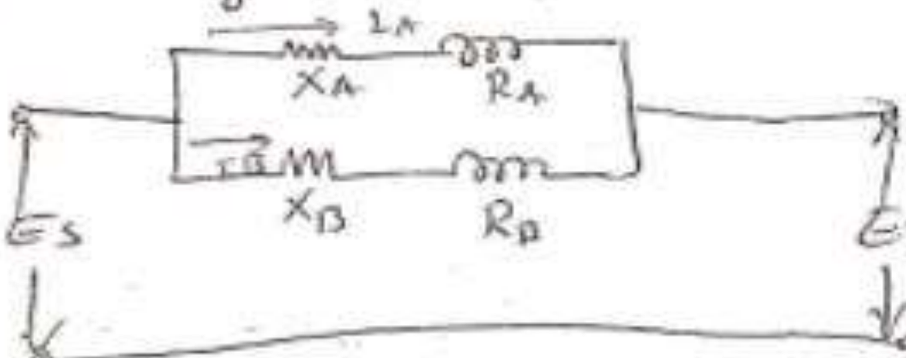
Lecture 8
Parallel Lines in Short Tx System

Parallel lines

Power Problems dealing with terminal conditions on 11 lines can be solved in exactly the same way as single line Problem on the assumption that the load current I now flows through an impedance $Z = R + jX$, where R is the joint resistance & X the joint reactance of the lines. if Z_A & Z_B represent the impedance of the lines and there is no electrical interaction the joint impedance is

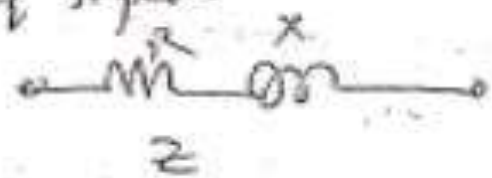


$Z_A = R_A + jX_A$

$Z_B = R_B + jX_B$

$$Z = \frac{Z_A Z_B}{Z_A + Z_B} \quad \underline{11}$$

Eq Impedance



$$Z = \frac{(R_A + jX_A)(R_B + jX_B)}{(R_A + R_B) + j(X_A + X_B)} * \frac{(R_A + R_B) - j(X_A + X_B)}{(R_A + R_B) - j(X_A + X_B)}$$

$$Z = \frac{R_A(R_B^2 + X_B^2) + R_B(R_A^2 + X_A^2)}{(R_A + R_B)^2 + (X_A + X_B)^2} + j\frac{X_A(R_B^2 + X_B^2) + X_B(R_A^2 + X_A^2)}{(R_A + R_B)^2 + (X_A + X_B)^2}$$

Impedance drop across A = Impedance drop across B = Impedance drop across eq Impedance

$$I_A Z_A = I_B Z_B = I Z$$

$$I_A Z_A = I Z \Rightarrow I_A = \frac{I}{Z_A} \frac{Z_A Z_B}{Z_A + Z_B} = I\left(\frac{Z_B}{Z_A + Z_B}\right)$$

Similarly $I_B = \frac{I Z_A}{Z_A + Z_B}$;

$\phi_r \to$ phase angle b/w I & E_r

ϕ_A = phase angle b/w I_A & E_r.

Total Power $P_T = E_r I \cos\phi_r$

$$E_r = \frac{P_T}{I \cos\phi_r}$$

$$P_A = \underline{E_r I_A} \cos\phi_A$$

$$P_A = P_r \frac{I_A \cos\phi_A}{I \cos\phi_r}$$ at p.f $\cos\phi_A$

$$P_B = P_r \frac{I_B \cos\phi_B}{I \cos\phi_r}$$ at Power factor ϕ_B.

Short T/L With Condition Known at One End (11)

This is explained with the help of Example

A 20 mile three phase line has the following Const/mile

$r = 0.295\ \Omega's$, $x = 0.557\Omega$

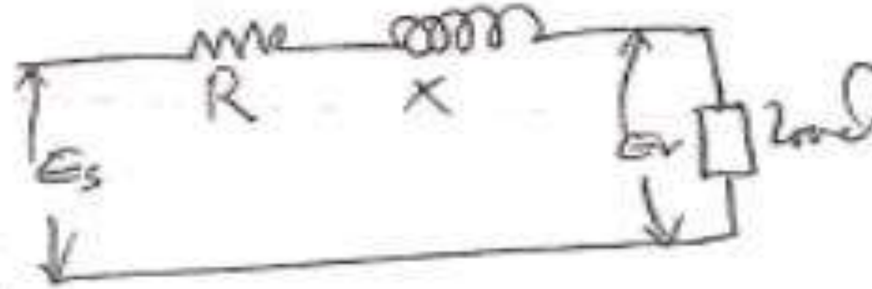
At the receiving end, the voltage is 30 KV & the load, 8000 KVA (7200 KW at 0.9 P.f lagging). Determine the sending end conditions, the η & Regulation.

Solution

Total Resistance $R = r*l = 0.295*20 = 5.9\Omega$

Total Reactance $= X' = x*l = 0.557*20 = 11.14\Omega$

<u>Condition at Receiving end</u>



KVA per Phase

$(KVA)_{phase} = \frac{8000}{3} = 2667$ KVA

$(KW)_{phase} = 7200/3 = 2400$ KW

$$E_{r(L-N)} = \frac{30{,}000}{\sqrt{3}} = 17320 \text{ Volts} = 17.32 \text{ kV}$$

$$I = \frac{(KVA)_{phase}}{E_r} = \frac{2667}{17320} = 154 \text{ Amp}$$

$IR = 154 * 5.9 = 900$ Volts

$IX = 154 * 11.14 = 1715$ Volts

$\cos\phi_r = 0.9 \Rightarrow \phi_r = \cos^{-1} 0.9 = 25.54°$

As $$E_s = E_r \sqrt{\left(\cos\phi_r + IR/E_r\right)^2 + \left(\sin\phi_r + \frac{IX}{E_r}\right)^2}$$

$$E_s = 17320 \sqrt{\left(0.9 + \frac{900}{17320}\right)^2 + \left(0.436 + \frac{1715}{17320}\right)^2}$$

$E_s = 18.92$ kV (to neutral)

& $\phi_s = \tan^{-1} \dfrac{\sin\phi_r + IX/E_r}{\cos\phi_r + IR/E_r}$ putting values

$\phi_s = 29.32^\circ$

$\cos\phi_s = 0.872$ lagging

$\text{Loss} = I^2 R = \dfrac{(154)^2 \times 5.90}{1000} = 140 \text{ kW}$

$\text{Efficiency} = \dfrac{P_{output}}{P_{input}} \times 100 = \dfrac{P_{output}}{P_{output} + \text{losses}} \times 100 = \left(\dfrac{2400}{2400+140}\right) \times 100$

$\eta = 94.5\%$

$\left(R = \dfrac{V_s - V_R}{V_R} \times 100\right)$

$\text{Regulation} = E_s - E_r = 18920 - 17320$

$R = 1600$ Volts

or % age Regulat $= \dfrac{1600 \times 100}{17320}\% = 9.24\%$

or can be derived the equation

$$\Rightarrow E_r^2 + E_r \underbrace{2I(R\cos\phi_r + X\sin\phi_r)}_{u} + \underbrace{I^2(R^2+X^2) - E_s^2}_{v} = 0$$

we replace them term by u, v

$$\boxed{x^2 a + bx + c = 0}$$

we get $E_r^2 + u E_r + v = 0$

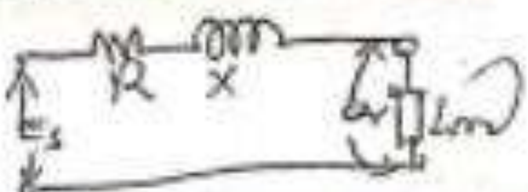
to get value E_r

$$E_r = \frac{-u \pm \sqrt{u^2 - 4v}}{2}$$

only +ve value chosen to get high E_r & high efficiency obtained.

Case-II $\quad$ P, E_s, ϕ_r are known

$P_r = E_r I \cos\phi_r$ is active power



$$I = \frac{P_r}{E_r \cos\phi_r} \quad \text{---(1)}$$

Reactive Power $Q_r = E_r I \sin\phi_r$ (KVAR)

$$I = \frac{Q_r}{E_r \sin\phi_r} \quad \text{---} \quad \text{II}$$

putting value in equation of $\underline{E_s}$

$$E_r^2 + E_r\left(2 \underline{\underline{I}} R\cos\phi_r + 2IX\sin\phi_r\right) + I^2(R^2+X^2) - E_s^2 = 0$$

$$E_r^2 + 2E_r\left[\frac{P_r}{E_r\cos\phi_r} \times R\cos\phi_r + \frac{Q_r}{E_r\sin\phi_r} X\sin\phi_r\right]$$

$$+\left(\frac{P_r}{E_r\cos\phi_r}\right)^2 (X^2+R^2) - E_s^2 = 0$$

$$\Rightarrow E_r^2 + 2(P_r R + Q_r X) - E_s^2 + \left(\frac{P_r}{\cos\phi_r}\right)^2\left(\frac{X^2+R^2}{E_r^2}\right) = 0$$

Multiply both side by E_r^2

$$\epsilon_r^4 + \epsilon_r^2\left[2(P_r R + Q_r X) - \epsilon_s^2\right] + \left(\frac{P_r}{\cos\phi_r}\right)^2 (X^2+R^2) = 0$$

Let $\epsilon_s^2 - 2(P_r R + Q_r X) = S$

$$\left(P_r/\cos\phi_r\right)^2 (X^2+R^2) = T$$

$$\Rightarrow (\epsilon_r^2)^2 - S\epsilon_r^2 + T = 0$$

Solution for ϵ_r

$$\epsilon_r = \sqrt{\left\{\frac{S \pm \sqrt{S^2 - 4T}}{2}\right\}}$$

The solution giving the higher value of ϵ_r and correspondingly the higher Tx η, is n. not smaller required and so n. + sign should be used.