NUMERICAL PROBLEMS

Problem 16.1 A 1.50cm length of pianos wire with a diameter of 0.25 cm is stretched by attaching 10 kg mass to one end. How far is wire stretched?

Solution

$\ell = 1.5$ cm $= 0.015$ m $\quad$ $d = 0.25$cm $= 2.5 \times 10^{-3}$ m

$r = \frac{d}{2} = 1.25 \times 10^{-3}$ m $\quad$ $m = 10$ Kg $\quad$ Extension e = ?

In this question, material of which wire is made, is not mentioned. So answer may not match with the textbook. Now, in order to solve it, we assume that piano wire is made up of steel whose Young's modulus is $Y = 20 \times 10^{10}$ Pa (textbook page 243).

By Definition, Young's modulus is

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F\ell}{A\,e}$$

Tensile Stress $= \frac{F}{A}$

Tensile Strain $= \frac{e}{\ell}$

Re-arranging, we get

$$e = \frac{F\ell}{AY} \text{ ------------------------------ (1)}$$

$F = m\,g = 10 \times 9.8 = 98$ N

$A = \pi r^2 = 3.14 \times (1.25 \times 10^{-3})^2 = 4.9 \times 10^{-6}$ m^2

Putting all the values in eq (1) we get

$$e = \frac{(98)(0.015)}{(4.9 \times 10^{-6})(20 \times 10^{10})} = 1.5 \times 10^{-6} \text{ m}$$

Problem 16.2 A cable has a length of 12 m and is stretched by 1.2×10^{-4} m when a stress of 8.0×10^8 N/m^2 is applied. What is the strain energy per unit volume in the cable when the stress is applied?

Solution

$\ell = 12$m $\quad$ $e = 1.2 \times 10^{-4}$ m $\quad$ $\sigma = 8 \times 10^8$ N/m^2 $\quad$ $E_v = ?$

Strain energy per unit volume $E_v = \frac{1}{2}$ (Stress x Strain)

$$E_v = \frac{1}{2}(\sigma \times \frac{e}{\ell}) = \frac{1}{2}(8 \times 10^8 \times \frac{(1.2 \times 10^{-4})}{12})$$

$$E_v = 4000 \text{ J m}^{-3}$$

Problem 16.3 A cylindrical steel rod 0.5 m long and 1 cm in radius is subjected to a tensile force of 1×10^4 N. Young's modulus for steel is 20×10^{10} Pa. (a) What is the tensile stress? (b) What is tensile strain? (c) By what amount does the rod stretch?

Solution

$\ell = 0.5$ m $\quad$ $r = 1$cm $= 0.01$m $\quad$ $F = 10^4$ N

(a) Stress $\sigma = ?$ $\quad$ (b) Strain $\varepsilon = ?$ $\quad$ (c) Extension e = ?

(a) Stress $\quad \sigma = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{10^4}{3.14 \times (0.01)^2}$

$$\sigma = 0.32 \times 10^8 \frac{N}{m^2}$$

Young's modulus $Y = 20 \times 10^{10}$ for steel (textbook page 243)

(b) Young's modulus $\quad Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon}$

$$\varepsilon = \frac{\sigma}{Y} = \frac{0.32 \times 10^8}{20 \times 10^{10}} = 1.6 \times 10^{-4}$$

(c) Strain $\varepsilon = \frac{e}{\ell}$

$$e = \varepsilon \times \ell = (1.6 \times 10^{-4})(0.50) = 8 \times 10^{-5}\ \text{m}$$

Problem 16.4 A cable has an un-stretched length of 12 m and it is stretched by 1.2 x 20^{-4} meter when a stress of 6.4 x 10^8 N/m² is applied. What is the strain energy per unit volume in the cable when this stress in applied?

Solution

$\ell = 12\text{m}$ $\quad$ $e = 1.2 \times 10^{-4}\ \text{m}$ $\quad$ if $\sigma = 6.4 \times 10^8 \frac{N}{m^2}$ $\quad$ then $E_v = ?$

Strain energy per unit volume $E_v = \frac{1}{2}(\text{stress} \times \text{strain})$

$$E_v = \frac{1}{2}\left(\sigma \times \frac{e}{\ell}\right)$$

$$E_v = \frac{1}{2}\left(6.4 \times 10^8 \times \frac{(1.2 \times 10^{-4})}{12}\right) = 3.2 \times 10^3\ \text{J m}^{-3}$$

Problem 16.5 Young's Modulus for a particular wood is 1 x 10^{10} N/m². A wooden chair has four legs each of length 42 cm and cross sectional area 2×10^{-3} m². Hamza has a mass of 100 kg. (a) What is the stress on each leg of the chair when Hamza stands on the chair? (b) By what amount do the chair legs shrink when Hamza stands on the chair?

Solution

$Y = 10^{10} \frac{N}{m^2}$ $\quad$ $\ell = 42\text{cm} = 0.42\text{m}$ $\quad$ $A = 2 \times 10^{-3}\ \text{m}^2$

$m = 100$ Kg $\quad$ (a) Stress on each leg $\sigma = ?$ $\quad$ (b) $e = ?$

(a) Stress on each leg is same as stress on all the four legs of chair. Hence

Stress $\quad \sigma = \frac{F}{A} = \frac{mg}{A} = \frac{100 \times 9.8}{2 \times 10^{-3}} = 5 \times 10^5 \frac{N}{m^2}$

(b) Young's modulus $\quad Y = \frac{\sigma}{\varepsilon} = \frac{\sigma \ell}{e}$ $\quad$ (as strain $\varepsilon = \frac{e}{\ell}$)

Re-arranging $\quad e = \frac{\sigma \ell}{Y} = \frac{(5 \times 10^5)(0.42)}{10^{10}} = 2.1 \times 10^{-5}\ \text{m}$

Problem 16.6 A force of 1.5 x 10^4 N causes a strain of 1.4 x 10^{-4} in a steel cable of cross-sectional area 4.8 x 10^{-4} m². (a) What is the Young's Modulus of steel cable? (b) Stress strain graph is linear for this cable. Calculate strain energy per unit volume stored in cable when cable has strain of 1x 10^{-4}.

Solution

$F = 1.5 \times 10^4\ \text{N}$ $\quad$ Strain $\varepsilon = 1.4 \times 10^{-4}$ $\quad$ $A = 4.8\times10^{-4}\ \text{m}^2$

(a) $Y = ?$ $\quad$ (b) $E_v = ?$ if strain $\varepsilon' = 10^{-4}$

(a) Young's modulus $\quad Y = \frac{stress}{strain} = \frac{\sigma}{\varepsilon} = \frac{F}{\varepsilon A} = \frac{1.5 \times 10^4}{(1.4 \times 10^{-4})(4.8 \times 10^{-4})} = 2.2 \times 10^{11} \frac{N}{m^2}$

(b) Strain energy per unit volume

$$E_v = \frac{1}{2}(\text{stress} \times \text{strain})$$

$$E_v = \frac{1}{2}\left(\frac{F}{A} \times \varepsilon'\right) = \frac{1}{2}\left(\frac{1.5 \times 10^4}{4.8 \times 10^{-4}} \times 10^{-4}\right) = 1.56 \times 10^3\ \text{J m}^{-3}$$