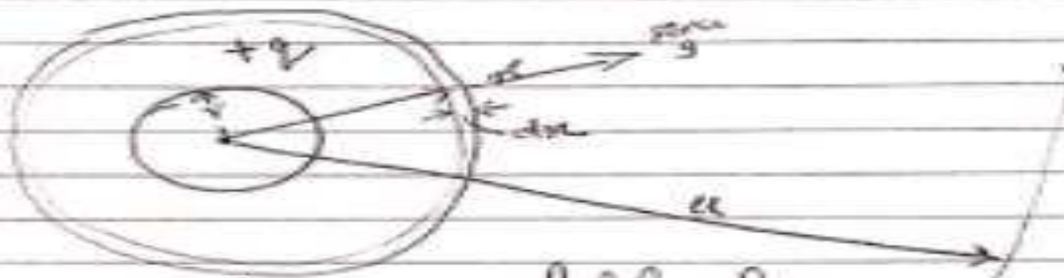
Lecture 6
Capacitance of Single and Three Phase

Capacitance of Conductors

Potential of Long Straight Conductor:-



The charge carried by an isolated long cylindrical Conductor is uniformly distributed over the surface and the equipotential surfaces are concentric with the conductor. The effect at any external point may be calculated as though the charge were concentrated on the conductor axis. Ref to above figure, if q is the charge in coulombs on an axial length of one metre, the Force at right angles to an equipotential surface of x metre radius is

$$\text{Force } g = \frac{q}{2\pi x \varepsilon_0} \text{ Volt/m}$$

ε_0 is the Permittivity of free space.

The Workdone in moving one Coulomb through a radial distance dx metre towards the conductor is $g\,dx$ joules. So the Potential difference between two points at radial distances of u and x metres respectively, where $u > x$, is

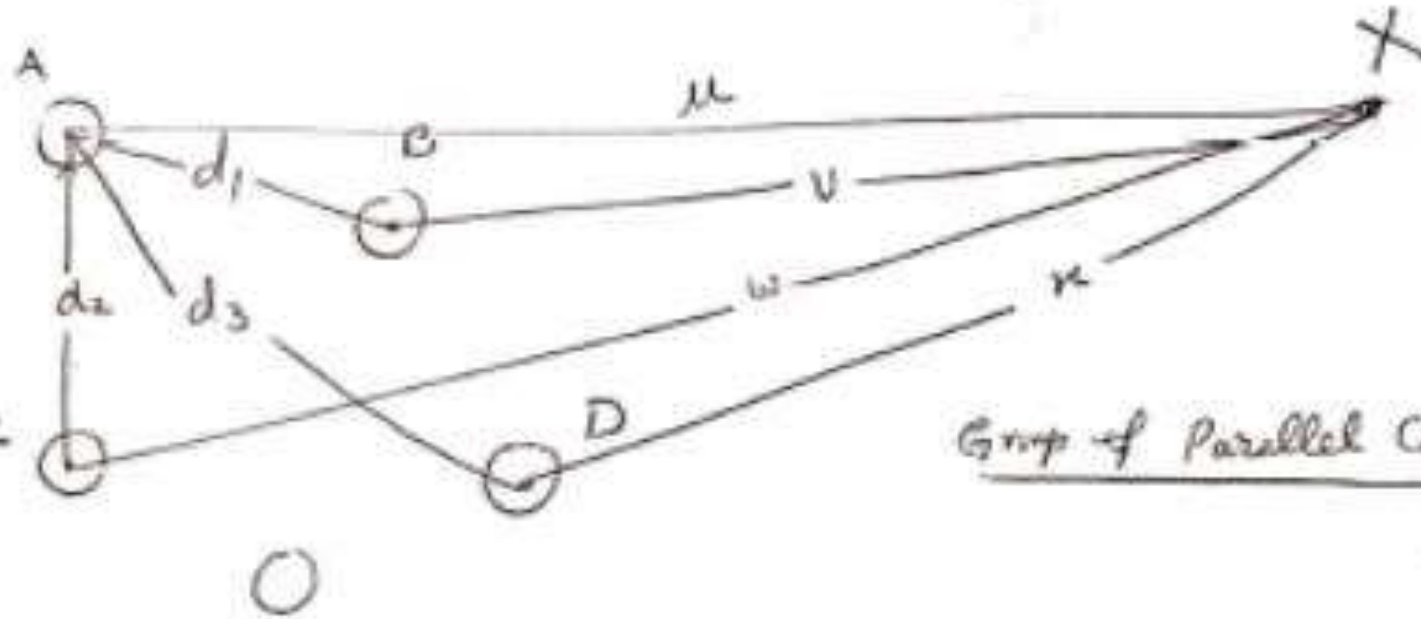
$$v = \int_x^u g\,dx = \frac{q}{2\pi\varepsilon_0}\int_x^u dx/x$$

$\rightarrow \dfrac{q}{2\pi\varepsilon_0}\ln\dfrac{u}{r}$ Volts

and since $\varepsilon_0 = 10^{-10}/3.6\pi$ Farad/meter.

$$\boxed{v = 1.8\times10^{10}\, q \ln\frac{u}{r}} \text{ Volts}$$

→ Electric Field of Parallel Cylindrical Conductors



Group of Parallel Conductors

Let the above figure represents a group of Parallel cylindrical conductor A, B, C, ... carrying charges $q_A \cdot q_B \cdot q_C$ ..... coulombs per metre respectively, the algebraic sum of the charges being zero.

X is a remote Parallel line distant u, v, w, ... metres respectively from the conductors.

In accordance with the conditions existing in practical overhead lines, the interaxial spacings of the conductors are large compared with their radii, so that the uniform distribution of charge on any of the conductors is not sensibly disturbed by the presence of neighbouring charges.

The Potential difference between Conductor A & the line X is

$$v = 1.8\times10^{10}\left(q_A \ln\frac{u}{r} + q_B \ln\frac{v}{d_1} + q_C \ln\frac{w}{d_2} + \cdots\right)$$

or rearranging

$$v = 1.8\times10^{10}\left(q_A \ln 1/r + q_B \ln 1/d_1 + q_C \ln 1/d_2 + \cdots\right) +$$

$$1.8\times10^{10}\left(q_A \ln u + q_B \ln v + q_C \ln w + \cdots\right)$$

As we know that $q_A = -(q_B + q_C + \cdots)$, the second term in eq B can be written as

$$1.8\times10^{10}\left(q_B \ln v/u + q_C \ln\frac{w}{u} + \cdots\right)$$

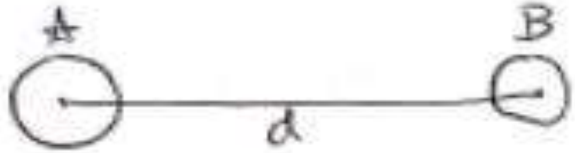
As X is moved farther away, the latter expression finally becomes zero, and the first term of Eq B then expresses the absolute potential of conductor A, so

$$v = 1.8\times10^{10}\left(q_A \ln 1/r + q_B \ln 1/d_1 + q_C \ln 1/d_2 + \cdots\right) \text{ Volts} \quad (C)$$

Capacitance of Single-Phase and Three-phase Lines

The above result can now be applied to

Single-phase lines



The figure represent two || cylindrical conductors A & B of radii r metres at an interaxial spacing of d metres, and q_A & q_B are the respective charges per metre where $q_A = -q_B = q$ coul.

from equation C, the potential of conductor A is

$$V_A = 1.8\times10^{10}\left(q_A \ln 1/r + q_B \ln 1/d\right)$$

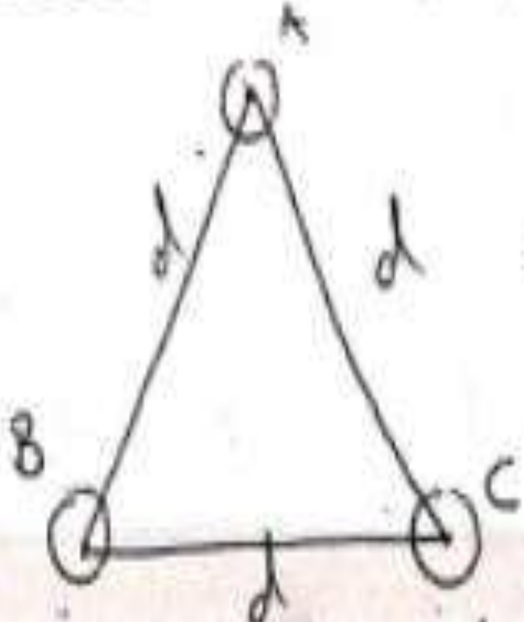
$$V_A = 1.8\times10^{10}\left(q_A \ln d/r\right) \text{ Volts} \quad (q_A = -q_B)$$

and so the capacitance of conductor A is

$$C_A = q_A/V_A = \frac{1}{1.8\times10^{10}\ln(d/r)} \text{ Farads/metre of cable}$$

This is the capacitance between conductor and neutral, and as conductor B has a similar value the capacitance of the loop formed by the two conductor is one half of this value.

Three-phase Line With Equal Spacing →



equal spacing d

(4)

In this Case, the three conductors A, B & C will occupy the Vertices 3 of an equilateral Triangle of Side d meters. Let the charges on the Conductors be q_A, q_B, & q_C respectively, the algebric sum of the charges being zero.

from Equation C

$$V_A = 1.8\times10^{10}\left(q_A \ln 1/r + q_B \ln 1/d + q_C \ln 1/d\right)$$

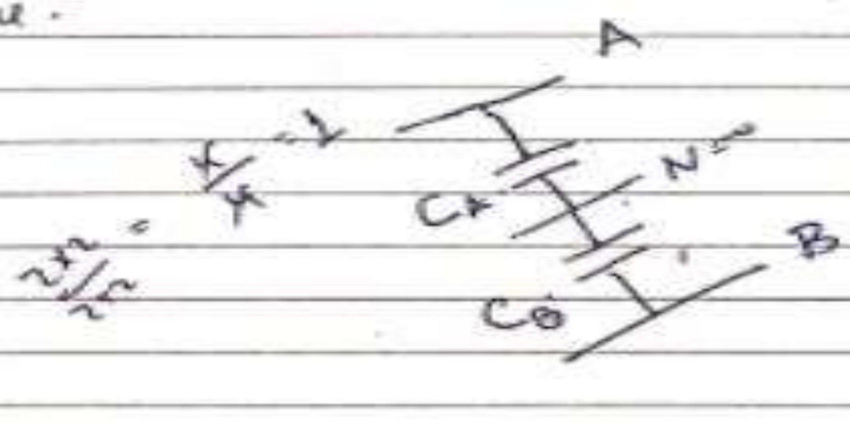
& substituting $-q_A$ for $q_B + q_C$

$$V_A = 1.8\times10^{10}\left(q_A \ln d/r\right) \text{ Volts}$$

So Capacitance of conductor A is

$$C_A = q_A/V_A = \frac{1}{1.8\times10^{10} \ln(d/r)} \text{ F/meter of ckt}$$

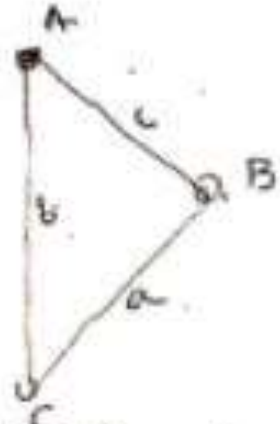
& by Symmetry B & C have the same value.

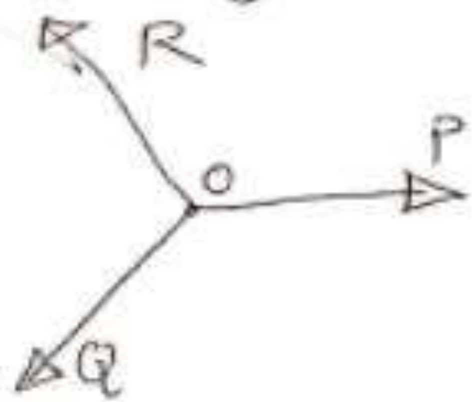


$q_A + q_B + q_C = 0$

$-q_B = q_A + q_C$

Three phase line with Irregular Spacing





When the line three line conductor are not symmetrically arranged the relative values of the charge Q must be known before the potential can be determined.

→ Using usually the electrostatic problem is one in which potentials of the cdt are given & these determine the value of the induced charges.

→ The Balanced 3-phase system is considered with unequal spacing.

A, B, C, the rms potentials of which are reduced by the vector OP, OQ, OR. if the potential of cdt A is $V_A = \overline{OP} = V$ Volts.

the Potential of B & C

$$V_B = OQ = V(-0.5 - j0.866)\ \text{Volts}$$
$$V_C = OR = V(-0.5 + j0.866)\ \text{Volts}$$

Using equations for V_A, V_B & V_C & $Q_A + Q_B + Q_C = 0$

after simplification getting the values of Q_A, Q_B & Q_C

Capacitance of A cdt

$$C_A = Q_A/V_A = \frac{3\ln\frac{a}{r} + j1.732\ln\frac{b}{c}}{1.8\times10^{10}\left(\ln\frac{bc}{ar}\ln\frac{ca}{br} + \ln\frac{ca}{br}\ln\frac{ab}{cr} + \ln\frac{ab}{cr}\ln\frac{bc}{ar}\right)}$$

Farad/metre of cdt

Similarly for C_B, C_C have similar denominator

$$C_B = \frac{3\ln\frac{b}{r} + j1.732\ln\frac{c}{a}}{\ \ }$$

$$C_C = \frac{3\ln\frac{c}{r} + j1.732\ln\frac{a}{b}}{\ \ }$$

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