NUMERICAL PROBLEMS

Problem 14.1 Two identical coils A and B of 500 turns each has parallel planes such that 70% of flux produced by one coil links with the other. A current of 6 A flowing in coil A produces a flux of 0.06mWb in it. If the current in coil A changes from 10 A to - 10 A in 0.03s, calculate (a) the mutual inductance and (b) the e.m.f induced in coil B.

Solution

$N_A = N_B = 500$ $\quad i_A = 6\text{ A}$ $\quad \Phi_A = 0.06\text{ m Wb}$

$\Phi_{eff} = 70\%$ of $\Phi_A = 0.70 \times 0.06\text{ m Wb} = 0.042 \times 10^{-3}\text{ Wb}$

$i_1 = 10\text{ A}$ $\quad i_2 = -10\text{ A}$ $\quad \Delta t = 0.03\text{ sec}$

(a) $M = ?$ $\quad$ (b) $\varepsilon = ?$

(a) Mutual inductance is given by

$$M = \frac{N_B\ \varphi_{eff}}{i_A} = \frac{500 \times (0.042 \times 10^{-3})}{6}$$

$$M = 3.5\text{ m H}$$

$\Delta i = i_2 - i_1$

$\Delta i = -10 - 10$

$\Delta i = -20\text{ A}$

(b) Induced emf produced in coil B due to change in flux in coil A is;

$$\varepsilon = -M\left(\frac{\Delta i}{\Delta t}\right)_p = -\frac{(3.5 \times 10^{-3})(-20)}{0.03}$$

$$\varepsilon = 2.33\text{ V}$$

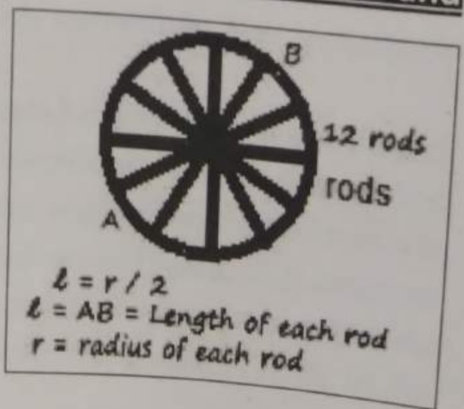
Problem 14.2 A wheel with 12 metal spokes each 0.6 m long is rotated with a speed of 180 r.p.m in a plane normal to earth's magnetic field at a place. If the magnitude of the field is 0.6 G, what is the magnitude of induced e.m.f. between the axle and rim of the wheel?

Solution

$\ell = 0.6\text{ m}$ $\quad r = \ell/2 = 0.3\text{ m}$

$\omega = 180\text{ r.p.m} = 180\left(\frac{2\pi}{60}\right) = 6\pi\text{ rad/sec}$

$B = 0.6\text{ G} = 0.6 \times 10^{-4}\text{ T}$ $\quad \varepsilon = ?$



Emf produced due to motion of conductor inside a magnetic field is

$$\varepsilon = B\ell v \text{ ------------------- } (1)$$

but $v = r\omega$

$\Rightarrow \quad \varepsilon = B\ell(r\omega)$

$\Rightarrow \quad \varepsilon = (0.6 \times 10^{-4})(0.6)(0.3 \times 6 \times 3.14)$

$\Rightarrow \quad \varepsilon = 2.035 \times 10^{-4}\text{ V}$

Problem 14.3 A circuit has 1000 turns enclosing a magnetic circuit $20 cm^2$ in section with 4 A current, the flux density is $1 Wb/m^2$ and with 9 A current, it is $1.4 Wb/m^2$. Find the mean value of the inductance between these current limits and the induced e.m.f. if the current falls from 9 A to 4 A in 0.05 s.

Solution N = 1000 $A = 20\ cm^2$

Flux density $\frac{\varphi_1}{A} = 1\ Wb/m^2$ when $i_1 = 4$ A

Flux density $\frac{\varphi_2}{A} = 1.4\ Wb/m^2$ when $i_2 = 9$ A

$\Delta t = 0.05$ sec $L = ?$ $\varepsilon = ?$

Since $$L = N\frac{\Delta \Phi}{\Delta i} \quad (1)$$

As $\frac{\varphi_1}{A} = 1$

$\Rightarrow$ $$\Phi_1 = 1 \times (20 \times 10^{-4}\ Wb) = 20 \times 10^{-4}\ Wb$$

Also $\frac{\varphi_2}{A} = 1.4\ Wb/m^2$

$\Rightarrow$ $$\Phi_2 = 1.4 \times (20 \times 10^{-4}\ Wb) = 28 \times 10^{-4}\ Wb$$

Change in flux $$\Delta\phi = \phi_2 - \phi_1 = (28 - 20) \times 10^{-4}$$

$$\Delta\phi = 8 \times 10^{-4}\ Wb \quad (2)$$

As $$\Delta i = i_2 - i_1 = 9\ A - 4\ A$$

$$\Delta i = 5 \quad (3)$$

Putting eq (2) and (3) in eq (1) we get

$$L = \frac{1000\ (8 \times 10^{-4})}{5} = 0.16\ H$$

Now induced emf is given by the relation

$$\varepsilon = L\ \frac{\Delta i}{\Delta t} = \frac{0.16 \times 5}{0.05} = 16\ V$$

Problem 14.4 A coil of resistance 100Ω is placed in a magnetic field of 1 mWb. The coil has 100 turns and a galvanometer of 400 Ω resistance is connected in series with it: find the average emf and the current if the coil is moved in 1/10th s from the given field to a field of 0.2 mWb.

Solution

R = 100 Ω $\Phi_1 = 1$ mWb N = 100 $R_g = 400\ \Omega$ (in series)

$\varepsilon = ?$ & $i = ?$ in $\Delta t = 0.1$ sec $\Phi_2 = 0.2$ mWb

According to Faraday's law of electromagnetic induction, induced emf "ϵ"

$$\varepsilon = -N\frac{\Delta \Phi}{\Delta t} \quad (1)$$

Where $$\Delta\Phi = \Phi_2 - \Phi_1 = 0.2 - 1 = -0.8\ mWb$$

Eq (1) => $\varepsilon = -100 \dfrac{-0.8 \times 10^{-3}}{0.1} = 0.8\ V$

Total resistance in series combination will be $R + R_g$

So the current $i = \dfrac{\varepsilon}{R+R_g} = \dfrac{0.8}{100+400} = 1.6\ mA$

Problem 14.5 A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5 m/s at right angles to the horizontal component of earth's magnetic field 0.30 x 10^{-4} Wb/m^2(a)What is instantaneous value of emf induced in wire? (b) What is the direction of the emf? (c) Which end of the wire is at the higher electrical potential?

Solution

$\ell = 10$ m $v = 5$ m/sec $B = 0.30 \times 10^{-4}$ Wb/m^2

$\theta = 90°$ (Induced emf in a conductor is maximum when it moves perpendicular both to its own length and to the B field)

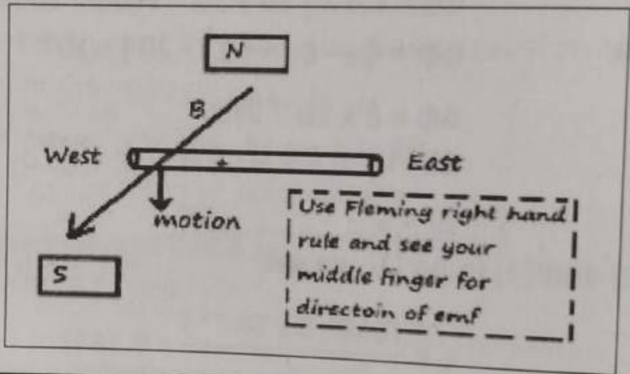
(a) ε = ? (b) Direction of ε = ? (c) Which end is at high potential?

(a) Motional emf in a conductor moving in a magnetic field is

$\varepsilon = B\ell v \sin\theta$

$\varepsilon = (0.30 \times 10^{-4})(10)(5)\sin 90°$

$\varepsilon = 1.5 \times 10^{-3}$ V



(b) Applying Fleming Right hand rule shows that direction of this induced emf is from east to west.

(c) The end towards east will be at higher electric potential.

Problem 14.6 Current in a circuit falls from 5 A to 0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit.

$i_1 = 5$ A $i_2 = 0$ A $\Delta t = 0.1$ s $\varepsilon = 200$ V L = ?

Solution

$$\varepsilon = -L\frac{\Delta i}{\Delta t}$$

$$\Rightarrow \quad L = -\varepsilon\frac{\Delta t}{\Delta i} = -\frac{200 \times 0.1}{(-5)} = 4\ H$$

Problem 14.7 A long solenoid with 15 turns per cm has a small loop of area 2 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily fron 2 A to 4 A in 0.1 s, what is the induced emf in the loop while the current is changing?

Solution

n = 15 per cm = 1500 per meter $\Delta A = 2\ cm^2 = 2 \times 10^{-4}\ m^2$

$\Delta i = 4 - 2 = 2$ A $\Delta t = 0.1$ sec ε = ?

Magnetic field inside a current carrying solenoid is

$$\Delta B = \mu_o n \Delta i = (4 \times 3.14 \times 10^{-7})(1500)(2) = 3.77 \text{ m T}$$

So emf ε for this single loop (N = 1) will be

$$\varepsilon = \frac{\Delta B\, A}{\Delta t} = \frac{(3.77 \times 10^{-3})(2 \times 10^{-4})}{0.1} = 7.54 \times 10^{-6} \text{ V}$$

Problem 14.8 A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to loop. What is emf developed across the cut if velocity of loop is 1 cm/sec in a direction normal to (a) longer side (b) shorter side of loop? For how long does induced voltage last in each case?

Solution

Length ℓ = 8cm = 0.08m Width b = 2cm = 0.02m N = 1

B = 0.3 T $\theta = 90°$ v = 1cm/s = 0.01 m/s

(a) emf ε= ? for Longer side (b) emf ε= ? shorter side $\Delta t_1 = \Delta t_2$ = ?

(a) Emf developed due to the motion of wire loop inside magnetic field is

$\varepsilon = B \ell v \sin\theta$ (ℓ for longer side)

$\varepsilon = (0.3)(0.08)(0.01) \sin 90°$

$\varepsilon = 2.4 \times 10^{-4}$ V

$\Delta\phi = BA = B(\ell b)$

$\Delta\phi = 0.3\,(0.08 \times 0.02)$

$\Delta\phi = 4.8 \times 10^{-4}$ Wb

As $\varepsilon = N \frac{\Delta \Phi}{\Delta t}$ re-arranging it, we get

$\Rightarrow$ $\Delta t_1 = N \frac{\Delta \Phi}{\varepsilon} = \frac{1 \times 4.8 \times 10^{-4}}{2.4 \times 10^{-4}} = 2$ sec

(b) Similarly emf developed due to motion of wire loop inside magnetic field

$\varepsilon = B\, b\, v \sin\theta$ (b is used for shorter side)

$\varepsilon = (0.3)(0.02)(0.01) \sin 90° = 0.6 \times 10^{-4}$ V

As $\varepsilon = N \frac{\Delta \Phi}{\Delta t}$

$\Rightarrow$ $\Delta t_2 = N \frac{\Delta \Phi}{\varepsilon} = \frac{1 \times 4.8 \times 10^{-4}}{0.6 \times 10^{-4}} = 8$ sec

Problem 14.9 A 90mm length of wire moves with an upward velocity of 35 m/s between the poles of a magnet. The magnetic field is 80 mT directed to the right. If the resistance in the wire is 5 mΩ what are the magnitude and direction of the induced current?

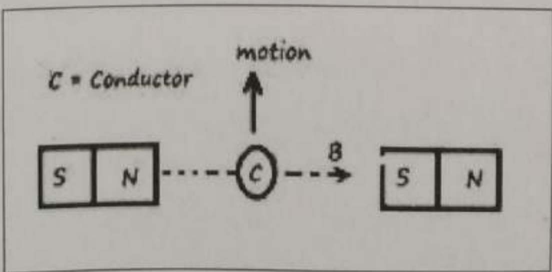
Solution

ℓ = 90 mm = 0.09 m v = 35 m/sec

B = 80 mT = 80 x 10^{-3} T

$\theta = 90°$ R = 5 mΩ = 0.005 Ω

i = ? (Magnitude and direction)



Using equation for motional emf is

$$\varepsilon = B\,\ell\,v\,\text{Sin}\,\theta$$

$$\varepsilon = (80 \times 10^{-3})\,(0.09)\,(35)\,\text{Sin}90^{\circ} = 0.252\text{ V}$$

The current is $$i = \frac{\varepsilon}{R} = \frac{0.252}{0.005} = 50.4\text{ A}$$

Direction of the current, according to Fleming Right hand rule, is into the plane of page (see figure).

Problem 14.10 A pair of adjacent coils has mutual inductance of 1.5 H. if current in one coil changes from 0 to 20A in 0.5 s, what is change of flux linkage with other coil?

Solution

$M = 1.5\text{ H}$ $\quad$ $i_1 = 0$ $\quad$ $i_2 = 20\text{A}$ in $\Delta t = 0.5$ sec

Change in current $\Delta i = i_2 - i_1 = 20\text{A}$ $\quad$ $N(\Delta\phi) = ?$

From Faraday's law of electromagnetic induction, magnitude of induced emf

$$\varepsilon = N\frac{\Delta \Phi}{\Delta t} \quad \text{-------------------------------- (1)}$$

Also magnitude of emf due to mutual induction is given by

$$\varepsilon = M\frac{\Delta i}{\Delta t} \quad \text{-------------------------------- (2)}$$

By comparing; we get; $$N\frac{\Delta \Phi}{\Delta t} = M\frac{\Delta i}{\Delta t}$$

Multiplying both sides by Δt $$N(\Delta\phi) = M(\Delta i)$$

Hence $$N(\Delta\phi) = 1.5 \times 20 = 30\text{ Wb}$$

Problem 14.11 Back emf in a motor is 120V when motor is turning at 1680 rev/min. what is back emf when the motor turns at 3360 rev/min?

Solution

Back emf $\varepsilon_1 = 120$ V $\quad$ When $\omega_1 = 1680\text{ r.p.m} = 1680 \times \frac{2\pi}{60} = 56\pi$ rad/sec

Back emf $\varepsilon_2 = ?$ $\quad$ When $\omega_2 = 3360\text{ r.p.m} = 3360 \times \frac{2\pi}{60} = 112\pi$ rad/sec

In motor, back emf produced is given by equation $\varepsilon = N\,A\,B\,\omega\,\text{Sin}\theta$

First case: $$\varepsilon_1 = N\,A\,B\,\omega_1\,\text{Sin}\,\theta \quad (1)$$

Second case: $$\varepsilon_2 = N\,A\,B\,\omega_2\,\text{Sin}\,\theta \quad (2)$$

Dividing eq. 1 by eq.2 and re-arranging, we get;

$$\varepsilon_2 = \frac{\omega_2}{\omega_1}\,\varepsilon_1 = \frac{112\pi}{56\pi} \times 120 = 240\text{ V}$$