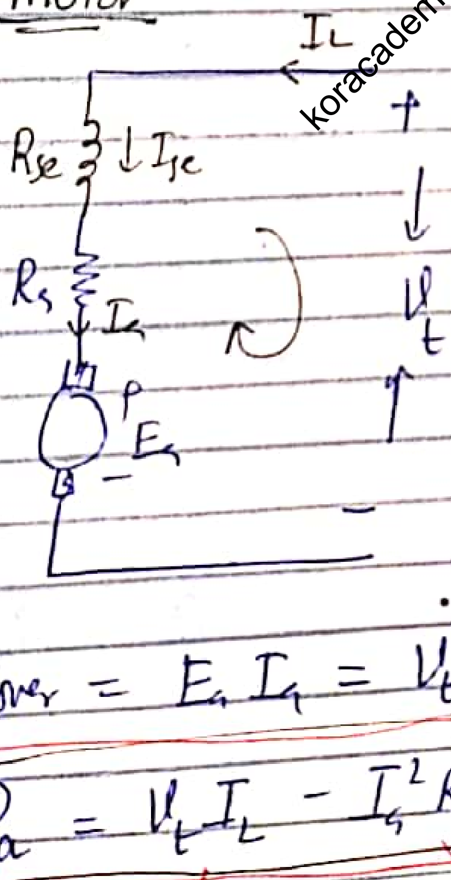
KUL → $\boxed{E_a = V_t + I_a(R_a + R_{se})}$

$$P = E_a I_a = V_t I_a + I_a^2 R_a + I_a^2 R_{se}$$

$$I_a = I_L = I_{se}$$

$$\boxed{\underbrace{P_a}_{\text{develped}} = \underbrace{V_t I_L}_{P_{out}} + \underbrace{I_a^2 R_a + I_{se}^2 R_{se}}_{\text{losses}}}$$

(5) Motor



$\phi_{se} \propto I_{se}$

$\boxed{I_{se} = I_L = I_a}$

$\boxed{E_a = V_t - I_a(R_a + R_{se})}$

$$P_{dev} = E_a I_a = V_t I_a - I_a^2 R_a - I_a^2 R_{se}$$

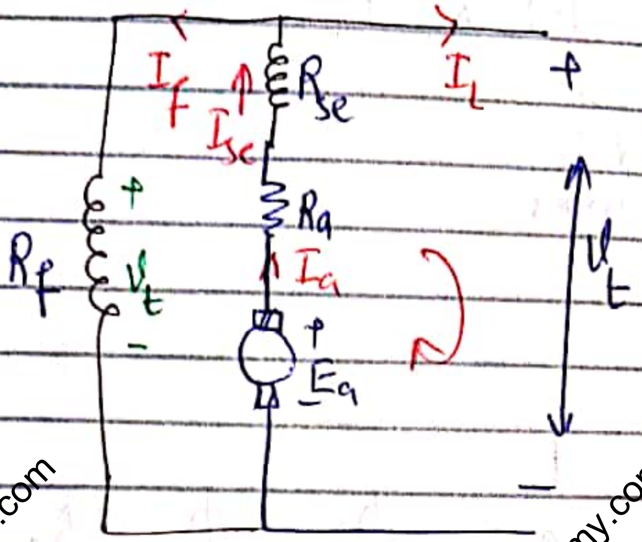
$$\boxed{\underbrace{P_a}_{o/p} = \underbrace{V_t I_L}_{P_{in}} - \underbrace{I_a^2 R_a - I_{se}^2 R_{se}}_{\text{losses}}}$$

DC Compound Machine

(i) Long Shunt DC machine

series + shunt

(a) Generator



Number of turns in series winding is less but the wire is thicker.

No. of turns in shunt winding are more but the wire is thinner.

$$\boxed{\phi_{total} = \phi_f \pm \phi_{se}}$$

$$\phi_f \propto I_f$$

$$\phi_{se} \propto I_{se}$$

$$I_f = \frac{V_t}{R_f}$$ V_t is always constant.

I_f = constant $\Rightarrow$ ϕ_f = constant.

$I_{se} = I_L \neq$ constant → depends on the load

$\Rightarrow \phi_{se} \neq$ constant $\Rightarrow \phi_{total} \neq$ constant

Induced voltage will be effected.

$$\boxed{I_a = I_{se} = I_f + I_L}$$

KVL →

$$\boxed{E_a = V_t + I_a R_a + I_a R_{se}}$$

$$\text{Power} = E_a I_a = V_t I_a + I_a^2 R_a + I_a^2 R_{se}$$

$$E_a I_a = V_t I_a + I_a^2 R_a + I_{se}^2 R_{se}$$

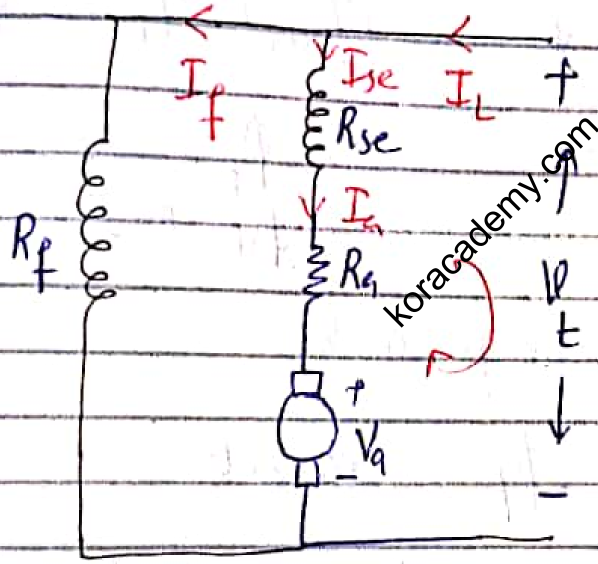
?

Put $I_a = I_f + I_L$

$$P_a = V_t I_L + \underset{\downarrow I_f R_f}{V_t} I_f + I_a^2 R_a + I_{se}^2 R_{se}$$

$$\boxed{\underbrace{P_a}_{\text{i/p pow (armature)}} = \underbrace{V_t I_L}_{\text{load}} + \underbrace{I_f^2 R_f + I_a^2 R_a + I_{se}^2 R_{se}}_{\text{losses}}}$$

(b) Motor



$$\phi_{total} = \phi_f + \phi_{se}$$

$$I_f = \frac{V_t}{R_f}$$

$$\boxed{I_a = I_{se} = I_L - I_f}$$

$$\boxed{E_a = V_t - I_a R_a - I_a R_{se}}$$

$$\text{Power} = E_a I_a = V_t I_a - I_a^2 R_a - I_{se}^2 R_{se}$$

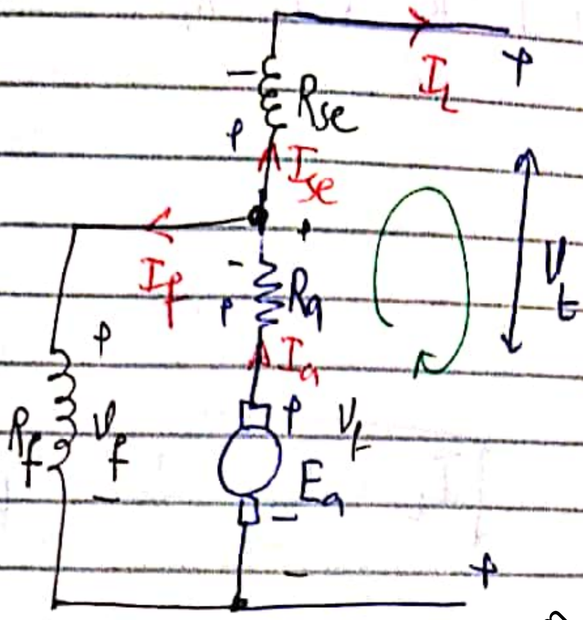
$$E_a I_a = V_t (I_L - I_f) - I_a^2 R_a - I_{se}^2 R_{se}$$

$$= V_t I_L - V_t I_f - I_a^2 R_a - I_{se}^2 R_{se}$$

$$\boxed{\underbrace{P_a}_{\text{o/p}} = \underbrace{V_t I_L}_{\text{i/p}} \underbrace{- I_f^2 R_f - I_a^2 R_a - I_{se}^2 R_{se}}_{\text{losses}}}$$

ii. Short Shunt DC machine

(a) Generator



$\phi_{total} = \phi_f + \phi_{se}$

$$\boxed{I_a = I_{se} + I_f}$$

$I_{se} = I_L$

↳ $\phi_{total} \neq$ constant

↳ depends on the load.

$E_a - I_a R_a - V_f = 0$

$$\boxed{V_f = E_a - I_a R_a}$$

(or)

$V_f - I_{se} R_{se} - V_t = 0$

$$\boxed{V_f = V_t + I_{se} R_{se}}$$

$$\boxed{I_f = \frac{V_f}{R_f} \neq \frac{V_t}{R_f}}$$

Complete KVL →

$$\boxed{E_a = V_t + I_a R_a + I_{se} R_{se}}$$

$P_{conv} = E_a I_a = V_t I_a + I_a^2 R_a + I_a I_{se} R_{se}$

$P_a = V_t (I_L + I_f) + I_a^2 R_a + (I_{se} + I_f) I_{se} R_{se}$

$P_a = V_t I_L + V_t I_f + I_a^2 R_a + I_{se}^2 R_{se} + I_{se} I_f R_{se}$

$P_a = V_t I_L + I_a^2 R_a + I_{se}^2 R_{se} + I_f (\underbrace{V_t + I_{se} R_{se}}_{= V_f})$

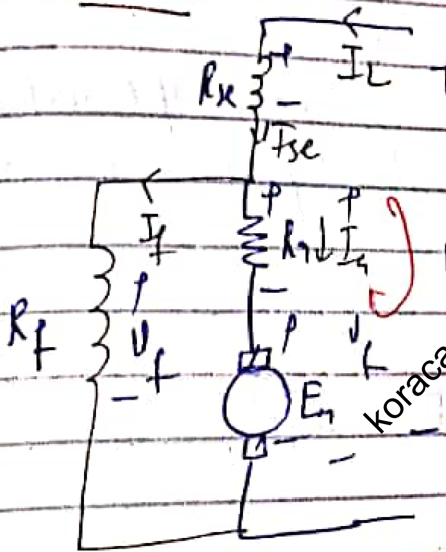
$$P_a = V_t I_L + I_a^2 R_a + I_{se}^2 R_{se} + V_f I_f$$

↳ $I_f = V_f / R_f$

$$\Rightarrow \boxed{P_a = V_t I_L + I_a^2 R_a + I_{se}^2 R_{se} + I_f^2 R_f}$$

i/p o/p losses

(b) Motor



$$\boxed{I_L = I_{se} = I_a + I_f}$$

$$E_a + I_a R_a - V_f = 0$$

$$\boxed{V_f = E_a + I_a R_a}$$

ⓐ

$$V_f + I_{se} R_{se} - V_t = 0$$

$$\boxed{V_f = V_t - I_{se} R_{se}}$$

$$\boxed{I_f = \frac{V_f}{R_f} \neq \frac{V_t}{R_f}}$$

KVL ⇒ $E_a + I_a R_a + I_{se} R_{se} - V_t = 0$

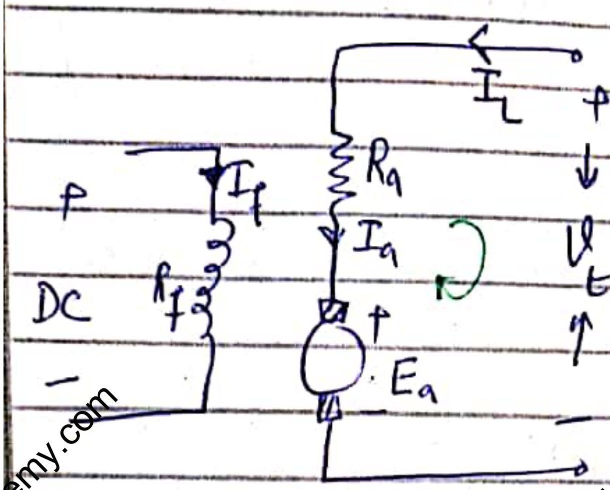
$$\Rightarrow \boxed{E_a = V_t - I_a R_a - I_{se} R_{se}}$$

$$\boxed{\text{Power} = P_a = V_t I_L - I_a^2 R_a - I_{se}^2 R_{se} - I_f^2 R_f}$$

o/p i/p

Questions

Q1. A separately excited DC motor has $R_a = 0.5\Omega$. It runs from 250V DC supply drawing a armature current of 20A at 1500 rpm. For the same field current (means flux n not changing), the torque developed in N-m for an armature current of 10A will be 15.278 Nm.



Case I

at 250 V , $I_{a1} = 20A$,

$N_1 = 1500$ rpm.

$$\boxed{T_a = K\phi I_a} \quad , \quad \boxed{T_a = \frac{P_g}{\omega_r} = \frac{E_a I_a}{\omega_r}}$$

$$K = \frac{PZ}{2\pi A}$$

ϕ = constant

Also As $T \propto \phi I_a$

$$\frac{T_2}{T_1} = \frac{\phi_2}{\phi_1}\frac{I_{a2}}{I_{a1}} \rightarrow \boxed{T_2 = \frac{I_{a2}}{I_{a1}} \times T_1}$$

where $\boxed{T_1 = \frac{E_{a1} I_{a1}}{\omega_{r1}}}$

KVL $E_{a1} = V_t - I_{a1}R_a = 250 - (20 \times 0.5)$

$E_{a1} = 250 - 10 = 240\ V$

$\rightarrow T_{a1} = \frac{(240)(20)}{2\pi \times 1500} \times 60 = 30.357$ N-m

Case II

$V_t = 250V$, $I_{a2} = 10A$, $T_2 = ?$

As $\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \Rightarrow T_2 = \frac{10}{20} \times 30.557$

$\Rightarrow T_2 = 15.28$ Nm.

or by another method.

$$\tau_2 = \frac{E_{a2} \cdot I_{a2}}{\omega_{r2}}$$

where $E_{a2} = V_t - I_{a2} R_a$

$\omega_{r2} = ?$

As $E_a = K\phi\omega_r \Rightarrow \frac{E_{a2}}{E_{a1}} = \frac{K\phi_2}{K\phi_1}\frac{\omega_{r2}}{\omega_{r1}}$

$\Rightarrow \frac{E_{a2}}{E_{a1}} = \frac{\omega_{r2}}{\omega_{r1}} \Rightarrow \omega_{r2} =$ ______.

Q2. A 4 pole dynamo wave bound armature has 51 slots containing 20 conductors in each slot. The induced emf is 357V and speed is 8500 rpm.
The flux per pole is 1.235mWb

$P = 4$, $A = 2$

No. of slots = 51

No. of conductors/slot = 20

$N_r = 8500$ rpm

$E = 357V$

$$E_a = K\phi\omega_r = \frac{PZ}{\cancel{2\pi} A}\cdot\phi_p \times \frac{\cancel{2\pi} N_r}{60}$$

$$\Rightarrow E_a = \frac{PZ}{A}\cdot\phi_p\cdot\frac{N_r}{60}$$

Z = ?

$$Z = (\text{No. of conductors/slot})(\text{total slots})$$

$$Z = 20 \times 51 = 1020$$

$$\phi_p = \frac{60 E_a A}{P Z N_r} = (\cancel{(60)(223}3 = \frac{(60)(357)(2)}{(8500)(4)(1020)}$$

$$\Rightarrow \phi_p = 1.235 \text{ mWb}$$

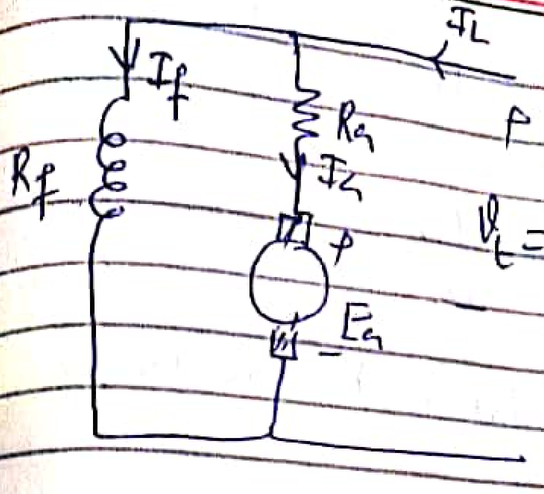
$$\text{No. of turns} = T = \frac{Z}{2} = \frac{1020}{2} = 510.$$

Q3. At 50% of full load, the armature current drawn by a DC shunt motor is 40A when connected to a 200V DC. By decreasing the field flux, its speed is raised by 20%, this also causes a 10% increase in torque. The R_a including the brush is 1Ω. The %age change in field current is ________. (Neglect saturation).

$$\%\text{age change} = \frac{\phi_2 - \phi_1}{\phi_1} \times 100$$

$$\text{or } \%\text{age change} = \frac{I_{f2} - I_{f1}}{I_{f1}} \times 100$$

$E_a = V_t \pm I_a R_a$ $\left(\begin{array}{l} T = K\phi I_a \\ E_a = K\phi\omega_r \end{array}\right)$



Case 1

$V_t = 200V$ $I_{a1}, E_{a1}, V_{t1},$

$N_{r1}, T_1, \phi_{f1}, I_{f1}$

$I_{a1} = 40A$

Case II

$W_{r2} = 1.2\, w_{r1}$, $T_2 = 1.1\, T_1$

$R_a = 1\Omega$, $I_{f2} = ?$ $\phi_2 = ?$

$$\frac{T_2}{T_1} = \frac{\cancel{K}\phi_2}{\cancel{K}\phi_1}\frac{I_{a2}}{I_{a1}} = \frac{\phi_2}{\phi_1}\times\frac{I_{a2}}{I_{a1}}$$

$$\Rightarrow \frac{1.1\cancel{T_1}}{\cancel{T_1}} = \frac{\phi_2}{\phi_1}\frac{I_{a2}}{I_{a1}}$$

$$\Rightarrow \frac{\phi_2}{\phi_1} = 1.1\left(\frac{I_{a1}}{I_{a2}}\right) = \frac{1.1\times 40}{I_{a2}}$$

$$\Rightarrow \boxed{I_{a2} = 44\times\frac{\phi_1}{\phi_2}} \text{ — Ⓐ} \quad \frac{\phi_1}{\phi_2} \rightarrow \text{let } x$$

$$\text{Now } \frac{E_{a2}}{E_{a1}} = \frac{K\phi_2}{K\phi_1}\times\frac{w_{r2}}{w_{r1}} = \frac{\phi_2}{\phi_1}\times\frac{1.2\,W_{r1}}{W_{r1}}$$

$$\boxed{\frac{E_{a2}}{E_{a1}} = 1.2\times\frac{\phi_2}{\phi_1}} \rightarrow \frac{1}{x}$$

As $E_{a1} = V_t - I_{a1}R_a = 200 - (40 \times 1) = 160$ V

we have;

$\frac{E_{a2}}{160} = \frac{1}{x}(1.2)$, $I_{a2} = 44x$ where $x = \frac{\phi_1}{\phi_2}$

$\frac{V_t - I_{a2}R_a}{160} = \frac{\overset{1.2}{\cancel{120}}}{x} \Rightarrow \frac{200 - (44x)(1)}{160} = \frac{1.2}{x}$

$\Rightarrow (1.2)(160) = 200x - 44x^2$

$\Rightarrow 44x^2 - 200x + 192 = 0$

$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{+200 \pm \sqrt{(-200)^2 - 4(44)(192)}}{2 \times 44}$

$\Rightarrow x = \frac{200 \pm 78.79}{88}$

$\Rightarrow x_1 = 3.16 = \frac{\phi_1}{\phi_2}$, ✓ $x_2 = \frac{\phi_1}{\phi_2} = 1.37$

Checking for the correct value of x.

Ⓐ ⇒ $I_{a2} = 139.04$ > full load current so x_1 X

Putting x_2

Ⓐ ⇒ $I_{a2} = 60.28$ < full load I ≈ 80.

$$\frac{\phi_1}{\phi_2} = 1.37 \quad \Rightarrow \frac{\phi_2}{\phi_1} = \frac{1}{1.37} = 0.72$$

$$\% \text{ change} = \frac{\phi_2 - \phi_1}{\phi_1} = \left(\frac{\phi_2}{\phi_1} - 1\right)$$

$$= -28\ \%$$

decrement ✓

Q4. A DC shunt generator delivers 60 kW at 240 V and 380 rpm. The armature and field resistences are 0.015 and 60 Ω. (Allowing 2 V per brush drop).

The speed of machine (rpm) when running as a shunt motor ~~and~~ taking 60 kW input at 240 V will be 343.16 rpm

Rating

DC Gen

eg 20 kW, 200 V

↓

El o/p power

V_t

DC Motor

eg 50 W, 100 V

↓

o/p | load | mech power

↳ V_t (i/p)

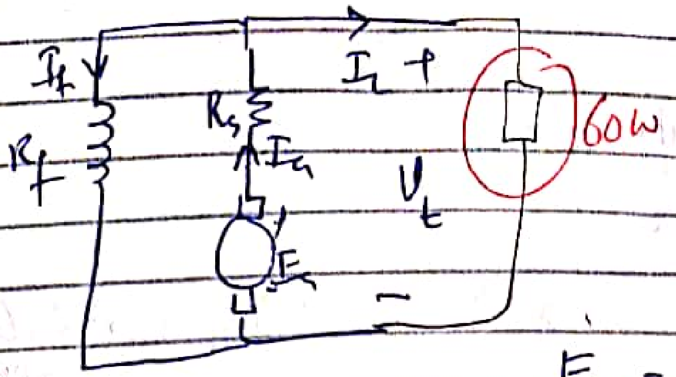
$$E_a = K\phi W_r$$

$$\boxed{\frac{E_{a2}}{E_{a1}} = \frac{\cancel{K}\phi_2}{\cancel{K}\phi_1}\frac{\omega_{r2}}{\omega_{r1}} = \frac{\phi_2}{\phi_1} \times \frac{N_{r2}}{N_{r1}}}$$

Case (i) Acting as DC Shunt Gen

$V_t = 240V, R_a = 0.015\Omega$

$R_f = 60\Omega$



$I_f = \frac{V_t}{R_f} = \frac{240}{60} = 4A$

$E_{a1} = V_t + I_{a1}R_a + \underset{\text{brushes}}{2V}$

$E_{a1} = 240 + I_{a1}(0.015) + 2$

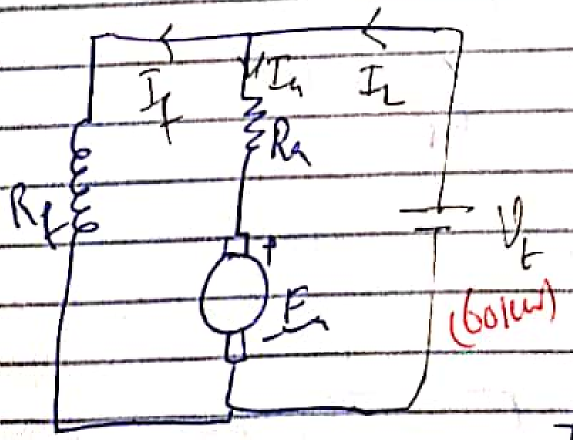
$60 = V_t \times I_L \Rightarrow I_L = \frac{60\times10^3}{240} = 250A$

$I_a = 250 + 4 = 254.$

$E_{a1} = 240 + (254)(0.015) + 2$

$\Rightarrow E_{a1} = 245.81 V$

If Acting as DC motor



$60 = V_t \times I_L$

$\Rightarrow I_L = 250A$

$I_f = \frac{V_t}{R_f} = \frac{240}{60} = 4A$

$I_{a2} = 250 - 4 = 246A$

$E_{a2} = V_t - I_{a2}R_a - 2V = 240 - 246(0.015) - 2$

$E_{a2} = 234.31\,V$

$N_{r2} = ?$

$$N_{r2} = \frac{E_{a2}}{E_{a1}} \times \frac{\phi_1}{\phi_2} \times N_{r1}$$

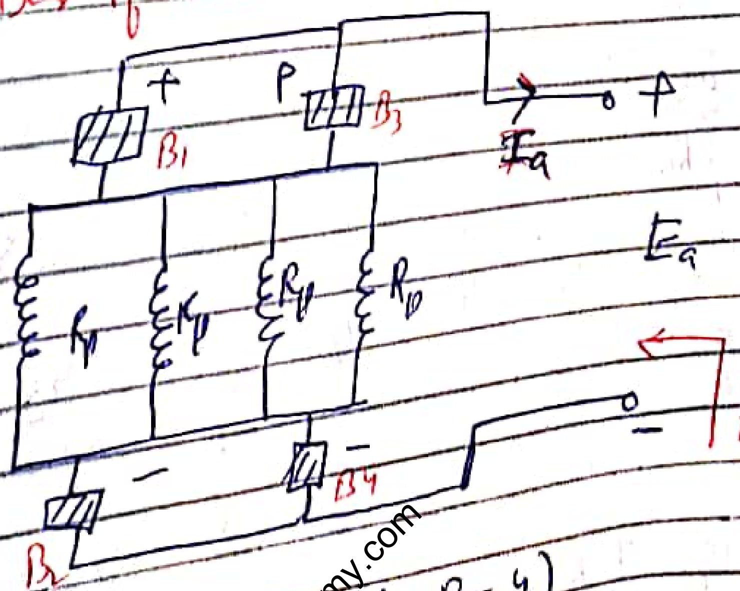
$$= \frac{234.31}{245.81} \times 1 \times 360 \Rightarrow N_{r2} = 343.16 \text{ rpm}$$

B/c $I_{f1} = I_{f2} \Rightarrow I_f = \text{constant}$

$\Rightarrow \phi_f = \text{constant}$

Q5. A 4 pole lap wound DC generator has developed power of P watts and voltage of E volts. Two adjacent brushes are removed as they are worn out. If the machine operates with the remaining brushes, the developed voltage and power that can be obtained are $P_a/4$, E.

Number of brushes = No. of parallel paths.

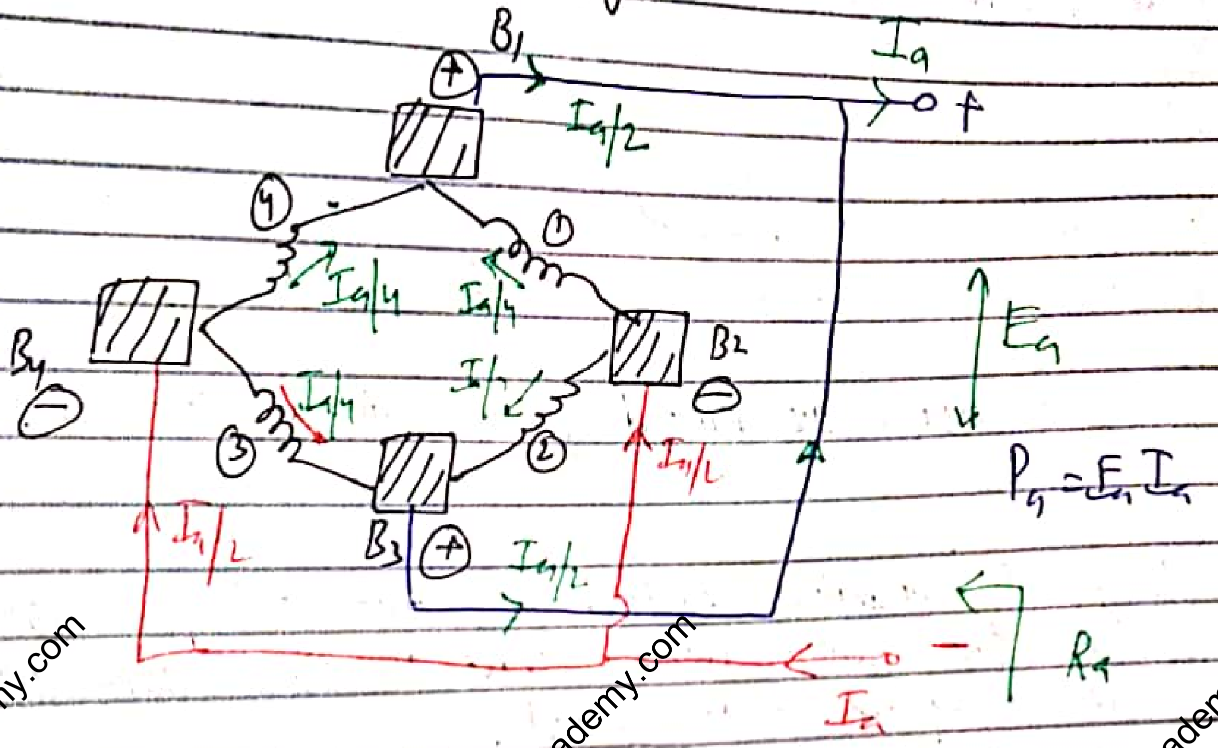


$R_a = \frac{R_p}{4}$

$(A = P = 4)$

$P = 4$

If drawn in another way.



⇒ If B1 is removed, the role of ① and ④ finishes.

The new current at the terminal $I_1' = I_a/2$

No effect on induced voltage i.e $E_a' = E_a$

$$\Rightarrow P' = E' \times I' = \frac{E_a I_a}{2} \Rightarrow \boxed{P' = \frac{P}{2}}$$

⇒ If two adjacent brushes are removed say B2 and B3.

the role of ①, ② and ③ coils finishes.

The new current is $I' = I_a/4$

No effect on voltage i.e $E_a' = E$

$$P' = E' \times I' = E_a \times I_a/4$$

$$\Rightarrow \boxed{P' = \frac{P}{4}}$$

→ If two opposite coils are removed.

$I_a' = 0 \rightarrow P' = 0$

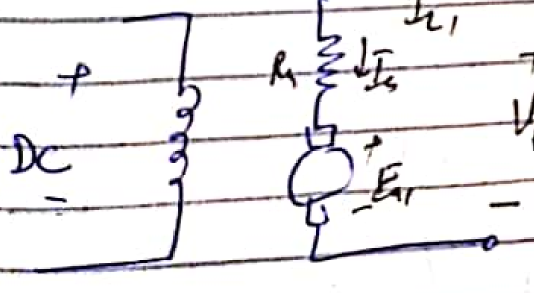
(Wave winding = permanently zero, 2 brushes s/c only two no. of parallel paths)

If n brushes are removed, n+1 coils are effected.

Q6. A separately excited DC shunt motor is driving a fan load whose torque is proportional to the square of its speed. When 100V is applied to the motor, the current taken is 8A, the speed being 500 rpm. Assume $R_a = 1\Omega$.
The applied voltage and current drawn by armature when speed reaches 750 rpm are 150V, 18A.

$\tau_a \propto N^2$

Case i $V_{t_1} = 100V$, $I_{L_1} = 8A$, $N_1 = 500rpm$
$R_a = 1\Omega$



Case II
$N_2 = 750 rpm$,
$V_t = ?$ $I_{a_2} = ?$

$E_{a_2} = V_{t_2} - I_{a_2} R_a$

ϕ = constant

(The two eqs $E_a = K\phi\omega_r$
$\tau_a = K\phi I_a$)

Variable $= I_a^2 R_a + I_{se}^2 R_{se} = I_a^2 (R_a + R_{se})$

Under maximum efficiency;
constant losses = variable losses

Q7. The R_a of permanent magnet DC motor is 0.8Ω. At no load, the motor draws 1.5 A from 25V and runs at 1500 rpm.
η while operating at 1500 rpm drawing current of 3.5A from the same source will be 48%.

Gen → No Load → $I_L = 0A$
Motor → No Load → Power is developed in armature but shaft power is zero (no power to load but developed power is available)

→ The power i/p is only utilized to compensate the losses inside the motor. → No load losses.

No load power = constant = Rotational losses

Case I

No Load

$$E_{a1} = V_{t1} - I_{a1} R_a = 25 - (1.5)(0.8)$$

$$\Rightarrow \boxed{E_{a1} = 23.8 V}$$

No load power $= E_{a1} I_{a1} = 23.8 \times 1.5$
no load loss $= 35.7 W$
rotational loss.

$N_1 = 1500$ rpm

Case # $V_t = 25V, \; 3\cdot5A$.

$$\eta_m = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + losses}$$

$$\eta_m = \frac{P_{in} - Losses}{P_{in}}$$

$$\eta_m = 1 - \frac{Losses}{P_{in}}$$

$P_{in} = 3\cdot5 \times 2\cdot5 =$

$Losses = 35\cdot7\,W$ + Variable losses.

$= 35\cdot7 + (3\cdot5)^2 (0\cdot8)$ $I_a^2 R_a$

$= 35\cdot7 + 9\cdot8$

$\Rightarrow Losses = 45\cdot5\,W$

$$\Rightarrow \eta_m = 1 - \frac{45\cdot5}{8\cdot75} = 48\%$$ Ans

If by another method.

$$\eta = \frac{P_{out}}{P_{in}}$$

$P_a = P_d = E_{a2} \times I_{a2} = (V_t - I_{a2} R_a) I_{a2}$

$P_d = (25 - (3\cdot5)(0\cdot8))\,3\cdot5 = 77\cdot7\,W$

$P_{sh} = P_o = P_d -$ No load losses $= 77\cdot7 - 35\cdot7$

$= 42\,W$

$$\eta = \frac{42}{8.75} = 0.48 = 48\%$$

Q8. In a DC motor, running at 2000 rpm, the hysteresis and eddy current loss are 500 W and 200 W respectively. If the flux remains constant, the speed at which the total iron loss are halved will be <u>1140 rpm</u>.

$P_h \propto B_m^{1.6} f \qquad P_e \propto B_m^2 f^2 \qquad P_i = P_e + P_h$

$B_m = \text{constant} \Rightarrow [P_h \propto f$ and $P_e \propto f^2]$

$P_h = a f$ and $P_e = b f^2$

Case i $\quad N_1 = 2000$ rpm , $P_{i_1} = P_{e_1} + P_{h_1} = 500+200$
$= 700\,W$

Case ii

$$P_{i2} = \frac{P_{i1}}{2} = \frac{700}{2} = 350\,W$$

$$\Rightarrow \underset{= K_2 N_2^2}{P_{e_2}} + \underset{= K_1 N_2}{P_{h2}} = 350 \text{ — } (A)$$

$$N = \frac{120 f}{P} \Rightarrow N \propto f$$

$\Rightarrow P_h = K_1 N \quad , \quad P_{e2} = K_2 N^2$

$P_{h1} = K_1 N_1 \quad , \quad P_{e2} = K_2 N_1^2$

$500 = K_1 \times 2000 \quad , \quad 200 = K_2 (2000)^2$

$K_1 = \frac{1}{4} \qquad\qquad K_2 = \frac{1}{2 \times 10^4}$

Ⓐ $\Rightarrow 350 = \frac{1}{2\times10^4} N_2^2 + \frac{1}{4} N_2$

$\Rightarrow \frac{N_2^2}{2\times10^4} + \frac{N_2}{4} - 360 = 0$

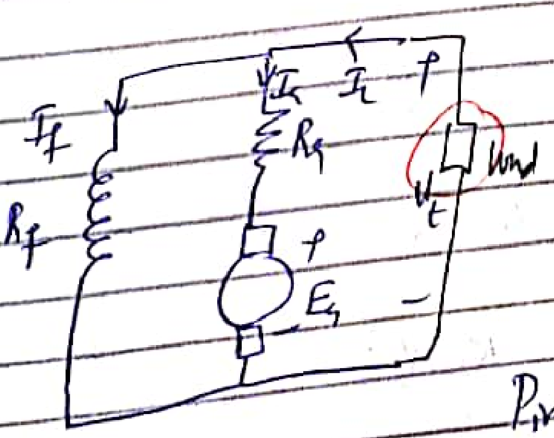
$$\Rightarrow N_2 = \frac{-\frac{1}{4} \pm \sqrt{\left(\frac{1}{4}\right)^2 + 4\left(\frac{1}{2\times10^7} \times 360\right)}}{2\times\left(\frac{1}{2\times10^4}\right)}$$

$N_2 = 1140$ rpm ✓, -6140 rpm

<u>Q9.</u> A 10KW, 250V DC shunt generator has total no load losses of 400 W. The armature circuit and shunt field resistances are 0.5 Ω and 250 Ω.

(i) Calculate the shaft power i/p and η at rated load.

(ii) Calculate the max η and corresponding power o/p.



At rated conditions;

$P_{out} = 10$ kw, $V_t = 250V$

$V_t = 250V$

$P_{NL} = 400W$.

$R_a = 0.5\Omega$, $R_f = 250\Omega$

$P_{in} = P_{out} + \text{Losses}$.

$P_{in} = 10kw + \text{No load losses} + I_a^2 R_a + I_f^2 R_f$

$P_{in} = 10kw + 400w + I_a^2 R_a + I_f^2 R_f$.

$V_t = 250\,V$ $\Rightarrow I_f = \frac{V_t}{R_f} = \frac{250}{250} = 1A.$

As $10\,KW = V_t \times I_L$

$\Rightarrow I_L = \left(\frac{10\times10^3}{250}\right) = 40A$

$I_a = I_L + I_f = 41A$

$\Rightarrow P_{in} = (10\times10^3) + 400 + (41)^2(0.5) + (1)(250)$

$\Rightarrow \boxed{P_{in} = 11490.5W = 11.49\,kW}$

↳ Electrical equivalent of mechanical.

Now

$\eta_{rated} = \frac{P_{out}}{P_{in}} = \frac{10\times10^3}{11.4\times10^3} \Rightarrow \boxed{\eta = 0.87 = 87\%}$

(ii) At maximum efficiency; constant losses = variable losses.

The i/p and o/p powers calculated will not be considered here b/c These are rated powers.

$\eta_{max} = \frac{P_{out}|_{\eta_{max}}}{P_{in}|_{\eta_{max}}} = \frac{P_{out(\eta_{max})}}{P_{out(\eta_{max})} + 2\times\text{constant losses}}$

variable loss $= I_a^2 R_a = 400 + I_f^2 R_f = 400 + 250$
$= 650$

$I_a = \sqrt{\frac{650}{R_a = 0.5}} \Rightarrow$ ~~$I_a = 65$~~

$\Rightarrow \boxed{I_a = 36.05\,A \Big|_{\eta_{max}}}$ $\Rightarrow \boxed{I_L = 35.05A}$

$$P_{out}/\eta_{max} = 250 \times 35.05 = \boxed{8763.8 W}$$

$$\eta_{max} = \frac{8763.8}{8763.8 + 2(650)}$$

$$\Rightarrow \boxed{\eta_{max} = 0.87 = 87\%}$$

I/P power at maximum efficiency;

$$P_{in} = 8763.87 + 2(650) = 10.06 kW$$

Armature Reaction

Armature is on the rotor.

The effect of armature flux (flux due to flow of I in armature conductor) on the stator flux is known as armature reaction.

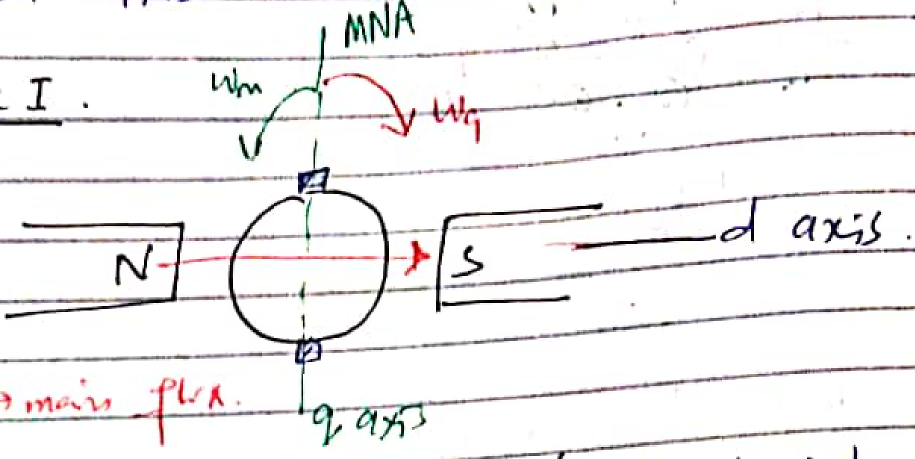
i. Cross magnetizing effect → ↓
ii. De magnetizing effect ↓ ↑

The commutation action (reversal of current) happens at the brush axis.

The axis at which induced voltage becomes zero is called magnetic neutral axis.
& here the axis of brushes.

→ Current reversal (x into •) takes place at the brush axis.

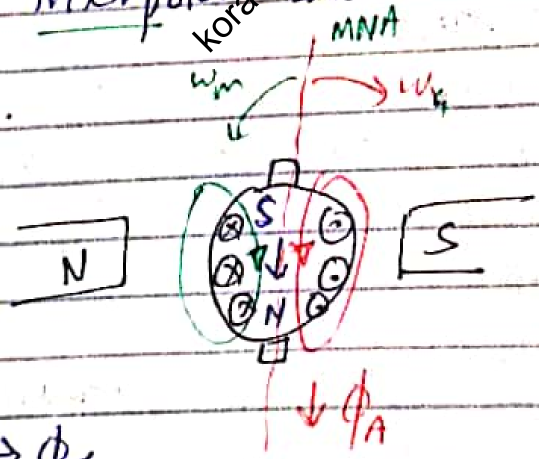
Case I.

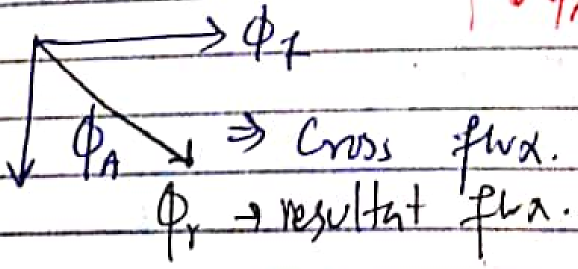


ϕ_f → main flux.

→ The axis of main flux (pole to pole axis)
→ is known as direct axis.
→ The axis perpendicular to d axis is called quadrature axis : geometrically neutral axis or interpolar axis

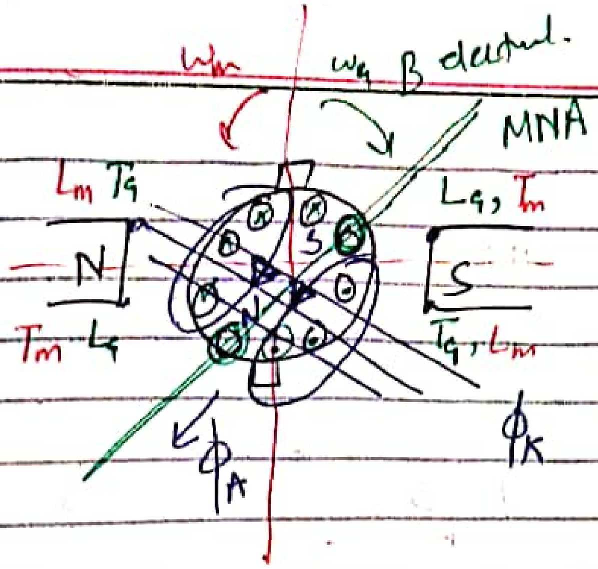
Case II.





ϕ_f = fixed, ϕ_A → depend on load.
↳ $I_a \rightarrow \phi_f$.

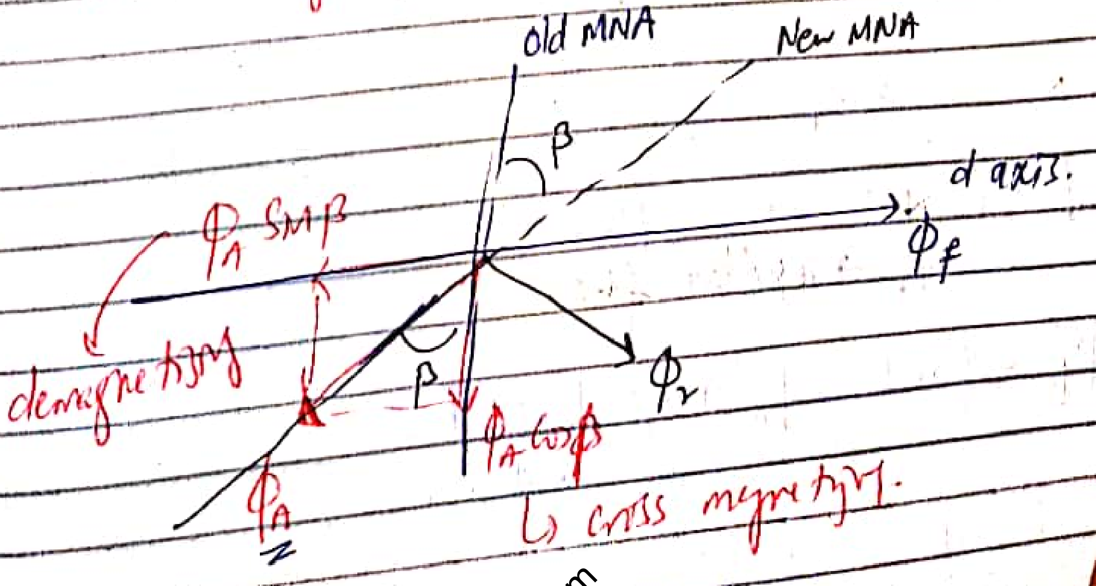
When velocity component of conductor is parallel to the magnetic lines, current is zero.



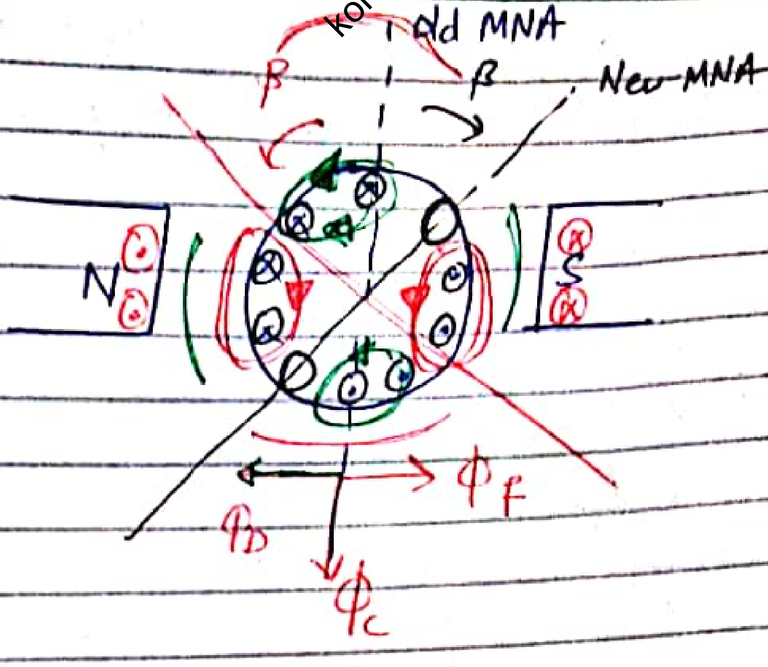
Armature reaction ⇒ Delayed commutation.
Now Brush ≠ Current reversal ⇒ sparking

Each N-S pole has two tips.
First after rotation is leading pole tip and the one to follow is trailing pole tip.

From armature reaction;
In Gen, weaking of flux will occur on the leading tips and strengthning at trailing tips.
In motor, strengthning at leading and weakning at trailing pole tips.



shifting ⇒ de magnetizing.



$P, Z, A, I_a.$ (A → no. of ∥ paths)

$$\boxed{\text{The peak value of armature mmf} = \frac{1}{2}\left[\frac{Z}{P}\cdot\frac{I_a}{A}\right]}$$

$$AT_d/\text{pole} = \frac{4\beta}{360^\circ}\left[\frac{1}{2}\,\frac{Z}{P}\cdot\frac{I_a}{A}\right]$$

$$AT_c/\text{pole} = \left(\frac{360^\circ - 4\beta}{360^\circ}\right)\frac{1}{2}\,\frac{Z}{P}\cdot\frac{I_a}{A}$$

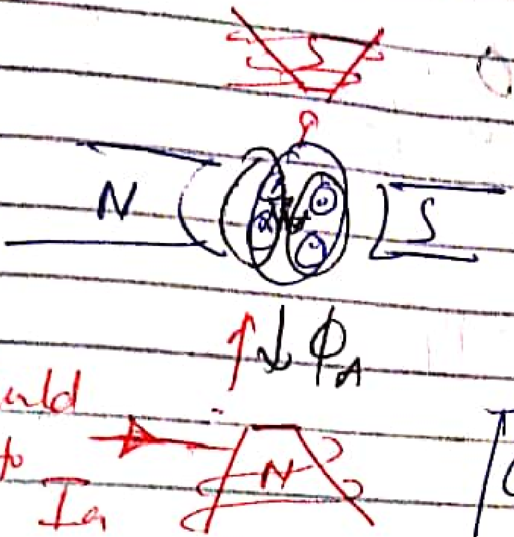
β° = electrical degree.

⇒ Compensating winding compensates for the cross magnetizing flux.
↓
dominant

$$AT_{\text{compensating}}/\text{pole} = \frac{1}{2}\left[\frac{Z}{P}\cdot\frac{I_a}{A}\right]\frac{\text{Pole arc}}{\text{pole pitch}}$$

Distance b/w two poles = 1 pole pitch

If an Interpole is used, the ϕ generated should be in opposite direction and equal to ϕ_A.



current should be equal to I_a

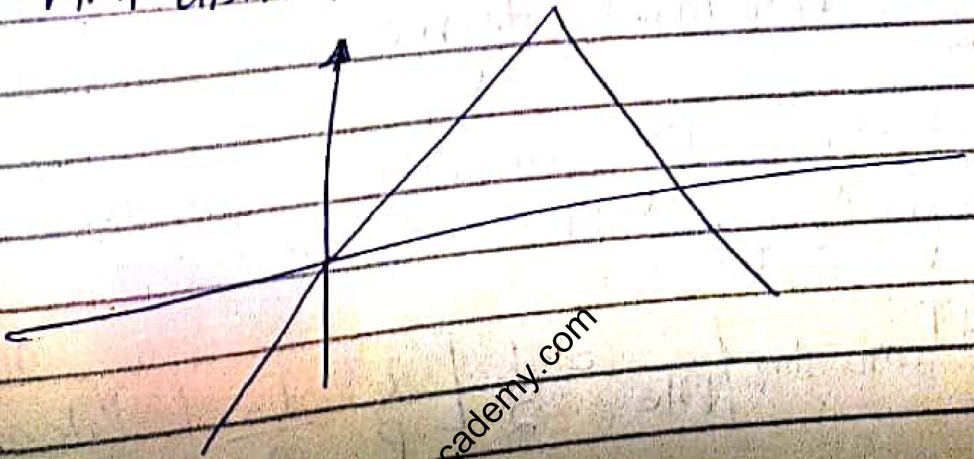
$$\boxed{\phi = \frac{1}{2}\frac{Z}{P}\frac{I_a}{A}}$$

If compensating winding is used along with interpole

$$AT_{interpole/pole} = \left(\frac{1}{2}\frac{Z}{P}\frac{I_a}{A}\right) - AT_{compensatn/pole}$$

$$\boxed{AT_{interpole/pole} = \frac{1}{2}\left(\frac{Z}{A}\frac{I_a}{P}\right)\left(1-\frac{\text{Pole arc}}{\text{Pole pitch}}\right)} + \frac{B_g l_g}{\mu}$$

MMF distribution



Q1. A compensated generator has 12000AT/pole. The ratio of the pole arc to pole pitch is 0.7. The length of interpole air gap is 1.25 cm and flux density of interpole region is 0.3T. Find AT/pole for compensating winding and interpole windings.

$$\text{Total AT/pole} = \frac{1}{2}\left[\frac{Z}{P}\cdot\frac{I_a}{A}\right] = 12000$$

$$\frac{\text{Pole arc}}{\text{Pole pitch}} = 0.7$$

Length $l_g = 1.25\,cm$, Flux density, $B_g = 0.3T$

As

$$AT_{interpole}/pole = \frac{1}{2}\left[\frac{Z}{P}\cdot\frac{I_a}{A}\right]\left[1 - \frac{\text{Pole arc}}{\text{Pole pitch}}\right] + \frac{B_g l_g}{\mu}$$

$$= (12000)(1-0.7) + \frac{(0.3)(1.25\times10^{-2})}{4\pi\times10^{-7}} \quad \text{for air}$$

$$= 6584.155\ AT.$$

$$AT_{compensating}/pole = 12000 - 6584.155$$
$$= 5415.84\ AT. \quad ✗$$

$$AT_{compensating}/pole = \frac{1}{2}\left[\frac{Z}{P}\cdot\frac{I_a}{A}\right]\left[\frac{\text{Pole arc}}{\text{Pole pitch}}\right]$$
$$= 8400. ✓$$

B/c $\frac{B_g l_g}{\mu}$ is not part of armature reaction. It is the reactance voltage (commutation part).

Q2. A 250V, 14.9kW, 8 pole DC machine has single turn coil. The armature is wave wound with 94 commutator segments. The motor effiency is 80%. Brushes are shifted by 2 segments at full load.
Calulate the cross magnetizing, demagnetizing and total AT.

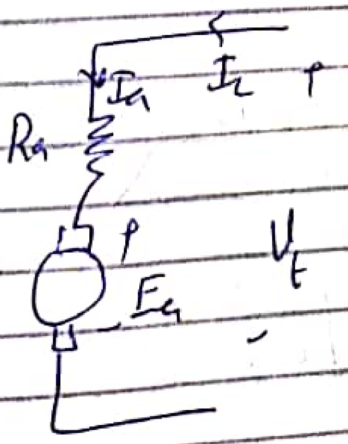
$A = 2$ $\quad V_t = 250V$, $P_{out(mech)} = 14.9kW$

$$\eta_{motor} = \frac{P_{out}}{P_{in}} = 0.8$$

$$\Rightarrow P_{in} = \frac{P_{out}}{0.8} = \frac{14.9\times10^3}{0.8} = 18.625 kW$$

Losses = o/p - i/p

By default separately excited;



$P_{in} = V_t \times I_a$

$18.625 = 250 I_a$

$\Rightarrow I_a = 74.5A$

$$AT_d/pole = \frac{4\beta}{360^\circ}\left[\frac{Z}{P}\frac{I_a}{A}\right]\left(\frac{1}{2}\right) \quad Ⓐ$$

$$AT_c/pole = \frac{360^\circ - 4\beta}{360^\circ}\left(\frac{1}{2}\right)\left[\frac{Z}{P}\frac{I_a}{A}\right] \quad Ⓑ$$

where $\frac{1}{2}\left(\frac{Z}{P}\right)\left(\frac{I_a}{A}\right)$ is the total AT (peak value)

$$AT = \frac{1}{2}\left[\frac{Z}{P}\;\frac{I_a}{A}\right] = \frac{1}{2}\left(\frac{Z}{8}\right)\left(\frac{74.5}{2}\right) \quad — ①$$

In DC machine;
No. of coils = No. of commutator segments = No. of slots

⇒ No. of coil = 94

⇒ $Z = 94 \times 2 = 188$

① ⇒ $AT = 437.80$

Now $\beta = ?$

94 slots (mech angle) = 360°

⇒ 1 slot (mech) = $360°/94 = 3.829°$

2 segment shift ⇒ $\beta_{mech} = 2 \times 3.829$
$= 7.659$

$\beta°(\text{elect}) = P/2\,(\beta°\text{ mech}) = \frac{8}{2} \times 7.659$

$\beta = 30.638°$

Ⓐ ⇒ $AT_a/\text{pole} = \frac{4(30.6)}{360°}(437.80) = 149.04\text{ AT}$

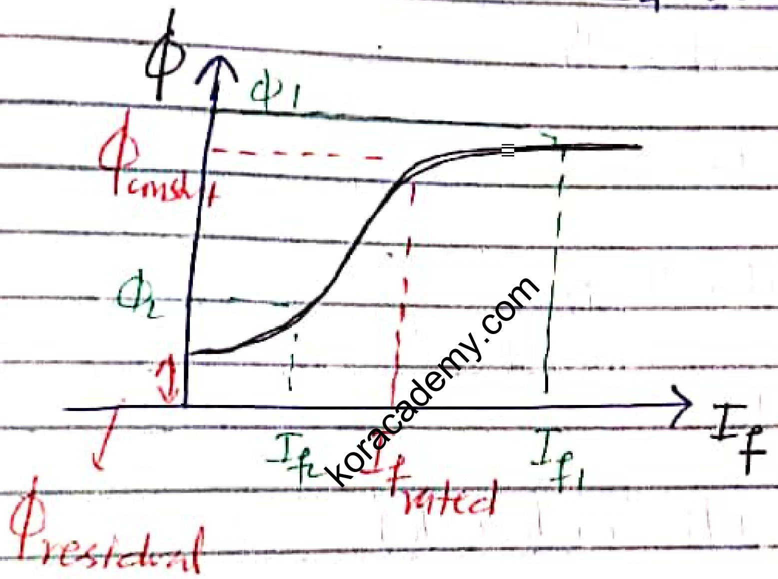
Ⓑ ⇒ $AT_c/\text{pole} = 288.76\text{ AT}$

No Load Characteristics of DC Machine

(ϕ vs I_f) also known as Magnetization characteristics or Saturation Xtics or Open Circuit Xtics.

let induced emf, $E_a = k\phi\omega_r$.

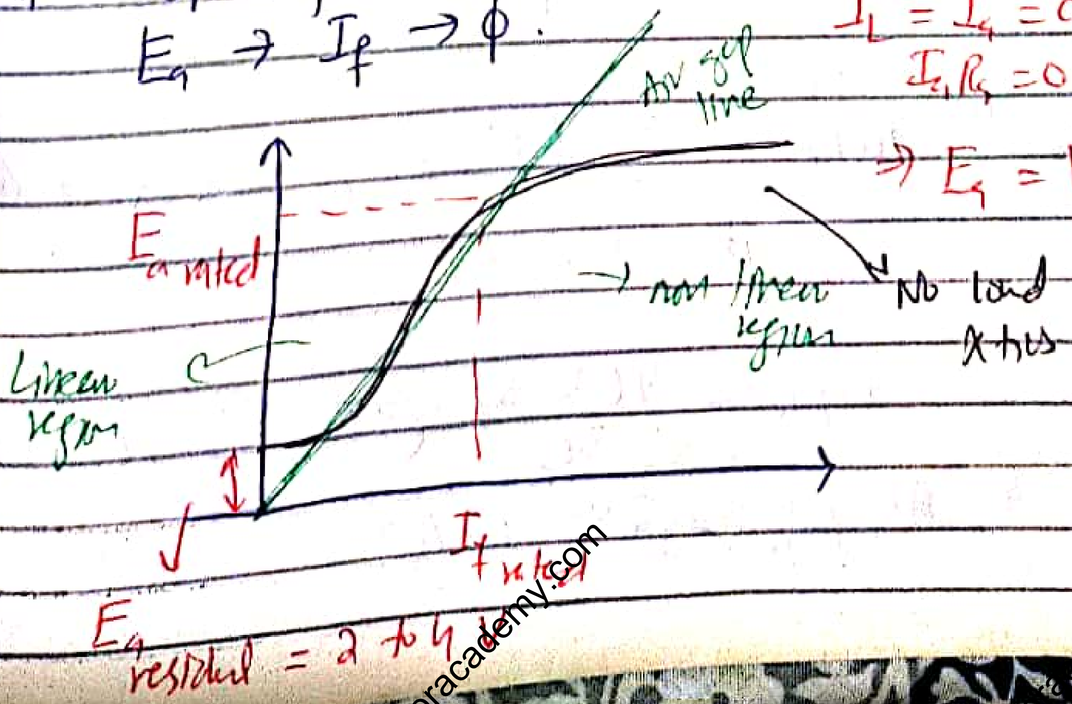
If ω_r = constant $\Rightarrow E_a \propto \phi$



weaking is very stronger than strengthing effect.

* ω_r = fixed.
$E_a \rightarrow I_f \rightarrow \phi$.

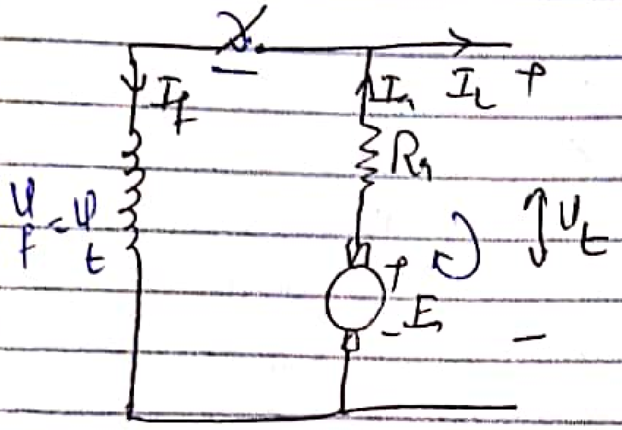
No load $\Rightarrow$
$I_L = I_a = 0$
$I_a R_a = 0$
$\Rightarrow E_a = V_t$

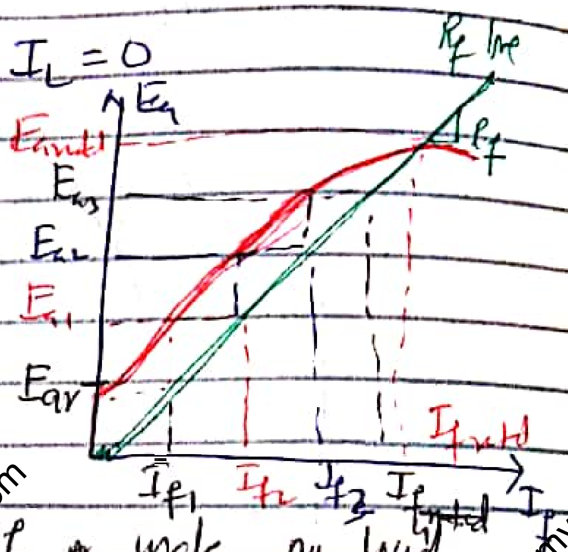


Air does not saturate. Self excited = residual flux

Voltage Build up In shunt Generator (self excited)

There should be a driving force to operate the machine.





Case i) when switch is off under no load condition

$E_a = E_{ar}$ = residual voltage

Switch is closed;

$$E_a = V_t + I_a R_a$$

↳ neglect → as smaller current due to no load

$$\rightarrow \boxed{E_a = V_t = V_f = I_f R_f}$$

$I_f = \dfrac{E_a}{R_f}$. As the switch is closed, the residual E_{ar} will flow a current I_{f1}.

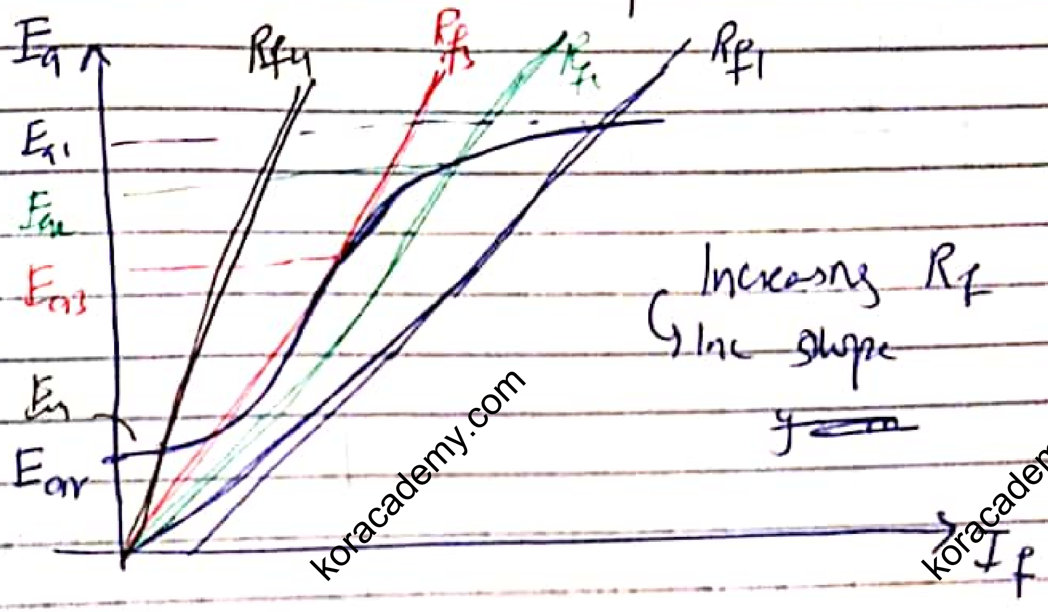
$E_a \propto \phi$ ↳ so induced emf increases.

After $I_{f\,rated}$, core is saturated.

The point when Xtics cut the R_f line is the operating point (rated V and I)

Case I

ω_r = constant. R_f = variable.



Increasing R_f
↳ Inc slope

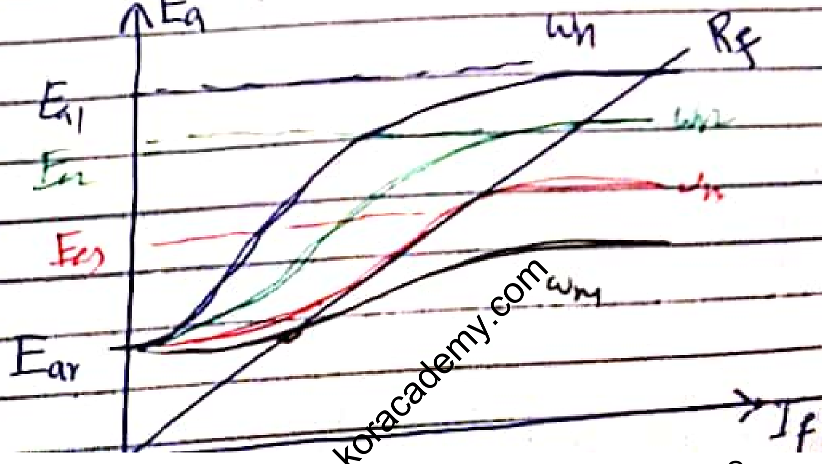
$\uparrow m = \frac{y}{x \downarrow}$ $\uparrow$

$R_{f4} > R_{f3} > R_{f2} > R_{f1}$

↓ critical

→ $\boxed{R_f < R_{f\,critical}}$ we will have build up process.

Case II R_f = constant ω_r = variable.



At ω_{r4} build up process will not be possible

$\omega_{r3} \rightarrow$ critical speed.

$\omega_{r4} < \omega_{r3} < \omega_{r2} < \omega_{r1}$

$\boxed{\omega_r > \omega_{r\,critical}}$ for build up process

① Speed Reversed.

$E_a \rightarrow \ominus$ No build up

$\phi \rightarrow \oplus$

② Flux reversed

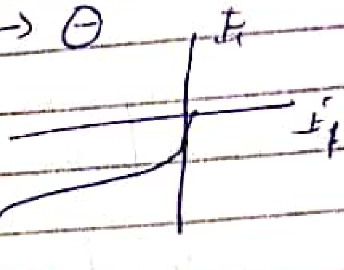
$\phi \rightarrow \ominus$ Still no build up.

$E_a \rightarrow \oplus$

③ -ve I_f and speed reversed.

$\phi \rightarrow \ominus$, $\omega_r \rightarrow \ominus$

$I_f \circlearrowright \overset{-}{\underset{+}{E_a}}$



$\boxed{R_f \text{ critical} \propto N}$

As $E_a = K\phi\omega_r \approx V_t$

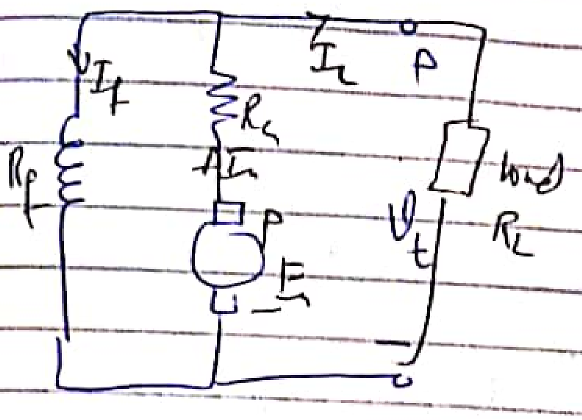
↳ as under no load condition neglected armature

$I_f R_f = K\phi\omega_r$

$$\boxed{\begin{array}{l} R_f \propto \omega_r \\ \Rightarrow R_f \propto N \end{array}}$$

External Xtics → V_t vs I_L → user

Internal Xtics → E_a vs I_a → manufacturer.



$$I_f = \frac{V_t}{R_f}$$

$$E_a = V_t + I_a R_a .$$

$$\uparrow I_L = \frac{V_t}{\downarrow R_L} \qquad \uparrow I_a = \uparrow I_L + I_f$$

$$\downarrow V_t = E_a - \uparrow I_a R_a \qquad \downarrow I_f = \frac{\downarrow V_t}{R_f} \Rightarrow \phi \downarrow$$

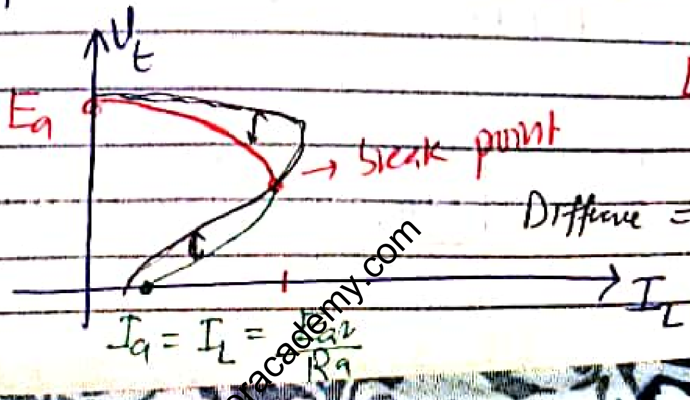
$$\downarrow E_a \propto \phi \downarrow \qquad \downarrow\downarrow V_t = \downarrow E_a - \uparrow I_a R_a$$

$$\downarrow\uparrow I_L = \frac{\downarrow V_t}{\downarrow R_L}$$ ↓ → dominating.

?

⇒ $I_a \uparrow \rightarrow \phi \downarrow$

↳ armature reaction also increases → reduce the flux more ⇒ $E \downarrow \Rightarrow V_t \downarrow \Rightarrow I_L \downarrow$

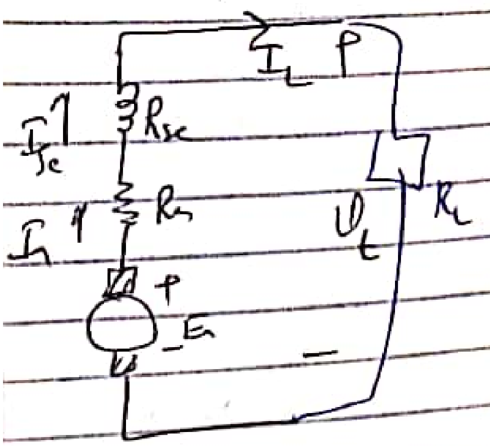


External xtics

Internal

Difference = $I_a R_a$ drop

Voltage Build Up In DC Series Generator.



$I_a = I_L = I_{se}$

$V_t = I_L R_L$

$E_a = V_t + I_a (R_a + R_{se})$

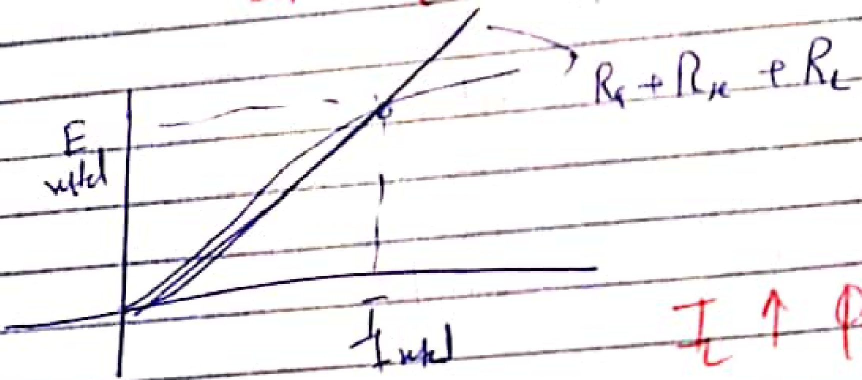
$E_a = I_L R_L + I_a (R_a + R_{se})$

$$\boxed{E_a = I_L (R_a + R_{se} + R_L)}$$

$$\boxed{I_L = I_a = I_{se} = \frac{E_a}{R_a + R_{se} + R_L}}$$

No load ⇒ $I_L = 0 = I_{se} = I_a$

$E_a = V_t = E_{ar}$

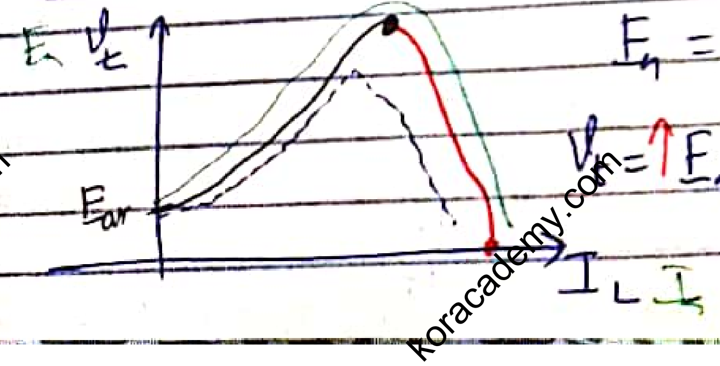


$I_L \uparrow \ \phi_{se} \uparrow \ E_a \uparrow$

Internal Xtics

Extenal Xtics

If armature rxn neglected



Booster region

$E_a = V_t + I_a (R_a + R_{se})$

$V_t = \uparrow E_a - \uparrow I_a (R_a + R_{se})$

$\downarrow$

$= I_L$

Value of R_{se} is small b/c it has to carry the load current!

R_{sh} is larger b/c it carries field current which is smaller.

where $V_t \approx$

↳

Synchronous generator is used as a booster in transmission line.

The drop $I_a(R_a + R_{se})$ is not included in internal chars.

Speed Control of DC Motor

$$E_a = K\phi\omega_r \quad , \quad T = K\phi I_a$$

$$E_a = V_t - I_a R_a .$$

$$K\phi\omega_r = V_t - I_a R_a$$

$$\boxed{\omega_r = \frac{V_t - I_a R_a}{K\phi}}$$

i. Flux control.
ii. Armature control
iii. Voltage control.

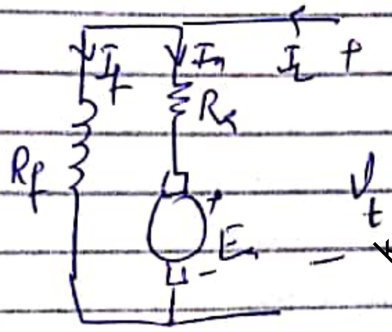
$$T_{load} = \text{constant}$$

As $T_e - T_L = J\,\frac{d\omega}{dt}$

At steady state; $\boxed{T_e = T_L}$

Base speed = Rated speed → Name plate

Q1. A 220V, 15kw, 1000rpm shunt motor with $R_a = 0.25A$ has rated line current of 68A and rated field current of 2.2A. The change in field flux required to obtain a speed of 1600rpm while drawing current of 52.8A and field current of 1.8A.



$I_{L1} = 68A$, $I_{f1} = 2.2A$

$N_1 = 1000$ rpm

$R_a = 0.25$, $V_t = 220V$

$N_2 = 1600$, $I_{L2} = 52.8$ $I_{f2} = 1.8$

As $E_a = k\phi\omega_r$.

$$\boxed{\frac{E_{a2}}{E_{a1}} = \frac{\phi_2}{\phi_1} \times \frac{N_2}{N_1}} \quad — Ⓐ$$

$$E_{a1} = V_t - I_{a1}R_a = 220 - (68-2.2)(0.25)$$
$$= 203.55V.$$

$$E_{a2} = V_t - I_{a2}R_a = 220 - (52.8-1.8)(0.25)$$
$$= 207.25V$$

Ⓐ $\Rightarrow \frac{207.25}{203.35} = \frac{\phi_2}{\phi_1} \times \frac{1600}{1000} \Rightarrow \frac{\phi_2}{\phi_1} = 0.6363$

$$\text{Chage} = \frac{\Phi_2 - \Phi_1}{\Phi_1} = \frac{\Phi_2}{\Phi_1} - 1$$

$$= 0.6363 - 1 \quad = -0.36 \quad = -36.36\,\%$$

↓
Value

TRANSFORMER

mmf = No. of turns x current (AT)

Flux, $\phi(t) = \frac{mmf}{reluctance}$ (wb)

Reluctance, $R = \frac{l}{\mu A}$

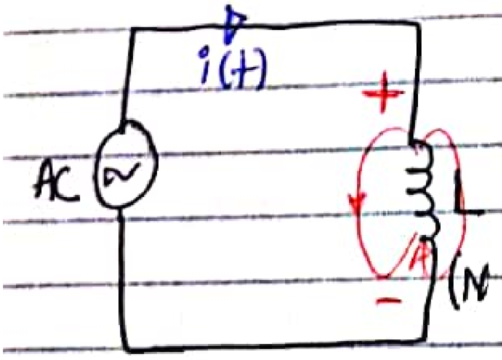
$\mu \rightarrow$ permeability $\sigma \rightarrow$ conductivity

Flux density, $B = \mu H$

[mmf ⇔ emf flux ⇔ current
reluctance ⇔ resistance]

Case 1

Single coil excited with AC supply.
L → inductance, N → no. of trn.



* AC voltage → i(t) → mmf = N·i(t)

$\phi(t) = \frac{Ni(t)}{R}$ ←

↳ induced voltage $e(t) = N\frac{d\phi}{dt}$

Say $i(t) = I_m \sin\omega t$

$\Rightarrow mmf = N \cdot I_m \sin\omega t$

$\Rightarrow \phi(t) = \frac{N \cdot I_m \sin\omega t}{R} = \phi_m \sin\omega t$

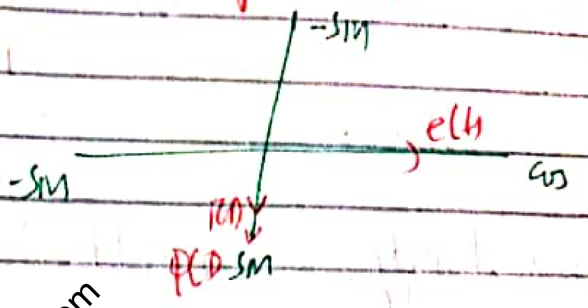
↳ Flux produced will be a function of current

$$\Rightarrow e(t) = N \times \frac{d}{dt}(\phi_m \sin wt)$$

$$\Rightarrow e(t) = N \phi_m w \cos wt = E_{max} \cos wt$$

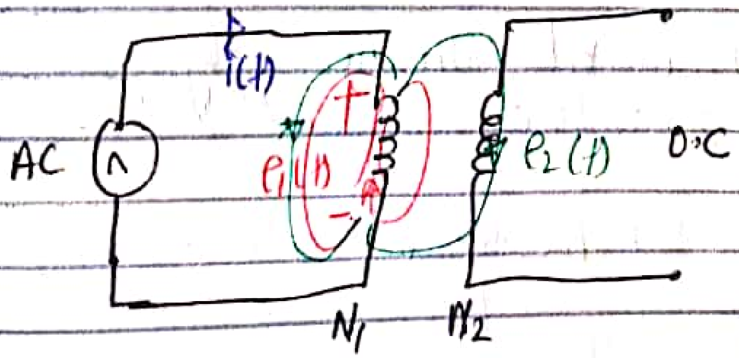
↳ self induced voltage.

In inductor, current always laggs voltage by 90°.



Case ii

when two coils are placed close to each other.



AC supply → i(t) → N_1 i(t) → $\phi(t) = \frac{N_1 i(t)}{R}$

$e_1(t) = N_1 \frac{d\phi}{dt}$ ←

Say $i(t) = I_m \sin wt$

$$e_1(t) = N_1 \times \frac{d}{dt}\left(\frac{N_1 I_m \sin wt}{R}\right)$$

$$e_1(t) = N_1 \phi_m \omega \cos\omega t = E_{1max} \cos\omega t$$

All the flux of N_1 will ~~cut~~ link the N_1 coil and also some part of it will cut N_2.

Total flux $\phi(t) = \phi_m \sin\omega t$

Say the flux linking 2 due 1 is;

$$\phi_{12} = \underset{\substack{\downarrow \\ \text{some percentage}}}{K} \phi_m \sin\omega t \qquad K < 1$$

$$\Rightarrow e_{12}(t) = N_2 \frac{d}{dt}(K\phi_m \sin\omega t)$$

$$\Rightarrow e_{12}(t) = K N_2 \omega \phi_m \cos\omega t$$

↳ mutually induced emf (due to another flux)

The polarity of $e_2(t)$ will be decided when we place a load.

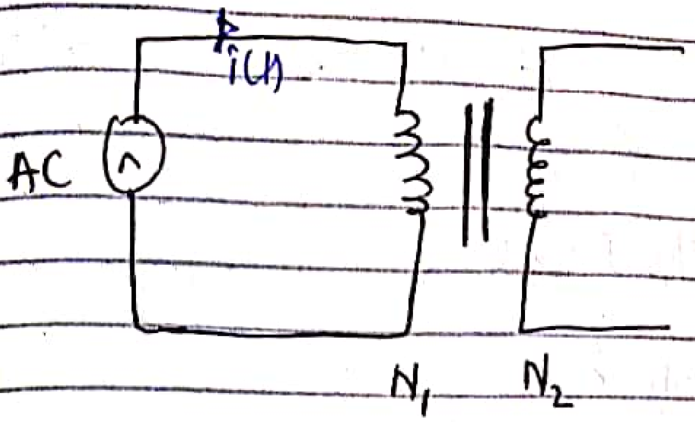
The ratio; $\dfrac{e_1(t)}{e_2(t)} = \dfrac{N_1}{KN_2}$

↳ This we are not studying transformer, we are simply talking of two coils placed together

Permeability of air is very low → restrict the flow of flux.

Case iii

Two coils placed on a core (transformer action).



Say $i(t) = I_m \sin\omega t$

$\phi(t) = \phi_m \sin\omega t$

$e_1(t) = N_1 \omega \phi_m \cos\omega t$

If the core material (||) is ideal, all the flux created in 1 will link coil 2.

A $\phi_{12}(t) = k \phi_m \sin\omega t$

Reluctance → 0 ⇐ Permeability → ∞

↳ All the flux will link coil 2 → $k = 1$

$\Rightarrow \phi_{12}(t) = \phi_m \sin\omega t$

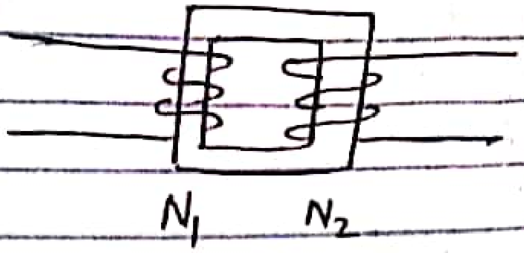
$\Rightarrow e_{12}(t) = e_2(t) = N_2 \frac{d}{dt} \phi_{12}(t) = N_2 \omega \phi_m \cos\omega t$

Now transformer turn ratio;

$$\boxed{\frac{e_1(t)}{e_2(t)} = \frac{N_1}{N_2}}$$

Ideal Transformer

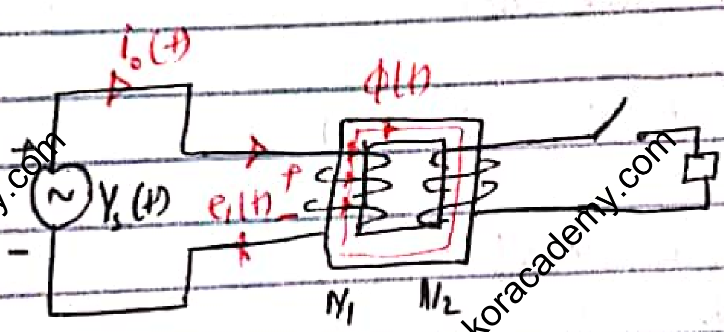
- → permeability of core tends to ∞.
  - → $R \to 0$ $R = \frac{l}{\mu A}$
- → No losses
  - → winding losses
  - → core losses



- → Leakage flux = 0, $K=1$
- → Under no load condition, current supplied by source will be zero.
  - ↳ but we will have flux.

Working

Case i Under no load.



Sw → open.

$\mu \to \infty$, $R \to 0$

$i(t) = I_m \sin\omega t$

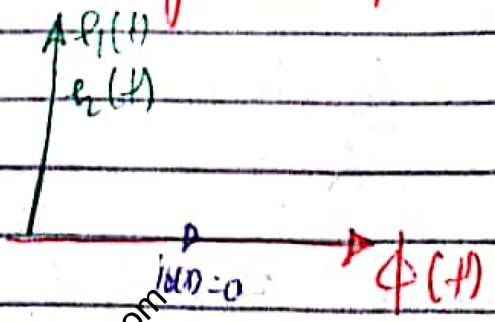
$\phi(t) = \phi_m \sin\omega t$

$$e_1(t) = N_1 \frac{d}{dt}\phi(t)$$

$e_1(t) = N_1 \omega \phi_m \cos\omega t$ → self induced

and $e_2(t) = N_2 \omega \phi_m \cos\omega t$ → mutually induced

$$\text{Ratio} = \frac{e_1(t)}{e_2(t)} = \frac{N_1}{N_2}$$ → turn ratio → only valid for ideal transformer



$N_1 > N_2$ → step down

$N_1 < N_2$ → step up

$N_1 = N_2$ → Isolation transformer

Case ii Under loaded condition

Xformer is a constant flux device
↳ The flux at loaded condition will be the same as no load condition.

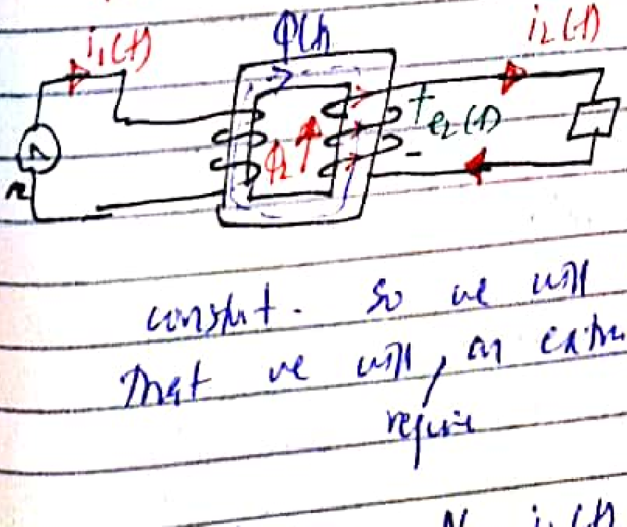
$$\phi(t) = \phi_m \sin \omega t$$

When switch is closed, current will start flowing through the load ($i_2(t)$).

↳ current has started flow in the winding N_2.
The direction should be such that it opposes its cause.

i_2 is developed due to $e_2 \rightarrow \phi(t)$.

The current direction in N_2 should be such that it oppose the flux $\phi(t)$.



The flux created due to I_2 will oppose the flux ϕ. (i.e. reduce it)
But we have to keep ϕ constant. So we will need to increase it. And for that we will require an extra current from the primary side.

$$\phi_2(t) = \frac{N_2 \cdot i_2(t)}{R}$$

$$\text{mmf} = N_1 i_1(t) = N_1 i_0(t) + N_2 i_2(t)$$

mmf balance equation

As per q-

$\mu \to \infty$ $R = \frac{l}{\mu A} \Rightarrow R \to 0$

$\phi(t) = \frac{N i(t)}{R_{\to 0}}$ and ϕ is finite $\Rightarrow i(t) \to 0$ $\frac{\to 0}{\to 0}$ = finite

As $i_0(t) = 0$

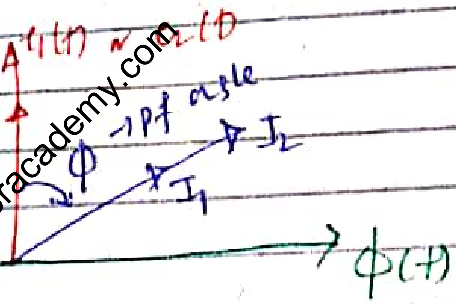
$$\Rightarrow \boxed{N_1 I_1 = N_2 I_2}$$

Ratio; $\boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1}}$ → I_1 and I_2 are in the same phase.

Polarity of $e_2(t)$ according to Lenz's law : the current will flow in such a direction that it oppose the flux ϕ_2.

(if decreasing → leave at + and enter at −)

say for a lagging load.



→ If permeability (μ) is finite (very high)

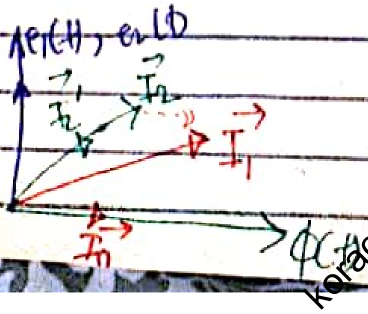
↳ I_0 will have some value (very small)

balance equation;

$$\Rightarrow N_1 I_1 = N_1 I_0 + N_2 I_2$$

$$\Rightarrow \boxed{I_1 = I_0 + \frac{N_2}{N_1} I_2}$$

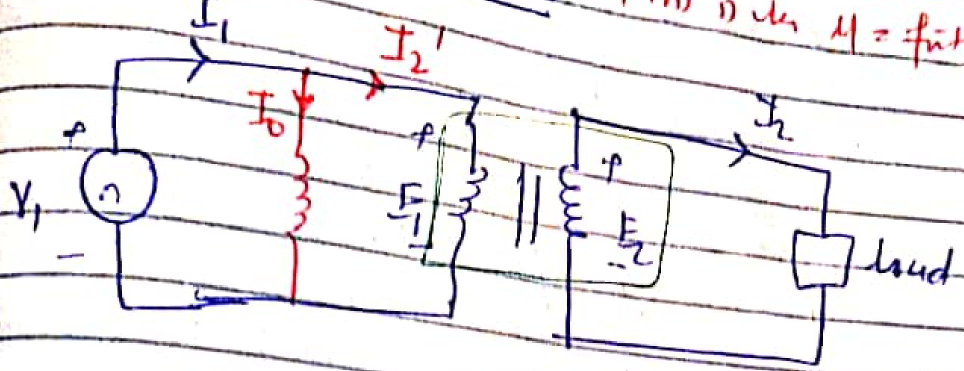
If $a = \frac{N_1}{N_2}$ $\Rightarrow I_1 = I_0 + \frac{I_2}{a} = I_0 + I_1'$



Here $\boxed{\frac{I_1}{I} = \frac{N_2}{N_1}}$ ✗

and $\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2}}$ ✓

Equivalent circuit; + TM is when μ = finite.



$$\vec{I_1} = \vec{I_0} + \vec{I_2'}$$ → so parallel circuit.

for infinite, $I_0 = 0$ ⇒ $I_1 = I_2' = I_2/a$

Current relation is valid for TM' one only;

ie $\frac{I_2'}{I_2} = \frac{1}{a}$

$e_1(t) = N_1 \omega \phi_m \cos\omega t$

$$e_1(rms) = \frac{(N_1 \times 2\pi f \times \phi_m)}{\sqrt{2}} \cos\omega t$$

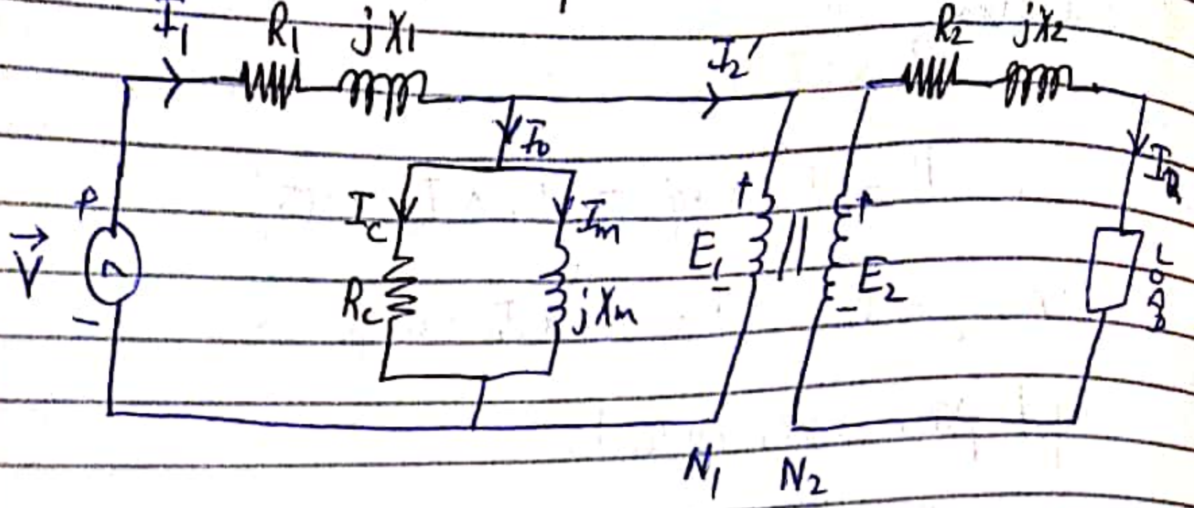
$$\boxed{\begin{aligned} e_1(rms) &= 4.44 f \phi_m N_1 \\ \text{and } e_2(rms) &= 4.44 f \phi_m N_2 \end{aligned}}$$

E_1 and E_2 will always be in the same phase.

I_2 and I_2' will always be in the same phase

↳ secondary current referred to primary side

Practical Transformer



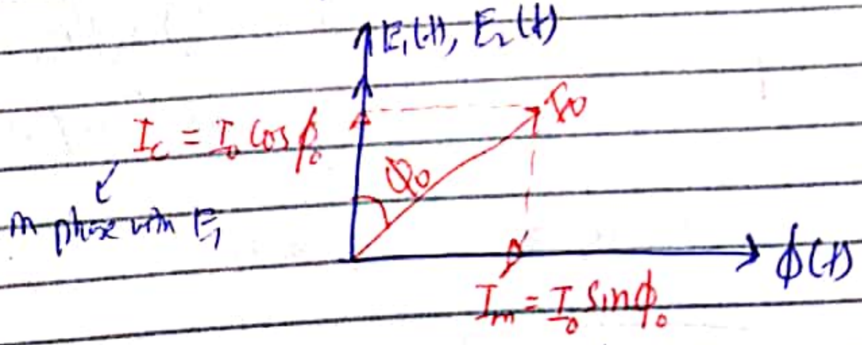
Case i No Load condition

$I_2 = 0A \Rightarrow I_2' = 0A \Rightarrow$ ~~I_0~~ I_c

$\Rightarrow I_1 = I_0 = I_m + I_c$ = no load current.
(2 to 5% of full load current)

Similarly $\frac{E_1}{E_2} = \frac{N_1}{N_2} = a$

current only in the core., this will develop a flux
$\phi_m \sin\omega t$ → no load flux



↳ inductor → will lagg from E_1 by 90°.

$I_0 = \sqrt{I_c^2 + I_m^2}$, $\bar{I_0} = \bar{I_c} + \bar{I_m}$

Core loss component $= R_c$

Core losses;

$$P_c = I_c^2 R_c$$

Ⓐ $P_c = VI\cos\phi = E_1 \times I_c \times \cos 0°$

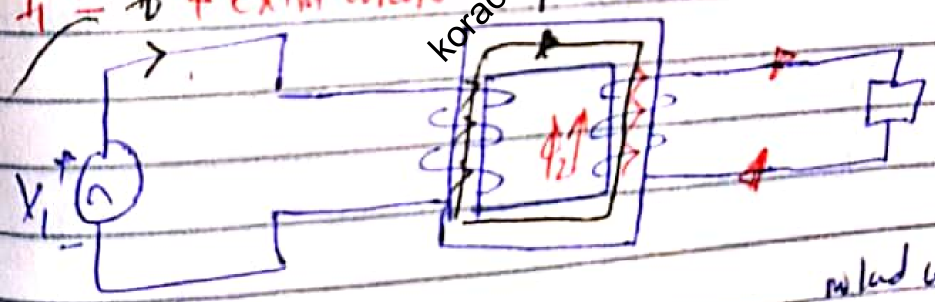
Ⓑ $P_c = VI\cos\phi = E_1 \times I_0 \times \cos\phi_0$

Input power under no load;

$$P_{in} = V_1 \times \underset{\hookrightarrow = I_0}{I_1} \times \cos\underset{\hookrightarrow \text{ angle b/w } V_1 \text{ and } I_0}{\phi_1}$$

Case ii Under loaded condition

$I_1 = I_0 +$ extra current $\phi \propto I_m$



$\rightarrow \underset{\substack{\downarrow \\ \text{core loss}}}{I_c}$ and $\underset{\substack{\downarrow \\ \phi}}{I_m}$

no load condition

mmf. $\underset{\hookrightarrow \text{total mmf develop at primary side}}{N_1 I_1} = \overset{?}{N_1 I_0} + \underset{\hookrightarrow \text{extra mmf due to load.}}{N_2 I_2}$

$$I_1 = I_0 + \frac{N_2}{N_1} I_2 = I_0 + \frac{I_2}{a}$$

$$\boxed{\bar{I_1} = \bar{I_0} + \bar{I_2}'}$$

The equations that we can deduce are;

i. $\overline{V_1} = I_1R_1 + jI_1X_1 + \overline{E_1}$

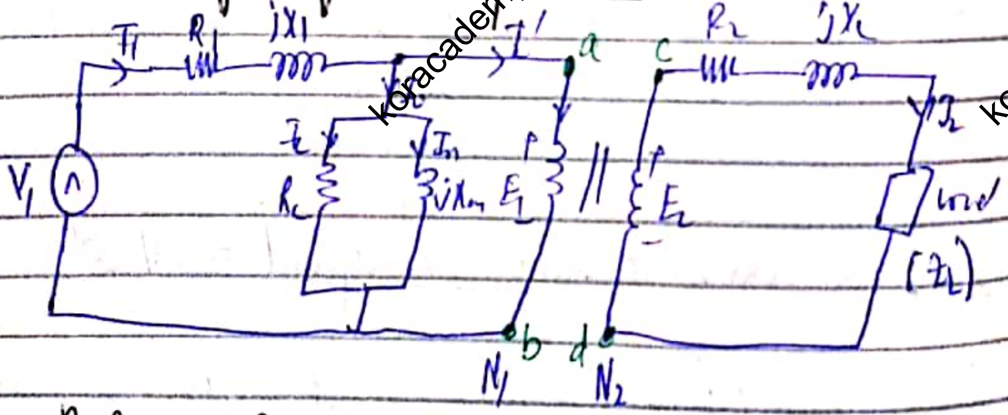
ii. $\overline{E_2} = \overline{V_2} + I_2R_2 + jI_2X_2$

iii. $\frac{E_1}{E_2} = a$ and $\frac{I_2'}{I_2} = \frac{1}{a}$

iv. $\overline{I_1} = \overline{I_0} + \overline{I_2}'$

v. $I_0 = \overline{I_c} + \overline{I_m}$

Modelling of Transformer



Referenced Circuit

$a = \frac{N_1}{N_2} = \frac{E_1}{E_2}$

↳ turn ratio is always defined from primary to secondary

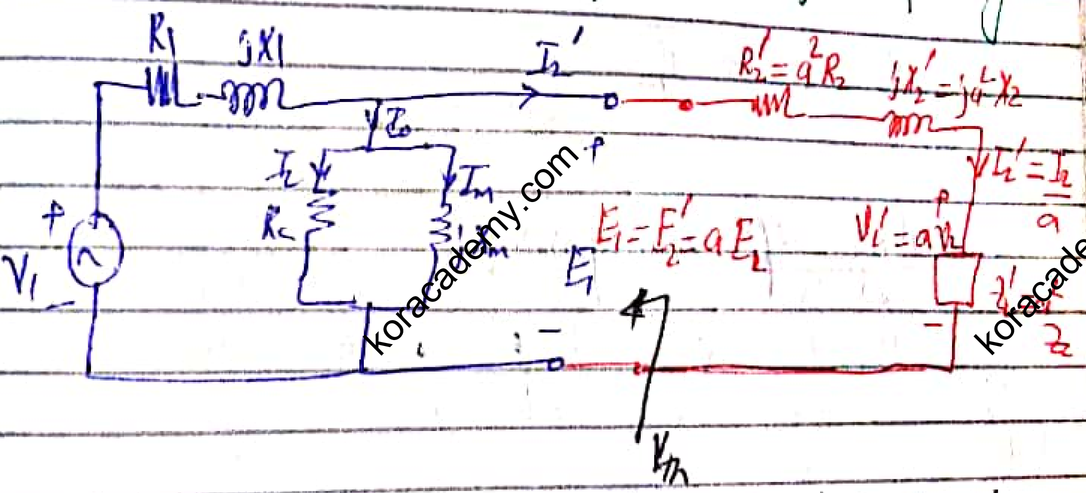
HV	LV
V ↑	V ↓ → ×a or $\frac{1}{a}$
I ↓	I ↑ → ×a or 1/a
Ω ↑	Ω ↑ → ×a^2 or $1/a^2$

Say primary is HV and secondary is LV ⇒ $\frac{N_1}{N_2} > 1$
⇒ $a > 1$ ⇒ step down transformer.

(i) Transformer equivalent circuit referred to HV side

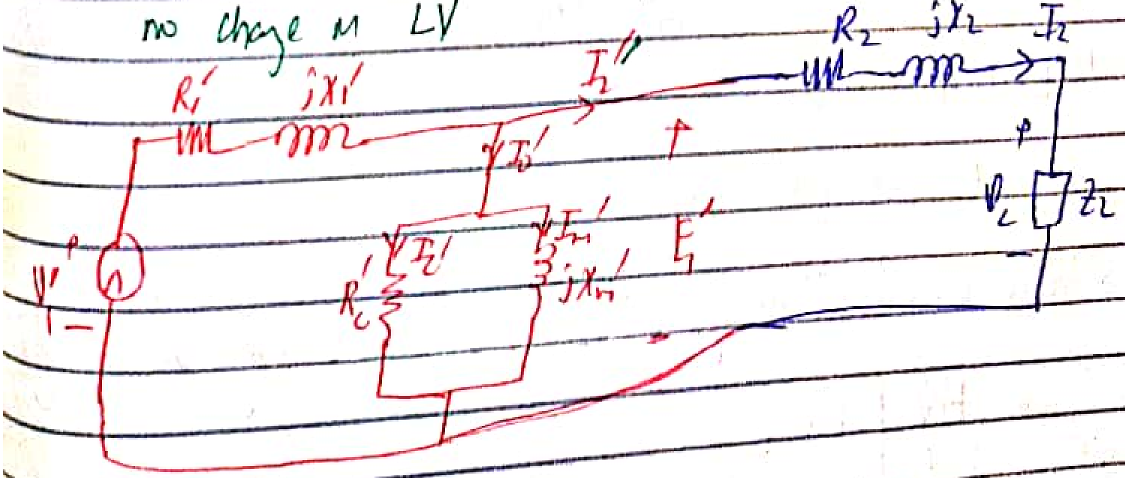
means I need all the circuit at HV side ie all the secondary data needs to be on the primary side.

All the changes will be in secondary and no changes in primary.



ii. Transformer equivalent circuit when referred to LV side

no change in LV



$V_1' = \frac{V_1}{a}$, $E_1' = \frac{E_1}{a} = E_2$, $R_1' = \frac{R_1}{a^2}$, $jX_1' = j\frac{X_1}{a^2}$

$R_c' = \frac{R_c}{a^2}$, $jX_m' = j\frac{X_m}{a^2}$

$I_1' = aI_1,\ I_c' = aI_c,\ I_m' = aI_m,\ I_0' = aI_0$

$I_2'' = I_2'a = \left(\frac{I_2}{a}\right)a \Rightarrow I_2'' = I_2$

Approximate equivalent circuit

Consider the transformer equivalent circuit referred to HV side.

If I need to find V_{Th} (as mentioned).

of course $V_{Th} \neq E_1$ b/c E_1 is the voltage when load 2 is connected but for V_{Th} we make open circuit.

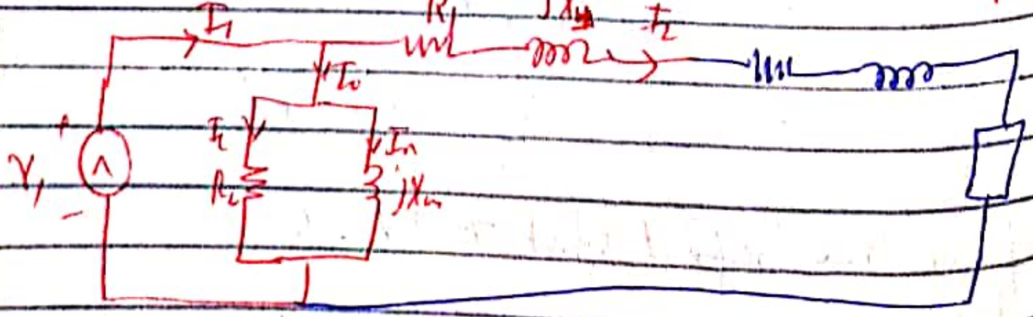
say $Z_1 = R_1 + jX_1$, $Z_{sh} = \frac{R_c \times jX_m}{R_c + jX_m}$

from voltage divider rule,

$$\boxed{V_{Th} = \frac{V_1 \times Z_{sh}}{Z_{sh} + Z_1}} \qquad \boxed{V_{Th} \approx V_1}$$

↳ drop across Z_1 is negligible

So we can connect the shunt branch across the supply.



↳ **First approximated equivalent circuit**

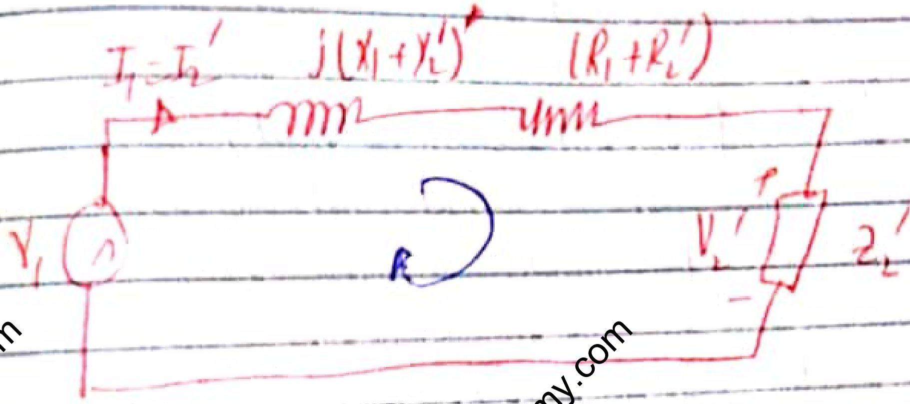
Now;

As The value of I_0 is only 2 to 5% of full load current.

↳ negligibly small

↳ so we can neglect the shunt Z.

The final circuit will be;



↳ Second / Final approximated model of single phase practical transformer

The equations will be as;

$$V_1 = V_2' + I_2'(R_1 + R_2') + j\,I_2'(X_1 + X_2')$$

say $Z_{eq} = R_{eq} + j\,X_{eq}$

$R_{eq} = R_1 + R_2'$, $X_{eq} = X_1 + X_2'$

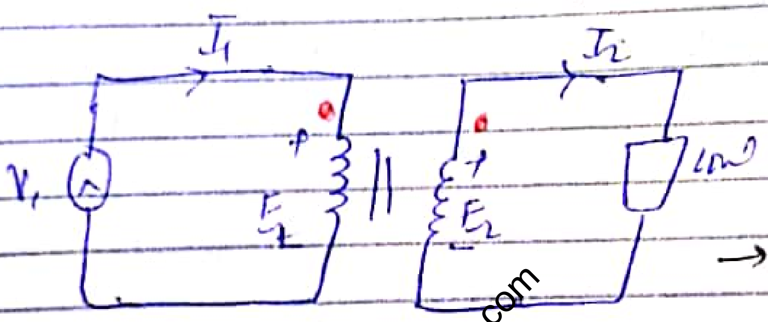
If we have referred circuit to LV side;

we have; V_1', R_1', R_2, X_1', X_2, V_2, Z_L

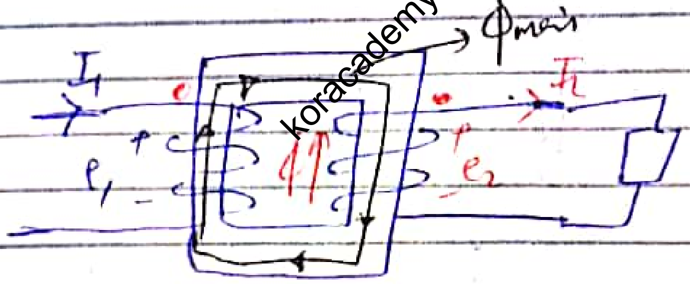
Dot Convention

$+ = \cdot$

If the current enters at dot in the primary and leaves at dot in the secondary, then flux will be opposing in nature → this is the transformer principle



→ reference circuit

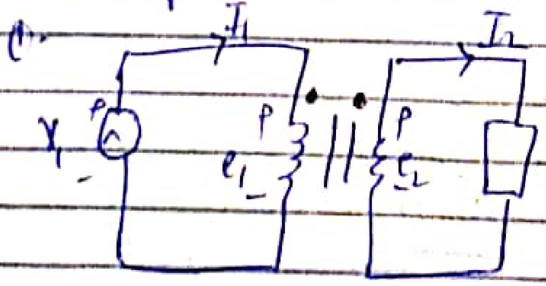


$$\frac{E_1}{E_2} = a$$

$$\frac{I_1}{I_2} = \frac{1}{a}$$

If current enters at both the dots, they will be additive in nature and this will not be in accordance with Lenz law and hence will not obey transformer principle

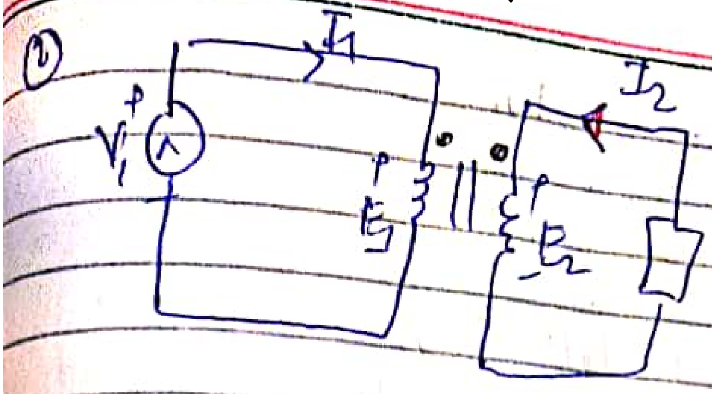
Examples



$$\frac{E_1}{E_2} = a \quad (+ve)$$

$$\frac{I_1}{I_2} = \frac{1}{a} \quad (+ve)$$

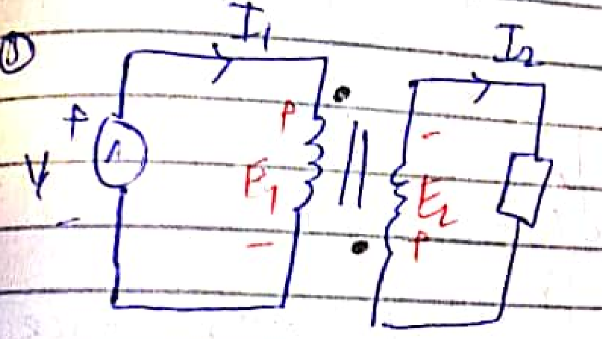
②



$$\frac{E_1}{E_2} = a \quad (\text{cel})$$

$$\frac{I_1}{I_2} = -\frac{1}{a}$$

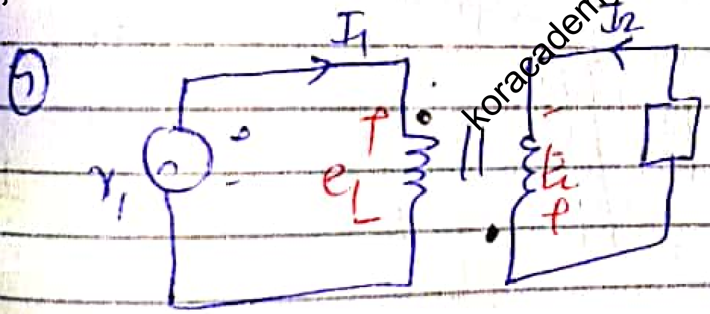
③



$$\frac{E_1}{E_2} = -a$$

$$\frac{I_1}{I_2} = -\frac{1}{a}$$

⑤



$$\frac{E_1}{E_2} = -a$$

$$\frac{I_1}{I_2} = \frac{1}{a}$$

Functions of transformer

1. Step up and step down voltage.
2. Impedance matching for maximum power transfer
3. Filtering of DC.
4. Isolation of circuit (primary and secondary are electrically isolated).

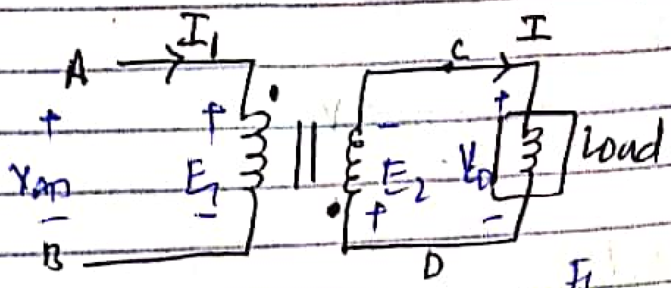
$\boxed{\text{Transformer is a constant power device}}$ ⇒ $E_1I_1 = E_2I_2$

only true for ideal Xfmr ← !

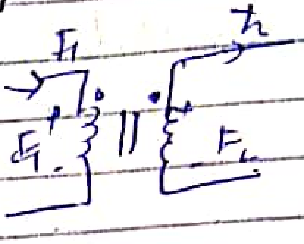
↳ $P_s = P_{\cdot}$

Questions

① The relation of I wrt V_{AB} = ?



The reference is as;



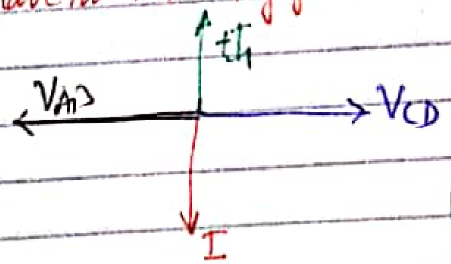
$\frac{E_1}{E_2} = a$

$\frac{I_1}{I_2} = \frac{1}{a}$

$\frac{V_{AB}}{V_{CD}} = \frac{E_1}{-E_2} = -a$

$\Rightarrow V_{AB} = -a\,(V_{CD})$

Current in an inductor will lags the voltage by 90°



(V_{AB} and V_{CD} are out of phase)

V_{AB} lags I by 90°

ⓞ I leads V_{AB} by 90°

Similarly if we talk about currents;

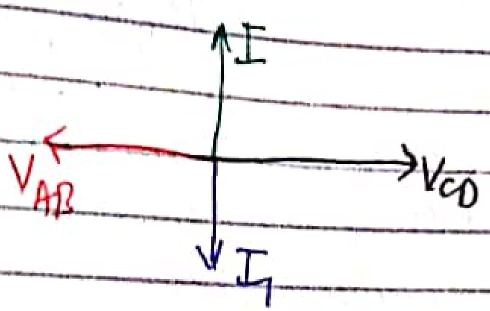
$\frac{I_1}{I} = -\frac{1}{a} \Rightarrow I_1 = -\left(\frac{1}{a}\right) I.$

I_1 lags V_{AB} by 90°

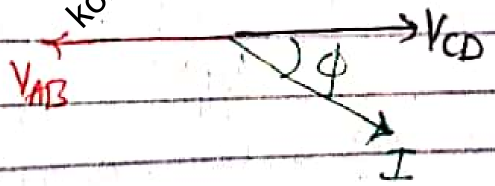
ⓞ V_{AB} leads I_1 by 90°.

If we have resistive load;

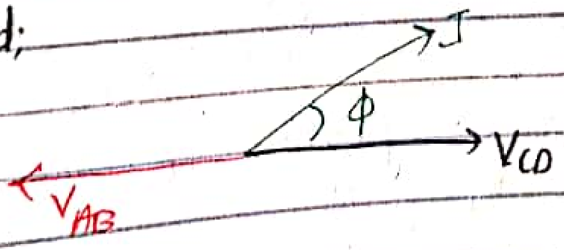
If capacitive load;



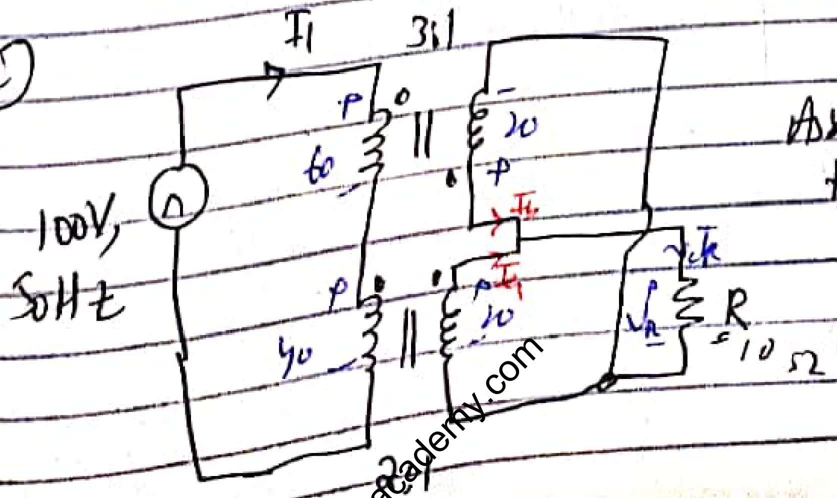
If R-L load;



If RC load;



②



Assuming ideal transformer-

Find I_1.

In such questions go for;

(i) Voltage balance eq (ii) mmf balance eq.

Power balance; $P_{in} = P_{out} = (VA)_1 \times (VA)_2$ ✓

in VA → rms values

$\Rightarrow 100 \times I_1 = V_R \times I_R$

As 100V is divided into 3:1 and 2:1;

$\Rightarrow 100 = 3x + 2x = 5x$

$\Rightarrow x = 20$

$\rightarrow V_R = 20V \rightarrow I_R = \frac{V_R}{R} = \frac{20}{10} = 2\,A$

$\rightarrow I_1 = \frac{20 \times 2}{100} = 0.4\,A.$

If by another method;

$I_R = I_a + I_b$

$2 = I_a + I_b$ —Ⓐ

Why did we not do phasor addition?

$\frac{I_b}{I_1} = \frac{3}{1} \Rightarrow I_b = 3I_1$

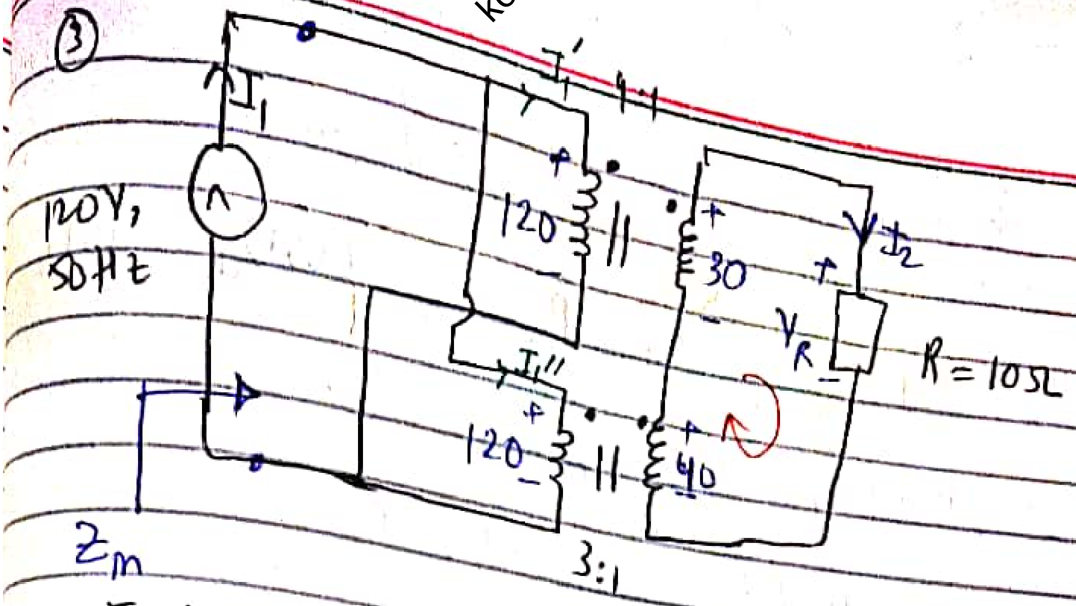
$\frac{I_a}{I_1} = \frac{2}{1} \Rightarrow I_a = 2I_1$

B/c This was simple resistive load and also ideal Xfmr (so no magnetizing branch) therefore no concept of phase shift so we did direct addition.

Ⓐ $\Rightarrow 2 = 3I_1 + 2I_1$

$\Rightarrow I_1 = \frac{2}{5} = 0.4\,A.$

③



Find I_1 and input impedance?

Power balance → $V_1 I_1 = V_R I_2$
→ for ideal only

→ $120 \times I_1 = V_R I_2$

$V_R = 30 + 40 = 70V$

→ $I_2 = \frac{V_R}{10} = \frac{70}{10} = 7A$

→ $I_1 = \frac{70 \times 7}{120} = 4.083 A.$

I/P impedance is the voltage at that terminal divided by the current through that terminal without any conditions.

→ $Z_{in} = \frac{120}{I_1} = \frac{120}{4.083} = 29.41 \Omega$

If solving by another method, (Using current)

$I_1 = I_1' + I_1''$

$\frac{I_1'}{I_2} = \frac{1}{4}$ → $I_1' = \frac{I_2}{4}$

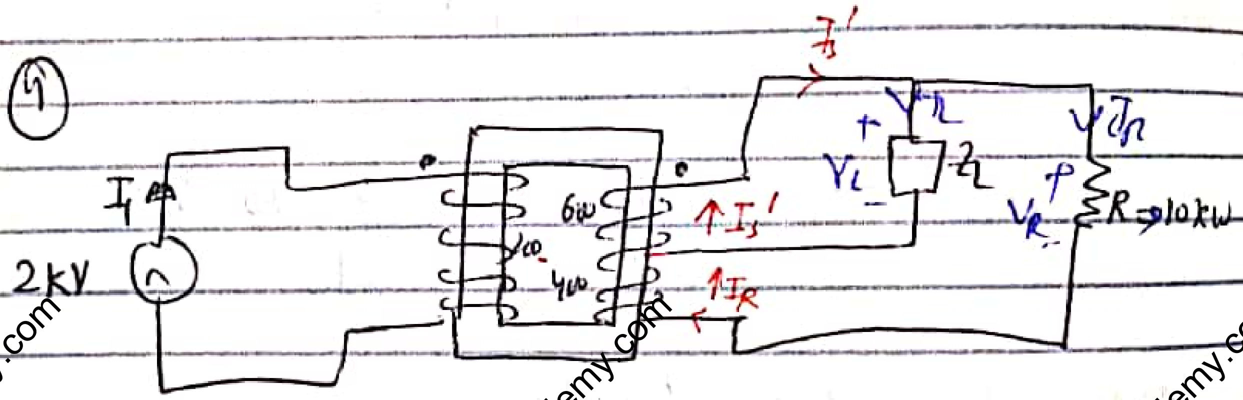
Also, $\frac{I_1''}{I_2} = \frac{1}{3} \Rightarrow I_1'' = \frac{I_2}{3}$

$\Rightarrow I_1 = \frac{I_2}{4} + \frac{I_2}{3} = \frac{7}{4} + \frac{7}{3} = 4.083\ A$

Volt Ampere and wattage value will be the same here s/c no magnetizing branch and no each reductance.

$120 \times 4.083 = \underline{\qquad}$ watt

④



$Z_L = 2000 \angle 45°\ \Omega \qquad I_1 = ?$

Power balancing $\Rightarrow \bar{V}_1 \bar{I}_1 = \bar{V}_2 \bar{I}_2 + \bar{V}_R \bar{I}_R$ —①

say $V_1 = 2KV \angle 0°$ = reference

$\frac{V_1}{V_R} = \frac{200}{1000} \Rightarrow V_R = \frac{1000}{200} \times 2K$

$\Rightarrow V_R = 10KV \angle 0°$

$\Rightarrow I_R = \frac{V_R}{R} = \frac{10kV}{R\ 10kW}$ ✗

As $V_R I_R = 10 \times 10^3 \Rightarrow I_R = 1 \angle 0° A$

no angle change while using turn ratio relation

~~$\bar{I}_1$~~ $\bar{V}_L = 10 \times 600 \angle 0° = 6 kV \angle 0°$

$$\bar{I}_2 = \frac{\bar{V}_L}{Z_L} = \frac{6 kV \angle 0°}{200 \angle 45°} = 3 \angle -45° \, A$$

① ⇒ $2 \times 10^3 \times \bar{I}_1 = (10 \times 10^3 \angle 0°) + (6 \times 10^3 \angle 0°) \times (3 \angle -45°)$

$$\bar{I}_1 = \frac{10 \angle 0° + 18 \angle -45°}{2}$$

⇒ $\bar{I}_1 = 5 \angle 0° + 9 \angle -45°$ ⇒ ~~$I_1 = 10.16$~~ ∠

⇒ $I_1 = 13.02 \angle 29.24$ A.

Complex power at i/p terminal;

$= V_1 I_1^* = 2 \times 10^3 \angle 0° \times 13.02 \angle +29.04$

= ______ .

(i) $P = VI\cos\theta$ → wattage power.

$= (2 \times 10^3)(13.02)\cos(29.24)$

$= 22.72$ kW.

(ii) $Q = VI\sin\theta$ → reactive power

$= (2 \times 10^3)(13.02)\sin(29.24)$

$= 12.719$ VAR

Also $S = \sqrt{p^2 + q^2}$

In mmf balance, we take current in the windings.

If solving this question by MMF balance equation;

$$N_1 I_1 = 600 \times I_3' + 400 \times I_R$$

$$\Rightarrow I_1 = \left(\frac{600}{200} \times I_3'\right) + \left(\frac{400}{200} \times I_R\right)$$

$$= 3 \times I_3' + 2 \times I_R$$

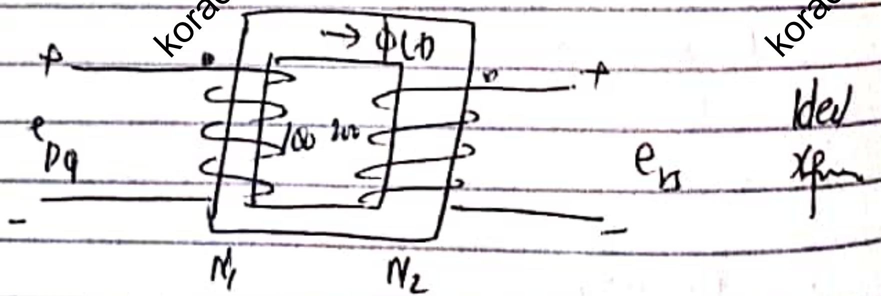
$$= 3\left(I_2 + I_R\right) + 2 I_R$$

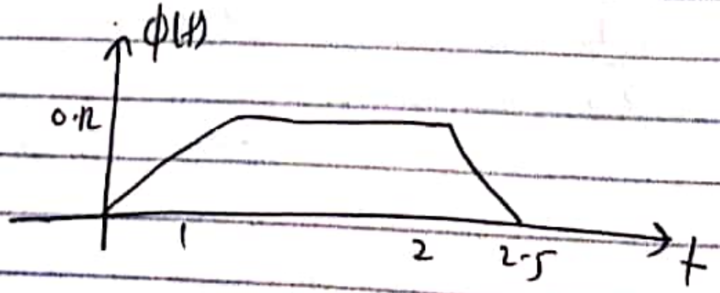
$$= 3\left(3\angle -45^\circ + 1\angle 0^\circ\right) + 2\left(1\angle 0^\circ\right)$$

$$= 9\angle -45^\circ + 3\angle 0^\circ + 2\angle 0^\circ$$

$$\Rightarrow I_1 = 13.02 \angle -29.24 \text{ A.}$$

⑤





Find the waveform of e_{pq} and e_{rs}. (induced voltage)

$$e(t) = N\frac{d\phi}{dt} = L\frac{di}{dt}$$

$$e_{pq} = N_1 \frac{d\phi}{dt} \qquad e_{rs} = N_2 \frac{d\phi}{dt}$$

e_{pq} ad e_{rs} are in the same phase.

$$\Rightarrow \frac{e_{pq}}{e_{rs}} = \frac{N_1}{N_2} = \frac{100}{200} = \frac{1}{2}$$

say

$$e_{rs} = 200 \times \frac{d}{dt}\phi(t)$$

(a) $0 < t < 1$

$$e_{rs} = 200 \times \frac{d}{dt}(0.12t) = 200 \times 0.12$$
$$= 24V$$

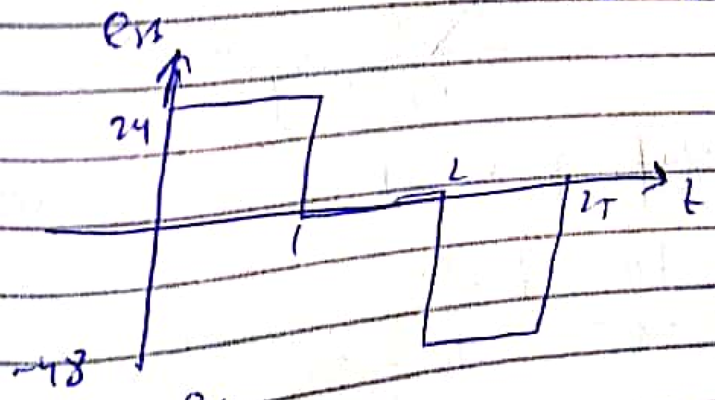
(b) $1 < t < 2$ $\Rightarrow e_{rs} = 0$

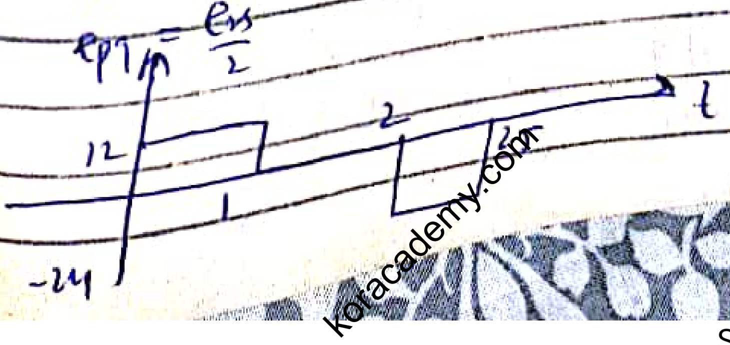
$\because \frac{d\phi}{dt} = 0$

(c) $2 < t < 2.5$

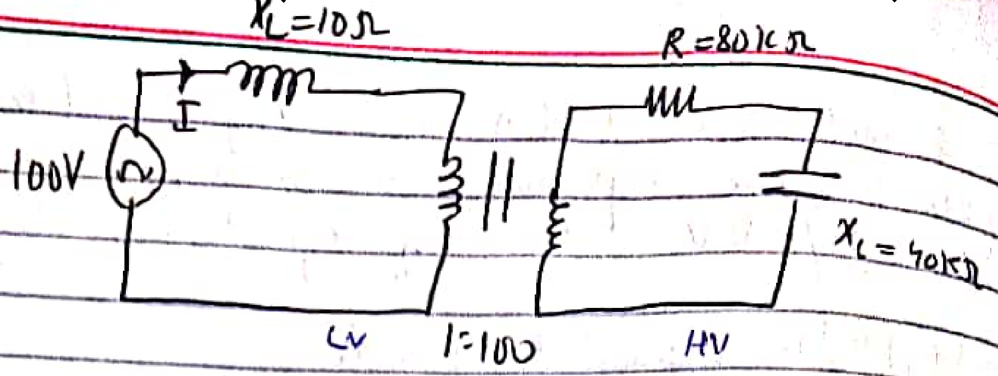
slope $= \frac{0.12}{0.5}$ $\Rightarrow e_{rs} = \frac{200 \times 0.12}{-0.5}$

$= ~~24V~~ = -48V$





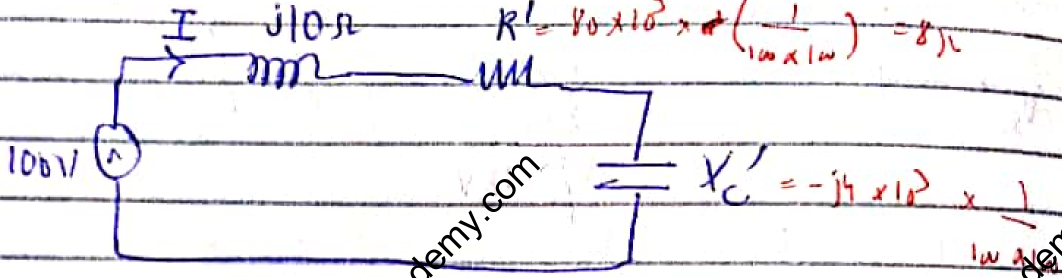
⑥



$I = ?$

$\frac{N_1}{N_2} = \frac{1}{100} = a$, $V_1 = 100V$

Refered to LV side.



Refeence; $\overline{V_1} = 100 \angle 0°$.

$\Rightarrow \overline{I} = \frac{100 \angle 0°}{8 + j6} = \frac{100 \angle 0°}{\sqrt{8^2 + 6^2} \angle \tan^{-1}\left(\frac{6}{8}\right)}$

$\Rightarrow \overline{I} = 10 \angle -36.86$

$\Rightarrow I = 10A$.

I/p power:

lost cause the only resistive R'

$P = VI\cos\theta$ → active power

$I^2R : (10)^2(8)$

$= 100 \times 10 \times \cos(36.86) = 800\,W$ $=80$

$Q = VI\sin\theta$ → reactive power

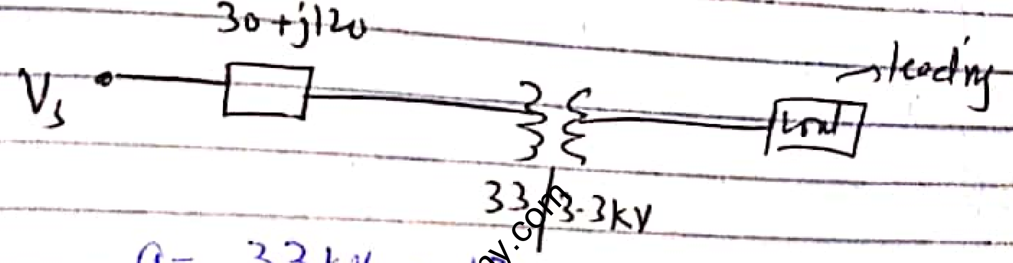
$= 100 \times 10 \times \sin(36.86) = 600\,VAR$

Cause of reactance

$600 = (10)^2(6) \leftarrow I^2X \leftarrow$ $jX_L + jX_c$

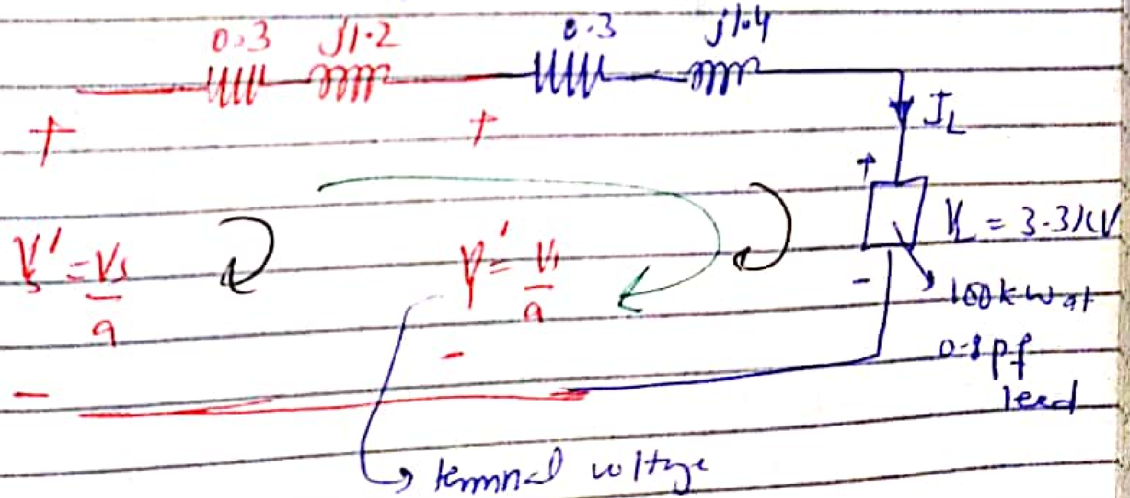
$j10 - j4 = j6$

⑦ A single phase load is fed thru a feeder (33kV) whose impedance is 30 + j120 Ω and a 33/3.3kV Xformer of equivalent impedance (0.3 + j1.4) Ω referred to LV side.
For a load of 100kW at 0.8pf leading at 3.3 kV.
compute (i) Sending end voltage
(ii) Voltage at primary terminal of Xformer



$$a = \frac{33kV}{3.3kV} = 10$$

Drawing the referred ckt to LV side;



KVL

$$\Rightarrow \bar{V_s'} = \bar{V_L} + \bar{I_L}(0.6 + j2.6) \quad — ①$$

V_L is my reference $3.3 \times 10^3 \angle 0°$

$I_L = ?$ $\quad$ $P = VI\cos\theta$

$100\times10^3 = 3.3\times10^3 \times I \times 0.8$

$I = 37.87\ A$

$\phi = \cos^{-1}(0.8) = 36.86.$

$\Rightarrow \bar{I} = 37.87 \angle 36.86^\circ$

Ⓐ $\Rightarrow \bar{V_s'} = (3.3\times10^2 \angle 0^\circ) + (37.87 \angle 36.86)(0.6 + j2.6)$

$\bar{V_s'} = 3260.53 \angle 1.619^\circ$

$\Rightarrow V_s = V_s' \times a = (3260.53 \angle 1.619)(10)$

$\Rightarrow \bar{V_s} = 32605.3 \angle 1.619$

Now for V_1'.

KVL $\Rightarrow$ $V_1' = V_L + \bar{I_L}(0.3 + j1.4)$

$= 3300\angle 0^\circ + 37.87 \angle 36.86 (0.3 + j1.4)$

$V_1' = 3277.68 \angle 0.855^\circ$

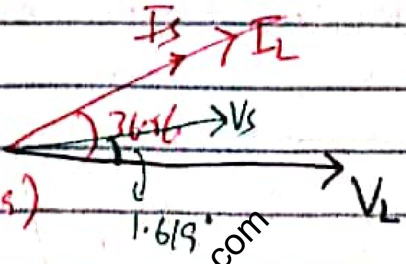
Now $V_1 = aV_1' = 10 \times V_1'$

$\Rightarrow V_1 = 32776.8 \angle 0.855^\circ$

$P_s = V_s \times I_s \times \cos\theta$

$= 32776 \times 3.787$

$\times \cos(36.16 - 1.619)$



$Q_s = V_s I \sin\theta$

$\Rightarrow P_s = 100.85\ kW$

Reactive pwr = $P \cdot \tan(\text{angle})$

DC is a constant signal.

When DC supply is given to a coil, there will be no core losses (hysteresis, eddy current losses $= 0$) but winding losses are very high.

Transformer Losses

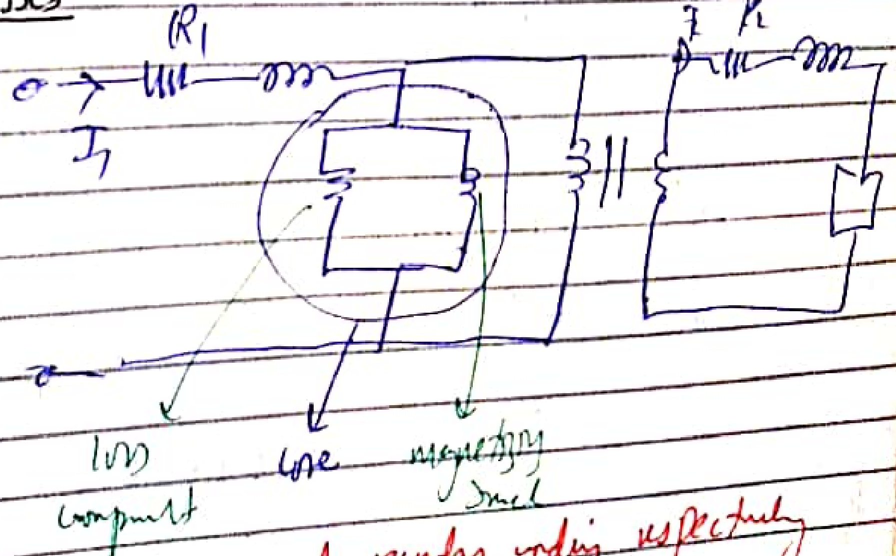
① Core loss (P_i) / Iron loss / Constant loss

(P_h) Hysteresis loss

(P_e) Eddy current loss

② Winding losses / variable losses / copper losses

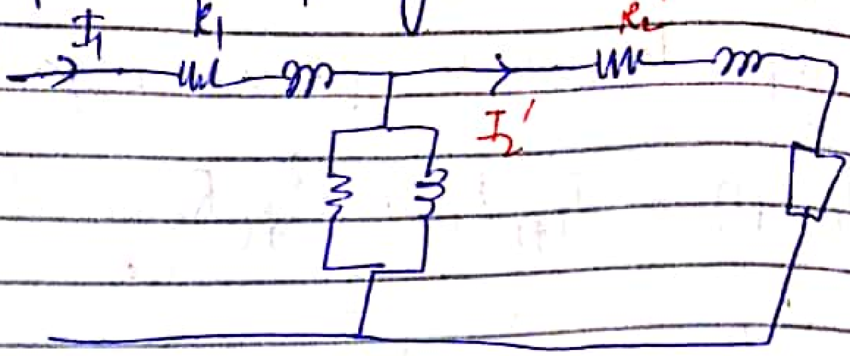
①. Winding losses



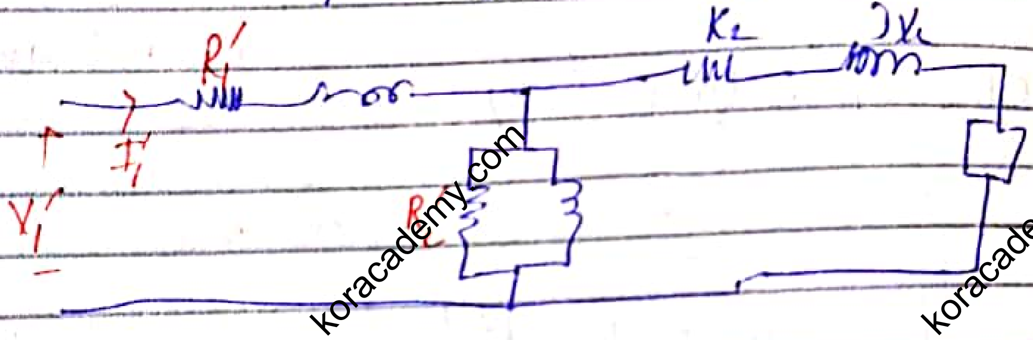
resistances of primary and secondary winding respectively are R_1 and R_2.

$$\boxed{\begin{aligned} P_{cux} &= I_1^2 R_1 + I_2^2 R_2 \\ &= I_1^2 R_1 + I_2'^2 R_2' \\ &= I_1'^2 R_1' + I_2^2 R_2 \end{aligned}}$$

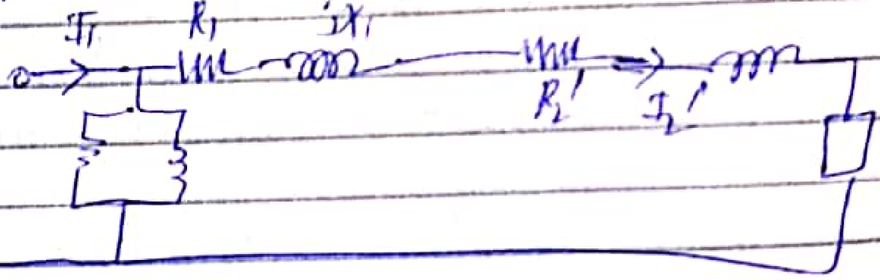
If refered to primary;



if refered from primary to secondary:



Approximate equivalent model (if refered to primary).



$$\boxed{\begin{aligned} P_{cu} &= I_2'^2 (R_1+R_2') \\ \text{or} \quad &= I_1^2 (R_1+R_2') \end{aligned}}$$

→ if the cur is neglected.

If refered to secondary:

$$\boxed{P_{cu} = I_2^2 (R_1'+R_2)}$$

If I_1 is full load current and I_2 (fl).
↳ So copper loss will be full load copper loss.

$$\boxed{P_{cu(fl)} = I_{1fl}^2 R_1 + I_{2fl}^2 R_2}$$

$$\boxed{P_{cux} = x^2 P_{cu}}$$ where $\boxed{x = \frac{I}{I_{fl}}}$

↳ fraction of loading

② Eddy current losses (P_e)

↳ due to induction process in the core.

$$\boxed{P_e = K_e f^2 B_m^2 t^2}$$

K_e → Steinmetz constant, f → frequency.
B_m → maximum flux density ; t → thickness of lamination.

$$E = V = 4.44 f \phi_m N = 4.44 f B_m A_c N$$

$$\Rightarrow B_m = \frac{V}{444 f A_c N} \quad \Rightarrow \quad \boxed{B_m \propto \frac{V}{f}}$$

Case 1 when B_m = constant

$\Rightarrow \frac{V}{f}$ = constant

proportionality constant

$$\Rightarrow \boxed{P_e \propto f^2} \quad \Rightarrow P_e = A f^2$$

Case ii when $B_m \neq$ constant

$$\Rightarrow \frac{V}{f} \neq \text{constant}$$

$$\Rightarrow P_e = K_1 K_e f^2 \left(\frac{V}{f}\right)^2 t^2$$

as $B_m \neq \frac{V}{f}$
it is proptl so K_1 constt.

$$\Rightarrow \boxed{P_e \propto V^2}$$

$$\Rightarrow P_e = K V^2$$

③ Hysteresis losses (P_h)

$$\boxed{P_h = K_h B_m^x f V_{core}}$$

K_h → steinmetz constant, V → volume of core.
x → steinmetz exponent
For ferromagnetic materials $\boxed{x = 1.6}$

Case i When B_m = constant $\Rightarrow \frac{V}{f}$ = constant

$$\Rightarrow \boxed{P_h \propto f}$$

$$\Rightarrow P_h = B f$$

Case ii when $B_m \neq$ constant $\Rightarrow \frac{V}{f} \neq$ constant

$$\Rightarrow P_h = K_h K_1 \left(\frac{V}{f}\right)^x \cdot f \cdot V_{core}$$

$$\Rightarrow \boxed{P_h \propto V^{1.6} f^{-0.6}}$$

$$\Rightarrow P_h = K' V^{1.6} f^{-0.6}$$

④ **Core loss** $P_i = P_e + P_h$

Case i $B_m = \text{constant}$

$$P_i = Af^2 + Bf$$

Case ii $B_m \neq \text{constant}$

$$P_i = kV^2 + k'V^{1.6}f^{-0.6}$$

ie core losses depend on whether the maximum flux density is constant or not.

Q1. A 1 ϕ Xformer 400V, 50 Hz has core loss = 5000W at rated condition. When operated at 200V, 25Hz the core loss is 2000W. When operated at 416 V, 52 Hz, the value of hysteresis loss divided by eddy current loss is ______.

$P_{i1} = 5000\,W$, $V_1 = 400V$, $f_1 = 50\,Hz$

$P_{i2} = 2000\,W$, $V_2 = 200V$, $f_2 = 25\,Hz$

$V_3 = 416V$, $f_3 = 52\,Hz$, $\frac{P_{h3}}{P_{e3}} = ?$

$$\frac{V_1}{f_1} = \frac{V_2}{f_2} = \frac{V_3}{f_3} = 8$$

$\Rightarrow B_m = \text{constant}$

$\Rightarrow P_e \propto f^2, P_h \propto f$

$\Rightarrow P_e = af^2, P_h = 5f$

$$\Rightarrow P_{i_1} = 5000 = P_{e1} + P_{h1} = a f_1^2 + b f_1 = a(50)^2 + b(50) \quad ①$$

$$P_{i2} = 2000W = P_{e2} + P_{h2} = a f_2^2 + b f_2 = a(25)^2 + b(25) \quad ②$$

Solving $\Rightarrow a = 0.8,\ b = 60$

$$\Rightarrow P_{e3} = a f_3^2 = (0.8)(52)^2 = 2163.2$$

$$P_{h3} = b f_3 = (60)(52) = 3120$$

Ratio, $\dfrac{P_{h3}}{P_{e3}} = 1.44$ ans

Q2 For a 1ϕ transformer, the supply voltage and frequency both have been increased by 10%. The percentage changes in the hysteresis and eddy current losses will be ____.

Say for $V_1, f_1 \rightarrow P_{i1} = P_{e1} + P_{h1}$, $\dfrac{V_1}{f_1}$

Now;

$$V_2 = 1.1 V_1, \quad f_2 = 1.1 f_1$$

$$\frac{V_2}{f_2} = \frac{V_1}{f} \Rightarrow B_m = \text{constant}$$

$\hookrightarrow P_e \propto f^2,\ P_h \propto f$

So $P_{i2} = P_{e2} + P_{h2}$

$$\left(\frac{1.1 f_1}{f_1}\right)^2 = \frac{P_{e2}}{P_{e1}} = \frac{f_2^2}{f_1^2}, \quad \frac{P_{h2}}{P_{h1}} = \frac{f_2}{f_1} = \frac{1.1 f_1}{f_1}$$

$$\Rightarrow \boxed{\begin{aligned} P_{e2} &= (1.1)^2 P_{e1} \\ P_{h2} &= 1.1 P_{h1} \end{aligned}}$$

$$\% \Delta P = \frac{P_2 - P_1}{P_1} \times 100\%$$

$$\% \Delta P_h = \frac{P_{h2} - P_{h1}}{P_{h1}} = \frac{1.1 P_{h1} - P_{h1}}{P_{h1}} \times 100\%$$

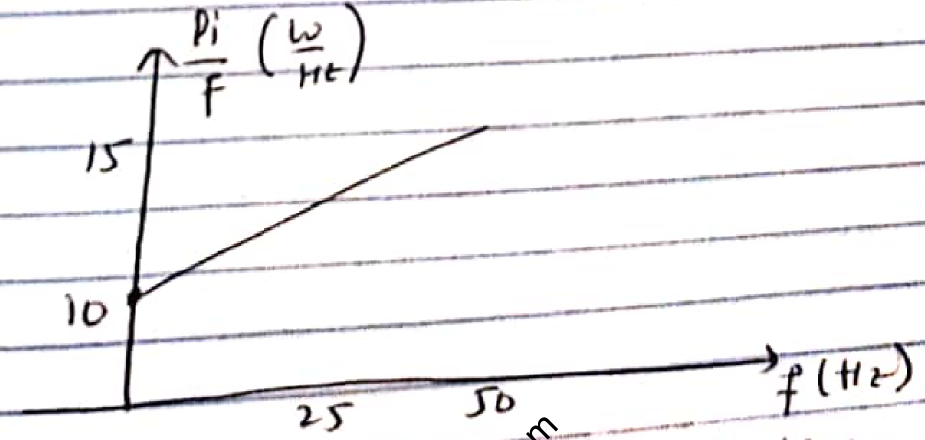
$$= \frac{0.1 \cancel{P_{h1}}}{\cancel{P_{h1}}} \times 100\% = \boxed{10\%}$$

↳ increment

$$\% \Delta P_e = \frac{P_{e2} - P_{e1}}{P_{e1}} = \frac{(1.1)^2 - 1}{1} \times 100\%$$

$$\boxed{= 21\%} \rightarrow \text{increment}$$

Q3. Given that V/f = constant.



The hysteresis and eddy current losses of the transformer at 25 Hz are ____ ?

B_m = constant ⇒ $P_e \propto f^2$, $P_h \propto f$

$P_e = af^2$, $P_h = bf$

⇒ $P_i = P_e + P_h = af^2 + bf$

⇒ $\boxed{\frac{P_i}{f} = af + b}$ $y = mx + c$

From graph.

i- when $f = 0$ Hz

$10 = (a \times 0) + b$ $\boxed{b = 10}$

⇒ $P_h = bf = (10)(25) = \underline{250W}$.

ii when $f = 50$ Hz.

$15 = (a \times 50) + \underset{=10}{b}$ ⇒ $\boxed{a = \frac{1}{10}}$

⇒ $P_e = af^2 = \left(\frac{1}{10}\right)(25)^2 = \underline{62.5W}$.

Q4. A 50Hz xformer having equal hysteresis and eddy current losses at rated excitation is operated at 45Hz at 90% of its rated voltage. Compare to rated operated point, the core loss under this condition, reduced by 14.5%.

(a)

V_1 → rated f_1 → rated = 50Hz

$P_{i1} = P_{h1} + P_{e1} = 2P_{e1}$ or $2P_{h1}$

b/c both are equal.

$f_2 = 45 Hz$, $V_2 = 0.9 V_1$

$$P_{i2} = P_{h2} + P_{e2}$$

Now

$$\frac{V_2}{f_2} = \frac{0.9V_1}{0.9f_1} = \frac{V_1}{f_1} \qquad \frac{f_2}{f_1} = \frac{45}{50} = 0.9$$

$$\Rightarrow f_2 = 0.9 f_1$$

$$\Rightarrow \frac{V}{f} = \text{constnt}$$

↳ $P_e \propto f^2$, $P_h \propto f$

$$\frac{P_{h2}}{P_{h1}} = \frac{f_2}{f_1} = \frac{45}{50} = 0.9$$

$$\Rightarrow \boxed{P_{h2} = 0.9 P_{h1}}$$

$$\frac{P_{e2}}{P_{e1}} = \frac{f_2^2}{f_1^2} = \left(\frac{45}{50}\right)^2 = (0.9)^2$$

$$\Rightarrow \boxed{P_{e2} = 0.81 P_{e1}}$$

Now;

$$P_{i2} = P_{e2} + P_{h2} = 0.9 P_{h1} + 0.8 P_{e1}$$

$$\Rightarrow P_{i2} = 0.9 P_{e1} + 0.8 P_{e1} = 1.71 P_{e1}$$

$$\% \Delta P_i = \frac{P_{i2} - P_{i1}}{P_{i1}} \times 100\% = \frac{1.71 P_{e1} - 2P_{e1}}{2P_{e1}}$$

$$= \frac{1.71 - 2}{2} \Rightarrow \boxed{\% \Delta P_i = -14.5\%}$$

reduced by 14.5 % ↵

No Load Test / Open Circuit Test

↳ under no load condition.

⇒ It is performed on L.V side (instruments)
↳ HV side is open. ⇒ $I_2 = 0\,A$

With this tests we find;

1. Core loss.
2. Shunt branch parameters (X_m and R_c)

It is done at rated voltage and frequency.

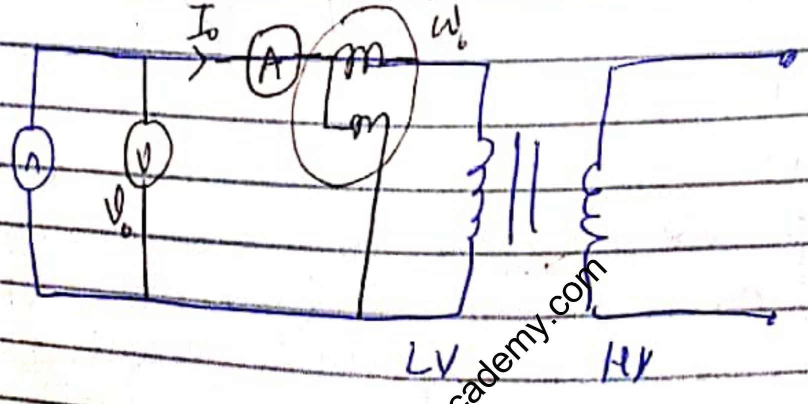
(It can also be performed on HV side but recommended is LV side).

$I_0 \to$ 2 to 5% of I_{FL}
↳ more precise ammeter will be required.
↳ voltmeter size will increase ⇒ cost

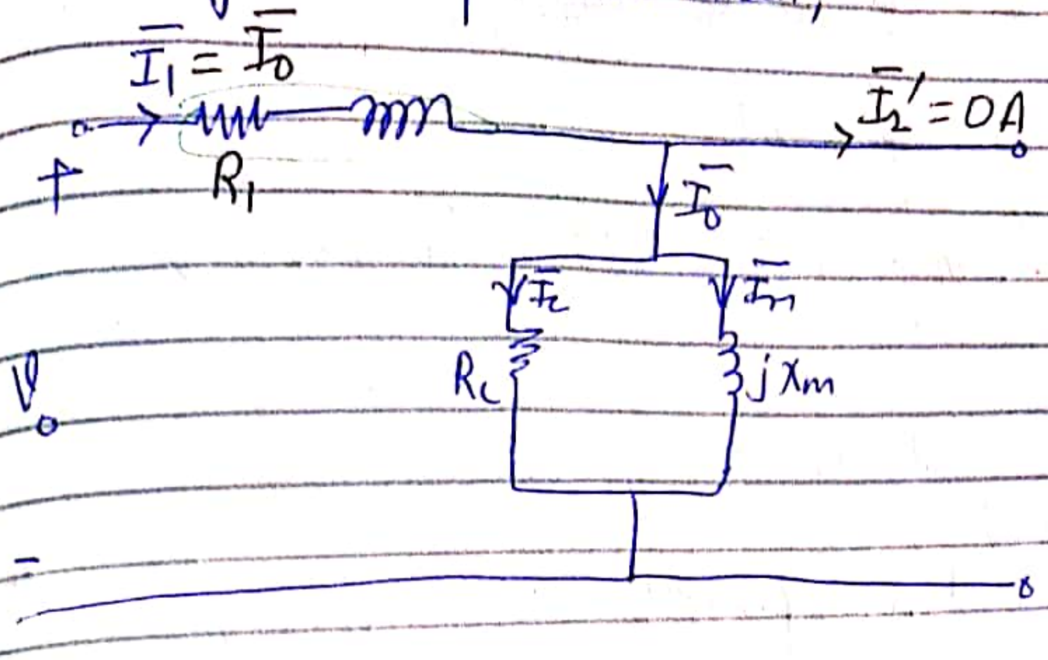
LV	HV
V ↓	V ↑
I ↑	I ↓

Three readings;

i. Voltmeter reading (V_0) → rated voltage.
ii. Ammeter reading (I_0) → 2 to 5% of full load current
iii. Wattmeter reading (W_0) → Iron losses (P_i)

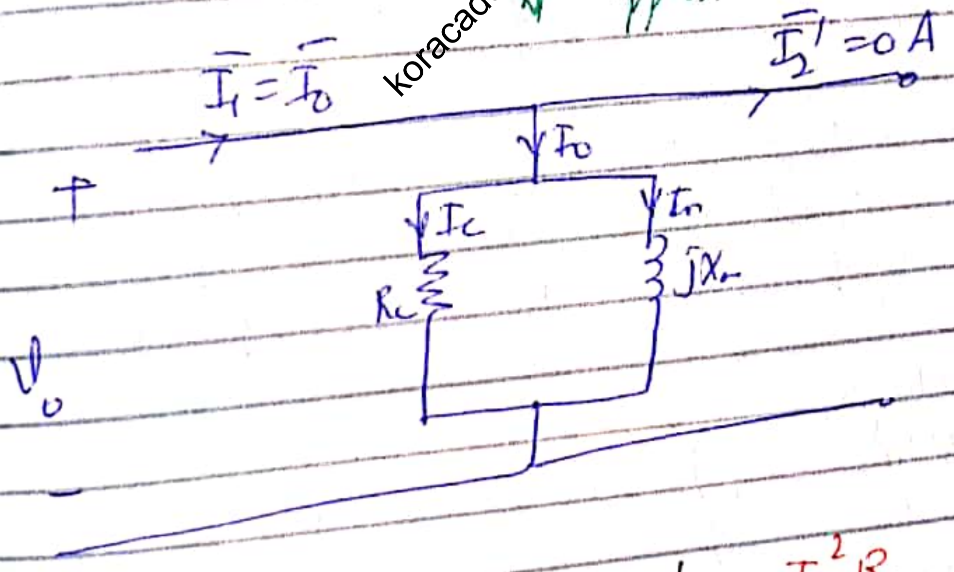


Drawing the equivalent circuit;



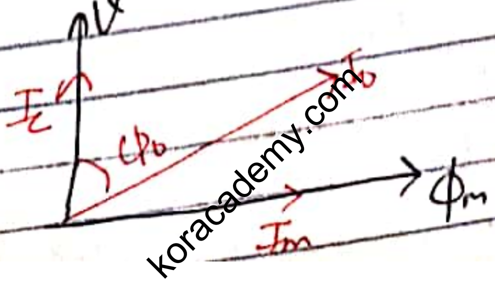
No load ⇒ I_0 is also very less ⇒ the drop will also be very less ⇒ neglect

⇓ approximation



(i) $W_0 = P_i = V_0 I_0 \cos\phi_0 = I_c^2 R_c$

i/p p.f ← where $\cos\phi_0 = \frac{W_0}{V_0 I_0}$



ii. $|I_c| = I_0 \cos\phi_0$.

$|I_m| = I_0 \sin\phi_0$

iii. $R_c = \frac{V_0}{I_c}$, $X_m = \frac{V_0}{I_m}$

If the winding resistance (R_1) is not neglected;

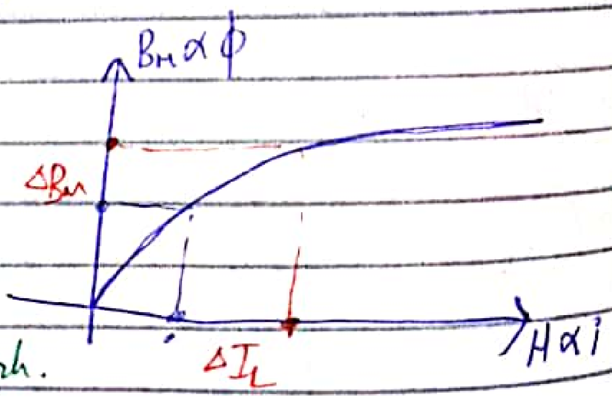
$$W_0 = P_i + I_0^2 R_1$$

Wattmeter will give us the total power taken from the input.

⇒ If no load test is performed at rated voltage and reduced frequency.

V = rated, $f \neq$ rated

$\uparrow B_m \propto \frac{V}{\downarrow f}$



ΔB_m will be very less due to saturation but to bring this change the ΔI will be very much.

If frequency is reduced than the rated, the core will move to deep saturation. → not advisable to reduce f than rated.

$\Rightarrow \uparrow I_m = \frac{V}{\downarrow jX_m} = \frac{V}{2\pi f\downarrow L}$ ⇒ demand of magnetising current from supply will increase

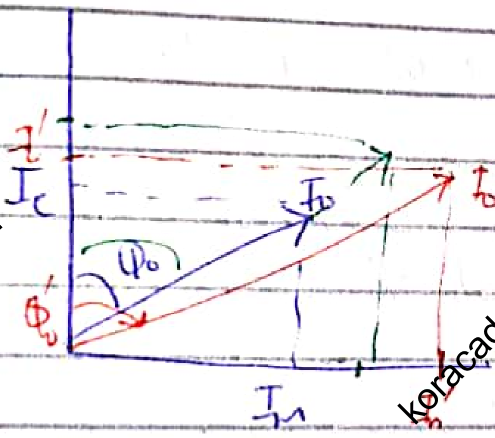
→ $B_m \neq$ constant.

$P_e \propto V^2$, $\uparrow P_h \propto \frac{V^{1.6}}{\downarrow f^{0.6}}$

↳ no effect of f is changed.

$$\uparrow P_i = P_c + \uparrow P_h'$$

$\uparrow P_i = \uparrow I_c^2 R_c$ → $\uparrow I_0 = \sqrt{\uparrow I_c^2 + \uparrow I_m^2}$



$I_0 \uparrow$
↳ 1. If I_c and I_m increase by the same rate.
↳ p.f angle will remain the same
which generally does not happen

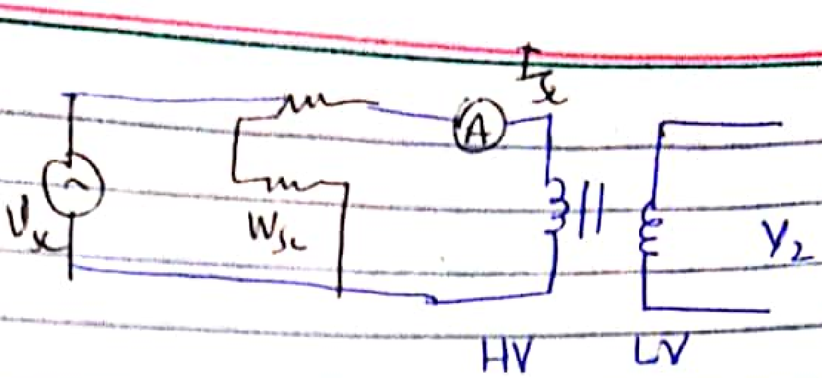
Usually I_m increases at a much faster rate than I_c.

$\phi_0' > \phi_0$ ⇒ p.f angle has increased.
⇒ $\cos\phi_0' < \cos\phi_0$ ⇒ pf has reduced.
(angle ↓ ⇒ cos ↑)
not good sign ✓

Short Circuit Test

↳ to know the series parameters.

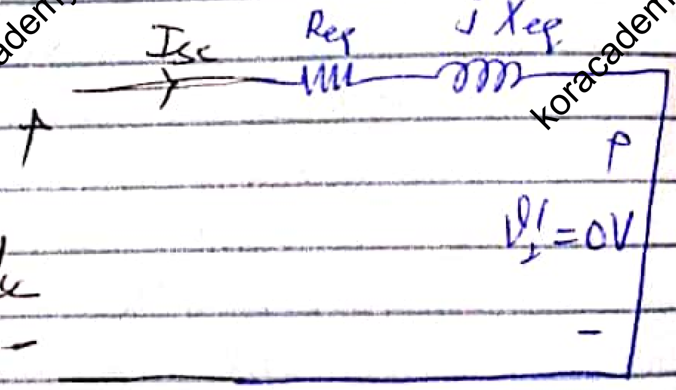
It is performed on HV side (recommended)



The readings will be;
W_{sc} , V_{sc} , I_{sc}.
↳ reduced voltage (a fraction of V_{rated})

Using the approximated equivalent circuit referring from secondary to primary -

$R_{eq} = R_1 + R_2'$
$X_{eq} = X_1 + X_2'$



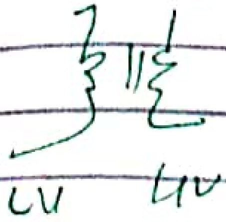
i) $Z_{eq} = \frac{|V_{sc}|}{|I_{sc}|}$

ii. $W_{sc} = I_{sc}^2 R_{eq} = V_{sc} I_{sc} \cos\phi_{sc}$.

i/p p.f at s.c ⇐ $\cos\phi_{sc} = \frac{W_{sc}}{V_{sc} I_{sc}}$

iii. $Z_{eq} = \sqrt{R_{eq}^2 + X_{eq}^2}$ ⇒ $X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$

If we change LV and HV.



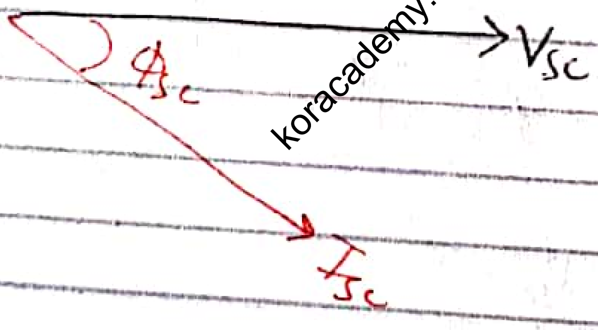
HV side short circuited.

Refer from primary to secondary.

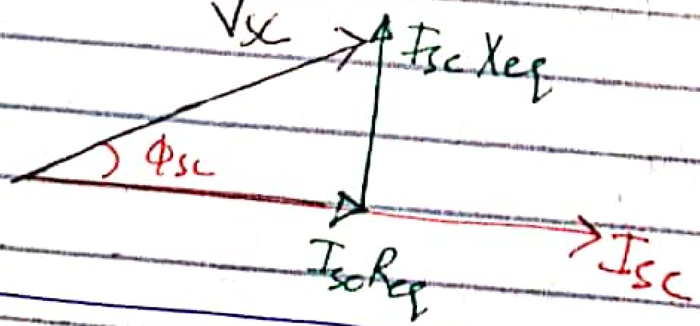
$\Rightarrow V_{sc}', \; I_{sc}', \; V_2, \; R_{eq} = R_1' + R_2$

$X_{eq} = X_1' + X_2$

Drawing phasor diagram for the equivalent circuit;



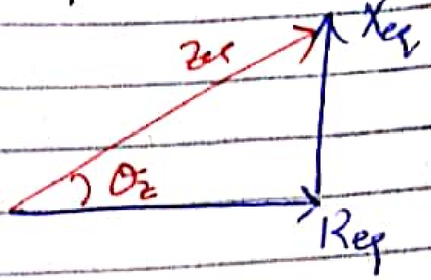
$$V_{sc} = I_{sc} R_{eq} + j I_{sc} R_{eq}$$



$$\boxed{\cos\phi_{sc} = \frac{I_{sc}R_{eq}}{V_{sc}}}$$

Talking of impedance triangle;

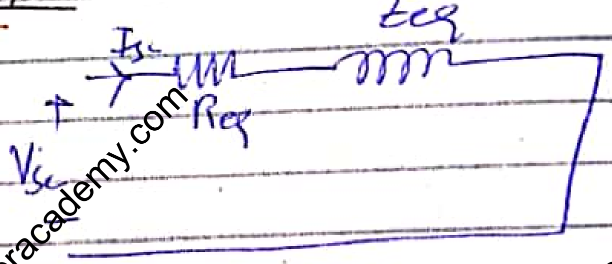
$$\boxed{\cos\theta_z = \frac{R_{eq}}{Z_{eq}}}$$



$$Z_{eq} = \frac{\bar{V}_{sc}}{\bar{I}_{sc}} = \frac{|V_{sc}| \angle \phi_{sc}}{|I_{sc}| \angle 0^\circ} = |Z_{eq}| \angle \theta_z$$

⇒ In case of S.C test; $\boxed{\phi_{sc} = \theta_z}$

Important Points.



1. Rated S.C current and Steady state sc current

(a) When V_{sc} is a reduced voltage

ie

$$\boxed{\begin{gathered} V_{sc(reduced)} < V_{sc(rated)} \\ \Rightarrow I_{sc(rated)} = \frac{V_{sc(reduced)}}{Z_{eq}} \end{gathered}}$$

(b) when V_{sc} is rated voltage.

$$\boxed{\begin{gathered} V_{sc} = V_{sc(rated)} \\ I_{sc(steady)} = \frac{V_{sc\ rated}}{Z_{eq}} \end{gathered}}$$

→ 10 to 15% times ~~greater than~~ ↑ Isc rated

→ Sc not advisable

$$\boxed{\begin{aligned} I_{sc\,(Stdy)\,pu} &= \frac{1}{Z_{eq\,(pu)}} \\ I_{sc\,rated\,(pu)} &= 1\,pu \end{aligned}}$$

ii Let V_{sc} = constant and $f \neq$ constant

If f is reduced ↓   If increased ↑.

$$\uparrow\downarrow Z_{eq} = \sqrt{R_{eq}^2 + \uparrow\downarrow X_{eq}^2} \quad , \quad \underset{= \text{no change}}{R_{eq}} \quad , \quad \uparrow\downarrow X_{eq} = 2\pi \overset{\uparrow}{f}\downarrow L_{eq}$$

$$\downarrow\uparrow \cos\phi_{sc} = \frac{R_{eq}}{\uparrow\downarrow Z_{eq}} \quad , \quad \downarrow\uparrow I_{sc} = \frac{V_{sc}}{\uparrow\downarrow Z_{eq}}$$

$$\phi_{sc} \downarrow \uparrow \qquad \downarrow\uparrow W_{sc} = \downarrow\uparrow I_{sc}^2 R_{eq}$$

P.f will improve

P.f will deteriorate.

Transformer Efficiency

$$\%\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$= \frac{P_{out}}{P_{out} + Losses} \times 100\%$$

$$\eta = \frac{VI\cos\phi}{VI\cos\phi + P_i + P_{cux}}$$

Generally for any load;

$$\boxed{\eta_{at\ any\ load} = \frac{x.(VI)_{rated}\overset{\text{pf of load}}{\cos\phi}}{x(VI)_{rated}\cos\phi + P_i + x^2 P_{cu}}}$$

x = fraction of loading , $x = I/I_{rated}$

$(VI)_{rated}$ = given Volt ampere rating of transformer.

$\Rightarrow$ also $x = \frac{VA}{(VA)_{rated}}$

$P_i = P_h + P_e$, P_{cu} = full load copper loss
$= (I_{Fl})^2 \times R_{eq}$

Under maximum efficiency;

$$\boxed{\frac{d\eta}{dI} = 0 \quad P_i = P_{cux} = x^2 P_{cu}}$$

copper loss should become equal to iron loss

$\therefore$ $P_{cux} = P_i \Rightarrow x^2 P_{cu} = P_i$

$$\Rightarrow \boxed{x = \sqrt{\frac{P_i}{P_{cu}}}}$$

ii. $$\boxed{VA\Big|_{\eta_{max}} = x.\,(VA)_{rated}}$$

iii. $$P_{cu} = (I_{rated})^2 R_{eq}$$

where $I_{rated} = \frac{VA_{rated}}{V_{rated}}$

Now:

$$P_{cu\,(pu)} = \frac{P_{cu}}{(P_{ower})_{base}} = \frac{I_{rated}^2 \cdot R_{eq}}{V_{rated} \times I_{rated}}$$

$$= \frac{R_{eq}}{V_{rated}/I_{rated}} = \frac{R_{eq}}{(\Omega)_{base}} = R_{eq}\,(pu)$$

$$\Rightarrow \boxed{P_{cu\,(pu)} = R_{eq\,(pu)}}$$

↳ only true at full load condition.

Efficiency in terms of pu.

As $$\eta = \frac{x.\,(VA)_{rated}.\,\cos\phi}{x.\,(VA)_{rated}\cos\phi + P_1 + x^2 P_{cu}}$$

$$\boxed{\eta_{pu} = \frac{x.\,1.\,\cos\phi}{x.\,1.\,\cos\phi + P_{1(pu)} + x^2 P_{cu(pu)}}}$$

where $P_{i(pu)} = \frac{P_{i\,actual}}{base = VA_{rated}}$

Q1. O.C and S.C test data are given below for a 1-ϕ 5 MVA, 200/400 V, 50 Hz Xformer.

O.C (LV) → 200V, 1.25 A, 150 W.
S.C (HV) → 20V 12.5A 175 W.

Transformer efficiency at 75% load at 0.7 pf

Given Xformer rating 200/400 V, VA_{rated} = 5MVA
LV HV

$V_0 = 200V$, $I_0 = 1.25A$, W_0 or $P_i = 150W$

$V_{sc} = 20V$, $I_{sc} = 12.5A$, W_{sc} or $P_{cu} = 175W$

$$\eta_x = \frac{x\,(VA)_{rated}\cos\phi}{x\,(VA)_{rated}\cos\phi + P_i + x^2 \boxed{P_{cu}}} \hookrightarrow ?$$

$$(I_{rated})_{HV} = \frac{(VA)_{rated}}{(V_{rated})_{HV}} = \frac{5\times10^6}{400} = 12.5$$

rated current corresponds full load copper loss

$\Rightarrow P_{cu} = 175 W$.

$$\Rightarrow \eta_x = \frac{(0.75)(5\times10^6)(0.7)}{(0.75)(5\times10^6)(0.7) + 150 + (0.75)^2(175)}$$

$\Rightarrow \eta = 91.35\%$ Ans.

At what load will we have maximum efficiency.

$P_i = x^2 P_{cu} \Rightarrow x\Big|_{\eta_{max}} = \sqrt{\frac{P_i}{P_{cu}}} = \sqrt{\frac{150}{175}} = 0.9$

Similarly $(VA)_{\eta_{max}} = x\,(VA)_{rated}$

$= 0.925 \times 5 = 4.62$ MVA

Q2 For the following data 10KVA, 500/250V Xfmr

O.C test → 250 V, 3 A, 200 W. (LV)
S.C test → 15 V, 30 A, 300 W. (HV)

Xfmr efficiency at full load at 0.8 pf is ____ ?

$V_0 = 250V$, $I_0 = 3A$, $P_i = 200W$.
$V_{sc} = 15V$, $I_{sc} = 30A$. $P_{cux} = 300\,W$ ← cu power at this current

Full load ⇒ $x = 1$ $VA_{rated} = 10 \times 10^3$

$\cos\phi = 0.8$, $P_i = 200W$

$$(I_{rated})_{hv} = \frac{(VA)_{rated}}{(V_{rated})_{hv}} = \frac{10 \times 10^3}{500} = 20A$$

$$P_{cux} = x^2 P_{cu} \Rightarrow 300 = \left(\frac{30}{20}\right)^2 P_{cu}$$

$\Rightarrow P_{cu} = 133.3\,W$.

$$\Rightarrow \eta = \frac{1 \times 10 \times 10^3 \times 0.8}{1 \times 10 \times 10^3 \times 0.8 + 200 + (1)^2 (133.3)}$$

$\Rightarrow \eta = 96\%$ Ans.

$Z_{eq} = \frac{V_{sc}}{I_{sc}} = \frac{15}{30} = \frac{1}{2}\,\Omega$.

→ $P_{cu} = I^2 R_{eq}$

$300 = (30)^2 R_{eq} \Rightarrow R_{eq} = \frac{1}{3}\ \Omega.$

$\Rightarrow X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = \sqrt{(\frac{1}{4})^2 - (\frac{1}{9})^2}$

$\Rightarrow X_{eq} = \sqrt{\frac{5}{36}} = 0.37\ \Omega$

→ $Z_{eq(pu)} = \frac{Z_{eq}}{Z_{eq(base)}} = \frac{1}{2 \times \frac{V_{rated}}{I_{rated}}}$

$= \frac{1}{2 \times \frac{500}{20}} \Rightarrow Z_{eq(pu)} = \frac{1}{50}$ pu.

→ $R_{eq(pu)} = \frac{1}{3 \times \frac{500}{20}} \Rightarrow R_{eq(pu)} = 0.013$ pu

Q3 A 1 kVA, 200/100 V, 50 Hz, 1 ϕ X-former have following results at 50 Hz

O.C (LV) → 100 V, 20 watt.
S.C (HV) → 5A, 25 watt.

It is assumed that no load loss component are equally divided. The above tests were then conducted on the same transformer at 40 Hz. The results were;

O.C (HV) → 160V $W_1 = ?$
SC (LV) → 10 A $W_2 = ?$

At 50 Hz , $P_{i1} = 20$ Watt

$(I_{rated})_{HV} = \frac{1\times10^3}{200} = \boxed{5A}$.

$\Rightarrow P_{cu(f1)} = 25$ watt.

Similarly $(I_{rated})_{LV} = \frac{1\times10^3}{100} = 10A$

$\Rightarrow W_2 = 25$ watt.

Full load copper loss is independent of frequency.

↳ copper loss will change but at full load it will not change

$P_{i1} = 20W = P_{e1} + P_{h1} = \underbrace{2P_{e1} \text{ or } 2P_{h1}}$

$f_1 = \frac{20}{2} = 10$ as equally divided

At 40 Hz,

Checking if B_m is constant or not.

$\frac{V_1}{f_1} = \frac{V_2}{f_2}$, $\frac{200}{50} = \frac{160}{40} \Rightarrow 4 = 4.$

$\Rightarrow B_m = \text{constant}$

$\Rightarrow P_e \propto f^2$, $P_h \propto f$

$\frac{P_{e2}}{P_{e1}} = \left(\frac{f_2}{f_1}\right)^2$ and $\frac{P_{h1}}{P_{h2}} = \frac{f_1}{f_2}$

$$\Rightarrow P_{e_2} = \left(\frac{40}{50}\right)^2 \times (10) = 6.4$$

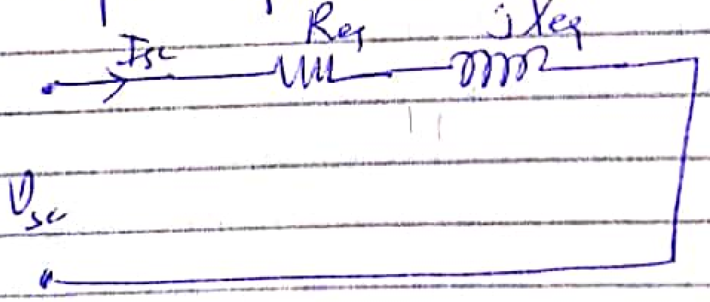
$$P_{h_2} = \left(\frac{40}{50}\right) \times (10) = 8 \text{ Watt}$$

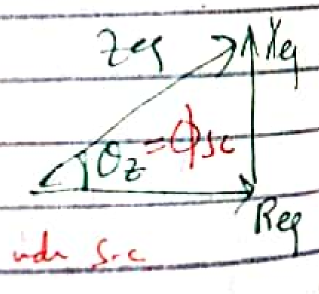
$$\Rightarrow P_{i_2} = P_{e_2} + P_{h_2} = 14.4 \text{ Watt}$$

Ans

Q4 A 50Hz 1ϕ transformer, draws a short circuit current of 30A at 0.2 pf lagg when connected to 16V, 50Hz source.
What will be the s.c current and its pf when the same Xformer is energized from 16V, 25Hz source.

The Xformer equivalent circuit is as:





$$\cos\phi_{sc} = \frac{R_{eq}}{Z_{eq}}$$

At 16V, 50Hz : $\cos\phi_{sc1} = 0.2$

$I_{sc_1} = 30A$, $V_{sc_1} = 16V$, $f_1 = 50Hz$

we know $I_{sc_1} = \dfrac{V_{sc1}}{Z_{eq1}}$

Now; $V_{sc2} = 96V$, $I_{sc2} = ?$, $\cos\phi_{sc2} = ?$

$f_2 = 25Hz$, $I_{sc2} = \frac{V_{sc2}}{Z_{eq2}}$

$$\frac{I_{sc2}}{I_{sc1}} = \frac{Z_{eq1}}{Z_{eq2}} = \left(\frac{Z_{eq1}}{R_{eq}}\right)\left(\frac{R_{eq}}{Z_{eq2}}\right)$$

$$\Rightarrow \frac{I_{sc2}}{I_{sc1}} = \frac{\cos\phi_{sc2}}{\cos\phi_{sc1}} \quad \text{—①}$$

Also $\tan\phi_{sc} = \frac{X_{eq}}{R_{eq}}$

$$\Rightarrow \frac{\tan\phi_{sc2}}{\tan\phi_{sc1}} = \frac{X_{eq2}}{X_{eq1}} = \frac{1}{2}$$

As $X_{eq} \propto f$

$$\Rightarrow \frac{X_{eq2}}{X_{eq1}} = \frac{f_2}{f_1} = \frac{25}{50} = \frac{1}{2}$$

$$\Rightarrow X_{eq2} = \frac{X_{eq1}}{2}.$$

$$\Rightarrow \tan\phi_{sc2} = \frac{1}{2} \times \tan\phi_{sc1} = \frac{1}{2}\tan(\cos^{-1} 0.2)$$

$$\Rightarrow \tan\phi_{sc2} = 2.448$$

$$\Rightarrow \phi_{sc2} = \tan^{-1}(2.448) = 67.78°$$

① ⇒ $I_{sc2} = 30 \times \left(\frac{\cos 67.78}{0.2}\right)$

⇒ $I_{sc2} = 57.72$ A

Also pf ⇒ $\cos\phi_{sc} = \cos(67.78) = 0.378$

Method 2

$\bar{I}_{sc1} = \frac{\bar{V}_{sc}}{Z_{eq}}$ ⇒ $Z_{eq} = \frac{\bar{V}_{sc}}{\bar{I}_{sc}} = \frac{16\angle 0^\circ}{30\angle -\cos^{-1}0.2}$

$$\Rightarrow Z_{eq} = \frac{16}{30}\angle 78.46 = \underset{R_{eq_1}}{\underset{\downarrow}{0.106}} + j\underset{X_{eq_1}}{\underset{\downarrow}{0.5225}}$$

Now

$I_{sc2} = \frac{V_{sc}}{Z_{eq_2}}$

$Z_{eq2} = R_{eq2} + jX_{eq_2} = 0.106 + j\left(\frac{0.5225}{2}\right)$

$$\Rightarrow I_{sc2} = \frac{16\angle 0^\circ}{0.106 + j\_\_} = 56.7\angle -67.78^\circ$$

$\downarrow$

$\cos 67.76 = 0.378$

Q5. A 1 φ X-fmer has no load loss of 64W as obtained from O.C test. When an S.C test is performed on it with 90% of rated current flowing both in LV and HV windings, the measured loss is 81W.

The X-fmer has maximum efficiency when operated at __80__ % of rated current

Under maximum efficiency condition,

$$P_i = P_{cua} = x^2 P_{cu}$$

$$\Rightarrow x = \sqrt{\frac{P_i}{P_{cu}}} = \sqrt{\frac{64}{100}} = 0.8$$

$$= 80\%$$

$$P_{cua} = x^2 P_{cu}$$

$$\Rightarrow P_{cu} = \frac{81}{(0.9)^2} = 100W.$$

Ans.

~~$\eta_{max} = \frac{x\,VA_{rated}\cos\phi}{x\,VA_{rt}\cos\phi + P_i + x^2 P_{cu}}$~~

Q6. A 10KVA, 400/200V 1 φ transformer with 10% impedance draws a steady state short circuit current of __250 A__.

→ 0.1 pu

$$\rightarrow I_{sc\,(steady\ state)} = \frac{V_{sc}}{Z_{eq}} = \frac{V_{sc\,rated}}{Z_{eq}}$$

$$\rightarrow I_{sc\ steady\ state}\,(p.u) = \frac{1}{Z_{eq}(pu)} = \frac{1}{0.1} = 10$$

$$I_{sc_{pu}} = \frac{I_{sc\,actual}}{I_{sc\,base}} \Rightarrow I_{sc_{Act}} = I_{sc_{pu}} \times I_{b}$$

rated current

$$I_{sc_{base}} = \frac{10 \times 10^3}{400} =$$

$$I_{sc} = \frac{10 \times 10 \times 10^3}{400} = \frac{100}{4} = 250\,A$$

method 2

$$Z_{base} = \frac{V_{rated}}{I_{rated}} = \frac{V^2_{rated}}{VA_{rated}}.$$

$$I_{sc_{steady\ state}} = \frac{V_{sc}}{Z_{eq}} = \frac{400}{0.1 \times \frac{(400)^2}{10 \times 10^3}}$$

$$\Rightarrow I_{sc} = 250\,A$$

Q7. An 11/0.4 kV, 25 Hz, 1 ϕ Xfmr has ohmic, hysteresis and eddy current losses of 1.8%, 0.8%, 0.3%, respectively.
The Xfmr is now operated at 22 kV, 50 Hz supply.
The current is assumed to remain same in both cases.
The %age increase in efficiency is ______.

$$P_{cu_{(pu)1}} = 0.018, \quad P_{h_1 pu} = 0.008, \quad P_{e_1 pu} = 0.003$$

$$V_1 = 11V, \quad f = 25\,Hz$$

Now, $V_2 = 22V$, $f_2 = 50Hz$

$\therefore \; P_w = I_{fl}^2 R_{eq} \rightarrow$ actual value

$$P_{w(pu)} = \frac{I_{fl}^2 R_{eq}}{V_{rated} \, I_{rated}} \Rightarrow P_w \propto \frac{1}{V_{rated}}$$

$$\Rightarrow \frac{P_{w_2 pu}}{P_{w_1 pu}} = \frac{V_{rated\,1}}{V_{rated\,2}} = \frac{11}{22} = \frac{1}{2}$$

$$\Rightarrow P_{w_2 pu} = \frac{P_{w_1 pu}}{2} = \frac{0.018}{2} = 0.009$$

$$\Rightarrow P_{i_1 pu} = 0.008 + 0.003$$

$\circ$ checking V/f ratio.

$$\frac{V_1}{f_1} = \frac{V_2}{f_2} \Rightarrow \frac{11}{25} = \frac{22}{50} = 0.44$$

$$\Rightarrow B_m = \text{constant}$$

$$\Rightarrow P_{e\,pu} \propto \frac{f^2}{VA} , \quad P_{h\,pu} \propto \frac{f}{VA}$$

current is also constant

$$\Rightarrow P_{e\,pu} \propto \frac{f^2}{V} , \quad P_{h\,pu} \propto \frac{f}{V}$$

$$\Rightarrow \frac{P_{e2_{pu}}}{P_{e1_{pu}}} = \left(\frac{50^2}{22\times10^3}\right) \times \frac{1}{\frac{(25)^2}{11\times10^3}} = 2$$

$$\Rightarrow P_{e2_{pu}} = 2\,P_{e1_{pu}} = \underbrace{2\,(0.003)}_{\text{true in pu.}}$$

$$\Rightarrow P_{e2_{pu}} = 0.006$$

Now for P_h.

$$\frac{P_{h2_{pu}}}{P_{h1_{pu}}} = \left(\frac{50}{22\times10^3}\right) \times \left(\frac{11\times10^3}{25}\right) = 1$$

$$\Rightarrow P_{h2_{pu}} = P_{h1_{pu}} = 0.008$$

↳ Hysteresis loss in pu will not change.
↳ The actual value will change however.

$$\eta_1 = \frac{n\,VA_{rated}\cos\phi}{n\,VA_{rated}\cos\phi + P_i + n^2 P_{cu}} = 97.18\ \%$$

$$\eta_{1_{pu}} = \frac{n\times1\times1}{n + 0.011 + \underbrace{n^2(0.018)}_{\text{assume } n=1.}}$$

$$\Rightarrow \eta_{1_{pu}} = 0.9718\ pu.$$

Now $$\eta_2 = \frac{1\times1}{1\times1 + (0.006+0.008) + 0.055}$$

$$= 0.977\,pu \qquad = 97.75\ \%$$

% age increase = ?

$$\% \eta = \frac{\eta_2 - \eta_1}{\eta_1} = \frac{0.9775 - 0.9718}{0.9718}$$

$$= 5.865 \times 10^{-3} \times 100\%$$

$$= 0.5865\ \%$$

Ans.

Name plate rating of transformer

eg 50 KVA , 220/110 V, 50 Hz

50 KVA → Rated KVA

220 → HV rated voltage

110 → LV rated voltage → At no load condition

50 Hz → operating frequency

Rated means at manufacturer end. It does not mean full load values.

Voltage regulation, $\boxed{V_R = \dfrac{V_{no\,load} - V_{load}}{V_{rated}}}$

and $\boxed{VR_{pu} = x\left[R_{eq_{pu}} \cos\phi \pm X_{eq_{pu}} \sin\phi\right]}$

+ → lag , − → lead

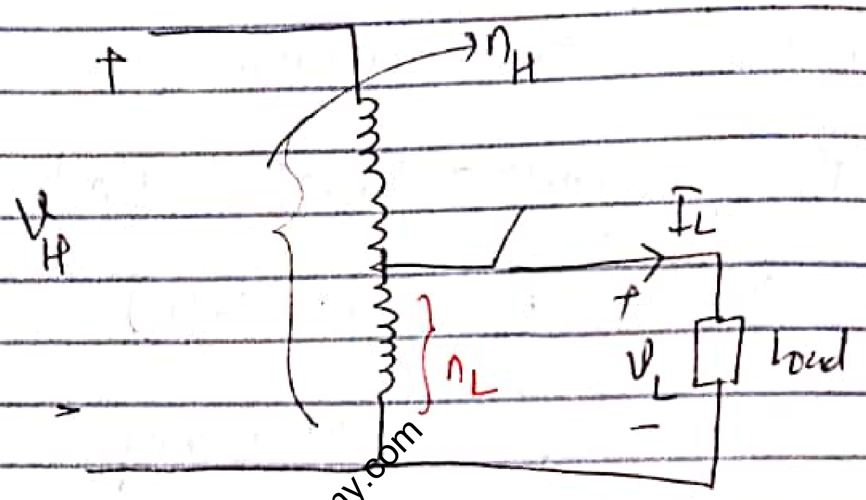
At the consumer end, we need the voltage to be constant (not fluctuating) but actually we have fluctuations due to voltage drops (depending on the loading condition). This change is called voltage regulation.

To study

Auto Transformer

In a simple two winding transformer, the windings (i.e. primary and secondary) are electrically isolated.

Whereas in auto transformer both the windings are electrically connected.

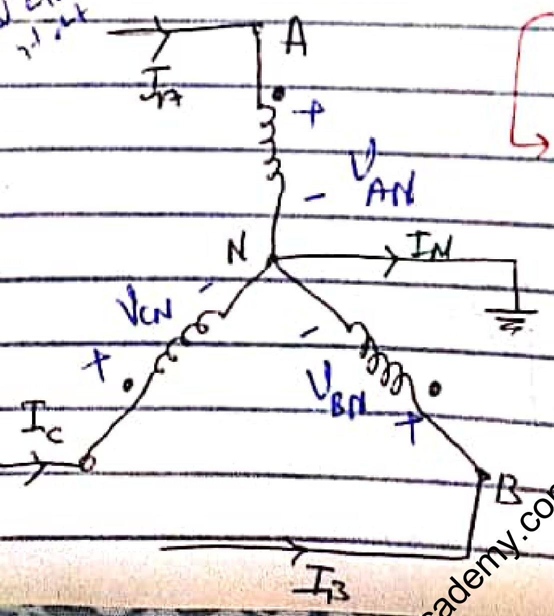


We can also convert a two winding Xformer into an auto Xformer. (There is a difference in a converted one and an independent auto Xformer so don't confuse it).

Three Phase Transformer

[illegible]

Star Connection



say phase sequence = ABC and balanced 3 Φ system.

Phase voltage = Line to neutral

→ magnitude is same while 120° apart.

$$\overline{V_{AN}} = V_p \angle 0^\circ = V_m \sin \omega t$$

→ ref

$$\overline{V_{BN}} = V_p \angle -120^\circ = V_m \sin(\omega t - 120^\circ)$$

$$\overline{V_{CN}} = V_p \angle -240^\circ = V_p \angle 120^\circ$$

$$= V_m \sin(\omega t - 240^\circ)$$

In star connection; $\boxed{I_L = I_p}$

and for a balanced system $\boxed{\bar{I}_A + \bar{I}_B + \bar{I}_C = 0}$

$\Rightarrow I_N = 0$

For an unbalanced system the summation may or may not be zero.

Line voltages (Line to line voltage)

$$\bar{V}_{AB} = \bar{V}_A - \bar{V}_B = V_m \sin\omega t - V_m \sin(\omega t - 120^\circ)$$

$$= V_m\left(\sin\omega t - \sin(\omega t - 120^\circ)\right)$$

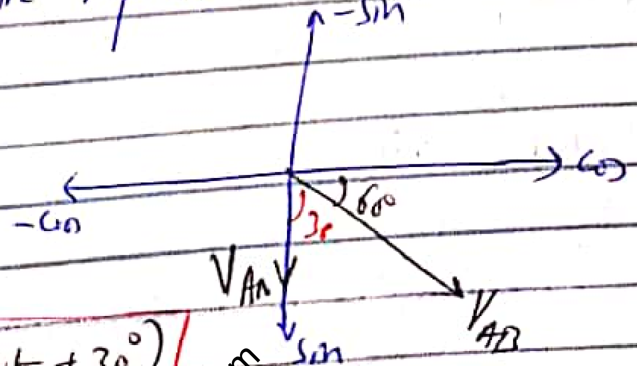
$$= 2 V_m \left[\cos\left(\frac{2\omega t - 120^\circ}{2}\right) \times \sin\left(\frac{120^\circ}{2}\right)\right]$$

$$\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$$

$$= \cancel{2} \times V_m \times \cos(\omega t - 60^\circ) \times \frac{\sqrt{3}}{\cancel{2}}$$

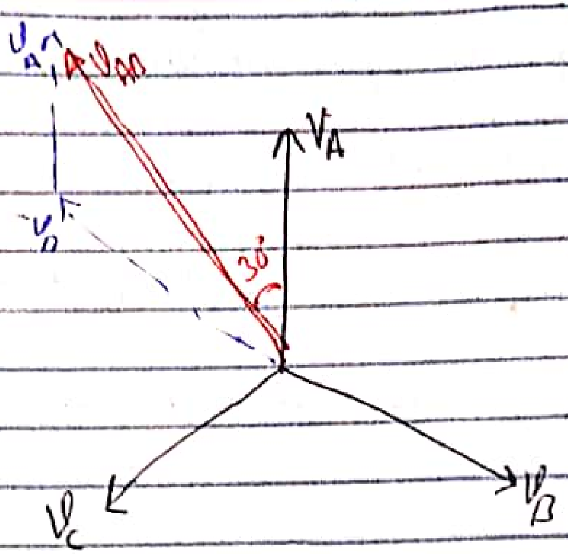
$$\bar{V}_{AB} = \sqrt{3}\, V_m \cos(\omega t - 60^\circ) = \sqrt{3}\, V_p \cos(\omega t - 60^\circ)$$

↳ Comparing with the reference:



$$\boxed{\bar{V}_{AB} = \sqrt{3}\, V_m \sin(\omega t + 30^\circ)}$$

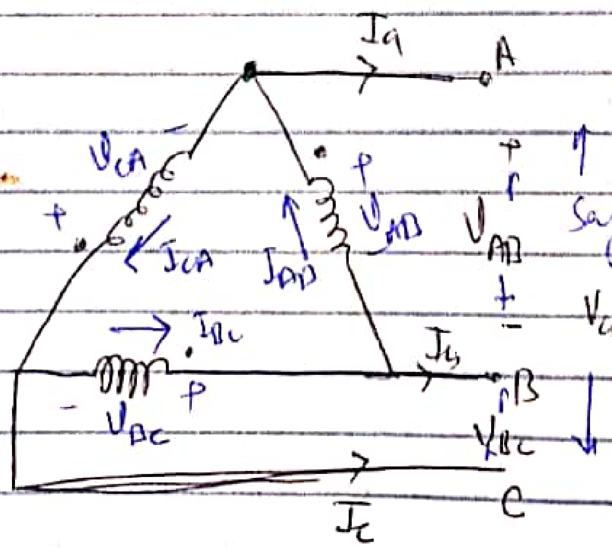
4 $$\boxed{\begin{aligned} V_{BC} &= \sqrt{3}\, V_{p,m} \sin(\omega t + 30 - 120^\circ) \\ V_{CA} &= \sqrt{3}\, V_p \sin(\omega t + 30^\circ + 120^\circ) \end{aligned}}$$



Line voltage will lead phase voltage by 30°

$$\boxed{|V_{AB}| = \sqrt{3}\, V_p}$$

Delta Connection



Assuming the delta connection

DC supply.

Current will leave at dot.

Say Phase sequence = ABC

balanced system

No neutral

~~Line voltages~~

(coil = phase) I_{AB}, I_{BC}, I_{CA} → phase currents

I_A, I_B, I_C → Line currents

In delta connection; $\boxed{V_L = V_p}$

Phase current

$$\boxed{\begin{aligned} I_{AB} &= I_p \angle 0^\circ = I_m \sin\omega t \\ I_{BC} &= I_p \angle -120^\circ = I_m \sin(\omega t - 120^\circ) \\ I_{CA} &= I_p \angle +120^\circ = I_m \sin(\omega t + 120^\circ) \end{aligned}}$$

↳ may be peak value or rms value ($I_{rms} = \frac{I_m}{\sqrt{2}}$)

Line currents

KCL ⇒ $I_A + I_{CA} = I_{AB}$

⇒ $I_A = I_{AB} - I_{CA} = I_m \sin\omega t - I_m \sin(\omega t + 120^\circ)$

⇒ $I_A = \sqrt{3} I_p \angle -30^\circ$

↳ lag phase current by 30°

$I_A = \sqrt{3} I_p \angle -30^\circ$

$I_B = \sqrt{3} I_p \angle -30^\circ - 120^\circ$

$I_C = \sqrt{3} I_p \angle -30^\circ + 120^\circ$

Power

$\boxed{\text{Instantaneous power} = 3 V_{p_{rms}} I_{p_{rms}} \cos\phi = P_{avg}}$

↳ does not vary w.r.t time.

$P_{avg} = \sqrt{3}\, V_{L_{rms}}\, I_{L_{rms}} \cos\phi$

or $= \sqrt{3}\, V_{p_{rms}}\, I_{p_{rms}} \cos\phi$

Reactive pwr:

$Q = \sqrt{3}\, V_{L_{rms}}\, I_{L_{rms}} \sin\phi$

or $= \sqrt{3}\, V_{p_{rms}}\, I_{p_{rms}} \sin\phi$

~~Comp~~ VA pwr $= \sqrt{3}\, V_{L_{rms}}\, I_{L_{rms}}$

Complex $= \sqrt{3}\, V_{L_{rms}}\, I_{L_{rms}}^{*}$

Connections of 3 φ Xformer

Group 1 → 12 ° clock → 0° → Yy_0 and Dd_0 → } Y-Δ

Group 2 → 6° clock → 180° → Yy_6 or Dd_6 } or Δ-Y

Group 3 → 1° clock → 30° lag → Yd_1 or Dy_1 } Y-Δ

Group 4 → 11 ° clock → 30° lead → Yd_{11} and Dy_{11} } or Δ-Y

Δ - Δ or Y - Y → even number

Δ - Y or Y - Δ → odd number

→ The degrees are always checked for secondary & primary is the reference.

eg 0° represents that secondary is shifted 0° w.r.t primary

180° " " " " 180° " ".

→ All the degrees are for line voltages. (not for phase voltages)

→ For any grp connection, the phase voltages in primary and secondary are always in phase.

$$\boxed{a = \frac{V_{p1}}{V_{p2}} = \frac{I_{p2}}{I_{p1}}}$$

↳ turns ratio always in terms of line voltages.

↳ In line voltage, it may or not be.

→ capital letter = HV   small letters = LV

→ Hour = LV, minute = HV

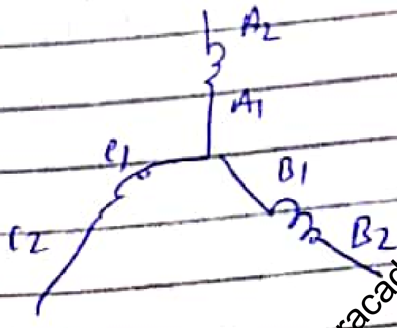
→ Phase sequence ABC, abc

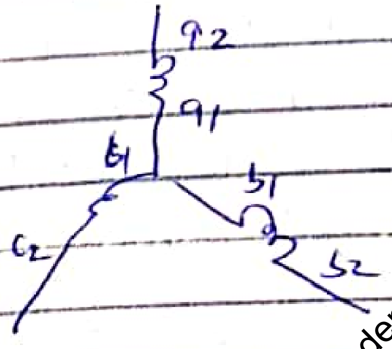
→ Y has neutral, Δ no neutral (may may).

→ All the terminals (i/p or o/p) will have a subscript 2.

$\left\{ \begin{matrix} A_2 \\ B_2 \\ C_2 \end{matrix} \right\}$ ↳ ip

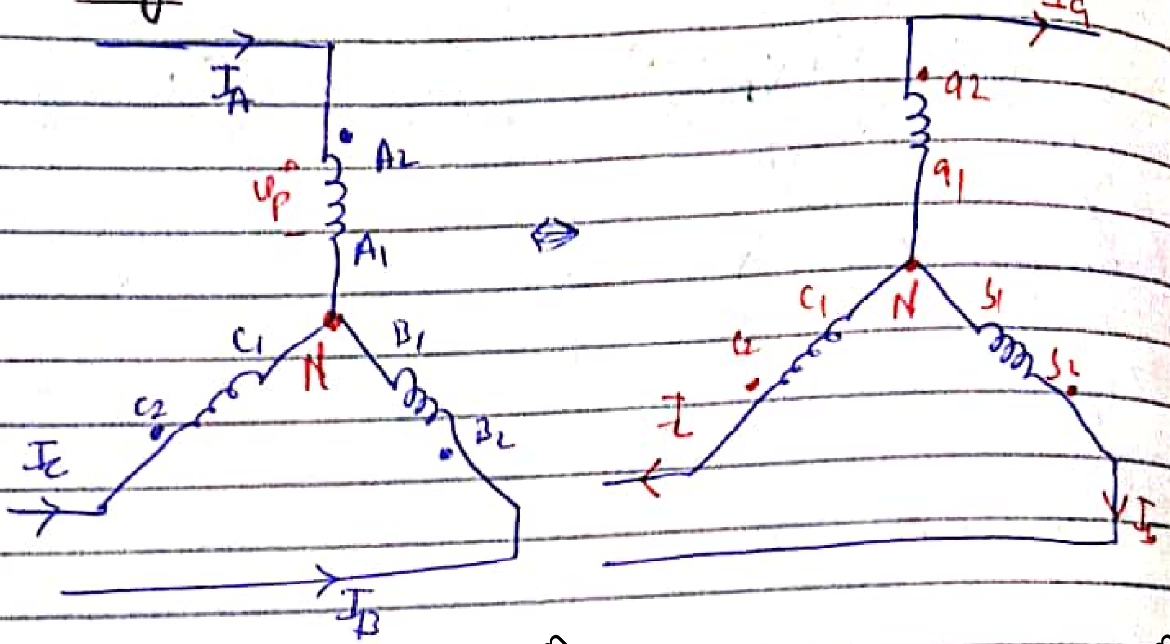
$\left\{ \begin{matrix} a_2 \\ b_2 \\ c_2 \end{matrix} \right\}$ → load end



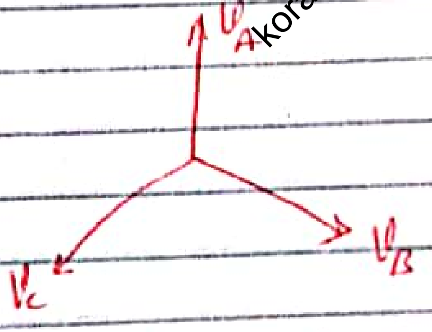


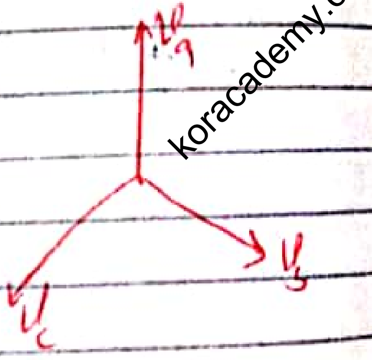
Group 1

① Yy0

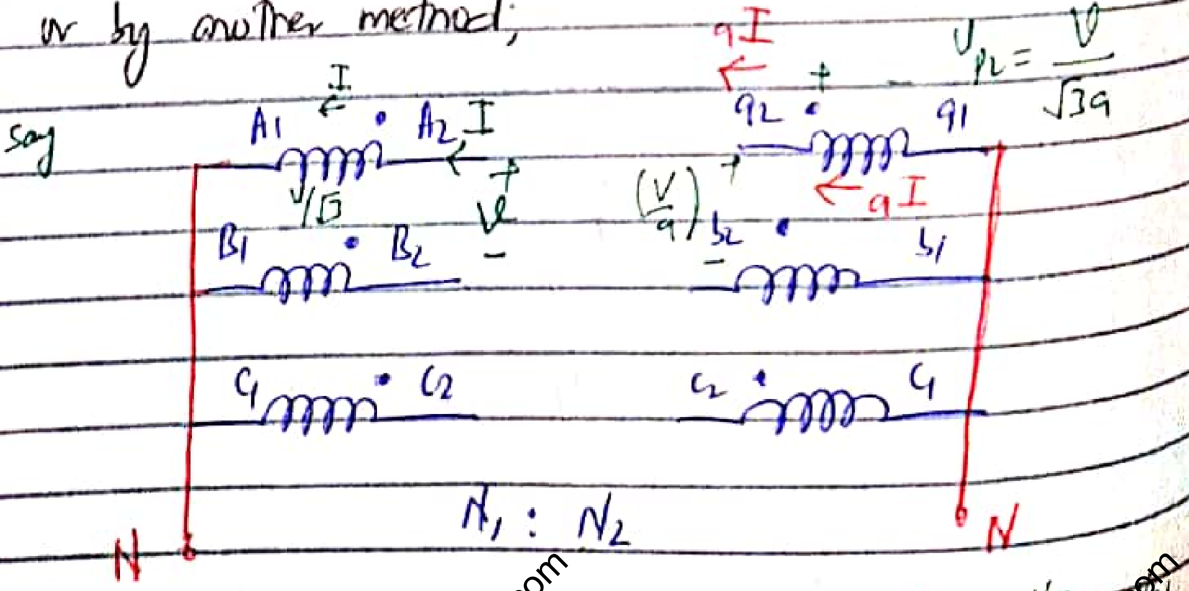


$V_p = V_{A2}V_{A1}$



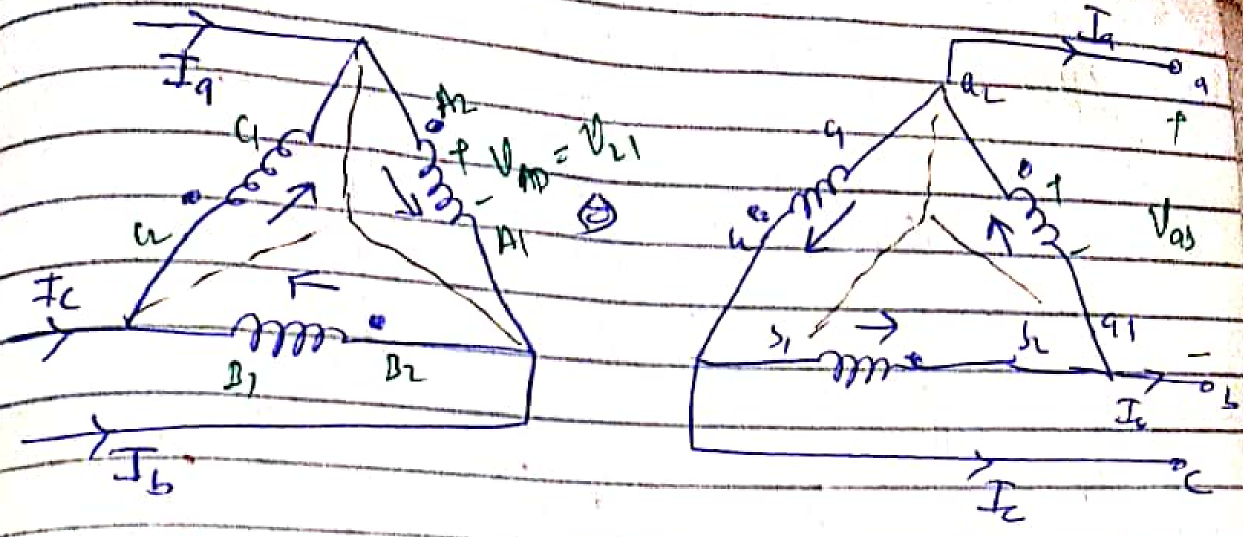


or by another method;

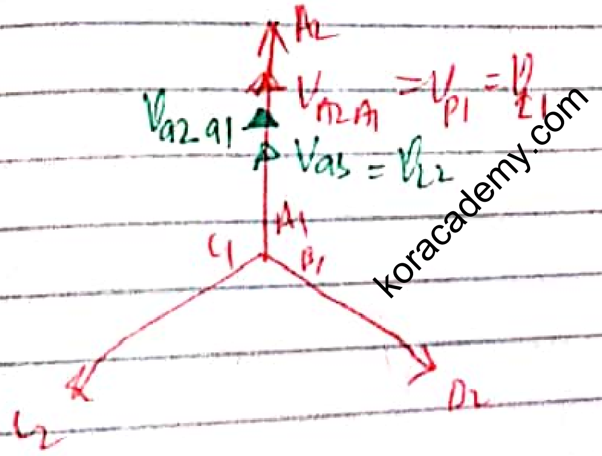


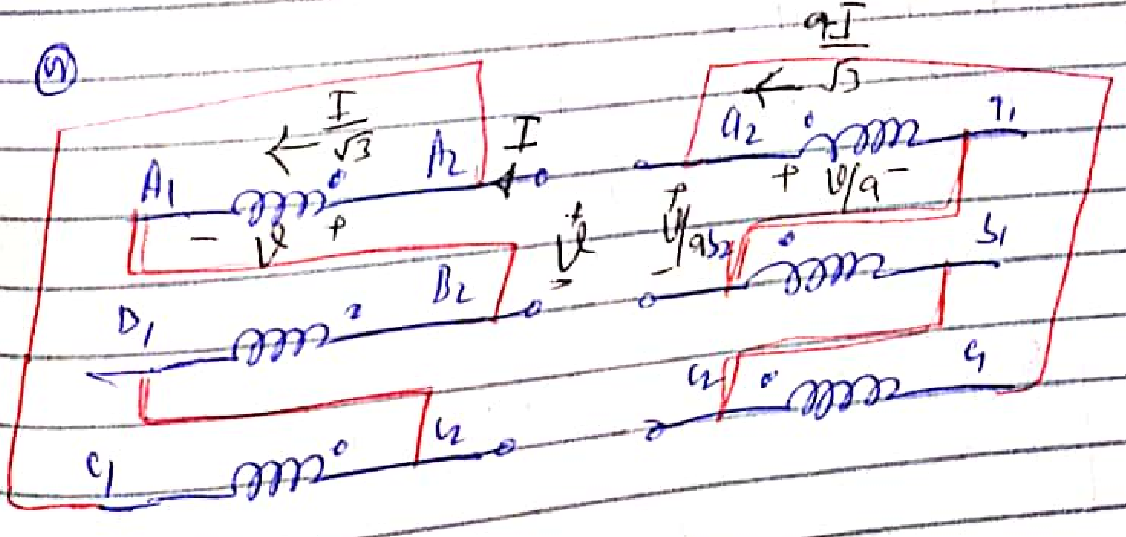
$a = \frac{V_2}{V_1} \Rightarrow V_p = \frac{V_{p1}}{a}$

② Dd0



$V_p = V_L = V_{A2A1} = V_{AB}$





undotted connected to dotted

$$\frac{V_{p2}}{V_{p1}} = \frac{1}{a} \quad V_{p2} = (\frac{1}{a}) V_{p1}$$

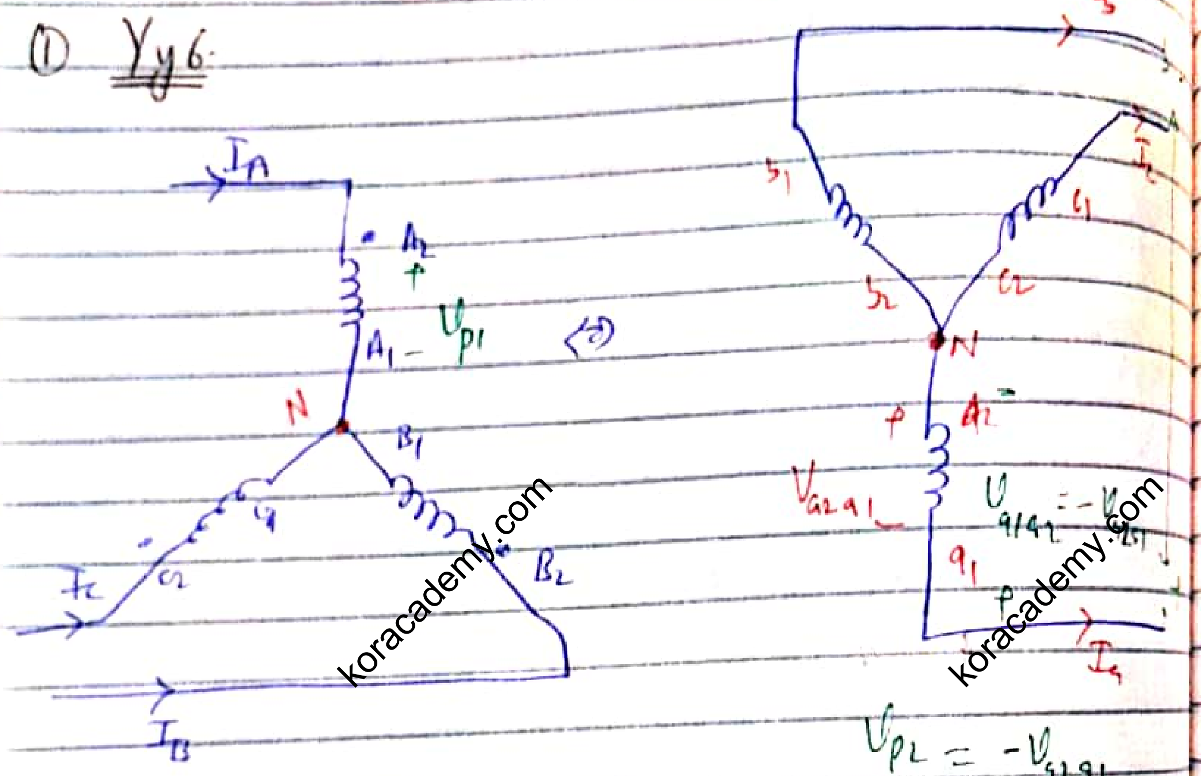
$$I_{p2} = a I_{p1}$$

Power, $P = \sqrt{3}\, V I$

$= 3 V_{ph} I_{ph} = 3 V . \frac{I}{\sqrt{3}}$

Group 2 (180° connection). ⅄ ↔ Y

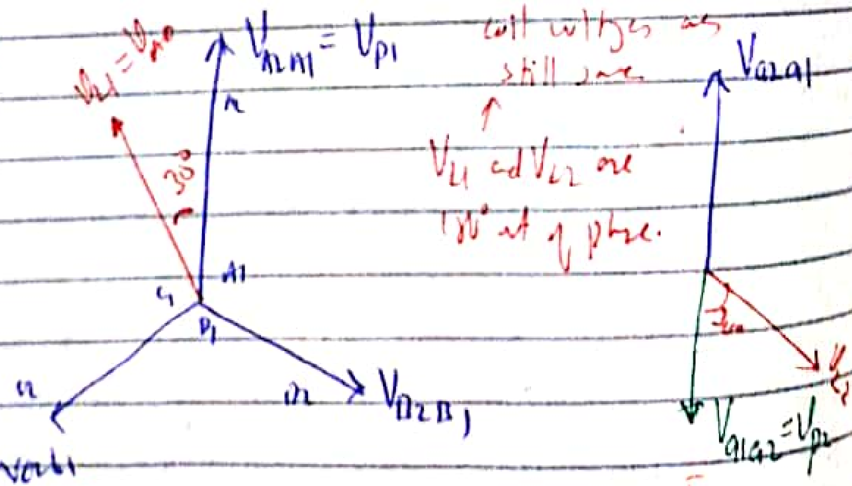
① Yy6.



$V_{p2} = -V_{a2a1}$

$V_{p1} = V_{A2A1}$

$V_{A2} = V_{L1}$

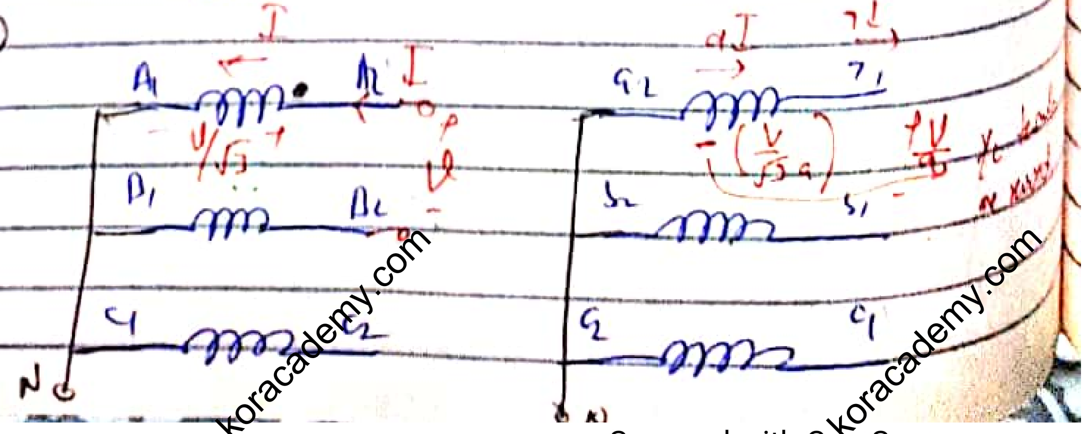


coil voltages are still same

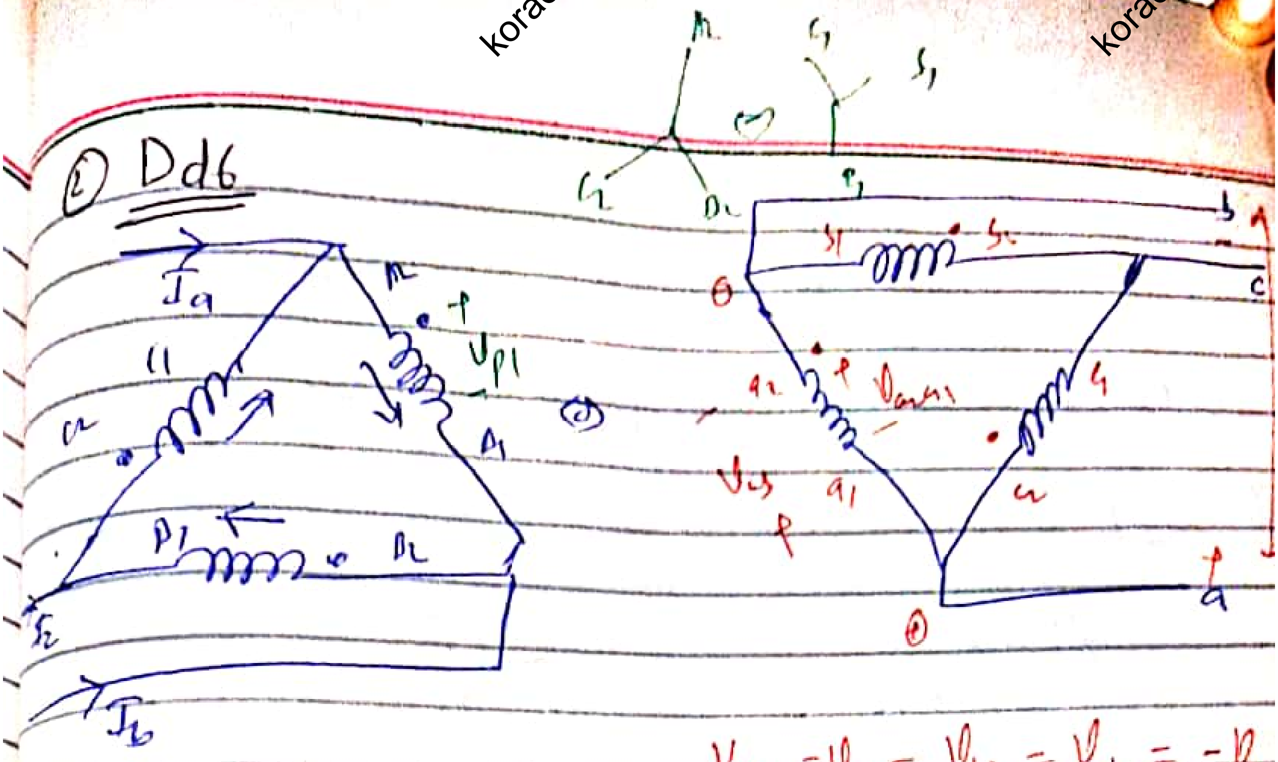
↑

V_{L1} and V_{L2} are 180° out of phase.

②



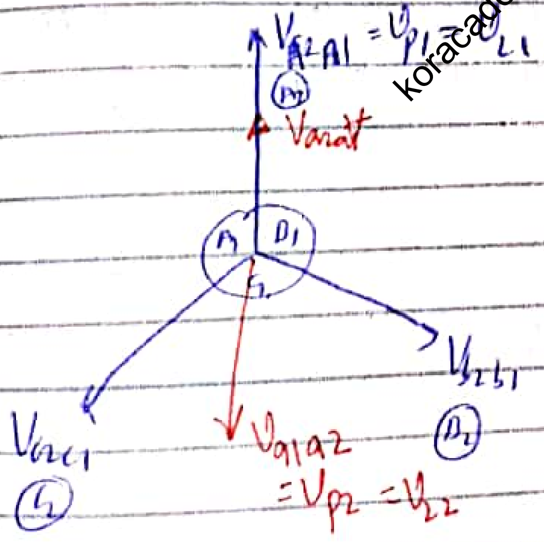
② Dd6



$V_{a1a2} = V_{p2} = V_{L2} = V_{a5} = -V_{a1}$

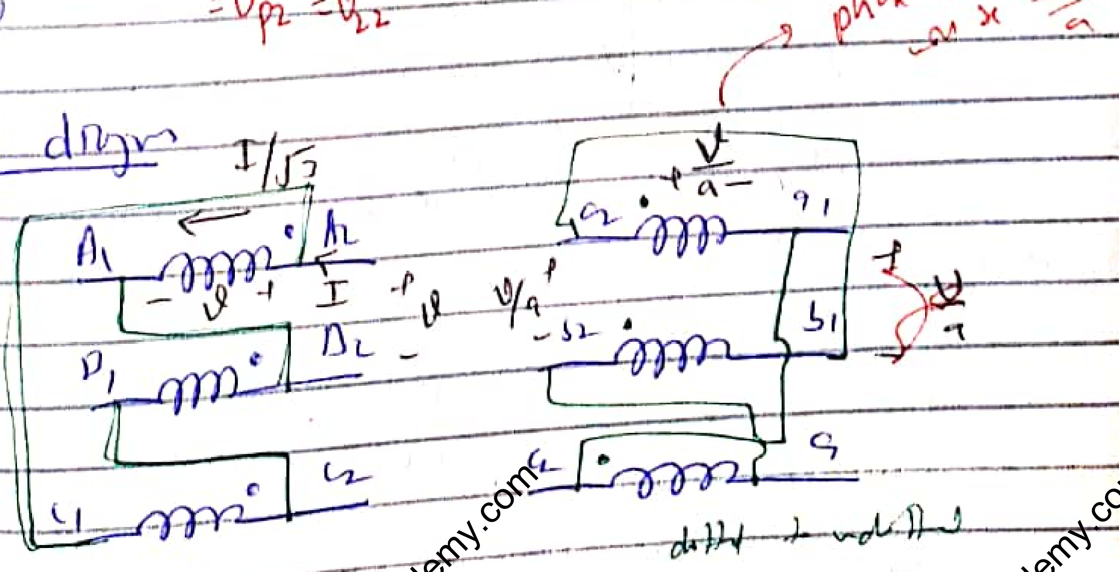
$V_{p1} = V_{a2a1}$.

$V_{AB} = V_{L1} = V_{a2a1}$

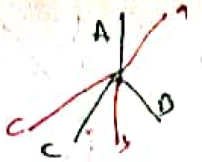


phase with ~~an~~ $= -\frac{V}{a}$

coil diagram

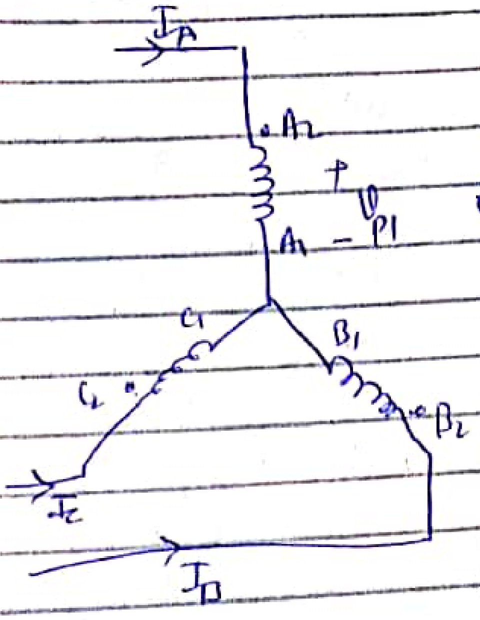


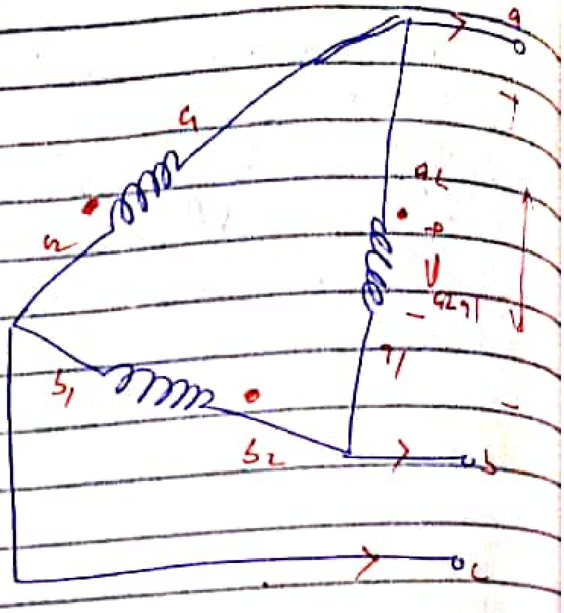
undotted to dotted.



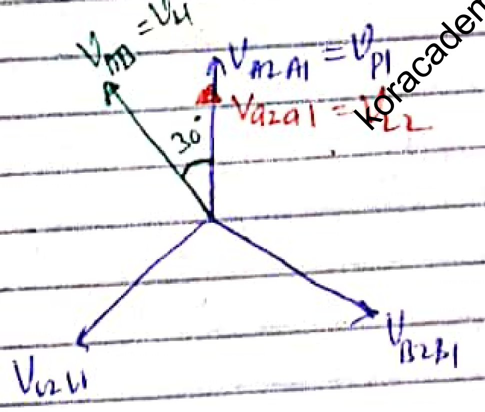
Group 3

① Yd_1





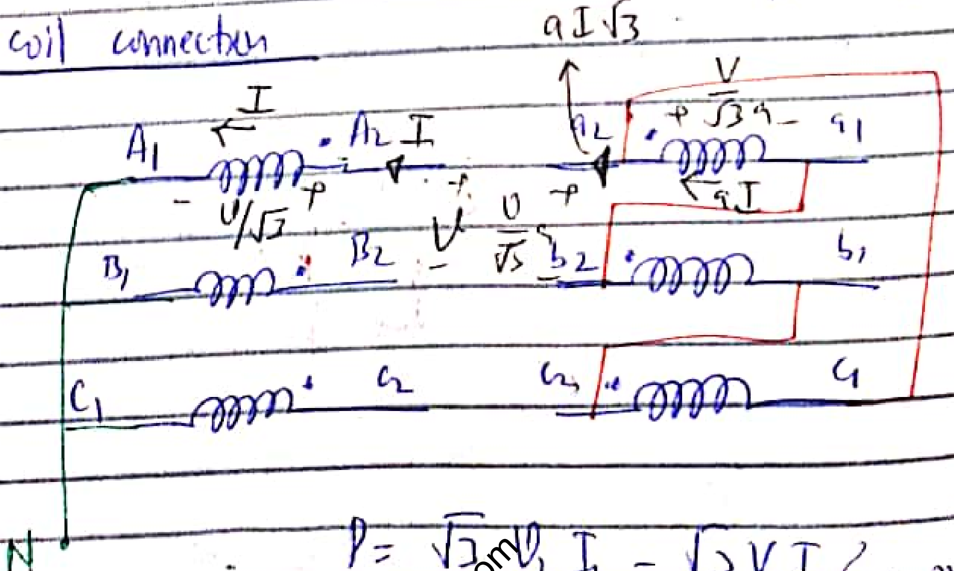
$V_{ab} = V_{L2} = V_{a2a1}$



$V_{P2} = \frac{1}{a} V_{P1}$

$I_{P2} = a I_{P1}$

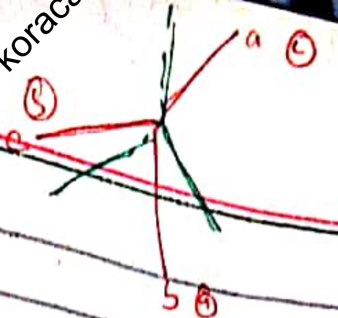
coil connection

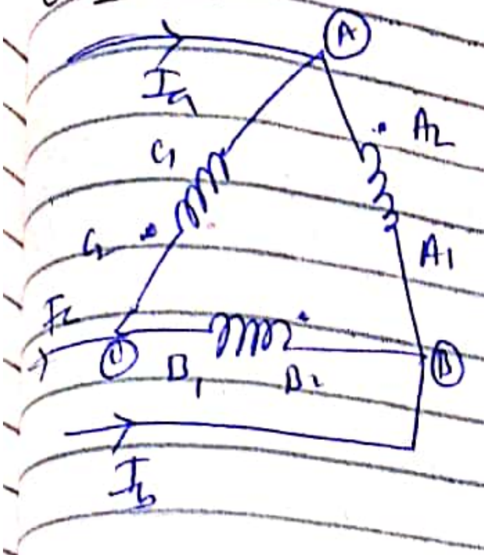


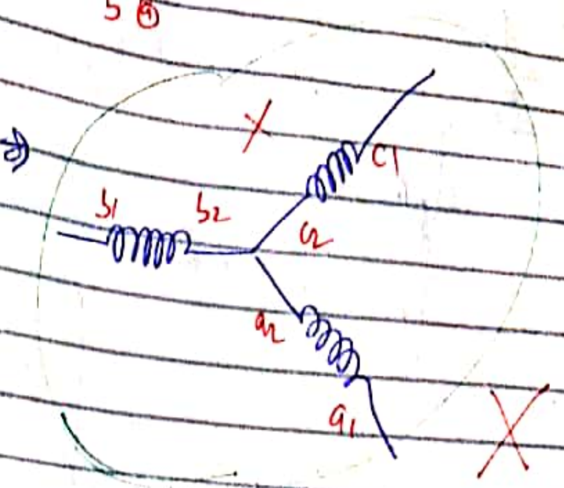
$P = \sqrt{3} V_2 I_2 = \sqrt{3} V I \Big\} \rightarrow$ phig

$= 3 V_{P2} \cdot I_{P2} =$

⑪ Dy1



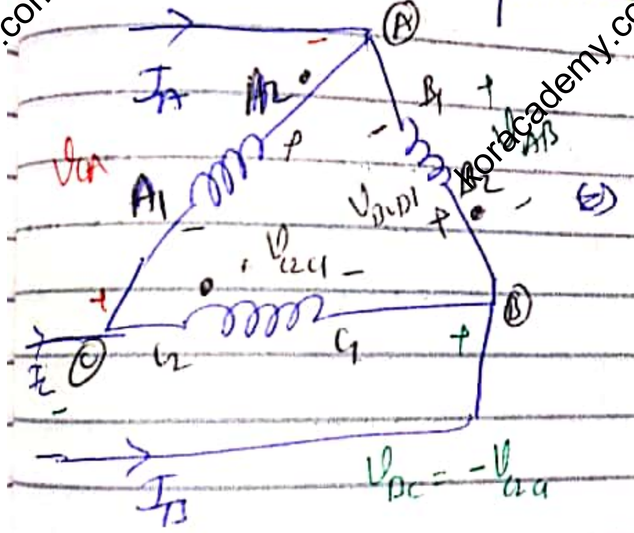


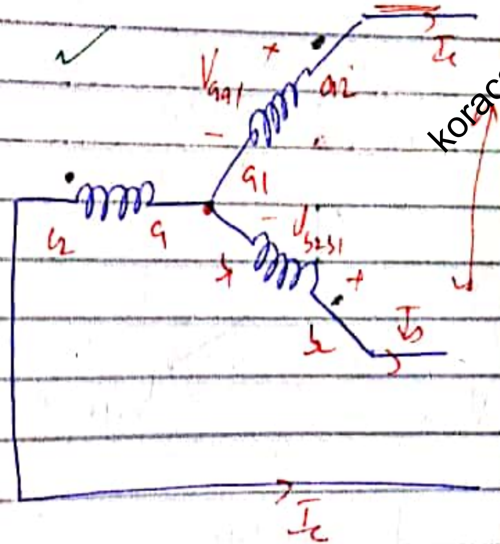


See the shift of ⓐ

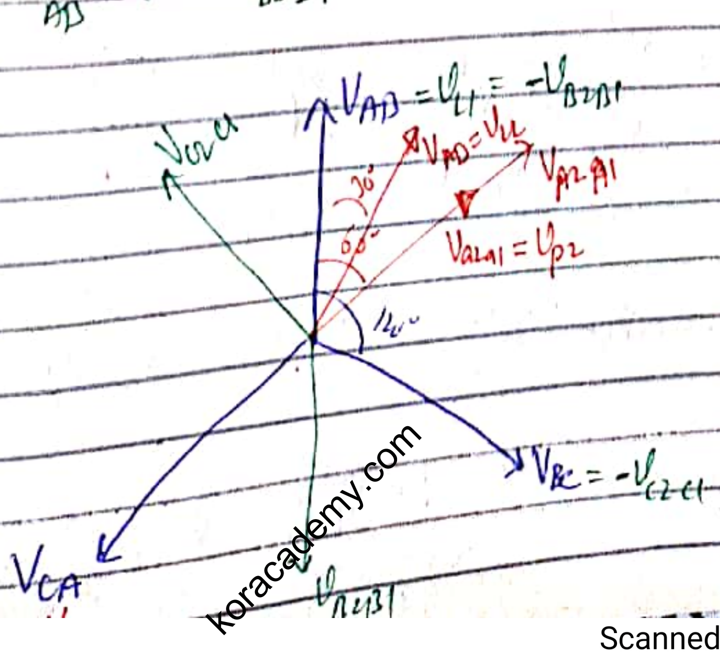
↳ Lagging by 30°..

So we do some manipulation in delta

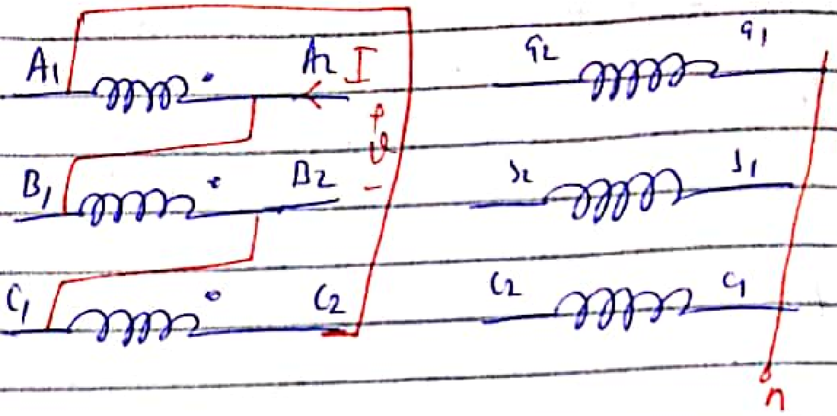




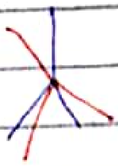
$$V_{P1} = V_{L1} = V_{AB} = V = -V_{B2B1}$$



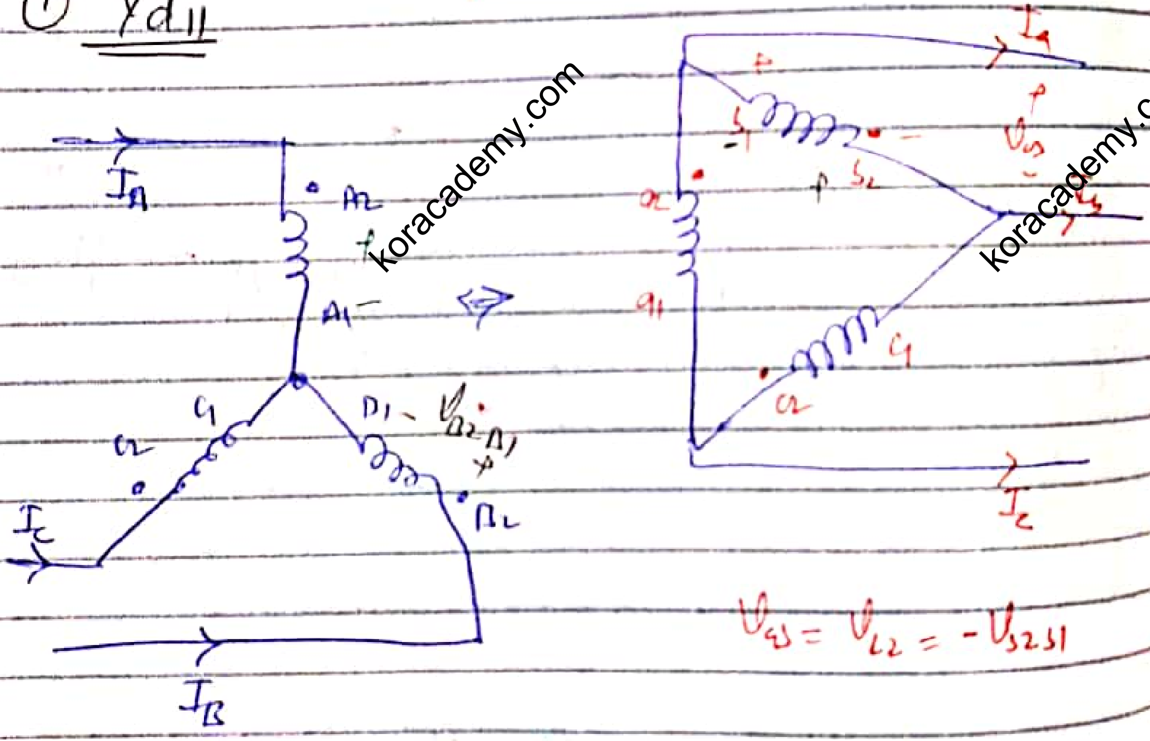
coil connection



Group 4



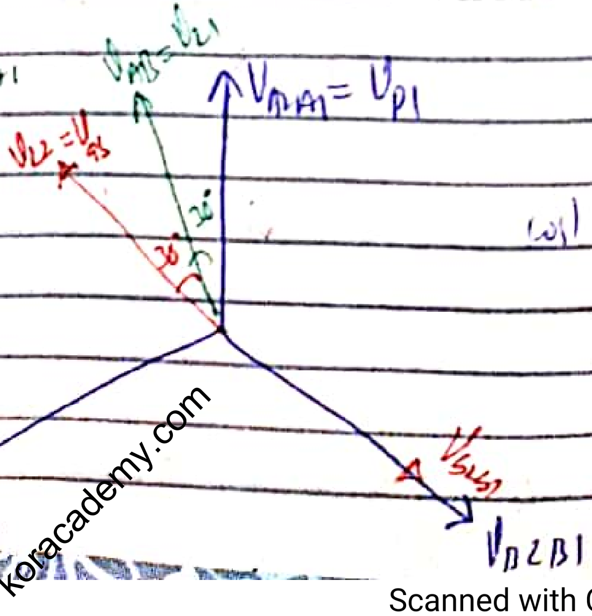
① Yd11



$V_{a3} = V_{L2} = -V_{s2s1}$

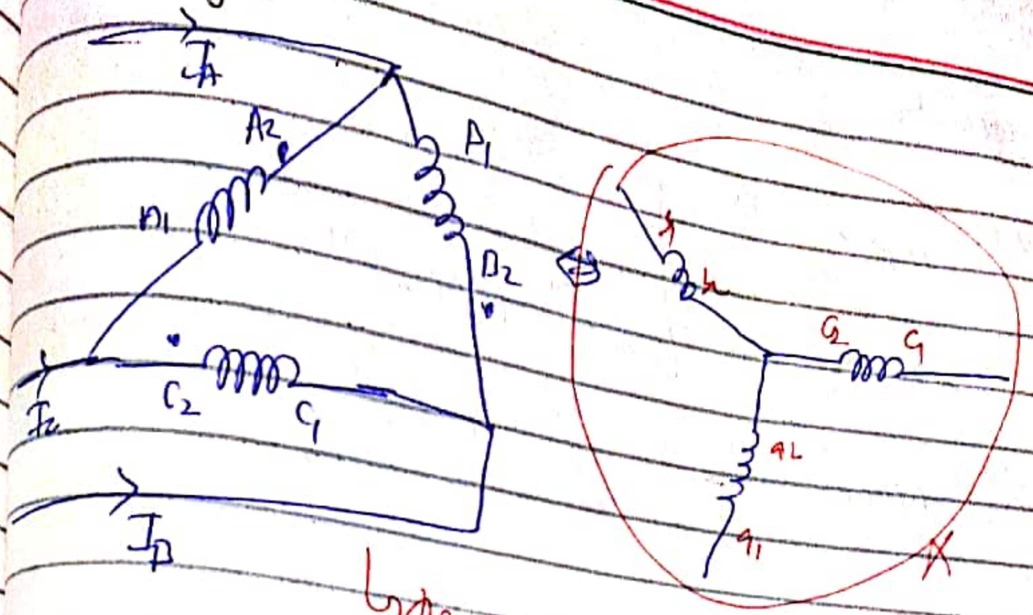
$V_{P1} = V_{AN} = V_{A2A1}$

V_{L2} leads V_{L1} by 30°



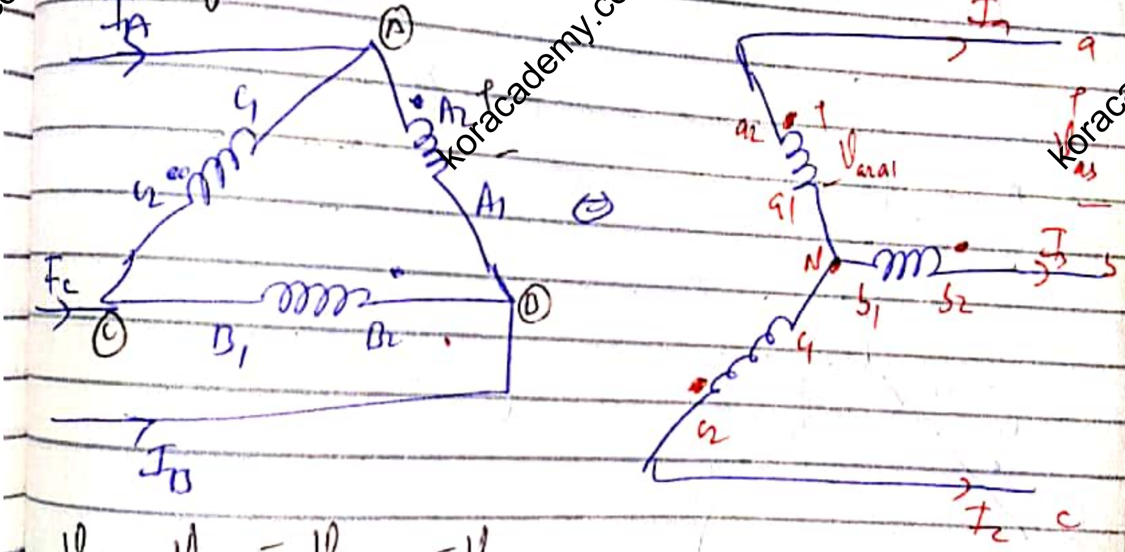
coil connection

② Dy 11



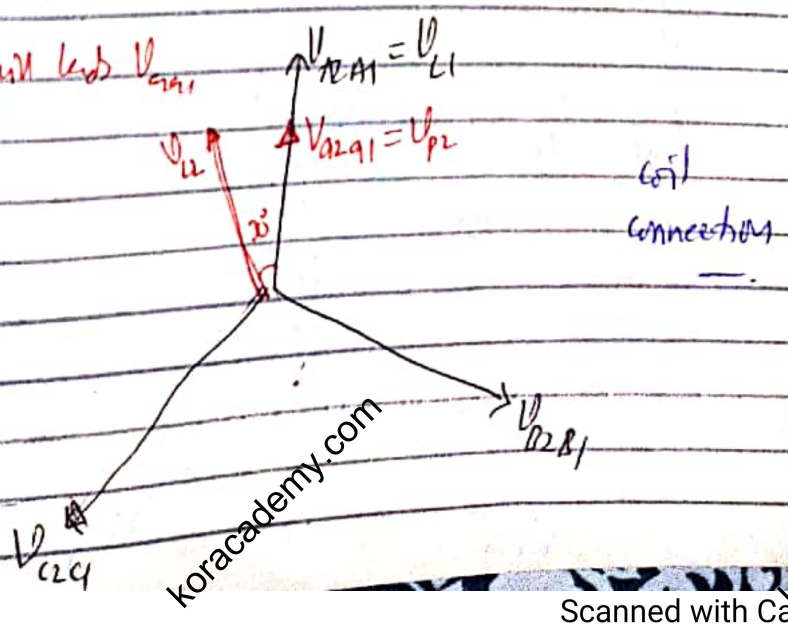
↳ this connectr any n Dy 1

Dy 11

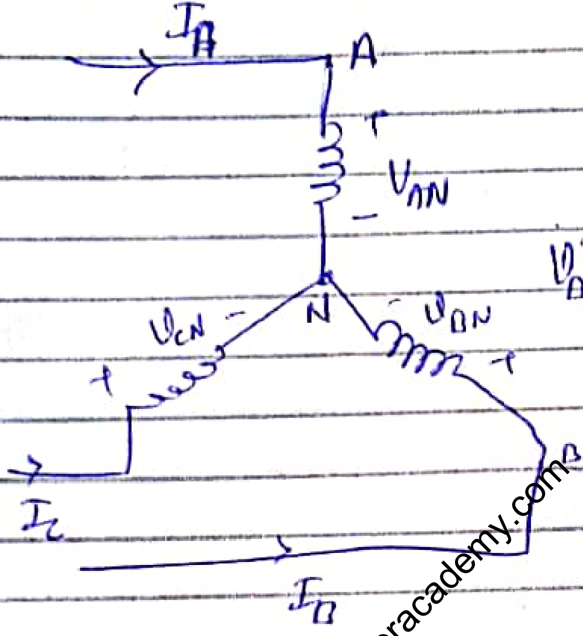


$V_{P1} = V_{L1} = V_{A2A1} = V_{AB}$

$V_{ab} = V_{L2}$ will lead V_{a2a1} by 30°



Q1 The line A to neutral voltage is $10\angle 15^\circ$ for a balanced 3ϕ Y connected load with phase sequence ABC.
The voltage of line B wrt C will be ____?



$V_{BC} = ?$

$$V_{AN} = \overset{\text{rms}}{10}\angle 15^\circ = 10\sqrt{2}\sin(\omega t + 15^\circ)$$

$$V_{BN} = 10\angle 15^\circ - 120^\circ$$
$$= 10\sqrt{2}\sin(\omega t + 15^\circ - 120^\circ)$$

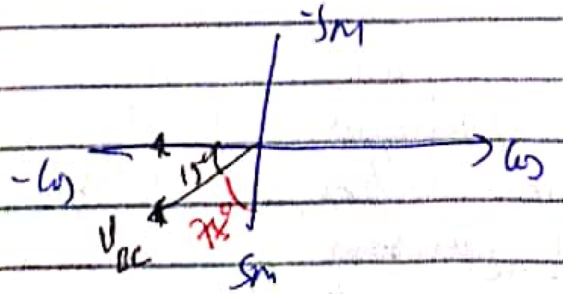
$$V_{CN} = 10\angle 15^\circ + 120^\circ$$
$$= 10\sqrt{2}\sin(\omega t + 15^\circ + 120^\circ)$$

$$\overline{V}_{BC} = \overline{V}_{BN} - V_{CN} = 10\sqrt{2}\sin(\omega t - 105^\circ) - 10\sqrt{2}\sin(\omega t + 135^\circ)$$

$$= 10\sqrt{2}\left[2\cos\left(\frac{\omega t - 105^\circ + \omega t + 135^\circ}{2}\right)\cdot\sin\left(\frac{-105^\circ - 135^\circ}{2}\right)\right.$$

$$\Rightarrow \overline{V}_{BC} = 20\sqrt{2}\cdot\sin(120^\circ)\cos(\omega t + 15^\circ)$$

$$\Rightarrow \overline{V}_{BC} = -24.49\cos(\omega t + 15^\circ)$$ Ans.



$$\Rightarrow V_{BC} = 24.49\sin(\omega t - 75^\circ)$$
$$= \frac{24.49}{\sqrt{2}}\angle -75^\circ$$

$$\Rightarrow V_{BC} = 17.32\angle -75^\circ$$

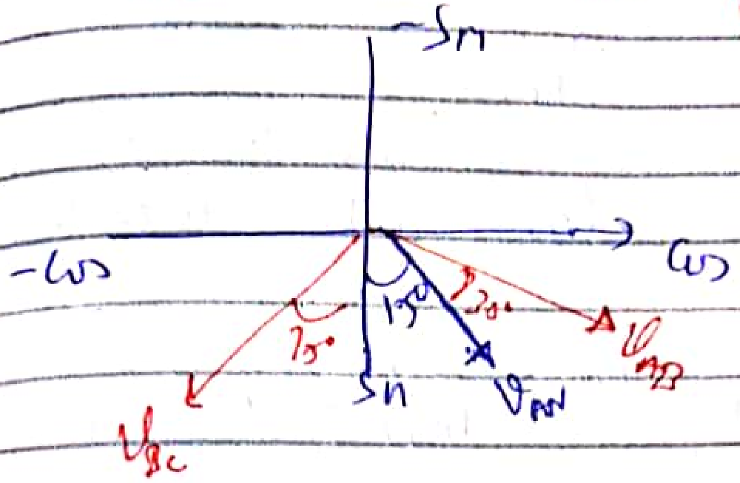
— Ans

method 2

$V_{AN} = 10\angle 15°$

$= 10\sqrt{2}\sin(\omega t + 15)$

In star connection;
V_{AB} will lead V_{AN} by 30°.



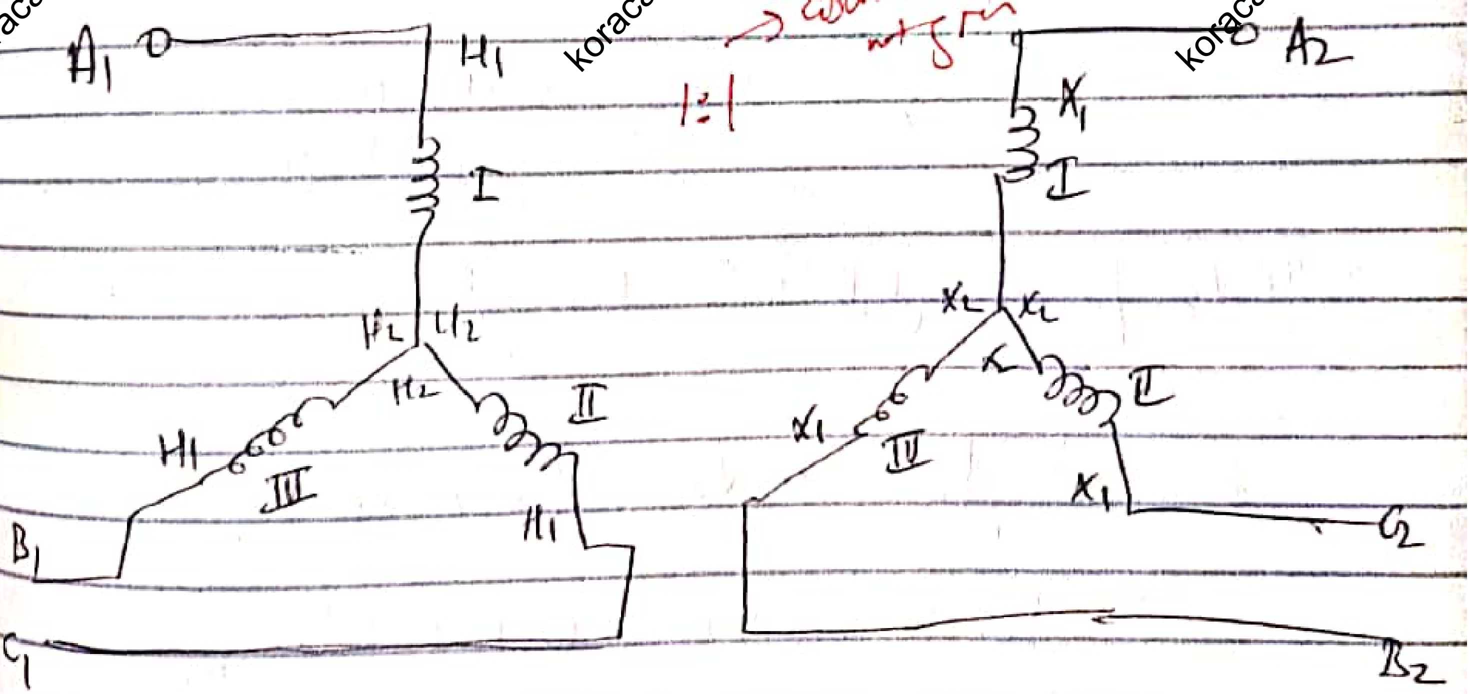
$|V_{AB}| = \sqrt{3}\, V_{p_{rms}}$

$= 10\sqrt{3}$

$\Rightarrow V_{BC} = 10\sqrt{3}\angle -75°$

Ans

Q2:



→ connected as not given

1:1

If the terminals H_1 and H_2 of phase I of the primary is reversed without changing the supply terminals A_1, B_1, C_1, then phase angle b/w line voltage V_{A2B2}, V_{C2A2} in secondary is 60°

Phs sq ACB (ccw ws) ACB

$V_{A1N} = V_p \angle 0° = V_{A2N}$

$V_{B1N} = V_p \angle -120° = V_{C2N}$

$V_{B1N} = V_p \angle 120° = V_{B2N}$

Now if terminals are reversed:

$V_{A1N} = -V_p \angle 0° = V_{A2N}$

$V_{C1N} = V_p \angle -120° = V_{C2N}$

$V_{B1N} = V_p \angle 120° = V_{B2N}$

$\Rightarrow V_{A2B2} = V_{A2n} - V_{B2n} = -V_p \angle 0° - V_p \angle 120°$

$\Rightarrow V_{A2B2} = V_p \angle -120°$

$V_{C2A2} = V_{C2n} - V_{A2n} = V_p \angle -120° - (-V_p \angle 0°)$

$\Rightarrow V_{C2A2} = V_p \angle -60°$

Q3. An 120KVA, 6000/400V, Y-Y 3φ, 50 Hz transformer has min loss of 1600W. The Xfmer's maximum efficiency occurs at 3/4 full load. Find the efficiency at full load unity power factor.

$P_i = 1600 W, \quad x\big|_{\eta_{max}} = \frac{3}{4}$

$$\eta_x = \frac{x VA \cos\phi}{x VA \cos\phi + P_i + x^2 \boxed{P_{cu}}} \quad —①$$

$\hookrightarrow ?$

at max effi.

As $P_i = P_{cu} = n^2 P_{cu}$
but η_{max}

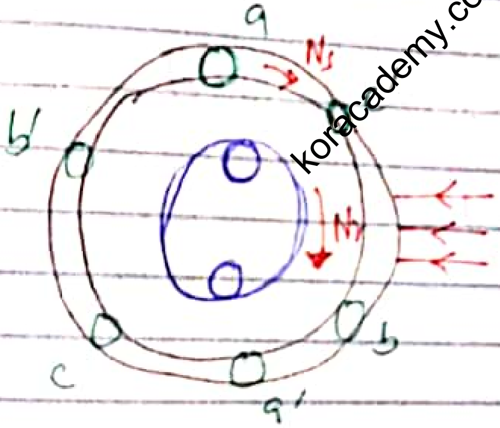
$n = \sqrt{\frac{P_i}{P_{cu}}}$ $\Rightarrow P_{cu} = \frac{P_i}{n^2} = \left(\frac{1600 \times 16}{9}\right)$

$P_{cu} = 2844.4 W$

Results

① $\Rightarrow$ $\eta = 95.57\%$ Ans

Operating Principle of 3 φ Induction motor



- stator
- rotor
- 3 φ conductors

If this is a motor we have a 3 Φ supply to the stator with f_s.

Stator windings are 120° displaced electrically.

(i) 3 φ supply → 3 φ stator winding → rotating magnetic field (stator RMF) rotating with a speed corresponding to stator supply frequency.
$N_s = \frac{120 f_s}{P}$ (say in clockwise direction. →

(ii) Stator winding is stationary and having a RMF (at N_s speed) around it.
Hence there will be a relative motion b/w stator conductor and stator RMF.

Speed of stator RMF wrt stator body = $N_s - 0 = N_s$

and induced emf $\propto$ Relative speed.

Hence there will be an induced emf in the stator conductor (winding).

$$\boxed{E_{ph\,(stator)} = E_{ph1} = 4.44\, f_s . \phi . T_{ph_1} K_{w1}}$$

at supply frequency.

[T_{ph} → no. of turns per phase.
K_{w_1} is a factor that represents that stator winding is a distributed winding.]

(iii) At starting or at stand still condition, the rotor is stationary. (b/c rotor requires torque and it is not yet present). $N_r = 0$.
Hence the rotor winding will also experience the cutting of stator RMF feild. (b/c the field developed in the stator is present in the air gap). and the rotor conductor will feel it as the stator winding does.

Speed of stator RMF w.r.t rotor at standstill condition $= N_s - 0 = N_s$

Hence induced emf $\propto$ relative speed.

$$\boxed{E_{ph\,rotor} = E_{ph\,2} = 4.44\, f_s . \phi . T_{ph2} . K_{w2}}$$

↳ its frequency $f_r = f_s$

b/c corresponding speed $= N_s$ [illegible] f_s

f_r → rotor frequency (or) frequency of induced voltage in rotor conductor.

(iv) Due to this induced emf at standstill condition, there will be a rotor current as rotor circuit is a closed circuit (assumed)

$$I_{r(\text{standstill})} = \frac{E_{ph_2}}{Z_r} \text{ amp.}$$

↳ AC current

(current in a conductor ⇒ flux)

(v) Here when current will flow, the rotor conductor will also produce MMF in such a direction that it opposes its cause. and this mmf will interact with the stator MMF in the air gap and produce torque on rotor body. (Electromagnetic torque or interaction torque).

Effect	Cause
Torque production	$\downarrow I_r \rightarrow \downarrow E_{ph_2}$ → Due to relative motion $= N_s - 0$

(to reduce relative speed) $N_s - N_r$ ↵
↳ clockwise

(vi) Now rotor will start rotation in clockwise direction to oppose its cause (Lenz law) and try to catch up the stator RMF speed (N_s)

Hence 3φ induction motor is a self starting machine.

(vii) Now under running condition

$\boxed{\text{Stator conductor}}$ — speed = 0

$\boxed{\text{Stator RMF}} \rightarrow N_s$

$\boxed{\text{Rotor}} \rightarrow N_r$

a. Speed of stator rmf wrt stator = N_s

b. Speed of stator RMF wrt rotor = $N_s - N_r$

Hence induced emf in rotor conductor,

$e \propto$ relative speed $\Rightarrow e \propto N_s - N_r$

$$E_{ph_2}|_{new} = 4.44\, f_r \cdot \phi \cdot T_{ph_2} \cdot K_{w_2}$$

Here will define a term slip (s) = $\frac{N_s - N_r}{N_s}$

(Ns will always be greater than Nr, b/c if they become equal ⇒ relative speed = 0 ⇒ induced voltage = 0 ⇒ torque = 0 ⇒ machine will not operate).

$\Rightarrow N_s - N_r = s N_s$

And frequency (f) ∝ relative speed.

⇒ At standstill condition; $f_r = f_s = \frac{PN_s}{120}$

At running condition; $f_r = \frac{P(N_s - N_r)}{120} = \frac{PsN_s}{120}$

$$\Rightarrow \boxed{f_r = s f_s}$$

$$\Rightarrow \boxed{E_{ph_2\,\text{running}} = 4.44\, s f_s \phi\, T_{ph_2} K_{w_2} = s E_{ph_2\,\text{Standstill}}}$$

As $0 < S < 1 \Rightarrow E_{ph_2\,\text{running}} < E_{ph_2\,\text{standstill}}$, $f_r < f_s$

⇒ Maximum emf will be induced in rotor when $N_r = 0$.

At standstill condition; $N_r = 0 \Rightarrow$ slip $= 1$

(viii). Under running condition, now induced voltage has been reduced and current will also reduce, but still rotor conductor have current hence there will be a rotor rotating magnetic field (RRMF).

Hence under running condition, there will be two mmfs in the air gap.

stator RMF ⟶ N_s → at f_s. (wrt stator)

rotor RMF ⟶ sN_s wrt stator

developed at rotor frequency $f_r = \frac{P(N_s - N_r)}{120}$

$s \cdot N_s$ → slip speed.

→ Speed of stator mmf wrt stator = N_s

→ Speed of rotor mmf wrt rotor = $sN_s = N_s - N_r$

→ Speed of rotor RMF wrt stator = $N_r + sN_s = N_s$

→ Speed of rotor RMF wrt stator mmf = $N_s - N_s = 0$

↳ Hence we will get a steady static torque and motor will continue its rotation at N_r always.

$(N_r < N_s)$.

Hence for any rotor speed, we will get steady state torque in 3ϕ induction motor.

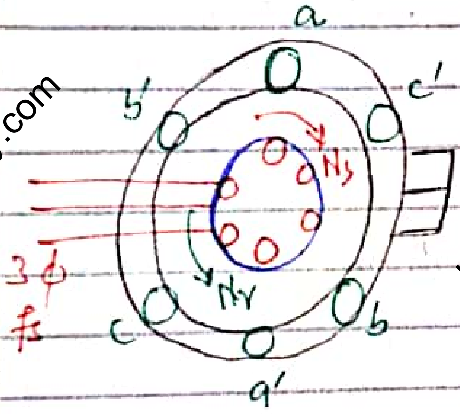
① Rotor RMF → $\downarrow sN_s$ = 50rpm

train ① Rotor body → ↓ $N_r = 1450$ rpm

train ② Stator RMF → ↓ $N_s = 1500$rpm

② Stator body → ↓ 0 rpm

Inverted Induction machine (Rotor Fed 3ϕ I.M)



3ϕ supply is given to rotor and stator winding is short circuited.

① Initially rotor is at rest condition and giving 3ϕ supply to 3ϕ rotor winding at supply freq.

3ϕ supply → 3ϕ (rotor winding) → rotor rotating magnetic field (RRMF) and rotating at supply frequency (f_s).

$$f_s = \frac{PN_s}{120} \longrightarrow N_s$$

$$\text{RRMF} \longrightarrow \downarrow N_s$$

Rotor conductor will experience a rate of change of flux in the air gap and hence there will be induced emf in the rotor conductor.

$e \propto$ relative speed $\propto N_s - 0 \propto N_s$

corresponding to fs ←

⇒ $\boxed{E_{ph_{rotor}} = 4.44 f_s . \phi . T_{ph_{rotor}} . K_{w_{rotor}}}$

* Air gap field → main feild flux → here RRMF

(ii) The stator conductor will also experience a change in flux and will also induce emf in stator conductor.

$e \propto$ relative speed $\propto N_s$ ↳ corresponding to fs.

$\boxed{E_{ph_{stator}} = 4.44 f_s . \phi . T_{ph_{stator}} K_{w_{stator}}}$

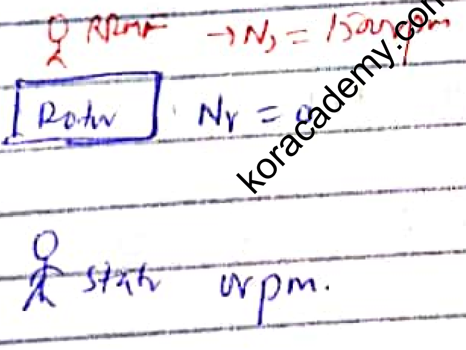
under this

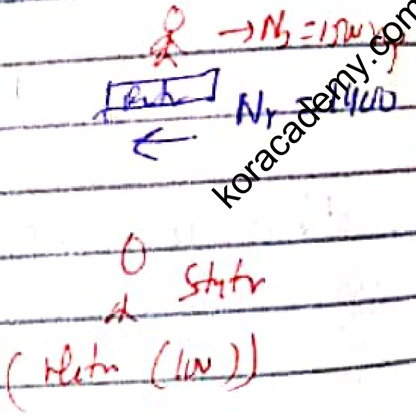
(iii). At standstill condition, the induced emf in stator conductor will drive a current as stator winding have a closed path through short circuit, hence will create its own MMF in such a direction that it opposes its cause;

Effect	Cause.
emf → I → stator magnetic field (stator RMF)	Relative motion b/w stator conductor and RRMF.

(Lenz's law) ⇒ Effect will oppose its cause will try to reduce the relative speed b/w stator conductor and RRMF.

⇒ But the stator cannot rotate, so the result will be that rotor will move in the opposite direction with speed N_r. ←





Now the speed of rotor RMF w.r.t stator will be
$N_s - N_R = s N_s$

Hence induced emf (e) ∝ $N_s - N_r$

⇒ $E_{ph\,stator\,new} = 4.44 f_r \phi T_{ph\,stator} K_{w\,stator}$

where f_r = running frequency = $\frac{P(N_s - N_r)}{120}$

$$\boxed{s f_s}$$

(IV.) Now the stator RMF will be generated f_r hence stator RMF will rotate at a $N_s - N_r = sN_s$ = slip speed speed in the same direction of RRMF.

→ Speed of rotor RMF wrt stator = $N_s - N_r = sN_s$
→ Speed of rotor wrt stator = N_r

→ Rotor RMF wrt rotor = N_s
= rotor RMM wrt stator + rotor speed wrt stator
$(N_s - N_r) + N_r = N_s$

→ Speed of stator RMF wrt stator = $N_s - N_r = sN_r$ in the same direction as RRMF.

→ Relative speed b/w rotor RMF and stator RMF = 0
Hence will produce steady state torque.

$S = \frac{N_s - N_r}{N_s} \Rightarrow N_r = (1-s)N_s$

$\Rightarrow \omega_r = (1-s)\omega_s$

$\omega = \frac{2\pi N}{60}$ and $N_s = \frac{120 f}{P}$

i. Rotor frequency; $f_r = s f_s$
frequency of rotor induced emf under running condition.
→ not the frequency of rotation of rotor

At standstill, $s = 1 \Rightarrow f_r = f_s$

ii. $E_{2r} = s E_2$ induced emf in rotor.

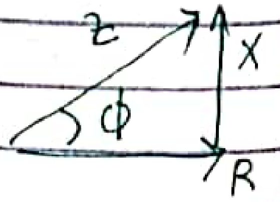
when rotor is not rotating ie $s=1 \Rightarrow E_{2r} = E_2$

iii. Resistance of rotor winding; $R_{2r} = R_2$
Reactance " " ; $X_{2r} = s X_2$

Now $Z_{2R} = R_{2r} + j X_{2r}$

$|Z_{2r}| = \sqrt{(R_{2r})^2 + (X_{2r})^2} = \sqrt{R_2^2 + (s X_2)^2}$

at standstill;
$|Z_2| = \sqrt{R_{2r}^2 + X_2^2}$



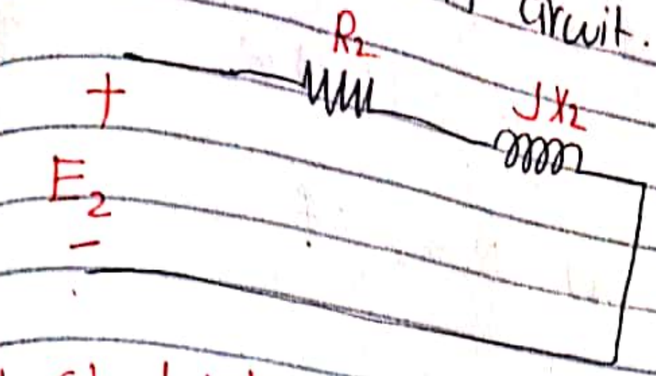
iv. P.f $\cos\phi = \frac{R}{Z}$

$\Rightarrow \cos\phi_2 = \frac{R_2}{Z_2}$

$\Rightarrow \cos\phi_{2r} = \frac{R_2}{Z_{2r}}$

Rotor equivalent circuit.

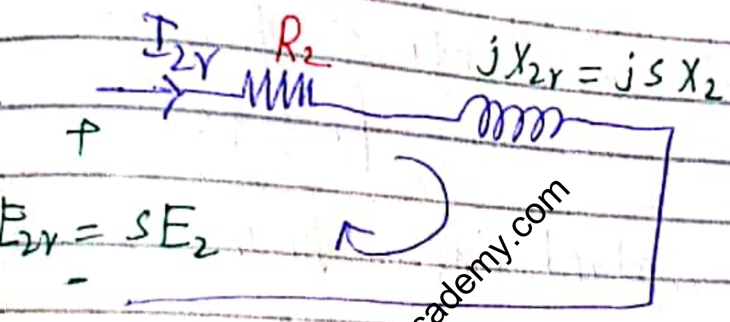
i.



At standstill and supply frequency

[$f_r = f_s$]

ii.



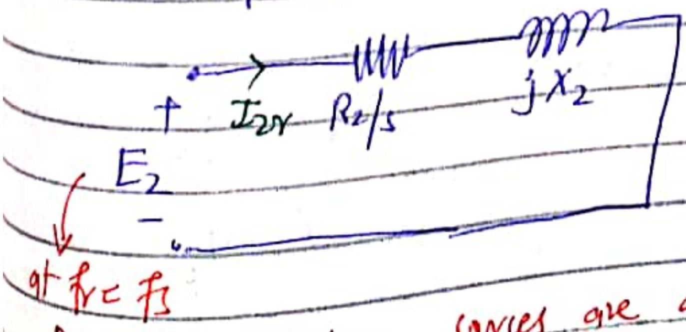
At running condition [$f_r = s f_s$]

KVL ⇒ $s\bar{E}_2 = \bar{I}_{2r} R_2 + j\,\bar{I}_{2r}\, s\, X_2$

divide by s

⇒ $\bar{E}_2 = \bar{I}_{2r} \frac{R_2}{s} + j\,\bar{I}_{2r}\, X_2$

→ the equivalent circuit:



Both circuits are representing the same equivalent circuit under running condition.

But the voltage sources are different which means that the frequency are different.

Power flow equation

Rotor i/p Power $= P_g = E_2 I_{2r} \cos\theta$

or $P_g = I_{2r}^2 \frac{R_2}{s}$ as that is a wattage value.

for 3φ ; $P_g = 3 I_{2r}^2 \frac{R_2}{s}$

$$I_{2r} = \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_2)^2}}$$

$$P_g = 3\left(\frac{E_2^2}{\left(\frac{R_2}{s}\right)^2 + (X_2)^2}\right) \cdot \frac{R_2}{s}$$

$$= 3 I_{2r}^2 \times \left(R_2 + R_2\left(\frac{1-s}{s}\right)\right)$$

$$= 3 I_{2r}^2 R_2 + 3 I_{2r}^2 R_2 \left(\frac{1-s}{s}\right)$$

↓ rotor winding loss (P_c)

↓ developed power in rotor (P_d)

$$\boxed{P_g = P_c + P_d}$$

$$\boxed{P_g : P_c : P_d = 1 : s : 1-s}$$

P_{in} electrical − (stator winding losses + core losses)

$= P_g$ (rotor i/p power or air gap power)

$P_g = P_c - P_d$

$P_d = P_g + P_g$

$$P_{out} = P_d - \text{rotational losses.}$$ → wattage

Torque relations

If stra r num, w need to deduct it ↑

$$T_{Load} = T_{shift} = T_{out} = \frac{P_{out}}{\omega_r} = \frac{P_d - \text{rot lo}}{\omega_r}$$

$$T_d = \text{developed torque} = \frac{P_d}{\omega_r} = \frac{(1-s)P_g}{(1-s)\omega_s} = \frac{P_g}{\omega_s}$$

$$\boxed{T_d = \frac{P_g}{\omega_s} = \frac{③}{\omega_s} \times I_{2r}^2 \times \frac{R_2}{s} = \frac{3}{\omega_s} \cdot \frac{E_2^2}{\left(\frac{R_2}{s}\right)^2 + X_2^2} \frac{R_2}{s}}$$

At maximum torque;

$$\boxed{s_m = R_2 / X_2}$$

$$\boxed{S \propto R_2 \propto \frac{1}{X_2} \propto \frac{1}{f}}$$

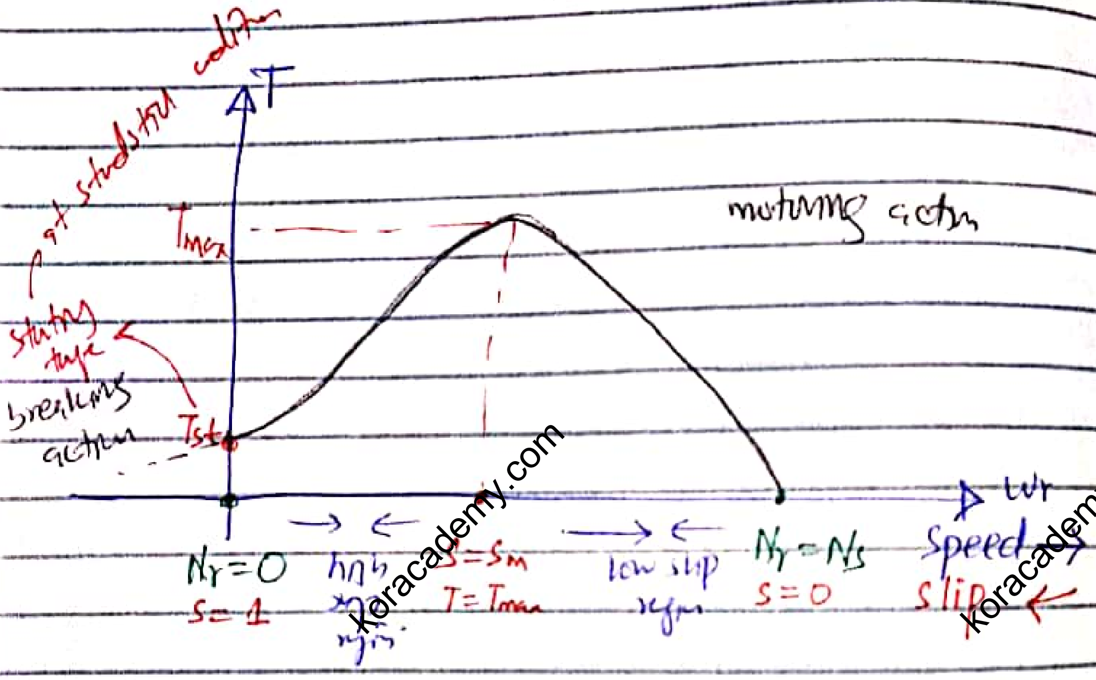
$$\boxed{T_{d_{max}} = \frac{3}{2\omega_s} \cdot \frac{E_2^2}{X_2}}$$ does not depend on rotor resistance

Full load torque; $s = s_{fl}$

$$\boxed{\frac{T_{fl}}{T_{max}} = \frac{2 s_m s_{fl}}{s_m^2 + s_{fl}^2}} \Rightarrow \boxed{\frac{T_{st}}{T_{max}} = \frac{2 s_m}{s_m^2 + 1}}$$ → starting torque (s=1)

$$\rightarrow \quad I_{2r} = \frac{V_1}{\sqrt{\left(\frac{R_2'}{s}\right)^2 + (X_2')^2}}$$

$$\rightarrow \quad T_d = \frac{3}{\omega_s} \cdot \frac{V_1^2}{\left(\frac{R_2}{s}\right)^2 + (X_2)^2} \cdot \frac{R_2}{s}$$



i- Low slip region. (operating / running region)

$$\uparrow\uparrow \frac{R_2}{\downarrow\downarrow s} \gg X_2 \quad \Rightarrow T_d = \frac{3}{\omega_s} \cdot \frac{s V_1^2}{R_2} \qquad T_d \propto s$$

linear

ii- High slip region. (starting region)

$$\downarrow\downarrow \frac{R_2}{\uparrow\uparrow s} \ll X_2 \quad \Rightarrow T_d = \frac{3}{\omega_s} \cdot \frac{V_1^2}{s X_2^2} \cdot R_2$$

$$T_d \propto \frac{1}{s} \rightarrow \text{non linear}$$

$$\Rightarrow \quad T_{st} = \frac{3}{\omega_s} \frac{V_1^2 R_2}{X_2^2}$$ depends on rotor resistance

$\frac{12}{5}$

Synchronous Machine

- It is an AC machine.

- We have two types of windings.
  1. Armature winding → It delivers the AC power to the load.
     → It absorbs AC power from the supply.

Armature of any machine always have AC voltage or AC power.

  2. Field winding It is supplied from a DC supply to create a fixed pair of poles.

Synchronous machine is a doubly excited machine.

In case of synchronous machine, the induced voltage is a dynamically induced voltage.
→ there is always a relative motion b/w conductor and field flux.

[Conductor in motion and field stationary → dynam.
(or) conductor fixed and flux rotating in nature → dynam]

[Conductor is stationary and flux is time varying ⇒ static]

→ Why not in transformer?
B/c coils were stationary but the flux was time varying in nature (no motion so no motional emf).

$e \propto$ relative motion.

Two produce relative motion, we have two cases;

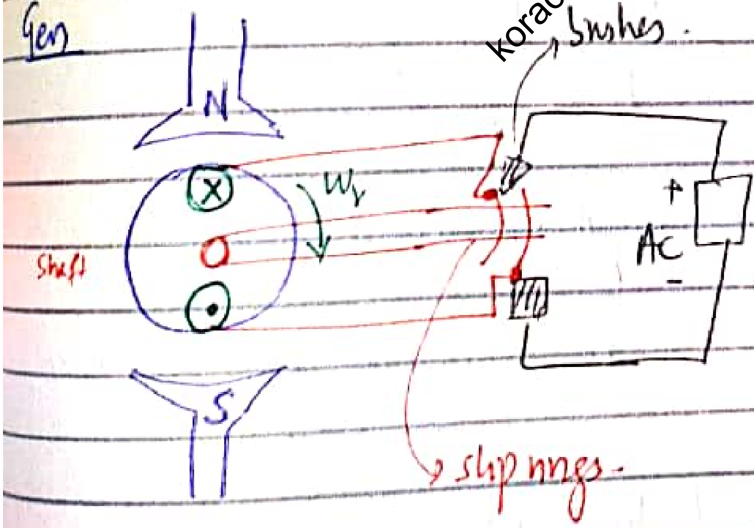
① Field winding on rotor and armature winding on stator. (Synchronous machine).

② Field winding on stator and armature winding in rotor. (DC machine)

→ A DC machine can behave as an AC machine if commutator is removed from its construction.

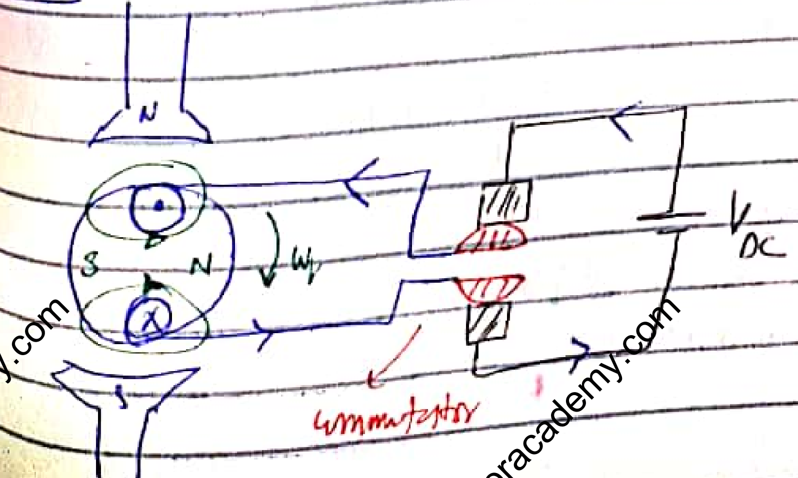
Consider the DC machine;

Gen



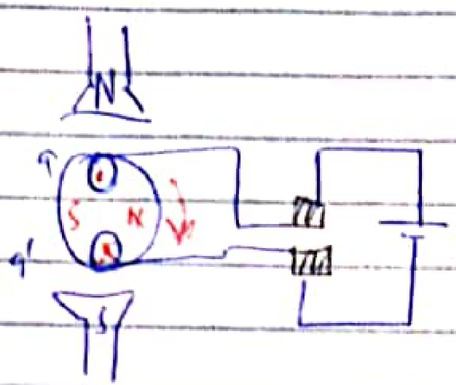
DC generator with a slip ring and brush assembly is providing an AC o/p hence behaving like AC generator.

motor



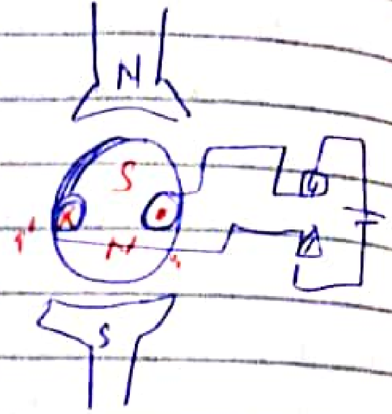
DC motor with commutator then it is converting DC supply into AC supply to the armature.

If the DC motor is used without a commutator;



Now after some rotation →

So no more rotation as N-S lock



In a synchronous machine;

Stator → armature winding → high power winding → AC

rotor → field winding → Low power winding → DC

↳ to develop working MMF or flux.

for high power applications.

There are number of advantages for doing so.

(i) It is very economized and efficient.
(slip rings and insulation cost)

(ii) Efficient cooling.

(iii) A large number of armature conductors can be accumulated on stator easily. or space requirement for rotor will become less, and hence rotor weight reduce.
↳ (moment of inertia, centrifugal forces etc).
mechanical strength

(iv) It will create more mechanical strength.

Synchronous Machine operation

① Rotor construction

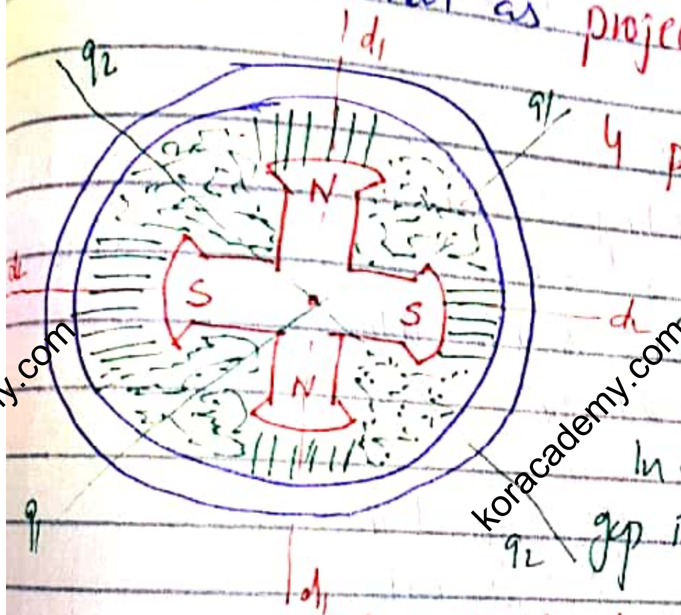
Two types.

i. Salient Pole type rotor → SP type SM. { → Gen, → Motor

ii. Cylindrical pole type rotor → C S M { → Gen, → motor

i. Salient Pole type rotor

It is also known as projected pole type rotor.



4 pole rotor (projected outward)

→ 90° electrical from d.

The air gap in nonuniform.

(because of poles)

In the d axis region, air gap is small ⇒ R ↓

$R = \frac{l}{\mu A}$

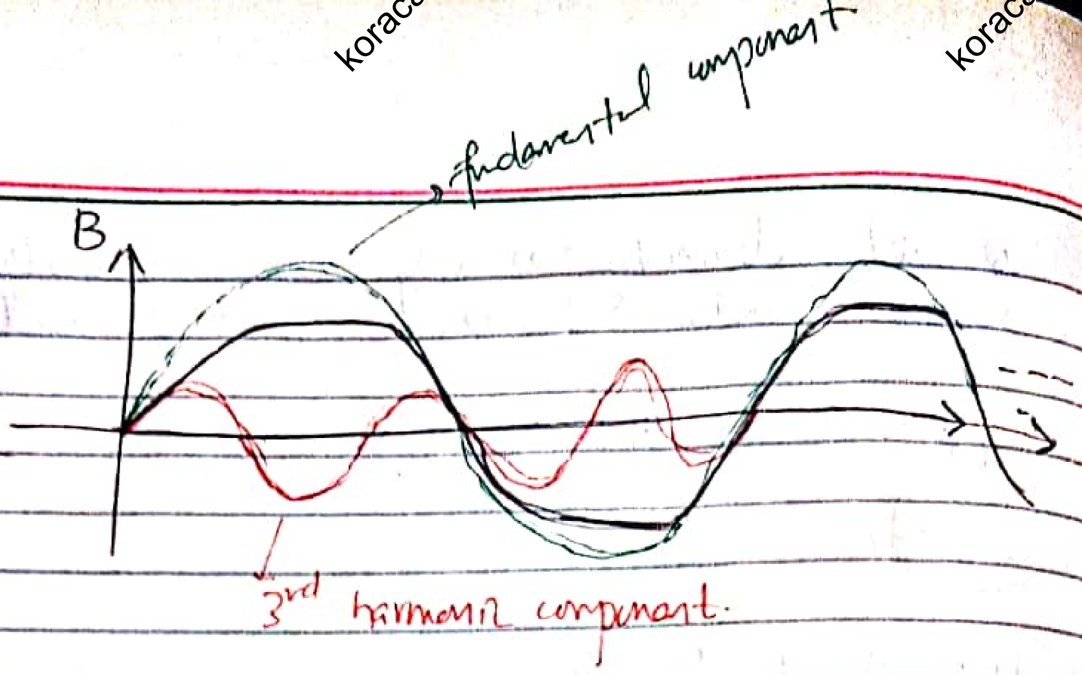
In the q axis region air gap is very large ⇒ R ↑

Hence the flux distribution will also be nonuniform.

as $\phi \propto \frac{1}{R}$

$$\boxed{\left(\theta_e = \frac{P}{2}\theta_m\right)}$$

Due to minimum air gap along d axis, the flux density will be maximum. and under the pole arc region the flux density is almost constant hence the flux density waveform is somewhat a flatt topped wave



Due to projection of the pole we have nonequal weight distribution ⇒ mechanical strength is poor.

So if it is rotated at a high speed, then the centrifugal forces will damage the rotor due to inequal distribution of force on the rotor body.

That's why it is used for low speed applications.

$$\boxed{\text{Speed range} = 50 \text{ to } 1000 \text{ rpm}}$$

For a 50Hz supply;

$$N_r = \frac{120f}{P} \Rightarrow P = \frac{120f}{N_r}$$

If speed, $N_r = 50 \Rightarrow P = 120$

If $N_r = 1000 \Rightarrow P_o = 6$

Hence pole requirement will be very high. (6 to 120)

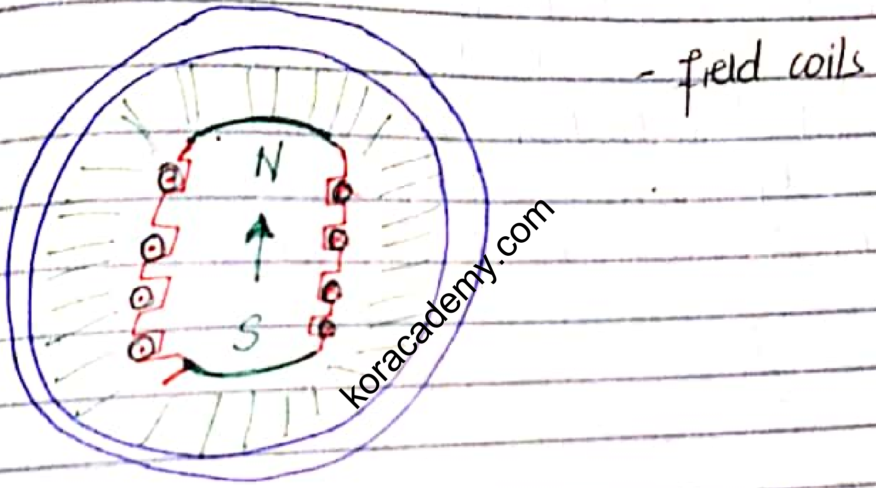
Such kind of rotor construction is used in hydro power plants as it requires low speed.

The rotor diameter is very large corresponding to stator diameter but axial length is small.

(poles ↑ ⇒ diameter ↑)

ii. Cylindrical rotor

i. In a cylindrical rotor, approximately $\frac{2}{3}$ rd area is used for the accomodation of field conductors and remaining portion is used as N or S pole.



ii. Here the poles have been induced with DC supply i.e it is not projected.

iii. In this air gap is almost uniform.
⇒ Reluctance will also be uniform.
↳ Flux distribution will also be uniform
and here air gap is low ⇒ Reluctance is low.

iv. Such kind of constructed is very useful for high speed applications.
At higher speed the centrifugal forces developed will be uniform over the rotor body. hence it will avoid mechanical damage.

v. Its mechanical stability is very high, hence mechanical strength is also good.

vi. Such construction is useful for;
1500 to 3000 rpm for 50 Hz supply
and 1800 to 3600 rpm for 60 Hz supply.
2–4 poles
2 to 4 poles

$$N_r = \frac{120f}{P}$$

$$\rightarrow 1500 = \frac{120(50)}{P} \Rightarrow P = 4$$

$$3000 = \frac{120(5)}{P} \Rightarrow P = 2$$

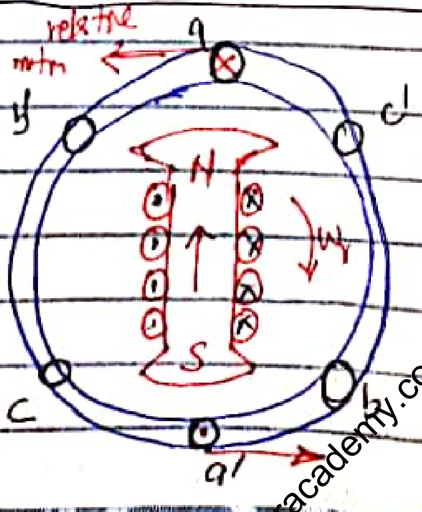
$$\rightarrow 1800 = \frac{120(60)}{P} \Rightarrow P = 4$$

$$3600 = \frac{120(60)}{P} \Rightarrow P = 2$$

vii. Here the rotor diameter is less as compared to salient machine and axial length will be high.

② θ_e and θ_m and induced emf

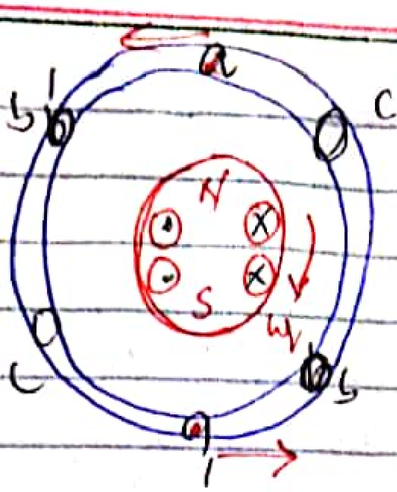
Gen



will have N turns ← aa' → phase a
N ← bb' → phase b
At t=0 N ← cc' → phase c

Now to understand the generation, taking one phase at a time full pitch ⇒ physical displacement = 180° electrical

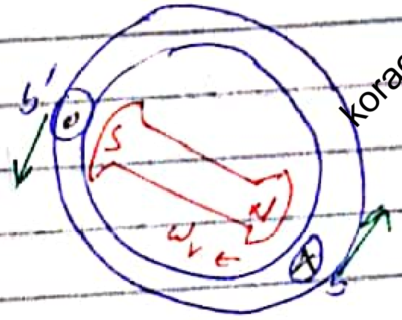
and



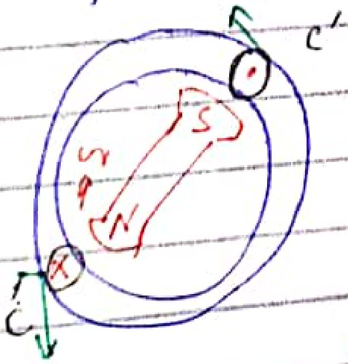
At t=0

Give rotation to the rotor at constant speed (Nr) → Excite the field winding (DC supply) → there will be relative motion b/w stator conductor and rotor flux.

If rotor rotates 120°



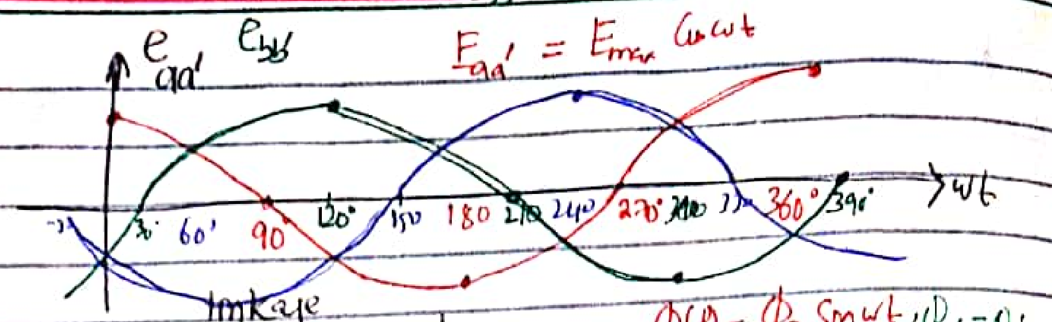
If rotates further 120°

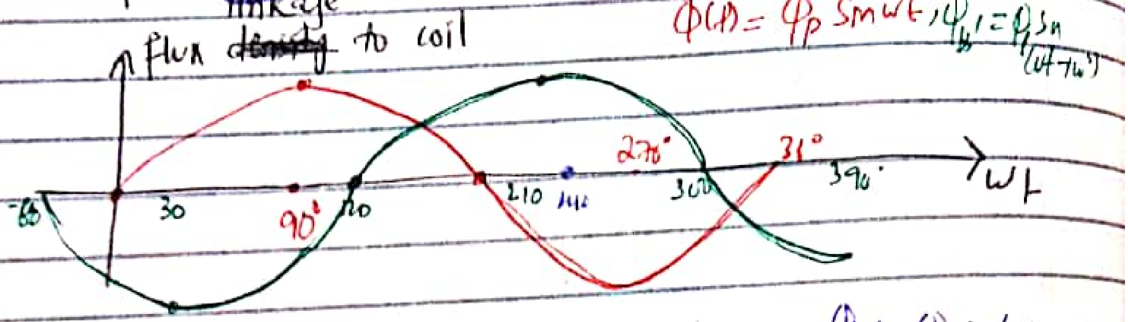


Induced voltage is maximum when the angle b/w flux and velocity component is 90°.

$E_{cc'} = E_{max}\cos(\omega t + 120^\circ)$

$E_{bb'} = E_{max}\cos(\omega t - 120^\circ)$





$\phi_{cc'} = \phi_p \sin(\omega t + 120^\circ)$

$$\theta_e = \frac{P}{2}\theta_m$$

$$\frac{d\theta_e}{dt} = \frac{P}{2}\frac{d\theta_m}{dt} \Rightarrow \omega_e = \frac{P}{2}\omega_r$$

$$\Rightarrow 2\pi f_e = \frac{P}{2}\frac{2\pi N_r}{60} \Rightarrow \boxed{N_r = \frac{120f}{P}}$$

constant speed for a particular frequency — Synchronous speed

→ Induced emf $e_{aa'} \propto \frac{d}{dt}\psi(t)$, $\psi(t) = N\phi(t)$

↓ flux linkage

$N = 120^\circ$

$$N_{ph} = \frac{\text{Total no. of turns}}{\text{no. of phases} = 3}$$

$$e_{aa'} = N_{ph}\frac{d}{dt} = -N_{ph}\frac{d\phi}{dt}$$

$$= -N_{ph}\frac{d}{dt}\phi_{aa'} = -N_{ph} \times \phi_p \times \omega\cos\omega t = -E_{max}\cos\omega t$$

$$\Rightarrow e_{aa'} = E_{max}\sin(\omega t - 90^\circ)$$

$$E_{a\,rms} = \frac{E_{ma}}{\sqrt{2}} = \frac{N_{ph}\phi_p 2\pi f}{\sqrt{2}}$$

$$\Rightarrow E_{a\,rms} = 4.44 f \phi_p N_{ph}$$

$$e_{aa'} = N_{ph}\phi_p \omega \sin(\omega t - 90^\circ)$$

$$e_{bb'} = N_{ph}\phi_p \omega \sin(\omega t - 210^\circ)$$

$$e_{cc'} = N_{ph}\phi \omega \sin(\omega t + 30^\circ)$$

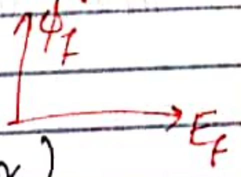
Hence each phase has same induced voltage as they have same number of turns and each voltage is displaced 120° electrical from each other.

$$\boxed{E_{a\,rms} = E_{b\,rms} = E_{c\,rms} = E_f = \frac{E_{ma}}{\sqrt{2}} = 4.44 f \phi N_{ph}}$$

$\Rightarrow \boxed{E_f = 4.44\ 4.44 f \phi N_{ph}}$ → true for full pitch winding

Induced emf due to field excitation only.

E_f will lag from ϕ_f by 90°



Types of Armature Winding (stator)

- (i) Single Layer
- (ii) Double layer.
- (iii) Full pitch.
- (iv) Short pitch
- (v) Concentrated
- (vi) Distributed

Conductor : The active length of wire under influence of field flux.

Turn : it is made with the use of conductors.

1 turn ⇔ 2 conductors.

T turns ⇔ $\frac{Z}{2}$ conductors ⇒ $Z = 2T$

Total no. of conductors per phase; $Z_{ph} = Z/3$

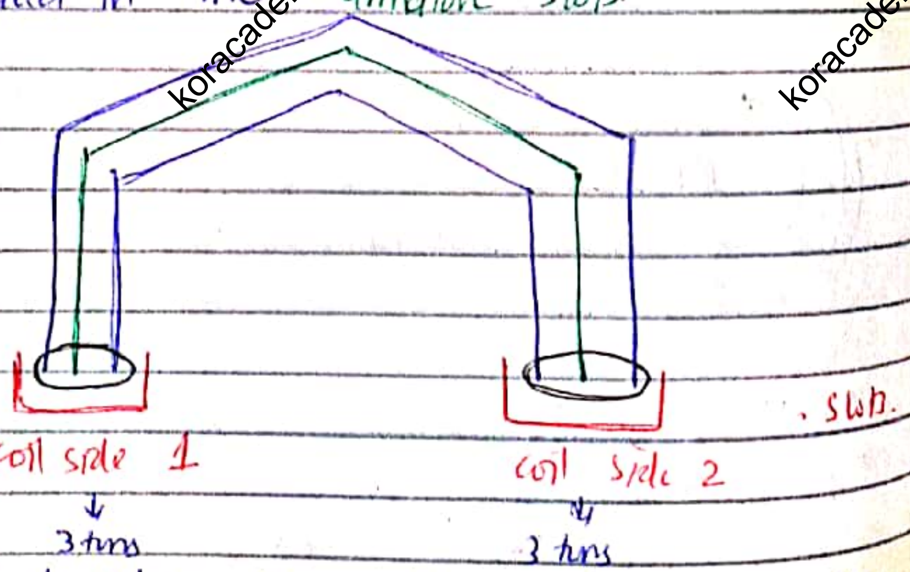
Total no. of turns per phase; $T_{ph} = N_{ph} = \frac{T}{3} = \frac{Z}{6} = \frac{Z_{ph}}{2}$

Coil A coil is formed by connecting number of turns.

1 coil ⇔ Minimum 1 turn ⇔ 2 conductors. → single turn coil.

Coil → N turns → 2 × N conductors → N turn coil.

Coil sides A coil always have 2 sides and these sides are placed in the armature slots.

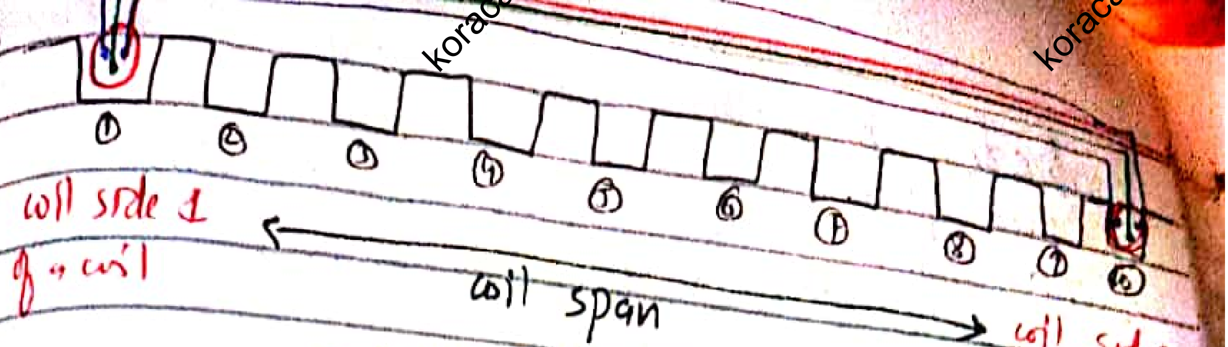


In both coil sides, the number of turns remain the same.

Total no. of conductors to make a 3 turn coil = 6.

(does not depend on coil side) [6 + 6] X

Coil span or Coil pitch It is the distance b/w two coil sides in terms of electrical degree or mechanical degree or number of slots present in b/w two sides.

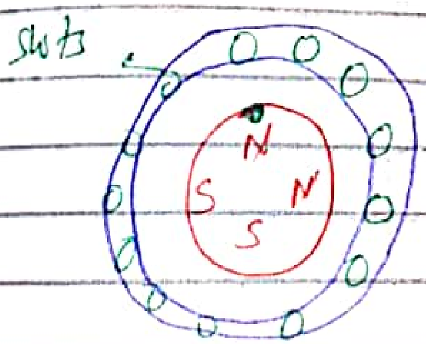


= 9 slots (finishing - starting)

Pole Pitch The centre to centre distance b/w two adjacent poles.

$$\boxed{1 \text{ Pole pitch} = \left(\frac{360^\circ}{P}\right)_{\text{mechn}}} = \frac{360^\circ}{P} \times \frac{P}{2} = \boxed{180^\circ \text{ electrical}}$$

Consider for a 4 pole machine;



$$1 \text{ pole pitch} = \frac{360^\circ}{4} = 90^\circ \text{ mechanical}$$

revolution

$$1 \text{ pole pitch} = 90^\circ \times \frac{4}{2} = 180^\circ \text{ electrical.}$$

In terms of slots;

$$1 \text{ pole pitch} = \frac{\text{No. of slots in stator}}{\text{No. of poles}} \qquad \left(\frac{13}{4}\right) \text{ here}$$

Slot angle (γ) The angle contributed by one slot.

$$\Rightarrow \boxed{\gamma = \frac{360^\circ}{\text{no. of slots}} \ (\text{mechanical})}$$

or

$$\boxed{\gamma = \frac{360^\circ}{\text{no. of slots}} \times \frac{P}{2} = \frac{180^\circ}{\text{No. of slots/pole}} \ (\text{electrical})}$$

As in an example ; 1 pole pitch = $\frac{13}{4}$ in terms of slots

$$\Rightarrow \gamma = \frac{360^\circ}{13} \text{ mechanical}$$

$$\Rightarrow 1 \text{ pole pitch} = \left(\frac{17}{4}\right)\left(\frac{360^\circ}{13}\right) = \frac{360^\circ}{4} \checkmark \text{ mechanical}$$

$$1 \text{ pole pitch} = \frac{360^\circ}{4} \times \frac{4}{2} = 180^\circ \text{ electrical}$$

On the basis of pitching the winding is classified into two categories.

①(i) Full Pitch coil winding

1 pole pitch = 180° electrical

or 1 pole pitch = $\frac{360^\circ}{P}$ mechanical

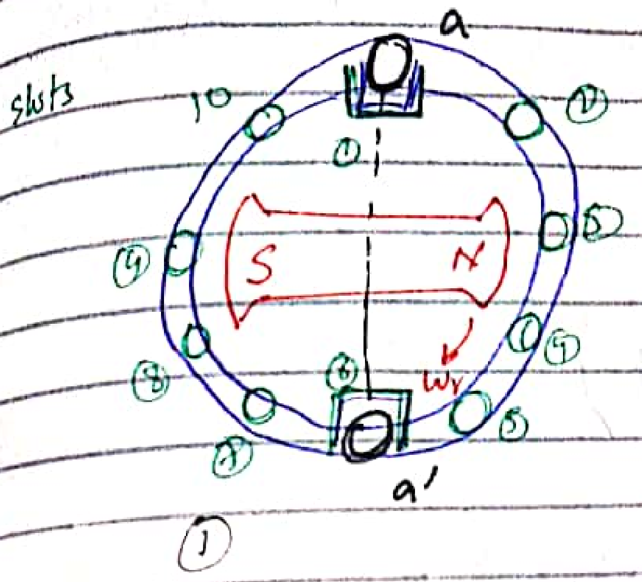
→ If the distance b/w two coil sides of a coil is 1 pole pitch, Then such a winding is known as full pitch winding.

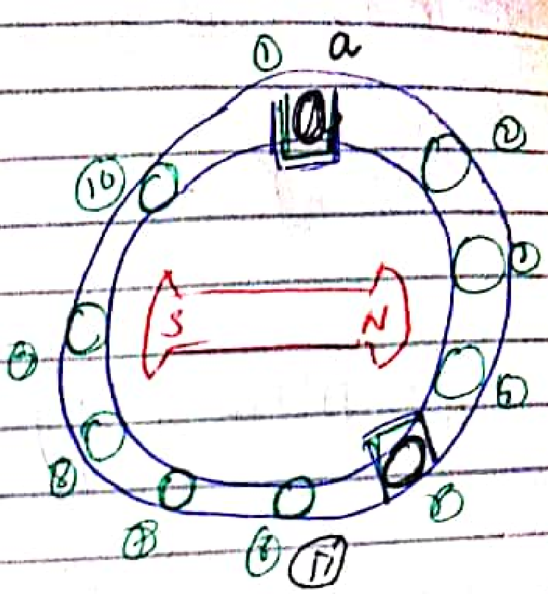
Coil span = 1 pole pitch = full pitch winding.

(ii) If the coil span is less than 1 pole pitch, Then the winding is known as short pitch winding.

coil span < 1 pole pitch

Consider a 2 pole machine;





Case i

For a full pitch coil $\phi_p = \frac{4 B_p l r}{P}$

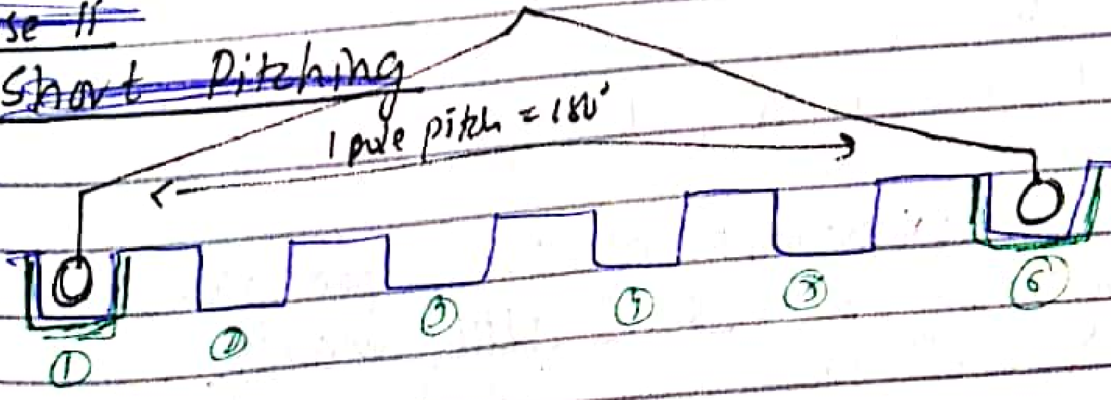
for $p = 2$
$\Rightarrow \phi_p = 2 B_p l r$

Induced emf for a full pitch coil:

$$E_f = 4.44 f \phi_p N_{ph}$$
↳ per phase rms value.

~~Case II~~

Short Pitching



1 pole pitch = 5 slots

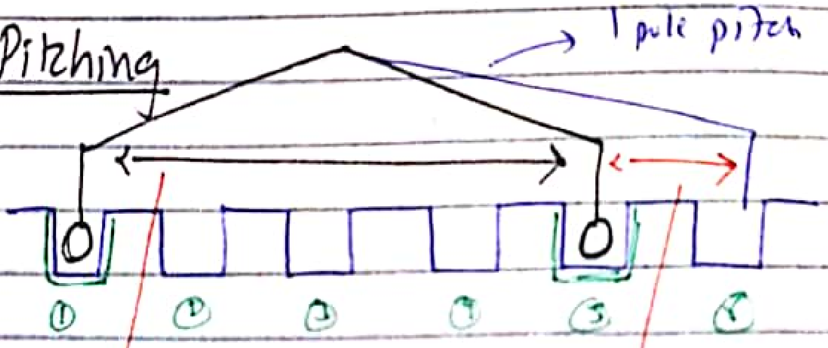
∴ total no. of slots = 5 × 2 = 10

Slot angle $\gamma = \frac{360°}{10} = 36°$ ~~electrical~~ mechanical

① $\gamma = \frac{36°}{2} \times \frac{P}{2} = 36°$ electrical

1 pole pitch $= 5 \times 36 = 180°$ electrical.

Case ii
Short Pitching



Short pitch coil span

ε = short pitch angle ↓ electrical angle.

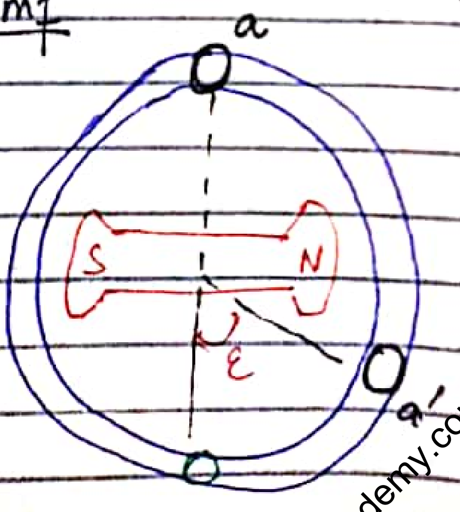
or

$\boxed{\text{short pitch coil span} = 180° - \varepsilon}$

eg here it has shorted by 1 slot ⇒ angle of 1 slot = 36°

⇒ $\varepsilon = 36°$ ⇒ short pitch coil span $= 180° - 36$
$= 144°$

Effect of short pitching on Flux per pole and induced emf



Due to short pitching the flux per pole (ϕ_p) will reduce.

The factor by which it reduces is known as pitch factor (K_p).

Consider the second figure for derivation purpose.

As $d\phi = B \cdot dA$

$$\int d\phi = \int B_p \sin\alpha \, (l r d\alpha)$$

$$\phi = \int_{\varepsilon/2}^{180^\circ - \varepsilon/2} B_p \cdot l \cdot r \left[-\cos\alpha\right]_{\varepsilon/2}^{180^\circ - \varepsilon/2}$$

$$= B_p l r \left[-\cos\left(180^\circ - \frac{\varepsilon}{2}\right) + \cos\frac{\varepsilon}{2}\right]$$

$$\Rightarrow \boxed{\phi_1 = 2 B_p l r \cos\frac{\varepsilon}{2}} \xrightarrow{} K_p$$

$$\boxed{\text{Pitch factor } (K_p) = \frac{\phi_p \text{ in full pitch winding}}{\phi_p \text{ in short pitch winding}}}$$

$$= \frac{2 B_p \cdot l \cdot r \cdot \cos\varepsilon/2}{2 B_p \cdot l \cdot r}$$

$$\Rightarrow \boxed{K_p = \cos\frac{\varepsilon}{2}} \qquad 0 \le K_p \le 1$$

If $k_p = 1 \Rightarrow$ full pitch coil.

$$\text{as} \quad \cos\frac{\varepsilon}{2} = 1 \Rightarrow \frac{\varepsilon}{2} = \cos^{-1}(1)$$

$$\Rightarrow \boxed{\varepsilon = 0}$$

Flux per pole for one pole area;

$$\frac{2\pi r l}{2} \longrightarrow 2B_p \cdot l \cdot r \cdot \cos\frac{\varepsilon}{2}$$

$$\boxed{\frac{2\pi r l}{P} \rightarrow \frac{2B_p l_, r \cos\varepsilon/2}{\pi r l} \times \frac{2\pi l r}{P} = \frac{4B_p l r}{P}\cos\frac{\varepsilon}{2}}$$

→ When coil is full pitch;

$$E_{f(fp)} = 4.44 f \left(\frac{4B_p l.r}{P}\right) N_{ph}$$

When coil is short pitched;

$$E_{f(sp)} = 4.44 f \left(\frac{4B_p l r}{P}\right) N_{ph} \cos\left(\frac{\varepsilon}{2}\right)$$

Ratio;

$$\boxed{\frac{E_{f(sp)}}{E_{f(fp)}} = \cos\left(\frac{\varepsilon}{2}\right) = k_p}$$

$$\boxed{E_{f(sp)} = 4.44 f \phi_p N_{ph} K_p}$$

$$\boxed{E_{f(sp)} < E_{f(fp)}}$$

② concentrated and distributed winding

Consider a 3 ϕ synchronous machine (assuming alternator) with full pitch winding.

No. of phases = 3

No. of poles (P) = 2

No. of slots available on stator for armature winding = 18

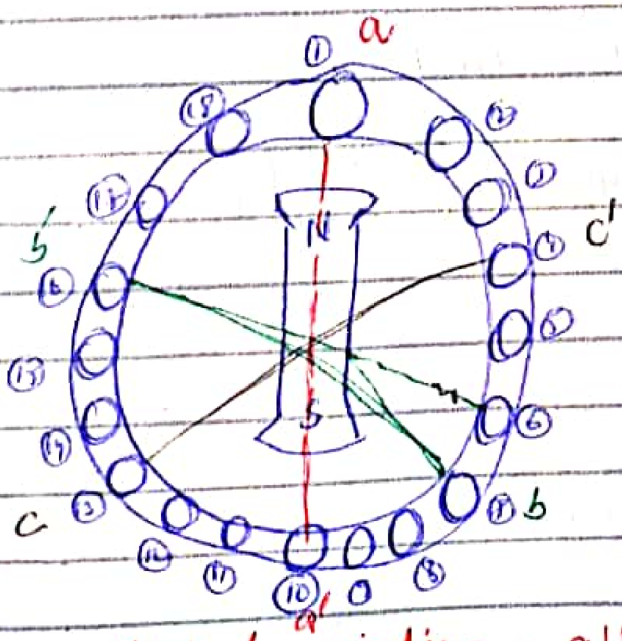
No. of slots per pole = pole pitch (in terms of slots)

$= \frac{18}{2} = 9 = 180^\circ$ electrical $= \frac{360^\circ}{2}$ mechanical

No. of slots/pole/phase = m = $\frac{\text{no. of slots}}{P \times \text{no. of phase}} = \frac{18}{2 \times 3} = 3$

Slot angle $(\gamma) = \frac{360^\circ}{18} = 20^\circ$ mechanical

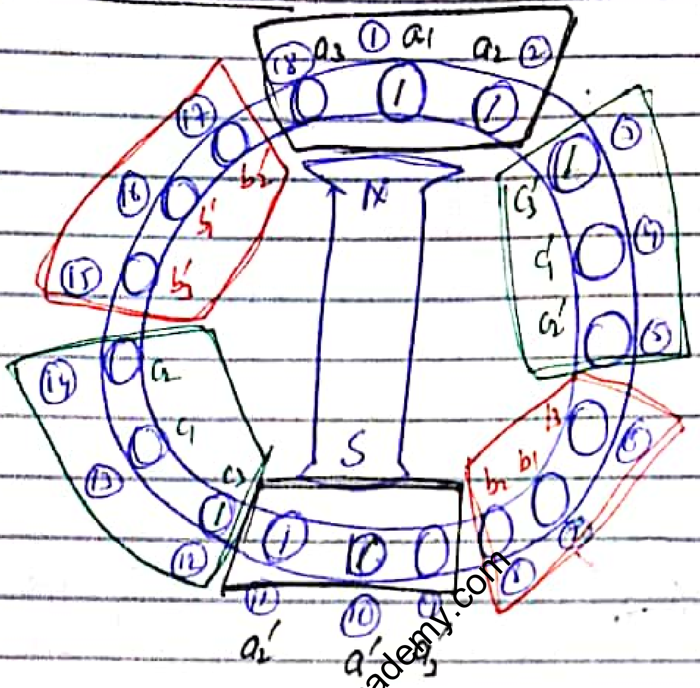
$= 20 \times \frac{2}{2} = 20^\circ$ electrical



In a concentrated winding, all the conductors or coils belonging to a phase are placed in one slot in every pole.

In concentrated winding the total induced emf is the algebraic sum of individual conductor emfs (coils)

→ **Distributed**



→ Phase belt @ phase spread

↳ The angle covered by one phase under one pole.

(here of 60°)

90-kn → 9

90 − 30
/3

In distributed winding, all the coils belonging to a phase are well distributed into m number of slots under every pole.

In distributed winding, the resultant emf is not algebraic summation of the individual coil emfs. That means the resultant emf will be calculated vectorially and induced emf will also get reduced due to distribution of the coil.

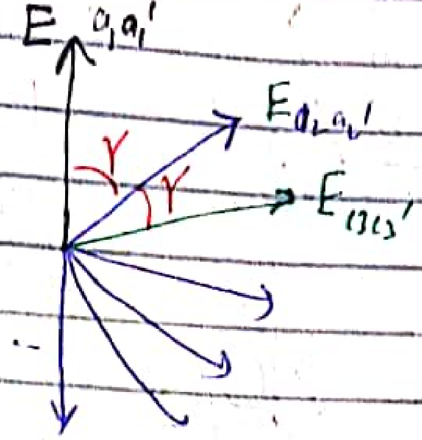
→ Assuming each coil in the 18 slots have same magnitude of emf as they have same number of turns.

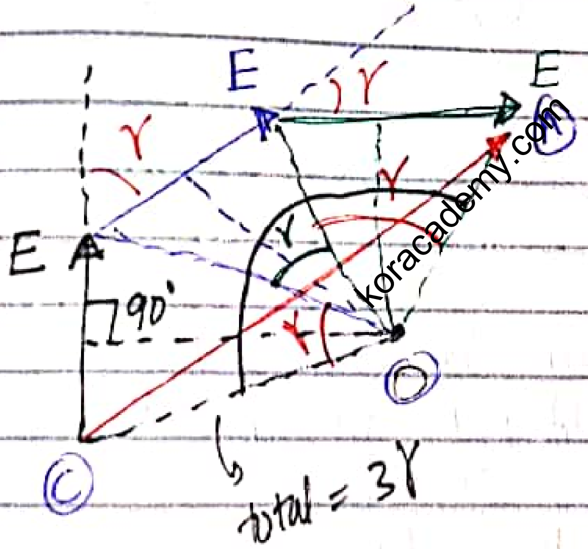
→ The projection of emf shown in the figure represents the total emf induced in a coil.

$$\left(e_{1\ \text{and}\ 10} = \underline{E} \right)$$

$$\left(e_a = +E/2 \ , \ e_{a'} = -\varepsilon/2 \right.$$
$$\left. \rightarrow e_{aa'} = e_a - e_{a'} = E \right)$$

and similarly

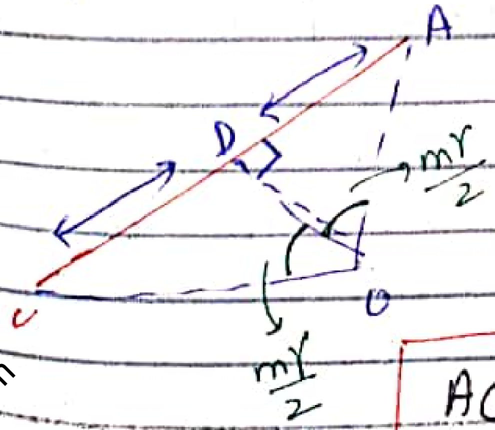




These all are lying in the circumference of circle with centre O

Similarly if all the coils are taken, γ will keep on adding

$\Rightarrow m\gamma$

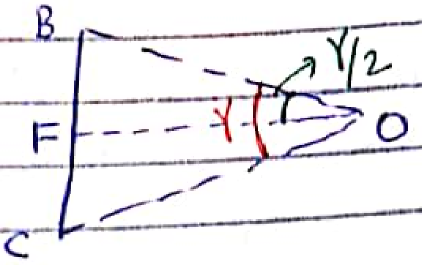


$$\sin\left(\frac{m\gamma}{2}\right) = \frac{AD}{OA}$$

$$\Rightarrow AD = OA \sin\left(\frac{m\gamma}{2}\right)$$

$$\boxed{AC = CA = 2AD = \frac{2}{OA}\sin\left(\frac{m\gamma}{2}\right)}$$

for algebric summation:



$\sin\left(\frac{\gamma}{2}\right) = \frac{BF}{OB} = \frac{BF}{OA}$

$\Rightarrow BF = OA \sin\left(\frac{\gamma}{2}\right)$

$\Rightarrow \boxed{BC = 2BF = 2\,OA \sin\frac{\gamma}{2}}$

Algebric summation

$= m \times 2\,OA \sin\frac{\gamma}{2}$

$K_d = \frac{\text{Vector sum of induced emf}}{\text{Algebric summation}} = \frac{2\,OA \sin\left(\frac{m\gamma}{2}\right)}{m\,2\,OA \sin\left(\frac{\gamma}{2}\right)}$

$\Rightarrow \boxed{K_d = \frac{\sin\left(\frac{m\gamma}{2}\right)}{m \sin\left(\frac{\gamma}{2}\right)}}$

↳ Distribution factor. or belt factor

also;

$\boxed{K_d = \frac{\text{Induced emf with distributed winding}}{\text{Induced emf with concentrated winding}}}$

$= \frac{2\,OA \sin\left(\frac{m\gamma}{2}\right)}{m\,2\,OA \sin\left(\frac{\gamma}{2}\right)} = \frac{\sin\left(\frac{m\gamma}{2}\right)}{m \sin\left(\frac{\gamma}{2}\right)}$

Now if we are using distributed winding:

$\boxed{E_{rms(ph)} = 4.44\, f\, \phi_p\, N_{ph}\, K_d}$

$d \leq 1$

$$\rightarrow K_d = \frac{\sin\left(\frac{m\gamma}{2}\right)}{m \sin\left(\frac{\gamma}{2}\right)}$$

m = no. of slots per pole per phase $= \frac{\text{No. of slots}}{P \times (\text{no. of phases})}$

$$\gamma = \frac{360^\circ}{\text{no. of slots}} \quad \text{(mechaical)}$$

$$\text{or} \quad \gamma = \frac{360^\circ}{\text{No. of slots}} \times \frac{P}{2} \quad \text{(electrical)}$$

$$m\gamma = \text{phase spread} = \delta = \left(\frac{\text{No. of slots}}{P(3)}\right)\left(\frac{360}{\text{No. of slots}}\right)\left(\frac{P}{2}\right)$$

$$\Rightarrow \delta = 60^\circ \text{ electrical}$$

$$\text{or} \left(\frac{120}{P}\right) \text{mechanical.}$$

If writing generally;

$$\delta = m\gamma = \frac{\text{No. of slots}}{P \times n} \times \frac{360^\circ}{\text{no. of slots}} = \frac{360^\circ}{P \times n} \quad \text{mechanical}$$

$$= \frac{360^\circ}{P \cdot n}\left(\frac{P}{2}\right) = \frac{180^\circ}{n} \quad \text{electrical}$$

n → no. of phases.

For a single ϕ system;

$$\delta = \frac{180^\circ}{1} = 180^\circ \text{ electrical}$$

For 2ϕ;

$$\delta_{2\phi} = \frac{180^\circ}{2} = 90^\circ \text{ electrical}$$

For 3φ.

$$\delta_{3\phi} = \frac{180^\circ}{3} = 60^\circ \text{ electrical}$$

$$\Rightarrow \boxed{K_d = \frac{\sin\left(\frac{\delta}{2}\right)}{m \sin\left(\frac{\delta}{2m}\right)}}$$

$m\gamma = \delta$

$\gamma = \left(\frac{\delta}{m}\right)$

in terms of phase spread.

Fractional slot winding

If m is an integer value ie (1, 2, 3, 4 ....)
↳ distributed winding.

if m is a fraction → fractional winding.

Here $m = \frac{S_K}{P_K}$

Here:

$$\boxed{K_d = \frac{\sin(\delta/2)}{S_K \cdot \sin\left(\frac{\delta}{2S_K}\right)}}$$

If $\underline{\underline{\gamma << 15^\circ}}$ ⇒ $\sin\left(\frac{\gamma}{2}\right) \simeq \frac{\gamma}{2}$ (radians)

$$\Rightarrow \boxed{K_d = \frac{\sin\left(\frac{m\gamma}{2}\right)}{m\frac{\gamma}{2}} = \frac{\sin\left(\frac{\delta}{2}\right)}{\frac{\delta}{2}}}$$

all angles are electrical.

↳ Uniformly distributed winding

where numerator must be in radians. (multiply π/180) ad $\boxed{E_{ph} = 4.44 f \phi_p N_{ph} K_d}$

Impact of harmonics on K_p, K_d ad ϕ_p and induced voltage.

(1) Flux / Pole

$$\phi_p = \frac{4 B_p l r}{P}$$ --- when there is no harmonics (fundamental flux)

$$\phi_{pn} = \frac{4 B_{pn} l r}{n.P}$$ → for n^{th} harmonic

For $n = 1 \Rightarrow \phi_{p1} = \frac{4 B_{p1} l.r}{P}$ → fundamental

$n = 3 \Rightarrow \phi_{p3} = \frac{4 B_{p3} l r}{3P}$ → 3rd harmonic component

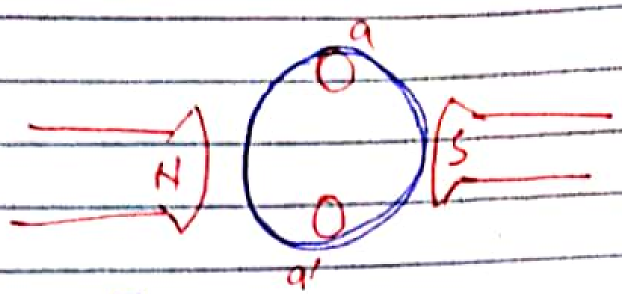
$n = 5 \Rightarrow \phi_{p5} = \frac{4 B_{p5} l r}{5P}$ → 5th harmonic component.

$$\frac{\phi_{p3}}{\phi_{p1}} = \frac{B_{p3}}{3/B_{p1}} = \left(\frac{B_{p3}}{B_{p1}}\right)\left(\frac{1}{3}\right)$$

$$\frac{\phi_{p5}}{\phi_{p1}} = \left(\frac{B_{p5}}{B_{p1}}\right)\frac{1}{5}$$

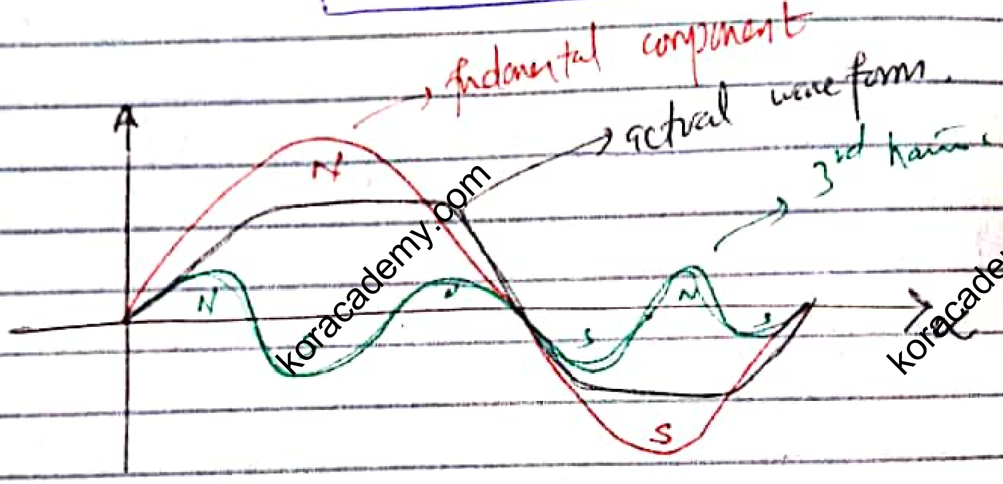
$$\boxed{\frac{\phi_{pn}}{\phi_{p1}} = \left(\frac{B_{pn}}{B_{p1}}\right) \times \frac{1}{n}}$$

Consider a two pole machine;



$$\Phi_p = \int_0^\pi B_p \sin\alpha \; l r \, d\alpha = B_p l r [-\cos]_0^\pi$$

$$\Rightarrow \boxed{\Phi_p = 2 B_p l r}$$



Considering the third harmonic only;

$$B = B_{p2} \sin 3\alpha$$

$$\Rightarrow \Phi_{p3} = \int_0^\pi B_{p2} \sin 3\alpha \cdot l \cdot r \, d\alpha$$

$$= \frac{B_{p3} \cdot l \cdot r}{3}(1+1) = \frac{2 B_{p3} l r}{3}$$

↳ divided by the harmonic number.

Generally, for any n and P;

$$\boxed{\Phi_{pn} = \frac{4 B_{pn} l r}{n P}}$$

ii. Pitch Factor (K_p)

For n^{th} harmonic,

$$K_{p_n} = \cos\left(\frac{n\varepsilon}{2}\right)$$

If $n=1$ ⇒ $K_{p_1} = \cos\left(\frac{\varepsilon}{2}\right)$ → due to fundamental harmonic in flux.

If $n=3$ ⇒ $K_{p_3} = \cos\left(\frac{3\varepsilon}{2}\right)$ → due to 3rd harmonic in flux

To eliminate harmonics; $\boxed{K_{p_n} = 0}$

⇒ $\cos\left(\frac{n\varepsilon}{2}\right) = 0$ ⇒ $\frac{n\varepsilon}{2} = \cos^{-1} 0$

⇒ $\frac{\varepsilon}{2} = \frac{\pi}{2n}$ ⇒ $\boxed{\varepsilon = \frac{\pi}{n}} = \left(\frac{180° \text{ electrical}}{n}\right)$

eg to eliminate 3rd harmonic:

$$\varepsilon = \frac{180°}{3} = 60°$$

⇒ $K_{p_3} = \cos\left(\frac{3 \times 60°}{2}\right) = \cos 90° = 0$

$K_{p_5} = \cos\left(\frac{5 \times 60°}{2}\right) = \cos 150°$ ⇒ 5th harmonic will exist.

Both cannot be eliminated at the same time. So it is better to eliminate 3rd as its amplitude is higher so it will have a higher impact...

(iii) Distribution factor (K_d)

$$K_{d_n} = \frac{\sin\left(\frac{mn\gamma}{2}\right)}{m \sin\left(\frac{n\gamma}{2}\right)} \rightarrow \text{Generalized expression}$$

If $\gamma << 15°$;

$$K_{d_n} = \frac{\sin\left(\frac{n\sigma}{2}\right)}{\frac{n\sigma}{2}}$$

For fractional slot winding;

$$K_{d_n} = \frac{\sin\left(\frac{n\sigma}{2}\right)}{S_K \sin\left(\frac{n\sigma}{2S_K}\right)}$$

V- Induced emf

$E_{ph} = 4.44 f \phi_p N_{ph}$ → fundamental for full pitch and concentrated winding.

Generalized in harmonics form;

$$\boxed{E_{ph_{(n)}} = 4.44 (nf) \phi_{p_n} (K_{p_n} K_{d_n}) N_{ph}}$$

$$\boxed{K_{wn} = K_{pn} \times K_{dn}}$$

↳ winding factor

If $n=1 \Rightarrow E_{ph_1} = 4.44 f \phi_{p1} K_{p1} K_{d1} N_{ph}$

If $n=3 \Rightarrow E_{ph_3} = 4.44 (3f) \phi_{p3} K_{p3} K_{d3} N_{ph}$

If $n=5 \Rightarrow E_{ph_5} = 4.44 (5f) \phi_{p5} K_{p5} K_{d5} N_{ph}$

If ratios;
$$\frac{E_{ph5}}{E_{ph1}} = \frac{5 \phi_{p5} K_{w5}}{\phi_{p1} K_{w1}}$$

Similarly
$$\frac{E_{ph3}}{E_{ph1}} = \frac{3 \phi_{p3} K_{w3}}{\phi_{p1} K_{w1}}$$

generalizing;
$$\boxed{\frac{E_{phn}}{E_{ph1}} = \frac{n \phi_{pn} K_{wn}}{\phi_{p1} K_{w1}}}$$

Total rms voltage:
$$\boxed{E_{ph_{rms}} = \sqrt{E_{ph_1}^2 + E_{ph_3}^2 + E_{ph_5}^2 + E_{ph_7}^2 + \cdots}}$$

As this is a 3ϕ system;

$E_{ph_a} = E_{m1} \sin\omega t + E_{m3} \sin 3\omega t + E_{m5} \sin 5\omega t + \cdots$

$E_{ph_b} = E_{m1} \sin(\omega t - 120°) + E_{m3} \sin(3\omega t - 3 \times 120°)$
$+ E_{m5} \sin(5\omega t - 5 \times 120°) + \cdots$

$E_{ph_c} = E_{m1} \sin(\omega t + 120°) + E_{m3} \sin(3\omega t + 3 \times 120) + E_{m5} (\ldots$
$\cdots + (\omega t + 5 \times 120°)$

$\rightarrow \sin(3\omega t \pm 3(120)) = \sin(3\omega t)$

$\rightarrow \sin(5\omega t \pm 5(120)) = \sin(5\omega t \pm 120^\circ)$

Case i Star connected armature winding.

There exists fundamental and 3,5,7,— harmonics in phase voltage

For line voltage,

$E_{ab} = E_a - E_b = [E_{m1}\sin\omega t - E_{m1}\sin(\omega t - 120^\circ)] + [E_{m3}\sin 3\omega t$

$- E_{m5}\sin(5\omega t + 120^\circ) + \text{------}$

$E_{bc} = E_b - E_c$

$E_{ca} = E_c - E_a$

In line voltage there will be no triplet harmonics (3rd, 9th ---)

$$\boxed{\begin{gathered} E_{ph_{rms}} = \sqrt{E_{ph_1}^2 + E_{ph_3}^2 + E_{ph_5}^2 + \text{---}} \\ E_{line_{rms}} = \sqrt{3} \times E_{ph\,rms} = \sqrt{3}\sqrt{E_{p1}^2 + E_{ph5}^2 + \text{---}} \end{gathered}}$$

Case ii In delta connection.

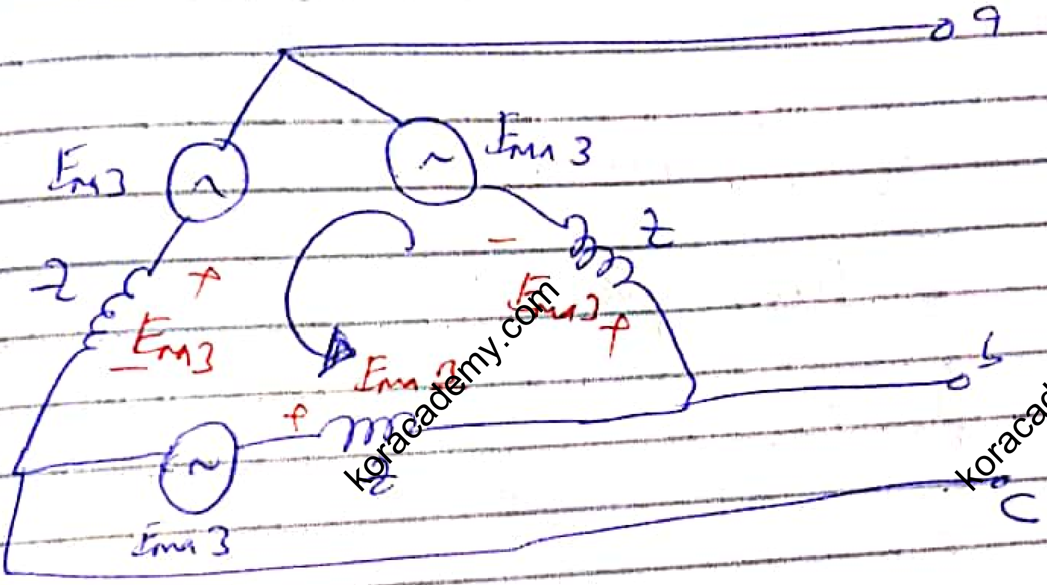
$E_a + E_b + E_c = 0 + 3E_{m3}\sin 3\omega t + 0$

$= 3E_{m3}\sin 3\omega t$

A current will circulate ; →

$$I_{c3} = \frac{3 E_{m3} \sin 3\omega t}{3Z}$$

$\therefore I_{c3} = \frac{E_{m3}}{Z}$ → due to 3rd harmonic.



$$\boxed{E_{ab} = 0, \quad E_{bc} = 0, \quad E_{ca} = 0}$$

In delta connection, no 3rd harmonic in line and phase voltage both (If 0 developed but would not exist s/c the circulating current will nullify ~~the~~ it).

$$\boxed{E_{ph_{rms}} = E_{line_{rms}} = \sqrt{E_{p1}^2 + E_{p5}^2 + E_{p7}^2}}$$