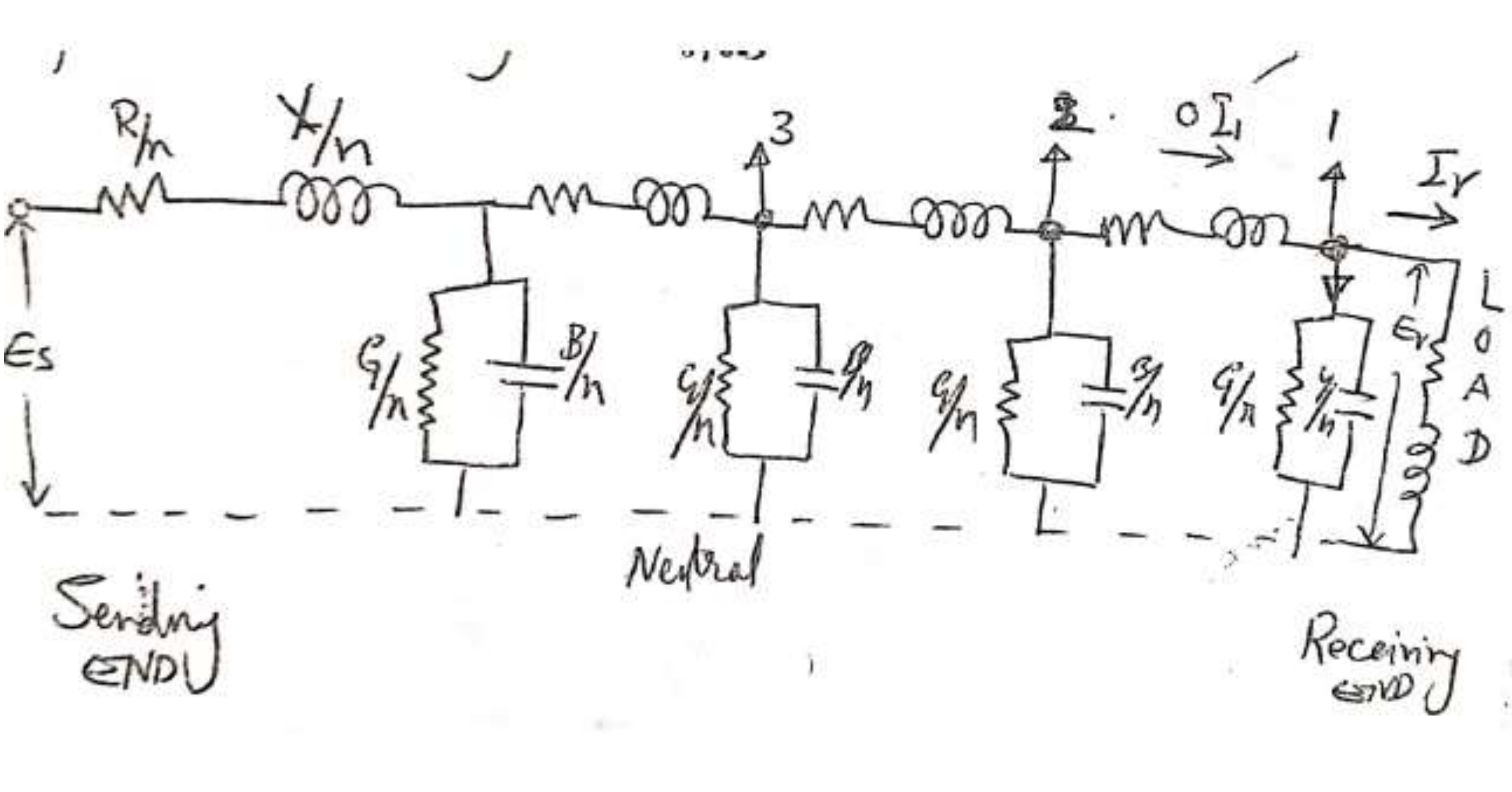
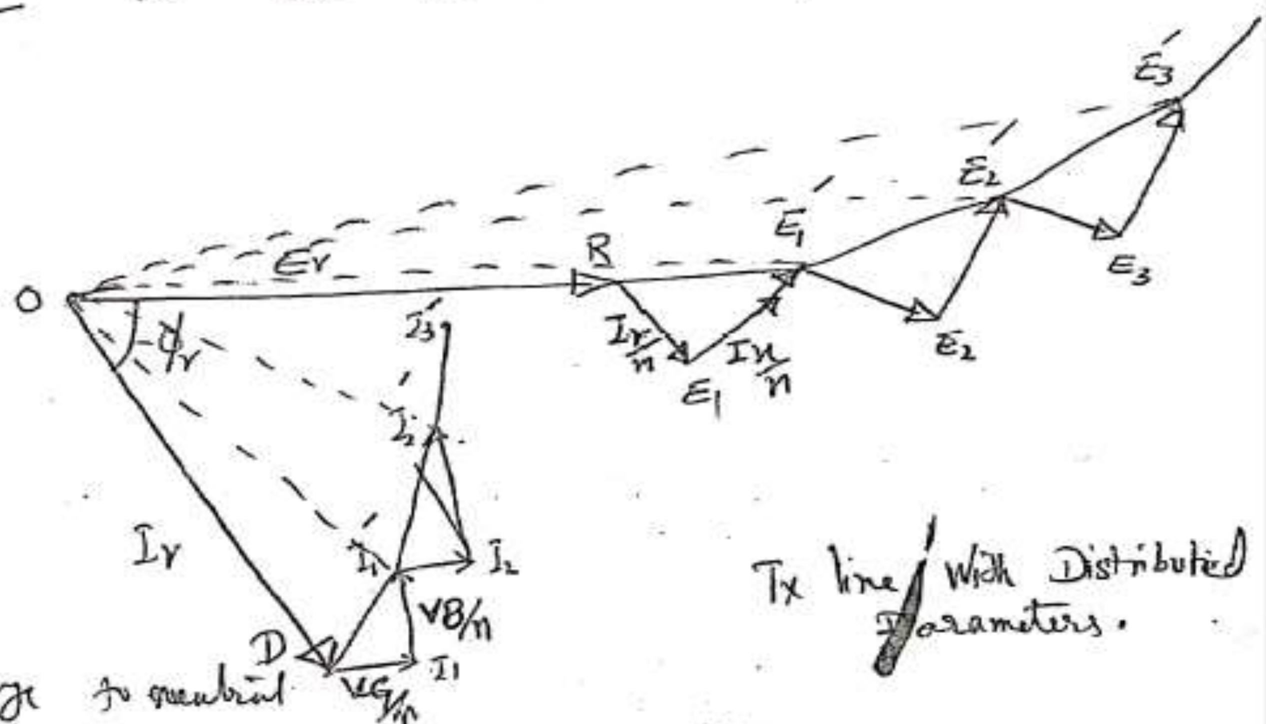
Performance of Long Tx lines (Exact Solution)

- For Tx lines less then 200 km, the line constants or parameters are taken as lumped. But for lines more then 200 km, distributed parameters are considered for accuracy of the analysis where voltage and current varies at each point on the transmission lines.

Transmission line with Distributed constants



Starting from one end of the line, the Voltage and
...nt at any other point can be found by taking
...h section in turn and constructing a diagram
...wing the Voltage consumed by the impedance, and
current flowing in the admittance, of each section.



Tx line with Distributed Parameters.

represent the Voltage to neutral at the receiving end of the line. the current at this end.

$I_y = VG/n = IR$
$I_b = VB/n = IR$

Step by Step

I_1 in G/n due to E_r (in phase to E_r)

$E_1 E_1'$ in X/n due to E_r (90° lead to E_r)

RE_1 drop in R/n } due to OI_1'

$E_1 E_1'$ drops in X/n }

$OE_1' \Rightarrow$ Voltage Drop at pt 2.

Similarly for 2nd section.

$I_1' I_2$ current in G/n (in phase to (E_1'))

$I_2 I_2'$ the current in B/n (lead by 90°)

$E_1' E_2$ drop in R/n → in phase with I_r OI_1'

$E_2 E_2'$ drop in X/n (lead current by 90°)

At pt 3 Voltage drop is OE_2'

Exp

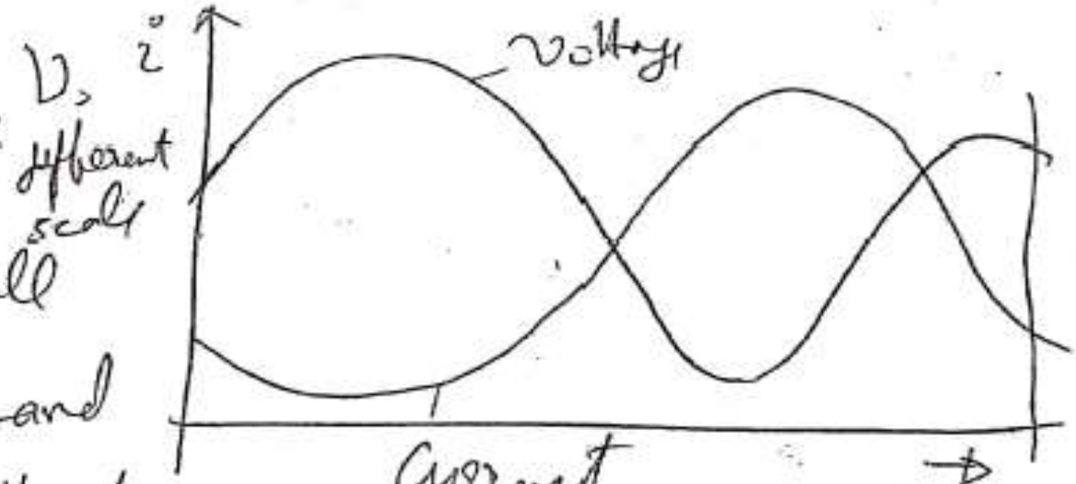
$r = 0.275\ \Omega/\text{mile}$ $\quad X = 0.769\ \Omega/\text{mile}$

$g = 0.15 \times 10^{-1}$ mho $\quad b = 552 \times 10^{-6}$ mhos.

$E_r = 50$ kV (l-n)

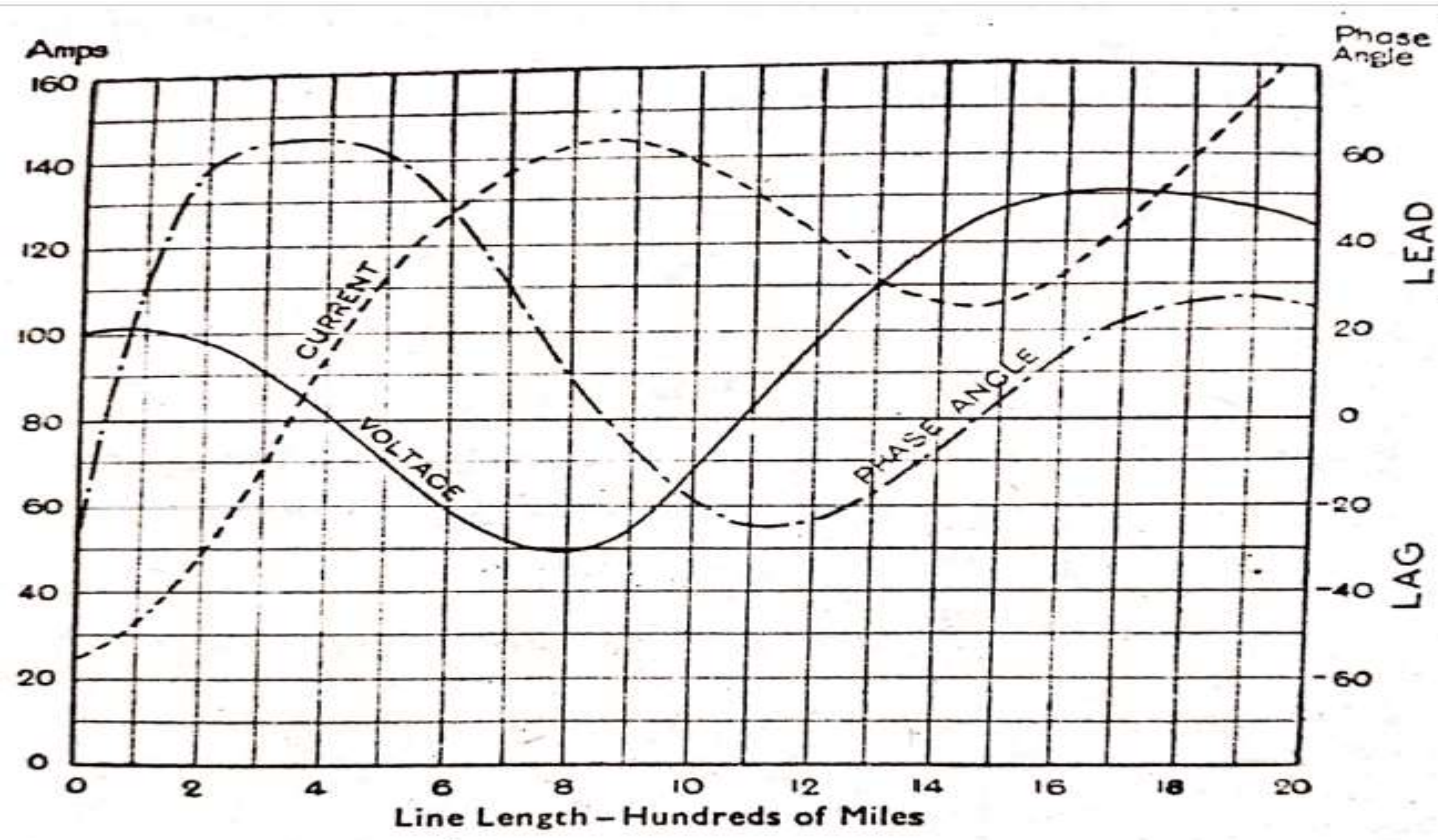
$I_r = 25$ Amp lagging 25° behind the Voltage

$l = 200$ miles.

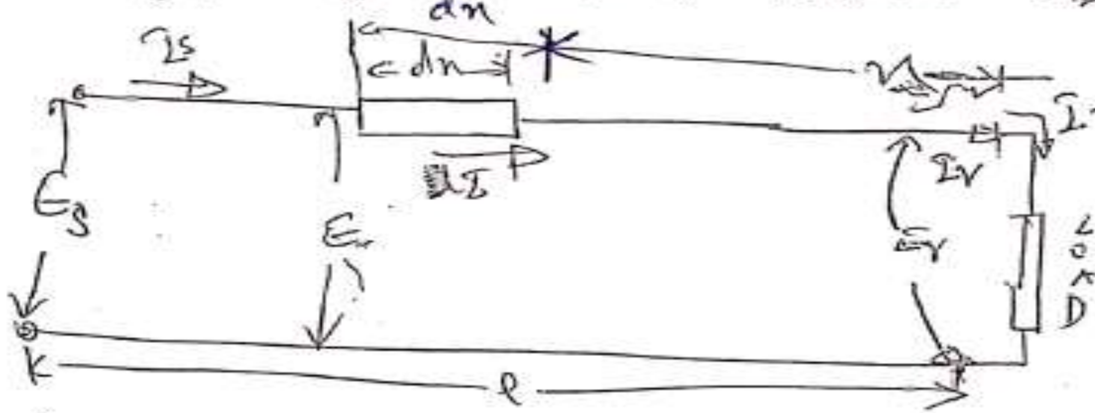


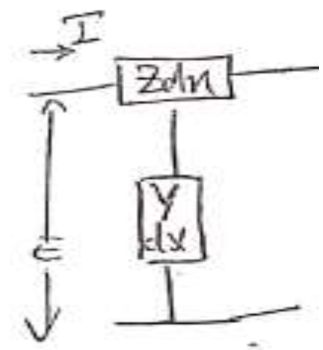
the Voltage & Current different scale
ernates rise & fall
longest the line and
gle of Phase different
nges periodically, between lag & lead

Voltage, Current & Phase angle in a long Tx line



Fundamental Differential Equation and Solution





Z → Impedance of Unit length of each cdt

Y → Admittance of unit length of each cdt

Take a Incremental length dx at x from receiving end.

Impedance of Incremental length = Zdx at distance of x from receiving end.

admittance of length dx of line is Ydx.

The Voltage Drop over length $dx = dE = I\,Z\,dx$

or $dE/dx = IZ \longrightarrow (1)$

the shunt current drawn from a incremental length dx of the line ; $dI = EY\,dx = \underline{VY}$

or $dI/dx = EY \longrightarrow (2)$

Taking derivative of (1) w.r.t x.

$$\frac{d^2E}{dx^2} = Z\,dI/dx = ZYE$$

$$d^2E/dx^2 - ZYE = 0 \longrightarrow (3)$$

$$d^2I/dx^2 = Y\,dE/dx = YIZ$$

☆

(4) $$d^2I/dx^2 - YIZ = 0 \longrightarrow (4)$$

③ $\rightarrow$ Aux Equation $\lambda^2 - YZ = 0 \Rightarrow \lambda = \pm\sqrt{YZ}$

Gen solution $\cancel{E=} A\, e^{\lambda_1 x} + B\, e^{\lambda_2 x}$

$$E = A\, e^{\sqrt{YZ}\,x} + B\, e^{-\sqrt{YZ}\,x} \quad \text{---} \quad (5)$$

(5) $\Rightarrow$ $$dE/dx = \sqrt{YZ}\left[A\, e^{\sqrt{YZ}\,x} - B\, e^{-\sqrt{YZ}\,x}\right]$$

as $dE/dx = IZ$, So

$$IZ = \sqrt{YZ}\left[A\, e^{\sqrt{YZ}\,x} - B\, e^{-\sqrt{YZ}\,x}\right]$$

$$I = \sqrt{\frac{Y}{Z}}\left[A\, e^{\sqrt{YZ}\,x} - B\, e^{-\sqrt{YZ}\,x}\right] \longrightarrow 6$$

find A & B : Apply boundary Conditions to 5, 6

receiving end $x=0$, $E = E_r$

$$E_r = Ae^0 + Be^0 = A+B$$

$$I_r = \sqrt{Y/Z}\,[A - Be^0] = \sqrt{Y/Z}\,(A-B)$$

$$E_r = A+B$$

$$I_r\sqrt{Z/Y} = A-B$$

Adding

$$A = \tfrac{1}{2}\left[E_r + \sqrt{Z/Y}\,I_r\right]$$

subtracting two Equations

$$B = \tfrac{1}{2}\left[E_r - \sqrt{Z/Y}\,I_r\right]$$

putting A & B values in Equation 5 & 6

$$E = \tfrac{1}{2}\left[E_r + \sqrt{Z/Y}\, I_r\right] e^{\sqrt{YZ}\,x} + \tfrac{1}{2}\left[E_r - \sqrt{Z/Y}\, I_r\right] e^{-\sqrt{YZ}\,x}$$

$$= E_r \left[\frac{e^{\sqrt{YZ}\,x} + e^{-\sqrt{YZ}\,x}}{2}\right] + \sqrt{\frac{Z}{Y}}\, I_r \left[\frac{e^{\sqrt{YZ}\,x} - e^{-\sqrt{YZ}\,x}}{2}\right]$$

$$\boxed{E = E_r \cosh \sqrt{ZY}\,x + \sqrt{Z/Y}\, I_r \sinh \sqrt{ZY}\,x} \quad (7)$$

As $\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$, $\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

For Current

$$I = \sqrt{Y/Z}\left[A e^{\sqrt{YZ}\,x} - B e^{-\sqrt{YZ}\,x}\right]$$

$$I = \sqrt{Y/Z}\left[\tfrac{1}{2}\left(E_r + \sqrt{Z/Y}\, I_r\right) e^{\sqrt{YZ}\,x} - \tfrac{1}{2}\left(E_r - \sqrt{\tfrac{Z}{Y}}\, I_r\right) e^{-\sqrt{YZ}\,x}\right]$$

$$\cancel{I = \sqrt{Y/Z}\left\{\tfrac{1}{2}\left(E_r + \sqrt{}\right.\right.}$$

cancel

$$I = \sqrt{Y/Z}\left[E_r\left(\frac{e^{\sqrt{YZ}\,x} - e^{-\sqrt{YZ}\,x}}{2}\right) + \sqrt{Z/Y}\, I_r\left(\frac{e^{\sqrt{YZ}\,x} + e^{-\sqrt{YZ}\,x}}{2}\right)\right]$$

$$\boxed{I = \sqrt{Y/Z}\, E_r \sinh \sqrt{YZ}\,x + I_r \cosh \sqrt{YZ}\,x} \quad (8)$$

For voltage at sending end put $x = l$ in Eq ⑦

$$E_s = E_r \cosh\sqrt{zy}\, l + \sqrt{z/y}\, I_r \sinh\sqrt{zy}\, l$$

$zl \rightarrow Z$ (total) , $Y = yl$ = total admittance

$$\sqrt{zy}\, l = \sqrt{zyl^2} = \sqrt{zl \cdot yl} = \sqrt{ZY}$$

$$\sqrt{z/y} = \sqrt{\frac{zl}{yl}} = \sqrt{Z/Y}$$

So

⑥

$$E_s = E_r \cosh\sqrt{ZY} + \sqrt{Z/Y}\, I_r \sinh\sqrt{ZY} \quad \text{---} (9)$$

Similarly

$$I_s = \sqrt{Y/Z}\, E_r \sinh\sqrt{YZ} + I_r \cosh\sqrt{YZ} \quad \text{---} 10$$

let

$$A = D = \cosh\sqrt{ZY}$$

$$B = \sqrt{Z/Y} \sinh\sqrt{ZY}$$

$$C = \sqrt{Y/Z} \sinh\sqrt{ZY}$$

then Equation ~~for Es & Is~~

$$E_s = A E_r + B I_r \quad \text{---} 9'$$

$$I_s = C E_r + D I_r \quad \text{----} 10'$$

A, B, C, D are constant, called general line constant (Distributed load constant) Depends upon freq etc.

Equivalent π & T circuit

Equivalent π & T circuit

A long transmission line may be replaced by a π or T circuit electrical equivalent in the same that the same relations exist between the terminal electrical condition in each case.

Ref to Figure (previous for M.T. line) assume that the ~~and the~~ shunt admittance and total series impedance of the π circuit are Z' and Y' respectively

Then the Voltage and current equations are:

$$E_s = E_r\left(1+\frac{Z'Y'}{2}\right) + I_r Z'$$

$$I_s = I_r\left(1+\frac{Z'Y'}{2}\right) + E_r Y'\left(1+\frac{Z'Y'}{4}\right)$$

While the corresponding equations of the long lines are

$$E_s = E_r \cosh\theta + I_r\sqrt{(Z/Y)}\sinh\theta$$

$$I_s = I_r \cosh\theta + E_r\sqrt{(Y/Z)}\sinh\theta$$

Where $\theta = \sqrt{ZY}$. for equivalence between the two circuits

$$Z' = \sqrt{\left(\frac{Z}{Y}\right)}\sinh\theta$$

& $1 + \frac{Z'Y'}{2} = \cosh\theta$ & substituting for Z' in the latter equation

$$1 + Y'/2\sqrt{(Z/Y)}\sinh\theta = \cosh\theta$$

$\theta = \sqrt{ZY}$

$\sqrt{\frac{YY}{YZ}} = \frac{Y}{\sqrt{YZ}} = \frac{Y}{\theta}$

So $$Y'/2 = \sqrt{(Y/Z)}\ \frac{\cosh\theta - 1}{\sinh\theta} = \frac{Y(\cosh\theta - 1)}{\theta\sinh\theta}$$

$$= Y/2\ \frac{\tanh(\theta/2)}{\theta/2}\ ---$$

The equivalent -π circuit is of importance since its constants, unlike the A, B, C, D constants, can form the basis of a physical model of the transmission line for ~~use~~ for use in the network analysis studies. It is ~~standard~~ should be however ~~lines for the~~ ~~used in network~~ analysis

Evaluation of General Line Constants or A, B, C, D Constants

Evaluation of General line Constants A, B, C & D (for Problems)

Method #1 The expension of Convergent Series of hyp form

$$\cosh x = 1 + x^2/2! + x^4/4! + x^6/6! + \cdots$$

$$\sinh x = x + x^3/3! + x^5/5! + x^7/7! + \cdots$$

$$\cosh\sqrt{ZY} = 1 + \frac{(\sqrt{ZY})^2}{2!} + (\sqrt{ZY})^4/4! \pm 1 + \frac{ZY}{2} + \frac{(ZY)^2}{4!}$$

Mostly $ZY < 0.5$, high Convergent Series, High values are rejected.

$$\sinh\sqrt{ZY} = \sqrt{ZY} + \frac{(\sqrt{ZY})^3}{3!} + (\sqrt{ZY})^5/5! + \cdots$$

So $A = D = 1 + ZY/2 + \dfrac{(ZY)^2}{4!}$ ——— 1

$$B = \sqrt{Z/Y}\left[\sqrt{ZY} + \frac{(\sqrt{ZY})^3}{6} + \frac{\sqrt{ZY}^5}{120}\right]$$

$\sqrt{ZY}$ common

$$B = Z\left[1 + \frac{ZY}{6} + \frac{(ZY)^2}{120}\right] \text{ ——— } 2$$

$$C = \sqrt{Y/Z}\left[\sqrt{ZY} + \frac{(\sqrt{ZY})^3}{6} + \frac{(\sqrt{ZY})^5}{120}\right]$$

$\sqrt{ZY}$ Com

$$C = Y\left[1 + \frac{ZY}{6} + \frac{(ZY)^2}{120}\right] \text{ - - - - } 3$$

Method

Using Hyperbolic Expansion

$\gamma = \sqrt{ZY} = \alpha + j\beta$

$A = \cosh\sqrt{ZY} = \cosh(\alpha + j\beta)$

$A = \cosh\alpha \cosh j\beta + \sinh\alpha \sinh j\beta$

$A = \cosh\alpha \cos\beta + j \sinh\alpha \sin\beta = D$

$\cosh j\beta = \cos\beta$
$\sinh j\beta = j\sin\beta$

$B = \sqrt{Z/Y}\,[\sinh(\alpha + j\beta)]$

$B = \sqrt{Z/Y}\,[\sinh\alpha \cosh j\beta + \sinh j\beta \cosh\alpha]$

$B = \sqrt{Z/Y}\,[\sinh\alpha \cos\beta + j\sin\beta \cosh\alpha]$

Similarly

$C = \sqrt{Y/Z}\,[\sinh(\alpha + j\beta)]$

$C = \sqrt{Y/Z}\,[\sinh\alpha \cos\beta + j\sin\beta \cosh\alpha]$