NUMERICAL PROBLEMS

P 20.1. Find the mass defect and binding energy for helium nucleus?

Solution

$\Delta m = ?$ $E_B = ?$ (for Helium nucleus)

Mass of proton $m_p = 1.00728$ a.m.u

Mass of neutron $m_n = 1.00867$ a.m.u

Mass of helium nucleus $M_{He} = 4.00125$ a.m.u

Helium has two protons (Z = 2) and its atomic mass A is 4. So it has (A - Z) neutrons i.e,

$$N = 4 - 2 = 2$$

Now mass defect can be determined by

$$\Delta m = (Z\, m_p - N\, m_n) - M_{He}$$

$$\Delta m = (2 \times 1.00728 - 2 \times 1.00867) - 4.00152$$

$$\Delta m = 0.03038 \text{ a.m.u}$$

To calculate binding energy E_B , since 1 a.m.u = 931.5 MeV

So $E_B = 0.03038 \times 931.5$ Mev = 28.3 MeV

P 20.2. A certain radioactive isotope has half-life of 8 hours. A solution containing 500 million atoms of this isotope is prepared. How many atoms of this isotope have not disintegrated after (a) 8 hours (b) 24 hours.

Solution

$T_{½}$= 8hrs N = 500Million (a) N_1 = ? (atoms left after 8 hrs)

(b) N_2 = ? (atoms left after 24 hrs)

(a) The given isotope has half-life of 8 hrs. So after 1st half-life, 250 Million will disintegrate and 250 Million will not disintegrate.

Hence N_1 = 250 Million are not disintegrated after 8 hrs.

(b) Now 24hrs means that 3 half-lives have passed. So the number N_2 of nuclei that are left un-decayed or those nuclei which are not disintegrated are $(\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) = \frac{1}{8}$ of the total number.

So $N_2 = \frac{1}{8} \times 500$ Million

N_2 = 62.5 Million are not disintegrated after 24 hrs

P 20.3. Write the nuclear equations for the beta decay of (a) $_{82}Pb^{210}$ (b) $_{83}Bi^{210}$ (c)

$_{90}Th^{234}$ (d) $_{93}Np^{239}$

Solution

Nuclides having excess no. of neutrons decay through negative beta emission.

$$N \rightarrow P + \beta^- + \bar{\gamma} \qquad \text{(Z increases by 1)}$$

Nuclides having greater no. of protons decay through positive beta emission

$$P \rightarrow N + \beta^+ + \gamma \qquad \text{(Z decreases by 1)}$$

For $_{82}Pb^{210}$ $Z = 82$ & $A = 210$ $\Rightarrow N = A - Z = 210 - 82 = 128$

Hence $N > Z$

For $_{83}Bi^{210}$ $Z = 83$ & $A = 210$ $\Rightarrow N = A - Z = 210 - 83 = 127$

Hence $N > Z$

For $_{90}Th^{234}$ $Z = 90$ & $A = 234$ $\Rightarrow N = A - Z = 234 - 90 = 144$

Hence $N > Z$

For $_{93}Np^{239}$ $Z = 93$ & $A = 239$ $\Rightarrow N = A - Z = 239 - 93 = 146$

Hence $N > Z$

Thus all the nuclides decay through negative beta emission.

$$_{82}Pb^{210} \longrightarrow {}_{83}Bi^{210} + {}_{-1}\beta^0 + \bar{\nu} + Q$$

$$_{83}Bi^{210} \longrightarrow {}_{84}Po^{210} + {}_{-1}\beta^0 + \bar{\nu} + Q$$

$$_{90}Th^{234} \longrightarrow {}_{91}Pa^{234} + {}_{-1}\beta^0 + \bar{\nu} + Q$$

$$_{93}Np^{239} \longrightarrow {}_{94}Pu^{239} + {}_{-1}\beta^0 + \bar{\nu} + Q$$

Where E shows the energy liberated and $\bar{\nu}$ are anti-neutrinos as they are always associated with negative beta emission.

P 20.4. Calculate the total energy released if 1kg of U^{235} undergoes fission? Taking the disintegration energy per event to be Q = 208 MeV.

Solution

E = ? Q = 208 MeV per event m = 1kg of U^{235}

Number of atoms in 1 kg of U^{235}

$$N = nN_A \qquad (1)$$

Number of moles

$$n = \frac{\text{Given mass " m "}}{\text{Atomic mass " M "}} = \frac{1\text{ kg}}{235\,\frac{\text{kg}}{\text{kmol}}}$$

$n = 4.255 \times 10^{-3}$ k mole putting in (1) we get

$$N = 4.255 \times 10^{-3}\text{ k mole} \times (6.023 \times 10^{23+3}\,\frac{\text{atoms}}{\text{kmol}})$$

$$N = 2.56 \times 10^{24}\text{ atoms}$$

Now total energy liberated is

$$E = N\,Q = 2.56 \times 10^{24}\,(208\text{ MeV})$$

$$E = 5.33 \times 10^{26}\ MeV$$

P 20.5. Find the energy released in the following fission reaction?

Solution

$Q = ?$ $\quad {}_0n^1 + {}_{92}U^{235} \longrightarrow {}_{36}Kr^{92} + {}_{56}Ba^{141} + 3{}_0n^1 + Q$

Total mass on reactants side m_r = mass of one neutron + mass of U^{235}

$$m_r = 1.00867 + 235.0439$$

$$m_r = 236.05257 \text{ a.m.u}$$

Now total mass on products side m_{pro} is given by

$$m_{pro} = \text{mass of } Kr^{92} + \text{mass of } Ba^{141} + \text{mass of 3 neutrons}$$

$$m_{pro} = 91.8973 + 140.9139 + 3\,(1.00867)$$

$$m_{pro} = 235.8372 \text{ a.m.u}$$

mass deficit $\quad \Delta m = m_r - m_{pro} = 236.05257 - 235.8372$

$$\Delta m = 0.21536 \text{ a.m.u}$$

And the energy released during the reaction is

$$Q = \Delta m \times 931.5\ MeV = 0.21536 \times 931.5\ MeV$$
$$Q = 201\ MeV$$

P 20.6. Find the energy released in the fusion reaction.

Solution

$Q = ?$ $\quad {}_1H^2 + {}_1H^3 \longrightarrow {}_2He^4 + {}_0n^1 + Q$

Total mass on reactants side is

$$m_r = \text{mass of deuterium } {}_1H^2 + \text{mass of tritium } {}_1H^3$$

$$m_r = 2.0141 + 3.01605$$

$$m_r = 5.03015 \text{ a.m.u}$$

Now total mass on products side is;

$$m_{pro} = \text{mass of one } \alpha\text{-particle} + \text{mass of one neutron}$$

$$m_{pro} = 4.00263 + 1.00867$$

$$m_{pro} = 5.0113 \text{ a.m.u}$$

$$\Delta m = m_r - m_{prod}$$

$$\Delta m = 0.01885 \text{ a.m.u}$$

Now the energy released during the given fusion reaction is;

$$Q = \Delta m \times 931.5\ MeV$$

$$Q = 0.01885 \times 931.5\ MeV$$

$$Q = 17.55\ MeV$$

P 20.7. Complete the following nuclear reactions.

$${}_7N^{14} + {}_2He^4 \longrightarrow {}_1H^1 + X_1$$

$${}_5B^{11} + {}_1H^1 \longrightarrow {}_6C^{11} + X_2$$

$${}_3Li^6 + X_3 \longrightarrow {}_4Be^7 + {}_0n^1$$

In all the equations, law of conservation of charge number (Z number) and law of conservation of mass number (A number) must be conserved.

First equation is

$${}_7N^{14} + {}_2He^4 \longrightarrow {}_1H^1 + X_1$$

At the left hand side of the above equation total Z number is 9 whereas A number is 18. Now Z number on right hand side is 1 whereas A number is also 1. Hence the missing nucleus X_1 will have Z number 9 - 1 = 8 while A number 18 - 1 = 17 which is of oxygen written as ${}_8O^{17}$

Now after putting the missing term the above equation becomes

$${}_7N^{14} + {}_2He^4 \longrightarrow {}_1H^1 + {}_8O^{17}$$

The second equation is

$${}_5B^{11} + {}_1H^1 \longrightarrow {}_6C^{11} + X_2$$

Here Z number on left hand side is 6 and A number is 12 whereas Z number on right side of equation is 6 and A number is 11. Hence the missing nucleus X_2 will have Z number 6 - 6 = 0 and A number 12 - 11 = 1 which is of neutron ${}_0n^1$. Now with this, the equation can be written as follows.

$${}_5B^{11} + {}_1H^1 \longrightarrow {}_6C^{11} + {}_0n^1$$

Third equation is

$${}_3Li^6 + X_3 \longrightarrow {}_4Be^7 + {}_0n^1$$

In this equation right hand side has total Z number 4 and A number 8 whereas left hand side has Z number 3 and A number 6. So the missing nucleus X_3 has Z number 4 - 3 = 1 and A number 8 - 6 = 2 which is of deuterium ${}_1H^2$. With this, the third equation is re-written as follows:

$${}_3Li^6 + {}_1H^2 \longrightarrow {}_4Be^7 + {}_0n^1$$

P 20.8. ${}_3Li^6$ is bombarded by deuterons. The reaction gives two α-particles along with release of energy equal to 22.3 MeV. Knowing masses of deuteron and α-particles determine mass of lithium isotope of ${}_3Li^6$.

Solution Equation can be written as

$${}_3Li^6 + {}_1H^2 \longrightarrow 2({}_2He^4) + 22.3\ MeV$$

$E = 22.3$ MeV

Using the equation $E = \Delta m\, c^2$

$$\Delta m = \frac{E}{c^2} = \frac{22.3\ MeV}{931.5\ \frac{MeV}{a.m.u}} = 0.023939\ a.m.u$$

Mass of deuteron $_1H^2$ is

$$m_d = 2.0141 \text{ a.m.u}$$

Mass of helium nucleus $_2He^4$ is

$$m_{He} = 4.00263 \text{ a.m.u}$$

Now the nuclear equation above can be interpreted as follows:

$$\text{Mass of } _3Li^6 + \text{Mass of } _1H^2 = 2(\text{mass of } _2He^4) + \text{mass defect } \Delta m$$

$$m_{Li} = 2(\text{mass of } _2He^4) + \text{mass defect } \Delta m - \text{mass of } _1H^2$$

$$m_{Li} = 2(4.00263) + 0.023939 - 2.0141 = 6.015 \text{ a.m.u}$$

P 20.9. Find the energy released when β-decay changes $_{30}Th^{234}$ into $_{91}Pa^{234}$

Solution

$A_{Th} = 234.0436u$ $A_{Pa} = 234.042762$ u. $Q = ?$

Equation for beta decay is shown as

$$_{90}Th^{234} \longrightarrow {}_{91}Pa^{234} + {}_{-1}\beta^o + Q$$

Energy released Q can be found by equation

$$Q = \Delta m \times 931.5 \text{ MeV} \text{ ------------------------------------ (1)}$$

$$\Delta m = \text{Mass of } _{90}Th^{234} - (\text{mass of } _{91}Pa^{234} + \text{mass of } _{-1}\beta^o)$$

$$\Delta m = 234.0436 - (234.042762 + 0.00055)$$

$\Delta m = 0.000288u$ put this in (1) we get

$$Q = 0.000288 \times 931.5 \text{ MeV} = 0.27 \text{ MeV}$$

P 20.10. Find out the K.E to which a proton must be accelerated to induce the following nuclear reaction.

Solution $Q = ?$ for the reaction Li^7 (p,n) Be^7

The equation can be written as follows

$$_3Li^7 + {}_1H^1 \longrightarrow {}_4Be^7 + {}_0n^1 + Q$$

Energy Q can be found using equation

$$Q = \Delta m \times 931.5 \text{ MeV} \text{ ------------------- (1)}$$

Mass of $_3Li^7 = 7.01823$ a.m.u

Mass of $_1H^1 = 1.00814$ a.m.u

Mass of $_4Be^7 = 7.015929$ a.m.u

Mass of $_0n^1 = 1.00867$ a.m.u

Now mass deficit Δm can be determined using the equation

$$\Delta m = (\text{mass of } _3Li^7 + \text{Mass of } _1H^1) - (\text{Mass of } _4Be^7 + \text{Mass of } _0n^1)$$

$$\Delta m = (7.01823 + 1.00814) - (7.015929 + 1.00867)$$

Δm = 0.001771 a.m.u putting this value in eq (1) we get

Q = 0.001771 x 931.5 MeV = 1.65 MeV