NUMERICAL PROBLEMS

18.1 The length of a space ship is measured to be exactly one-third of its proper length. What is the speed of the spaceship relative to the observer?

Solution Proper length = L_o Measured length $L = \frac{L_o}{3}$ v = ?

Formula for Lorentz length contraction is

$$L = L_o \sqrt{1-(\frac{v^2}{c^2})}$$

According to given condition ($L = \frac{L_o}{3}$) above equation can be written as

$$\frac{L_o}{3} = L_o \sqrt{1-(\frac{v^2}{c^2})}$$

⇒ $\frac{1}{3} = \sqrt{1-(\frac{v^2}{c^2})}$ Squaring both sides & solving for v

⇒ $\frac{1}{9} = 1 - (\frac{v^2}{c^2}) \Rightarrow \frac{v^2}{c^2} = \frac{8}{9}$

$v = 0.9428\ c$

18.2 The time period of a pendulum is measured to be 3s in inertial frame of the pendulum. What is the period when measured by an observer moving with a speed of 0.95c with respect to the pendulum?

Solution $t_o = 3$ sec t = ? If speed v = 0.95 c

Equation for time dilation is

$$t = \frac{t_o}{\sqrt{1-(\frac{v^2}{c^2})}} = \frac{3}{\sqrt{1-(\frac{(0.95\,c)^2}{c^2})}} = \frac{3}{\sqrt{1-(0.9025)}}$$

$t = 9.6$ sec

18.3 An electron, which has a mass 9.11x10 $^{-31}$ kg, moves with a speed of 0.75c. Find its relativistic momentum and compare this value with the momentum calculated from classical expression.

Solution

$m_e = 9.11 \times 10^{-31}$ kg $\quad v = 0.75\,c$

Relativistic momentum P_r = ? and compare with $P_{classical}$

$$P_r = m\,v = \frac{m_o v}{\sqrt{1-(\frac{v^2}{c^2})}}$$

$$\frac{P_r}{P_{classical}} = 1.51$$
$$P_r = 1.51\,P_{classical}$$

$$P_r = \frac{(9.11 \times 10^{-31})(0.75 \times 3 \times 10^8)}{\sqrt{1-(\frac{(0.75\,c)^2}{c^2})}}$$

$P_r = 3.1 \times 10^{-22}$ kg m s^{-1}

$P_{classical} = m_o\,v$

$P_{classical} = (9.11 \times 10^{-31})\,(0.75 \times 3 \times 10^8)$

$P_{classical} = 2.05 \times 10^{-22}$ kg m s^{-1}

For comparison, we find % difference as $(\frac{P_r - P_{classical}}{P_{classical}}) \times 100\% = 51\,\%$

18.4 An electron moves with a speed of v = 0.85c. Find its total energy and K.E in electron volt.

Solution $\quad v = 0.85c \quad$ T.E = ? (in eV) $\quad$ E = ? (in eV)

At the given speed (0.85c) the relativistic mass m is

$$m = \frac{m_o}{\sqrt{1-(\frac{v^2}{c^2})}} = \frac{9.11 \times 10^{-31}}{\sqrt{1-\frac{(0.85\,c)^2}{c^2}}} = 1.73 \times 10^{-30}\text{ kg}$$

Now Total energy T.E is

T.E = $mc^2 = (1.73 \times 10^{-30})\,(3 \times 10^8)^2$

T.E = 1.56×10^{-13} J

divide by 1.6×10^{-19} J / eV

T.E = 0.973 MeV

$E_o = m_o\,c^2 = (9.11 \times 10^{-31})\,(3 \times 10^8)$

$E_o = 8.199 \times 10^{-31}$ J $\quad$ divide by "e"

$E_o = 0.5124$ MeV

Now total energy T.E and kinetic energy K.E are related to rest energy E_o

T.E = K.E + E_o

K.E = T.E - E_o

K.E = (0.973 - 0.5124) MeV

K.E = 0.461 MeV

18.5 Rest mass of a proton is 1.67x10^{-27}kg. At what speed would the mass of the proton be tripled?

Solution

$m_o = 1.67 \times 10^{-27}$ kg $\quad$ v = ? at which $m = 3m_o$

Relativistic mass of proton is given by

$$m = \frac{m_o}{\sqrt{1-(\frac{v^2}{c^2})}}$$

According to the given condition ($m = 3m_o$)

$$3\,m_o = \frac{m_o}{\sqrt{1-(\frac{v^2}{c^2})}}$$ taking reciprocal

$$\frac{1}{9} = 1 - (\frac{v^2}{c^2}) \Rightarrow \frac{v^2}{c^2} = \frac{8}{9}$$

Taking square root

$$v = 0.9428\,c$$

18.6 At what fraction of speed of light must a particle move so that its K.E is one and a half times its rest energy?

Solution

$V = ?$ when $K.E = \frac{3}{2} E_o = \frac{3}{2}(m_o c^2)$

Total energy E = kinetic energy K.E + rest energy E_o

$$mc^2 = \frac{3}{2}(m_o c^2) + (m_o c^2)$$ divide by c^2

$$m = \frac{3}{2} m_o + m_o = \frac{5}{2} m_o$$

$$\frac{m_o}{\sqrt{1-(\frac{v^2}{c^2})}} = \frac{5}{2} m_o$$ As $m = \frac{m_o}{\sqrt{1-(\frac{v^2}{c^2})}}$

$$\frac{1}{\sqrt{1-(\frac{v^2}{c^2})}} = \frac{5}{2}$$ Take reciprocal and square

$$1 - (\frac{v^2}{c^2}) = \frac{4}{25}$$

$$\frac{v^2}{c^2} = \frac{21}{25}$$

Taking square root & solving for "v"

$$v = 0.916c$$

18.7 A metal, whose work function is 3.0 eV, is illuminated by light of wavelength 3×10^{-7}m. Calculate (a) The threshold frequency, (b) The maximum energy of photoelectrons (c) The stopping potential.

Solution

$\Phi = 3eV = 3 \times 1.6 \times 10^{-19} = 4.8 \times 10^{-19}$ J

$\lambda = 3 \times 10^{-7}$m (a) $f_o = ?$ (b) $K.E_{max} = ?$ (c) $V_o = ?$

(a) Work function is the minimum amount of energy required for ejecting electrons from a metal surface

$$\Phi = h f_o$$

$$f_o = \frac{\varphi}{h} = \frac{4.8 \times 10^{-19}}{(6.63 \times 10^{-34})} = 0.72 \times 10^{15}\ Hz$$

(b) According to the Einstein's equation of photoelectric effect

$$E = \Phi + K.E_{max}$$

$$K.E_{max} = hf - \phi$$

Since

$$f = \frac{c}{\lambda} = \frac{3 \times 10^{8}}{(3 \times 10^{-7})} = 10^{15}\ Hz$$

$$K.E_{max} = (6.63 \times 10^{-34})(10^{15}) - (4.8 \times 10^{-19})$$

$$K.E_{max} = 1.83 \times 10^{-19}\ J \qquad \text{divide by } 1.6 \times 10^{-19}$$

$$K.E_{max} = 1.14\ eV$$

(c) $K.E_{max} = eV_o$ where V_o is stopping potential

$\Rightarrow$ $$V_o = \frac{K.E_{max}}{e} = \frac{1.14 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$\Rightarrow$ $$V_o = 1.14\ V$$

18.8 The thermal radiation from the sun peaks in the visible part of the spectrum. Estimate the temperature of the sun.

Solution

T = ? For visible part of spectrum

For visible part of spectrum, wavelength is 500 nm

$\lambda_{max} = 500$ nm

Using Wien's displacement law

$$\lambda_{max} T = 0.2898 \times 10^{-2}\ m\ K$$

$$T = \frac{0.2898 \times 10^{-2}}{\lambda_{max}} = \frac{0.2898 \times 10^{-2}}{500 \times 10^{-9}}$$

$$T = 5796\ K \approx 5800\ K$$

18.9 A 50 keV X-ray is scattered through an angle of 90°. What is the energy of X-ray after Compton scattering?

Solution

Initially $E = 50\ keV = 50{,}000 \times 1.6 \times 10^{-19}\ J = 8 \times 10^{-15}\ J$

$\theta = 90^\circ$ Energy after scattering $E' = ?$

Energy of X-ray photon after scattering can be found by equation

$$E' = \frac{hc}{\lambda'} \qquad (1)$$

λ' is wavelength of scattered photon and is given by the equation

$$\lambda' = \lambda + \frac{h}{m_o c}(1 - \cos\theta) \qquad (2)$$

λ is the only unknown at the right hand side for which we use

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{8 \times 10^{-15}}$$

$\lambda = 2.49 \times 10^{-11}$ m putting this value in eq (2), we get;

$$\lambda' = 2.49\times10^{-11} + \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(3\times10^{8})}(1-\cos 90)$$

$\lambda' = 2.73\times10^{-11}$ m put this in eq (1) we get

$$E' = \frac{(6.63\times10^{-34})(3\times10^{8})}{2.73\times10^{-11}}$$

$E' = 7.28\times10^{-15}$ J divide by "e"

$E' = 45.5$ keV

18.10 Calculate the wavelength of de Broglie waves associated with electrons accelerated through a potential difference of 200 V.

Solution

λ = ? (for electron) $V_o = 200$ V

De-Broglie's wavelength associated with a moving electron is given by

$$\lambda = \frac{h}{mv} \quad (1)$$

Kinetic energy $\frac{1}{2}mv^2 = eV_o$

$$v = \sqrt{\frac{2eV_o}{m}} = \sqrt{\frac{2(1.6\times10^{-19})(200)}{9.11\times10^{-31}}}$$

$v = 8.38\times10^{6}$ m/s

put this value in eq (1) we get $\lambda = \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(8.38\times10^{6})} = 0.87$ A°

18.11 An electron is accelerated through a potential difference of 50V. Calculate its de Broglie Wavelength.

$V_o = 50$ V λ = ? (for electron)

Solution

De-Broglie's wavelength associated with a moving electron is given by

$$\lambda = \frac{h}{mv} \text{ ----------------------------- (1)}$$

Since $\frac{1}{2}mv^2 = eV_o$ Re-arranging them we get

$$v = \sqrt{\frac{2eV_o}{m}}$$

$$v = \sqrt{\frac{2(1.6\times10^{-19})(50)}{9.11\times10^{-31}}} = 4.19\times10^{6} \text{ m/s}$$

Put this value in eq (1) we get $\lambda = \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(4.19\times10^{6})} = 1.74$ A°

18.12 The speed of an electron is measured to be 5×10^{3} m/s to an accuracy of 0.003%. Find the uncertainty in determining the position of this electron.

Solution

Accuracy $A_c = 0.003\% = \frac{0.003}{100} = 3\times10^{-5}$

$v = 5\times10^{3}$ m/s

Uncertainty in position Δx = ? (for electron)

Uncertainty in velocity $\Delta v = 5 \times 10^3 \times A_c$

$\Delta v = 5 \times 10^3 \times (3 \times 10^{-5}) = 0.15$ m/s

Now according to Heisenberg's uncertainty principle,

$$\Delta x \, \Delta P = h \qquad (\text{As } \Delta P = m\,\Delta v)$$

$$\Delta x = \frac{h}{m\,\Delta v} = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(0.15)} = 4.85 \times 10^{-3} \text{ m}$$

18.13 The life time of an electron in an exited state is about 10^{-8} sec. What is its uncertainty in energy during this time?

Solution

$\Delta t = 10^{-8}$ sec $\qquad \Delta E$ = ?

By Heisenberg's uncertainty principle,

$$\Delta E \, \Delta t = h$$

$$\Delta E = \frac{h}{\Delta t} = \frac{6.63 \times 10^{-34}}{10^{-8}} = 6.63 \times 10^{-26} \text{ J}$$