the form of wavy curves.

NUMERICAL PROBLEMS

15.1. The peak voltage of an AC supply is 300 V. What is the rms voltage?

Solution

$V_o = 300$ V $\quad V_{rms} = ?$

$$V_{rms} = V_o / \sqrt{2} = 300 / \sqrt{2} = 212 \text{ V}$$

15.2. Rms value of current in an AC circuit is 10A. What is the peak current?

Solution

$i_{rms} = 10$ A $\quad i_o = ?$

$$i_o = i_{rms} \times \sqrt{2} = 14.1 \text{ A}$$

15.3. The A.C voltage across a 0.5 μF capacitor is 16 Sin (2×10^3 t)V. Find (a) the capacitive reactance (b) the peak value of current through capacitor.

Solution

(a) $X_c = ?$ $\quad$ (b) $i_o = ?$ $\quad$ $C = 0.5 \times 10^{-6}$ F

$$V = 16 \text{ Sin } (2000t) \quad (1)$$

(a) Equation for voltage is;

$$V = V_o \text{ Sin } (\omega t) \quad (2)$$

Comparing (1) and (2), we get;

$$\omega = 2000 \text{ Hz}$$

Now as capacitive reactance

$$X_c = \frac{1}{\omega C} = \frac{1}{2000 \times 0.5 \times 10^{-6})} = 1000 \ \Omega$$

(b) Again comparison of eq (1) and (2) shows

$$V_o = 16\text{V}$$

$$i_o = \frac{V_0}{Xc} = \frac{16}{1000} = 16 \text{ mA}$$

15.4. Voltage across a 0.01 μF capacitor is 240 Sin (1.25×10^4t—30°) V. Write the mathematical expression for the current through it.

Solution $\quad C = 0.01\mu$F

$$V = 240 \text{ Sin}(1.25 \times 10^4 t - 30°) \quad (1)$$

Expression for i = ? just like eq (1)

We need i_o , ω and Φ to represent current " i " as an equation;

We have equation for sinusoidally varying voltage as

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$$V = V_o \sin(\omega t + \Phi) \quad (2)$$

Compare (1) and (2) shows that

$$\omega = 2\pi f = 1.25 \times 10^4 \text{ rad/sec}$$

$$f = \frac{\omega}{2\pi} = \frac{1.25 \times 10^4}{2\,(3.14)} = 1990.4 \text{ Hz}$$

$$\Rightarrow \quad X_c = \frac{1}{2\pi f C} = \frac{1}{2\,(3.14)(1990.4)\,(0.01 \times 10^{-6})} = 8000\ \Omega$$

$$i_o = \frac{Vo}{Xc} = \frac{240}{8000} = 0.03 \text{ A}$$

For Φ: In AC capacitive circuit, current leads voltage by 90°. So the phase constant for current will be equal to

$$\Phi = -30^\circ + 90^\circ = 60^\circ$$

Now our required parameters are $i_o = 0.03$ A, $\Phi = 60^\circ$ whereas "ω" will remain the same for current and voltage.

Now our required expression for instantaneously varying current is;

$$i = i_o \sin(\omega t + \Phi)$$

Putting all the values

$$i = 0.03 \sin(1.25 \times 10^4 t + 60^\circ)$$

15.5. An inductor with an inductance of 100 μH passes a current of 10 mA when its terminal voltage is 6.3 V. Calculate the frequency of A.C supply.

Solution

$L = 100\ \mu H$ $\quad i = 10\text{mA}$ $\quad V = 6.3$ V $\quad f = ?$

We have relation for inductive reactance as

$$X_L = 2\pi f L$$

As $\quad X_L = \frac{V}{i} \quad$ So $\Rightarrow \frac{V}{i} = 2\pi f L$

$$f = \frac{V}{2\pi L i} = \frac{6.3}{2\,(3.14)(100 \times 10^{-6})(10 \times 10^{-3})} = 10^6 \text{ Hz}$$

15.6. Calculate the inductive reactance of a 3 mH inductor, when 60 Hz and 10 kHz AC voltages are applied. (b) What is rms current at each frequency if the applied rms voltage is 120 V?

(a) $X_L = ?$ for f_1 & f_2 $\quad L = 3 \times 10^{-3}$H $\quad f_1 = 60$Hz $\quad f_2 = 10$ kHz

(b) $i_{rms\,1} = ?$ & $i_{rms\,2} = ?$ If $V_{rms} = 120$ V

Solution

$$X_{L1} = 2\pi f_1 L$$

$$X_{L1} = 2\,(3.14)\,(60)\,(3 \times 10^{-3}) = 1.13\ \Omega$$

$$X_{L2} = 2\pi f_2 L$$

$$X_{L2} = 2\,(3.14)\,(10 \times 10^3)\,(3 \times 10^{-3}) = 188.4\ \Omega$$

(b) $\quad i_{rms\,1} = \frac{V}{X_{L\,1}} = \frac{120}{1.13} = 106$ A

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Similarly $i_{rms\,2} = \dfrac{V}{X_{L\,2}} = \dfrac{120}{188.4} = 0.637\ A$

15.7. For the same RLC series circuit having a 40 Ω resistor, a 3 mH inductor and a 5 μF capacitor: (a) Find resonant frequency. (b) Calculate i_{rms} at resonance if V_{rms} is 120 V.

R = 40Ω L = 3 m H C = 5 μ F

(a) f_o = ? (b) i_{rms} = ? at resonance if V_{rms} = 120 V

Solution

(a)
$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2(3.14)\sqrt{(3\times10^{-3})(5\times10^{-6})}} = 1300\ Hz$$

(b) At resonance impedance Z = R

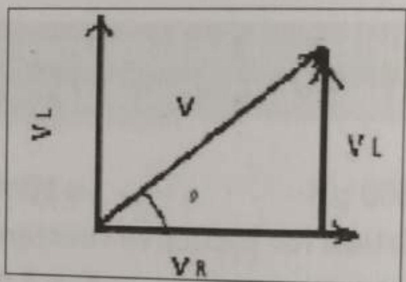
$$i_{rms} = \frac{V_{rms}}{R} = \frac{120}{40} = 3A$$

15.8. A coil of pure inductance 318 mH is connected in series with a pure resistance of 75Ω. The voltage across resistor is 150 V and the frequency of power supply is 50 Hz. Calculate the voltage of power supply and phase angle.

L = 318mH R = 75 Ω V_R = 150 V

f = 50 Hz (a) V = ? (b) Φ = ?

Solution



(a)
$$V_R = i\,R$$

⇒
$$i = \frac{150}{75} = 2\ A$$

In series current " i " remains same for each component

$$V_L = i\,X_L = i\,(2\pi f L)$$
$$V_L = 2\,(2 \times 3.14 \times 50 \times 318 \times 10^{-3})$$
$$V_L = 199.7\ V \ \ldots\ldots\ldots\ldots (1)$$

From figure, using Pythagoras theorem

$$V^2 = V_R^2 + V_L^2 = \sqrt{(150)^2 + (199.7)^2} = 250\ V$$

(b) From figure, $\cos\theta = \dfrac{V_R}{V} = \dfrac{150}{250} = 0.6$

$$\theta = 53.1^\circ$$

15.9. A resistor of resistance 30 Ω is connected in series with a capacitor of capacitance 79.5 μF across a power supply of 50 Hz and 100 V. Find (a) impedance (b) current (c) phase angle and (d) equation for the instantaneous value of current.

R = 30 Ω C = 79.5 μ F f = 50 Hz V = 100 V

(a) Z = ? (b) i = ? (c) ϕ = ? (d) Equation for i = ?

Solution

(a) In RC series circuit, $Z = \sqrt{R^2 + Xc^2}$ (1)

Capacitive reactance $X_c = \dfrac{1}{2\pi f C} = \dfrac{1}{2(3.14)(50)(79.5\times10^{-6})} = 40\ \Omega$

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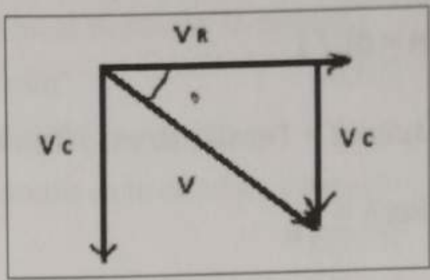
Now putting R and X_c in equation (1)

$$Z = \sqrt{(30)^2 + (40)^2} = 50\ \Omega$$

(b) $i = \frac{V}{Z} = \frac{100}{50} = 2\ A$

(c) From figure $\sin\Phi = \frac{V_c}{V} = \frac{i X_c}{i Z}$



$\sin\Phi = \frac{40}{50} = 0.8$

⇒ $\Phi = \sin^{-1}(0.8) = 53.1°$

(d) $i = i_o \sin(\omega t + \Phi)$ (2)

So we need to find i_o, ω and Φ

$i_o = i\sqrt{2} = 2\sqrt{2} = 2.828$

Since $\omega = 2\pi f = 2(3.14)50 = 314$ rad/sec

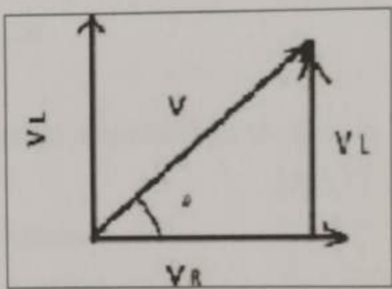
Now eq (2) ⇒ $i = 2.828 \sin(314t + 53°)$

15.10. A coil having a resistance of 7 Ω and an inductance of 31.8 mH is connected to 230 V, 50 Hz supply. Calculate (a) the circuit current (b) phase angle (c) power factor and (d) power consumed.

R = 7Ω L = 31.8mH V = 230 V

f = 50 Hz (a) i = ? (b) Φ = ?

(c) Cos Φ = ? (d) P = ?



Solution

(a) Impedance " Z " for RL series circuit is given by

$$Z = \sqrt{R^2 + X_L^2} \quad (1)$$

$X_L = 2\pi f L = 2(3.14)(50)(31.8 \times 10^{-3}) = 10\ \Omega$

Eq (1) ⇒ $Z = \sqrt{(7)^2 + (10)^2} = 12.2\ \Omega$

As $i = \frac{V}{Z} = \frac{230}{12.2} = 18.8\ A$

(b) From figure $\sin\phi = \frac{V_L}{V} = \frac{i X_L}{i Z} = \frac{10}{12.2} = 0.820$

$\Phi = \sin^{-1}(0.820) = 55°$

(c) Power factor for RL series circuit is defined as

$\cos\phi = \cos(55°) = 0.574$

(d) Power consumed in RL series circuit is

$P = i V \cos\phi = (18.8)(230)(0.574)$

$P = 2481$ Watt

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