Course Overview – Part II

Week #	*Contact Hours*	*Topics*	*Assignment/ Quiz*
10	03	**Momentum and Forces in Fluid Flow:** Pressure due to jet of water on fixed flat plates, pressure on moving plates, pressure on fixed curved plates	
11	03	Flow over radial vanes, jet propulsion	
12	03	**Hydraulic Machinery – Turbines:** Classification of turbines, reaction and impulse turbines, Specific speed of turbines, the governing of turbines	A–4 Q–4
13	03	Pelton wheel and Francis turbine	
14	03	Water Hammer	
15	03	**Hydraulic Machinery – Pumps:** Axial flow and Centrifugal pumps, head developed by pumps,	A–5 Q–5
16	03	Performance characteristics at constant and different speeds and sizes, Specific Speed,	
17	03	Selection of pumps, pump installations	A–6 Q–6
18	**FINAL TERM EXAM**		

Chapter 5

Dynamic Force and Momentum

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Contents

- Pressure due to Jet of Water on Fixed Flat Vertical Plate
- Pressure due to Jet of Water on Fixed Flat Inclined Plate
- Pressure due to Jet of Water on Moving Flat Plates
- Pressure on Fixed Curved Plate
- Flow Over Radial Vanes
- Jet Propulsion

Rate of Flow or Discharge (Q)

- It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel.

- For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second.

- For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section. Thus,

  i. For liquids the units of Q are m^3/s or liters/s
  ii. For gases the units of Q is kgf/s or Newton/s

- Consider a liquid flowing through a pipe in which

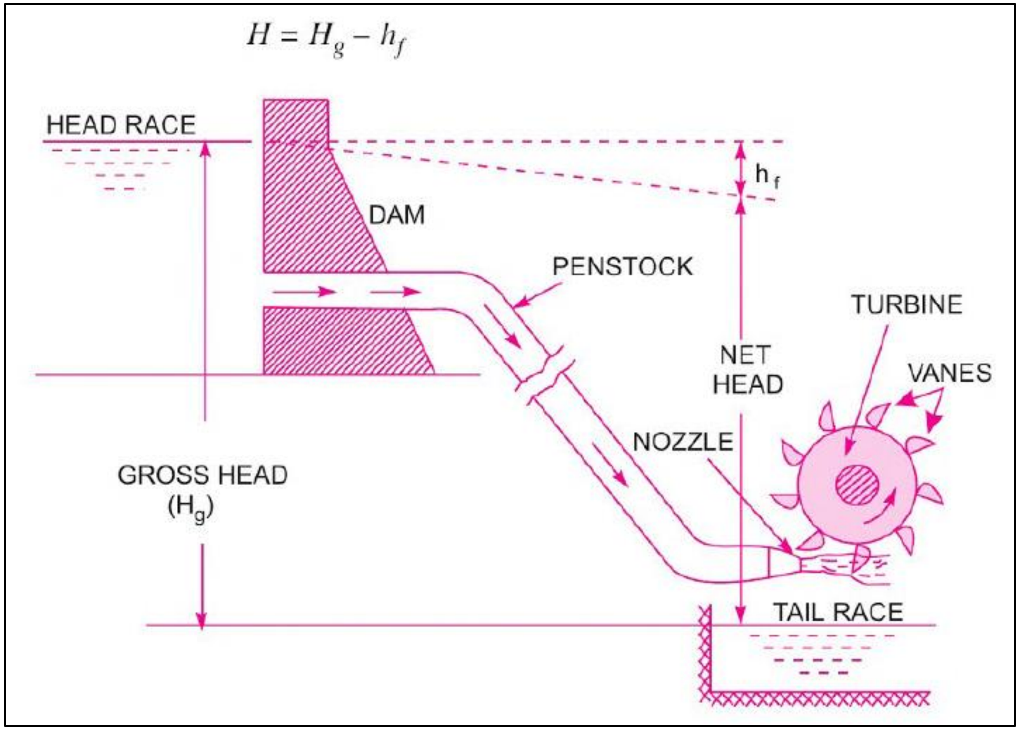
  A = Cross-sectional area of pipe

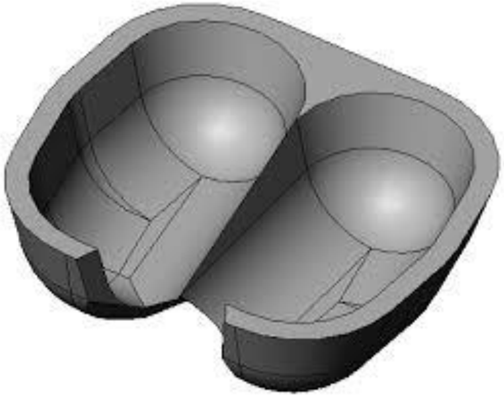
  V = Average velocity of fluid across the section

  Then discharge,

  $Q = A \times V$

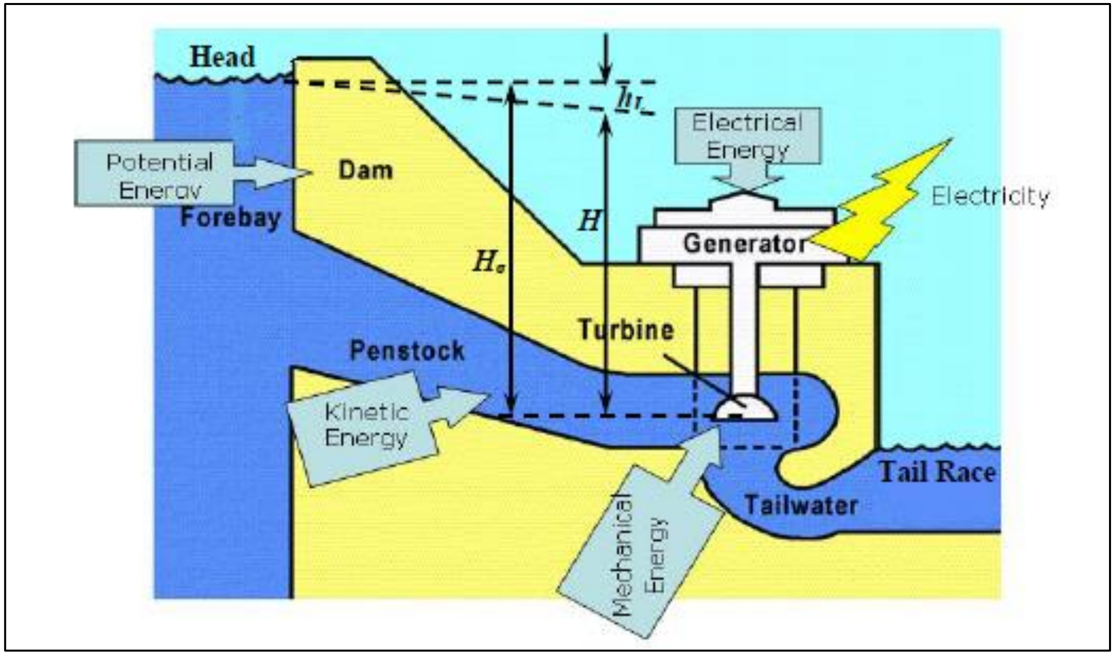
Pelton Wheel Turbine

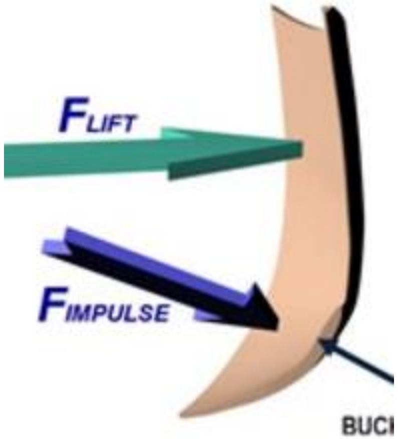




Pelton Bucket

Francis Turbine





Francis Vane

Pressure due to Jet of Water on Fixed Flat Vertical Plate

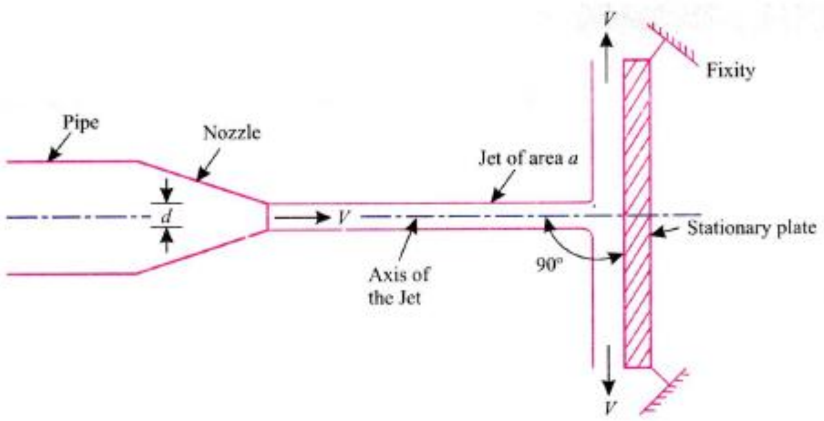
- The impact of jet coming out from a pipe's nozzle means the "force" by which the jet strikes the plates/vanes of a turbine.
- If some plate (or vane) which may be fixed or moving, is placed in the path of the jet, a force is exerted by the jet on the plate.
- As per Newton's Second Law of Motion , this force is equal to the rate of change of momentum of the jet, or to the change of momentum per second, and may be obtained from the impulse-momentum equation.

$$F = ma$$

$$F = m\frac{\Delta V}{t}$$

$$F = \frac{\Delta p}{t}$$

$F.t = m.\Delta V$ (Imp-Mom Eq.)



- It has been observed that if the plate is not fixed, then the plate starts moving in the direction of the jet, because of the force.
- The jet strikes the plate and leaves it tangentially so that all its momentum in a direction normal to the plate is destroyed.

Pressure due to Jet of Water on Fixed Flat Vertical Plate

- Consider a jet of water coming out from a nozzle and strikes a fixed vertical plate as shown in the Figure below. Let,

  V = Velocity of the jet (m/s),

  ∀ = volume of water (m^3), Q = rate of flow or discharge of water (m^3/s)

  a = Cross-sectional area of the jet (m^2),

  γ = specific weight (kN/m^3)

  Mass of water striking the plate = m = W/g = γ ∀/g = γ Qt/g = γ aVt/g

  Mass of water striking per second = γ aV/g = ρaV (kg)

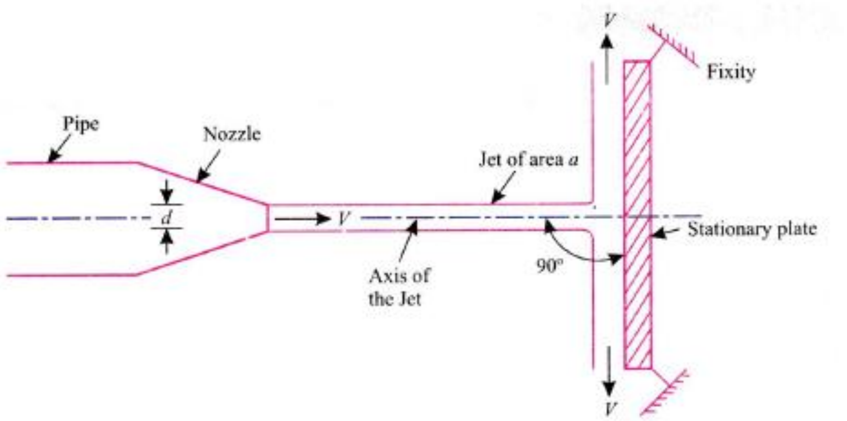
- Since, the final velocity of jet, in its original direction, is reduced to zero after the impact (as the plate is fixed).
- Therefore, the force exerted by jet on the plate as per impulse-momentum eq:

  F = Mass of water flowing/s x Change of velocity

$$F = \frac{\gamma a V}{g} \times (V_i - V_f) == \frac{\gamma a V}{g} \times (V_i - 0)$$

$$F = \frac{\gamma a V^2}{g}$$

$$\boldsymbol{F_{jet} = \frac{\gamma a V^2}{g}} \textbf{ (kN)}$$



Pressure due to Jet of Water on Fixed Flat Inclined Plate

- Consider a jet of water impinging on an inclined fixed plate as shown.

  Let V = Velocity of the jet in m/s, and

  a = Cross-sectional area of the jet in m^2

  θ = Angle at which the plate is inclined with the jet

  Mass of water flowing per second = γ aV/g (kg)

- Since, the force exerted by the jet in its original direction,

$$\mathbf{F_{jet}} = \frac{\gamma a V^2}{g}\,\textbf{(kN)}$$

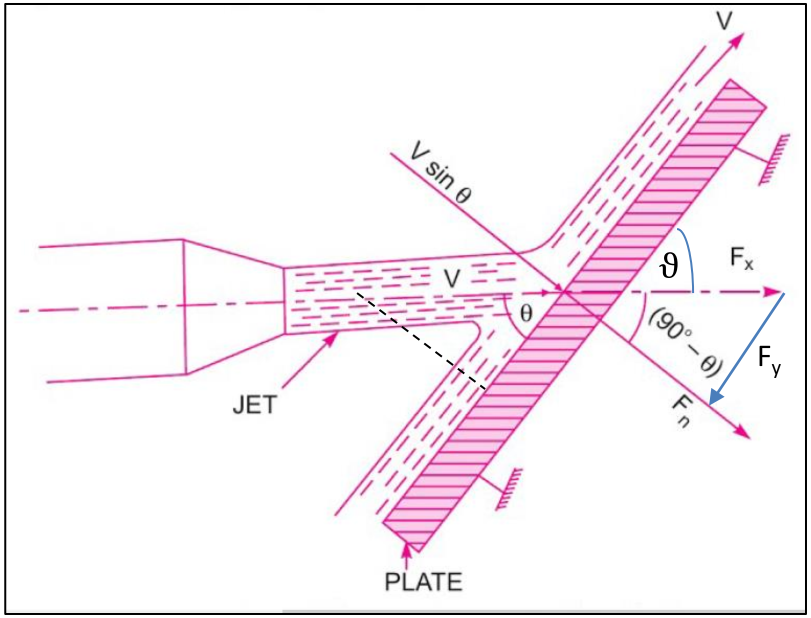
- Therefore, the force of jet normal to the plate,

$$\mathbf{F_n} = \frac{\gamma a V^2}{g} Sin\theta$$

- and, its component in the direction of flow,

$$F_x = F_n \cos(90 - \theta) = \frac{\gamma a V^2}{g} Sin\theta \times Sin\theta$$

$$F_x = \frac{\gamma a V^2}{g} Sin^2\theta$$



- Similarly, the component of F_n perpendicular to the flow,

$$F_y = F_n \sin(90 - \theta) = \frac{\gamma A V^2}{g} Sin\theta \times Cos\theta$$

$$F_y = \frac{\gamma a V^2}{2g}(2sin\theta cos\theta) = \frac{\gamma a V^2}{2g}(sin2\theta)$$

cos(90 – θ) = (cos90)(cosθ) + (sin90)(sinθ) = sinθ
sin(90 – θ) = (sin90)(cosθ) – (cos90)(sinθ) = cosθ
$Sin2\theta = 2Sin\theta Cos\theta$

Example:

A jet of water of 100 mm diameter, moving with a velocity of 20 m/s strikes a stationary plate. Find the pressure on the plate in the direction of the jet, when (a) the plate is normal to the jet, and (b) the angle between the jet and plate is 45°.

Solution:

d = 100 mm = 0.10 m
V = 20 m/s
θ = 45°

$$a = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.10)^2 = 7.854 \times 10^{-3}\ m^2$$

(a) Pressure on the plate when it is normal to the jet:

$$F = \frac{\gamma a V^2}{g} = \frac{9.81 \times 7.854 \times 10^{-3} \times (20)^2}{9.81} = \mathbf{3.142\ kN}$$

(b) Pressure on the plate when it is inclined to the jet:

$$F_x = \frac{\gamma a V^2}{g} \times sin^2\theta = \frac{9.81 \times 7.854 \times 10^{-3} \times (26)^2}{9.81} \times sin^2 45 = \mathbf{1.57\ kN}$$

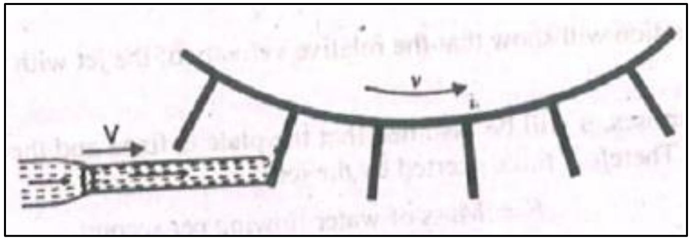
Pressure due to Jet of Water on Moving Flat Plates

- Consider a jet of water impinging normally on a plate and as a result of the impact of jet the plate move in the direction of the jet as shown in Figure.

  Let, V = velocity of jet

  a = cross-sectional area of the jet

  u = velocity of the plate due to impact of jet

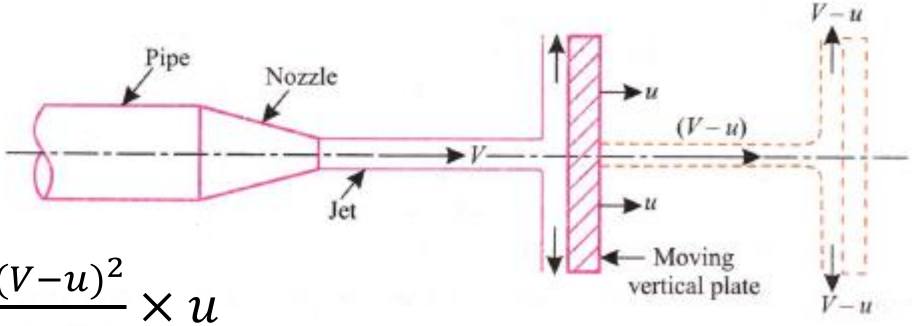


- The relative velocity of the jet with respect to the plate is (V – u) i.e. the velocity at which the jet strikes the plate.
- Let us assume that the plate is fixed, and the jet is moving with a velocity of (V – u), therefore, force exerted by the jet,

$$F = \frac{\gamma a(V-u)}{g} \times [(V-u) - 0]$$

$$F = \frac{\gamma a(V-u)^2}{g}$$

Work Done by Jet per sec = $\frac{F\,S}{t} = \frac{\gamma a(V-u)^2}{g} \times u$



- This would not be possible in practice as there would be continually lengthening jet.
- Now, consider a continuous series of plates at a fixed distance apart radially around the circumference of a large wheel and all moving in the same direction as the jet with a velocity of **u** as shown above, the force of jet on the moving plates is,

$F = \frac{\gamma a V}{g} \times (V-u)$ Work Done by Jet = $\frac{F\,S}{t} = \frac{\gamma a V(V-u)}{g} \times u$

Problem:

A jet of water of 50 mm diameter, moving with a velocity of 26 m/s is impinging normally on a plate. Determine the pressure on the plate, when (a) it is fixed and (b) it is moving with a velocity of 10 m/s in the direction of the jet.

Solution:

d = 50 mm = 0.05 m
V = 26 m/s
u = 10 m/s

$$a = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.05)^2 = 1.96 \times 10^{-3}\ \mathrm{m}^2$$

(a) Pressure on the plate when it is fixed:

$$F = \frac{\gamma a V^2}{g} = \frac{9.81 \times 1.96 \times 10^{-3} \times (26)^2}{9.81} = \mathbf{1.31\ kN}$$

(b) Pressure on the plate when it is moving:

$$F = \frac{\gamma a (V - u)^2}{g} = \frac{9.81 \times 1.96 \times 10^{-3} \times (26 - 10)^2}{9.81} = \mathbf{0.50\ kN}$$

Pressure on Fixed Curved Plates

1. *When the Jet of Water Strikes at the Center of a Symmetrical Plate:*

- Let,

  V = Velocity of the jet in m/s, and

  a = Cross-sectional area of the jet in m^2
- The velocity of jet is not changed in magnitude while flowing over the vane (if plate is smooth, no energy loss) but only the direction is changed.
- We know that the force of the jet in the direction of the jet,

  F_x = Mass of water flowing per second x Change of velocity

$$F_x = \frac{\gamma a V}{g}[V - (-Vcos\theta)] = \frac{\gamma a V}{g}(V + Vcos\theta)$$

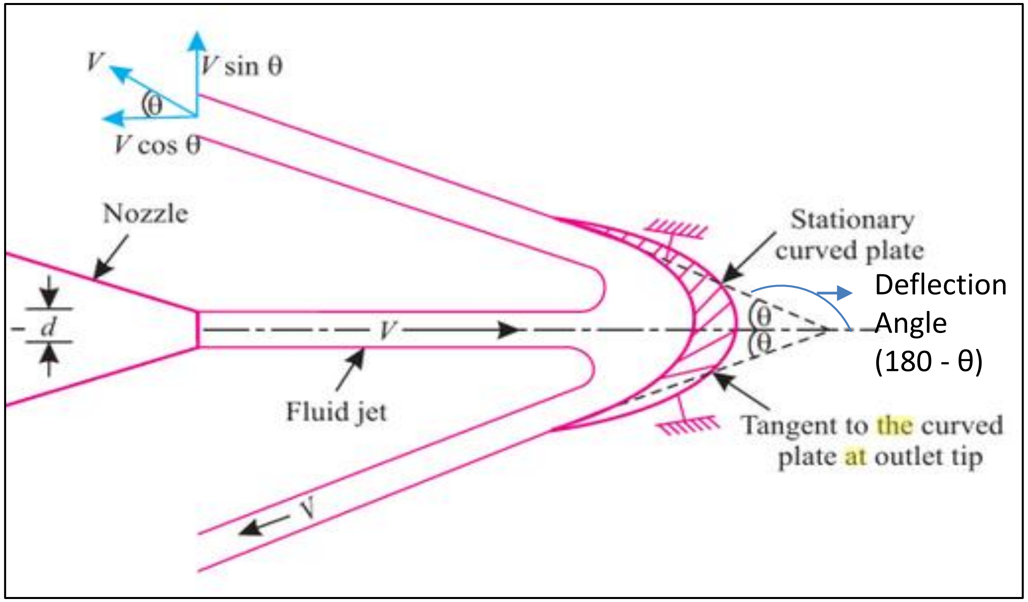
$$F_x = \frac{\gamma a V^2}{g}(1 + cos\theta)$$

- Force exerted by jet on curved fixed plate in vertical direction,

$$F_y = \frac{\gamma a V}{g}(0 - Vsin\theta)$$

$$F_y = \frac{-\gamma a V^2}{g}sin\theta$$

- **-ve** sign means that force is acting in the downward direction on the plate.
- Angle of deflection of the jet = (180° - θ)



Pressure on Fixed Curved Plates

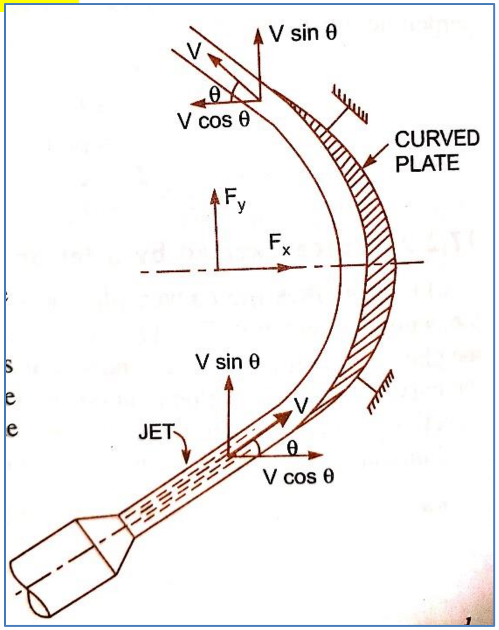
2. *When the Jet of Water Strikes Tangentially at One End of Symmetrical Plate:*

Let,

V = Velocity of the jet in m/s, and

a = Cross-sectional area of the jet in m^2

θ = inlet angle of the jet with x-axis at inlet tip



- The velocity of jet is not changed in magnitude while flowing over the vane (if plate is smooth, no energy loss) but only the direction is changed.
- We know that the force of the jet in x direction (normal force),

F_x = Mass of water flowing per second x Change of velocity in x direction

$$F_x = \frac{\gamma a V}{g}[Vcos\theta - (-Vcos\theta)] = \frac{\gamma a V}{g}(Vcos\theta + Vcos\theta)$$

$$F_x = \frac{2\gamma a V^2}{g}cos\theta$$

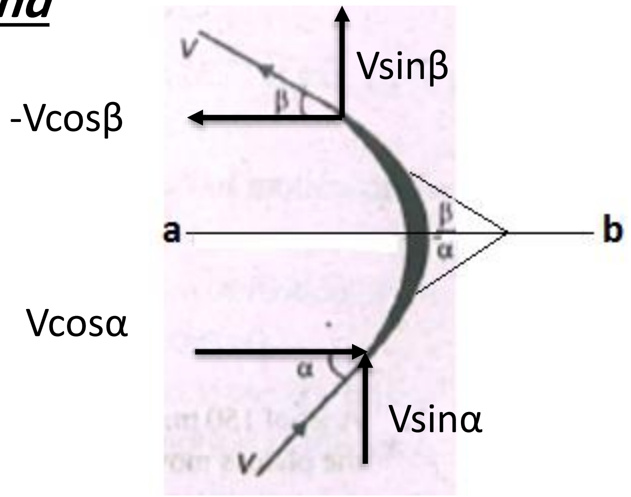
- Similarly, force of the jet in y direction

$$F_y = \frac{\gamma a V^2}{g}(sin\theta - sin\theta) = 0$$

Angle of deflection = 180 - 2θ

Pressure on Fixed Curved Plates

3. *When the Jet of Water Strikes Tangentially at One End of Unsymmetrical Plate:*



Let, V = Velocity of the jet in m/s, and

a = Cross-sectional area of the jet in m^2

α = inlet angle of the jet with x-axis at inlet tip

β = outlet angle of the jet with x-axis at outlet tip

ab = normal at the center of the vane

- It is clear that the vane deflects the jet through an angle 180 – (α + β).
- The velocity of jet is not changed in magnitude while flowing over the vane (if plate is smooth, no energy loss) but only the direction is changed.
- This force may be determined by finding out the components of the force along and perpendicular to ab (normal to the vane).
- We know that the force of the jet along ab (i.e. normal force),

F_x = Mass of water flowing per second x Change of velocity along ab

$$\mathbf{F_x = \gamma\, aV / g\, [V \cos\alpha - (-V \cos\beta)] = \gamma\, aV/g\, (V \cos\alpha + V \cos\beta)}$$

- Similarly, force of the jet perpendicular to ab

$$\mathbf{F_y = \gamma\, aV/g\, (V \sin\alpha - V \sin\beta)}$$

Pressure on Fixed Curved Plates

- Sometimes the total angle through which the jet is deflected is given instead of the inlet and outlet angles of the jet.

- In such a case, assuming α = β and θ = 180° - (α + β) the force of the jet along ab,

$$F_x = \frac{\gamma a V}{g}\, 2V \sin{}^{\theta}/_{2}$$

$$F_x = \frac{2\gamma a V^2}{g} \sin{}^{\theta}/_{2}$$

- If the vane is semicircular (hemispherical), the angles are each equal to zero (i.e. α = β = 0, and θ = 180° then, the force on vane in direction ab,

$$F_x = \frac{2\gamma a V^2}{g}$$

- The force is thus twice as great as that on a flat plate.

- This is due to the fact that, with a semicircular vane the leaving fluid also exerts the same force as that of the entering fluid.

- This principle is made use of in the Pelton Wheel turbine.

Problem:

A jet of water 40 mm diameter enters a fixed curved vane with a velocity of 50 m/s at an angle of 20° to the horizontal. Find the normal and tangential forces exerted by the jet, if it leaves the vane at an angle of 15° to the horizontal.

Solution:

d = 40 mm = 0.04 m

V = 50 m/s

α = 20°

β = 15°

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.04)^2 = 1.257 \times 10^{-3} \text{ m}^2$$

Normal force exerted by the jet:

$$F_x = \frac{\gamma a V (V\cos\alpha + V\cos\beta)}{g} = \frac{9.81 \times 1.257 \times 10^{-3} \times 50(50\cos 20 + 50\cos 15)}{9.81} = 6.0 \text{ kN}$$

Tangential force exerted by the jet:

$$F_y = \frac{\gamma a V (V\sin\alpha - V\sin\beta)}{g} = \frac{9.81 \times 1.257 \times 10^{-3} \times 50(50\sin 20 - 50\sin 15)}{9.81} = 0.262 \text{ kN}$$

Flow Over a Radial Vane

- The force exerted by the jet of water on a single plate (which may be flat or curved) is not practically feasible as it is a theoretical case.
- In actual practice, a large number of plates are mounted on the circumference of a wheel at a fixed distance apart as shown.
- The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving and the second plate mounted on the wheel appears before the jet, which again exerts the force on the second plate.
- Thus each plate appears successively before the jet and the jet exerts force on each plate.
- The jet starts moving at a constant speed.
- Consider the blade to be one of the series of blades fixed radially to the rim of a rotating wheel as shown in Figure.
- Let R_1 = radius of wheel at inlet of the vane

  R_2 = radius of wheel at exit of the vane

  ω = angular velocity of wheel

  V_1 = velocity of jet at vane inlet

  V_2 = velocity of jet at vane exit

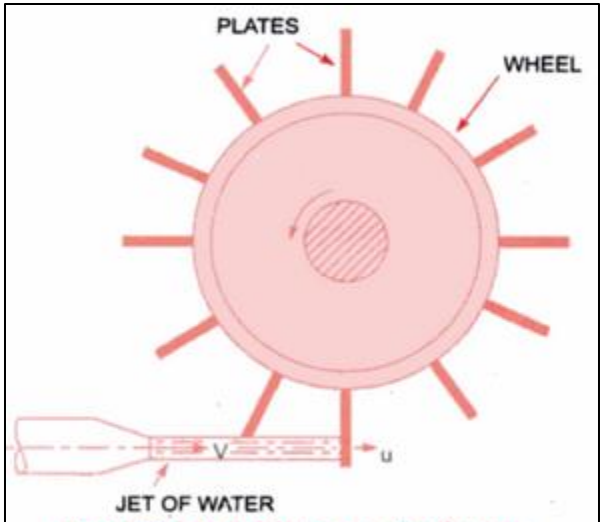
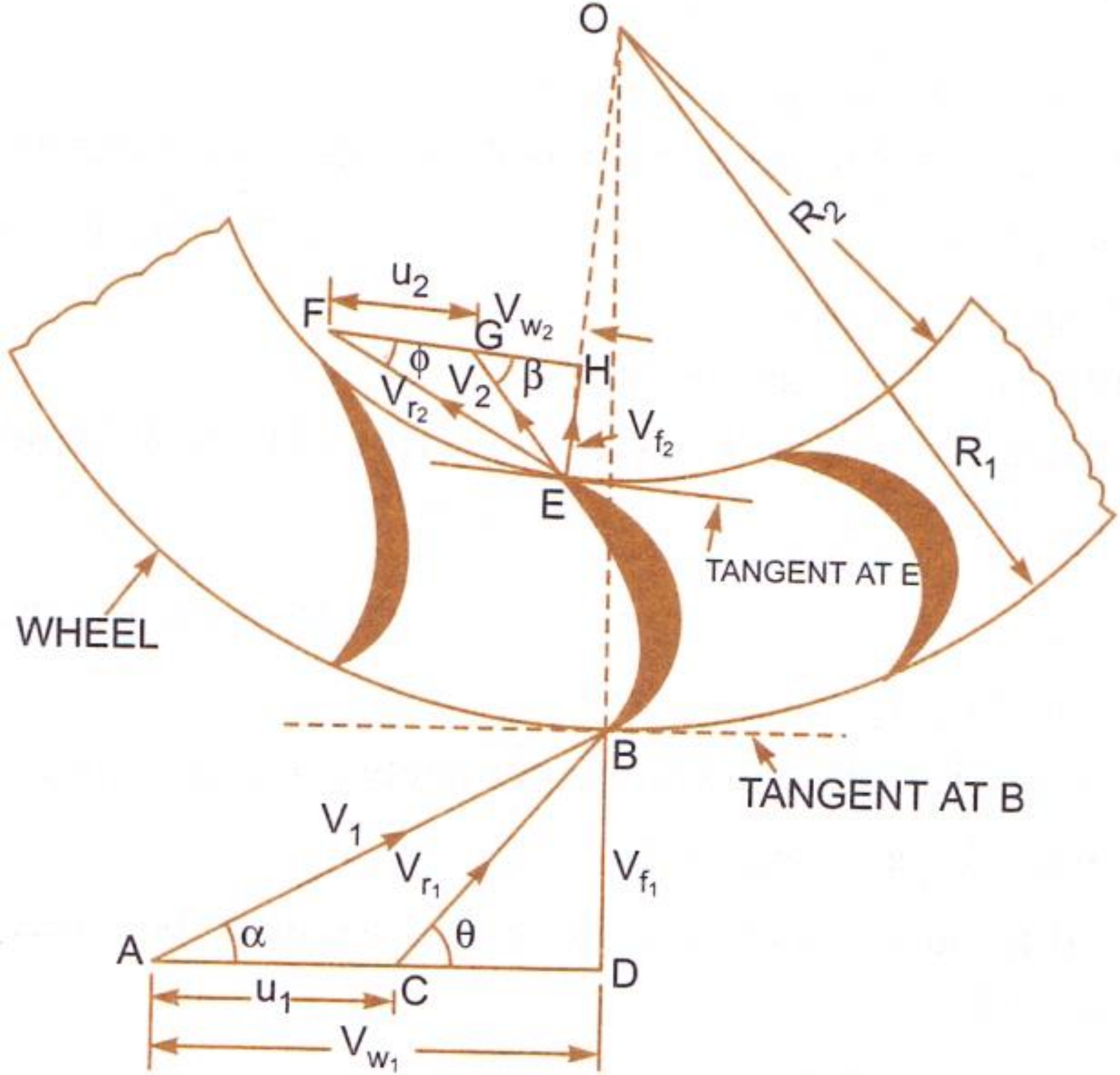


Fig. 17.22 *Jet striking a series of vanes.*


O
R_2
R_1
F
u_2
G
V_{w_2}
H
ϕ
β
V_2
V_{r_2}
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E
TANGENT AT E
WHEEL
B
TANGENT AT B
V_1
V_{r_1}
V_{f_1}
A
α
θ
C
D
u_1
V_{w_1}

Flow Over a Radial Vane

V_1 = absolute velocity of water at inlet

V_2 = absolute velocity of water at exit

u_1 = tangential velocity of vane tip at inlet (also called spherical velocity)

u_2 = tangential velocity of vane tip at exit

V_{r_1} = relative velocity of jet and vane at inlet

V_{r_2} = relative velocity of jet and vane at exit

V_{w_1} = velocity of whirl at vane inlet (horizontal component of V_1 i.e. $V_1 \cos\alpha$)

V_{w_2} = velocity of whirl at vane exit (horizontal component of V_2 i.e. $V_2 \cos\beta$)

V_{f_1} = velocity of flow at vane inlet (vertical component of V_1 i.e. $V_1 \sin\alpha$)

V_{f_2} = velocity of flow at vane exit (vertical component of V_2 i.e. $V_2 \sin\beta$)

α = angle of the jet with the direction of motion of vane at inlet

β = angle of the jet with the direction of motion of vane at exit

θ = vane angle at inlet

ϕ = vane angle at exit

a = cross-sectional area of the jet

- Then

$u_1 = \omega R_1$ and $u_2 = \omega R_2$

- The velocity triangles at inlet and outlet are drawn as shown in the Figure.

Flow Over a Radial Vane

- The mass of water striking per second for a series of vanes

  = mass of water coming out from nozzle per second = $\frac{\gamma a V_1}{g}$

- Momentum of water striking the vanes in the tangential direction per second at inlet (p/sec = m/sec x V = F)

  F = mass of water per second x component of V_1 in tangential direction

  $F = \frac{\gamma a V_1}{g} \times V_{\omega_1}$

- Similarly, momentum of water per second at vane exit

  F = mass of water per second x component of V_2 in tangential direction

  $F = \frac{\gamma a V_1}{g} \times (-V_2 cos\beta) = -\frac{\gamma a V_1}{g} \times V_{\omega_2}$

- Now, angular momentum per second at inlet

  L = momentum at inlet x radius at inlet = $\frac{\gamma a V_1}{g} \times V_{\omega_1} \times R_1$ $\because$ L = P x r

- And, angular momentum per second at exit

  L = momentum at exit x radius at exit = $-\frac{\gamma a V_1}{g} \times V_{\omega_2} \times R_2$

Flow Over a Radial Vane

- Torque exerted by water on the wheel is given by impulse-momentum theorem as the rate of change of angular momentum

$$\mathrm{T} = \left(\frac{\gamma a V_1}{g} \times V_{\omega_1} \times R_1\right) - \left(-\frac{\gamma a V_1}{g} \times V_{\omega_2} \times R_2\right)$$

$$\mathrm{T} = \frac{\gamma a V_1}{g}\left(V_{\omega_1} \times R_1 + V_{\omega_2} \times R_2\right)$$

- Work done per second on the wheel = Torque x Angular velocity = T x ω

$$\mathrm{WD/s} = \frac{\gamma a V_1}{g}\left(V_{\omega_1} \times R_1 + V_{\omega_2} \times R_2\right) \times \omega$$

$$\mathrm{WD/s} = \frac{\gamma a V_1}{g}\left(V_{\omega_1} \times R_1 \times \omega + V_{\omega_2} \times R_2 \times \omega\right)$$

$$\mathrm{WD/s} = \frac{\gamma a V_1}{g}\left(V_{\omega_1} u_1 + V_{\omega_2} u_2\right)$$

Since, $u_1 = \omega R_1$ and $u_2 = \omega R_2$

- In the above case, always the velocity of whirl at outlet is given by both magnitude and direction as $V_{\omega_2} = V_{r_2} cos\emptyset - u_2$
- If the discharge is radial at outlet (i.e. β = 90°), then $V_{\omega_2} = 0$ and hence the equation reduces to

$$\frac{\mathrm{WD}}{\mathrm{s}} = \mathrm{P} = \frac{\gamma a V_1}{g} V_{\omega_1} u_1 = \rho Q V_{\omega_1} u_1$$

$$\mathrm{KE/s} = \frac{1}{2} m {V_1}^2 = \frac{1}{2} \frac{\gamma a {V_1}^3}{g}$$

Flow Over a Radial Vane

- Efficiency of the wheel is

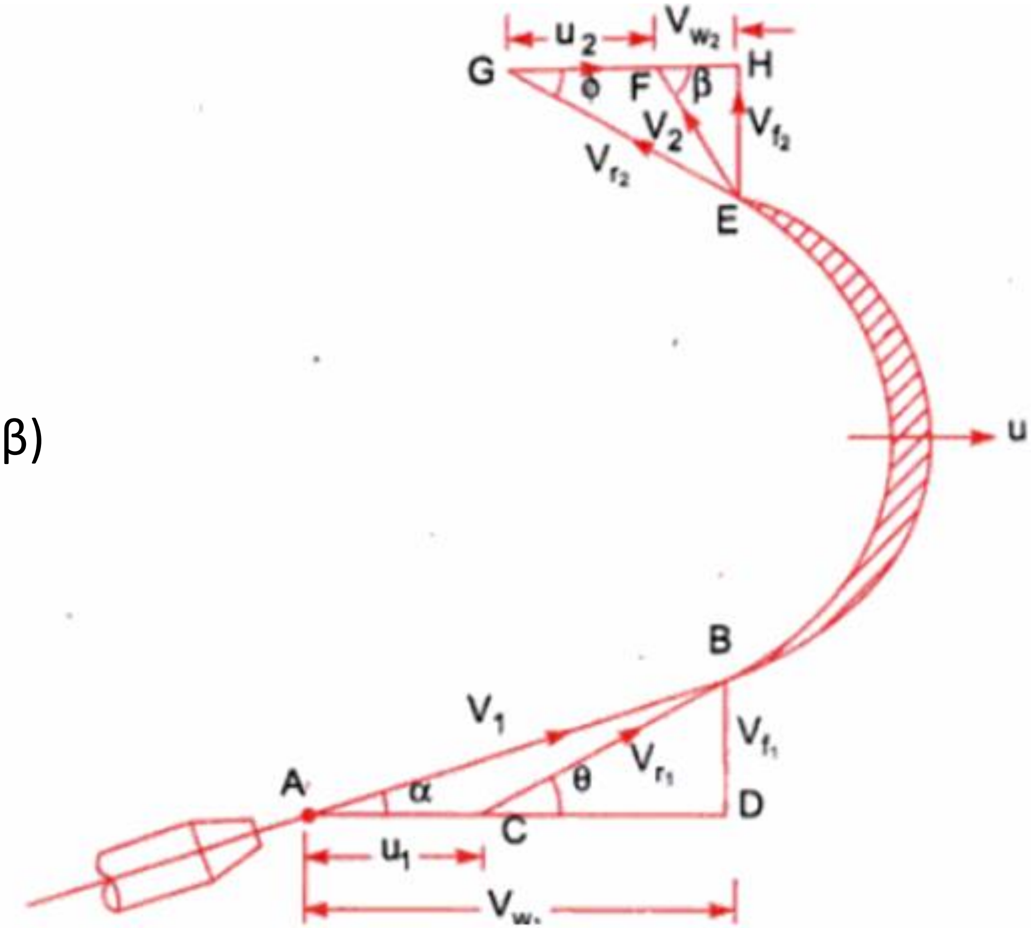
$$\eta = \frac{output}{input} = \frac{WD/s}{KE/s}$$

$$\eta = \frac{\frac{\gamma a V_1}{g}\left(V_{\omega_1} u_1 + V_{\omega_2} u_2\right)}{\frac{1}{2}\frac{\gamma a V_1^{\,3}}{g}}$$

$$\eta = \frac{2\left(V_{\omega_1} u_1 \pm V_{\omega_2} u_2\right)}{V_1^{\,2}}$$

(-ve sign for obtuse angle β)

- For maximum efficiency,
  - V_2 should be minimum
  - V_{ω_2} should be maximum
  - ϕ should be minimum



Jet striking a moving curved vane at one of the tips.

Problem:

A jet of water having a velocity of 30 m/s impinges on a series of vanes with a velocity of 15 m/s. The jet makes an angle of 30º to the direction of motion of vanes when entering and leaves at an angle of 120º. Sketch velocity triangles at entrance and exit, determine the vane angles so that the water enters and leaves without shock, and the work done per unit mass.

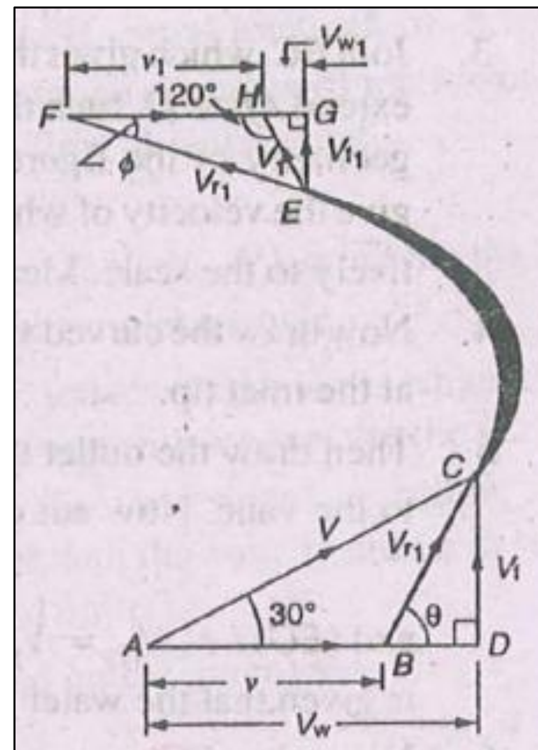
Solution:

V = 30 m/s

u = 15 m/s

α = 30º

β = 120º



1. First of all draw a horizontal line in the direction of motion of the vanes and cut off AB equal to 15 m/s to some suitable scale, to represent the velocity of vanes (u).
2. Draw a line at an angle of 30o i.e. the angle of entering jet and cut off AC equal to 30 m/s to the scale to represent velocity of the jet (V).
3. Join BC, which gives the relative velocity (V_r) to the scale. Now extend AB to D such that DC is perpendicular to AB. The length of AD gives the velocity of whirl (V_w) and that of DC gives the velocity of flow (V_f).
4. Now draw the curved vane such that V_r is tangential to the vane at the entrance tip.

5. Then draw a velocity triangle at exit with V_{r1} tangential to the vane. Now cut off EF i.e. $V_{r1} = V_r$. Through F draw a horizontal line and cut off FH equal to 15 m/s to the scale to represent the velocity of vanes at exit (v_1).
6. Join EH which will make an angle of 120° with FH as the jet leaves the vane at an angle of 120°. Now draw EG perpendicular to FH.

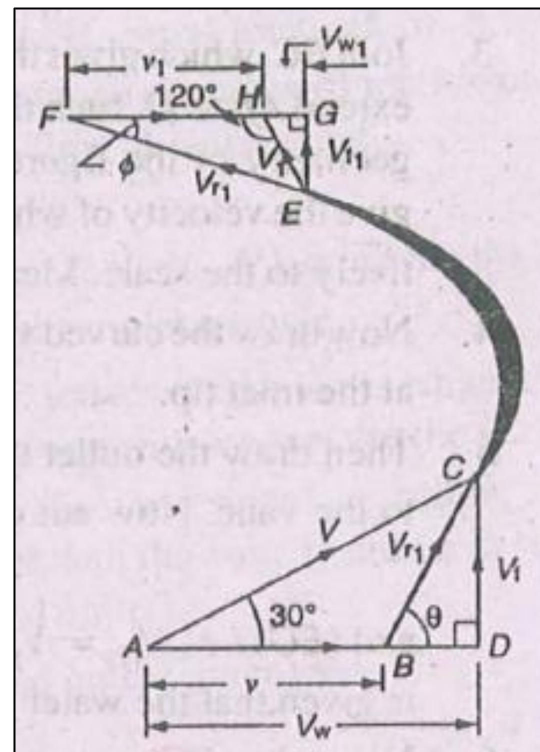
$$V_{\omega_1} = V_1 cos\alpha = 30cos30^o = 25.98\, m/s$$

$$V_{f_1} = V_1 sin\alpha = 30sin30^o = 15\, m/s$$

$$tan\theta = \frac{V_{f_1}}{V_{\omega_1} - u_1} = \frac{15}{25.98 - 15} = 1.366$$

$$\theta = 53.8^o$$

$$V_{r_1} = \frac{V_{f_1}}{sin\theta} = \frac{15}{sin53.8^o} = 18.59\, m/s$$



- From the outlet triangle we find from the sine rule

$$\frac{u_1}{\sin(60 - \varphi)} = \frac{V_{r_1}}{sin\beta}$$

$$\sin(60 - \varphi) = \frac{15 \times sin120^o}{18.59} = 0.6988$$

$$60 - \varphi = 44.3^o$$

$$\varphi = 15.7^o$$

$$WD = (V_{\omega_1} + V_{\omega_2})u = [25.98 + (V_{r_1} cos\varphi - u)] \times 10$$

$$WD = [25.98 + (18.59cos15.7 - 10)] \times 10 = 338.76\, \frac{N - m}{kg - s}$$

Experiment of Impact of Jet on Curved Plate