

NUMERICAL PROBLEMS

P 20.1. Find the mass defect and binding energy for helium nucleus?

Solution

$\Delta m = ?$ $E_B = ?$ (for Helium nucleus)

Mass of proton $m_p = 1.00728$ a.m.u

Mass of neutron $m_n = 1.00867$ a.m.u

Mass of helium nucleus $M_{He} = 4.00125$ a.m.u

Helium has two protons ($Z = 2$) and its atomic mass A is 4. So it has $(A - Z)$ neutrons i.e,

$$N = 4 - 2 = 2$$

Now mass defect can be determined by

$$\Delta m = (Z m_p - N m_n) - M_{He}$$

$$\Delta m = (2 \times 1.00728 - 2 \times 1.00867) - 4.00152$$

$$\Delta m = 0.03038 \text{ a.m.u}$$

To calculate binding energy E_B , since $1 \text{ a.m.u} = 931.5 \text{ MeV}$

So $E_B = 0.03038 \times 931.5 \text{ MeV} = 28.3 \text{ MeV}$

P 20.2. A certain radioactive isotope has half-life of 8 hours. A solution containing 500 million atoms of this isotope is prepared. How many atoms of this isotope have not disintegrated after (a) 8 hours (b) 24 hours.

Solution

$T_{1/2} = 8 \text{ hrs}$ $N = 500 \text{ Million}$ (a) $N_1 = ?$ (atoms left after 8 hrs)

(b) $N_2 = ?$ (atoms left after 24 hrs)

(a) The given isotope has half-life of 8 hrs. So after 1st half-life, 250 Million will disintegrate and 250 Million will not disintegrate.

Hence $N_1 = 250 \text{ Million}$ are not disintegrated after 8 hrs.

(b) Now 24hrs means that 3 half-lives have passed. So the number N_2 of nuclei that are left un-decayed or those nuclei which are not disintegrated are $(\frac{1}{2}) (\frac{1}{2}) (\frac{1}{2}) = \frac{1}{8}$ of the total number.

So $N_2 = \frac{1}{8} \times 500 \text{ Million}$

$N_2 = 62.5 \text{ Million}$ are not disintegrated after 24 hrs

P 20.3. Write the nuclear equations for the beta decay of (a) ${}_{82}\text{Pb}^{210}$ (b) ${}_{83}\text{Bi}^{210}$ (c)

${}_{90}\text{Th}^{234}$ (d) ${}_{93}\text{Np}^{239}$

Solution

Nuclides having excess no. of neutrons decay through negative beta emission.



Nuclides having greater no. of protons decay through positive beta emission



For ${}_{82}\text{Pb}^{210}$ $Z = 82$ & $A = 210$ $\Rightarrow N = A - Z = 210 - 82 = 128$

Hence $N > Z$

For ${}_{83}\text{Bi}^{210}$ $Z = 83$ & $A = 210$ $\Rightarrow N = A - Z = 210 - 83 = 127$

Hence $N > Z$

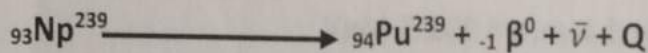
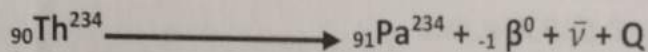
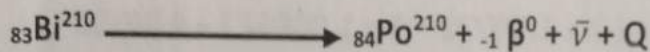
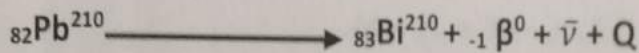
For ${}_{90}\text{Th}^{234}$ $Z = 90$ & $A = 234$ $\Rightarrow N = A - Z = 234 - 90 = 144$

Hence $N > Z$

For ${}_{93}\text{Np}^{239}$ $Z = 93$ & $A = 239$ $\Rightarrow N = A - Z = 239 - 93 = 146$

Hence $N > Z$

Thus all the nuclides decay through negative beta emission.



Where E shows the energy liberated and $\bar{\nu}$ are anti-neutrinos as they are always associated with negative beta emission.

P 20.4. Calculate the total energy released if 1kg of U^{235} undergoes fission? Taking the disintegration energy per event to be $Q = 208 \text{ MeV}$.

Solution

$E = ?$ $Q = 208 \text{ MeV per event}$ $m = 1\text{kg of } \text{U}^{235}$

Number of atoms in 1 kg of U^{235}

$$N = nN_A \quad (1)$$

Number of moles

$$n = \frac{\text{Given mass "m"}}{\text{Atomic mass "M"}} = \frac{1 \text{ kg}}{235 \frac{\text{kg}}{\text{kmol}}}$$

$n = 4.255 \times 10^{-3} \text{ k mole}$ putting in (1) we get

$$N = 4.255 \times 10^{-3} \text{ k mole} \times (6.023 \times 10^{23+3} \frac{\text{atoms}}{\text{kmol}})$$

$$N = 2.56 \times 10^{24} \text{ atoms}$$

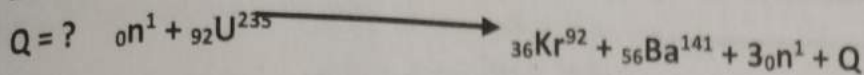
Now total energy liberated is

$$E = N Q = 2.56 \times 10^{24} (208 \text{ MeV})$$

$$E = 5.33 \times 10^{26} \text{ MeV}$$

P 20.5. Find the energy released in the following fission reaction?

Solution



Total mass on reactants side m_r = mass of one neutron + mass of U^{235}

$$m_r = 1.00867 + 235.0439$$

$$m_r = 236.05257 \text{ a.m.u}$$

Now total mass on products side m_{pro} is given by

$$m_{\text{pro}} = \text{mass of Kr}^{92} + \text{mass of Ba}^{141} + \text{mass of 3 neutrons}$$

$$m_{\text{pro}} = 91.8973 + 140.9139 + 3 (1.00867)$$

$$m_{\text{pro}} = 235.8372 \text{ a.m.u}$$

mass deficit

$$\Delta m = m_r - m_{\text{pro}} = 236.05257 - 235.8372$$

$$\Delta m = 0.21536 \text{ a.m.u}$$

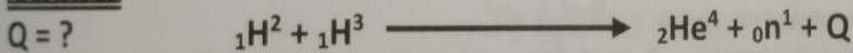
And the energy released during the reaction is

$$Q = \Delta m \times 931.5 \text{ MeV} = 0.21536 \times 931.5 \text{ MeV}$$

$$Q = 201 \text{ MeV}$$

P 20.6. Find the energy released in the fusion reaction.

Solution



Total mass on reactants side is

$$m_r = \text{mass of deuterium } {}_1^2\text{H} + \text{mass of tritium } {}_1^3\text{H}$$

$$m_r = 2.0141 + 3.01605$$

$$m_r = 5.03015 \text{ a.m.u}$$

Now total mass on products side is;

$$m_{\text{pro}} = \text{mass of one } \alpha\text{-particle} + \text{mass of one neutron}$$

$$m_{\text{pro}} = 4.00263 + 1.00867$$

$$m_{\text{pro}} = 5.0113 \text{ a.m.u}$$

$$\Delta m = m_r - m_{\text{prod}}$$

$$\Delta m = 0.01885 \text{ a.m.u}$$

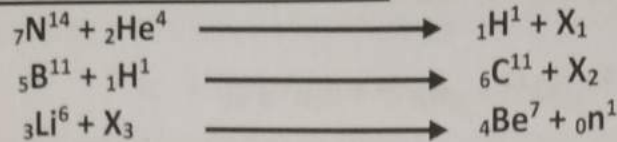
Now the energy released during the given fusion reaction is;

$$Q = \Delta m \times 931.5 \text{ MeV}$$

$$Q = 0.01885 \times 931.5 \text{ MeV}$$

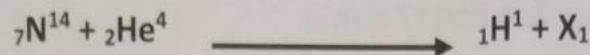
$$Q = 17.55 \text{ MeV}$$

P 20.7. Complete the following nuclear reactions.



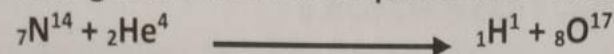
In all the equations, law of conservation of charge number (Z number) and law of conservation of mass number (A number) must be conserved.

First equation is

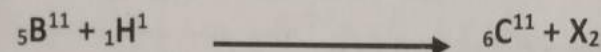


At the left hand side of the above equation total Z number is 9 whereas A number is 18. Now Z number on right hand side is 1 whereas A number is also 1. Hence the missing nucleus X_1 will have Z number $9 - 1 = 8$ while A number $18 - 1 = 17$ which is of oxygen written as ${}_8\text{O}^{17}$

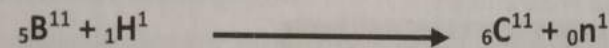
Now after putting the missing term the above equation becomes



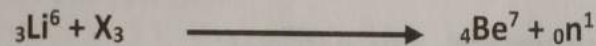
The second equation is



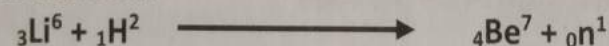
Here Z number on left hand side is 6 and A number is 12 whereas Z number on right side of equation is 6 and A number is 11. Hence the missing nucleus X_2 will have Z number $6 - 6 = 0$ and A number $12 - 11 = 1$ which is of neutron ${}_0\text{n}^1$. Now with this, the equation can be written as follows.



Third equation is

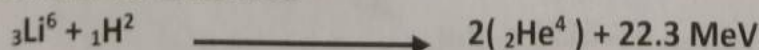


In this equation right hand side has total Z number 4 and A number 8 whereas left hand side has Z number 3 and A number 6. So the missing nucleus X_3 has Z number $4 - 3 = 1$ and A number $8 - 6 = 2$ which is of deuterium ${}_1\text{H}^2$. With this, the third equation is re-written as follows:



P 20.8. ${}_3\text{Li}^6$ is bombarded by deuterons. The reaction gives two α -particles along with release of energy equal to 22.3 MeV. Knowing masses of deuteron and α -particles determine mass of lithium isotope of ${}_3\text{Li}^6$.

Solution Equation can be written as



$$E = 22.3 \text{ MeV}$$

Using the equation $E = \Delta m c^2$

$$\Delta m = \frac{E}{c^2} = \frac{22.3 \text{ MeV}}{931.5 \frac{\text{MeV}}{\text{a.m.u}}} = 0.023939 \text{ a.m.u}$$

Mass of deuteron ${}_1\text{H}^2$ is

$$m_d = 2.0141 \text{ a.m.u}$$

Mass of helium nucleus ${}_2\text{He}^4$ is

$$m_{\text{He}} = 4.00263 \text{ a.m.u}$$

Now the nuclear equation above can be interpreted as follows:

$$\text{Mass of } {}_3\text{Li}^6 + \text{Mass of } {}_1\text{H}^2 = 2(\text{mass of } {}_2\text{He}^4) + \text{mass defect } \Delta m$$

$$m_{\text{Li}} = 2(\text{mass of } {}_2\text{He}^4) + \text{mass defect } \Delta m - \text{mass of } {}_1\text{H}^2$$

$$m_{\text{Li}} = 2(4.00263) + 0.023939 - 2.0141 = 6.015 \text{ a.m.u}$$

P 20.9. Find the energy released when β -decay changes ${}_{90}\text{Th}^{234}$ into ${}_{91}\text{Pa}^{234}$

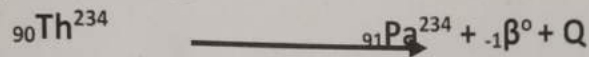
Solution

$$A_{\text{Th}} = 234.0436 \text{ u}$$

$$A_{\text{Pa}} = 234.042762 \text{ u.}$$

$$Q = ?$$

Equation for beta decay is shown as



Energy released Q can be found by equation

$$Q = \Delta m \times 931.5 \text{ MeV} \text{ ----- (1)}$$

$$\Delta m = \text{Mass of } {}_{90}\text{Th}^{234} - (\text{mass of } {}_{91}\text{Pa}^{234} + \text{mass of } {}_{-1}\beta^0)$$

$$\Delta m = 234.0436 - (234.042762 + 0.00055)$$

$$\Delta m = 0.000288 \text{ u} \quad \text{put this in (1) we get}$$

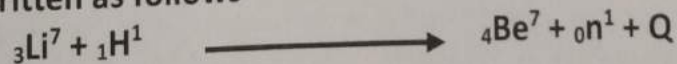
$$Q = 0.000288 \times 931.5 \text{ MeV} = 0.27 \text{ MeV}$$

P 20.10. Find out the K.E to which a proton must be accelerated to induce the following nuclear reaction.

Solution

$$Q = ? \text{ for the reaction } \text{Li}^7 (p, n) \text{Be}^7$$

The equation can be written as follows



Energy Q can be found using equation

$$Q = \Delta m \times 931.5 \text{ MeV} \text{ ----- (1)}$$

$$\text{Mass of } {}_3\text{Li}^7 = 7.01823 \text{ a.m.u}$$

$$\text{Mass of } {}_1\text{H}^1 = 1.00814 \text{ a.m.u}$$

$$\text{Mass of } {}_4\text{Be}^7 = 7.015929 \text{ a.m.u}$$

$$\text{Mass of } {}_0\text{n}^1 = 1.00867 \text{ a.m.u}$$

Now mass deficit Δm can be determined using the equation

$$\Delta m = (\text{mass of } {}_3\text{Li}^7 + \text{Mass of } {}_1\text{H}^1) - (\text{Mass of } {}_4\text{Be}^7 + \text{Mass of } {}_0\text{n}^1)$$

$$\Delta m = (7.01823 + 1.00814) - (7.015929 + 1.00867)$$

$\Delta m = 0.001771 \text{ a.m.u}$ putting this value in eq (1) we get

$$Q = 0.001771 \times 931.5 \text{ MeV} = 1.65 \text{ MeV}$$