Lecture # 12

Improving Transient Response

We have learned that transient performance of a system can be improved by changing the plant's parameters; ζ and ω_n, since the percent overshoot depends on ζ, whereas the rise time and settling time depends on both ζ and ω_n. Incorporating such changes can shift or reshape the root locus to pass through the desired closed-loop pole. But as discussed in the previous lecture, shifting the root locus to the left will improve the transient performance but will deteriorate the steady-state performance. On the other hand shifting the root locus to the right is expected to improve the steady-state performance, however, at the expense of transient performance. Thus improving the transient-response deteriorates the steady-state performance to some extent. A compromise between the improvement of transient and steady-state performance is therefore essential. We know that incorporating changes in the system without affecting the plant's transfer function for improvement of transient and steady-state response can be accomplished by subsystems known as compensators or controllers. The compensator usually adds poles and zeros to the transfer function, therefore the transfer function of the overall system is modified for gaining optimum performance so that the compensated system has a root locus that goes through the desired pole location for some value of gain. This modification of a system to reshape its root locus in order to improve system performance is called compensation or stabilization.

A compensator is commonly placed in series with the plant is referred to as series or cascade compensation. A compensator can also be placed in the feedback path of the system will modify the shape of the loci with inserting additional poles and zeros and as such the method is referred to as feedback or parallel compensation. The good thing about cascade compensation is that additional poles and zeros can be added at the input side, before the plant so that it does not interfere with the power output of the plant. Compensators and controllers for electrical systems can be fabricated with passive components (*RL*, *RC* or *RLC*) or an active network incorporating operational amplifiers with passive components.

Lead Compensator Design

Lead compensation approximates the function of Proportional Derivative (PD) control and acts mainly to speed up a response by lowering rise time and to decrease the overshoot. The general transfer function of the lead compensator is given as: $G_C(s) = \dfrac{K_C(s+z_C)}{(s+p_C)}$.

Let us say that we desire to keep the percentage overshoot at 10% and try to reduce the settling time to get speedy transient response, so that the system rejects disturbances quickly as possible. In this case we need to reduce settling time.

For 10% overshoot the value of ζ can be obtained using: $\%OS = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$ or $OS = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$. If we put OS = 0.1 (10%) in this expression the value of ζ = 0.6. For designing a lead compensator, we follow certain steps in sequence. Let us re-consider the system in Example 1 of the previous lecture (11) in which case the system open-loop transfer function was:

$KG(s)H(s) = \dfrac{K}{(s+2)(s+4)(s+6)}$, for which the root locus has already been drawn (please refer to notes of lecture 11).

Step 1: The root locus has to be drawn, which has already been drawn in the previous lecture so we don't redraw it here.

Step 2: Draw the damping line with angle ϕ, subtended with respect to horizontal axis and with origin as center. This angle can be calculated using:

$$\phi = 180^0 - \cos^{-1}(\zeta) = 180^0 - \cos^{-1}(0.6) = 127^0$$

Step 3: The point of intersection between the root locus and the damping line is the present operating point P_1 of the system. Remember any point on the damping line will have same 10% overshoot with value of $\zeta = 0.6$. The coordinated of P_1 from the root locus plot are approximated to:

$$P_1: \quad -2.1 + j2.7 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$$

From which: $2.1 = \zeta\omega_n$. The settling time is then: $T_S = \dfrac{4}{\zeta\omega_n} = \dfrac{4}{2.1} = 1.9$ seconds. Let us say we desire to have it as 1.33 seconds, with overshoot maintained at 10%. This gives us: $\zeta\omega_n = 3$. With $\zeta = 0.6$, ω_n comes out to be about 5 rad/sec. Thus: $\omega_n\sqrt{1-\zeta^2} = 3.8$ approximately. Thus the new operating point to achieve settling time of 1.33 seconds while maintaining 10% overshoot is P_2. Thus:

$$P_2: \quad -3 + j3.8 = -\zeta\omega_n^{/} + j\omega_n^{/}\sqrt{1-\zeta^2}$$

Step 4: Once the parameters are known, we proceed to design the lead compensator. Draw a horizontal line at P_2 as shown in root locus diagram. The angle OP_2Q is 127° according to geometry. Bisect this angle, so that with respect to the bisector, the angle is 127/2 = 63.5° on either side.

Step 5: Let us calculate the angle deviation from 180° at P_2, since it does not lie on the root locus. This angle is supposed as θ, and is obtained from considering the open loop transfer function of the plant with s replaced by $-3 + j3.8$ and the using angle criteria. Thus:

$$\theta = \left.\frac{K}{(s+2)(s+4)(s+5)}\right]_{-3+j3.8} = -231.7^0$$

Not caring about the negative sign, simply subtract 180° from it that is:

$$\psi = 231.7 - 180 = 51.7$$

Now draw lines at an angle of 51.7/2 = 26° on either side of the bisector.

Step 6: Extend the lines so that it cuts the real axis as shown. The first cutting point from the origin is where the compensator zero is and in this case it is at –2.8. The second line cutting point is where the compensator pole is, which in this case is: –7.2. Thus at this stage the transfer function of compensator is:

$$G_C(s) = \frac{K_C(s+2.8)}{(s+7.2)}$$

Step 7: In this step we calculate the gain K of the open loop transfer function of plant by substituting the point P_1: $s = -2.1 + j2.7$ in the transfer function: $\dfrac{K}{(s+2)(s+4)(s+6)}$ and using the magnitude criterion, we can find K that is: $\left.\dfrac{K}{(s+2)(s+4)(s+5)}\right]_{-2.1+j2.7} = 1$. Once $s = -2.1 + j2.7$ is substituted in each term, then the magnitude of each term is taken that is:

$$\frac{K}{(-2.1+j2.7+2)(-2.1+j2.7+4)(-2.1+j2.7+6)} = 1$$

Or
$$\frac{K}{(-0.1+j2.7)(1.9+j2.7)(3.9+j2.7)} = 1$$

Or
$$K = 2.7 \times 3.3 \times 4.74 = 42.23$$

Step 8: Gain of overall compensated system with transfer function:

$$G = \frac{K_C K(s+2.8)}{(s+2)(s+4)(s+6)(s+7.2)}$$

Or
$$G = \frac{K_1(s+2.8)}{(s+2)(s+4)(s+6)(s+7.2)}$$

The gain is evaluated by putting P_2: $s = -3 + j3.8$ in the above expression and using magnitude criterion. The gain is then: $K_1 = 115.34$

Step 9: The gain of the compensator is then:

$$K_C K = K_1$$

Or
$$K_C = \frac{115.34}{42.23} = 2.73$$

Thus the transfer function of the compensator is: $G_C(s) = \dfrac{2.73(s+2.8)}{(s+7.2)}$ and the transfer function of the compensated system is: $G = \dfrac{115.34(s+2.8)}{(s+2)(s+4)(s+6)(s+7.2)}$. This system will have 10%

overshoot with settling time of transients 1.33 seconds. If you draw the root locus it will now pass through point P_2.

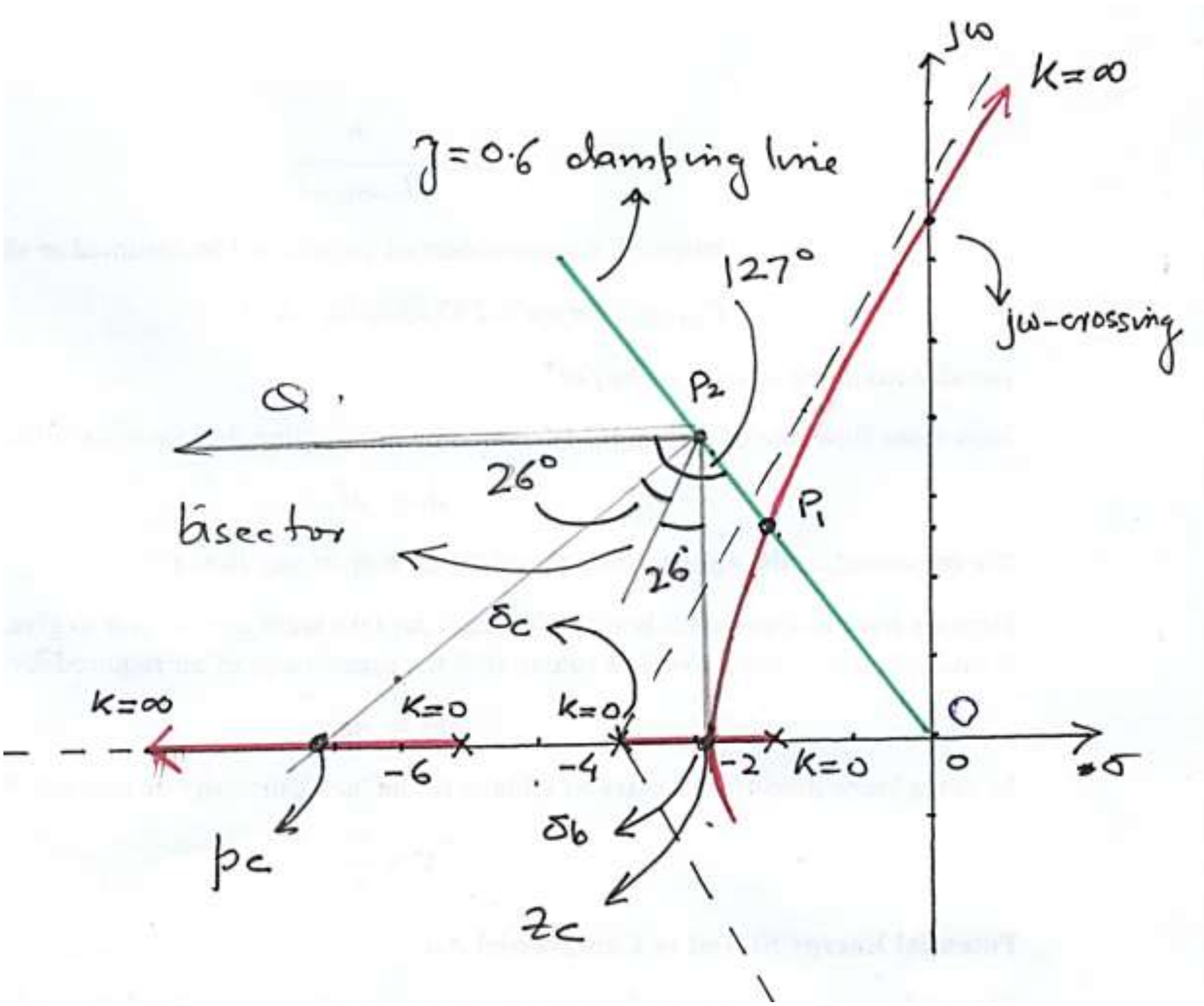

Figure (1) Root locus diagram with illustrations

It can be noted that when compensator is included the overall system becomes a 4^{th} order system. The original system is 3^{rd} order. In order to maintain the order, we can select compensator zero such that it cancels the system finite pole which is nearest to the origin. Thus if we choose compensator zero at –2, then the numerator will have a term (s + 2), which cancels with the pole in the term (s + 2) in the denominator. The system order in this case is maintained. Try this yourself as a practice problem by following the steps. Everything else remaining unchanged, from step 1 to 5. We start at step 6 and proceed as follows:

Step 6: As illustrated in Figure (2), join P_2 to a point –2 on the real axis. This becomes the zero of the compensator. Thus the zero of the compensator is at s = –2. Now draw a line subtending an angle of 51.7° with respect to the line connecting P_2 and the zero on the real axis that is –2.

Extend this line to cut the real axis at a point which then becomes the compensator's pole. In the present problem, this line cuts the real axis at point –6.8. Thus the compensator pole is at s = –6.8. The transfer function of the compensator up to this stage is:

$$G_C(s) = \frac{K_C(s+2)}{(s+6.8)}$$

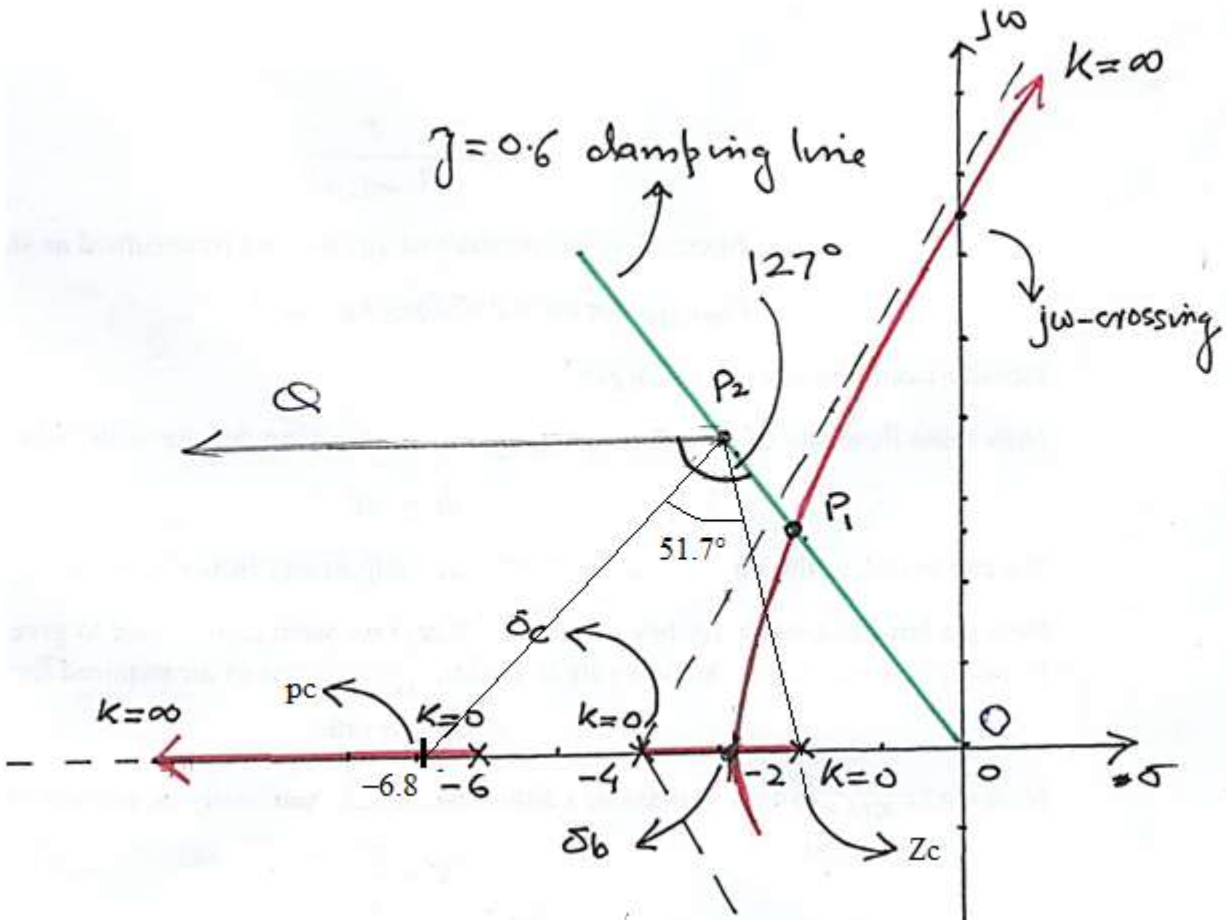

Figure 2: Illustration of alternative technique

Step 7: Same as step 7 already described.

Step 8: Once the compensator pole and zero are selected, the next task is to find out the gain *K*c of the compensator and get the complete expression. Before that we have to compute the gain of the overall compensated system using the desired dominant closed-loop pole: $s = -3 + j3.8$. The overall compensated system will have a transfer function:

$$G = \frac{K_C K(s+2)}{(s+2)(s+4)(s+6)(s+6.8)}$$

Or $$G = \frac{K_1}{(s+4)(s+6)(s+6.8)}$$

The gain is evaluated by putting P_2: $s = -3 + j3.8$ in the above expression and using magnitude criterion that is:

$$G = \frac{K_1}{(-3+j3.8+4)(-3+j3.8+6)(-3+j3.8+6.8)} = 1$$

Or $$\frac{K_1}{(1+j3.8)(3+j3.8)(3.8+j3.8)} = 1$$

Obtaining the magnitude of each of the bracketed term of the denominator and cross multiplying, we have:

$$K_1 = 3.93 \times 4.84 \times 5.37 = 102.14$$

The gain is then: $K_1 = 102.14$

Step 9: The gain of the compensator is then: $K_C K = K_1$ Or $K_C = \frac{102.14}{42.23} = 2.42$

Thus the transfer function of the compensator is: $G_C(s) = \frac{2.42(s+2)}{(s+6.8)}$ and the transfer function of the compensated system is: $G = \frac{102.14}{(s+4)(s+6)(s+7.2)}$. This system will have 10% overshoot with settling time of transients 1.33 seconds. If you draw the root locus it will now pass through point P_2. This technique maintains the system order, however, at the expense of slight reduction in gain, but that does not matter. The first method is preferred for accuracy.

Important Note:

Consider the root locus of the plant as quoted in example 1 in one of the previous lectures with open-loop transfer function: $KG(s)H(s) = \frac{K}{(s+2)(s+4)(s+6)}$. For simplicity only the upper portion of the root locus is re-drawn as in Figure (3) on the next page.

1. It must be remembered that root locus starts at the open-loop poles and ends at open-loop zeros but these are not included in the root locus. All points on the root locus are the closed-loop poles of the system.
2. The gain at open-loop poles is $K = 0$ and that at open-loop zero is $K = \infty$. Thus as we move away from the open-loop pole, the gain has a finite value other than zero and it increases as we go further from the open-loop pole.
3. The root locus possesses conjugate symmetry that is w.r.t the real axis, the root loci above and below is the mirror images.

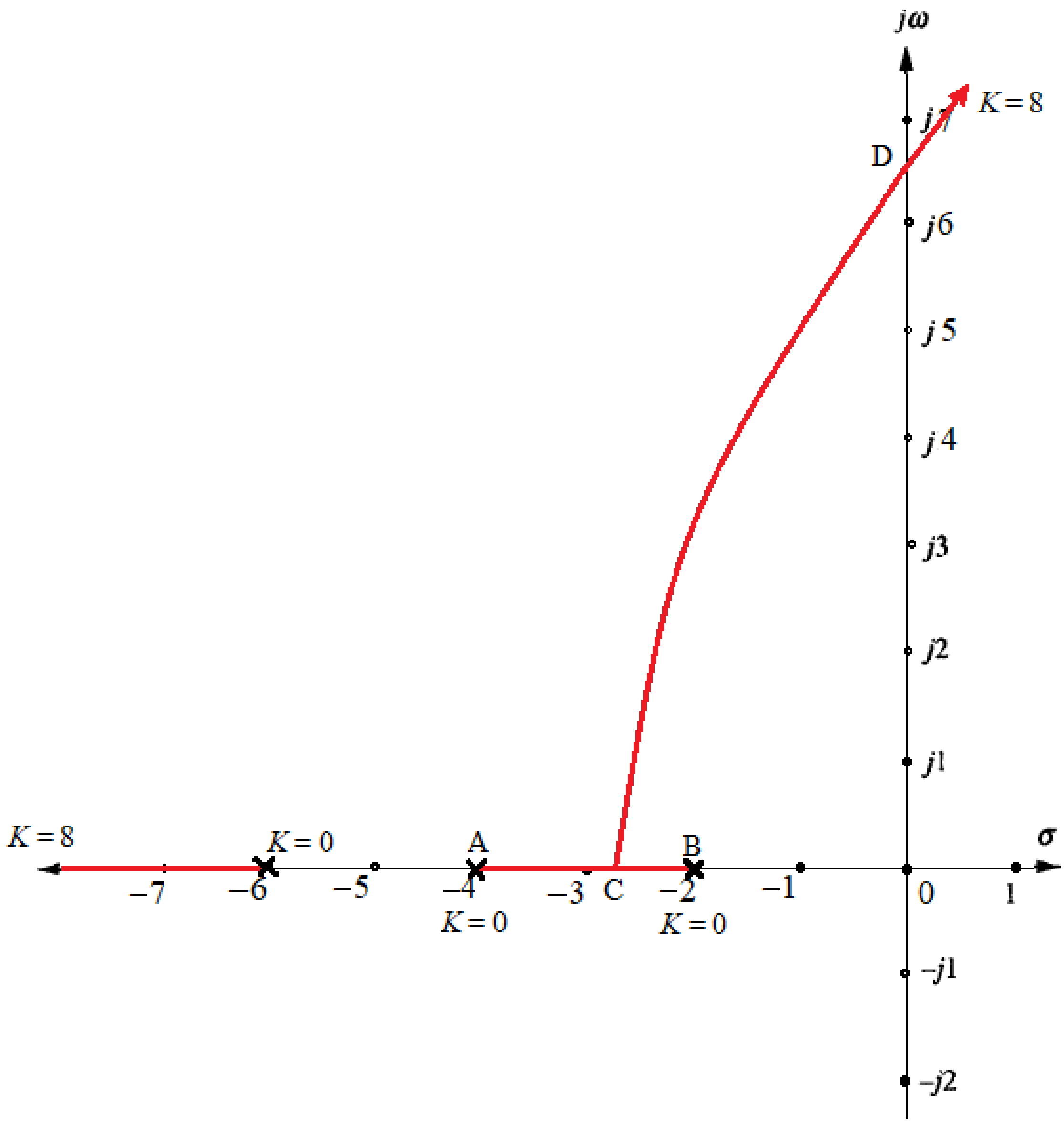

Figure 3: Root locus of example 1 (only upper portion)

4. Consider the real axis loci between –2 and –4. The system gain increases above zero and as we move out of the open-loop poles, we enter the territory of closed-loop poles. On the real axis the closed loop poles of the system are real and unequal and the system is over-damped with $\zeta > 1$. Thus at points A and B (just out of open-loop poles) the system is highly over-damped, this nature gradually fades out as we move nearer to the breakaway point at C. At C the system roots are real and equal and behaves as critical damping system with $\zeta = 1$, with the real axis gain maximum.
5. At the breakaway point the system departs from the real axis and move into the complex region with all the closed loop poles occurring in the form of complex conjugate pairs. Here the general expression for the step response of the system will have the form:

$$c(t) = Ae^{-\zeta wn}\cos(\omega_d t + \phi)$$

Where: A is the amplitude and ϕ is the phase angle. Thus the system possesses a damped-oscillatory nature and the system becomes under-damped with $0 < \zeta < 1$. The value of zeta is near to 1 (but less than 1) near the breakaway point in the complex region and gradually decreases and approaches zero as the root locus moves from C towards D. The step response of the system thus becomes more oscillatory as it approaches D.

6. At point D the damping coefficient $\zeta = 0$ and the system becomes pure oscillatory with no damping. At point D on the $j\omega$-axis the system characteristic roots (closed-loop poles) are purely imaginary with no real part and system possesses un-damped nature with sustain oscillations. The above expression for zeta equals 0 is:

$$c(t) = A\cos(\omega_d t + \phi)$$

7. When the root locus crosses D it then enters into the unstable region (right-half plane). The system oscillations keep on increasing with zeta less than 0 ($\zeta < 0$) or negative, so that in the general expression given above, the exponential term becomes a rising quantity that is for negative zeta:

$$c(t) = Ae^{\zeta wn}\cos(\omega_d t + \phi)$$

Note: Please check the calculations.