CIRCUIT ANALYSIS II

Sir Amjad Khattak

) Engineering Circuit Analysis by J David Irwin 10th edi

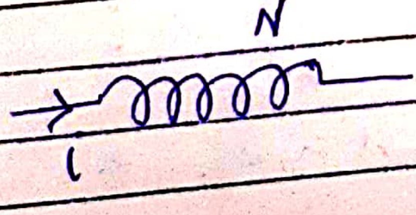
Alternating Current Circuits by K Y Tang 3rd edition.

Course Outline

1. Magnetically Coupled Circuits.
2. 2 Port networks.
3. Laplace Transform and Its applications.
4. Transfer Functions / Poles / Zero diagrams.
5. Non sinusoidal waves analysis / Applications.
6. Filters.

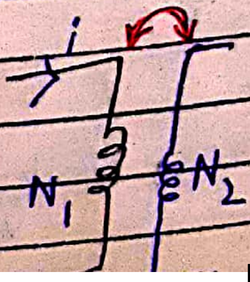
Magnetically Coupled Circuits

Ampere's law → current through a coil produces a magnetic field.



Induced

magnetic flux induced due to changing current in a coil → self induced → self inductance → L ⇒ unit : Henry



Flux in second coil due to first → mutual inductance.

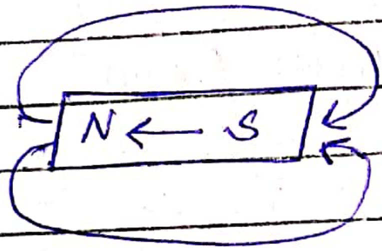
$M_{12} = M_{21} = M$

Ampere's law:

When there is a flow of current, it will always be assosiated by a magnetic field.

Faraday's law

If we have a magnetic field linked with a circuit (time varying magnetic field) it will induce emf in that circuit.



All the magnetic field lines complete their journey N-N independently.

Self induction And Mutual induction.

Flux And Flux Linkage (ϕ and λ)

$$\boxed{\lambda = N\phi}$$

We know that $\phi \propto i$

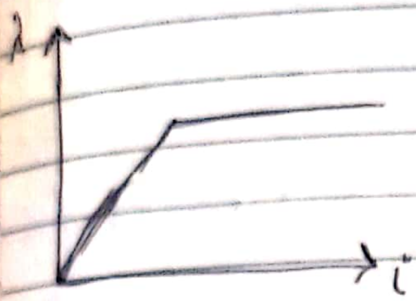
$$\Rightarrow \lambda \propto i \Rightarrow \lambda = Ki$$

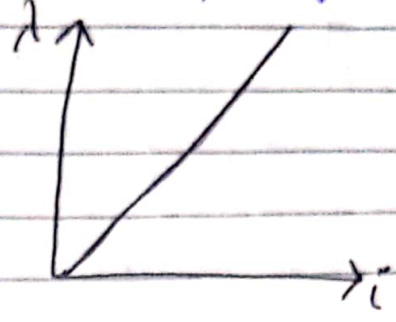
$$K = \frac{\lambda}{i} = \frac{N\phi}{i}$$

K → inductance $\boxed{L = \frac{N\phi}{i}}$

unit = Henry (H)

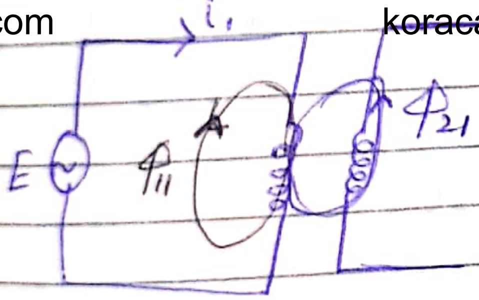
Non magnetic materials are those not attracted strongly by a magnet.
eg wood, paper, water, copper, glass, gold and silver.





Magnetic materials are those which are strongly attracted by a magnetic eg iron, cobalt, nickel and their alloys.

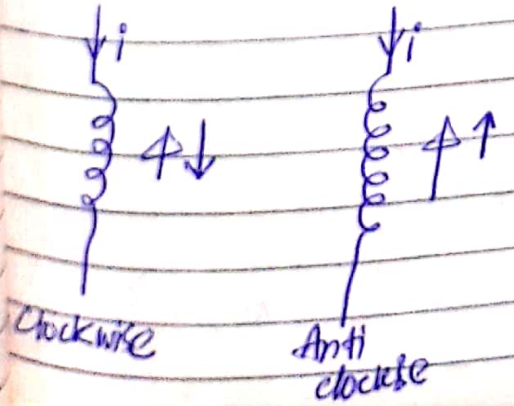
Mutual Flux And Leakage Flux



$$\phi_1 = \phi_{11} + \phi_{21}$$

$\phi_{11} \rightarrow$ leakage flux.
$\phi_{21} \rightarrow$ mutual flux

Direction of flux depends on;
i) direction of current.
ii) orientation of winding.



Self Inductance

$$L_1 = \frac{N_1 \phi_1}{i_1} \qquad L_2 = \frac{N_2 \phi_2}{i_2}$$

Mutual Inductance

$$M = \frac{N_2 \phi_{21}}{i_1} \text{ in coil 2.}$$

$$M = \frac{N_1 \phi_{12}}{i_2}, \quad \text{in coil 1.}$$

Coefficient of Coupling. (k)

It is the measure of magnetic proximity of two coils.

$$k = \frac{\text{Mutual flux}}{\text{Total flux.}} = \frac{\phi_{21}}{\phi_1} = \frac{\phi_{12}}{\phi_2}$$

$$0 \leq k \leq 1$$

$$\phi_1 = \phi_{11} + \phi_{21}$$

if $\phi_{11} = 0 \Rightarrow \phi_1 = \phi_{21}$

0.98 = power transformers $\leftarrow k = 1$

Communication transformer $k = 0.5$

↳ Here we want leakage flux.

Relationship B/w k, M, L_1, L_2

$$M = \frac{N_1 \phi_{12}}{i_2} \rightarrow (1) \qquad M = \frac{N_2 \phi_{21}}{i_1} \rightarrow (2)$$

Multiply (1) and (2)

$$M^2 = \left(\frac{N_1 \phi_{12}}{i_2}\right)\left(\frac{N_2 \phi_{21}}{i_1}\right)$$

$$M^2 = \left(\frac{N_1 k \phi_2}{i_2}\right)\left(\frac{N_2 k \phi_1}{i_1}\right)$$

$$M^2 = k^2 \left(\frac{N_1 \phi_1}{i_1}\right)\left(\frac{N_2 \phi_2}{i_2}\right) \Rightarrow M^2 = k^2 L_1 L_2$$

$$\boxed{M = K\sqrt{L_1 L_2}}$$

multiply by ω.

$$\omega M = K\omega\sqrt{L_1 L_2} = K\sqrt{\omega^2 L_1 L_2}$$

$$\omega M = K\sqrt{\omega L_1 \omega L_2}$$

$$X_m = K\sqrt{X_1 X_2}$$

Example 1.

coil 1 of a pair of coupled coils has a constant current of 5A. The corresponding fluxes ϕ_{11} and ϕ_{21} are 20,000 and 40,000 maxwell. If the turn ratios are $N_1 = 500$ and $N_2 = 1500$

Find L_1, L_2, M, K.

$$\boxed{1\ Wb = 10^8\ Maxwell}$$

$$L_1 = \frac{N_1 \phi_1}{i_1}$$

As $\phi_1 = \phi_{11} + \phi_{21}$
$= 60,000$ maxwell $= 60000 \times 10^{-8}$ wb

$L_1 = 0.06$ Henry.

$$K = \frac{N\phi_{21}}{\phi_1} = \frac{40,000}{60,000} = \frac{4}{6} = \frac{2}{3}$$

$$M = \frac{N_2 \phi_{21}}{i_1} = 0.12H$$

$$M = K\sqrt{L_1 L_2}$$

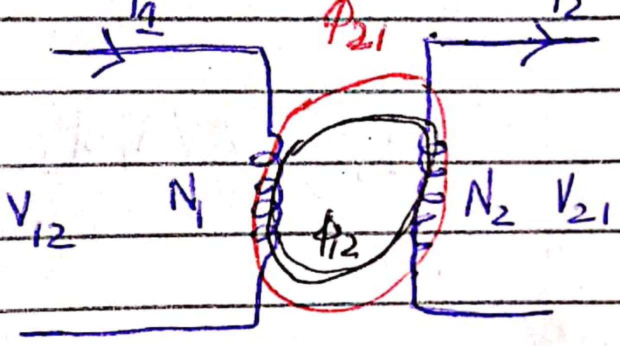
$L_2 = 0.535H$.

Example 2

Two coupled coils of $L_1 = 0.8H$, $L_2 = 0.2$, have a $k = 0.9$. Find M and $\frac{N_1}{N_2}$.

Lecture 2

Voltage of mutual Inductance



Faradays law $e = N\frac{d\phi}{dt}$

$$V_{21} = N_2 \frac{d\phi_{21}}{dt} \rightarrow (1)$$

$$V_{12} = N_1 \frac{d\phi_{12}}{dt} \rightarrow (2)$$

$M_{12} = M_{21} = M = \frac{N_2 \phi_{21}}{i_1} \Rightarrow M i_1 = N_2 \phi_1 \rightarrow$ differentia

$$M \frac{di_1}{dt} = N_2 \frac{d\phi_1}{dt} \; V_{21} \Rightarrow \boxed{V_{21} = M \frac{di_1}{dt}} \rightarrow (3)$$

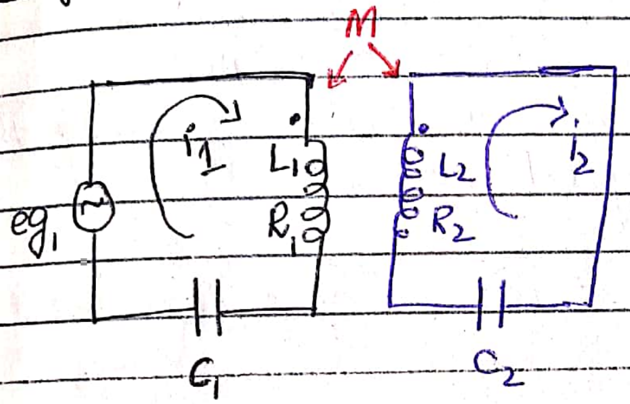
Similarly $M = \frac{N_1 \phi_{12}}{i_2} \Rightarrow M i_2 = N_1 \phi_{12} \rightarrow$ Different

$$M \frac{di_2}{dt} = N_1 \frac{d\phi_{12}}{dt} \; V_{12} \Rightarrow \boxed{V_{12} = M \frac{di_2}{dt}} \rightarrow (4)$$

Dot Rule

If both currents are directed towards the dot or away from the dot, then sign of the M term will be the same as that of the L terms.

Voltage Equation of Circuit With R, L, C, M



Instantanous Equations

loop 1

$$eg_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + \frac{1}{C}\int i_1 dt - M\frac{di_2}{dt}$$

loop2

$$0 = R_2 i_2 + L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int i_2 dt - M\frac{di_1}{dt}$$

Steady state form

(4) $$Eg_1 = R_1 I_1 + j\omega L_1 I_1 + \frac{1}{j\omega C_1} I_1 - j\omega M I_2$$

$$Eg_1 = \left(R_1 + j\omega L_1 + \frac{1}{j\omega C_1}\right) I_1 - j\omega M I_2 \rightarrow (1)$$

$$0 = R_2 I_2 + j\omega L_2 I_2 + \frac{1}{j\omega C_2} I_2 - j\omega M I_1 \rightarrow (2)$$

$$0 = (-j\omega M) I_1 + \left(R_2 + j\omega L_2 + \frac{1}{j\omega C_2}\right) I_2 \rightarrow (3)$$

Eq (1) $\Rightarrow$ $Z_{11} I_1 - Z_m I_2 = Eg_1$

Eq (3) $\Rightarrow$ $-Z_m I_1 + Z_2 I_2 = 0$

$$\begin{bmatrix} Eg_1 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & -Z_m \\ -Z_m & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

If we have n coupled circuits;

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & Z_{23} & \cdots & Z_{2n} \\ \vdots & & & & \\ Z_{n1} & & \cdots & & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} E \\ E \\ \vdots \\ E \end{bmatrix}$$

Driving Point ~~voltage~~ Impedence $Z_{in} = \frac{E_{g1}}{I_1}$

It is also known as input impedence

It is defined as the ratio of voltage current at the specified points

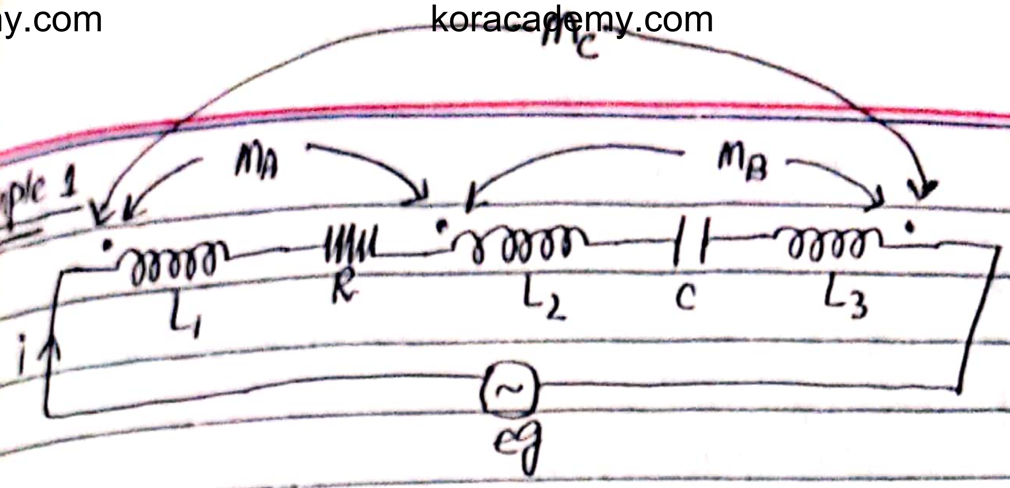
$$I_1 = \frac{\begin{bmatrix} E_{g1} & -Z_m \\ 0 & Z_{22} \end{bmatrix}}{Z_{11}Z_{22} - Z_m^2} \Rightarrow Z_{in} = E_{g1} \times \frac{Z_{11}Z_{22} - Z_m^2}{E_{g1} Z_{22}}$$

$$\boxed{Z_{in} = Z_{11} - \frac{Z_m^2}{Z_{22}}}$$

Transfer Impedence

Ratio of voltage in one part of circuit to current in the other part of circuit

Example 1



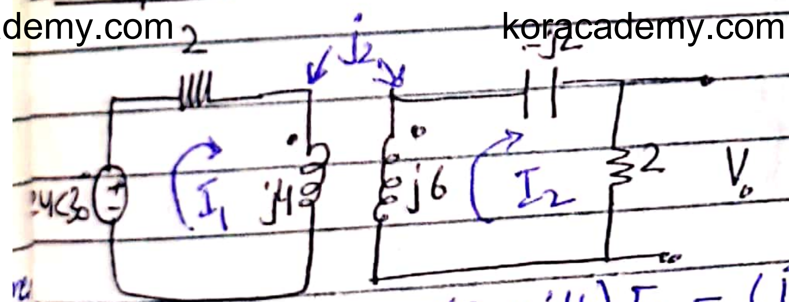
Instantanous eqs

$$eg = Ri + L_1\frac{di}{dt} + L_2\frac{di}{dt} + \frac{1}{C}\int i\,dt + L_3\frac{di}{dt} + M_A\frac{di}{dt}$$

$$- M_C\frac{di}{dt} + M_A\frac{di}{dt} - M_B\frac{di}{dt} - M_C\frac{di}{dt} - M_B\frac{di}{dt}$$

Steady state

Example 2



loop 1 $\quad 24\angle 30^\circ = (2+j4)I_1 - (j2)I_2 \rightarrow ①$

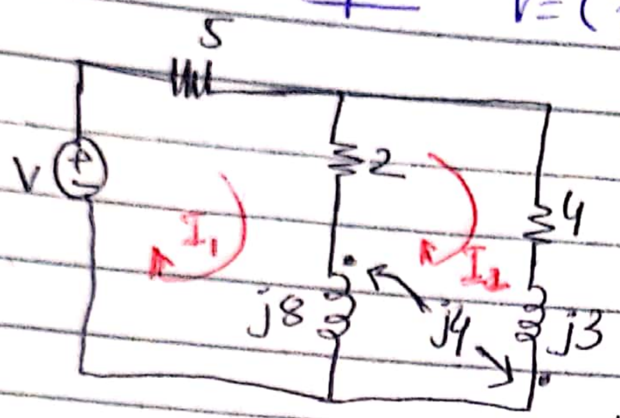
loop2 $\quad 0 = (-j2)I_1 + (2+j4)I_2 \rightarrow ②$

$$\begin{bmatrix} 2+j4 & -j2 \\ -j2 & 2+j4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 24\angle 30 \\ 0 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 2+j4 & 24\angle 30^\circ \\ -j2 & 0 \end{vmatrix}}{\begin{vmatrix} 2+j4 & -j2 \\ -j2 & 2+j4 \end{vmatrix}}$$

$V_o = 2I_2 = 5.36\angle 3.43^\circ$

Example 3



loop 1

$$V = (7+j8)I_1 - (2+j8)I_2 - (j4)I_2$$

$$V = (7+j8)I_1 - (2+j12)I_2$$

Loop 2

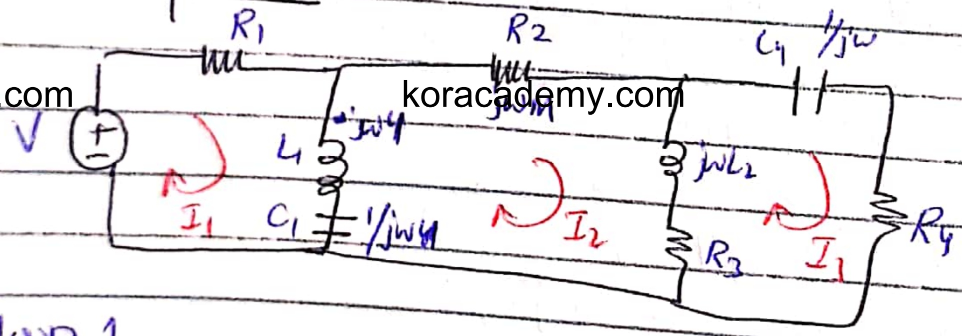
$$0 = (6+j11)I_2 - (2+j8)I_1 + j4I_2 + j4(I_2 - I_1) \rightarrow ②$$

$$0 = (-2-j8-j4)I_1 + (6+j11+j4+j4)I_2$$

$$0 = (-2-j12)I_1 + (6+j19)I_2 \rightarrow ②$$

$$\begin{bmatrix} 7+j8 & -(2+j12) \\ -(2+j12) & (6+j19) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V \\ 0 \end{bmatrix}$$

Example 4



loop 1

$$V = \left(R_1 + j\omega L_1 + \frac{1}{j\omega C_1}\right)I_1 + \left(-j\omega L_1 - \frac{1}{j\omega C_1}\right) + j\omega M(I_2 \ldots$$

$$V = \left(R_1 + j\omega L_1 + \frac{1}{j\omega C_1}\right)I_1 + \left(-j\omega L_1 - \frac{1}{j\omega C_1} + j\omega M\right)I_2 - j\omega M I_3 \rightarrow ①$$

loop 2

$$0 = \left(R_2 + R_3 + j\omega C_1 + j\omega C_2 + \frac{1}{j\omega C_1}\right)I_2 + \left(-j\omega L_1 - \frac{1}{j\omega C_1}\right)$$

$$+(-R_3 - j\omega L_2)I_3 - j\omega M(I_2 - I_3) - j\omega M(I_2 - I_1)$$

$$0 = \left(-j\omega L_1 - \frac{1}{j\omega C_1} + j\omega M\right) I_1 + \left(R_2 + R_3 + j\omega L_1 + j\omega L_2 + \frac{1}{j\omega C_1} - 2j\omega M\right) I_2 + \left(-R_3 - j\omega L_2 + j\omega M\right) I_3 \rightarrow (2)$$

loop 3

$$0 = \left(R_3 + R_4 + j\omega L_2 + \frac{1}{j\omega C_4}\right) I_3 + \left(-j\omega L_2 \cancel{+} R_3\right) I_3 + j\omega M (I_2 - I_1)$$

$$0 = -j\omega M I_1 + \left(-R_3 - j\omega L_2 + j\omega M\right) I_2 + \left(R_3 + R_4 + j\omega L_2 + \frac{1}{j\omega C_4}\right) I_3 \rightarrow (3)$$

Two Port networks:

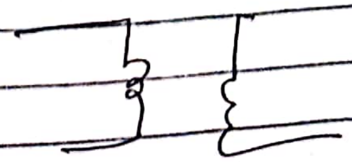
Z_{11} → open circuit input impedance } open circuit
Z_{22} → open circuit output impedance } driving impedance

Z_{12} and Z_{21} → Open circuit transfer impedance.

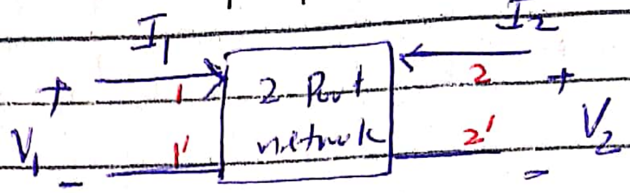
For any passive, linear and bilateral element
$$Z_{12} = Z_{21}$$

linear elements are those elements whose value does not change with changes in current or voltage.

Example 1



Determination of Y parameters.

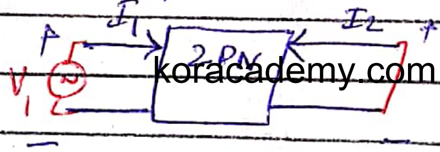


$$I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (1)$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 \rightarrow (2)$$

Short Circuit test

(i) Short output (2-2') and apply a source at (1-1') and find V_1, I_1, V_2, I_2

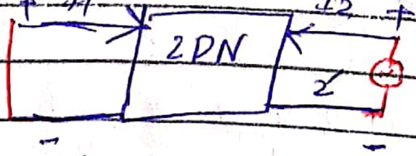
$V_2 = 0$



$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

(ii) short input (1-1') and apply source at output (2-2')



$V_1 = 0$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

$Y_{12} = Y_{21}$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{|Y|} = \frac{Y_{22}}{|Y|} I_1 - \frac{Y_{12}}{|Y|} I_2$$

$V_1 = Z_{11} I_1 + Z_{12} I_2$

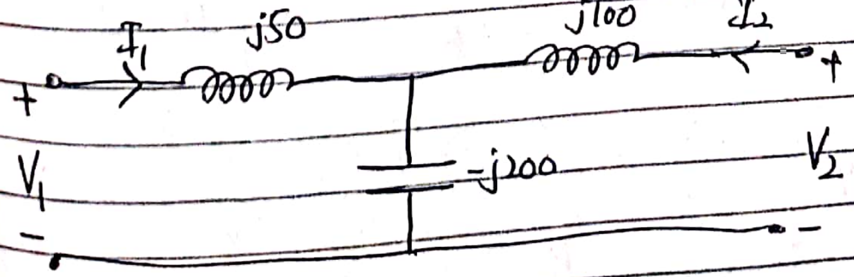
$$\Rightarrow Z_{11} = \frac{Y_{22}}{|Y|} \qquad Z_{22} = -\frac{Y_{12}}{|Y|}$$

Similarly $Z_{22} = \frac{Y_{11}}{|Y|}, \quad Z_{21} = \frac{-Y_{21}}{|Y|}$

ABCD parameters.

Lecture 5

Find (a) ABCD parameters. (b) Write expressions for the input values in terms of output values. (c) Write expressions for output values in terms of input values (d) Show that $AD - BC = 1$.



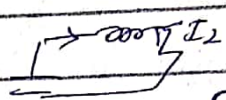
(b) $V_1 = AV_2 - BI_2$
$I_1 = CV_2 - DI_2$

(c) $V_2 = DV_1 - BI_1$

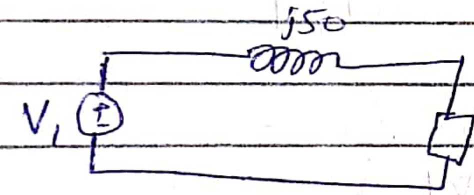
$I_2 = CV_1 - AI_1$

(a) $A = \frac{V_1}{V_2}\Big|_{I_2=0} \Rightarrow A = \frac{V_1}{V_1 \times \frac{-j200}{j50-j200}} \Rightarrow \boxed{A =}$

$C = \frac{I_1}{V_2}\Big|_{I_2=0} = \frac{I_1}{I_1(-j200)} \Rightarrow \boxed{C = \frac{j}{200}\ S}$



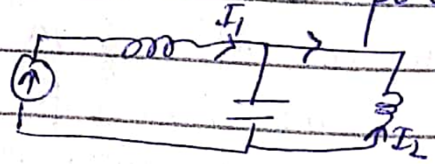
$B = \frac{-V_1}{I_2}\Big|_{V_2=0} \Rightarrow B = \frac{-V_1}{-I_1 \times \frac{-j200}{-j200+j1}}$



$I_1 = \frac{V_1}{Z_t} = \frac{V_1}{j50 + (-j200 \,\|\, j100)} = \frac{}{j50 +}$

$= \frac{V_1}{j50+j200} = \frac{V_1}{j250} \Rightarrow B = \frac{V_1}{2 \times \frac{V_1}{j250}} = \frac{j250}{2} \Rightarrow$

$D = \frac{-I_1}{I_2}\Big|_{V_2=0}$ Current source at input



$D = \frac{-I_1}{-I_1 \times \frac{-j200}{-j200+j100}} = \frac{-j100}{-j200} \Rightarrow \boxed{D = \frac{1}{2}}$

(d) $\left(\frac{3}{4}\right)\left(\frac{1}{2}\right) - (j125)\left(\frac{j}{200}\right)$

$= \frac{3}{8} + \frac{125}{200} = \frac{3}{8} + \frac{5}{8}$

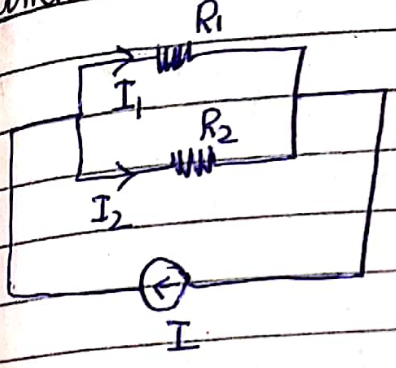
$= \frac{8}{8} = 1$

proved.

Series → current is same
Series → voltage divides.

Parallel → current divides
Parallel → voltage is same.

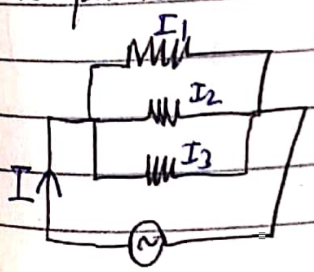
Current divider Rule



$$I_1 = I \frac{R_2}{R_1 + R_2}$$

$$I_2 = I \frac{R_1}{R_1 + R_2}$$

If three resistors are connected in parallel.

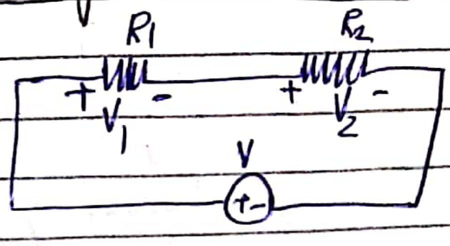


$$I_1 = I \times \frac{(R_2 \| R_3)}{R_1 + (R_2 \| R_3)}$$

$$I_2 = I \times \frac{(R_1 \| R_3)}{R_2 + (R_1 \| R_3)}$$

$$I_3 = I \times \frac{(R_2 \| R_1)}{(R_1 \| R_2) + R_3}$$

Voltage Divider Rule.



$$V_1 = V \frac{R_1}{R_1 + R_2}$$

$$V_2 = V \frac{R_2}{R_1 + R_2}$$

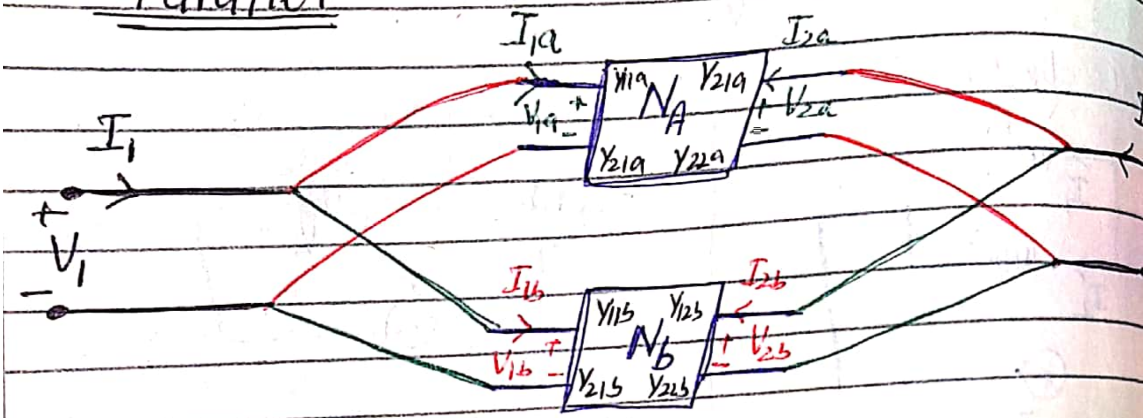
If more than two resistors are connected in series;

$$V_3 = V \frac{R_3}{R_1 + R_2 + R_3}$$

and so on.

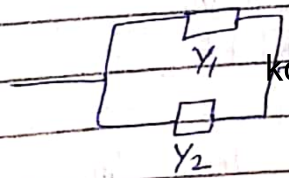
Interconnection of Two Port Networks

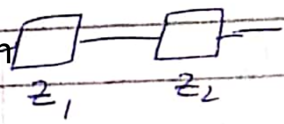
Parallel



$V_{1a} = V_1$ $\qquad$ $V_{2a} = V_2$ $\qquad$ $I_1 = I_{1a} + I_{1b}$

$V_{1b} = V_1$ $\qquad$ $V_{2b} = V_2$ $\qquad$ $I_2 = I_{2a} + I_{2b}$



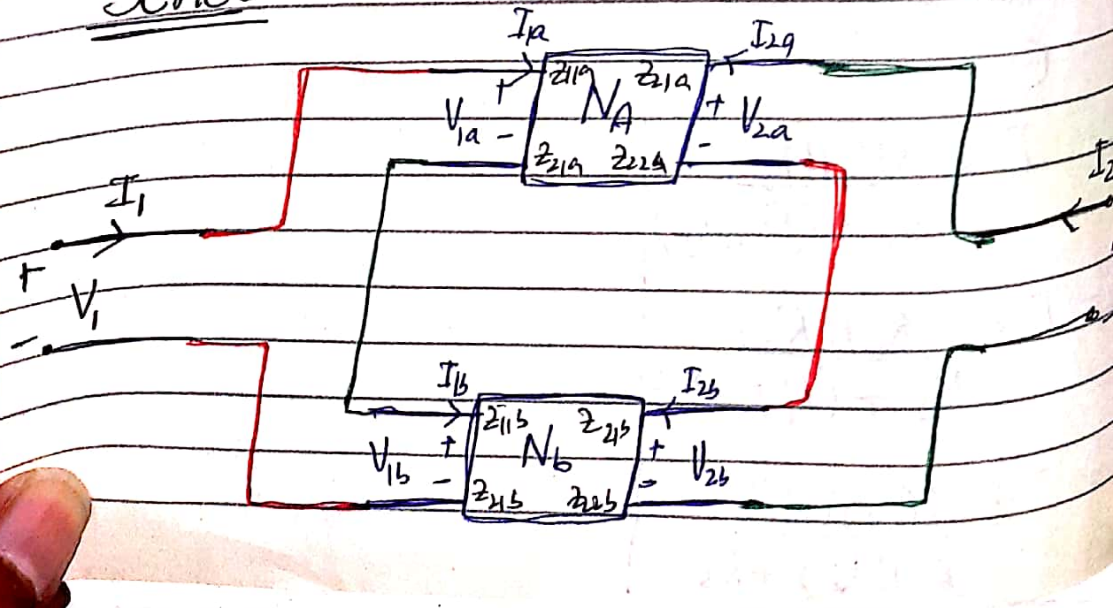


$Z = Z_1 + Z_2$.

$Y = Y_1 + Y_2$

So $\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11a} + Y_{11b} & Y_{12a} + Y_{12b} \\ Y_{21a} + Y_{21b} & Y_{21a} + Y_{22b} \end{bmatrix}$

Series

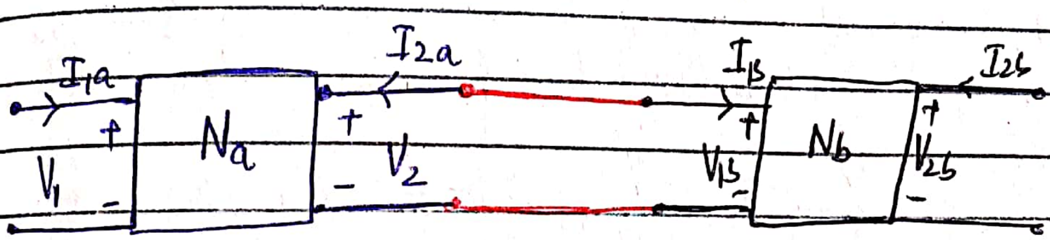


$$V_1 = V_{1a} + V_{1b} \qquad V_2 = V_{2a} + V_{2b}$$

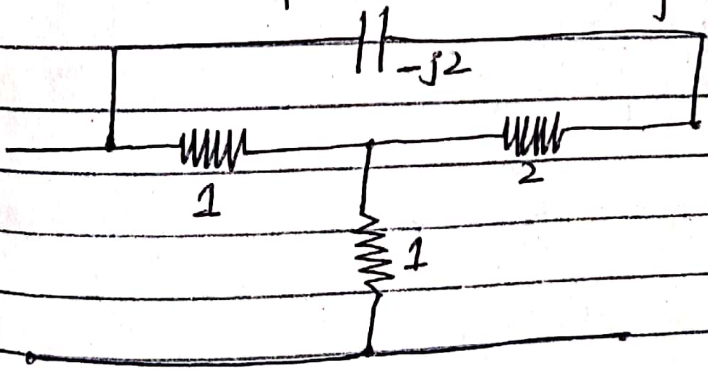
$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_{11a} + z_{11b} & z_{12a} + z_{12b} \\ z_{21a} + z_{21b} & z_{22a} + z_{22b} \end{bmatrix}$$

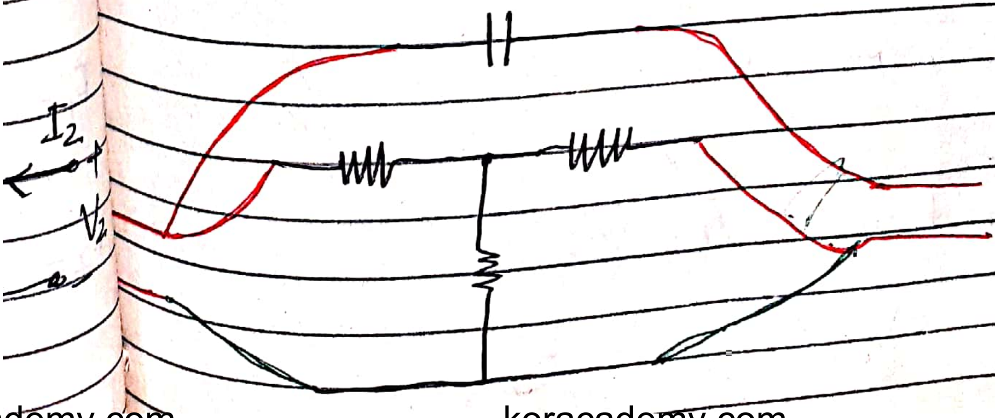
Cascade

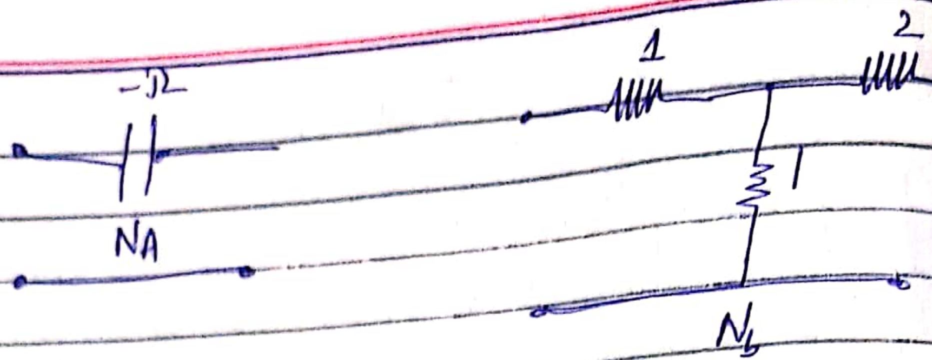
Where we connect output of one network with input of another network.
for cascade we use ABCD parameters



Example 16.5 Find the Y parameters.
Consider it a parallel combination of two different networks.







Nb

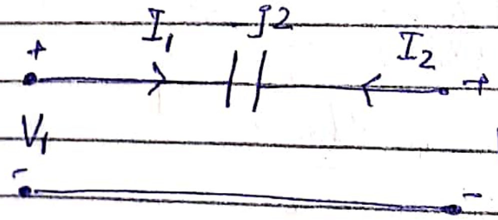
$$Y_{11b} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{V_1}{V_1\left(1+\frac{1\times2}{1+2}\right)} \Rightarrow Y_{11b} =$$

$$Y_{21b} = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{-I\times\frac{1}{3}}{V_1} = \frac{-V_1}{3V_1\left(1+\frac{1\times2}{1+2}\right)}$$

$Y_{21b} = -1/5$ seimen Also $Y_{12b} = \frac{-1}{5}$

$$Y_{22b} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{I_2}{I_2\left(2+\frac{1\times1}{1+1}\right)} \Rightarrow Y_{22b} = \frac{}{5}$$

Na



$$Y_{11a} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{V_1}{V_1(-j2)} = \frac{j}{2}\ S$$

Symmetrical network so $Y_{11} = Y_{22a} = j/2$ S.

$$Y_{12a} = \frac{I_1}{V_2}\Big|_{V_1=0} = \frac{-V_2}{V_2(-j2)} = \frac{-j}{2}$$

$$Y_{12a} = Y_{21a} = \frac{-j}{2}\ S$$

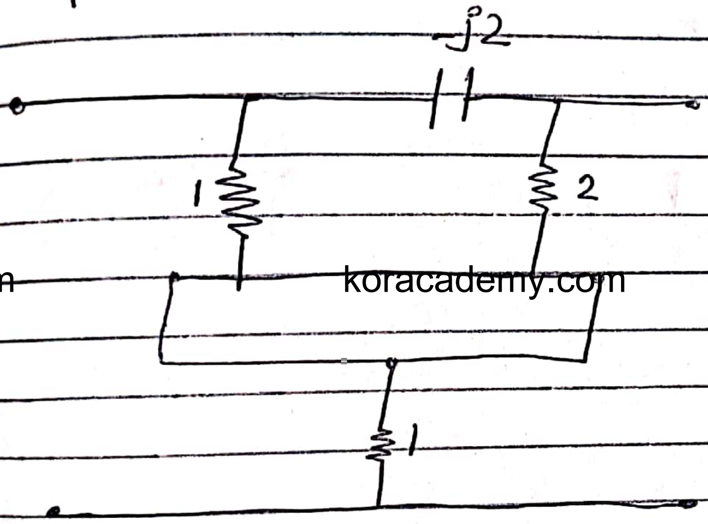
$Y_{11} = Y_{11a} + Y_{11b} = \left(j/2 + 3/5\right) S$

$Y_{12} = -\left(1/5 + j/2\right) S$

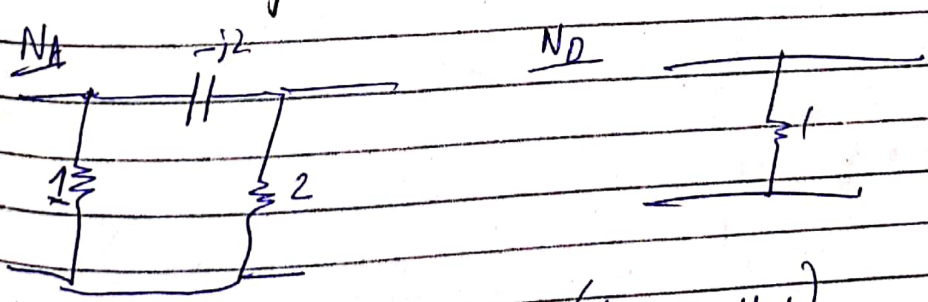
$Y_{21} = -\left(1/5 + j/2\right) S$

$Y_{22} = \left(j/2 + 3/5\right) S.$

Example 16.6



Find Z parameters by considering it to be series interconnection of two networks.



$\underline{N_A}$ $\quad Z_{11a} = \left.\dfrac{V_1}{I_1}\right|_{I_2=0} = \dfrac{I_1\left((2-j2)\,\|\,1\right)}{I_1}$

$Z_{11a} = \dfrac{2-j2}{3-j2}$

$$z_{21a} = \frac{V_2}{I_1}\bigg|_{I_2=0} = \frac{\cancel{I_1} \times \frac{1}{3-j2} \times 2}{\cancel{I_1}}$$

$$z_{21a} = \frac{2}{3-j2}\ \Omega$$

linear, bilateral, passive so; $z_{12} = z_{21}$

with the same at output

$$z_{22a} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

$$= \frac{\cancel{V_2}}{\cancel{V_2}/2 \parallel (1-j2)} \qquad z_{22} = \frac{2-j4}{3-j2}$$

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N_b

$z_{11b} = z_{22b} = 1$

$z_{12b} = z_{21b} = 1$

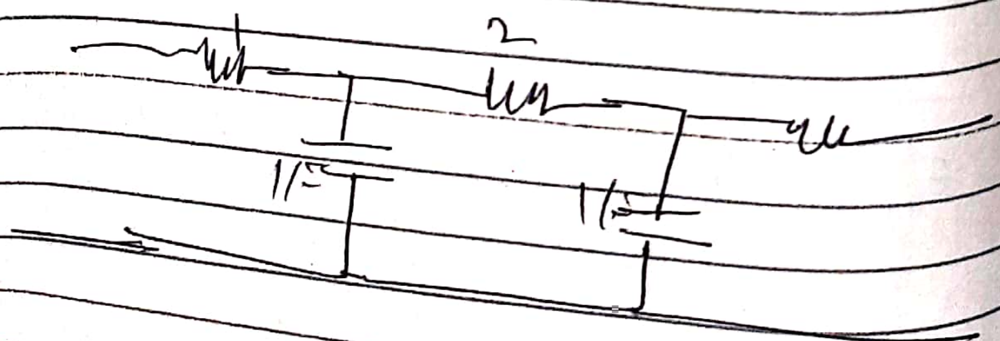
total $z_{11} = z_{11a} + z_{11b} = 5-j4/3-j2$

$z_{12} = z_{12a} + z_{12b} = 5-j2/3-j2$

$z_{21} = z_{21a} + z_{21b} = 5-j2/3-j2$

$z_{22} = z_{22a} + z_{22b} = 5-j6/3-j2$

Example 16.7



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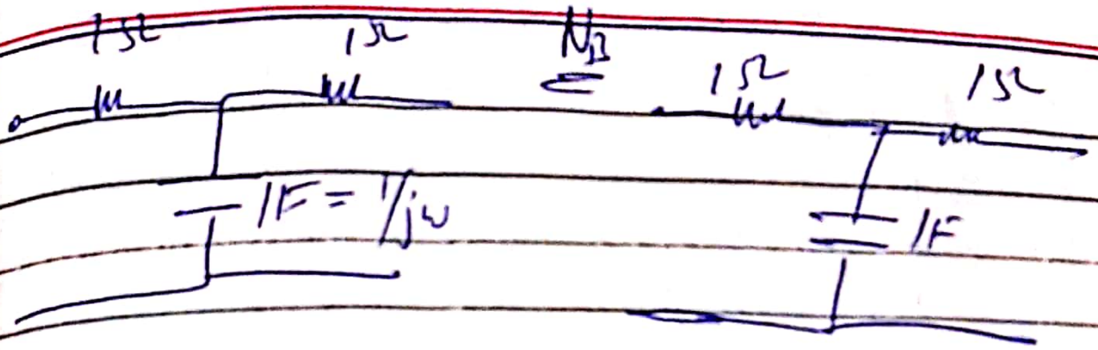

$A_4 = V_1/V_2 \Big|_{I_2=0} = 1+j\omega = A_3$

$B_4 = \frac{-V_1}{I_2}\Big|_{V_2=0} = 2+j\omega = B_3$

$C_4 = \frac{I_1}{V_2}\Big|_{I_2=0} = j\omega = C_3$

$D_4 = \frac{-I_1}{I_2}\Big|_{V_2=0} = 1+j\omega = D_3$

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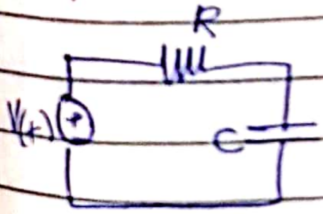
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_4 & B_4 \\ C_4 & D_4 \end{bmatrix}\begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1+4j\omega-2\omega^2 & 4+6j\omega-2\omega^2 \\ 2j\omega-2\omega^2 & 1+4j\omega-2\omega^2 \end{bmatrix}$$

lecture 6

Laplace Transforms



Time domain analysis. $f(t)$

$F(s)$ Frequency domain

$y = f(t)$

$F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt = L f(t)$

→ functions integrable in a specific time range.

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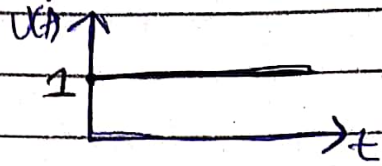
Unit step function is a function that;



$U(t) = 0 \quad t<0$

$U(t) = 1 \quad t>0$

laplace transform $\quad F(s) = \int_0^{\infty} U(t) e^{-st} dt$

$= \int_0^{\infty} 1 e^{-st} dt = -\frac{1}{s} e^{-st} \Big|_0^{\infty} = -\frac{1}{s}(e$

$\Rightarrow F(s) = \frac{1}{s}$

If time is from 0 to a.

$\int_0^{\infty} 1 \cdot e^{at} \cdot e^{-st} dt \Rightarrow F(s) = \frac{1}{s+a}$

Time Shift

$f(t) = u(t+a) \rightarrow$ ① Advance

$f(t) = u(t-a) \rightarrow$ ② Delay

$\int_a^{\infty} 1 e^{-st} dt = \frac{e^{-at}}{s}$

Inverse laplace transform. $\quad F(s) \rightarrow f(t)$

Partial fraction.

$F(s) = \frac{N_r(s)}{D_r(s)} = \frac{\text{polynomial}}{\text{polynomial}}$

$m<n$

$$= \frac{s^m + a_1(m-1)s^{m-1} + a_2(m-2)s^2 + \cdots\cdots + a_0}{s^n + b_1(n-1)s^{n-1} + b_2(n-2)s^2 + \cdots + b_0}$$

$$F(s) = \frac{N_r}{(s-s_1)(s-s_2)(s-s_3)\cdots\cdots}$$

$$F(s) = \frac{k_1}{(s-s_1)} + \frac{k_2}{(s-s_2)} + \frac{k_3}{(s-s_3)} + \cdots$$

$$k_1 = F(s)(s-s_1)\Big|_{s=s_1}$$

$$k_2 = F(s)(s-s_2)\Big|_{s=s_2}$$

$$\Rightarrow f(t) = L^{-1} F(s)$$

eg. Distinct roots.

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$$F(s) = \frac{s}{s^2+5s+6} = \frac{k_1}{s+3} + \frac{k_2}{s+2}$$

$$k_1 = F(s)(s+3)\Big|_{s=-3} = \frac{s}{\cancel{(s+3)}(s+2)}\cancel{(s+3)}$$

$$\frac{-3}{-3+2} \rightarrow k_1 = 3$$

$$k_2 = \frac{s}{\cancel{(s+2)}(s+3)}\cancel{(s+2)}\Big|_{s=-2} = \frac{-2}{-2+3} \Rightarrow k_2 = -2$$

$$L^{-1} F(s) = L^{-1}\left(\frac{3}{s+3} - \frac{2}{s+2}\right)$$

$$f(t) = 3e^{-3t} - 2e^{-2t}.$$

eg <u>same roots.</u>

$$F(s) = \frac{s+2}{s(s+1)^2(s+3)}$$

$\hookrightarrow s=-1$
$s=-1$

$$k_4 = \frac{d}{ds}\left[F(s)\,(s+1)^2\right]\Big|_{s=-1}$$

$$\frac{d}{ds}\left[\frac{(s+2)}{s(s+1)^2(s+3)} \times (s+1)^2\right]_{s=-2}$$

$$f(t) = \frac{2}{3} + \frac{1}{12}e^{-3t} - \frac{3}{4}e^{-t}$$

<u>Complex Roots:</u>

$$F(s) = \frac{s}{s^2+2s+5} \rightarrow s = 1 \pm j2 \Rightarrow \begin{array}{l} s_1 = 1+j2 \\ s_2 = 1-j2 \end{array}$$

$$F(s) = \frac{k_1}{s+\underbrace{1+j2}_{s_1}} + \frac{k_2}{s+\underbrace{1-j2}_{s_2}}$$

$$k_1 = F(s)\,(s+1+j2)\Big|_{s=-1-j2} \qquad k_2 =$$

$$e^{-t}\left(\frac{e^{j2t}+e^{-j2t}}{2} - \frac{1}{2} \times \frac{e^{j2t}-e^{-j2t}}{2j}\right)$$

$$e^{-t}\left(\cos 2t - \frac{1}{2}\sin 2t\right)$$

Laplace transform is an important technique in that for a given set of initial conditions it will yield the total response of a circuit consisting of both the natural and forced responses in one operation.

Using the laplace transform we transform the circuit problem from time domain into complex frequency domain and finally we transform the solution to time domain.

laplace transform is an integral transform that convert a set of linear, simultaneous, integro differential to a set of simultaneous algebric equations.

complementary function → natural / transient response
particular integral → forced / steady state response

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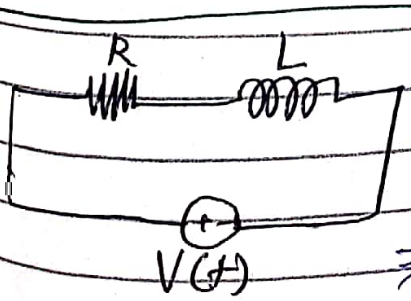
$$\boxed{\begin{aligned} L\left(\frac{d}{dt} f(t)\right) &= sF(s) - f(0^+) \\ L\left(\int f(t)\,dt\right) &= \frac{F(s)}{s} + \frac{f^{-1}(0^+)}{s} \end{aligned}}$$



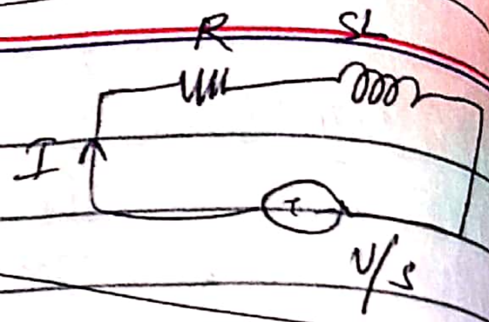
$$V(t) = Ri(t) + L\frac{di(t)}{dt}$$

$$\frac{V}{s} = RI(s) + L\left(sI(s) - i(0^+)\right)$$

$$\frac{V}{s} = (R+Ls)\,I(s).$$

$$I(s) = \frac{V/s}{R+sL} = \frac{V(s)}{Z(s)}$$

Draw s domain circuit



$$I(s) = \frac{V/L}{s(s+R/L)} = \frac{K_1}{s} + \frac{K_2}{s+R/L}$$

$$K_1 = \frac{V}{R} \qquad K_2 = \frac{-V}{R}$$

$$I(s) = \frac{V/R}{s} + \frac{-V/R}{s+R/L}$$

$$i(t) = \frac{V}{R} - \frac{V}{R} e^{-R/L\, t}$$

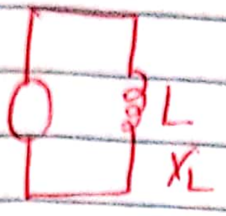
$$V_L = L\frac{di}{dt} = L\left(0 - \frac{V}{R}\left(-\frac{R}{L}\right)e^{-R/L\, t}\right)$$

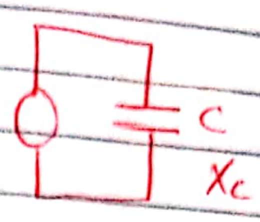
$$V_L = Ve^{-R/L\, t} = Ve^{-t/\tau}$$

$$\tau = \frac{L}{R}$$

Final

Lecture 1



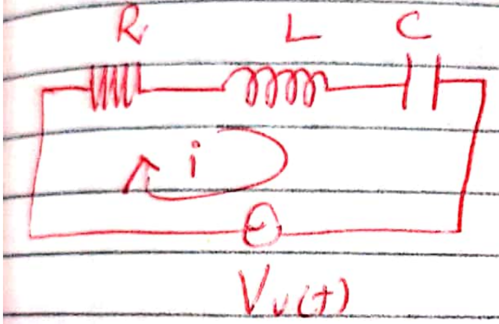


$V = IR$ $\qquad V = L\frac{di}{dt}$ $\qquad V_c = \frac{1}{C}\int i\,dt$

$Z = R + YL$ $\qquad Z = R + YC.$

1st order equations.

Non linear behaviour.



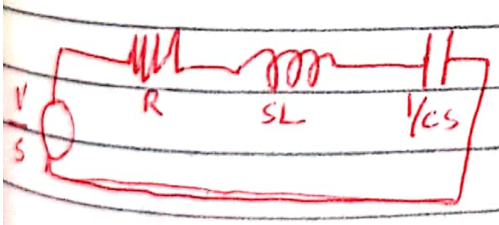
Time domain eq; KVL

$$V_v(t) = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Frequency response

Generally $j\omega \equiv S$ domain.

$$V(t) \Rightarrow \frac{V}{S} = SLI(S) + \frac{1}{CS}I(s) + R(I(s)$$



$$\frac{V}{S} = I(s)\left\{R + SL + \frac{1}{CS}\right\}$$

$$I(s) = \frac{V/s}{\left(R + SL + \frac{1}{CS}\right)}$$

$$I(s) = \frac{V/L}{\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)} \rightarrow \text{laplace transform}$$

$$S = \frac{-R/L \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\left(\frac{1}{LC}\right)}}{2}$$

$$S = \alpha \pm \beta$$

① $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

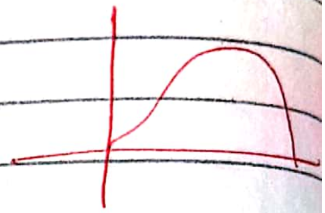
$$I(s) = \frac{k_1}{s-(\alpha+\beta)} + \frac{k_2}{s-(\alpha-\beta)}$$

Inverse laplace;

$$L^{-1}\, I(s) = i(t) = k_1 e^{(\alpha+\beta)t} + k_2 e^{(\alpha-\beta)t}$$

$$i(t) = e^{\alpha t}\left(k_1 e^{\beta t} + k_2 e^{-\beta t}\right)$$

overdamped



② $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$ underdamped imaginary

③ $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ critically damped

$$R^2 = \frac{4L^2}{LC} \Rightarrow R_{cr} = 2\sqrt{\frac{L}{C}}$$

Damping ratio ξ zeeta $= \frac{R}{R_c}$

or $\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$

showing it in terms of frequency;

$$2\omega_n \xi = 2\frac{R}{2}\sqrt{\frac{C}{L}} \times \frac{1}{\sqrt{LC}}$$

↳ natural frequency.

$$2\omega_n \xi = \frac{R}{L}$$

As the characteristic equation is;

$$s^2 + \frac{R}{L}s + \frac{1}{}$$

Showing in terms of natural frequency

$$s^2 + 2\omega_n \xi s + \omega_n^2 = 0$$

$$s = \frac{-2\omega_n \xi \pm \sqrt{4\omega_n^2 \xi^2 - 4\omega_n^2}}{2}$$

$$s_1 = -\omega_n \xi + \omega_n \sqrt{\xi^2 - 1}$$

$$s_2 = -\omega_n \xi - \omega_n \sqrt{\xi^2 - 1}$$

$$\Rightarrow i(t) = K_1 e^{(-\omega_n \xi + \omega_n \sqrt{\xi^2-1})t} + K_2 e^{(-\omega_n \xi - \omega_n \sqrt{\xi^2-1})t}$$

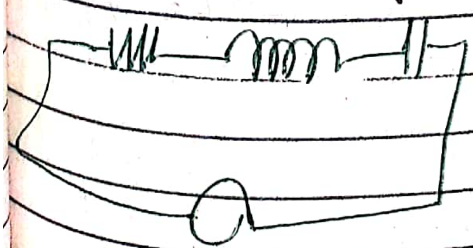
i) $\xi > 1$ overdamped.

ii) $\xi < 1$ underdamped

iii) $\xi = 0$ Oscillating

iv) $\xi = 1$ critically damped.

Q. A series RLC ckt with $R = 100\,\Omega$ $L = 0.1H$ $C = 100\mu F$ has a DC voltage source of 200V applied at $t=0$. Find the expression for transient current by assuming initially relaxed condition.



relaxed $\Rightarrow$ $i_L(0) = 0$
$V_c(0) = 0$

$$V(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i\,dt$$

$$200\,u(t) = 100\,i(t) + 0.1\frac{di}{dt} + \frac{10^6}{100}\int i\,dt$$

laplace

$$\frac{200}{s} = 100\,I(s) + 0.1\left[s\,I(s) - i(0)\right] + 10^{4}\left[\frac{I(s)}{s} + \frac{q(0)}{s}\right]$$

$$\frac{200}{s} = \left(100 + 0.1s + \frac{10^4}{s}\right) I(s)$$

$$I(s) = \frac{200/s}{0.1s + 100 + 10^4/s} = \frac{200}{0.1s^2 + 10\ldots}$$

$$I(s) = \frac{2000}{s^2 + 1000s + 10^5}$$

$\rightarrow s_1 = -112.7$

$\rightarrow s_2 = -887.3$

$$I(s) = \frac{2000}{(s+112.7)(s+887.3)} = \frac{K_1}{s+112.7} + \frac{K_2}{s+887.3}$$

$$K_1 = I(s)(s+112.7)\Big|_{s=-112.7} = \frac{2000}{s+887.3}\Big|_{s=-112.7}$$

$$K_1 = 2.58$$

$$K_2 = I(s)(s+887.3)\Big|_{s=-887.3} = \frac{2000}{s+112.7}\Big|_{s=-887.3}$$

$$K_2 = -2.58$$

Overdamped

$$I(s) = \frac{2.58}{s+112.7} - \frac{2.58}{s+887.3}$$

Inverse

$$i(t) = 2.58e^{-112.7t} - 2.58e^{-887.3t}$$

Example 2 $C = ?$ in previous example will make the system critically damped → Also find the transient current using this value of C.

For critical $\frac{R^2}{4L^2} = \frac{1}{LC}$

$C = \frac{4L^2}{R^2 L} \Rightarrow C = \frac{4L}{R^2} = \frac{4 \times 0.1}{(100)^2}$

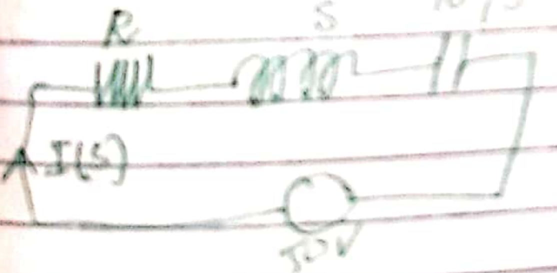
$$\boxed{C = 40\mu F}$$

$$i(t) = 2000t\, e^{-500t} \quad \text{Ans.}$$

Example 3 Series RLC with $R = 300\,\Omega$ $L = 1H$ and $C = 100\mu F$ $V = 50V$ applied at $t = 0$.

Find the max value of current by assuming zero initial conditions.

$10^4/s = \frac{1}{Cs} = \frac{10^6}{100s}$



$I(s) = \frac{V(s)}{Z(s)}$

$= \frac{50/s}{300 + s + 10^4/s}$

$I(s) = \frac{50}{s^2 + 300s + 10^4}$ $\quad s = -38.19, -261.8$

$I(s) = \frac{50}{(s+38.19)(s+261.8)} = \frac{K_1}{s+38.19} + \frac{K_2}{s+261.8}$

$K_1 = 0.2236$ $\quad K_2 = -0.2236$

$$I(s) = \frac{0.2236}{s+38.19} - \frac{0.2236}{s+261.8}$$

$$i(t) = 0.2236\,e^{-38.19t} - 0.2236\,e^{-261.8t}$$

$$i(t) = 0.2236\left[e^{-38.19t} - e^{-261.8t}\right]$$

derivate $= 0 \Rightarrow$ time $\rightarrow$ in function $\rightarrow$ for max

$$\Rightarrow \frac{di(t)}{dt} = 0$$

$$\Rightarrow 0.2236\left[e^{-38.19t}(-38.19) - e^{-261.8t}(-261.8)\right]$$

$$\Rightarrow 0 = -38.19\,e^{-38.19t} + 261.8\,e^{-261.8t}$$

$$38.19\,e^{-38.19t} = 261.8\,e^{-261.8t}$$

$$e^{-38.19t+261.8t} = 261.8/38.19$$

$$e^{223.61t} = 6.855$$

$$223.61\,t = \ln(6.855)$$

$$\Rightarrow \boxed{t = 0.00861\ s}$$

$$\Rightarrow \boxed{i(t)_{max} = 0.137\ A}$$

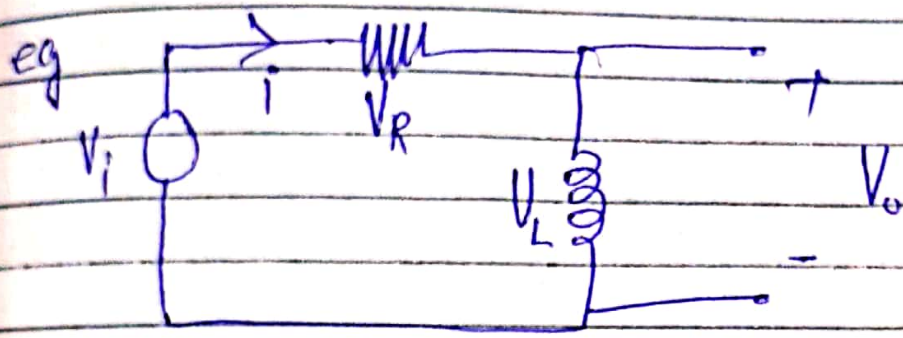
Lecture 2

Transfer functions / System Response / Behavior etc

$$T.F = \frac{\text{Laplace transform of output}}{\text{Laplace transform of input}}$$

with the initial conditions as zero.

eg



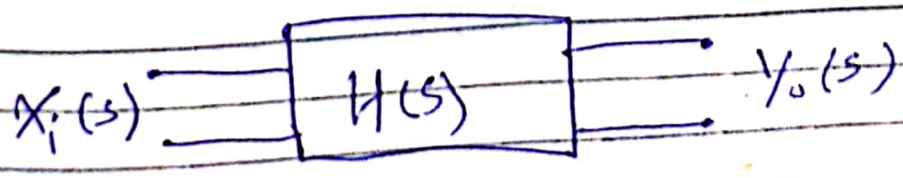
$$T.F = \frac{V_o}{V_i} \quad \text{voltage gain.}$$

$$T.F. = \frac{I_o}{I_i} \quad \text{current gain}$$

$$\frac{V_o}{I_i} = \text{Transfer impedance}$$

$$\frac{I_o}{V_i} = \text{Transfer admittance}$$

$$H(s) = \frac{\text{output}}{\text{input}} = \frac{Y_o(s)}{X_i(s)}$$



$$\frac{Y_o(s)}{X_i(s)} = \frac{a_m s^n + a_{m-1} s^{n-1} + a_{m-2} s^{m-2} + \cdots + a_1 s [illegible]}{b_n s^n + b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \cdots + b_1 s + [illegible]}$$

The roots;

$$= \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_m)}$$

Two extremes

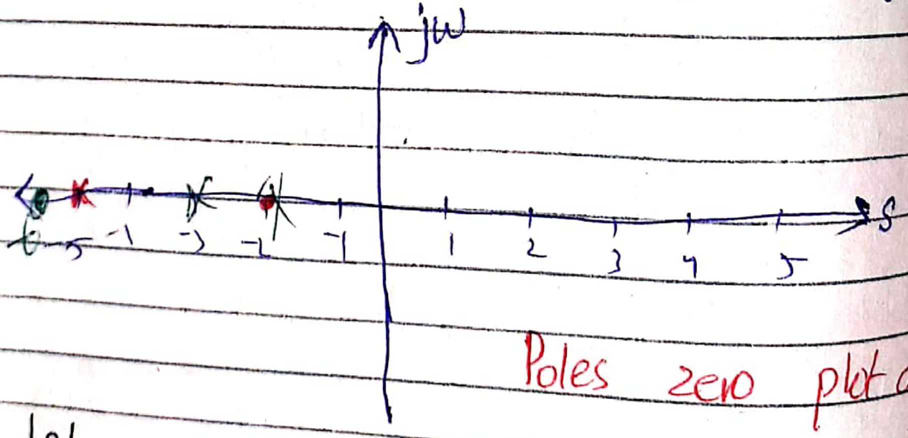
$z \rightarrow s$ zeros of the system (no respon[illegible]

$p \rightarrow s$ Poles of the system

let $H(s) = \dfrac{s+2}{s^2+5s}$

Zero $= -2 = -2 + 0jw$

Poles $= X = 0, -5 = 0, -5 + 0jw$



Poles zero plot dia[illegible]

let a system with

$$H(s) = \frac{s+6}{(s+2)(s+3)}$$

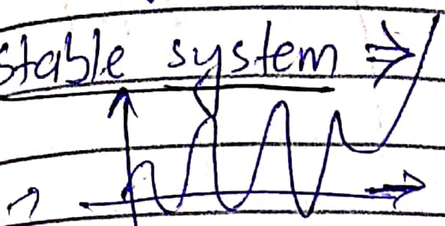
Zero $= -6$

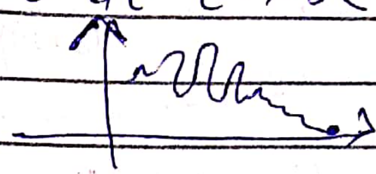
Poles $= -2, -3$

Pole zero diagram deals with the stability of the system.

Stability?
How system reacts to events.

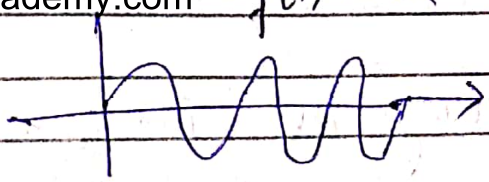
→ Stable system ⇒ $f(t) \to 0$ at $t \to \alpha$





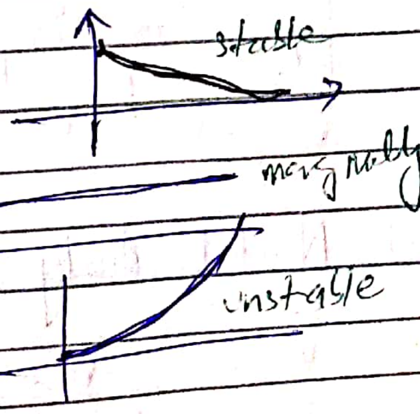
→ Unstable system
⇒ $f(t) \to \alpha$ at $t \to \alpha$

→ Marginally stable
oscillating system $0 < f(t) < \alpha$



$$H(s) = \frac{1}{s+a}$$

$$L^{-1}[H(s)] = h(t) = e^{-at}$$

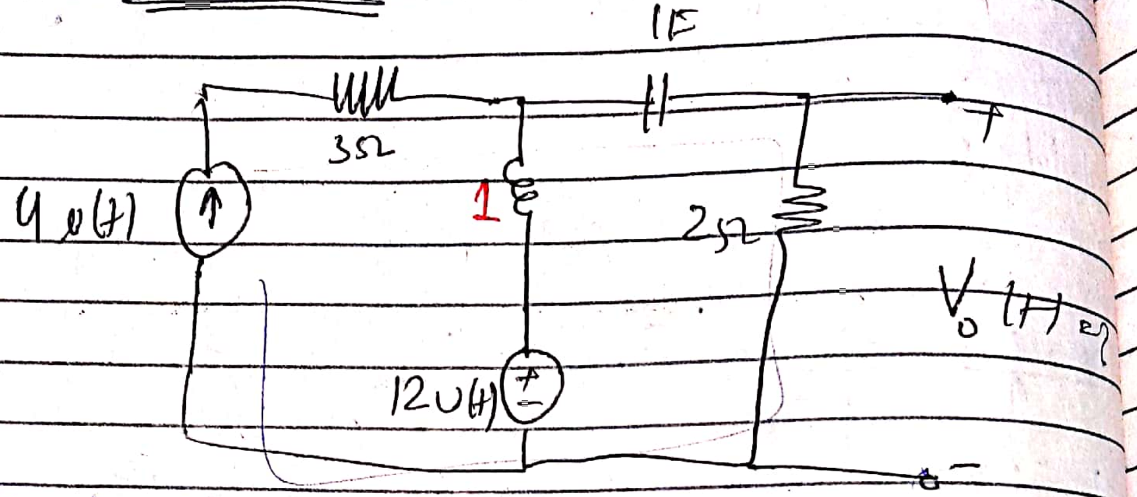


if $a > 0$ $h(t) = e^{-at}$

if $a = 0$ $h(t) = 1$

if $a < 0$ $h(t) = e^{at}$

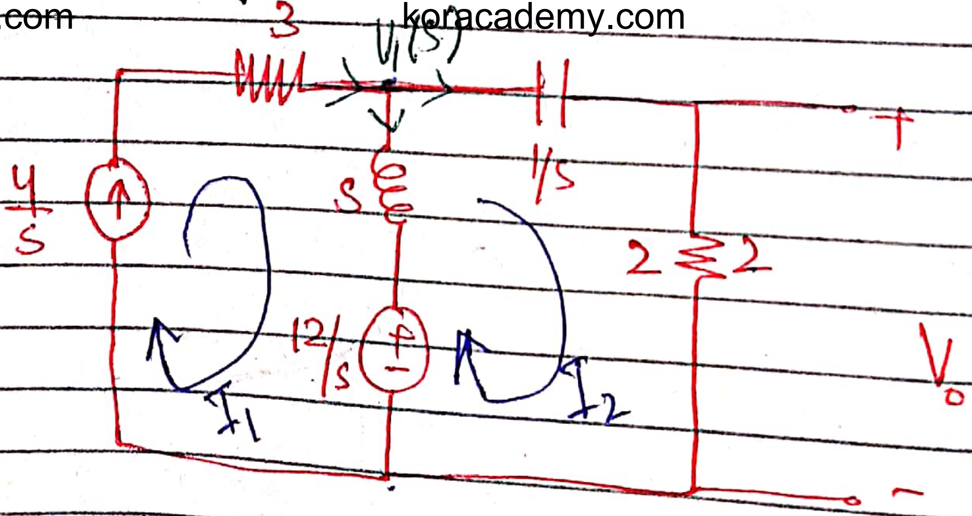
Ex 14.3



For $t<0$ $\quad i_L(0) = 0 \quad V_C(0) = 0$

$u(t)$ means start at $t=0$.
$u(t-2)$ means start at $t=2$. → delay
$u(t+2)$ means start at $t=-2$ → advance

Drawing the circuit is s domain;



① Nodal Analysis

KCL

$$\frac{4}{s} = \frac{V_1(s)}{2 + 1/s} + \frac{V_1(s) - 12/s}{s}$$

$$\frac{4}{s} = V_1(s)\left(\frac{1}{2+1/s} + \frac{1}{s}\right) - \frac{12}{s^2}$$

$$\frac{4}{s} + \frac{12}{s^2} = V_1(s)\left(\frac{s}{2s+1} + \frac{1}{s}\right)$$

$$\frac{4s+12}{s^{\cancel{2}}} = V_1(s)\left(\frac{s^2+2s+1}{\cancel{s}(2s+1)}\right)$$

$$V_1(s) = \frac{(4s+12)(2s+1)}{s(s^2+2s+1)}$$

$$V_0(s) = V_1(s) \times \frac{2}{2+1/s}$$

$$= \frac{4(s+3)\cancel{(2s+1)}}{s(s^2+2s+1)} \times \frac{2}{\cancel{(2s+1)}}$$

$$V_0(s) = \frac{8(s+3)}{s^2+2s+1}$$

$$V_0(s) = \frac{8(s+3)}{(s+1)^2}$$

Mesh is the smallest possible loop.

② Mesh Analysis

$$I_1 = 4/s$$

$$\frac{12}{s} = -sI_1 + \left(s+2+\frac{1}{s}\right)I_2$$

$$\frac{12}{s} = -s \times \frac{4}{s} + \left(\frac{s^2+2s+1}{s}\right)I_2$$

$$I_2 = \frac{4(s+3)}{(s+1)^2}$$

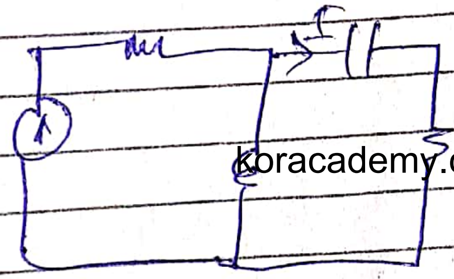
$$V_0(s) = 2I_2(s)$$

$$\Rightarrow V_0(s) = \frac{8(s+3)}{(s+1)^2}$$

(3) Superposition theorem

If you have more than one sources in a circuit; consider only one source at a time and make all other sources zero (I source = open ckt and V source = short circuit) and then finally the answer (output) will be sum of all the answers (outputs).

(a)

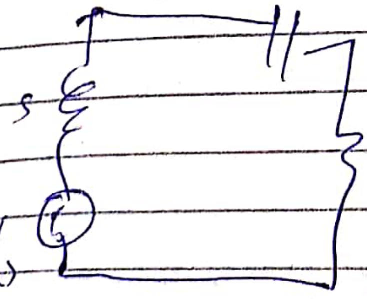


$$V_0 = 2I_1$$

$$= \frac{2\times 4}{s} \times \frac{s}{s+2+1/s}$$

$$V_{0_1} = \frac{8s}{s^2+2s+1}$$

(b)



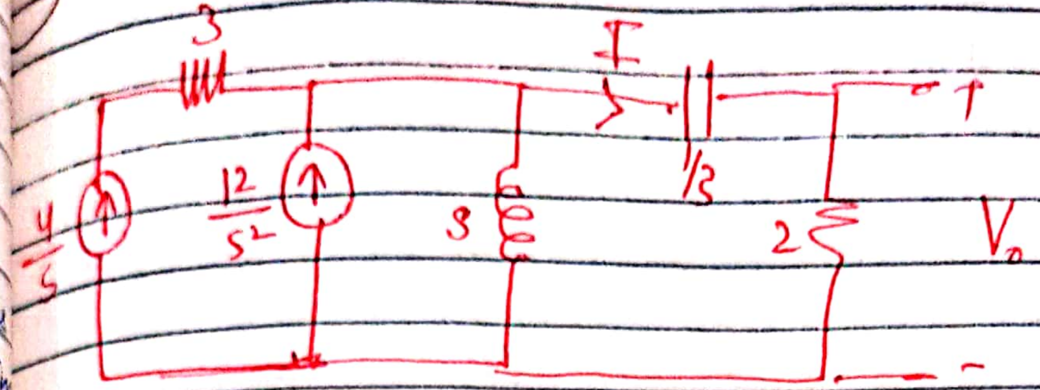
$$V_0(s) = \frac{12}{s} \times \frac{2}{s+2+1/s}$$

$$= \frac{24s}{s(s^2+2s+1)}$$

$$V_{0_2}(s) = \frac{24}{s^2+2s+1}$$

$$V_0(s) = V_{0_1}(s) + V_{0(2)}s = \frac{8(s+3)}{(s+1)^2}$$

4) Source Conversion



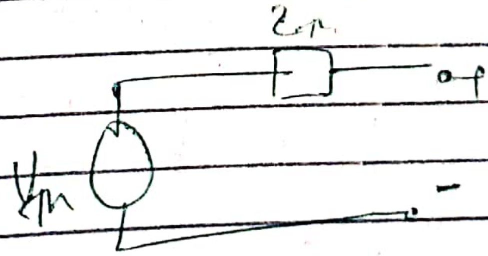
$$V_o(s) = 2I$$

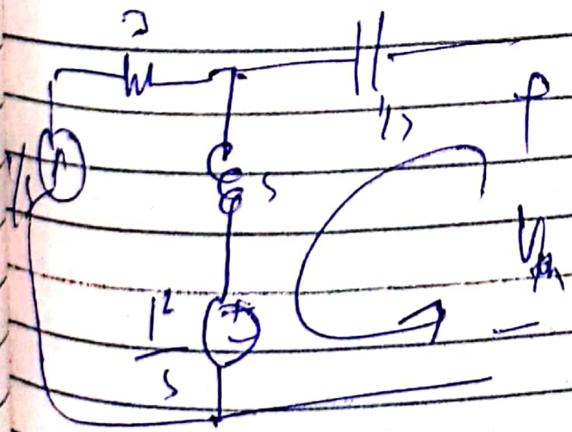
$$= 2\left(\frac{4}{s} + \frac{12}{s^2}\right)\left(\frac{s}{s+2+1/s}\right)$$

$$= 2\left(\frac{4s+12}{\cancel{s^2}}\right)\left(\frac{\cancel{s^2}}{s^2+2s+1}\right)$$

$$V_o(s) = \frac{8(s+3)}{(s+1)^2}$$

5) Thevinin theorem



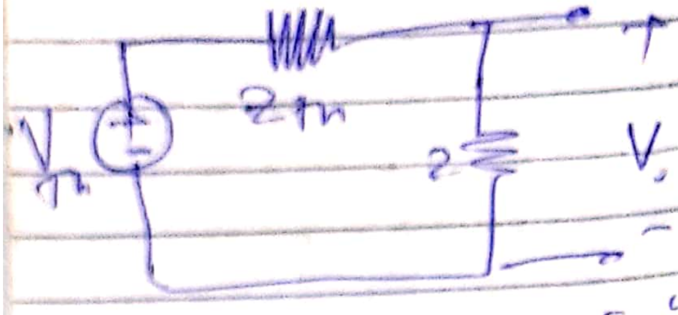


$$V_{th} = s \times \frac{4}{s} + \frac{12}{s}$$

$$V_{th} = 4 + \frac{12}{s}$$

$$V_{th} = \frac{4s+12}{s}$$

$$Z_{th} = s + \frac{1}{s} = \frac{s^2+1}{s}$$



$$V_o = V_{th} \times \frac{2}{2+Z_{th}}$$

$$= \frac{4s+12}{s} \times \frac{2}{2+\frac{s^2+1}{s}}$$

$$= \frac{4s+12}{s} \times \frac{2s}{s^2+2s+1}$$

$$V_o(s) = \frac{8(s+3)}{(s+1)^2}$$

Norton then

Changing to time domain

$$V_o(s) = \frac{8(s+3)}{(s+1)^2} = \frac{K_1}{(s+1)^2} + \frac{K_2}{(s+1)}$$

$$K_1 = V_o(s) \cdot (s+1)^2 \Big|_{s=-1}$$

$$= 8(s+3) \quad \Rightarrow K_1 = 16$$

$$K_2 = \frac{1}{1!} \times \frac{d}{ds}\left(V_o(s) \cdot (s+1)^2\right)\Big|_{s=-1}$$

$$K_n = \frac{1}{(n-1)!} \frac{d^{n-1}}{ds^{n-1}}\left(V_o(s) \cdot (s+k)^n\right)\Big|_{s=-k}$$

$$k_2 = \frac{d}{ds}\left(8(s+3)\right)\Big|_{s=-1} \Rightarrow k_2 = 8$$

$$V_o(s) = \frac{16}{(s+1)^2} + \frac{8}{(s+1)}$$

$$V_o(t) = \left(16te^{-t} + 8e^{-t}\right)u(t) \text{ volts.}$$

Ex 14.5   Ex 14.13

Lecture 3

$$H(s) = \frac{s+10}{s^2+4s+8}$$

$$V_o(s) = H(s)\,V_i(s) = \frac{s+10}{s(s^2+4s+8)} = \frac{s+10}{s(s+2-j2)(s+2+j2)}$$

$$V_o(s) = \frac{k_1}{s} + \frac{k_2}{s+2-j2} + \frac{k_3}{s+2+j2}$$

$$k_1 = 5/4 \quad k_2 = -5/8 + j3/8 \quad k_3 = k_2^* = -5/8 - j3/8$$

$$V_o(s) = \frac{5/4}{s} + \frac{-5/8 + j3/8}{s+2-j2} + \frac{(-5/8 - j3/8)}{s+2+j2}$$

$$V_o(t) = \frac{5}{4} + \left(\frac{-5}{8} + j\frac{3}{8}\right)e^{-(2-j2)t} + \left(\frac{-5}{8} - j\frac{3}{8}\right)e^{-(2+j2)t}$$

$$= \frac{5}{4} - e^{-2t}\left[\frac{5}{8}e^{j2t} - j\frac{3}{8}e^{j2t} + \frac{5}{8}e^{-j2t} + j\frac{3}{8}e^{-j2t}\right]$$

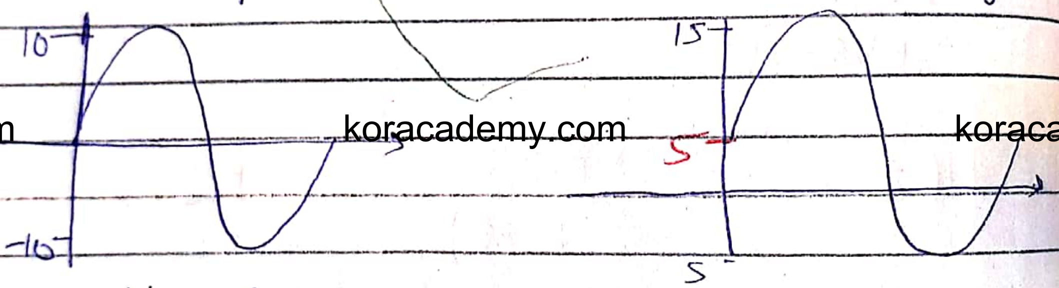
$$= \frac{5}{4} - e^{-2t}\left[\frac{5}{4}\left(\frac{e^{j2t}+e^{-j2t}}{2}\right) + \frac{3}{4}\left(\frac{e^{j2t}-e^{-j2t}}{j2}\right)\right]$$

$$V_0(t) = \left[\frac{5}{4} + e^{-2t}\left\{\frac{5}{4}\cos 2t + \frac{3}{4}\sin 2t\right\}\right]u(t)$$

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$

$\downarrow(\overline{\cos})$

Synthesis of wave Building up of a wave from its harmonics is called synthesis



$V = 10\sin\omega t$

Fundamental + DC → unsymmetrical
Fundamental + even → unsymmetrical
Fundamental + odd harmonics → symmetrical
For symmetrical wave $a_0 = 0$ even harmonics $= 0$

Average Value of Non sinusoidal EMF or current

$$e = E_0 + E_{m1}\sin(\omega t + \phi_1) + E_{m2}\sin(\omega t + \phi_2) + \cdots$$

$$\boxed{\text{avg}(e) = E_0} + 0 + 0 + \cdots$$

Average value of any parameter is the constant (dc) part of it.

Effective or RMS value of Non sinosidal waveform

$E = \text{Rms}(e)$

$$E = \sqrt{\frac{1}{T}\int_0^T e^2 dt} \quad \text{or} \quad E = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} e^2 d\omega t}$$

$$E = \frac{1}{2\pi}\int_0^{2\pi}\sqrt{\left(E_0 + E_{m1}\sin(\omega t + \phi_1) + E_{m2}\sin(\omega t + \phi_2) + \dots\right)^2} \, d\omega t$$

$$\Rightarrow \boxed{E = \sqrt{E_0^2 + E_1^2 + E_2^2 + \dots\dots}}$$

$$E_1 = \frac{E_{m1}}{\sqrt{2}}, \quad \frac{E_{m2}}{\sqrt{2}} = E_2, \dots\dots$$

Example

$i = 10 + 100\sqrt{2}\sin\omega t + 50\sqrt{2}\sin(3\omega t + 60^\circ) - 10\sqrt{2}(\sin 7\omega t - 210)$

$I_{avg} = ?$ $\quad I_{rms} = ?$ $\quad I_{avg} = 10\text{ A}$

$$I_{rms} = \sqrt{I_0^2 + I_1^2 + I_2^2 + \dots}$$

$$= \sqrt{(10)^2 + (100)^2 + (50)^2 + (-10)^2}$$

$$\Rightarrow I_{rms} = 112.7\text{ A}.$$

Power Calculation

$e = E_0 + E_{m1}\sin(\omega t + \phi_1) + E_{m2}\sin(2\omega t + \phi_2) + E_{m3}\sin(3\omega t + \phi_3 + \dots\dots)$

$i = I_0 + I_{m1}\sin(\omega t + \theta_1 + \beta_1) + I_{m2}\sin(2\omega t + \theta_2 + \beta_2) + I_{m3}\sin(3\omega t + \theta_3 + \beta_3) + \dots\dots$

Instantaneous power ; $p = ei$

Reading of voltmeter = Average power.

$$P = \frac{1}{2\pi}\int_0^{2\pi} p\,d\omega t$$

or $$P = \frac{1}{2\pi}\int_0^{2\pi} ei\,d\omega t$$

$P = VI\cos\phi$

If voltage and current both sources are sinusoidal ⇒

Applying superposition theorem;

$$P = E_0 I_0 + E_1 I_1 \cos\beta_1 + E_2 I_2 \cos\beta_2 + E_3 I_3 \cos\beta_3 + \ldots$$

$$E_1 = \frac{E_{m1}}{\sqrt{2}} \qquad I_1 = \frac{I_{m1}}{\sqrt{2}} \quad \ldots$$

Example

$e = 20 + 100\sqrt{2}\sin\omega t + 50\sqrt{2}\sin(3\omega t + 90) - 10\sqrt{2}\sin(5\omega t$

$i = 2 + 10\sqrt{2}\sin(\omega t - 30^\circ) + 5\sqrt{2}\sin(5\omega t - 270^\circ)$

$P = (20 \times 2) + (100 \times 10 \cos 30^\circ) + (0) + ((-10)(5)\cos(-$

$\Rightarrow \underline{P = 881\ W}$

*** Analysis of Waves By Fourier Series.**

Generally Fourier series is represented as;

$$f(\omega t) = a_0 + a_1\cos\omega t + a_2\cos 2\omega t + \ldots + a_n\cos n\omega t$$
$$+ b_1\sin\omega t + b_2\sin 2\omega t + \ldots + b_n\sin n\omega t$$

Analytical method.

Determination of constant term (a_0)

Multiply eq ① with $d\omega t$ and integrate over period i.e. $\int_0^{2\pi}$

$$\int_0^{2\pi} f(\omega t)\, d\omega t = \int_0^{2\pi} a_0\, d\omega t + \int_0^{2\pi} a_1 \cos\omega t\, d\omega t + \int_0^{2\pi} a_2 \cos 2\omega t\, d\omega t + \cdots + \int_0^{2\pi} a_n \cos n\omega t\, d\omega t + \int_0^{2\pi} b_1 \sin\omega t\, d\omega t + \int_0^{2\pi} b_2 \sin 2\omega t\, d\omega t + \cdots + \int_0^{2\pi} b_n \sin n\omega t\, d\omega t$$

average value of Sin and Cos = 0.

$$\int_0^{2\pi} f(\omega t)\, d\omega t = a_0 (2\pi) \Rightarrow \boxed{a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\omega t)\, d\omega t}$$

<u>Determination of cos terms (a_n)</u>

multiply eq (1) with ($\cos n\omega t\, d\omega t$) and integrate $\int_0^{2\pi}$.

$$\int_0^{2\pi} f(\omega t) \cos n\omega t\, d\omega t = \int_0^{2\pi} a_0 \cos n\omega t\, d\omega t + \int_0^{2\pi} a_1 \cos\omega t \cdot \cos n\omega t\, d\omega t + \cdots + \int_0^{2\pi} a_n \cos^2 n\omega t\, d\omega t + \int_0^{2\pi} b_1 \sin\omega t \cos n\omega t\, d\omega t$$

$$+ \int_0^{2\pi} b_2 \sin 2\omega t \cos n\omega t\, d\omega t + \cdots + \int_0^{2\pi} b_n \sin n\omega t \cos n\omega t\, d\omega t$$

$$\int_0^{2\pi} f(\omega t) \cos n\omega t\, d\omega t = a_n \int_0^{2\pi} \left(\frac{1 + \cos 2n\omega t}{2} \right) d\omega t$$

As $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ and $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$\rightarrow = \frac{a_n}{2}\left((\omega t)\Big|_0^{2\pi} + 0 \right) = \frac{a_n}{\cancel{2}}(\cancel{2}\pi) = a_n(\pi)$$

$$\boxed{a_n = \frac{1}{\pi}\int_0^{2\pi} f(\omega t) \cos(n\omega t)\, d\omega t}$$

Determination of sine terms (b_n)

Xing eq(1) with $\sin n\omega t \, d\omega t$ and integrate $\int_0^{2\pi}$

$$\int_0^{2\pi} f(\omega t).\sin n\omega t \, d\omega t = \int_0^{2\pi} a_0 \sin n\omega t \, d\omega t + \int_0^{2\pi} a_1 \cos\omega t \sin n\omega t \, d\omega t$$

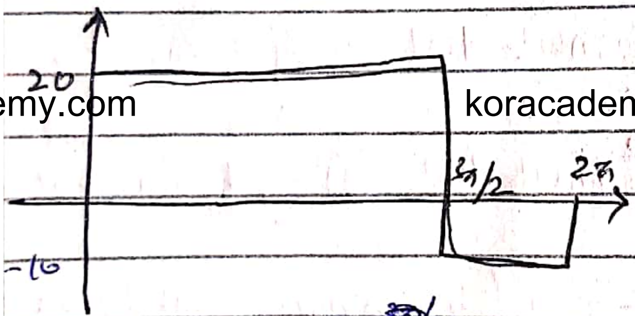
$$\cdots + \int_0^{2\pi} a_n \cos n\omega t \sin n\omega t \, d\omega t + \cdots + \int_0^{2\pi} b_n \sin^2 n\omega t \, d\omega t$$

$$\Rightarrow \int_0^{2\pi} f(\omega t) \sin n\omega t \, d\omega t = \frac{b_n}{n} \int_0^{2\pi} (1 - \cos 2n\omega t) \, d\omega t$$

$$= \frac{b_n}{2} (2\pi - 0) = b_n(\pi)$$

$$\boxed{b_n = \frac{1}{\pi} \int_0^{2\pi} f(\omega t) \sin n\omega t \, d\omega t}$$

Q. One cycle of a periodic rectangular wave is shown in figure. Find the first few harmonics.



Mathematical model = ?

$$f(\omega t) = \begin{cases} 20, & 0 \le \omega t < 3\pi/2 \\ -10, & 3\pi/2 < \omega t < 2\pi \end{cases}$$

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$$a_0 = \frac{1}{2\pi} \left(\int_0^{2\pi} f(\omega t) \, d\omega t \right)$$

$$a_0 = \frac{1}{2\pi} \left(\int_0^{3\pi/2} 20 \, d\omega t + \int_{3\pi/2}^{2\pi} (-10) \, d\omega t \right)$$

$$= \frac{1}{2\pi} \left(20 (\omega t) \Big|_0^{3\pi/2} - 10 (\omega t) \Big|_{3\pi/2}^{2\pi} \right)$$

$$= \frac{1}{2\pi} \left(20 \left(\frac{3\pi}{2} - 0 \right) - 10 \left(2\pi - \frac{3\pi}{2} \right) \right) = \frac{1}{2\pi} (30\pi - 5$$

$$\boxed{a_0 = 12.5}$$

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$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(wt)\cos nwt \cdot dwt$$

a_n

$$a_n = \frac{1}{\pi}\left(\int_0^{3\pi/2} 20\cos nwt\, dwt + \int_{3\pi/2}^{2\pi} (-10)\cos nwt\, dwt\right)$$

$$= \frac{1}{\pi}\left(20\frac{\sin nwt}{n}\Big|_0^{3\pi/2} - 10\frac{\sin nwt}{n}\Big|_{3\pi/2}^{2\pi}\right).$$

$$= \frac{1}{n\pi}\left(20\sin n(3\pi/2) - \cancel{\sin 0}\right) - 10\left(\cancel{\sin n(2\pi)} - \sin n\,3\pi/2\right)$$

$$= \frac{1}{n\pi}\left(30\sin\frac{n3\pi}{2}\right)$$

$n = 2,4,6,$ -- even $\quad a_n = 0$.

$n = 1,3,5$ -- odd $\quad a_1 = -9.54 \quad a_3 = 3.18$

b_n

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(wt)\sin nwt\, dwt$$

$$b_n = \frac{1}{\pi}\left[\int_0^{3\pi/2} 20\sin nwt\, dwt + \int_{3\pi/2}^{2\pi} (-10)\sin nwt\, dwt\right]$$

$$= \frac{1}{\pi}\left[\frac{-20}{n}\cos nwt\Big|_0^{3\pi/2} + \frac{10}{n}\cos nwt\Big|_{3\pi/2}^{2\pi}\right]$$

$$= \frac{1}{n\pi}\left[-20\left(\cos n\frac{3\pi}{2} - 1\right) + 10\left(1 - \cos\left(n\,3\pi/2\right)\right)\right].$$

$$= \frac{1}{n\pi}\left[-20\cos\left(\frac{n3\pi}{2}\right) + 20 + 10 - 10\cos\left(\frac{n3\pi}{2}\right)\right].$$

$$b_n = \frac{1}{n\pi}\left[30 - 30\cos\left(\frac{n3\pi}{2}\right)\right].$$

$$b_1 = \frac{1}{\pi}(30) = 30/\pi$$

$$b_2 = \frac{1}{2\pi}(30 + 30) = \frac{30}{\pi}.$$

$$b_3 = \frac{10}{\pi}.$$

Example 23.6

Effect of Symmetry On Fourier Series

① **Constant term = 0**

If a wave has equal area above and below the horizontal axis [in a] complete cycle then a_0 will be zero.

② **Half wave symmetry** $f(\omega t) = -f(\omega t + \pi)$

$a_0 = 0$ and all the even harmonics will be zero

③ **Odd function** $f(\omega t) = -f(-\omega t)$

eg Sin wave is an odd function. If a given function is odd then a_0 and all cosine terms will be zero.

④ **Even function** $f(\omega t) = f(-\omega t)$ eg cos function.

(An odd function must always pass through the origin).

If a function is even, then all of sine terms are zero.

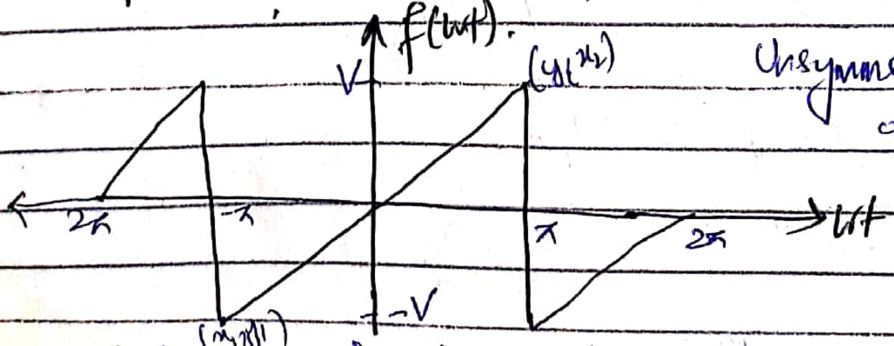
The limits of integration are taken over one time period (not necessarily 0 to 2π, eg may be $-\pi$ to π).

If we want to integrate w.r.t T;

$$a_0 = \frac{1}{T}\int_0^{t+T} f(\omega t)\,dt \qquad a_n = \frac{2}{T}\int_0^{t+T} f(\omega t)\cos n\omega t\,dt$$

$$b_n = \frac{2}{T}\int_t^{t+T} f(\omega t)\cdot \sin n\omega t\, d\omega t$$

Example Find the fourier series of the sawtooth wave



Unsymmetrical.
odd function
$a_0 = 0$
$a_n = 0$

Taking the interval $-\pi$ to π. ie $-\pi \le \omega t \le \pi$

$$f(\omega t) = mx + c$$

$$= \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(\omega t) + c$$

$$= \frac{V-(-V)}{\pi-(-\pi)}(\omega t) + 0 \Rightarrow \boxed{f(\omega t) = \frac{V}{\pi}(\omega t)}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\omega t)\cdot \sin n\omega t \, d\omega t$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{V}{\pi}(\omega t)\sin(n\omega t)\, d\omega t$$

$$= \frac{V}{\pi^2}\int_{-\pi}^{\pi}(\omega t)\sin n\omega t \, d\omega t \qquad \int ab = a\int b - \int a'\int b$$

$$= \frac{V}{\pi^2}\left[-(\omega t)\cdot\frac{\cos n\omega t}{n} + \int\frac{\cos n\omega t}{n}\, d\omega t\right]_{-\pi}^{\pi}$$

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$$b_n = \frac{V}{n\pi^2}\left[-(\omega t)\cos n\omega t + \sin\left(\frac{n\omega t}{n}\right)\right]\Big|_{-\pi}^{\pi}$$

$$b_n = \frac{V}{n\pi^2}\left[\left(-\pi\cos n\pi + \cancel{\sin\frac{n\pi}{n}}^{0}\right) - \left(\pi\cos n\pi - \cancel{\frac{\sin n\pi}{n}}^{0}\right)\right]$$

$$b_n = \frac{V}{n\pi^2}\left[-\pi\cos n\pi - \pi\cos n\pi\right] = \frac{V}{n\pi^2}(-2\pi\cos n\pi)$$

$$b_n = \frac{-2V}{n\pi}\cos n\pi$$

$n = 1, 3, 5, \ldots$ odd $\Rightarrow b_n = \frac{2V}{n\pi}$

$n = 2, 4, 6 \ldots$ even $\Rightarrow b_n = -\frac{2V}{n\pi}$

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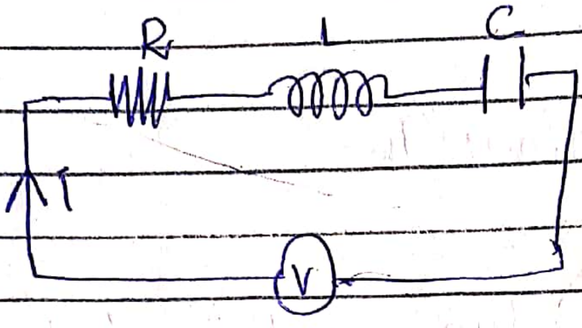
Lecture 4

Application of Non Sinusidal waves to linear cir

Superposition theorem is only for linear elements.

→ **Relationship for voltage in RLC series circuit if current is non sinosidal.**



No dc component current (i_0) $\frac{1}{2}$ c of

$$i = i_1 + i_2 + i_3 + \dots$$

$$i = I_{m1} \sin(\omega t + \theta_1) + I_{m2} \sin \dots + \dots + I_{mn} \sin(m\omega t + \dots$$

$X_L = 2\pi f L$ $\quad X_C = 1/2\pi f C$ $\quad Z = \sqrt{R^2 + (X_L - X_C)^2}$

$V = IZ$ $\quad \beta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

$V\angle\phi = I\angle\theta \; Z\angle\beta \Rightarrow V\angle\phi = IZ\angle\theta + \beta$

$$V = I_{m1} Z_1 \sin(\omega t + \theta_1 + \beta_1) + I_{m2} Z_2 \sin(2\omega t + \theta_2 + \beta_2) + \dots + I_{mn} Z_n \sin(n\omega t + \theta_n + \beta_n)$$

$$Z_1 = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad \beta_1 = \tan^{-1}\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

$$Z_2 = \sqrt{R^2 + \left(2\omega L - \frac{1}{2\omega C}\right)^2} \qquad \beta_2 = \tan^{-1}\frac{\left(2\omega L - \frac{1}{2\omega C}\right)}{R}$$

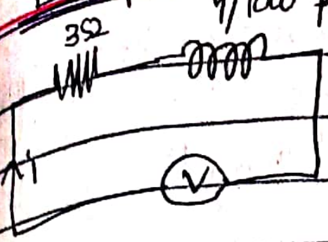
$$Z_n = \sqrt{R^2 + \left(n\omega L - \frac{1}{n\omega C}\right)^2} \qquad \beta_n = \tan^{-1}\frac{\left(n\omega L - \frac{1}{n\omega C}\right)}{R}$$

$X_C > X_L \Rightarrow$ capacitive $\Rightarrow$ current leads voltage

$X_L > X_C \Rightarrow$ inductive $\Rightarrow$ voltage leads current.

If for the first harmonic, circuit is inductive then it will be inductive for all the rest harmonics

Example



$i = 10 + 100\sin(100t + 30^\circ) + 50\sin(300t + 90^\circ)$

$\omega = 1000 \, r/s$ (a) Find V

(b) Determine I_{rms}, V_{rms} (c) $P_{avg} = ?$

$V = 10Z_0 + 100Z_1 \sin(100t + 30^\circ + \beta_1) + 50Z_2 \sin(300t + 90^\circ + \beta_2)$

$Z_0 = R = 3\Omega$ $\quad Z_1 = \sqrt{R^2 + \omega L^2} = \sqrt{3^2 + 4^2} = 5\Omega$

$\beta_1 = \tan^{-1} \omega L / R = 4/3 = 53.13^\circ$ $\quad Z_3 = \sqrt{R^2 + (3\omega L)^2} = \sqrt{3^2 + (3\times4)^2}$

$\beta_3 = \tan^{-1} \frac{3\times4}{3} = 76^\circ$ $\quad = 12.4\Omega$

$V = 10\times3 + 100\times5\sin(100t + 30^\circ + 53.13^\circ) + 50\times12.4\sin(300t + 90^\circ + 76^\circ)$

$V = 30 + 500\sin(100t + 83.13^\circ) + 620\sin(300t + 166^\circ)$ Ans

(b) $I_{rms} = \sqrt{I_0^2 + I_1^2 + I_3^2}$

$= \sqrt{(10)^2 + (100/\sqrt{2})^2 + (50/\sqrt{2})^2} \Rightarrow I_{rms} = 79.7 \, A$

$V_{rms} = \sqrt{V_0^2 + V_1^2 + V_3^2}$

$= \sqrt{(30)^2 + (500/\sqrt{2})^2 + (620/\sqrt{2})^2} = 565 \, W.$

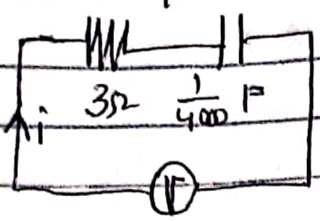
(c) $P_{avg} = I_{rms}^2 R$ or $\frac{V_{rs}^2}{R}$ $\quad$ ($I_{rs}^3 V_{rs}^2$ gives apparent power)

$P_{avg} = 79.7^2 \times 3 = 19kW$

(or) $P_{avg} = (10)(30) + \left(\frac{500}{\sqrt{2}}\right)\left(\frac{100}{\sqrt{2}}\right)\cos 53.13 + \left(\frac{620}{\sqrt{2}}\right)\left(\frac{50}{\sqrt{2}}\right)\cos 76^\circ$

$= 19kW.$

Example



$i = 100\sin(100t + 30^\circ) + 50\sin(300t + \ldots$

$\omega = 1000$ r/s

(a) $V = ?$ (b) V_{rms}, $I_{rms} = ?$ (c) $P_{avg} = ?$

$$Z_1 = \sqrt{R^2 + \frac{1}{(\omega C)^2}} = 5\ \Omega \qquad \beta = \tan^{-1}\left(\frac{-1/\omega C}{R}\right) = -53.1^\circ$$

$-ve$ s/c $X_L - X_C$ and $X_L = 0$

$$Z_3 = \sqrt{R^2 + \left(\frac{1}{3\omega C}\right)^2} = 3.3\ \Omega \qquad \beta_3 = \tan^{-1}\left(\frac{-1/3\omega C}{R}\right) =$$

$$V = (100)(5)\sin(100t + 30^\circ - 53.1^\circ) + (50)(3.3)\sin(300t + \ldots$$

$$V = 500\sin(100t - 23.1^\circ) + 165\sin(300t + 66^\circ)\ V$$

(b) $$V_{rms} = \sqrt{\left(\frac{500}{\sqrt{2}}\right)^2 + \left(\frac{165}{\sqrt{2}}\right)^2} = 373\ V.$$

$$I_{rms} = \sqrt{\left(\frac{100}{\sqrt{2}}\right)^2 + \left(\frac{50}{\sqrt{2}}\right)^2} = 79\ A.$$

(c) $$P_{avg} = I_{rms}^2 R = 79^2 \times 3 = 18.7\ kW.$$

(or)

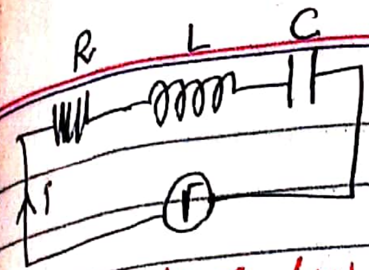
$$P_{avg} = \frac{500}{\sqrt{2}} \times \frac{100}{\sqrt{2}} \cos 53.1^\circ + \frac{165}{\sqrt{2}} \times \frac{50}{\sqrt{2}} \cos 24^\circ$$

$$= 18.7\ kW.$$

Current due to Non sinusoidal voltage

Current cannot have a dc term even if voltage has.

$-\beta$



$$Z_n = \sqrt{R^2 + \left(n\omega L - \frac{1}{n\omega C}\right)^2}$$

$$\beta_n = \tan^{-1}\left(\frac{n\omega L - \frac{1}{n\omega C}}{R}\right) + \theta_n$$

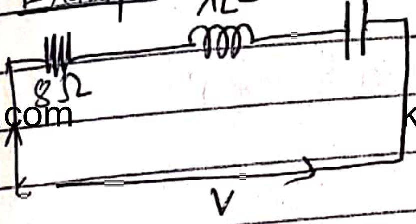
$$V = V_0 + V_{m1}\sin(\omega t + \theta_1) + V_{m2}\sin(2\omega t + \theta_2) + \dots + V_{mn}\sin(n\omega t$$

$$i = \frac{V_{m1}}{Z_1}\sin(\omega t + \theta_1 - \beta_1) + \frac{V_{m2}}{Z_2}\sin(2\omega t + \theta_2 - \beta_2) + \dots$$

$$\dots + \frac{V_{mn}}{Z_n}\sin(n\omega t + \theta_n - \beta_n)$$

$$i_0 = 0A \quad Z_0 = \infty\ \Omega$$

<u>Example</u>



$\omega = 1000\ r/s$

$V = 20 + 100\sin 100t + 50\sin(300t + 60^\circ)$

$I = ?$

$I_0 = 0A$ as $Z_0 = \infty\ \Omega$

$$Z_1 = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{8^2 + (2-8)^2} = 10\,\Omega$$

$$\beta_1 = \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right) = \tan^{-1}\left(\frac{2-8}{8}\right) = -36.8^\circ$$

capacitive

$$Z_3 = \sqrt{R^2 + \left(3\omega L - \frac{1}{3\omega C}\right)^2} = 8.65\,\Omega$$

$$\beta_3 = \tan^{-1}\left(\frac{3\omega L - 1/3\omega C}{R}\right) = 22.6^\circ \rightarrow \text{inductive}$$

$$i = \frac{100}{Z_1}\sin(100t - \beta_1) + \frac{50}{Z_3}\sin(300t + 60^\circ - \beta_3)$$

$$i = \frac{100}{10}\sin(100t + 36.8) + \frac{50}{8.65}\sin(300t + 60^\circ - 22.6^\circ)$$

$$i = 10\sin(100t + 36.8^\circ) + 5.78\sin(300t + 38.4^\circ)$$

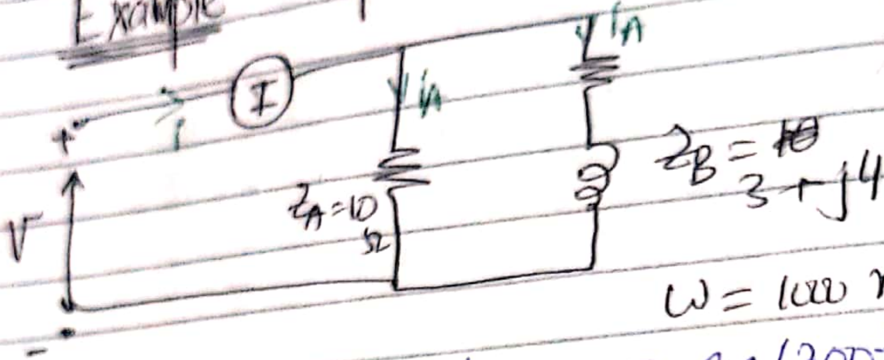
lead lagg Ans

Resonance in RLC series. $Z_n = R$ $I_n =$

$n\omega L = \frac{1}{n\omega C}$

AC ammeter reads RMS value

Example (parallel network)



$V = 20\sin(100t+10^\circ) + 100\sin(300t+30^\circ)$

$Z_B = 3 + j4$

$i = i_A + i_B$

$\omega = 100\ r/s$

$i_A = \frac{20}{10}\sin(100t+10^\circ) + \frac{100}{10}\sin(300t+30^\circ)$

$i_A = 2\sin(100t+10^\circ) + 10\sin(300t+30^\circ)$

For i_B

$Z_1 = \sqrt{3^2+4^2} = 5\Omega$ $\quad \beta_1 = \tan^{-1} 4/3 = 53.2^\circ$

$Z_3 = \sqrt{R^2 + (3\omega L)^2} = 12.4\Omega$ $\quad \beta_3 = \tan^{-1}\frac{3\omega L}{R} = \tan^{-1}\frac{3\cdot 4}{3}$

$= 76^\circ$

$i_B = \frac{20}{5}\sin(100t + 10^\circ - 53.2^\circ) + \frac{100}{12.4}\sin(300t + 30^\circ - 76^\circ)$

$i_B = 4\sin(100t - 43.2^\circ) + 8.08\sin(300t - 46^\circ)$

$i = i_A + i_B$

$i = 2\sin(100t+10^\circ) + 10\sin(300t+30^\circ) + 4\sin(100t-43.2^\circ)$
$+ 8.08\sin(300t - 46^\circ)$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$i = 48.9\sin 100t - 23.8\cos 100t + 14.28\sin 300t - 0.81\ldots$

$i = 54.4\sin(100t - 26^\circ) + 14.3\sin(300t - 3.25^\circ)$

$$I_{rms} = \sqrt{I_1^2 + I_3^2} = \sqrt{\left(\frac{54.4}{\sqrt{2}}\right)^2 + \left(\frac{14.3}{\sqrt{2}}\right)^2}$$

$$\boxed{I_{rms} = 30.7\,A}$$

$\rightarrow a_n \cos n\omega t + b_n \sin n\omega t = C_n \sin(n\omega t + \theta_n)$

$C_n = \sqrt{a_n^2 + b_n^2}$, $\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$.

Lecture 5

Filters

Electrical filter : frequency

① Active filters.

R, L, C, Op Amplifier.

② Passive filters.

R, L, C, diode.

- gain is low $A = 1$
- shaping of waves.
- Frequency selectivity.

Types (ideal)

→ Low pass filter

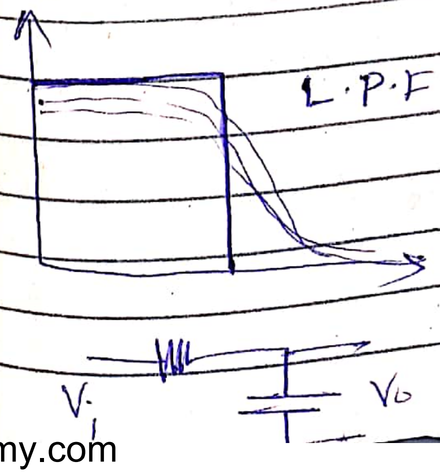
→ High pass filter.

→ Band pass filter

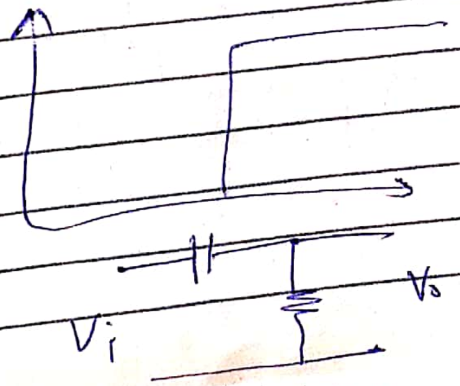
→ Band stop filter.

First order filters

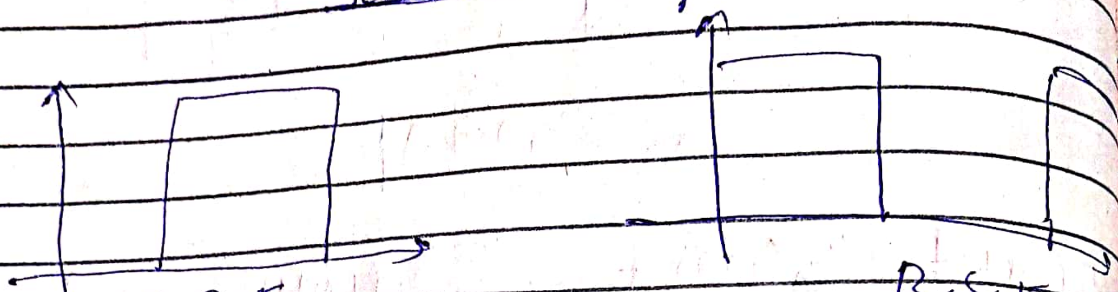
L.P.F.



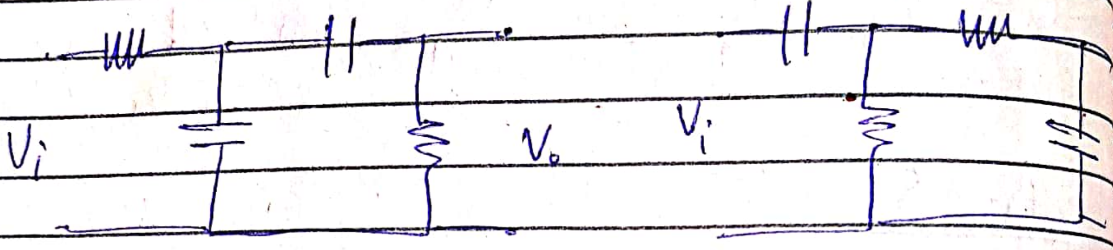
H.P.F



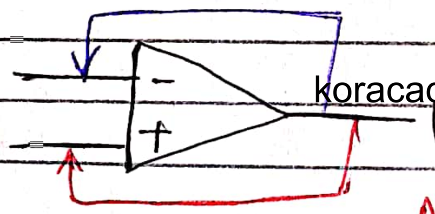
Second order filters



B.P.F B.S.F



Operational Amplifier (Op-Amp)

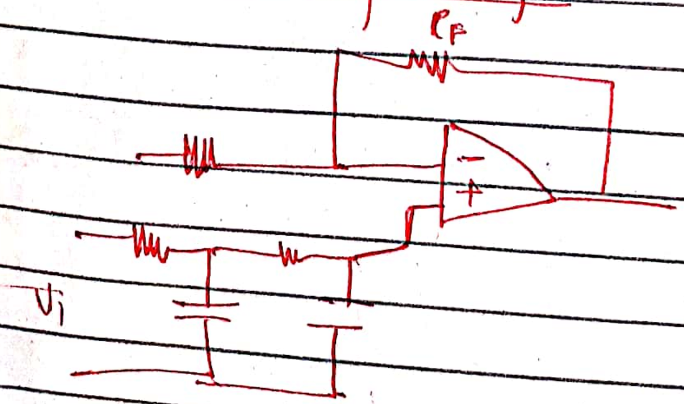


Inverting configuration $V_i = -V_o$

Non inverting op amp $V_o = V_i$

Characteristics
1) I/P impedence is α.
2) O/P impedence is 0.
3) Gain is infinity.

Active low pass filter.



K-1

using gain

$R = R_F$

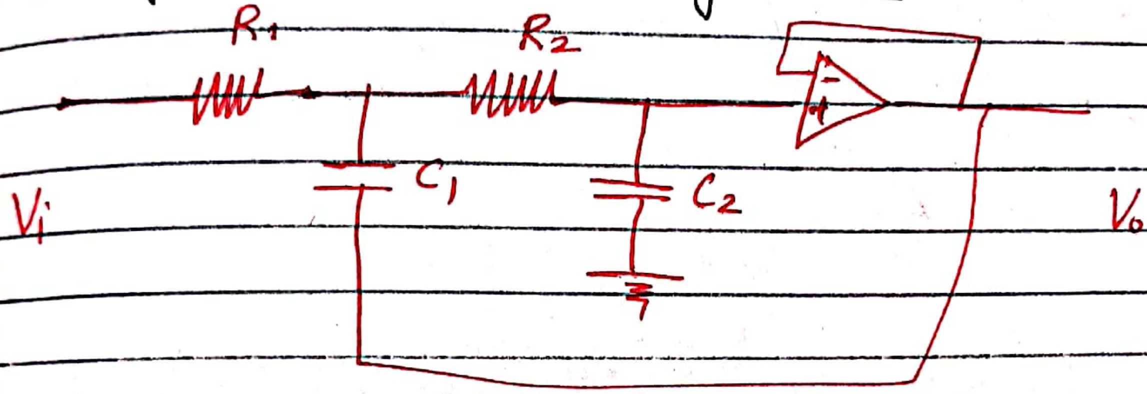
Irwin

Butterworth   Chybeshev   Inverse chybsheve

elliptic   Bessel

Lecture 6 :

Unity Gain Sallen Key Filter.

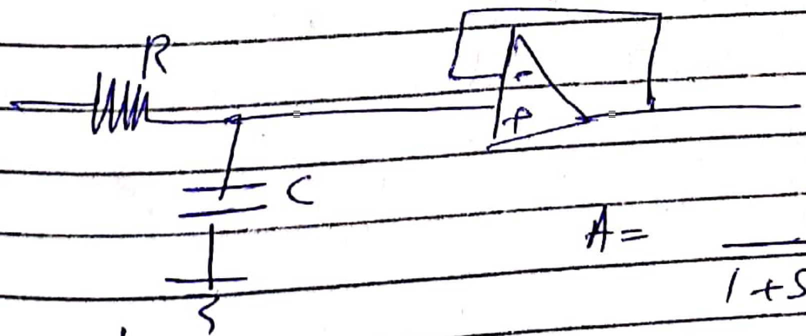


Normalization

$w = 1\ r/s$ $R_1 = R_2 = 1\,\Omega$.

$C_1 = 2Q$ $C_2 = \dfrac{1}{2Q}$

$$A = \frac{1}{s^2 + \frac{1}{Q}s + 1} \rightarrow S.O.F$$



$$A = \frac{1}{1 + SRC}$$

$W = \dfrac{1}{RC}$ Normalization $R = 1\,\Omega$, $w = 1\ r/s$.

$C = 1F$

$$A = \frac{1}{S+1} \rightarrow F.O.F$$

T.O.F. $A = \dfrac{1}{(s+1)\left(s^2+\frac{1}{Q}s+1\right)}$.

Fourth order F. $A = \dfrac{1}{\left(s^2+\frac{1}{Q_1}s+1\right)\left(s^2+\frac{1}{Q_2}s+1\right)}$

5th order F. $A = \dfrac{1}{(s+1)\left(s^2+\frac{1}{Q_1}s+1\right)\left(s^2+\frac{1}{Q_2}\right.}$

$$A = \frac{1}{B_n(s)}$$

$$B_n(s) = s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_1$$

n polynomials

1. $s+1$
2. $s^2+\sqrt{2}s+1$
3. s^3+2s^2+2s+1
4. $s^4 + 2.61313s^3 + 3.4142s^2 + 2.63105 + 1$

Characteristics

1. Poles lies on unity circle
2. The poles will have a angular separation $180°/n$, where n is order of the filter