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# CONTROL SYSTEMS

**System**

A combination of different components which is supposed to accomplish a certain function.

The o/p must be in a certain limit → control system

↳ always stable

objective function → despite any drastic changes in the i/p quantity.

For any system;

i/p, o/p, components (different configurations/arrangements).

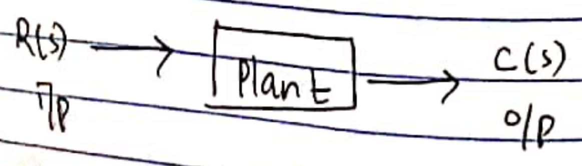


control systems need something special.

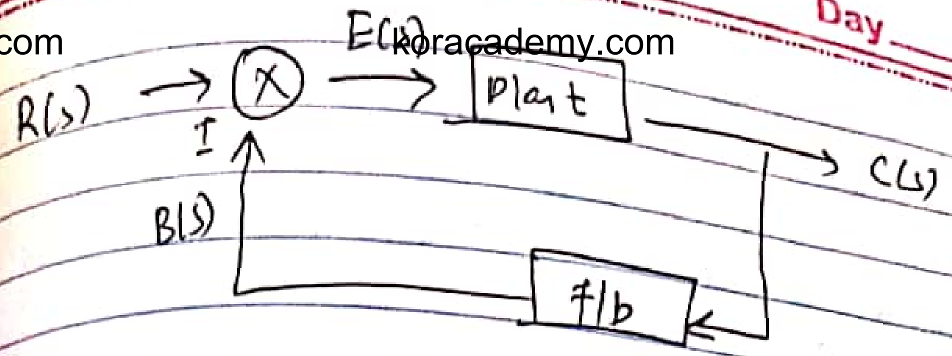
| i/p | o/p | components |
|---|---|---|
| $r(t)$ | $c(t)$ | Impulse response |
| $R(s)$ | $C(s)$ | transfer function |

**Types:**

1) Open loop control systems. → no communication b/w i/p & o/p.
2) Closed loop " " . → communication b/w i/p and o/p through feedback element.

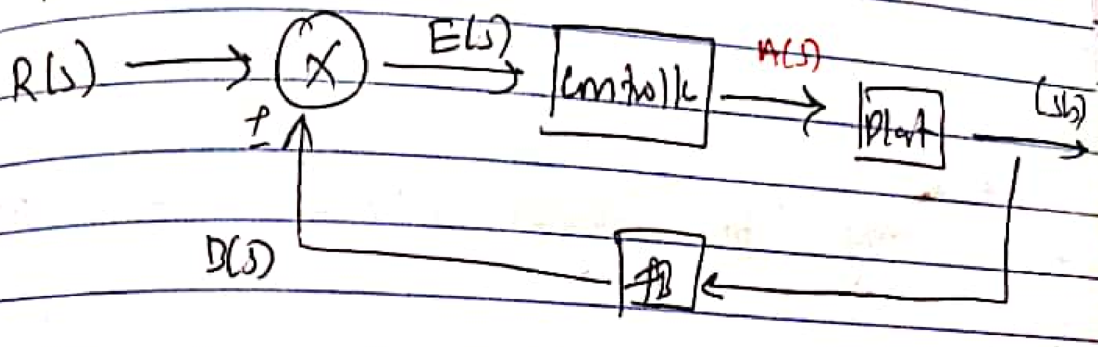



open loop




E(s) → error signal , B(s) → feedback signal.

The control and the plant can also be shown separately;






A(s) → actuating signal → signal that is driving the plant

→ Whether a system is open loop or closed loop, it has to do a certain task.

The relation b/w i/p and o/p is given by the transfer function of the system. (it is always in the laplace transform mode).

$$\boxed{T(s) = \frac{K\, P(s)}{Q(s)}}$$

→ We will be studying causal LTI systems with single i/p and single o/p.

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K = system gain $0 < K < 1 \rightarrow$ attenuator.

$K > 1 \rightarrow$ amplifier $K < 0 \rightarrow$ inverter $K = 1 \rightarrow$ buffer

Consider a general differential equation defining system;

$$a\frac{d^2y(t)}{dt^2} + b\frac{dy(t)}{dt} + cy(t) = \frac{d}{dt}x(t) + 2x(t).$$

Taking laplace transform.

$$a s^2 Y(s) + b s Y(s) + c Y(s) = s X(s) + 2X(s)$$

 we usually define  ratio of o/p & i/p 

$$\frac{Y(s)}{X(s)} = \frac{s+2}{a\left(s^2 + \frac{b}{a}s + \frac{c}{a}\right)} = \frac{1/a\,(s+2)}{s^2 + As + B}$$

the highest degree term will always have coefficient 1.

$\Rightarrow K = 1/a.$

P(S) → polynomial expression of the numerator.

P(s) = 0 → equation.

eg here $s+2 = 0$

solving gives us a root called as zero of the system. (0)

ie putting this will make entire TF = 0.

Babar Register

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$Q(s) \rightarrow$ characteristic polynomial.

make $Q(s) = 0 \Rightarrow$ characteristic equation.

solving will give us roots called poles of the system (x)

The number of poles and zeros must be always balance.

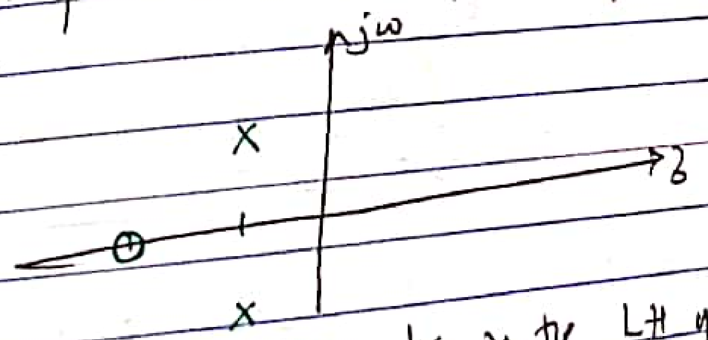
eg here if we have 1 zero and 2 poles so we consider the other pole to be at infinity.

The poles and zeros can be real, imaginary or complex quantities.

→ when they are complex quantities, they always occur in conjugate pairs ←



we can also plot the s-plane and determine the stability.



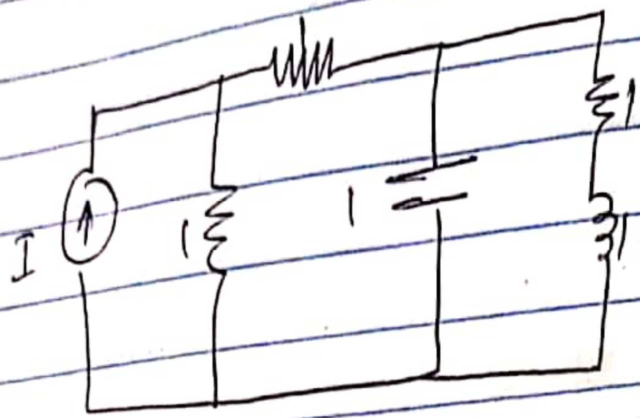

If the closed loop poles lies in the LH of s plane the system is stable.

**Order of a system** The order of a system is always defined as the degree of polynomial $Q(s)$

→ Three types of system;
Electrical, Mechanical, Electromechanical system.

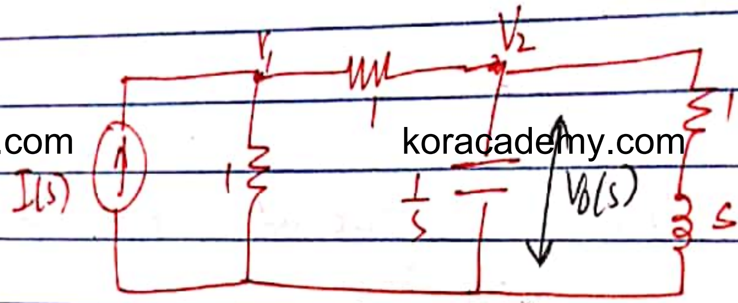
Data:

Consider an electrical system;




The first step to get T.F is to transform this circuit into s domain i.e. to take the laplace transform.

$C \rightarrow \frac{1}{SC}$  $L \rightarrow SL$  $R \rightarrow R$






KCL

node $V_1 \Rightarrow I_s = \frac{V_1}{1} + \frac{(V_1 - V_2)}{1} = 2V_1 - V_2 \rightarrow ①$

node $V_2 \Rightarrow 0 = \frac{(V_2 - V_1)}{1} + \frac{V_2}{(1/s)} + \frac{V_2}{(s+1)} \rightarrow ②$

we want $V_0 / I_s$.

$V_2 = V_0$  $② \Rightarrow 0 = V_0 - V_1 + sV_0 + \frac{V_0}{s+1}$

$\Rightarrow V_1 = V_0\left(1 + s + \frac{1}{s+1}\right)$

Put this $V_1$ in ①

$I_s = 2V_0\left(1 + s + \frac{1}{s+1}\right) - V_0$

Babar Regist...

$\Rightarrow I_s = V_0 \left[ 2\left(1+s+\frac{1}{s+1}\right) - 1 \right]$

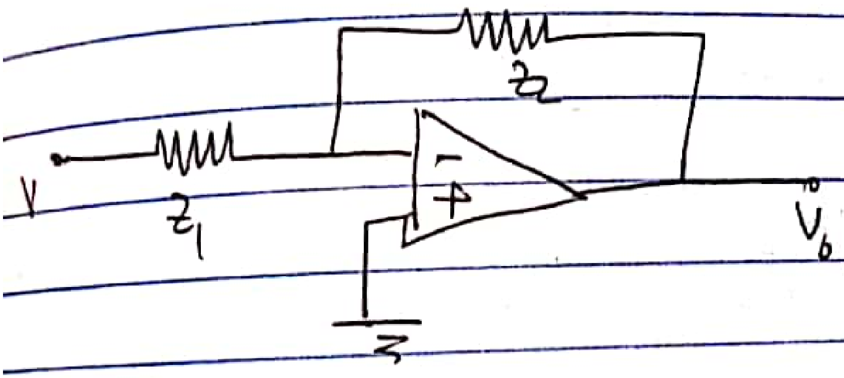
$\Rightarrow$ T.F $\frac{V_0}{I_s} = \frac{1}{\left(2+2s+\frac{2}{s+1}\right)-1}$ .......

## Op Amp

Most of the sub systems of control system are composed of op amp.



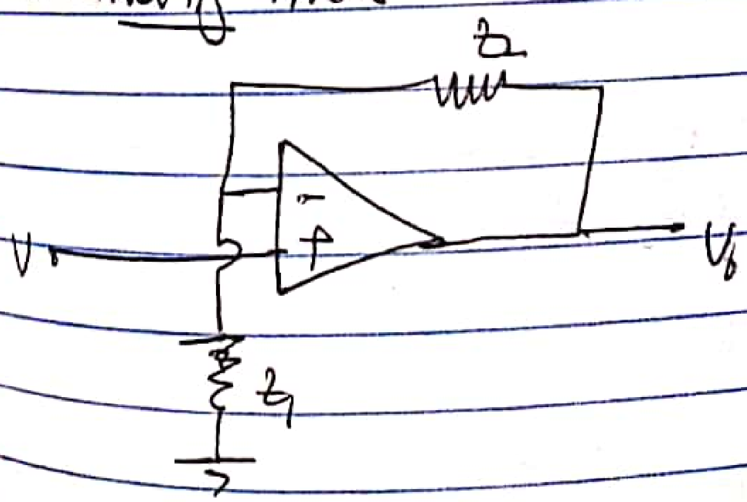
Inverting mode op amp




T.F $= -\frac{Z_2(s)}{Z_1(s)} = \frac{V_0}{V}$

-ve only shows inverting mode.

non inverting mode




T.F $= \frac{Z_1(s)+Z_2(s)}{Z_1(s)} = \frac{V_0}{V}$

# Lecture 2

## Mechanical systems

In electrical systems;
R, L, C , V(t), i(t).

$$v(t) = i(t) \cdot R = I(s) \cdot R$$

$$v(t) = L \frac{di(t)}{dt} = sL\,I(s)$$

$$v(t) = \frac{1}{C} \int i(t)\,dt = \frac{I(s)}{sC}$$

$$i(t) = \frac{v(t)}{R} = \frac{V(s)}{R}$$

$$i(t) = \frac{1}{L} \int v(t)\,dt = \frac{V(s)}{sL}$$

$$i(t) = C \frac{dv(t)}{dt} = sC\,V(s)$$

Mechanical system can be;

1) Translational → driving force = force, $f(t)$.
2) Rotational → driving force = torque, $T(t)$.

Translational

Three components;

in terms of newton force.

- Mass (M) $f = Ma = M\,dv/dt = M\,d^2x/dt^2$
- Damper (D). $f = Dv = D\,dx/dt$ → Damper's constant
- Spring (k). $f = kx$ hook's law

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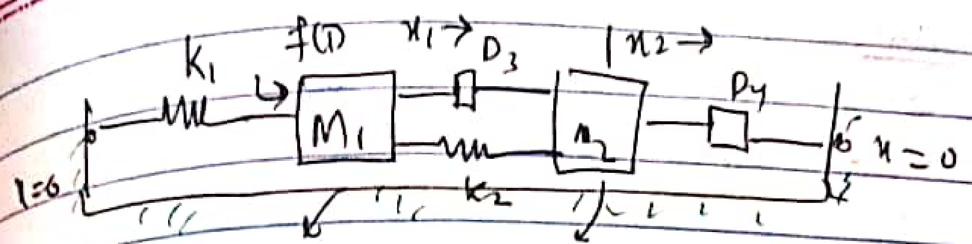


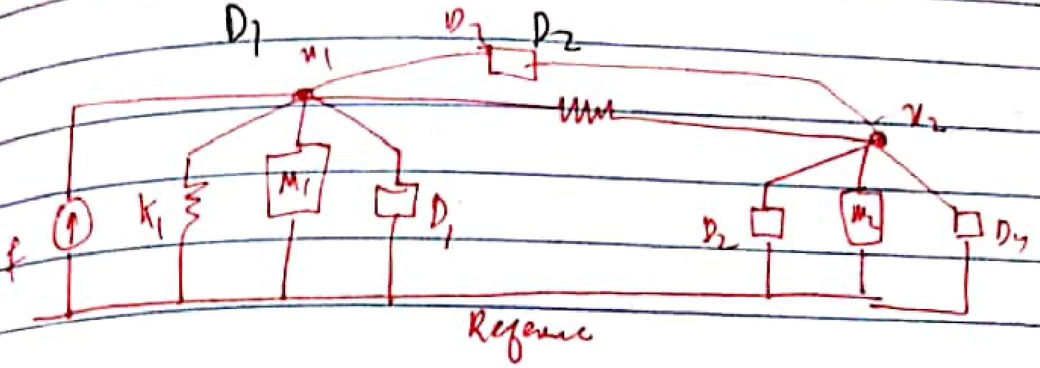


$$f = M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + D_1 \frac{dx_1}{dt} + K_2(x_1 - x_2) + D_3 \frac{d}{dt}(x_1 - x_2)$$

Take laplace trsf.



$$F = (s^2 M_1 + K_1 + sD_1 + K_2 + sD_3) X_1 - X_2 (K_2 + sD_3) \rightarrow (1)$$

Similarly

$$0 = M_2 \frac{d^2 x_2}{dt^2} + D_2 \frac{dx_2}{dt} + D_4 \frac{dx_2}{dt} + K_2(x_2 - x_1) + D_3 \frac{d}{dt}(x_2 - x_1)$$

$$0 = (s^2 M_2 + sD_2 + sD_4 + K_2 + sD_3) X_2 - X_1 (K_2 + sD_3) \rightarrow (2)$$

$$T.F = \frac{X_2(s)}{F(s)}$$

Put $X_1$ in (1) and then solve for T.F.

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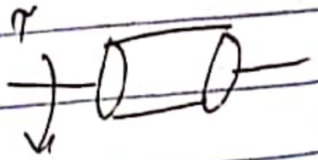
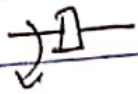
$\theta \to$ displacement.

## Rotational



J (inertia) rotating quantity.

$$T = J\alpha = J\frac{d\omega}{dt} = J\frac{d^2\theta}{dt^2}$$

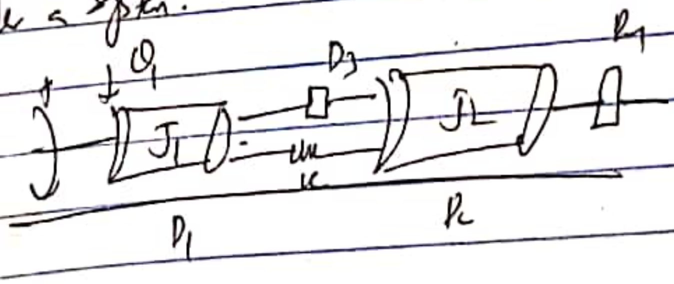
Damper  like clutching of automobile.

$$T = D\omega = D\frac{d\theta}{dt}$$

 Torsional spring $\to$ rotating of a shaft



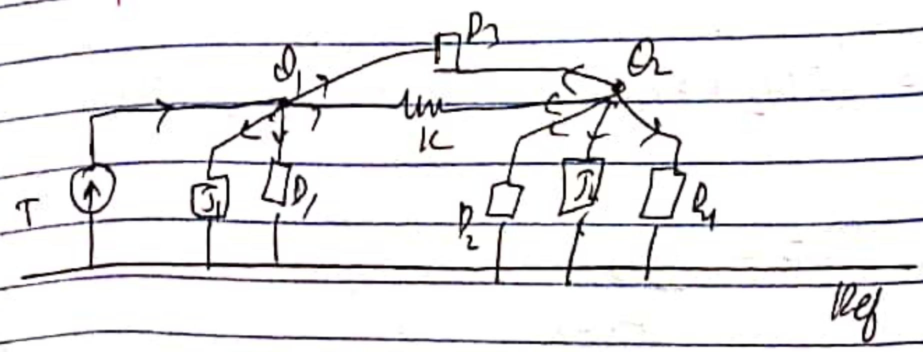
Consider a system.



$D_1$ and $D_2$ are rolling frictions.

Equivalent circuit (mechanical)

$\theta_1$ and $\theta_2$ will act as nodes.



$$T = s^2 J_1 \theta_1 + sD_1\theta_1 + k(\theta_1 - \theta_2) + sD_3(\theta_1 - \theta_2)$$

$$\Rightarrow T = (s^2 J_1 + sD_1 + k + sD_3)\theta_1 - \theta_2(k + sD_3) \rightarrow ①$$

Similarly;

$$0 = (s J_2 + sD_2 + sD_4 + k + sD_3)\theta_2 - \theta_1 (k + sD_3)$$

② ←

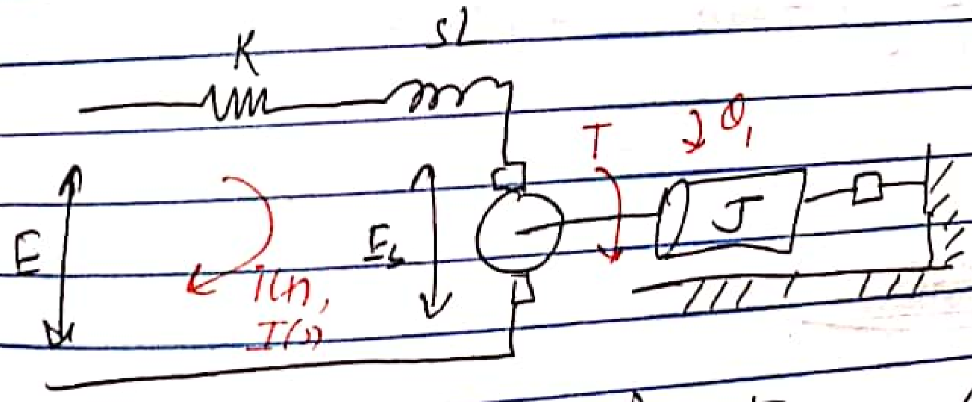
we want T.F, $\frac{\theta_1}{T} = ?$

Put $\theta_2$ from ② in ① and solve for T.F.

## Electromechanical systems



Consider a motor;




KVL $\Rightarrow$ $E = I(R + sL) + E_b$ — ① electrical side eq

$$E_b = k_1 \omega = k_1 \frac{d\theta}{dt} = s k_1 \theta$$

$$E = I(R + sL) + s k_1 \theta \quad — ②$$

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Now coming to mechanical side.

$T \propto I$

$T = K_2 I$ — (3)

(2) ⇒ $E = \frac{I}{K_2}(R + sL) + sK_1\theta$ — (4)

On mech; $T = s^2 J\theta + sD\theta$

$T = (s^2 J + sD)\theta$ — (5)

$T.F = \frac{\theta}{E} = ?$

(5) in (4) ⇒ $E = \frac{(s^2 J + sD)\theta}{K_2}(sL + R) + sK_1\theta$

$= \left\{\frac{1}{K_2}\left[(s^2 J + sD)(sL + R)\right] + sK_1\right\}\theta$

and solve for T.F.

# Lecture 3

## State Space Representation

→ time domain → differential eq
→ frequency (s) domain → transfer function.

eg $2\frac{d^3 y(t)}{dt^3} + 3\frac{d^2 y(t)}{dt^2} + 4\frac{dy}{dt} + y = u$

forcing function $= u$ Response $= y$

$\Rightarrow 2s^3Y(s) + 3s^2Y(s) + 4sY(s) + Y(s) = X(s)$

$\Rightarrow \dfrac{Y(s)}{X(s)} = \dfrac{1}{2s^3 + 3s^2 + 4s + 1}$ ....

we have another time domain → modern time domain based on state space.

state → system responses in terms of change of states in the i/p

@ how an o/p behaves under a certain change of state.

Spaces (actually represents matrices/vectors)



State variables $x_1(t), x_2(t), \ldots$

State derivatives $\dfrac{dx_1(t)}{dt}, \dfrac{dx_2(t)}{dt}, \ldots$

↳ $\dot{x}_1(t), \dot{x}_2(t)$

In state space we are interested in the first derivative through the generalized eqs which are a set comprising of two equations.

State space represent
$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \rightarrow \text{state derivative eq} \\ c(t) = Cx(t) + Du(t) \rightarrow \text{o/p eq} \end{cases}$$

The state derivative consists of all these derivative in a vector form. → column vector.

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$$\begin{bmatrix} x_1^{\circ} \\ x_2^{\circ} \\ \vdots \\ x_n^{\circ} \end{bmatrix} = \begin{bmatrix} \\ n \times n \\ (\text{square}) \\ \\ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + B \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

all state variables    i/p symbols

A → system matrix
B → u(t) → o/p

$$C(t) = \begin{bmatrix} & C & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + [D]$$

C → o/p matrix / o/p row vector
D → direct transmittance matrix → zero in most cases
↳ not well defined → since not defined properly.

Ex. Consider the D.E,

$$2\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y = x.$$

Represent in terms of state space.

(1) To know what is o/p and i/p.

y = o/p = response function.
x = i/p = forcing function.

let $y = x_1$

This means that the response is due to state 1 of the system.

Differentiate ⇒ $\frac{dy}{dt} = \frac{dx_1}{dt} = x_1^{\circ} = x_2$

second derivative ⇒ $\frac{d^2y}{dt_2} = \frac{d^2}{dt_2}x_1 = x_2^\circ = x_3$

third derivative ⇒ $\frac{d^3y}{dt_3} = \frac{d^3x_1}{dt_3} = x_3^\circ = x_4$

Highest derivative | order $= 3$ so no more derivative.

(2) $2D^3y = -3D^2y - 4Dy - y + x$

$\Rightarrow D^3y = -\frac{3}{2}D^2y - 2Dy - \frac{1}{2}y + \frac{1}{2}x$

$\Rightarrow x_3^\circ = -\frac{3}{2}x_3 - 2x_2 - \frac{1}{2}x_1 + \frac{1}{2}x$ —



we will right the derivative a the LHS only (the highest order) and will not write with the RHS.

Similarly

$x_2^\circ = x_3$ — we have a set

$x_1^\circ = x_2$ —

$$\begin{bmatrix} x_1^\circ \\ x_2^\circ \\ x_3^\circ \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1/2 & -2 & -3/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + x \begin{bmatrix} 0 \\ 0 \\ 1/2 \end{bmatrix}$$

Now for C matrix ($c = o/p = y$ in this case)

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

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we can calculate the transfer function also.

Take laplace transform of the set of state variables eqs;

$$s X(s) = A X(s) + B R(s)$$
$$C(s) = C X(s) + D R(s)$$

In control systems, we take our i/p as a reference i.e represented by $r(t)$.

$$s X(s) - A X(s) = B R(s)$$
$$\Rightarrow X(s)(sI - A) = B R(s)$$
$$\Rightarrow X(s) = (sI - A)^{-1} B R(s)$$

$I \rightarrow$ identity matrix $\rightarrow$ diagonal $= 1$, others $= 0$

Put value of $X(s)$ in $C(s)$ eq;

$$\Rightarrow C(s) = c(sI - A)^{-1} B R(s) + D R(s)$$

$$TF = \frac{C(s)}{R(s)} = c(sI - A)^{-1} B + D$$

$\downarrow$
$\times$

→ State variables are those which is bringing about a change or exhibiting a change.

→ State derivatives have to do with derivatives of the state variables. → (change) imparting this property → energy storage devices.

In electrical system ⇒ L, C

$$V_L(t) = L\frac{di_L(t)}{dt}, \quad i_c(t) = C\frac{dV_c(t)}{dt}$$

The state variables for an RLC circuit are $i_L$ and $V_c$ - why b/c they are responsible for initial conditions.
(involves first derivative).

State variables are those which cares to put initial conditions in s system.

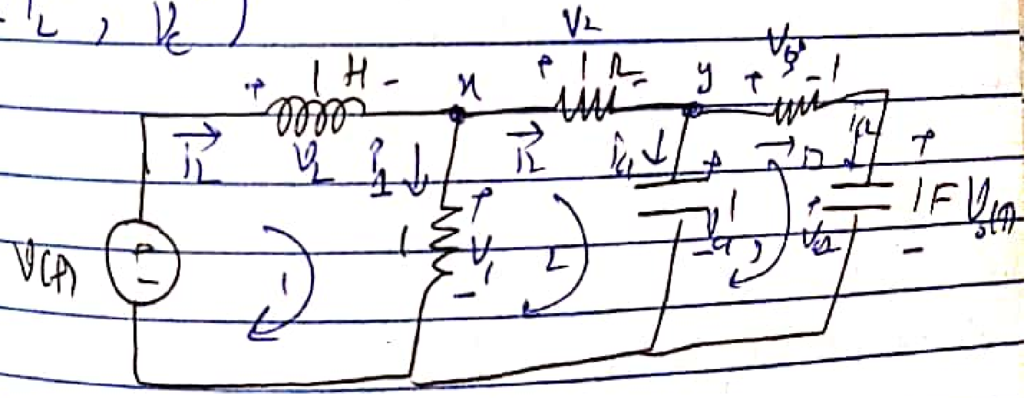
Of course we have the forcing function (that may be a current or voltage).
↳ driving forces



The state derivatives are $\frac{di_L}{dt}$, $\frac{dV_c}{dt}$.

$(i_L^0, V_c^0)$

Ex




Three loops, two nodes.
i/p = driving force = V(t).
Say o/p = $V_o$ = capacitor voltage

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State variables $i_L$, $V_{c1}$, $V_{c2}$, $v(t)$

State derivatives $\frac{di_L}{dt}$, $\frac{dV_{c1}}{dt}$, $\frac{dV_{c2}}{dt}$

Do All the labelling.

$i_3 = i_{c2}$

loop 1

$v(t) = V_L + V_1 = V_L + i_1$ — ①

loop 2

$V_1 = V_2 + V_{c1} = i_2 + V_{c2}$ — ②

loop 3

$V_{c1} = V_3 + V_{c2} = i_3 + V_{c2} = i_{c2} + V_{c2}$ — ③

Node x

$i_L = i_1 + i_2$ — ④

node y

$i_2 = i_{c1} + i_3 = i_{c1} + i_{c2}$ — ⑤

Branch current = Potential difference b/w two nodes / value of the element

$\Rightarrow i_2 = \frac{V_1 - V_{c1}}{1}$

⑤ $\Rightarrow V_1 - V_{c1} = i_{c1} + i_{c2}$ —

I want to get value of $V_1$

① $\Rightarrow V_1 = v(t) - V_L$

$\Rightarrow v(t) - V_L - V_{c1} = i_{c1} + i_{c2}$

$\Rightarrow v(t) - \frac{di_L}{dt} - V_{c1} = \frac{dV_{c1}}{dt} + \frac{dV_{c2}}{dt}$ — (6)

(5) $\Rightarrow V_{c1} = i_{c2} + V_{c2} = \frac{dV_{c2}}{dt} + V_{c2}$

$\Rightarrow \frac{dV_{c2}}{dt} = V_{c1} - V_{c2}$ — (A)

(A) in (6) $\Rightarrow v(t) - \frac{di_L}{dt} - V_{c1} = \frac{dV_{c1}}{dt} + V_{c1} - V_{c2}$

$\Rightarrow \frac{dV_{c1}}{dt} + \frac{di_L}{dt} = -V_{c1} - V_{c1} + V_{c2} + v(t)$ $\hookrightarrow$ (7)



I need something like independent $dV_{c1}$ and independent $di_L$.

(4) $\Rightarrow i_L = i_1 + i_2 = v(t) - V_L + \frac{V_1}{2} - V_{c2}$

$\Rightarrow i_L = v(t) - V_L + i_1(1) - V_{c2}$

$= v(t) - V_L + v(t) - V_L - V_{c2}$

$i_L = 2v(t) - 2V_L - V_{c2}$

$\Rightarrow 2V_L = -i_L - V_{c2} + 2v(t)$

$\Rightarrow V_L = -\frac{1}{2} i_L - \frac{1}{2} V_{c2} + v(t)$

$\Rightarrow \frac{di_L}{dt} = -\frac{1}{2} i_L - \frac{1}{2} V_{c2} + v(t)$ — (B)

(B) in (7)

$$\Rightarrow \frac{dV_{c1}}{dt} - \frac{1}{2} i_L - \frac{1}{2} V_{c2} + \cancel{V(t)} = -2V_{c1} + V_{c2} + \cancel{V(t)}$$

$$\rightarrow \frac{dV_{c1}}{dt} = \frac{1}{2} i_L - 2V_{c1} + \frac{3}{2} V_{c2} \quad \text{— (C)}$$

In matrix form;

$$\begin{bmatrix} \dot{i_L} \\ \dot{V_{c1}} \\ \dot{V_{c2}} \end{bmatrix} = \begin{bmatrix} -1/2 & 0 & -1/2 \\ 1/2 & -2 & 3/2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_L \\ V_{c1} \\ V_{c2} \end{bmatrix} + V(t) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

The other set is of o/p



$$V_o(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ V_{c1} \\ V_{c2} \end{bmatrix}$$

## Lecture 4

### Block Diagrams

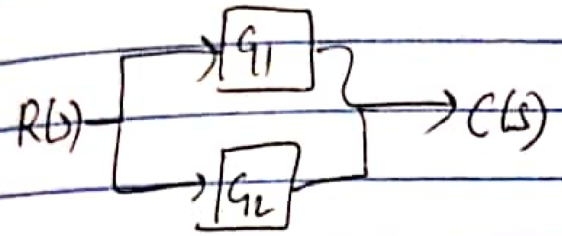
It is a pictorial / schematic representation of different components of a control system.

→ We can obtain the T.F by looking at the block diagram and try to simplify steps one and try to bring that to a single block with the I/P on one side and O/P on the other.

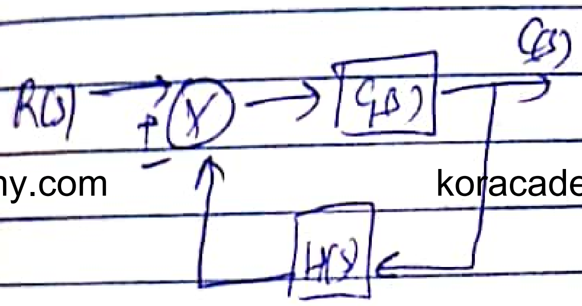
$R(s) \rightarrow \boxed{T/F} \rightarrow C(s)$

The components can be connected in 3 ways
- Cascade/series - Parallel - Loop

$R(s) \rightarrow \boxed{G_1} \rightarrow \boxed{G_2} \rightarrow C(s) \equiv R(s) \rightarrow \boxed{G_1 G_2} \rightarrow C(s)$



$\equiv R(s) \rightarrow \boxed{G_1 + G_2} \rightarrow C(s)$



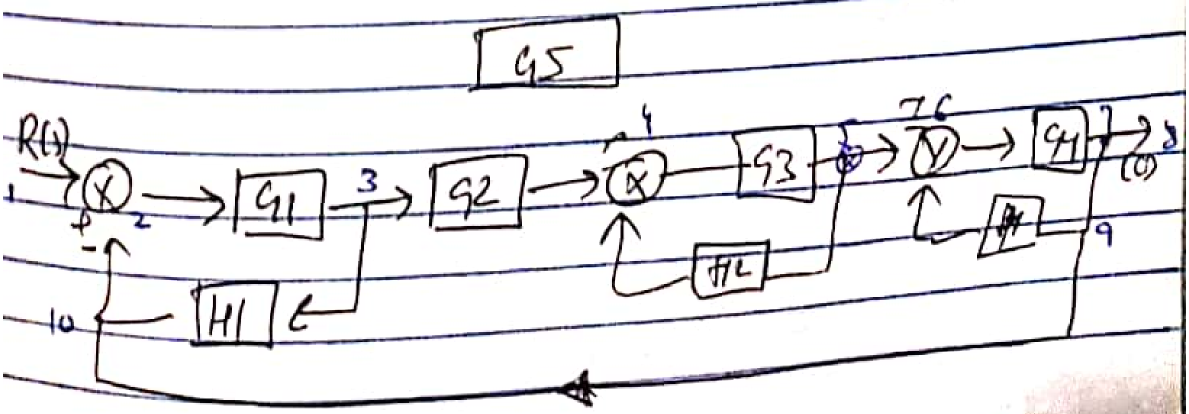


$\equiv R(s) \rightarrow \boxed{\dfrac{G(s)}{1+H(s)G(s)}} \rightarrow C(s)$

→ Arrow head from i/p to o/p ⇒ forward block
" " o/p to i/p ⇒ feedback.

Transmittance → type of T.F but we do not use a block in this case.

Ex consider the figure.



# Signal flow graph

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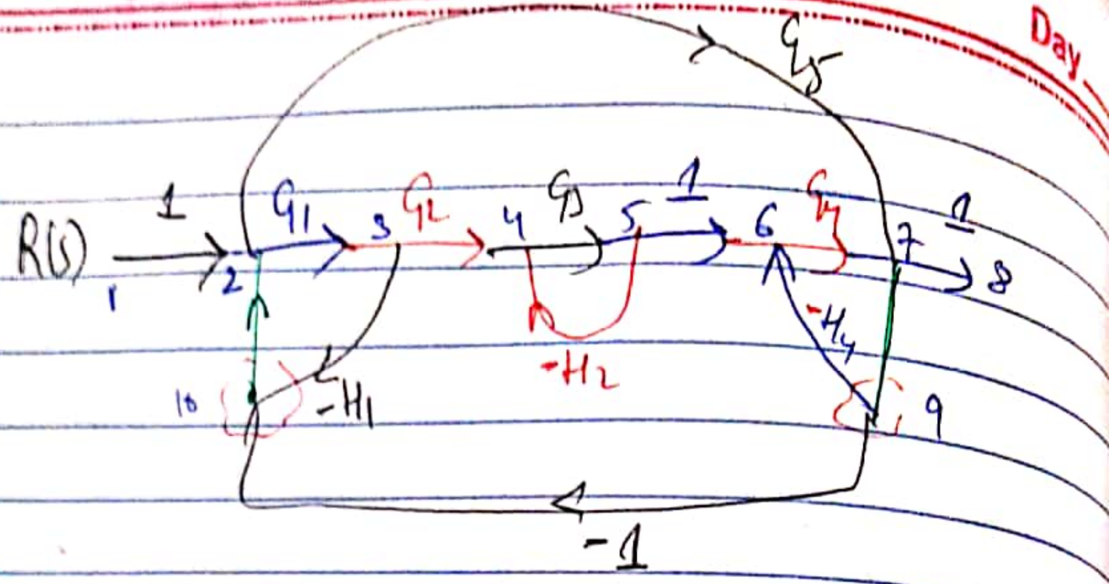


these two nodes can be absorbed to simplify.

Now collective transmittance;

2 forward paths $P_1 = G_5$



$P_2 = G_1 G_2 G_3 G_4$

A single node must be only traversed once.

Independent loops.

$L_1 = -G_1 H_1$

$L_2 = -G_3 H_2$

$L_3 = -G_4 H_4$

$L_4 = -G_1 G_2 G_3 G_4$

$L_5 = -G_5$

Pair of two non touching loops.

↳ they should'nt have any node in common.

$L_1 L_2 = G_1 G_3 H_1 H_2$

$L_1 L_3 = G_1 G_4 H_1 H_4$

$L_2 L_3 = G_3 G_4 H_2 H_4$

$L_2 L_5 = G_3 G_5 H_2$

set of 3 non touching loops;

$\rightarrow L_1 L_2 L_3 = -G_1 G_3 G_4 H_1 H_2 H_4$

If the set of two non touching loops exceed 3; there is always a possibility of 7 such set.

**Mason's Rule** of getting T.F through a signal flow graph

$$\boxed{T/F = \frac{\Sigma P_i \Delta_i}{\Delta}}$$

$\Delta = 1 -$ ($\Sigma$ of all transmittance of independent loops)

$+$ ($\Sigma$ of product of two non touching loops)

$-$ ($\Sigma$ of product of 3 non touching loops)

$- - - - + -$

Here in our example)

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_1 L_2 + L_1 L_3 + L_2 L_3 + L_2 L_5) - (L_1 L_2 L_3)$$

$$\Delta = 1 + G_1 H_1 + G_3 H_2 + G_4 H_4 + G_1 G_2 G_3 G_4 + G_5 + G_1 G_3 H_1 H_2 + G_1 G_4 H_1 H_4 + G_3 G_4 H_2 H_4 + G_5 G_3 H_2 + G_1 G_3 G_4 H_1 H_2 H_4$$

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Here $T/F = \dfrac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$

$\Delta_1 \to$ cofactor of the forward path $P_1$.

Remove the nodes that are included in the forward path.

Once they are removed, see how many loops are left behind in proper shape.

only $L_2$ remains. (in expression of $\Delta$)

$\Rightarrow \Delta_1 = 1 + G_3H_2$

Similarly for $\Delta_2$ if we remove, no nodes are left behind

$\Delta_2 = 1$



$$\Rightarrow T/F = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_5(1 + G_3H_2) + G_1G_2G_7G_4(1)}{\Delta}$$

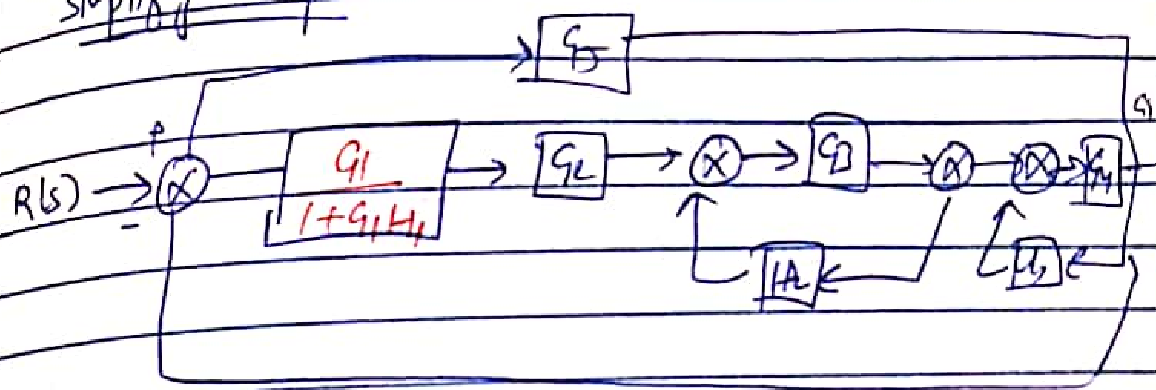
$$\Rightarrow T/F = \frac{G_5 + G_3G_5H_2 + G_1G_2G_7G_4}{\Delta}$$

Ans

We can also do it without using Mason's rule, but that will be a problem in many cases.

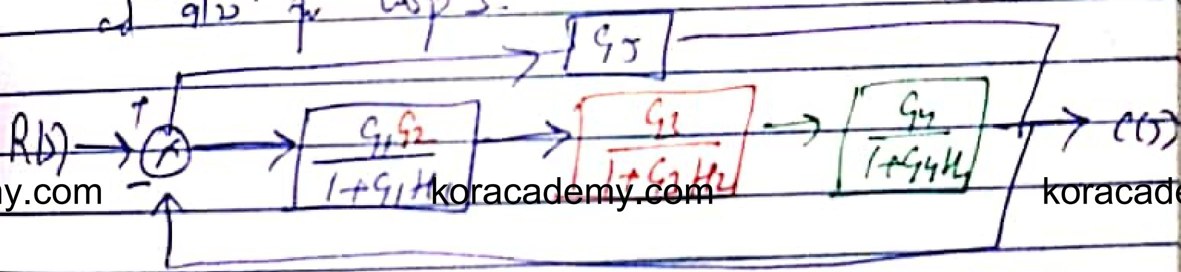
Say first we check for the inner loops.

Simplify loop 1.



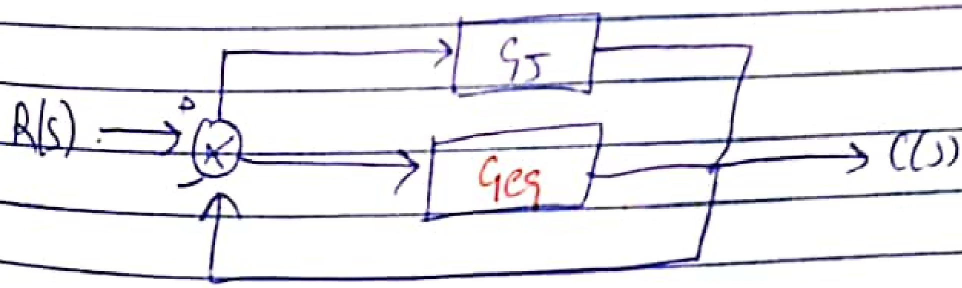

Now $\frac{G_1}{1+G_1H_1}$ and $G_2$ are in series.

Similarly loop 2 is replaced by a single block and also for loop 3.



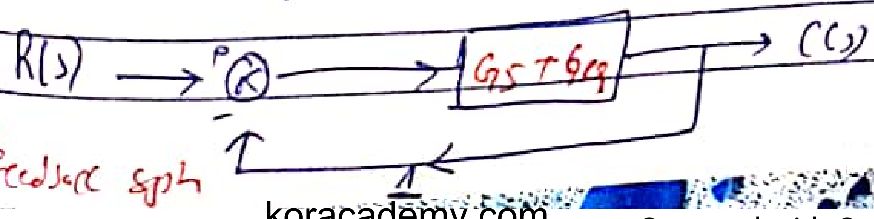



Now the three blocks are in series.




where $G_{eq} = \frac{G_1G_2G_3G_4}{(1+G_1H_1)(1+G_3H_2)(1+G_3H_1)}$

Now $G_5$ and $G_{eq}$ are in parallel.




This is a unity feedback system

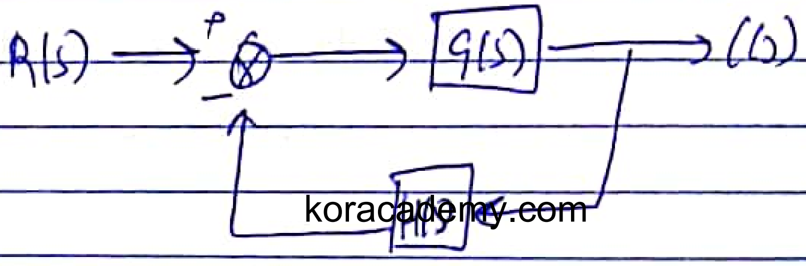
$R(s) \rightarrow \boxed{G_0} \rightarrow C(s)$

where $G_0 = \dfrac{G_5 + G_6}{1 + G_5 + G_6}$ as $H = 1$

where $G_0$ would be exactly the same as previously obtained T/F i.e. $\dfrac{G_5 + G_3 G_5 H_2 + G_1 G_2 G_3 G_4}{\Delta}$.

## Unity Feedback loop / system.

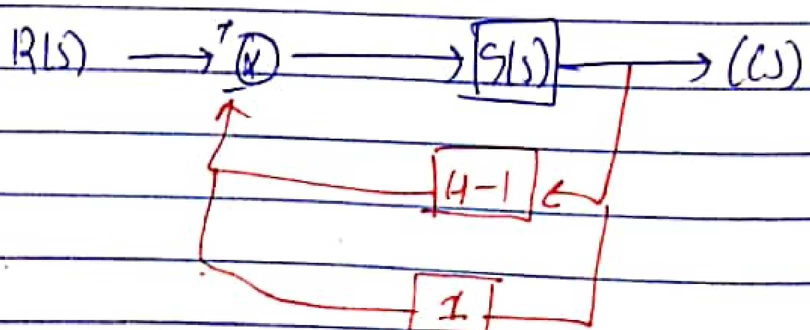
As



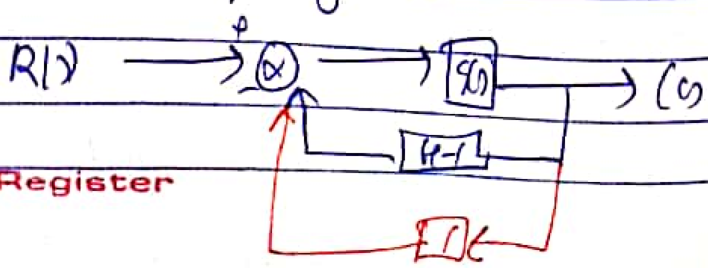



say $H(s) = H(s) - 1 + 1 = (H(s) - 1) + 1$

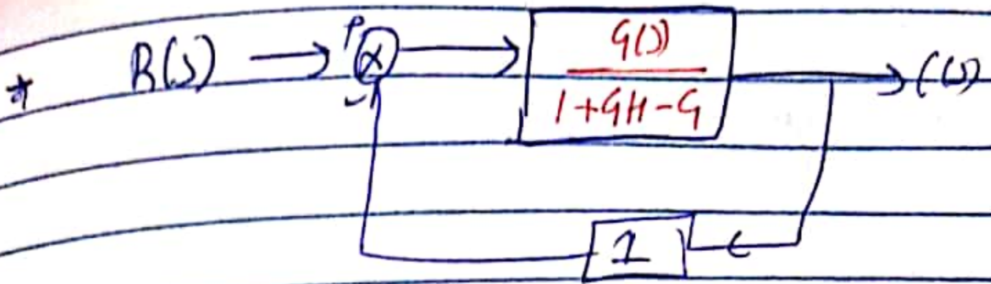
This means they are two different components to be connected in parallel.




Or it can be drawn separately as:

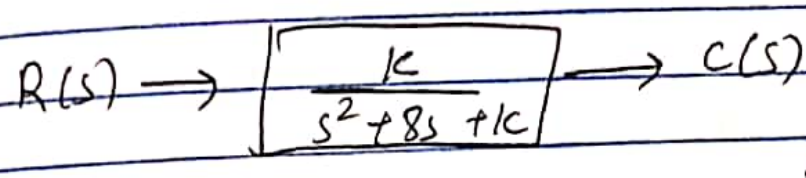


The Sure accuracy + loops finale;




one can be replaced by a single arrow.

consider the T.F as single block.




convert this into an equivalent unity feedback block.

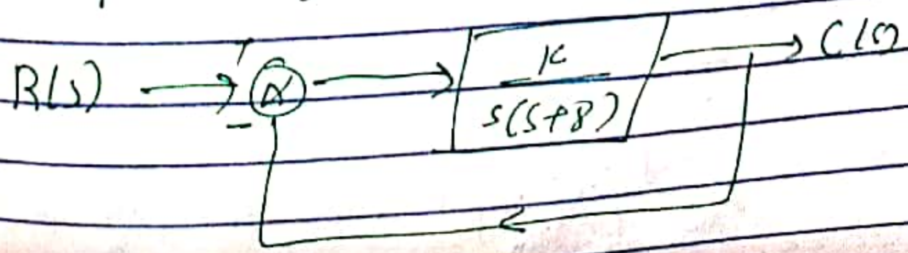


Take the first two terms common

$$\Rightarrow \frac{K}{s^2+8s+K} = \frac{K}{s^2+8s\left(1+\frac{K}{s^2+8s}\right)}$$

$$= \frac{K/s^2+8s}{1+K/s^2+8s} = \frac{G}{1+G} \quad (H=1)$$

So the equivalent unity feedback is;



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# Lecture 5

## Control System Response

systems → $1^{st}$ order
→ $2^{nd}$ order

### Standard i/ps

1. Unit step signal. $u(t) = 1 \leftrightarrow 1/s$
2. Unit ramp, $t$. $\leftrightarrow 1/s^2$
3. Parabolic. $\frac{1}{2}t^2 \leftrightarrow 1/s^3$

Relations

$$\int u(t)\,dt = \int 1\,dt = t = \text{ramp}$$



$$\int t\,dt = \frac{1}{2}t^2 = \text{parabola}.$$

and the reverse with differentiation.

A $1^{st}$ order system will generally have a TF as;

$$\boxed{\frac{C(s)}{R(s)} = \frac{K}{s+a}}$$

$$\Rightarrow C(s) = R(s) \cdot \frac{K}{s+a}$$

Unit step response

$$R(s) = \frac{1}{s} \rightarrow C(s) = \frac{1}{s} \cdot \frac{K}{s+a} = \frac{K}{s(s+a)}$$

In order to find the response, we decompose this into Partial fraction

$$= \frac{A}{s} + \frac{B}{s+a}$$

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solve for A and B, and then take inverse laplace transform and you will get the desired response in time domain.

Similarly for unit ramp $R(s) = 1/s^2$ ..

## 2nd order systems

The general form of a second order system is as;

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{K}{s^2 + 2\zeta\omega_n s + K}.$$

where $\omega_n$ = undamped natural frequency.
$\zeta$ = damping coefficient (damping ratio).
ratio of actual damping to critical damping



Also $\omega_d = \omega_n\sqrt{1-\zeta^2}$ → damping frequency.

2nd order systems are generally classified in 4 different categories;

1. Critically damped → Roots are real and equal. No oscillations

The characteristic eq is $s^2 + 2\zeta\omega_n s + \omega_n^2$
(fast acting) Equate this to zero and find roots.

$$s = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

Solving

$$\Rightarrow s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

poles

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2. Underdamped systems — Accompanied by oscillations in the transient portion (i.e. damped oscillations).

→ Roots are complex in nature.

→ Poles are complex conjugate of each other.

(most practical systems are underdamped)

3. Over damped system — No oscillations. (slow settling).

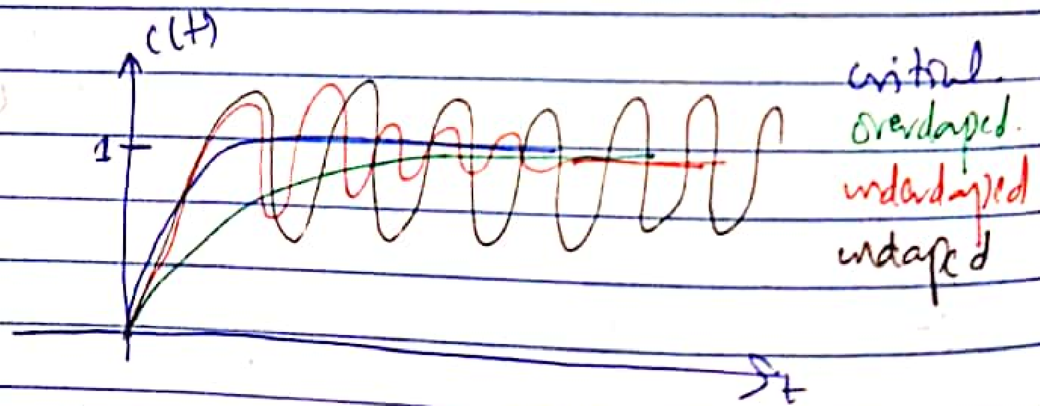
→ Roots are real and unequal.

4. Undamped system

Roots are purely imaginary (they lie on the jw axis).

It is accompanied with sustained oscillations of fixed magnitude and fixed frequency.






→ The solution of differential equation gives us response of the circuit

$$c(t) = C_t + C_{ss}$$

$C_t$ → transient portion / complementary part / natural response

$C_{ss}$ → steady state portion / particular part / forced response

The overshoot depends on value of $\zeta$ (zeta).

For

| | |
|---|---|
| CDS | $\zeta = 1$ |
| UDS | $0 < \zeta < 1$ |
| ODS | $\zeta > 1$ |
| UnDS | $\zeta = 0$ |

$\zeta \to 0 \Rightarrow$ more overshoot
$\zeta \to 1 \Rightarrow$ less overshoot

Consider the general $2^{nd}$ order T/F.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



For unit step response $R(s) = 1/s$

$$\Rightarrow C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

Decompose into partial fractions.
↓
inverse laplace transform. ---

The response of a general second order system is;

$$c(t) = \underbrace{1}_{\text{steady state value.}} + \underbrace{\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cos(\omega_n\sqrt{1-\zeta^2}\, t - \alpha)}_{\text{transient response.}}$$

↳ we have an exponential and oscillatory (cos) term.

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System settles down to the steady state value following the decay of transients.

As $\omega_d = \omega_n \sqrt{1-\zeta^2}$

$$\Rightarrow c(t) = 1 + \frac{\omega_n}{\omega_d} e^{-\zeta\omega_n t} \cos(\omega_d t - \alpha)$$

Consider $\frac{C(s)}{R(s)} = \frac{100}{s^2 + 10s + 100}$

$\zeta = ?$ $\omega_n = ?$

Comparing with standard eq $\Rightarrow \omega_n^2 = 100$

$\Rightarrow \omega_n = 10$ rad/s



Also $2\zeta\omega_n = 10s$

$\Rightarrow \zeta\omega_n = 5$ $\Rightarrow \zeta = 0.5$

Ans.

## Transient Response

we need two important parameters $\rightarrow \omega_n, \zeta$

time specifications

1. Rise time $T_r = \frac{\pi - \cos^{-1}\zeta}{\omega_d}$
2. Time to peak $T_p = \pi / \omega_d$
3. * %age overshoot. $\%OS = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100$

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4. Settling time $T_s = \frac{4}{\zeta \omega_n}$

Systems behave normally under 10 - 20% overshoot.

less than 10% overshoot → over stressed → X

Consider a closed loop TF of a 3rd order system,

$$\frac{C(s)}{R(s)} = \frac{1}{s^3 + 3s^2 + 7s + 5}$$

$$= \frac{1}{(s+1)(s^2 + 2s + 5)}$$

$$= \frac{1}{(s+1)(s+1+j2)(s+1-j2)}$$



3 poles one real, two complex conjugate

This system is basically a cascade of a first order and second order systems

$$= \frac{1}{(s+1)} \left( \frac{1}{s^2 + 2s + 5} \right) \left( \frac{5}{5} \right)$$

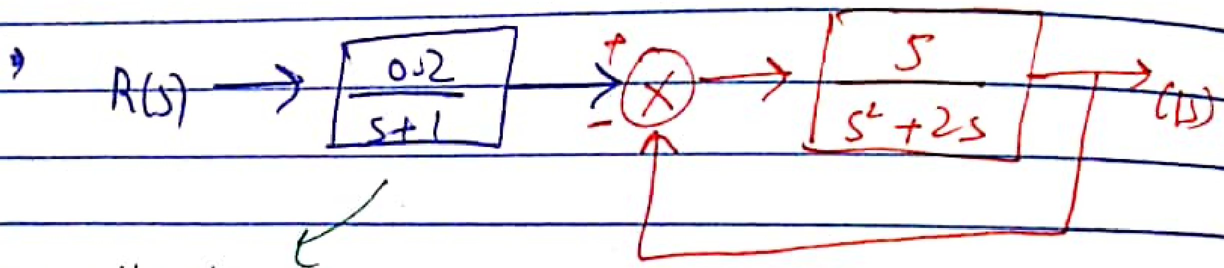
$$= \left( \frac{0.2}{s+1} \right) \left( \frac{5}{s^2 + 2s + 5} \right)$$

$$R(s) \rightarrow \boxed{\frac{0.2}{s+1}} \rightarrow \boxed{\frac{5}{s^2 + 2s + 5}} \rightarrow C(s)$$

Now $\frac{5}{(s^2+2s)\left(1+\frac{5}{s^2+2s}\right)} = \frac{5}{s^2+2s+5}$

$= \frac{5/(s^2+2s)}{1+5/(s^2+2s)}$ which is TF. for $\frac{G(s)}{1+G(s)}$ and $H(s) = 1$

so unity feedback system

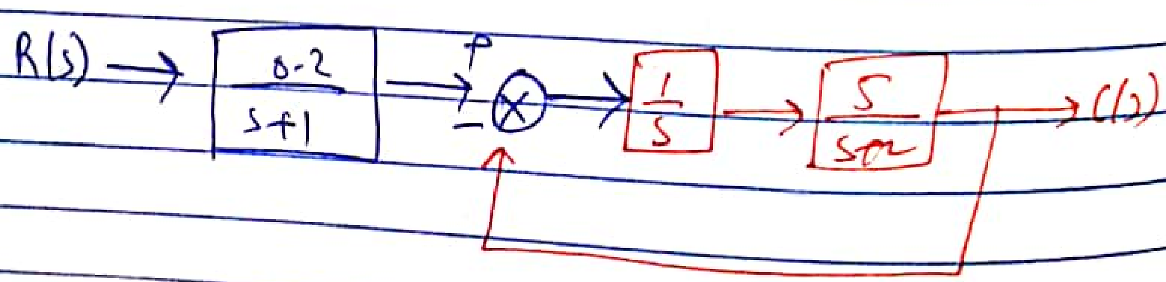



i/p transducer



$\frac{5}{s^2+2s} = \frac{5}{s(s+2)} = \left(\frac{1}{s}\right)\left(\frac{5}{s+2}\right)$

cascaded




$1/s$ → controller (integrator)

$5/s+2$ → plant

Inorder to find the transient behavior, we only consider the actual system,
(ie don't consider the i/p transducer)

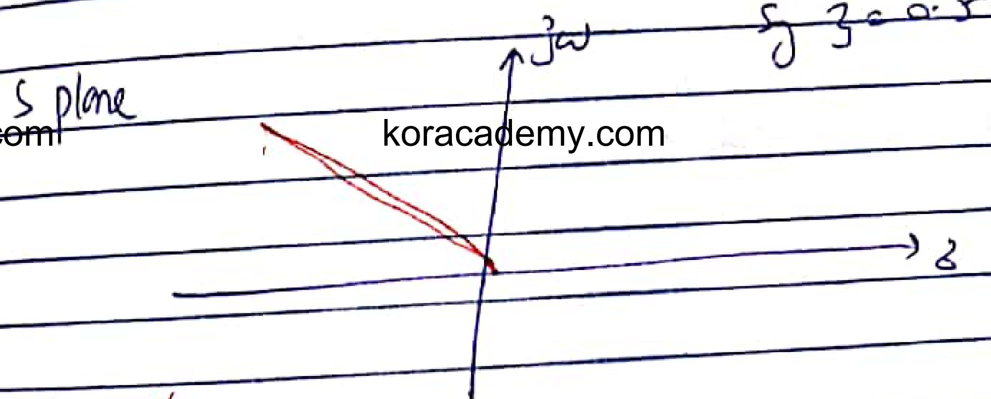
$$\frac{5}{s^2+2s+5}$$

$\Rightarrow \omega_n^2 = 5 \Rightarrow \omega_n = \sqrt{5}$

Smly find $\zeta$, $\omega_d$, $T_r$, $T_p$, %OS, $T_s$.

$\zeta$ is very important in deciding the % OS.

$\zeta \downarrow \Rightarrow OS \uparrow$






$180° - \cos^{-1}\zeta$
↳ damping line → $\zeta$ is constant
↳ $\omega_n$ changes.

If a system is operated on the damping line irrespective of whatever operating point you will choose, the system overshoot will remain the same.

→ we are interested in %OS and $T_s$ ← . $T_s = \frac{4}{\zeta \omega_n}$

$\omega_n \uparrow \Rightarrow T_s \downarrow$

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Lecture 6

## Stability

stable = bonded i/p ⇒ bonded o/p.

→ we can also comment on stability through the closed loop TF.

as $$T(s) = \frac{G(s)}{1 \pm H(s)G(s)}$$

G(s) → plant   H(s) → feedback

+ → -ve feedback
↳ stable

- → +ve feedback



so the CLTF for a stable system,

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

The CLTF for a unity feedback system,

$$T(s) = \frac{G(s)}{1 + G(s)} = \frac{P(s)}{Q(s)} \text{ generally}$$

Q(s) → characteristic polynomial = 0
↳ roots = CL poles

The poles are plotted in complex s plane.

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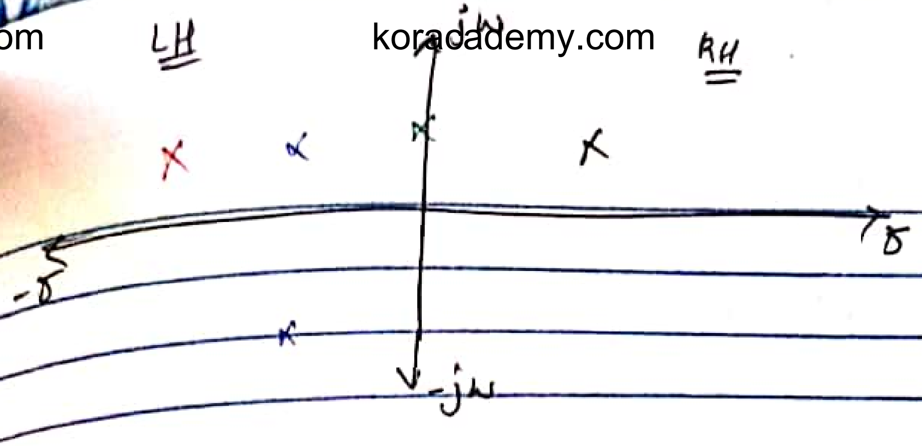


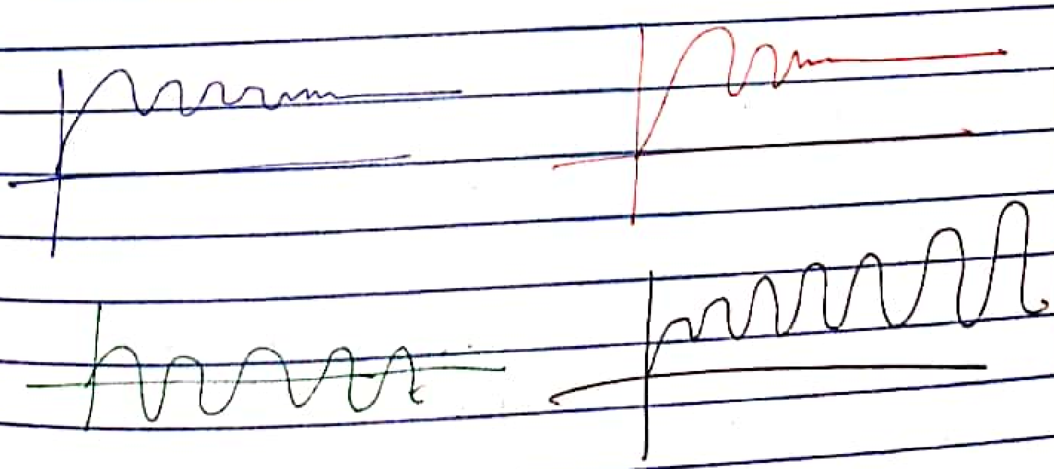

If the CL poles are located in LH of s plane, the system is stable.

If any pole lie in the RH → unstable

Taking jω axis as reference, the more the poles are to the left, the more stable the system is.

If the poles lie on the jω axis → marginally stable.
→ the response is the form of sustained oscillations with a particular frequency





## Routh Hurwitz Criteria

Routh table → array

↳ Deals with the number of poles that are placed in the right half of the s plane.

## Routh table

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 2 | 6 |
| $s^3$ | 3 | 1 | 0 |
| $s^2$ | 5/3 | 6 | 0 |
| $s^1$ | -9.8 | 0 | 0 |
| $s^0$ | 6 | 0 | 0 |

$$\frac{(3\times2)-(1\times1)}{3} = \frac{5}{3} \qquad \frac{(3\times6)-(1\times0)}{3} = 6$$



$$\frac{(5/3)(1)-(3\times6)}{5/3} = \frac{5/3-18}{5/3} = -9.8$$

$$\frac{(-9.8\times6)-0}{-9.8} = 6$$

After completing the Routh table, we apply the Routh Hurwitz criteria. → elements of first column.
↳ signs

No. of sign changes in first column of routh table = no. of closed loop poles located in RH of s plane.

1, 3, 5/3, -9.8, 6.
① ②

2 sign changes ⇒ 2 poles in RH plane ⇒ unstable system

The Routh table is to be formed from the characteristic equation

Ex Consider the closed loop TF.

$$T(s) = \frac{10}{s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10}$$

$$Q(s) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$$

Routh table

| | | | |
|---|---|---|---|
| $s^5$ | 1 | 2 | 11 |
| $s^4$ | 2 | 4 | 10 |
| $s^3$ | | | |
| $s^2$ | | | |
| $s^1$ | | | |
| $s^0$ | | | |



Consider $Q(s) = s^4 + 2s^3 + 11s^2 + 18s + 18$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 11 | 18 |
| $s^3$ | 2 | 18 | 0 |
| $s^2$ | 2 | 18 | 0 |
| $s^1$ | 0 | 0 | 0 |
| | 4 | 0 | 0 |
| $s^0$ | 18 | 0 | 0 |

All zero row?

for $s^1$ row we make an auxiliary polynomial.

$$A(s) = 2s^2 + 18$$

s/c the $s^2$ next can go to $s^0$.

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(A(s) from immediately above row).

Differentiating $\Rightarrow \frac{dA(s)}{ds} = 4s + 0$

No changes in sign $\Rightarrow$ all poles lying in LHP
$\Rightarrow$ stable system

* Consider $s^4 + 7s^3 + 15s^2 + (25+k)s + 2k$.

what range of k is necessary for stability?

k → gain of controller



Routh table

| | | | |
|---|---|---|---|
| (+) $s^4$ | 1 | 15 | 2K |
| (+) $s^3$ | 7 | 25 + K | 0 |
| (+) $s^2$ | $80-K/7$ | 2K | 0 |
| (+) $s^1$ | $-K^2-43K+2000 / 80-K$ | 0 | 0 |
| (+) $s^0$ | 2k | 0 | 0 |

$\left(\frac{80-K}{7}\right) > 0 \Rightarrow 80 - K > 0 \Rightarrow -K > -80$

$\boxed{K < 80}$

$\frac{-K^2 - 43K + 2000}{80 - K} > 0 \Rightarrow -K^2 - 43K + 2000 > 0$

$\Rightarrow \boxed{\begin{array}{l} K < 28.12 \\ K < -71.2 \end{array}}$

$2K > 0 \Rightarrow \boxed{K > 0}$

$\Rightarrow \boxed{0 < K < 28.12}$

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→ at $K = 28.12$ the system is marginally stable.

→ poles are located on the $j\omega$ axis.

→ If we want to find auxiliary equation

make $\dfrac{-K^2 - 43K + 200}{80 - K} = 0$

This happens at $K = 28.12$

Now $\left(\dfrac{80-K}{7}\right)s^2 + 2K = A(s) = 0$

⇒ $\left(\dfrac{80-28.12}{7}\right)s^2 + 2(28.12) = 0$

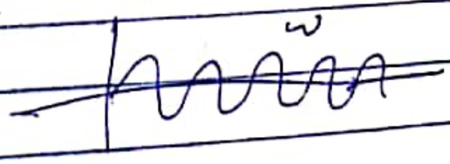


Solving ⇒ $s = \pm j2.75$

⇒ $j\omega = j2.75$

⇒ $\omega = 2.75$ rad/s

frequency of oscillation.




## Lecture 7

### Steady state error

Error → Deviation from the desired quantity.

Accuracy of a system is measured in terms of error.

Error $\propto \dfrac{1}{\text{Accuracy}}$

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System response consists of two parts;

$c(t) =$ transient + steady state response.

transient ↓ diminish after some time

steady state response ↓ remains.

Talking of the steady state response, we need to first determine which i/p is desired for a system.

Conversely we can say what is the acceptance sy conversely we can say what is the acceptance sy the system concerning these i/p quantities which are also referred as test i/ps.

These are standard i/ps;

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i) unit step, $u(t) = 1 \rightarrow LT = 1/s \rightarrow T0$

ii) Unit ramp, $t\,u(t) = t \rightarrow 1/s^2 \rightarrow T1$

iii) Unit parabolic, $\frac{1}{2}t^2 u(t) = \frac{1}{2}t^2 \rightarrow 1/s^3 \rightarrow T2$

(switched at $t=0$)

By definition, $s = \frac{d}{dt}$, $\frac{1}{s}$ = simple integrator

System types on the basis of steady state error:

Type 0, Type 1, Type 2,

Type 0 ↓ constant position system

Type 1 ↓ constant velocity system

Type 2 ↓ constant acceleration system

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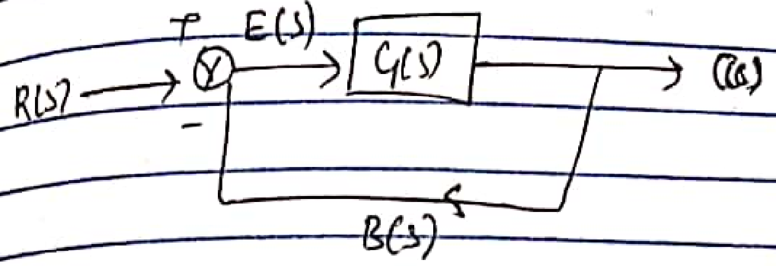
lets first generally define steady state error.

consider a unity feedback system;




$$E(s) = R(s) - B(s)$$

As $B(s) = C(s)$ (due to unity feedback)

$$\Rightarrow \quad E(s) = R(s) - C(s)$$

the o/p by definition, $C(s) = G(s)\,E(s)$



$$\Rightarrow \quad E(s) = R(s) - E(s)\,G(s)$$

$$\Rightarrow \quad E(s)\,(1 + G(s)) = R(s)$$

$$\Rightarrow \quad \boxed{E(s) = \frac{R(s)}{1+G(s)}} \quad \text{— (1)}$$

Xly b.s of (1) with s and take lmt $s \to 0$

$$\Rightarrow \quad \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} \frac{s\,R(s)}{1+G(s)}$$

Applying final value theorem of laplace transform

$$\boxed{e(\infty) = \lim_{s\to 0} \frac{s\,R(s)}{1+G(s)}} \quad \text{— (2)}$$

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(i) $R(s) = 1/s$

$$\Rightarrow e(\infty) = \lim_{s \to 0} \frac{1}{1+G(s)} = \frac{1}{1+\lim_{s\to 0} G(s)}$$

By definition, $\lim_{s\to 0} G(s) = K_p \rightarrow$ static error constant for type 0 system

$\rightarrow$ constant position

② in terms of $K$ $\Rightarrow$ $\boxed{e(\infty) = \frac{1}{1+K_p}}$

ii. $R(s) = 1/s^2$

$$e(\infty) = \lim_{s\to 0}\left(\frac{1}{s+sG(s)}\right) = \frac{1}{\lim_{s\to 0} sG(s)}$$



By definition, $\lim_{s\to 0} sG(s) = K_v \rightarrow$ static error const for type 1 system

$\rightarrow$ constant velocity.

$$\Rightarrow \boxed{e(\infty) = \frac{1}{K_v}}$$

iii $R(s) = 1/s^3$

$$e(\infty) = \lim_{s\to 0}\left(\frac{1}{s^2+s^2G(s)}\right) = \frac{1}{\lim_{s\to 0} s^2G(s)}$$

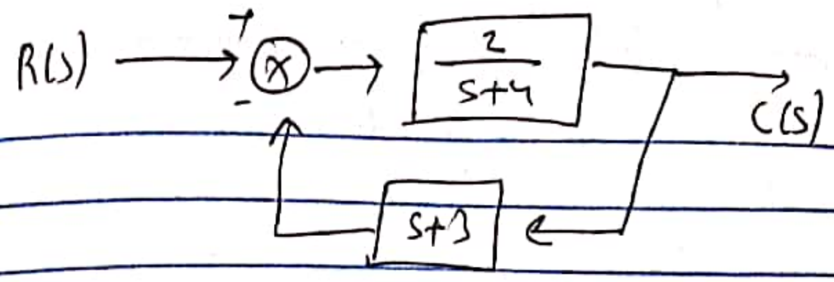
By definition

$\lim_{s\to 0} s^2 G(s) = K_a \rightarrow$ static error constant for

constant acceleration $\leftarrow$ type 2 system

$$\Rightarrow \boxed{e(\infty) = \frac{1}{K_a}}$$

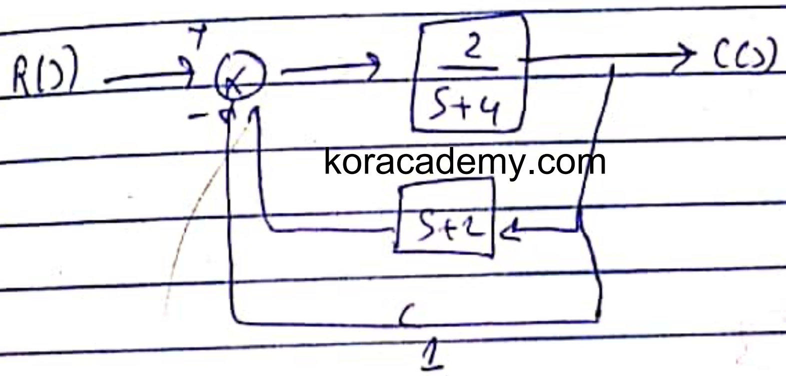
Ex: Consider




As we have derived all the formulas for a unity feedback system, so lets first convert the given system into a unity feedback sys.

$$s+3 = s+3+1-1 = (s+2)+1$$

parallel






Inner loop is a -ve feedback system which can be simplified as;

$$\frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\frac{2}{s+4}}{1+(s+2)\left(\frac{2}{s+4}\right)} = \frac{2}{(s+4)+2(s+2)} = \frac{2}{3s+8}$$

$$= \frac{2}{3(s+8/3)} = \frac{2/3}{s+8/3}$$

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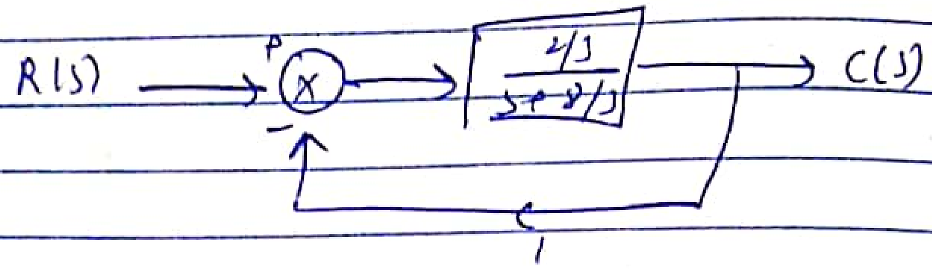
⇒ The eq unity feedback system is;




Now $G(s) = \dfrac{2/3}{s + 8/3}$

i. $\lim_{s\to 0} G(s) = \dfrac{2/3}{8/3} = \dfrac{2}{8} = \dfrac{1}{4} = 0.25 = K_p$

⇒ $e(\infty) = \dfrac{1}{1+K_p} = \dfrac{1}{1+0.25} =$

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ii. $\lim_{s\to 0} sG(s) = \lim_{s\to 0} \dfrac{s\,2/3}{s+8/3} = 0 = K_v$

here $e(\infty) = \dfrac{1}{K_v} = \dfrac{1}{0} = \infty = NaN$

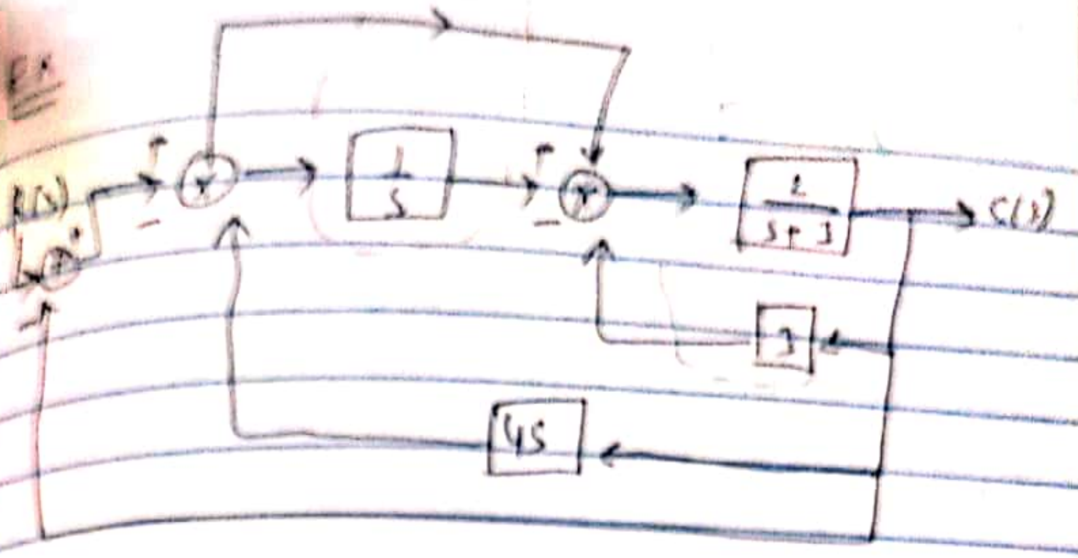
not a number

iii. $\lim_{s\to 0} s^2G(s) = \lim_{s\to 0} \dfrac{2/3\, s^2}{s+8/3} = 0 = K_a$

$e(\infty) = \dfrac{1}{K_a} = \dfrac{1}{0} = \infty$

⇒ This system is a type 0 system.

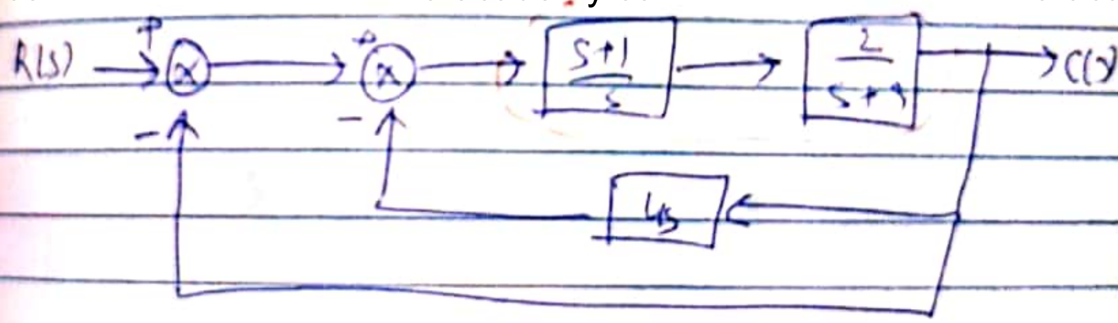
Ex



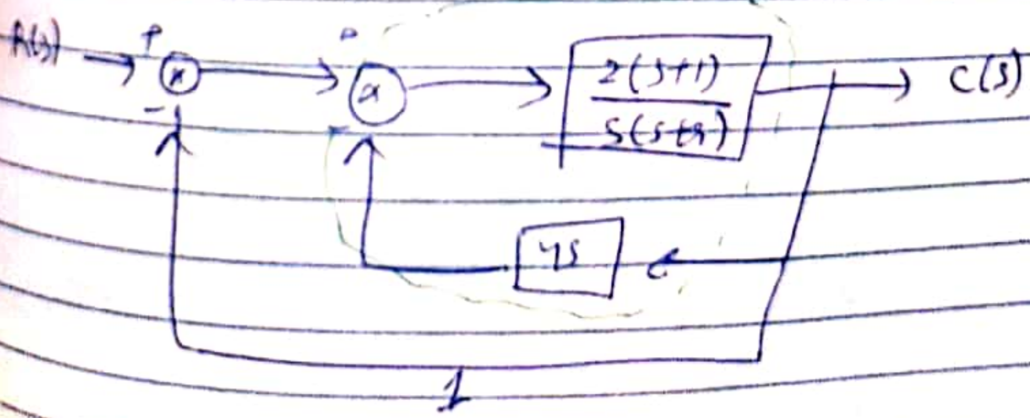

imp $\dfrac{G(s)}{1+G(s)H(s)} = \dfrac{2/(s+3)}{1+6/(s+3)} = \dfrac{2}{s+9}$

parallel $1 + \dfrac{1}{s} = \dfrac{s+1}{s}$






Series $\left(\dfrac{s+1}{s}\right)\left(\dfrac{2}{s+9}\right) = \dfrac{2}{s}\left(\dfrac{s+1}{s+9}\right)$



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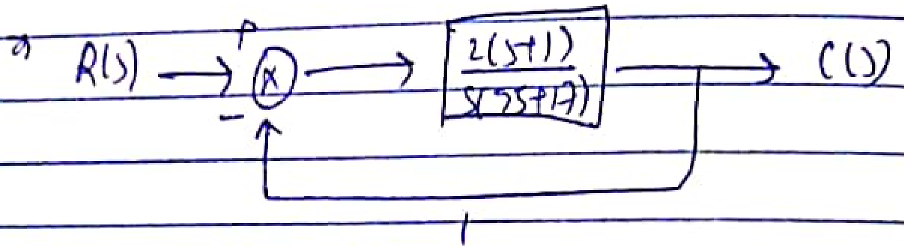
Inner loop $\dfrac{G(s)}{1+G(s)H(s)}$

$$\frac{\dfrac{2(s+1)}{s(s+9)}}{1+(4s)\left(\dfrac{2(s+1)}{s(s+9)}\right)} = \frac{\dfrac{2(s+1)}{\cancel{s(s+9)}}}{\dfrac{s+9+8s+8}{\cancel{(s+9)}}} = \frac{2(s+1)}{s(9s+17)}$$




This is the eq unity feedback configuration.

with $G(s) = \dfrac{2(s+1)}{s(9s+17)}$



i. $\lim_{s\to 0} G(s) = \lim_{s\to 0}\left[\dfrac{2(s+1)}{s(9s+17)}\right] = \infty = NaN$

$K_p = \infty$

$\Rightarrow e(\infty) = \dfrac{1}{\infty} = 0$

ii. $\lim_{s\to 0} sG(s) = \lim_{s\to 0} \dfrac{\cancel{s}\, 2(s+1)}{\cancel{s}(9s+17)} = \dfrac{2}{17} = K_v$

$\Rightarrow e(\infty) = \dfrac{1}{K_v} = \dfrac{17}{2}$ = finite error.

$\Rightarrow e(\infty) = 15 \times \dfrac{17}{2} = 127.5$

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ii. $\lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} \left( \frac{2s(s+1)}{9s+17} \right) = 0$

$\Rightarrow e(\infty) = \frac{1}{0} = \infty = NaN$

→ This is a type 1 system.

Once we identify the type of system, we can apply the i/p and find out steady state error.

$e(\infty) = 15 \times \frac{17}{2} = 127.5$

Single integration in forward path → Type 1 system
No pure integrator " " → Type 0
Two pure integrator " " → Type 2



FINAL

Lecture 8

Root Locus

Roots of an equation.
Here roots will refer to either poles or zeros.

Zeros are roots of numerator expression in the transfer function where poles are roots of the characteristic eq (denominator).

We are interested in the poles → closed loop poles
Locus → forming a continuous curve by the collection of different points (ie roots of the equation) by changing the gain of the system

Basically we are drawing a graph.

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Generally a characteristic eqn of the form:

$$1 + K\,G(s)H(s) = 0$$

Solving ⇒ closed loop poles.

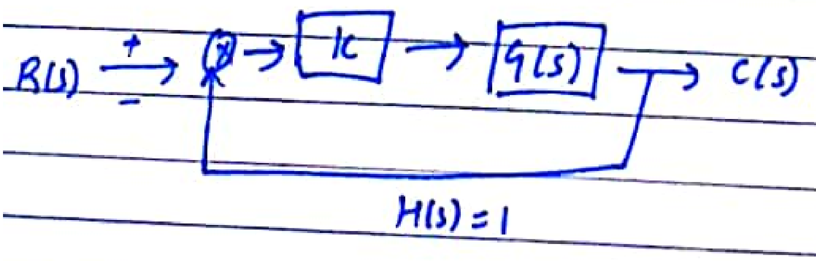
For stable system, all closed loop poles lie in left half of the s plane.

Any pole in the right half plane gives instability.

The Routh stability criteria gives you absolute stability of the system. (no. of sign changes = no. of roots in right half) while the root locus gives us relative stability.

Mostly we use a unity feedback system;






Consider a closed loop transfer function of second order;

$$T(s) = \frac{K}{s^2 + 3s + K}$$

solving ⇒ $s = \dfrac{-3 \pm \sqrt{9-4K}}{2}$

depends on value of K (0 to ∞).

We would generally be speaking of a finite positive value of K.

$4K < 9 \rightarrow$ real roots.

when $4K > 9 \rightarrow$ roots will be complex conjugate pair.

Finding values of s for different values of K is the root locus.

We cannot do it for all possible values of K, so we take a certain limit.

say $K < 10$ or $0 < K < 10$.

## Characteristics of root locus

(i) We need open loop T/F of the system.

$$\text{say } \frac{K}{s^3+s^2+4s+5+K} \rightarrow \text{CL TF}$$



To convert to OL, leave K and take others common:

$$\frac{K/s^3+s^2+4s+5}{1+ K/s^3+s^2+4s+5}$$

$K\,G(s)H(s)$ → the OL TF.

$$= \frac{G(s)}{1+G(s)} = \frac{K\,G(s)}{1+K\,G(s)} = \frac{K\,G(s)}{1+K\,G(s)\,\cancel{H(s)}}$$

(ii) RL starts at OL pole and ends at OL zero. ($K=0$ at OL pole, $K=\infty$ at OL zero).

(iii) Any point on RL is a CL pole and obey the angle magnitude criterion of the OL TF.

i.e. $K\,G(s)H(s) = 1\angle 180°$

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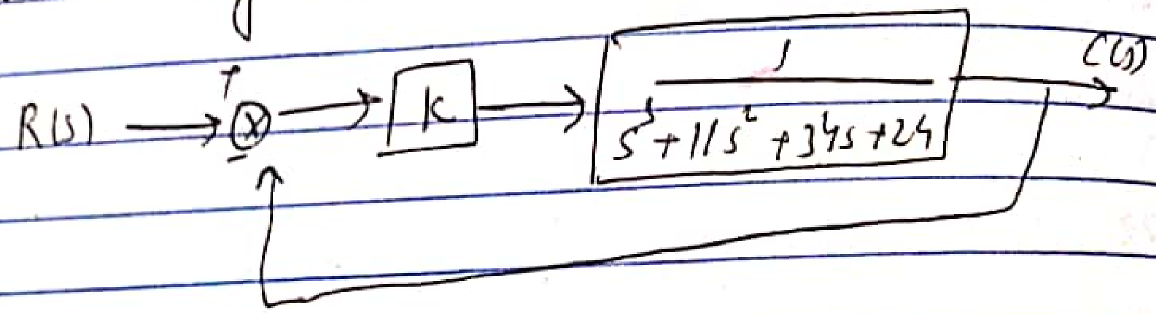
(iv) It possesses conjugate symmetry.

(v) RL will be finite if OL poles = OL zeros in number

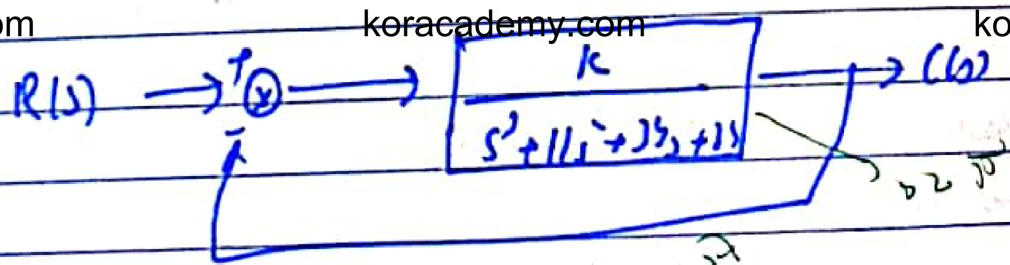
infinite RL if no. of OL zeros lie at infinity

Consider a system;




Combine (two blocks in cascade). $H(s) = 1$






factorize it

$$G(s)H(s) = \frac{K}{s^3+11s^2+34s+24} = \frac{K}{(s+1)(s+4)(s+6)}$$

**Step 1** Count number of poles and zeros (OL)

OL poles = 3 OL Z = 3 at $\infty$.

**Step 2** RL does not consist of a single curve.

No. of loci = No. of OL poles = 3

**Step 3** Plot OLP, −1, −4, −6

Draw the real axis loci.

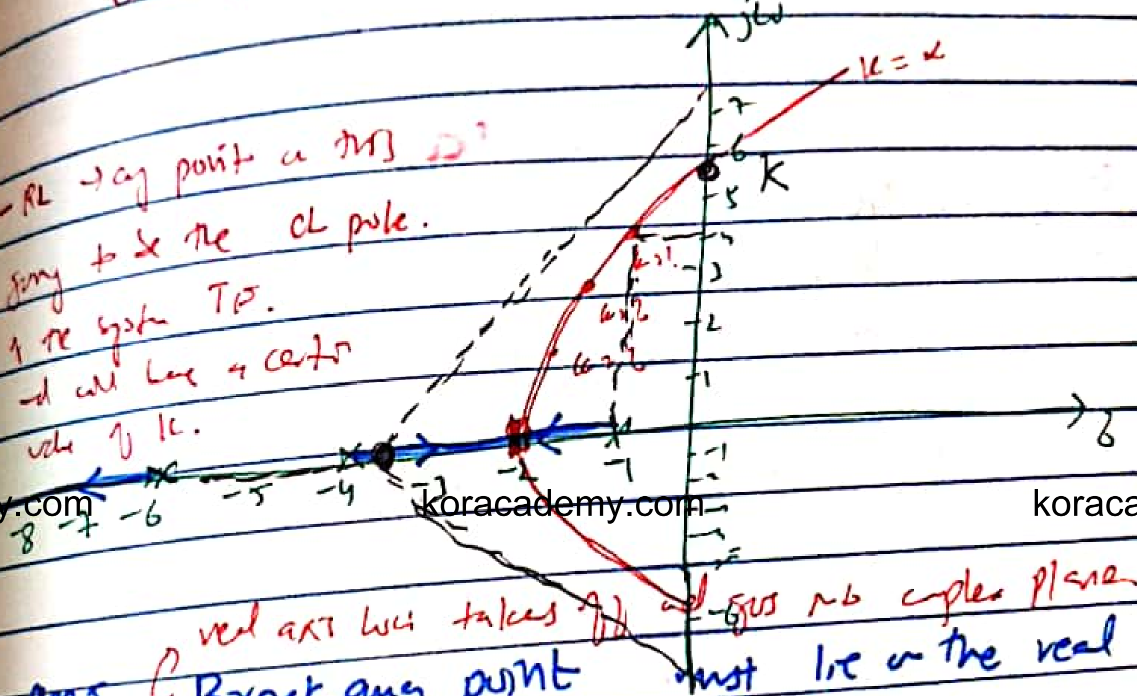
Step 4

(1) Real axis loci always lie b/w two adjacent OL poles.

(2) Real axis loci always lie b/w two adjacent OL zeros.

(3) Real axis loci lie b/w a OL zero and a OL pole.

-RL → any point on this is going to be the OL pole. of the system TF. and will have a certain value of K.






real axis loci takes off and goes into complex plane

Step 5 Breakaway point must lie on the real axis and b/w two adjacent OL poles.

There may be a breakin point. Always b/w two adjacent OL zeros.

As you move gain increases.
At breakaway point, gain is maximum.

$1 + K\,G(s)H(s) = 0$

$K\,G(s)H(s) = -1 \Rightarrow K = \dfrac{-1}{G(s)H(s)}$

$\dfrac{dK}{ds} = \dfrac{d}{ds}\left(\dfrac{-1}{\ldots}\right) = 0$

In our example, $K = -(s^3 + 11s^2 + 34s + 24)$

$$\frac{dK}{ds} = -(3s^2 + 22s + 34) = 0$$

$$\Rightarrow 3s^2 + 22s + 34 = 0 \quad \text{— (1)}$$

The roots are; $s = -2.21$ and $s = -5.12$

$\sigma_b = 2.21$ — no real axis loci

**Step 6** How the root loci will take off? (will be plotted in the complex plane).

We require angles and centre of asymptotes.

$$\theta_m = \frac{(2m+1)180^\circ}{P-Z} \; ; m = 0, 1, 2, 3 \ldots$$



asymptote → guide path for RL in complex plane.

$m=0 \Rightarrow \theta_0 = \frac{180^\circ}{3} = 60^\circ$

$m=1 \Rightarrow \theta_1 = \frac{3(180^\circ)}{3} = 180^\circ$

$m=2 \Rightarrow \theta_2 = \frac{5(180^\circ)}{3} = 300^\circ$

$m=3 \Rightarrow \theta_3 = \frac{7(180^\circ)}{3} = 420^\circ$ is the same as $\theta_1 = 60^\circ$ so we stop at $\theta_2$.

centre of asymptotes; $\sigma_c = \frac{\Sigma P - \Sigma Z}{P - Z}$

$$\sigma_c = \frac{(-1-4-6) - 0}{3} = \frac{-11}{3} = -3.66$$

Draw the angles here ↵

The root locus cannot touch the asymptotes

Step 7 jw axis crossing point

This can be extracted from the characteristic eq → for that we go back to CL TF.

ie $T(s) = \dfrac{K / s^3 + 11s^2 + 34s + 24}{1 + K / s^3 + 11s^2 + 34s + 24}$

characteristic polynomial:



$$Q(s) = s^3 + 11s^2 + 34s + 24 + K = 0$$

Put $s = jw$

$$-jw^3 - 11w^2 + j34w + 24 + K = 0 \quad — (2)$$

$$-jw^3 + j34w = 0$$

$\Rightarrow w^2 = 34 \Rightarrow w = \pm 5.83$ rad/sec

$$\boxed{jw = \pm j5.83}$$

Conjugate symmetry in the RL.

let say we want the value of K for which the system becomes marginally stable. ω oscillatory in nature
↳ when CL poles lie on the jw axis.

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Back to ②: consider real parts only.

$$② \Rightarrow -11w^2 + 24 + K = 0$$
$$-11(5.83)^2 + 24 + K = 0$$
$$\Rightarrow K = 349.8$$

Consider:

$s_1 = -1.3 + j3.5$ → dominant CL pole

This value of s when substituted in the OL TF and using the ~~angle~~ gain magnitude criteria;

$$\text{OL TF}: \frac{K}{s^3 + 11s^2 + 34s + 24} = 1$$



$$\Rightarrow |K| = s^3 + 11s^2 + 34s + 24$$



Now use that value of $s_1$

$$|K| = |(-1.3 + j3.5)^3 + 11(-1.3 + j3.5)^2 + 34(-1.3 + j3.5) + 24|$$

$$K = \sqrt{Re^2 + Im^2}$$

$s_1 = -1.3 + j3.5$

comparing with standard generalized form of CL pole;

$$= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

From this dominant CL pole you can find wrt K from the OL TF given.

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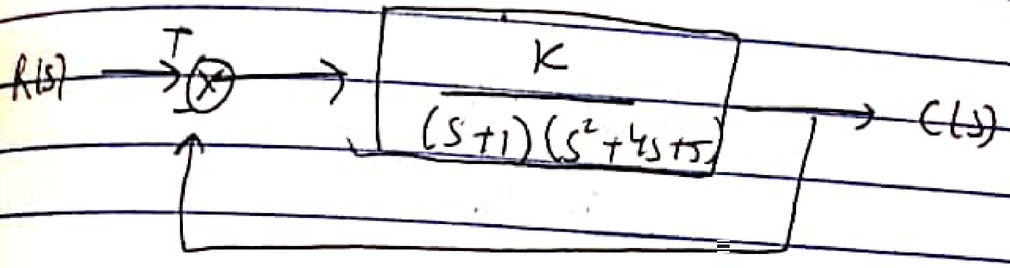
and then we can find $\zeta$, $\omega_n$ by comparing.

Then we can go back to the transient time specification;

$$\omega_d = \left(\sqrt{1-\zeta^2}\right)\omega_n$$

$T_r$, $T_p$, $T_s$, %OS

<u>Example</u>




OL TF $\quad kG(s)H(s) = \dfrac{K}{(s+1)(s^2+4s+5)}$ —(1)



CL TF $\quad T(s) = \dfrac{K}{s^3+5s^2+9s+5+K}$ —(2)

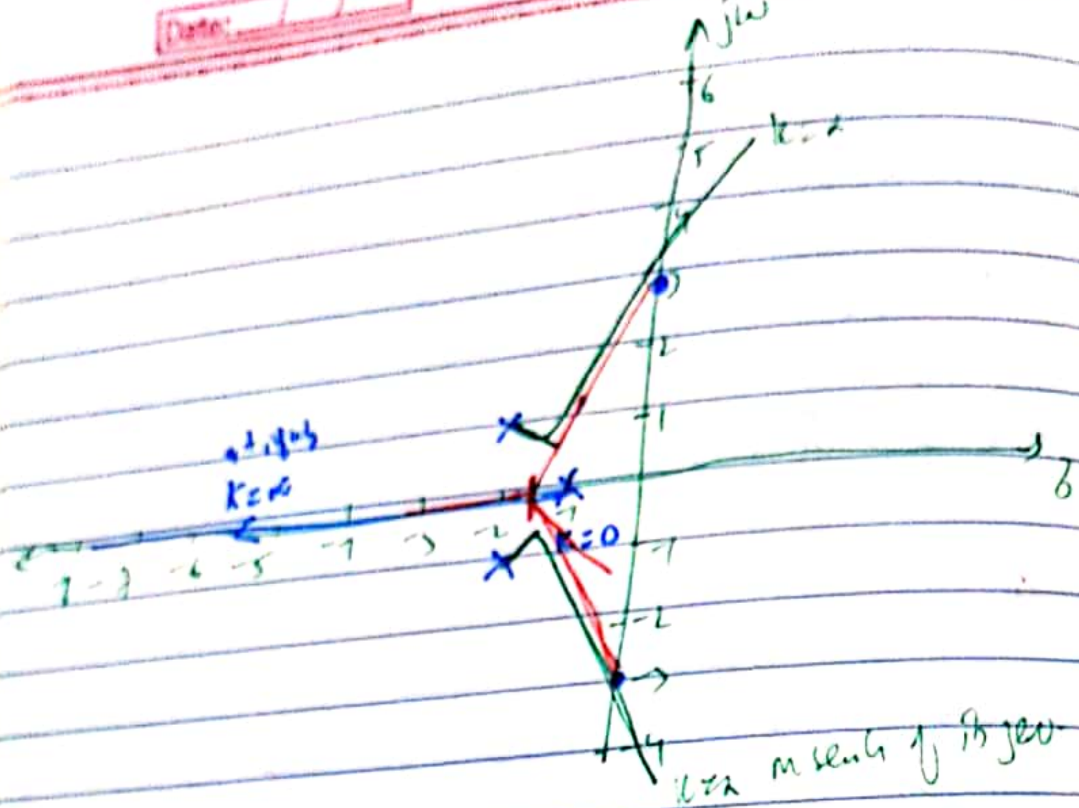
we need OL TF for RL, so we factorize;

$$kG(s)H(s) = \frac{K}{(s+1)(s+2-j1)(s+2+j1)}$$

TA

No. of OL poles = 3, OL zeros = 0

$p = -1, -2+j1, -2-j1$ $\quad$ ↳ 3 zeros at infinity.




Any point on the RL is a CL pole of the system TF.
RL does not contain the OL poles or zeros; it starts from there (it does not include them).



No. of loci = No. of OL poles = 3

Angle of departure; $\theta_D = 180 + \angle[KG(s)H(s)]'$

$[G(s)H(s)]' = ?$

In this expression (1), we have to exclude that particular pole which we are dealing with.

Consider $-2+j1$

In (1A) remove its factor;

ie $[KG(s)H(s)]' = \dfrac{K}{(s+1)(s+2+j1)}$

Put s equal to that pole ie $-2+j1$.

$$\frac{\angle 0^\circ}{(-2+j1+1)(-2+j1+2+j1)}$$

$$= \frac{\angle 0^\circ}{(-1+j1)(j2)}$$

Angle $\frac{\angle 0^\circ}{\angle 135^\circ \angle 90^\circ}$

$\Rightarrow \angle[KG(s)H(s)]' = -225^\circ$

$\Rightarrow \theta_D = 180^\circ - 225^\circ = -45^\circ$

Similarly $\theta_D$ for the other pole ie $s = -2-j1$ will be $+45^\circ$.



Centre of asymptote;

$$\sigma_c = \frac{\Sigma P - \Sigma Z}{P - Z} = \frac{-1-2+j1-2-j1}{3}$$

$$= -5/3 = -1.67$$

Angle: $\theta_m = \frac{(2m+1)180^\circ}{P-Z}$

$\theta_0 = 60^\circ$, $\theta_1 = 180^\circ$, $\theta_2 = 300^\circ$

$j\omega$ axis crossing point;

$$s^3 + 5s^2 + 9s + 5 + K = 0$$

Put $s = j\omega$

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$\Rightarrow -j\omega^3 - 5\omega^2 + j9\omega + 5 + k = 0$

$\Rightarrow -j\omega^3 + j9\omega = 0$

$\omega^2 = 9$

$\Rightarrow \omega = \pm 3$

$j\omega = 3, \quad j\omega = -3 \rightarrow$ two crossing points

## Lecture 9

## Design Using Root Locus

Example

Consider a OL TF.

$$kG(s)H(s) = \frac{k(s^2+2s+2)}{s^2+8s+12}$$



Factorizing:

$$kG(s)H(s) = \frac{k(s+1+j1)(s+1-j1)}{(s+2)(s+6)}$$

No. of $p = 2 = -2, -6$

No. of $z = 2 = -1+j1, -1-j1$

- Real axis loci $\rightarrow$ b/w $-2$ and $-6$

$\hookrightarrow$ breakaway point.

Using angle & magnitude criteria;

$$\frac{k(s^2+2s+2)}{s^2+8s+12} = -1$$

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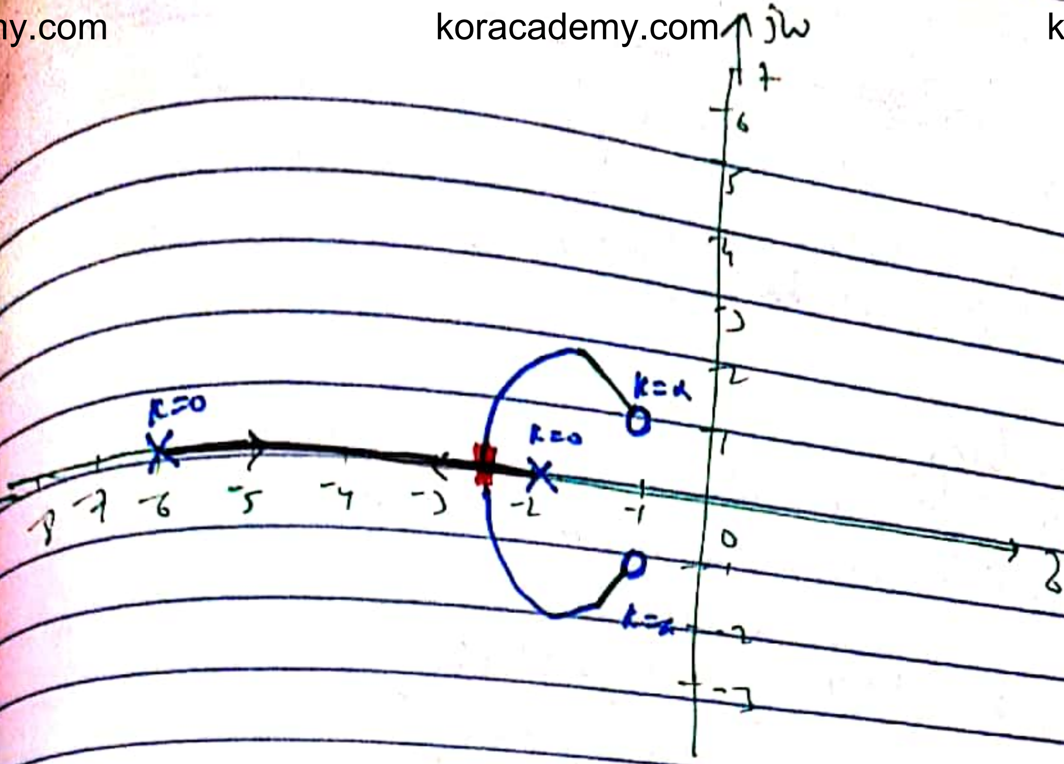


$\Rightarrow k = -\frac{(s^2+8s+12)}{s^2+2s+2}$

$\frac{dk}{ds} = \frac{d}{ds}\left[\frac{-(s^2+8s+12)}{s^2+2s+2}\right] = 0$



Quotient rule;

$\Rightarrow \quad s = -0.46 \; , \; s = -2.86$ ✓ the RAL.

$\sigma_b = -2.86$

At $\sigma_b$, the locus will depart from this particular point and will move into the complex plane.

As $P - Z = 0 \Rightarrow$ we can't find $\sigma_c$ and $\theta_a$ (physically RL is finite in this case).

Assymptotes are supposed to be defined for that RL which is infinite ie it has to look for zeros lying at infinity.

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But we need the angle of arrival at the complex zeros.

Here we have $(KGH)''$.

considering $z=-1+j1$.

The factor for this zero is; $s+1-j1$

exclude this factor in the TF.

$$\Rightarrow (KGH)'' = \frac{k(s+1+j1)}{(s+2)(s+6)}$$

Now angle; $\angle KGH\Big|_{s=-1+j1} = 33.7°$



Angle of arrival, $\theta_A = 180° - \angle(KGH)''$

$= 180° - 33.7° = 146.3°$

Similarly if we consider the other zero,
$s=-1-j1$

The factor for this is $(s+1+j1)$

Excluding

$$\Rightarrow (KGH)'' = \frac{k(s+1-j1)}{(s+2)(s+6)}$$

Now angle; $\angle KGH\Big|_{s=-1-j1} =$

$\Rightarrow \theta_A = -146.3°$

No jw crossing

complex poles ⇒ angle of departure
complex zeros ⇒ angle of arrival

**Example**

**Design of a simple gain controller for a desired performance.**

Say: the % OS should not exceed 10%.

$\Rightarrow O.S = 0.1$

$$KG(s)H(s) = \frac{K}{s^3 + 11s^2 + 34s + 24}, \quad K > 0$$

$$= \frac{K}{(s+1)(s+4)(s+6)}$$



No. of OL $p = 3$ No. of physical OL zeros = 0

$p = -1, -4, -6$

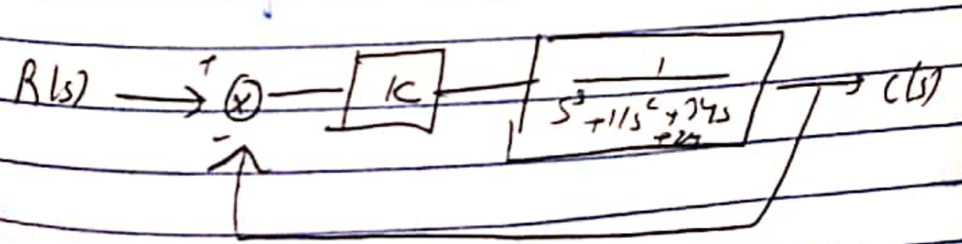
Real axis loci, b/w $-1$ and $-4$ and b/w $-6$ and $\infty$.

Breakaway point, $\delta_B = -2.21$

Centre of asymptotes; $\delta_c = -3.66$

Angle of asymptotes; $60°, 180°, 300°$.

$j\omega$ crossing = $\pm j\,5.83$



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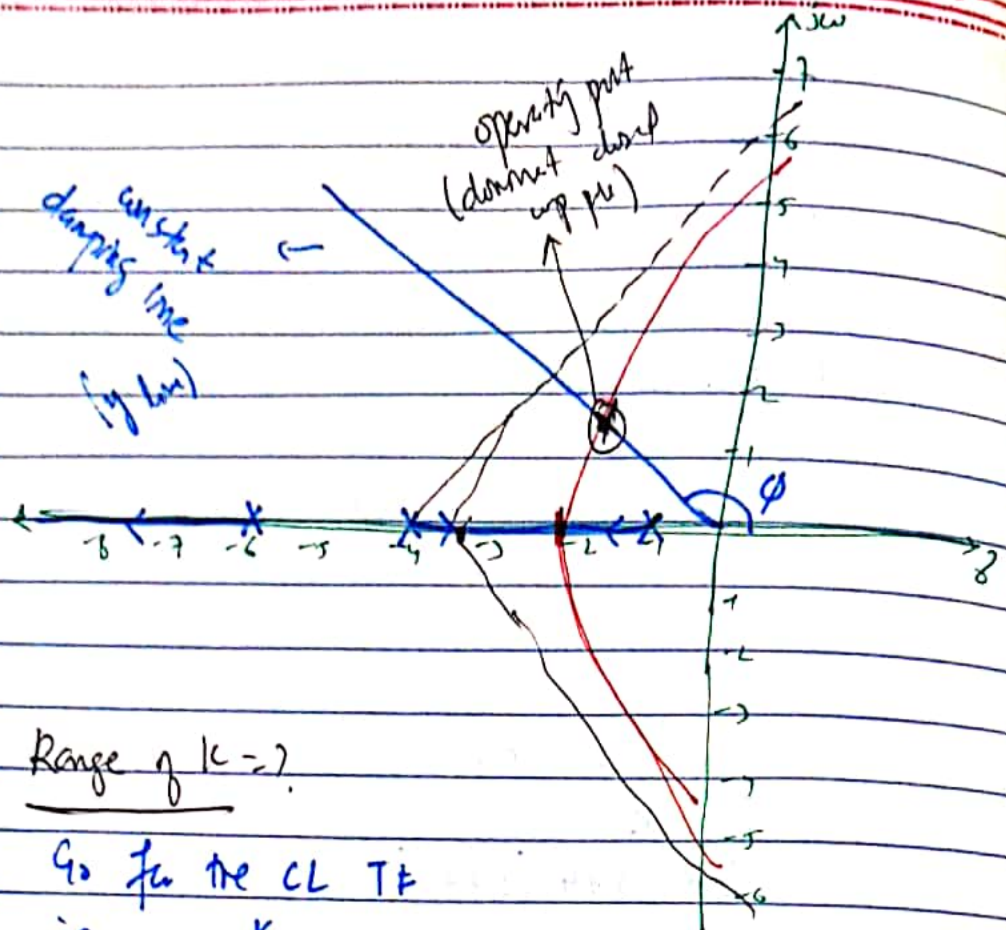


Range of K = ?

Go for the CL TF

i.e. $\dfrac{K}{s^3+11s^2+34s+24+K} = T(s)$



Xtristic eq = $\phi(s) = s^3+11s^2+34s+24+K$

The Routh table;

| | | |
|---|---|---|
| $s^3$ | 1 | 34 |
| $s^2$ | 11 | $24+K$ |
| $s^1$ | $\frac{350-K}{11}$ | 0 |
| $s^0$ | $24+K$ | 0 |

For stability $24+K>0 \Rightarrow K>-24$

Also $\dfrac{350-K}{11}>0 \Rightarrow K<350$

$\Rightarrow$ Range = $\boxed{0<K<350}$

what value of k to choose?

Root locus makes things easier.

$M_p$ %.OS = 10%.

%.OS only depends on value of $y$. $y$ is damping coefficient.

$$OS = - e^{-y\pi/\sqrt{1-y^2}}$$

$$\Rightarrow 0.1 = e^{-y\pi/\sqrt{1-y^2}}$$

Take natural log

$$\Rightarrow -2.3 = -\frac{y\pi}{\sqrt{1-y^2}}$$

$$\Rightarrow y = 0.6$$

we need to draw an angle at the origin ($\phi$)

$$\phi = 180^\circ - \cos^{-1} y$$



$$\Rightarrow \phi = 127^\circ.$$

y line Any point on this line will have value = 0.6 and will refer to 10% overshoot.

Dominant CL pole, $\otimes = -1.8 + j1.8$

Finding value of k.

Refer to OL TF (in factor form)

$$= \frac{K}{(s+1)(s+3)(s+6)}$$

Angle magnitude criteria; $K\,G(s)\,H(s) = -1 = 1\angle 180^\circ$

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Apply magnitude criterion;

$$K\,G(s)\,H(s) = 1$$

Evaluate at $s = -1 + j1.8$

$$\Rightarrow \frac{|K|}{(-1.8 + j1.8 + 1)(-1.8 + j1.8 + 4)(-1.8 + j1.8 + 6)} = 1$$

say $\frac{|K|}{M_1 M_2 M_3} = 1$

$$\Rightarrow K = M_1 M_2 M_3$$



## Lecture 10

### Compensation and controllers

transient response is measured in terms of %OS and settling time ($T_s$).

Steady state response is measured in terms of the steady state error. which depends on the system type. (type 0, type 1, type 2)

constant position, constant velocity, constant acceleration

$K_p$ Type 0 : gives finite s.s error with unit step i/p.

$K_v$ Type 1 : finite error with ramp function.

$K_a$ Type 2 : " " " " with parabolic function.

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For type 0: $e(\infty) = \frac{1}{1+K_p}$

For type 1; $e(\infty) = \frac{1}{K_v}$

For type 2; $e(\infty) = \frac{1}{K_a}$

---

Say $T_s = 2s$ but we want $T_s = 1s$.

CL pole.

$T_s = \frac{4}{\zeta \omega_n}$

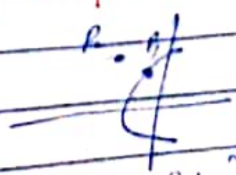
$s = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$



If $\zeta$ = constant, we only will vary $\omega_n$.

A system has to be operated on any point which must lie on the Root Locus but maybe now our desired point does not lie on the RL. Therefore this is not an optimum point to operate the control system on.

If you want to reduce the $T_s$, shift the RL towards the left.



At this point we can't ~~dear~~ operate your already designed system (s/c it is on RL.)
So we need to compensate the system with the help of certain subsystem called compensators and then they act as controller.

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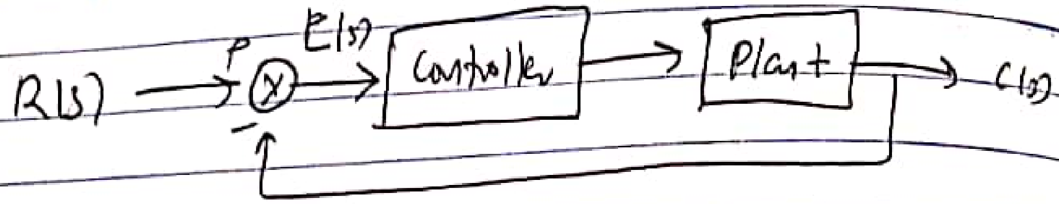
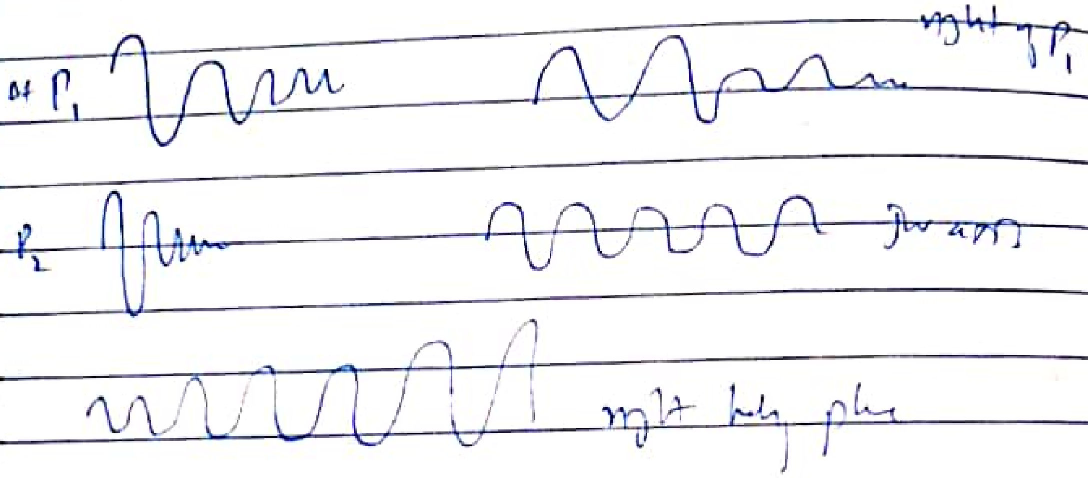
for the plant.




Example ①

Consider OL TF of the plant (complete system including the controller).

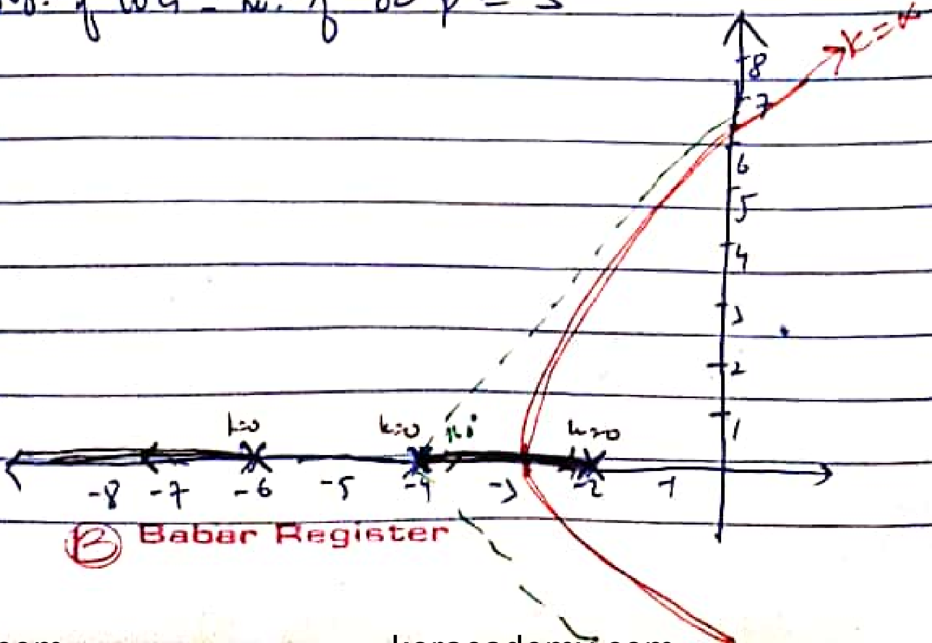
$$KG(s)H(s) = \frac{K}{(s+2)(s+4)(s+6)}$$

(K → controller, rest → plant)

$P=3, \quad Z=0$

$-2, -4, -6 \qquad 3$ at infinity. $\qquad P-Z=3$

No. of loci = no. of OL P = 3




Babar Register

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Real axis lies : b/w −2 and −4 and b/w −6 and −∞.

Angle magnitude criterion; $KG(s)H(s) = -1$

$$\frac{K}{(s+2)(s+4)(s+6)} = -1$$

$$\Rightarrow K = -(s+2)(s+4)(s+6)$$

$$\frac{dK}{ds} = -\frac{d}{ds}\left[(s+2)(s+4)(s+6)\right] = 0$$

$$s = -2.8 = \sigma_B$$

Centre of asymptotes, $\sigma_c = \dfrac{\Sigma P - \Sigma Z}{P - Z} = \dfrac{-2-4-6-0}{3}$



$$\Rightarrow \sigma_c = -4$$

Angle of asymptodes; $\theta_c = \dfrac{180(2m+1)}{P-Z}$

Putting m ⇒ $\theta_c = 60°, 180°, 300°$.

For jω crossing point, first obtain CL TF for the given OL TF.

$$\boxed{CL\ T/F = \frac{KG(s)}{1+KG(s)H(s)}}$$

$$= \frac{K}{(s+2)(s+4)(s+6)+K} = \frac{K}{s^3+12s^2+44s+48+K}$$

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$Q(s) = s^3 + 12s^2 + 44s + 48 + K$

Put $s = j\omega$ (FT)

$Q(j\omega) = -j\omega^3 - 12\omega^2 + j44\omega + 48 + K = 0$

$Im = 0 \Rightarrow -j\omega^3 + j44\omega = 0 \Rightarrow \omega^2 = 44$

$\Rightarrow \omega = \pm 6.63$

$\Rightarrow j\omega = \pm j\, 6.63$

<u>Conclusion</u> The RL goes into unstable region crossing the $j\omega$ axis. → critical gain.

The gain of system is maxim at $j\omega$ and for the stability. After that gain moves let the system becomes unstable



we should operate our system such that the gain should not be approached to the critical gain point. (should be left). There should be adequate margin.

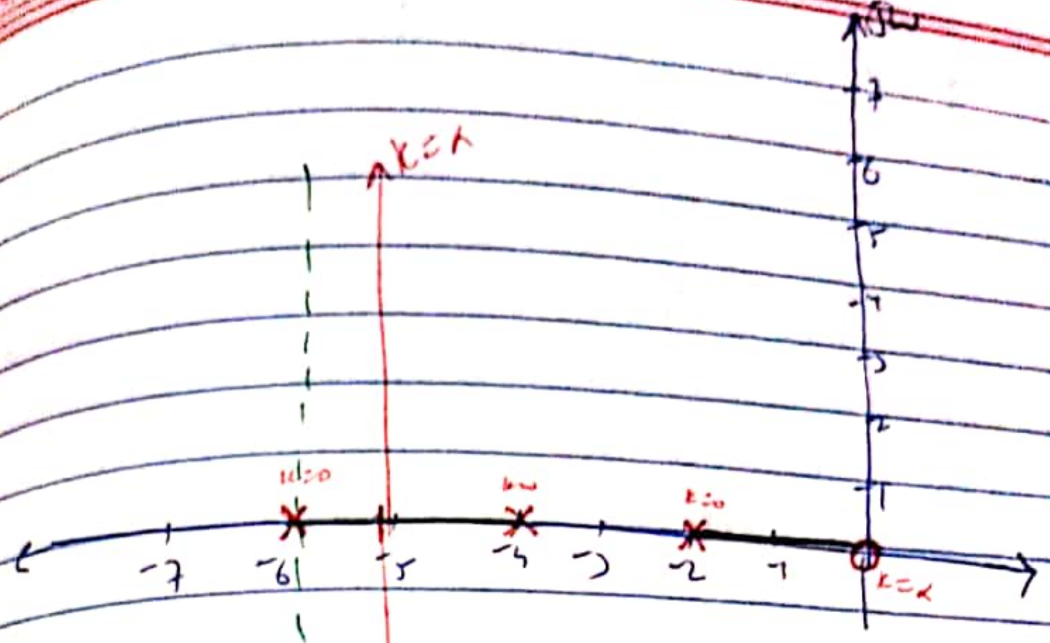
Lets do some changes in T/F and see what happens

<u>Example</u> ②

$$OL\ TF = \frac{sK}{(s+2)(s+4)(s+6)}$$

$P = -2, -4, -6$ ③ $\quad Z = 0$, one at origin. ①

$P - Z = 2$




Breakaway point, $\sigma_b = -5.06$

$$\sigma_c = \frac{\sum P - \sum Z}{P - Z} = \frac{-4-2-6-0}{2} = -6$$



$$\theta_c = \frac{180(2m+1)}{P-Z} = 90^\circ, 270^\circ$$

(No. of $\theta_c = P - Z$)

Now the system is not going to an unstable region. (always be stable)

We found a system that is very close to minimum phase system and always stable system.

Ex (1) → RL was toward, the right

Ex (2) → RL shifted toward the left. ($\sigma_b$).

Now the settling time will be small as compared to (1). (speedy transient response).

Also far from jω axis → adequate gain margin incorporated.

This everything happened b/c of multiplying a s.

What is s?

$s = \frac{d}{dt}$ we are differentiating the system.

In other words the controller is a pure gain differentiator (sK).

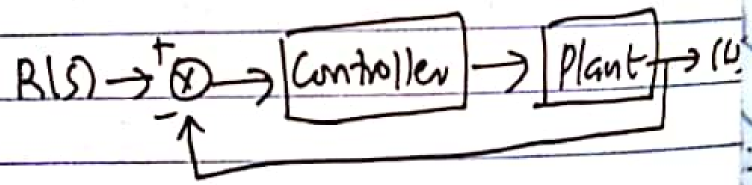
↳ Providing a derivative control to the plant. i.e. $(s+2)(s+5)(s+6)$.

The plant is governed by a control signal which is extracted from the controller.

By adding a differentiator factor in the original TF (in with the plant) we can improve the transient behaviour of the system in a sense that RL is shifted towards the left making the transient response quicker or speedy transient response with less settling time.



## Lecture 11




Ex 3

Consider:

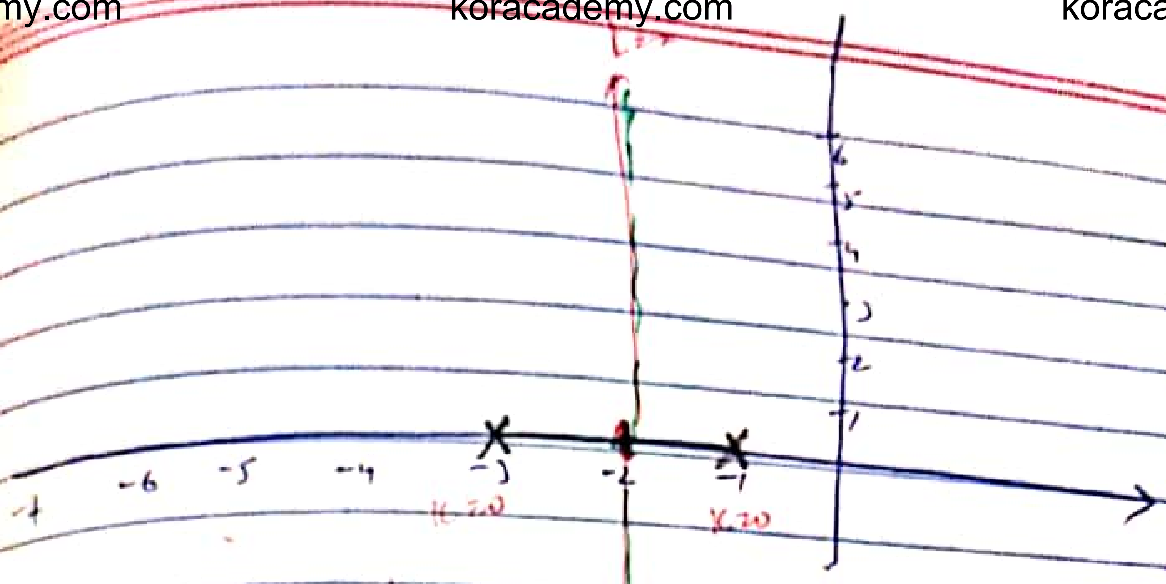
$$OL\ TF = \frac{K}{(s+1)(s+3)} = KG(s)H(s)$$ second order system

↳ always stable

No. of p = 2   No. of zero = 0

$p = -1, -3$ , $z = \infty$ at infinity.

$p - z = 2$   No. of LoCs = no. of OL p = 2




Real axis loci is b/w -1 and -3.

For breakaway point, we use angle magnitude criteria.

$$KG(s)H(s) = -1$$

$$\frac{K}{s^2+4s+3} = -1 \Rightarrow K = -(s^2+4s+3)$$



$$\frac{dK}{ds} = -\frac{d}{ds}(s^2+4s+3) = 0$$

$$2s+4=0 \Rightarrow s=-2 \Rightarrow \sigma_b = -2$$

For centre and angle of asymptotes;

$$\sigma_c = \frac{\Sigma P - \Sigma Z}{P-Z} = \frac{-1-3-0}{2} = -2$$

$$\theta_c = \frac{(180^\circ)(2m+1)}{P-Z} = 90^\circ, 270^\circ$$

Second order equation is always stable.

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lets do a little modification; Adding a pole at the right.

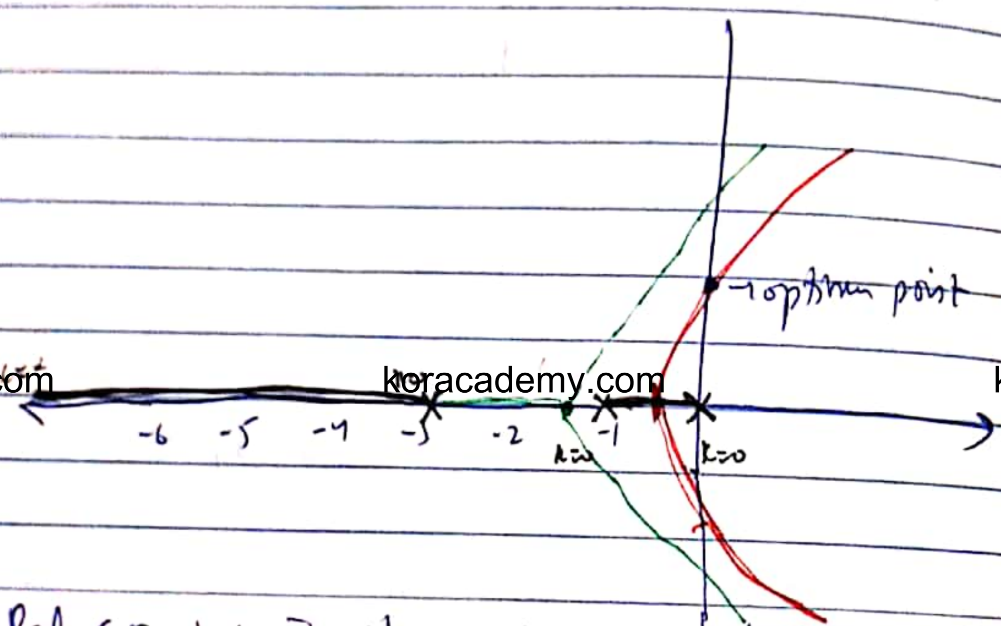
Ex (4)

$$\text{OL TF} = \frac{K}{s(s+1)(s+3)} = KG(s)H(s)$$

$p = -1, -3, 0$ $\quad z = +3$ at infinity.

$p - z = 3 - 0 = 3$

No. of loci = No. of OL poles = 3






Red axis lies b/w 0 and -1 and b/w $-\infty$ and -3.

$Z_b$ AM criteria $KG(s)H(s) = -1$

$$K = -s(s^2+4s+3) = -(s^3+4s^2+3s)$$

$$\frac{dK}{ds} = -(3s^2+8s+3) = 0$$

$3s^2 + 8s + 3 = 0 \Rightarrow s = -0.45, -2.21$

$\Rightarrow Z_b = -0.45$

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$$\sigma_c = \frac{\Sigma P - \Sigma Z}{P - Z} = \frac{-1-3+0+0}{3} = \frac{-4}{3} = -1.33$$

$$\theta_c = \frac{180^\circ (2m+1)}{P-Z} = 60^\circ, 180^\circ, 300^\circ$$

For $j\omega$ crossing point, we need CL TF of the given OL TF.

$$CL\ TF = \frac{KG(s)}{1+KG(s)H(s)}$$

$$= \frac{K}{s(s^2+4s+3)+K} = \frac{K}{s^3+4s^2+3s+K}$$

$$Q(s) = s^3+4s^2+3s+K$$



$s=j\omega \Rightarrow -j\omega^3 - 4\omega^2 + j3\omega + K = 0$

Xteristic eq in terms of FT.

Imag $\Rightarrow -j\omega^3 + j3\omega = 0 \qquad \omega^2 = 3$

$\omega = \sqrt{3} \Rightarrow \omega = \pm 1.732$

$j\omega = \pm j1.732$

Comparing $\Rightarrow$

Ex(3) was a stable system. But by incorporating a simple or pole at the origin the scenario has changed.

Ex(4) is going to unstable region and also it is shifted towards the right as to (3). (from $\sigma_c$).

$\Rightarrow$ The transient response is deteriorated (disturbed).

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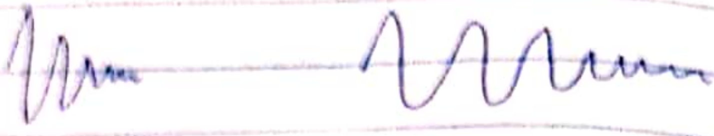
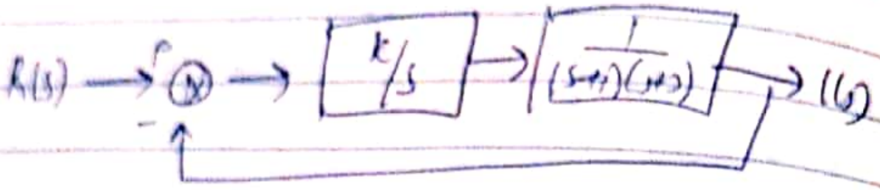
The system becomes slow in getting rid of the transients (disturbances).



① 




We incorporated an integral controller with the plant.

Something good about integral controller?

When we shift RL towards right, the



we increasing [illegible] steady state response will improve.

Derivative control → improves transient response i.e shifting RL towards the left. → more speedy in rejecting the disturbances.

Integral control → deteriorating transient behaviour of the system, in terms of slower acting system in getting rid of the disturbances. But improving the steady state response.

However pure gain derivative and integral controllers are not practically available.

Practically we have proportional controllers.

Proportional derivative controller (PD).

with TF $sK_D + K$

Proportional integral controller (PI)

with TF $\frac{K_I}{s} + K$

The combination of both will improve both transient and steady state responses called PID controller. with TF $sK_D + \frac{K_I}{s} + K$

The original TF $K/(s+1)(s+3)$ → type 0

no integration in forward path.

Add pole; (integration in the forward path) $K/s(s+1)(s+3)$ → type 1



3rd order not always stable

The number of integrations you use increases the type order.

Adding 2 integrators ; $K/s^2(s+1)(s+2)$ → type 2.

The no. of integrations in the forward path governs the system type.

3rd order system are theoretically always stable.

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# Lecture 12

## Lead Compensator

~~acts as a proportional derivative controller.~~

has a TF : $G_c(s) = \dfrac{K_c(s+z_c)}{s+p_c}$

$K_c \rightarrow$ compensator gain.
$z_c \rightarrow$ zero of the compensator
$p_c \rightarrow$ pole of the compensator.

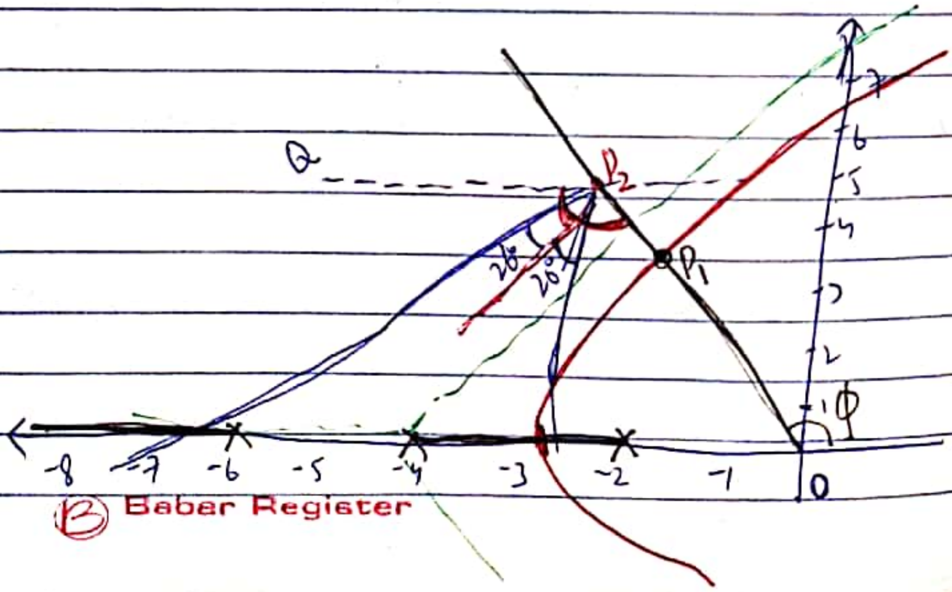
A simple lead compensator is a simple first order system.

Consider the previously discussed example;



$$R(s) \rightarrow \bigotimes \rightarrow \boxed{\frac{K}{(s+2)(s+4)(s+6)}} \rightarrow C(s)$$

$P = -2, -4, -6$        $z = 3 \rightarrow$ to infinity    $P - Z = 3$
No. of loci $= 3$
Real axis loci is b/w $-2$ and $-4$ and b/w $-6$ and $-\infty$

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$\sigma_a = \frac{-2-8}{\ldots}$ , $\sigma_a = -4$ , $\theta_a = 60^\circ, 180^\circ, 300^\circ$.

$j\omega$ crossing point $= \pm 6.68$

Our requirement from the system?

say we want to reduce the settling time of transients keeping the % OS the same.

say $O.S = 10\%$

$$O.S = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \qquad 0.1 = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$\ln(0.1) = \frac{-\pi\zeta}{\sqrt{1-\zeta^2}} \Rightarrow \zeta = 0.6$$



where is operating point lying at the moment?

$\Rightarrow$ draw the damping line. $\phi = 180^\circ - \cos^{-1}\zeta$

$$\phi = 180^\circ - \cos^{-1}(0.6) \Rightarrow \phi = 127^\circ$$

$P_1$ is the current operating point

↳ dominant closed loop pole.

$$s = -2.1 + j2.7 = P_1$$

Comparing with general closed loop pole of the form:

$$-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$$

$$\Rightarrow \zeta\omega_n = 2.1$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{2.1} = 1.9 \text{ sec}$$

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At the moment system is optimized with 10% OS and $T_s = 1.9s$.

we want to reduce the settling time.
Any point on the damping line will have the same 10% OS ie $\zeta = 0.6$.

If a point P is moved wrt to $P_1$ towards the left, $T_s$ reduces.

Consider the point $P_2$;

we expect the same 10% OS and $T_s < 1.9s$

$P_2 = -3 + j3.8$
Comparing with $-\zeta\omega_n' + j\omega_n'\sqrt{1-\zeta^2}$

($\zeta$ will be constant b/c OS is the same)

$$T_s' = \frac{4}{\zeta\omega_n'} = \frac{4}{3} = 1.33 \text{ sec}$$
↓
we are happy with this

lets proceed to the design of lead compensator.
$P_2$ is to be selected as an optimum point in this case b/c it is an desired point.

→ Draw a horizontal line → this line will subtend
→ angle as shown → it will also be equal to 127°.
→ Bisect this angle

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Now some calculations based on selecting a proper poles and zeros.

For that purpose we have to look for the amount of deviation of angle when we place a CL pole at $P_2$ s/c it is out of the RL.

Any point on the RL must satisfy the angle magnitude criteria.

$$KG(s)H(s) = -1 = 1\angle 180^\circ$$

Now in $P_2$, the angle is not $180^\circ$ s/c it is not at RL.

How much is the deviation from $180^\circ$ at $P_2$?

Consider the OL TF; $\dfrac{k}{(s+2)(s+4)(s+6)}\Big|_{s=-3+j3.8}$

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$$= \frac{\angle 0}{\angle\theta_1 \angle\theta_2 \angle\theta_3} \qquad \theta = -231.7^\circ$$

$$\text{Deviation} = \psi = 231.7 - 180^\circ = 51.7^\circ$$

$$\text{Divide by } 2 \Rightarrow \frac{\psi}{2} = 26^\circ$$

$$\angle OP_2Q = 127^\circ \qquad \text{bisect} = 63.5^\circ$$

The first line will cut real axis at $-2.8$

$$z_c = -2.8$$

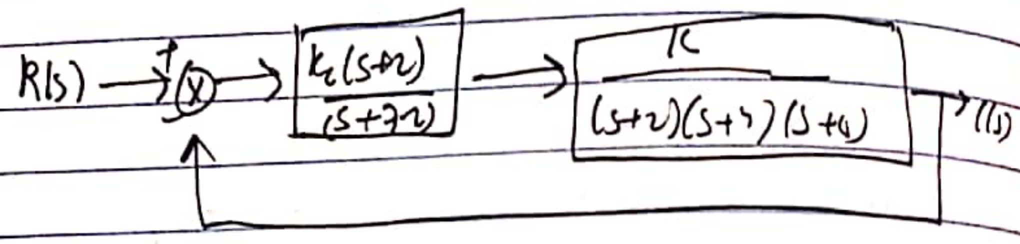
The second at $-7.2$ $\quad P_c = -7.2$

The TF of compensator will take the form;

$$G_c(s) = \frac{K_c(s+2.8)}{(s+7.8)}$$

we don't have to use -ve sign

In order to find $K_c$, lets form the total forward TF of the compensated system;




$$G'(s) = \frac{K_c K (s+2.8)}{(s+2)(s+4)(s+6)(s+7.2)}$$



Say $K_c K = K_1 = ?$

Applying the magnitude criterion only;

$$\left| G'(s) = \frac{K_1(s+2.8)}{(s+2)(s+4)(s+6)(s+7.2)} \right|_{s=-3+j3.8} = 1$$

$$\Rightarrow K_1 = 115.34$$

K is calculated from original (uncompensated) syst.

$$\left| \frac{K}{(s+2)(s+4)(s+6)} \right|_{s=-2.1+j2.7} = 1 \Rightarrow K = 42.23$$

↳ of previous (original) op.

$$\Rightarrow \boxed{K_c = 2.73}$$

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∴ Our compensator has a TF;

$$\boxed{\frac{2.73\,(s+2.8)}{(s+7.2)}}$$

Now the RL will pass through the operating point $P_2$.

Conclusion

The overall TF of the compensated system is as;

$$G'(s) = \frac{115.34\,(s+2.8)}{(s+2)(s+4)(s+6)(s+7.2)}$$

Now it is a 4th order TF. S/c no of $p=4$.

(transformed 3rd → 4th).

↳ This may not be desirable sometimes.

What if we need to keep order the same?

↳ cancellation of one of the OL poles (the one near to the origin). s+2 in this case.

If you can this pole (s+2), it will be much better in shifting the RL far towards the left.

How to do that?

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Select a zero of compensator at −2.

$G_c$ will take a form; $\frac{K_c(s+2)}{(s+P_c)}$

It will cancel the OL pole when incorporated in series with the original TF of the plant.

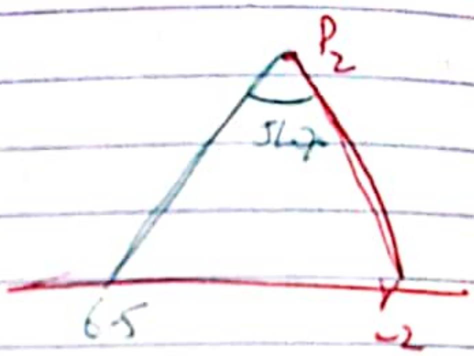
Now where should be $P_c$?

−2 to $P_2$; $Z_c = -2$

draw $\psi = 51.7°$

and line changed to real axis

gives $P_c = -6.5$






So $G_c = \frac{k_c(s+2)}{(s+6.5)}$

Incorporate $G_c$ in series with the plant;

$$G'(s) = \frac{K_c K \cancel{(s+2)}}{\cancel{(s+2)}(s+4)(s+6)(s+6.5)}$$

$$G'(s) = \frac{k_c k}{(s+4)(s+6)(s+6.5)}$$

the 3 poles are -4 and -6. (s+6)? 1.87?

so the order of the system remain the same.

Similarly repeat for $k_c k = k_1$, (K) and then find $k_c$.

will not change 11.23

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# Lecture 13

## Lag Compensator

$$KG(s)H(s) = \frac{K}{(s+2)(s+4)(s+6)}$$

$K = 47$, 10 % OS $\zeta = 0.6$

$s = -2 + j2.8$

Type 0 system.

Static error constant, $K_p = \lim_{s \to 0} G(s) = \lim_{s \to 0} \left[ \frac{47}{(s+2)(s+4)(s+6)} \right]$

$= \left[ \frac{47}{(2)(4)(6)} \right] \Rightarrow K_p = 0.98$



Steady state error $= \frac{1}{1+K_p} \Rightarrow e(\infty) = 0.5$

$= 50\%$

Say we need to reduce this error to 10%.
On this basis we calculate new value of $K_p$.

Steps in sequence to design a lag compensator.

1. Draw the RL given the OL TF of the system.
2. Based on your desired transient behaviour i.e 10% OS, evaluate value of $\zeta$.
3. Get the angle $\phi$ to draw damping line.
   $\phi = 127^\circ$.

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Get coordinates of point P (where damping line cuts the RL.

$P = s = -2 + j2.8$ for 10% OS.

(4) Find out $\omega_n$ by comparing P with general form of CL pole.

$= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

(5) Find value of static error constant.

(6) Calculate the error.

(7) To find the new value of Kp for the desired steady state response.



$10\% \Rightarrow 0.1$

$$e'(\infty) = \frac{1}{1+K_p'} = 0.1$$

$\Rightarrow K_p' = 9.9$ for 10% error

(8) What will TF of a lag compensator look like.

$$G_c(s) = \frac{K_c(s+z_c)}{(s+P_c)}$$

$K_c \rightarrow$ compensator gain

Difference in lead and lag compensator TFs?

(1) $K_c \sim 1$ eg 0.98, 0.99, 1.11 etc.

(2) $P_c < Z_c$

(B) Babar Register

(9) How much should be $z_c$ and $P_c$?

This is arbitrary.

You can choose any value but it should be very close to the origin.

$$\boxed{\begin{aligned} &\text{Say} \quad z_c = 0.01 \quad P_c = \frac{z_c}{\beta} = 0.001 \\ &\text{where} \quad \beta = \frac{K_p'}{K_p} = \frac{9.9}{0.98} \approx 10 \end{aligned}}$$

$$\Rightarrow G_c(s) = \frac{k_c(s + z_c)}{(s + z_c/\beta)}$$

$$\Rightarrow \boxed{G_c(s) = \frac{k_c(s+0.01)}{(s+0.001)}}$$



$k_c$ in most cases is unity.

(10) To find deviation of angles from 180°.

Any point on the RL contributes 180° angle.

That deviation should be very small.

Angle Criteria;

$k_c$ = constant $\Rightarrow$ angle is zero.

We need to know the deviation based on our operating point i.e. $-2 + j2.8$

$$\frac{\angle -2 + j2.8 + 0.01}{\angle -2 + j2.8 + 0.001} = -0.012^\circ$$

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The angle deviation must be within $\pm 5^\circ$.

Find gain of compensator for the overall TF:

$$\frac{K_c K (s+0.01)}{(s+0.001)(s+2)(s+4)(s+6)}$$

Apply the controller in series with the plant

Say $K_c K = K_1$

Using magnitude criterion;

$$\left| \frac{K_1 (s+0.01)}{(s+0.001)(s+2)(s+4)(s+6)} \right|_{s=-2+j2.8} = 1$$

$$\Rightarrow K = 47.16$$



Now $K_c = \frac{K_1}{K} = \frac{47.16}{47} \rightarrow K_c = 1.003 \sim 1$

The compensator we have designed is looking like this;

$$\boxed{\frac{1.003 (s+0.01)}{(s+0.001)}}$$

Now lets see the performance with the compensator,

The overall TF is;

$$G_1^p(s) = \frac{47.16 (s+0.01)}{(s+0.001)(s+2)(s+4)(s+6)}$$

$$K_p' = \lim G_1(s) = 9.82$$

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$e'(s) = \dfrac{\quad}{1+9.82} = 0.092$

$= \boxed{9.2\ \%}$

objective achieved ←

Lead compensator → PD controller.
Lag compensator → P.I controller

You can use a combination as well.
First design a lead compensator, fix your transient response specifications and then follow it by a lag compensator having $K_c = 1$ which is now correcting your steady state error.

The combined effect of PID controller (lead lag compensator).



## Lecture 14

### Frequency Response

r/o of the system to some i/ps that are frequency dependent. eg sinusoids.

$$v(t) = Y_m \sin(\omega t \pm \phi)$$

→ What will happen when this sinusoide passes through a system?
The magnitude and phase angle will change but the frequency will remain the same.

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In time domain we were interested in three basic standard i/ps;

step, ramp, parabola.

test i/ps → mathematical expression is known.

Two types of responses;

Transient response (natural)

Steady state response (forced).

↳ must have the same nature as i/p.

→ $T_r$, $T_p$, $T_s$, % OS

$T_s \downarrow \zeta\omega_n$, % OS $\downarrow \zeta$

Underdamped $0 < \zeta < 1$



when $\zeta \to 0$, the system will oscillate more violently which mean O.S will be much larger.

$\zeta\omega_n$ → real portion of complex conjugate pole of a second order system.

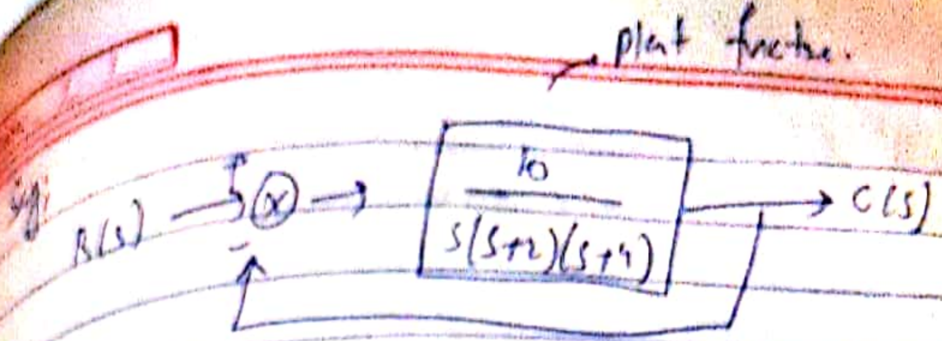
Transient response tells us about nature of the system (how a system behaves initially when you close the switch).

After that system will come to a steady state, where we are interested in finding error. That error depends on static error constant.

($K_p$, $K_v$, $K_a$). (type 0, type 1, type 2)

↳ depends on no. of pure integrators in the forward path.

eg:




single integration (s) (one OL pole at origin)
↳ Type 1.

If no s → type 0    If $s^2$ → type 2

S.S response deals with what type of a system are you dealing with.
That depends on static error constant.
Reciprocal of ↙ = steady state error.



To correct S.S error → compensators and controllers.

lead compensator → transient response
lag compensator → S.S error

PDC    PIC

→ In time domain we were more interested in laplace transform. why b/c we end up with a governing equation that was a differential equation.

Solution is more convenient using LT.

→ In frequency response, we use Fourier transform
$s \rightarrow j\omega$

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If $G(s) = \dfrac{10}{s(s+2)(s+4)}$

FT $\Rightarrow G(j\omega) = \dfrac{10}{j\omega(j\omega+2)(j\omega+4)}$

(Generally ignore initial conditions).

Now we need the magnitude and phase.

If you multiply inside first:

$G(s) = \dfrac{10}{s^3 + 6s^2 + 8s} \Rightarrow G(j\omega) = \dfrac{10}{-j\omega^3 - 6\omega^2 + j8\omega}$



For magnitude, group the real and imaginary parts.

$|G(j\omega)| = \left| \dfrac{10}{-6\omega^2 + j(8\omega - \omega^3)} \right|$

$\Rightarrow |G(j\omega)| = \dfrac{10}{\sqrt{36\omega^4 + (8\omega - \omega^3)^2}}$

Phase $= \phi = \angle G(j\omega) = -\operatorname{Tan}^{-1}\left(\dfrac{8\omega - \omega^2}{6\omega^2}\right)$

b/c it is in denominator

and when it goes up, it will serve -ve.

Talking of the frequency response of any system we talk of both magnitude and phase in relation with the frequency.

Frequency response is a two part analysis;
- Magnitude frequency analysis.
- Phase frequency analysis.

Why frequency response?
B/c there are several systems in real life (electronics, communication) etc which includes a frequency element.
So for that system we cannot use i/ps like step, ramp etc. We use a sinusoidal.



Sinusoidal is characterized by frequency, phase and amplitude.

Frequency Response specifications;
1. Gain margin $K_g$ or GM
2. Phase margin PM
3. Resonance frequency.
4. Resonance peak (peak magnitude) $M_p$.
5. Bandwidth. BW

Gain margin gives us information that how much gain should be added to the system so that it is shifted to the right half plane. (to produce instability).
Phase margin is that how much phase shift is required in a system to go to the unstable region.

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Resonance frequency is a single ~~place~~ frequency that gives you the peak value.

Bandwidth is those frequencies where the power is one half. or the magnitude/amplitude is 0.707 of the maximum value. $\left(\frac{1}{\sqrt{2}}\right)$

In FR, the magnitude is often converted to dBs.
dB is log to the base 10.

When we talk of magnitudes, say in case of an amplifier;

$$V_1 \rightarrow \boxed{Amp} \rightarrow V_2$$

So we talk of dBs.



$$dB = 20 \log_{10}\left(\frac{V_2}{V_1}\right)$$

Talking of frequencies, we have to deal with a lot of them, starting from 0 and going to larger values approaching ~~infinity~~. (kHz, MHz, GHz).

So simple graph paper does not work.
Special graphs to plot FR: Semi log graph paper.

dBs give you relative magnitude wrt some reference quantity.
As in amp, reference is $V_1$ as it is known.

log (0.707) = −3 dB.

Bandwidth is all the frequency for which the magnitude is −3 dB.

Consider $|G(j\omega)| = \dfrac{10}{\sqrt{36\omega^4 + (8\omega - \omega^3)^2}}$ — ①

and $\angle G(j\omega) = -\tan^{-1}\left(\dfrac{8\omega - \omega^3}{\cancel{36}\omega^{\cancel{4}}}\right)$ — ②

Also $T(s) = \dfrac{10}{s^3 + 6s^2 + 8s + 10}$ — ③ CLTF of the same G(s).



$|T(j\omega)| = \dfrac{10}{\sqrt{(10 - 6\omega^2)^2 + (8\omega - \omega^3)^2}}$

Step ① To convert the given G(s), (OLTF) of unity feedback system to FT expression; getting its magnitude and phase.

Step ② Consider Xtstc eq (denominator polynomial of CLTF)

$s^3 + 6s^2 + 8s + 10 = 0$

FT ⇒ $-j\omega^3 - 6\omega^2 + j8\omega + 10 = 0$

Group imaginary parts and equate to 0.

$-\omega^3 + 8\omega = 0$

⇒ $-\omega^2 + 8 = 0$ ⇒ $\boxed{\omega_\phi = 2.83 \text{ rad/s}}$

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phase crossover frequency

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Step (3) To put $\omega_\phi$ in $|G(j\omega)|$

$$(1) \Rightarrow |G(j\omega)| = \frac{10}{\sqrt{36(2.83)^4 + [8(2.83 - 2.83^3)]^2}}$$

$$\Rightarrow |G(j\omega)| = 0.58$$

Step (4) Gain margin $= \dfrac{1}{|G(j\omega)|} = \dfrac{1}{0.58} = 1.72$

This is not a very good system b/c there is a very small gain margin.

mostly we are interested in a system with a GM b/w 5 to 10.

 (or system may go to unstable state) 

Now for phase margin;

Equate (1) equal to 1.

$$|G(j\omega)| = \frac{10}{\sqrt{36\omega^4 + (8\omega - \omega^3)^2}} = 1$$

Square both sides.

$$\Rightarrow 100 = 36\omega^4 + (8\omega - \omega^3)^2$$

$$0 = \omega^6 + 20\omega^4 + 64\omega^2 - 100$$

let $\omega^2 = x$

$$\Rightarrow x^3 + 20x^2 + 64x - 100 = 0$$

select a +ve real root

$$x = 1.136$$

$$\Rightarrow \omega_g = \sqrt{1.136} = 1.06 \text{ rad/sec}$$

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Gain crossover frequency.

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Now put $w_g = 1.06$ rad/s in phase eq (2) to get angle for $G(j\omega)$.

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{8(1.06)-1.06^3}{6(1.06)^2}\right)$$

$$\Rightarrow \angle G(j\omega) = -132.84^\circ$$

Phase margin $= 180^\circ + \angle G(j\omega)$
$= 180^\circ - 132.84$

$\Rightarrow$ P.M $= 42.15$.

PM could be in the range of about 40 to 60 degrees.

Some say like 50, 55 is a much better figure.



Resonance frequency is only one frequency in the entire spectrum.

Obtain eq (3)

Take derivative of $|.|$ wrt $\omega$.

$$\frac{d}{d\omega}|T(j\omega)| = \frac{d}{d\omega}\left[\frac{10}{\sqrt{(10-6\omega^2)^2+(8\omega-\omega^3)^2}}\right]$$

$$0 = \frac{d}{d\omega}\left[10\left[(10-6\omega^2)^2+(8\omega-\omega^3)^2\right]^{-1/2}\right]$$

$\downarrow$

$$3\omega^4 + 40\omega^2 - 56 = 0$$

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Say $w^2 = x$

$\Rightarrow 3x^2 + 40x - 56 = 0$

Select +ve real root; ~~x = 3.56~~

$x = 1.277$

$w_r = \sqrt{x}$ $\Rightarrow$ $\boxed{w_r = 1.13}$ rad/s

For Resonance peak;
Consider the mag of CLTF ie $|T(j\omega)|$ and
put $w = w_r$

$|T(j\omega)|$ is max when $w = w_r$



$$M_p = |T(jw_r)| = \frac{10}{\sqrt{(10 - 6(1.13)^2)^2 + (8(1.13) - (1.13)^3)^2}}$$

$\Rightarrow M_p = 1.025$

For Bandwidth

eq (3)

$$|T(j\omega)| = \frac{10}{(10-6w^2)^2 + (8w - w^3)^2} = \frac{1}{\sqrt{2}}$$

$200 = (10 - 6w^2)^2 + (8w - w^3)^2$

$w^6 + 20w^4 + 56w^2 - 100 = 0$

let $w^2 = x$

$\Rightarrow x^3 + 20x^2 - 56x - 100 = 0$

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Solving $\Rightarrow x = 3.566$

$\Rightarrow \omega = \sqrt{x} = \sqrt{3.566}$

$\Rightarrow \boxed{\omega_b = 1.88}$ rad/s

$\Rightarrow$ The systems that we are dealing with under frequency response are filters basically.

Consider $G(s) = \dfrac{10}{s(s+4)}$

CLTF $T(s) = \dfrac{10}{s^2+4s+10}$



$T(j\omega) = \dfrac{10}{-\omega^2+j4\omega+10}$

$|T(j\omega)| = \dfrac{10}{\sqrt{(10-\omega^2)^2+16\omega^2}}$ — 1

$\angle T(j\omega) = -\tan^{-1}\left(\dfrac{4\omega}{10-\omega^2}\right)$ — 2

Check for two extremes, $\omega = 0$ and $\omega = \infty$

$\omega = 0$ 1 $\Rightarrow$ $|T(j\omega)| = 1$ low frequency region

$\omega = \infty$ 1 $\Rightarrow$ $|T(j\omega)| = 0$ corresponding magnitude of the high frequency region is being attenuated.

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$\Rightarrow$ It looks like a low pass filter.

apparently