

...ing heating and so on.

Numerical problems

1. What is the magnitude of the force of attraction between an iron nucleus bearing charge $q = 26e$ and its innermost electron, if the distance between them is 1×10^{-12} m. (6×10^{-3} N)
2. Charges $2 \mu\text{C}$, $-3 \mu\text{C}$, and $4 \mu\text{C}$ are placed in air at the vertices of an equilateral triangle of sides 10 cm. what is the magnitude of resultant force acting on $4 \mu\text{C}$ charge?

~~15.7 N~~
(9.5 N) ~~(15.7 N)~~

3. A charge q is placed at the centre of the line joining the two charges, each of magnitude Q . Prove that the system of three charges will be in equilibrium if $q = -Q/4$.

4. Two equal and opposite charges of magnitude 2×10^{-7} C are placed 15 cm apart. What is the magnitude and direction of electric intensity (E) at a point mid-way between the charges? What force would act on a proton (charge = $+1.6 \times 10^{-19}$ C) placed there?

($.64 \times 10^6$ N/C along AB, 1.024×10^{-13} N along AB)

5. Two positive point charges of 15×10^{-10} C and 13×10^{-10} C are placed 12 cm apart. Find the work done in bringing the two charges 4 cm closer.
(7.31×10^{-8} J)

6. A hollow sphere is charged to $14 \mu\text{C}$. Find the potential (a) at its surface (b) inside the sphere (c) at a distance of 0.2 m from the surface. The radius of the sphere is 0.3 m.

(42×10^4 V, 42×10^4 V, 25.2×10^4 V)

7. If 280 J of work is done in carrying a charge of 2 C from a place where the potential is -12 V to another place where potential is V, calculate the value of V.
(128 V)

8. Calculate the electric potential at the surface of a silver nucleus having radius 3.4×10^{-14} m. The atomic number of silver is 47 and charge on a proton = 1.6×10^{-19} C.
(1.99×10^6 V)

9. The electric field at a point due to a point charge is 26 N/C and the electric potential at that point is 13 J/C. Calculate the distance of the point from the charge and magnitude of charge.
(0.5 m, 0.722×10^{-9} C)

10. Two point charges of $8 \mu\text{C}$ and $-4 \mu\text{C}$ are separated by a distance of 10 cm in air. At what point on the line joining the two charges is the electric potential zero?

(6.6 cm from $8 \mu\text{C}$ and, 3.3 cm from $-4 \mu\text{C}$ charge)

11. An electron with an initial speed of $29 \times 10^5 \text{ ms}^{-1}$ is fired in the same direction as a uniform electric field with a magnitude of 80 NC^{-1} . How far does the electron travel before being brought to rest momentarily and turned back?

$$K.E = q \Delta V$$

$$K.E = qEd$$

$$\frac{1}{2}mv^2 = qEd \quad (.299\text{m})$$

12. Two capacitors of capacitance $4 \mu\text{F}$ and $8 \mu\text{F}$ are first connected (a) in series and then (b) in parallel. In each case external source of voltage is 200 V . Calculate in each case the total capacitance, the potential drop across each capacitor, and the charge on each capacitor.

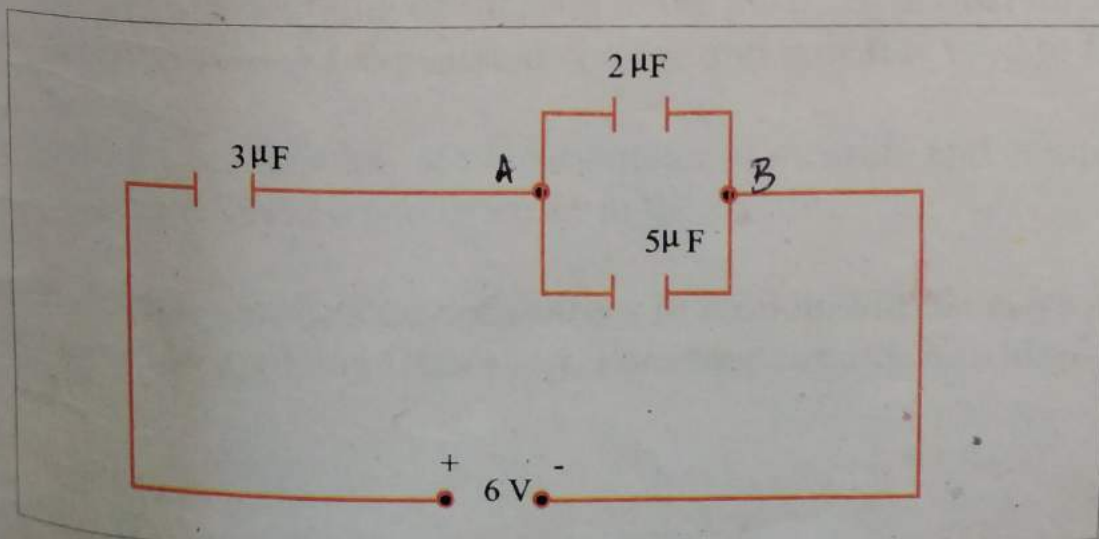
$$(2.66\mu\text{F}, 5.33 \times 10^{-4}\text{C}, 133.2\text{V}, 66.6\text{V}, 12\mu\text{F}, 200\text{V}, .08\mu\text{C}, .16\mu\text{C})$$

13. Three capacitors of capacitance $4\mu\text{F}$, $6\mu\text{F}$ and $8\mu\text{F}$ respectively are connected in series to a 250V d.c. supply. Find (i) the total capacitance (ii) charge on each capacitor and (iii) P.D. across each capacitor.

$$(1.84 \mu\text{F}, 460 \times 10^{-6} \text{C}, 115 \text{V}, 76.6\text{V} \text{ and } 57.7 \text{V})$$

14. If $C_1 = 14\mu\text{F}$, $C_2 = 20 \mu\text{F}$, $C_3 = 12\mu\text{F}$ and the insulated plate of C_1 be at potential of 100V , one plate of C_3 being earthed, what is the potential difference between the plates of C_2 , three capacitors being connected in series? $(24.4 \mu\text{V})$

15. Find the charge on $5\mu\text{F}$ capacitor in the circuit shown in Fig. .



$$(9 \mu\text{C})$$

16. Two parallel plate capacitors A and B having capacitance of $2 \mu\text{F}$ and $6 \mu\text{F}$ are charged separately to the same potential of 120V . Now positive plate of A is connected to the negative plate of B and the negative plate of A is connected to the positive of B. Find the final charge on each capacitor.

($120 \mu\text{C}$, $360 \mu\text{C}$)

17. A $6 \mu\text{F}$ capacitor is charged to a P.D. of 120V and then connected to an un-charged $4 \mu\text{F}$ capacitor. Calculate the P.D. across the capacitors.

(72V)

18. Two capacitor of capacitance $8 \mu\text{F}$ and $10 \mu\text{F}$ respectively are connected in series across a P.D. of 180V . The capacitors are disconnected from the supply and are reconnected in parallel with each other. Calculate the new P.D. and charge on each capacitor.

(88.8V , $710 \mu\text{C}$, $888 \mu\text{C}$)

Numerical Problems

1. Solution: Charge on iron nucleus, $q_1 = 26e$, Charge on inner most electron, $q_2 = e$
Where $e = 1.6 \times 10^{-19} \text{ C}$, Separation, $r = 1 \times 10^{-12} \text{ m}$.

Coulomb's constant for vacuum, $k = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$,

According to Coulomb's Law:

$$F = k \frac{q_1 q_2}{r^2} = 6 \times 10^{-3} \text{ N}$$

2. Solution: $q_1 = 2 \mu\text{C}$, $q_2 = -3 \mu\text{C}$, $q_3 = 4 \mu\text{C}$

Separation between any two charges, $r = 10 \text{ cm} = 0.1 \text{ m}$,

Magnitude of resultant force on $q_3 = F = ?$

Let F_1 be the force on q_3 due to q_1 and is given by,

$$F_1 = k \frac{q_1 q_3}{r^2} = 7.2 \text{ N}$$

similarly, Let F_2 be the force on q_3 due to q_2 and is given by,

$F_2 = k \frac{q_2 q_3}{r^2} = -10.8 \text{ N}$, the negative sign shows that the force is attractive, and its direction is correctly represented in the diagram.

Now resolve F_1 and F_2 into its components.

For F_1 :-

$$F_{1x} = F_1 \cos 60^\circ = 3.6 \text{ N}$$

$$F_{1y} = F_1 \sin 60^\circ = 6.24 \text{ N}$$

For F_2 :-

$$F_{2x} = F_2 \cos 60^\circ = 5.4 \text{ N}$$

$$F_{2y} = F_2 \sin 60^\circ = 9.35 \text{ N}$$

Then, in the light of the diagram, $F_x = F_{1x} + F_{2x} = 9 \text{ N}$

and $F_y = F_{1y} - F_{2y} = -3.11 \text{ N}$

Now magnitude of $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(9)^2 + (-3.11)^2} = \sqrt{90.67} = 9.52 \text{ N}$

3. Solution: Let two equal charges of magnitude Q , are placed apart $2r$, at points A and C, as shown.

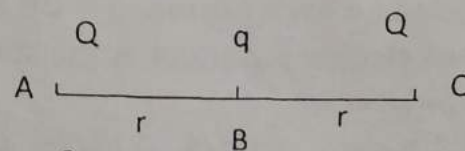
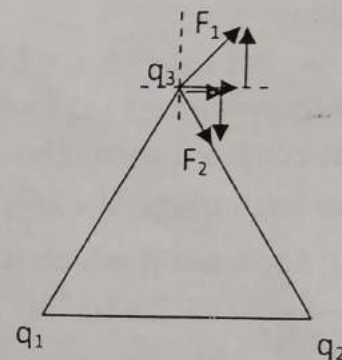
Let q is a charge placed at mid point B. In order to prove that $q = -\frac{Q}{4}$, for which the system of these charges is in equilibrium, let we focus on charge placed at point A. Let F_1 be the force on Q (at A) due to q (at B). Let F_2 be the force on Q (at A) due to Q (at C).

Then, according to the statement of the problem, $F_1 + F_2 = 0 \rightarrow (1)$

To find F_1 and F_2 , we use coulomb's law.

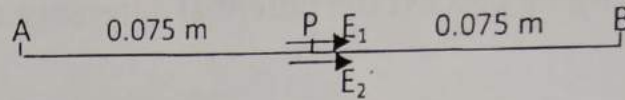
$$F_1 = k \frac{Qq}{r^2}, \text{ and } F_2 = k \frac{QQ}{(2r)^2}$$

Therefore, Eq. 1 $\Rightarrow k \frac{Qq}{r^2} + k \frac{QQ}{(2r)^2} = 0 \Rightarrow k \frac{Qq}{r^2} = -k \frac{QQ}{(2r)^2} \Rightarrow q = -\frac{Q}{4}$ Proved



4. Solution:-

$q_1 = +2 \mu\text{C}$ and $q_2 = -2 \mu\text{C}$, separation between the two charges = $15 \text{ cm} = 0.15 \text{ m}$
Let P be the mid-point. Let E.F. intensity at P due to q_1 and q_2 respectively are E_1 and E_2 . Their correct directions are represented in the diagram below.



$$\text{Thus, } E_1 = k \frac{q_1}{r^2} = 3.2 \times 10^5 \text{ N/C}$$

$$\text{and } E_2 = k \frac{q_2}{r^2} = 3.2 \times 10^5 \text{ N/C} \text{ (the negative sign has compensated by representing the direction)}$$

As E_1 and E_2 are in same direction therefore, net E.F. Intensity,

$$E = E_1 + E_2 = 6.4 \times 10^5 \text{ N/C} = 0.64 \times 10^6 \text{ N/C, from A to B}$$

To find the magnitude of force on a proton placed at mid-point, we know that charge on proton is $q = 1.6 \times 10^{-19} \text{ C}$

Let F be the net force, then $F = Eq = 1.024 \times 10^{-13} \text{ N}$ from A to B

5. Solution:

$q_1 = 15 \times 10^{-10} \text{ C}$, $q_2 = 13 \times 10^{-10} \text{ C}$, Initially separated by a distance, $r_2 = 12 \text{ cm} = 0.12 \text{ m}$
final separation between charges, $r_1 = 8 \text{ cm} = 0.08 \text{ m}$

As we know that in an electric field work done appears as change in electric P.E (ΔU), therefore,

$$W = (\Delta U) = k q_1 q_2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = 7.31 \times 10^{-8} \text{ J}$$

6. Solution:

Charge on the sphere, $q = 14 \mu\text{C} = 14 \times 10^{-6} \text{ C}$, Radius of the sphere, $r = 0.3 \text{ m}$

The solution for part (a) and part (b) is the same because we know that electric potential remains constant throughout the volume of a conducting sphere.

$$\text{Thus for both parts, } V = k \left(\frac{q}{r} \right) = 42 \times 10^4 \text{ V}$$

Part (c): For a point which is $r_1 = 0.2 \text{ m}$ away from the surface of the sphere

$$V = k \left(\frac{q}{r + r_1} \right) = 25.2 \times 10^4 \text{ V}$$

7. Solution:

Charge to be moved, $q = 2 \text{ C}$, Electric potential at point B, $V_B = -12 \text{ V}$

Electric potential at point A, $V_A = V = ?$, W. Done from B to A, $W_{B \rightarrow A} = 280 \text{ J}$

As we know that P.d between A and B is, $\Delta V = \frac{W_{B \rightarrow A}}{q} = \frac{280}{2} = 140 \text{ V}$

As, P.d = (High voltage) - (low voltage) = $V_A - V_B$

$$\text{or } V_A = \Delta V + V_B = 140 + (-12) = 128 \text{ V}$$

8. Solution:

Radius of silver nucleus, $r = 3.4 \times 10^{-14} \text{ m}$, Atomic number of silver, $Z = 47$

charge on each proton, $q = 1.6 \times 10^{-16} \text{ C}$, total charge on silver nucleus, $Q = Zq = 75.2 \times 10^{-19} \text{ C}$

Thus electric potential at the surface of the silver nucleus, $V = (kQ)/r = 1.99 \times 10^6 \text{ V}$

9. Solution:

$$\text{(a) } E = V/r, \quad r = V/E = 13/26 = 0.5 \text{ m}$$

$$\text{(b) As we know, } V = (kq)/r \Rightarrow q = (Vr)/k = 0.722 \times 10^{-9} \text{ C}$$

10. Solution:

$q_1 = 8 \mu\text{C}$, $q_2 = -4 \mu\text{C}$, separation between the charges, $r = 10 \text{ cm} = 0.1 \text{ m}$

Let the distance of the point of zero potential is $x \text{ m}$ from q_1 and $(r - x) \text{ m}$ from q_2 on the line between them. Let $V_1 =$ electric potential at that point due to q_1

$V_2 =$ electric potential at that point due to q_2

Thus according to the statement of the question, $V_1 + V_2 = 0$

$$[(kq_1)/x] + [(kq_2)/(r-x)] = 0 \Rightarrow [(kq_1)/x] = -[(kq_2)/(r-x)]$$

$$\frac{q_1}{x} = -\frac{q_2}{(r-x)} \Rightarrow \frac{8 \mu\text{C}}{x} = -\frac{(-4 \mu\text{C})}{(r-x)} \Rightarrow \frac{2}{x} = \frac{1}{r-x} \Rightarrow x = 2r - 2x \Rightarrow 3x = 2r$$

$$\Rightarrow x = 2r/3 = 0.0666 \text{ m} = 6.6 \text{ cm}$$

hence the point is 6.6 cm away from q_1 and $(r-x) = 10 - 6.6 = 3.3 \text{ cm}$ from q_2 on the line joining them.

11. Solution:

Loss in K.E = w. done By electric field = $qV = qEd$

$$\Rightarrow d = (\text{loss in K.E}) / qE = \left(\frac{1}{2} m v^2\right) / qE = 0.299 \text{ m}$$

12. Solution:

$C_1 = 4 \mu\text{F}$, $C_2 = 8 \mu\text{F}$, $V = 200 \text{ V}$

(a) $C_{\text{eq}} = ?$ in series

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = 2.6 \times 10^{-6} \text{ F}$$

$$Q = C_{\text{eq}} V = 5.33 \times 10^{-4} \text{ C}$$

As we know that in series combination, p.d across different capacitors is different.

$$V_1 = Q / C_1 = 133.2 \text{ V}$$

$$V_2 = Q / C_2 = 66.6 \text{ V}$$

(b) $C_{\text{eq}} = ?$ in parallel

$$C_{\text{eq}} = C_1 + C_2 = 12 \times 10^{-6} \text{ F}$$

in parallel the p.d. across each capacitor is same, i.e. 200 V

in parallel charge on each capacitor will be different

$$Q_1 = C_1 V = 8 \times 10^{-4} \text{ C}$$

$$Q_2 = C_2 V = 16 \times 10^{-4} \text{ C}$$

13. Solution:

$C_1 = 4 \mu\text{F}$, $C_2 = 6 \mu\text{F}$, $C_3 = 8 \mu\text{F}$, $V = 250 \text{ V}$

(a) $C_{\text{eq}} = ?$ in series.

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{4} + \frac{1}{6} + \frac{1}{8} = \frac{6+4+3}{24} = \frac{13}{24}$$

$$\Rightarrow C_{\text{eq}} = 1.846 \times 10^{-6} \text{ F}$$

(b) in series combination charge on each capacitor is same, as total charge

$$Q_1 = Q_2 = Q_3 = Q = C_{\text{eq}} V = 462 \times 10^{-6} \text{ C}$$

(c) in series the p.d. across each capacitor is different.

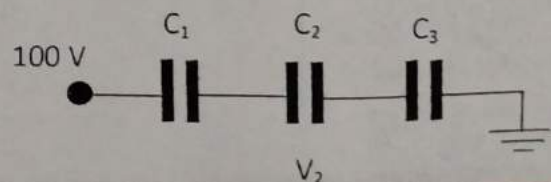
$$V_1 = Q / C_1 = 115.5 \text{ V}$$

$$V_2 = Q / C_2 = 77 \text{ V}$$

$$V_3 = Q / C_3 = 57.7 \text{ V}$$

14. Solution:

$C_1 = 14 \mu\text{F}$, $C_2 = 20 \mu\text{F}$, $C_3 = 12 \mu\text{F}$, $V = 100 \text{ V}$,



p.d. across C_2 , $V_2 = ?$

In series combination, p.d. across each capacitor is different. therefore,

$$V_2 = \frac{Q_{total}}{C_2} \rightarrow (1)$$

To find total charge, first we need to determine the C_{eq} .

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow C_{eq} = 4.88 \times 10^{-6} \text{ F}$$

$$\text{Now, } Q_{total} = C_{eq} V = 488 \times 10^{-6} \text{ C}$$

$$\text{thus from eq. (1), } V_2 = (488 \times 10^{-6} \text{ C}) / (4.88 \times 10^{-6} \text{ F}) = 24.4 \text{ V}$$

15. solution

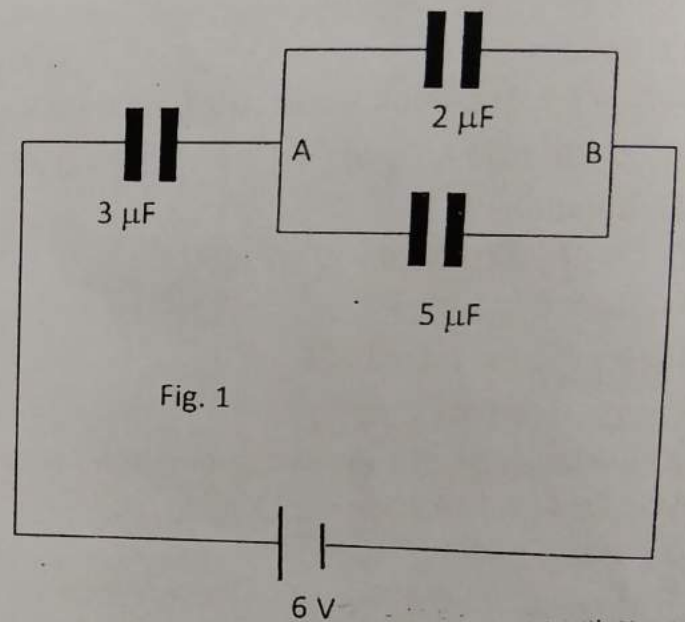
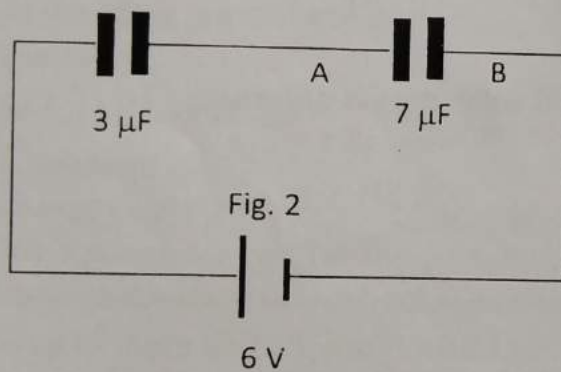
Let, $C_1 = 3 \mu\text{F}$

$C_2 = 2 \mu\text{F}$ and $C_3 = 5 \mu\text{F}$

Let C_{AB} the net/total capacitance between A and B, in fig. 1. Then,

$$C_{AB} = 7 \mu\text{F}$$

Now the circuit can be re-draw, as shown in fig.2.



Let C_{eq} be the net capacitance of fig.2, then $C_{eq} = \frac{C_1 C_{AB}}{C_1 + C_{AB}} = \frac{3 \times 7}{3 + 7} = \frac{21}{10} = 2.1 \mu\text{F}$

Let Q be the total charge stored in circuit shown in fig.2

$$Q = C_{eq} V = 12.6 \mu\text{C}$$

Thus charge stored on C_{AB} will also be 12.6 μC . Now p.d. between A and B is,

$$V_{AB} = \frac{Q}{C_{AB}} = \frac{12.6}{7} = 1.8 \text{ V}$$

Now charge on C_3 , $Q_3 = C_3 V_{AB} = 5 \mu\text{F} \times 1.8 \text{ V} = 9 \mu\text{C}$

$$q_3 = 9 \mu\text{C}$$

11.16 Two parallel plate capacitors A and B having capacitance of $2 \mu\text{F}$ and $6 \mu\text{F}$ are charged separately to the same potential of 120 V . Now positive plate of A is connected to the negative plate of B and the negative plate of A is connected to the positive of B. Find the final charge on each capacitor.

Solution

$$C_1 = 2 \mu\text{F}$$

Initial charge on A;

Initial charge on B;

So net capacitance C_{eq} is

$$C_2 = 6 \mu\text{F}$$

$$V = 120 \text{ V}$$

$$Q_1 = Q_2 = ?$$

$$q_1 = C_1 V = (2 \times 10^{-6}) (120) = 240 \mu\text{C}$$

$$q_2 = C_2 V = (6 \times 10^{-6}) (120) = 720 \mu\text{C}$$

$$C_{\text{eq}} = C_1 + C_2 = 8 \mu\text{F}$$

When they are connected, net charge

$$Q = q_2 - q_1 = (720 - 240) \mu\text{C}$$

$$Q = 480 \mu\text{C}$$

New potential difference

$$V_n = \frac{Q}{C_{\text{eq}}} = \frac{480 \times 10^{-6}}{8 \times 10^{-6}} = 60 \text{ V}$$

Now final charge on A;

$$Q_1 = C_1 V_n = (2 \times 10^{-6}) (60) = 120 \mu\text{C}$$

And final charge on B;

$$Q_2 = C_2 V_n = (6 \times 10^{-6}) (60) = 360 \mu\text{C}$$

11.17 A $6 \mu\text{F}$ capacitor is charged to a P.D. of 120 V and then connected to an uncharged $4 \mu\text{F}$ capacitor. Calculate the P.D. across the capacitors.

Solution

$$C_1 = 6 \times 10^{-6} \text{ F} \quad V_1 = 120 \text{ V}$$

$$C_2 = 4 \times 10^{-6} \text{ F} \quad V = ? \text{ (Parallel)}$$

$$Q = C_1 V_1 = (6 \times 10^{-6}) (120) = 720 \mu\text{C}$$

When capacitors are connected in parallel then total capacitance

$$C_{\text{eq}} = C_1 + C_2 = (6 + 4) \mu\text{F} = 10 \mu\text{F}$$

As both are in parallel so P.D. across both will be same, given by equation

$$V = \frac{Q}{C} = \frac{720 \times 10^{-6}}{10 \times 10^{-6}} = 72 \text{ V}$$

11.18 Two capacitors of capacitance $8 \mu\text{F}$ and $10 \mu\text{F}$ respectively are connected in series across a P.D. of 180 V . The capacitors are disconnected from the supply and

are reconnected in parallel with each other. Calculate the new P.D. and charge on each capacitor.

Solution

$$C_1 = 8 \times 10^{-6} \text{ F} \quad C_2 = 10 \times 10^{-6} \text{ F} \quad V = 180 \text{ V (Series)}$$

$V' = ?$ $q_1 = ?$ and $q_2 = ?$ when connected in parallel & supply removed

In series combination, equivalent capacitance C_{eq} is

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{8 \times 10 \times 10^{-12}}{(8 + 10) 10^{-6}} = 4.4 \mu\text{F}$$

Now total charge

$$Q = C_{eq} \times V = (4.4 \times 10^{-6}) \times 180 \approx 800 \mu\text{C}$$

Since they are connected in series so

$$q_1 = q_2 = Q = 800 \mu\text{C}$$

Total charge q_{tot} in parallel combination is sum of charges q_1 & q_2

So

$$q_{total} = q_1 + q_2 = (800 + 800) \mu\text{C} = 1600 \mu\text{C}$$

When capacitors are connected in parallel, equivalent capacitance is

$$C_{eq} = C_1 + C_2 = (8 + 10) \mu\text{F} = 18 \mu\text{F}$$

Total charge q_{tot} in parallel combination is distributed between two capacitors to have a common P.D. so new potential difference (V') across the capacitors is

$$V' = \frac{\text{total charge } (q_{tot})}{\text{total capacitance } (C_{eq})} = \frac{1600 \times 10^{-6}}{18 \times 10^{-6}} = 88.89 \text{ V}$$

New charge on $8 \mu\text{F}$ capacitor is

$$Q_1 = C_1 V' = (8 \mu\text{F}) (88.89) = 711 \mu\text{C}$$

New charge on $10 \mu\text{F}$ capacitor

$$Q_2 = C_2 V' = (10 \mu\text{F}) (88.89) = 888.9 \mu\text{C}$$