NUMERICAL PROBLEMS

19.1 Find the shortest wavelength photon emitted in the Lyman series of hydrogen?

Solution

λ_{min} = ? (for Lyman series)

Lyman series occurs due to transition in n = 1 from any upper energy level. Shortest wavelength λ_{min} will occur if transition occurs from n = ∞ to p = 1 and can be determined by.

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n^2} - \frac{1}{p^2} \right)$$

⇒ $$\frac{1}{\lambda_{min}} = (1.097 \times 10^7) \left(\frac{1}{1^2} - \frac{1}{\infty^2} \right)$$

⇒ $$\lambda_{min} = 91.1 \text{ nm}$$

19.2 What is the wavelength of the second line of Paschen series?

Solution

λ = ? (2^{nd} line of Paschen series)

Paschen series occurs when electrons from any upper state "n" jump down to p = 3. Now 2^{nd} line will occur for n = 5 transition. So we use equation

$$\frac{1}{\lambda} = R_H \left(\frac{1}{p^2} - \frac{1}{n^2} \right)$$

For 2^{nd} line of Paschan series, n = 5 and p = 3

$$\frac{1}{\lambda} = (1.097 \times 10^7) \left(\frac{1}{(3)^2} - \frac{1}{(5)^2} \right)$$

$$\frac{1}{\lambda} = (1.097 \times 10^7) \left(\frac{25-9}{225} \right)$$

$$\lambda = 1.282 \times 10^{-6} \text{ m} = 1282 \text{ nm}$$

19.3 Calculate the longest wavelength of radiation for the Paschen series?

Solution

Paschen series occurs when electrons from any upper state n, jump down to p = 3. Now longest wavelength or in other words, shortest energy will occur for n = 4 to p = 3 transition. So we use equation

$$\frac{1}{\lambda} = R_H \left(\frac{1}{p^2} - \frac{1}{n^2} \right)$$ R_H is Rydberg's constant

$$\frac{1}{\lambda_{min}} = (1.097 \times 10^7) \left(\frac{1}{(3)^2} - \frac{1}{(4)^2} \right)$$

$$\frac{1}{\lambda_{min}} = (1.097 \times 10^7) \left(\frac{16-9}{144} \right)$$

$$\lambda_{min} = 1.875 \times 10^{-6} \text{ m} = 1875 \text{ nm}$$

19.4 The series limit wavelength of the Balmer series is emitted as the electron in the hydrogen atom falls from n = ∞ to the n = 2 state. What is the wavelength of this line. Where ΔE = 3.40 eV.

Solution

$\lambda = ?$

We know that $\Delta E = 3.40\ eV = 3.40\ (1.6 \times 10^{-19}) = 5.44 \times 10^{-19}\ J$

$$\Delta E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{5.44 \times 10^{-19}} = 366\ nm$$

19.5 A photon is emitted from hydrogen atom which undergoes a transition from n = 3 state to n = 2 state. Calculate (a) energy (b) wavelength and (c) frequency of photon.

Solution

(a) $E = ?$ (b) $\lambda = ?$ (c) $f = ?$ from $n = 3$ to $p = 2$

(a) Energy emitted by photon due to transition from $n = 3$ to $p = 2$ is equal to the energy difference between the given two levels.

Since $E = E_n - E_p$

Where $E_n = -\frac{E_o}{n^2}$ & $E_p = -\frac{E_o}{p^2}$ & $E_o = 13.6\ eV$

$$E = E_3 - E_2 = (-\frac{E_o}{(3)^2}) - (-\frac{E_o}{(2)^2}) = (-\frac{E_o}{9}) - (-\frac{E_o}{4})$$

$$E = \frac{E_o}{4} - \frac{E_o}{9} = 13.6\ (\frac{5}{36}) = 1.89\ eV$$

(b) Wavelength λ corresponding to the transition $n = 3$ to $p = 2$ is

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{1.89 \times (1.6 \times 10^{-19})} = 658\ nm$$

(c) Frequency

$$f = \frac{c}{\lambda}$$

$$f = \frac{3 \times 10^{8}}{658 \times 10^{-9}} = 4.56 \times 10^{14}\ Hz$$

19.6 Find the longest wavelength of light capable of ionizing a hydrogen atom. How much energy in needed to ionize a hydrogen atom?

Solution

$\lambda_{max} = ?$ (for ionizing hydrogen atom)

$E_{min} = ?$ (for ionizing hydrogen atom)

Longest wavelength corresponds to minimum energy and minimum energy to ionize hydrogen atom means transition from $p = 1$ to $n = \infty$

Now as $E_n = -\frac{E_o}{n^2}$ $E_o = 13.6\ eV$

So $E_{min} = E_n - E_p = E_\infty - E_1$

$$E_{min} = -\frac{E_o}{\infty} - (-\frac{E_o}{(1)^2})$$

$$E_{min} = \frac{E_o}{1} - 0 = 13.6\ eV$$

Now this E_{min} means maximum wavelength λ_{max} given by equation

$$\lambda_{max} = \frac{hc}{E_{min}}$$

$$\lambda_{max} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{13.6 \times (1.6 \times 10^{-19})} = 91.4 \text{ nm}$$

19.7 Calculate radius of the innermost orbital level of the hydrogen atom?

Solution

$r_1 = ?$ $m_e = 9.11 \times 10^{-31}$Kg $q = e = 1.6 \times 10^{-19}$C

For hydrogen atom, radii of the orbits can be determined by the equation

$$r_n = \frac{n^2 h^2}{4\pi^2 k m e^2}$$

For innermost shell, n = 1, putting values of other constants we get

$$r_n = \frac{(1)^2 (6.63 \times 10^{-34})^2}{4(3.14)^2 (9 \times 10^9)(9.11 \times 10^{-31})(1.6 \times 10^{-19})^2} = 5.3 \times 10^{-11} \text{ m}$$

19.8 (a) Determine energy associated with innermost orbit of the hydrogen atom (n=1). (b) Determine the energy associated with second orbit of the hydrogen atom, (c) What energy does an incoming photon possess to raise an electron from first to the second allowed orbit of the hydrogen atom?

Solution

(a) E_n = ? (For innermost orbit of hydrogen i.e, n=1)

(b) E_n = ? (For 2nd orbit of hydrogen i.e, n=2)

(c) E = ? (For transition from n = 2 to p = 1)

(a) Energy of quantized hydrogen atom in any orbit is given by

$$E_n = -\frac{E_o}{n^2} \qquad E_o = 13.6 \text{ eV (ground state energy)}$$

For innermost orbit of hydrogen atom, n = 1

$$E_1 = -\frac{13.6}{(1)^2} = -13.6 \text{ eV}$$

(b) For 2nd orbit of hydrogen atom, n = 2. So we write the same equation

$$E_2 = -\frac{13.6}{(2)^2} = -3.4 \text{ eV}$$

(c) Energy needed to raise an electron from 1st orbit to 2nd orbit is equal to the difference of energies between both energy levels.

Since

$$E = E_2 - E_1$$

$$E = -3.4 - (-13.6) = 10.2 \text{ eV}$$

19.9 An electron drops from the second energy level to the first energy level within an excited hydrogen atom, determine (a) the energy of the photon emitted (b) the frequency of the photon emitted and (c) the wavelength of the photon emitted.

Solution

(a) E = ? (For transition from n = 2 to p = 1 in hydrogen atom)

(b) f = ? (c) λ = ?

(a) Energy emitted by an electron of hydrogen atom in making transition from 2^{nd} orbit to 1^{st} orbit, is equal to the difference of energies between both energy levels.
Mathematically $E = E_2 - E_1$

$$E = -\frac{E_0}{(2)^2} - \left(-\frac{E_0}{(1)^2}\right)$$

$$E = \frac{E_0}{(1)^2} - \frac{E_0}{(2)^2} = E_0\left(\frac{4-1}{4}\right) = \frac{3E_0}{4} = \frac{3(13.6)}{4}$$

$$E = 10.2 \text{ eV}$$

(b) As
$$E = hf$$

So
$$f = \frac{E}{h}$$

$$f = \frac{10.2 \times (1.6 \times 10^{-19})}{6.63 \times 10^{-34}} = 2.46 \times 10^{15} \text{ Hz}$$

(c) Frequency and wavelength are related to each other by equation $f = \frac{c}{\lambda}$

OR
$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{2.46 \times 10^{15}} = 1.22 \times 10^{-7} \text{ m}$$

19.10 An electron is in the first Bohr orbit of hydrogen. Find (a) the speed of the electron, (b) the time required for the electron to circle the nucleus.

Solution

(a) speed V = ? (for 1^{st} Bohr orbit in hydrogen)
(b) T = ? (time to complete one round trip by electron in hydrogen)
(a) According to 2^{nd} postulate of Bohr's model of hydrogen atom

$$m v_n r_n = \frac{nh}{2\pi}$$

$$v_n = \frac{nh}{2\pi m r_n} \qquad \text{for } 1^{st} \text{ Bohr's orbit, } n = 1$$

$$v_1 = \frac{(1)h}{2\pi m r_1}$$

$$v_1 = \frac{(1)(6.63 \times 10^{-34})}{2(3.14)(9.11 \times 10^{-31})(5.3 \times 10^{-11})}$$

$$v_1 = 2.19 \times 10^6 \text{ m/s}$$

(b) We know that relation between linear and angular velocity is

for 1^{st} orbit n = 1

$$V = r\omega$$

As $\omega = \frac{2\pi}{T}$

$$V_1 = r_1 \omega_1$$

$$V_1 = r_1\left(\frac{2\pi}{T_1}\right) \qquad \text{re-arranging the equation we get}$$

$$T_1 = \frac{2\pi r_1}{v_1} = \frac{2(3.14)(5.3 \times 10^{-11})}{2.19 \times 10^6} = 1.52 \times 10^{-16} \text{ sec}$$

19.11 Electrons in an X-ray tube are accelerated through a potential difference of 3000 V. if these electrons were slowed down in a target, what will be the minimum wavelength of X-rays produced.

Solution

$V_o = 3000\ V$ $\lambda_{min} = ?$

When an electron in x-ray tube accelerated by potential difference V_o, provides all its energy hf_{max} in a single collision to the target then

$$e V_o = h f_{max} = \frac{h c}{\lambda_{min}}$$

$$\lambda_{min} = \frac{h c}{e V_o}$$

$$\lambda_{min} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{(1.6 \times 10^{-19})(3000)} = 4.14\ A^{\circ}$$

19.12 Compute the potential difference through which an electron must be accelerated in order that the short-wave limit of continuous X-ray spectrum shall be exactly 0.1 nm

Solution

$V_o = ?$ $\lambda_{min} = 0.1\ nm$

When an electron in X-ray tube is accelerated by potential difference V_o provides all its energy hf_{max} in a single collision to the target then;

$$e V_o = h f_{max} = \frac{h c}{\lambda_{min}}$$

$$V_o = \frac{h c}{e \lambda_{min}} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{(1.6 \times 10^{-19})(0.1 \times 10^{-9})} = 12431\ V$$