Lecture # 10

Generation of High Voltage DC

High voltage DC is mainly used for pure scientific research work and for testing equipment employed especially in High Voltage Direct Current (HVDC) transmission systems. Besides, HVDC are extensively used in physical science laboratories such as in electron microscopy, particle acceleration, production of X-rays. In communications and electronics, HVDC is used in broadcast transmitters, television and phototubes. HVDC are generally obtained by means of rectification of AC voltage. Voltage doubler circuits in desired numbers are used in cascade for the multiplication of the DC voltage.

High Voltage Rectifiers

Half-wave and full-wave rectifiers are commonly used in high voltage testing laboratories to generate HVDC through rectification of AC voltages. The rectifier mainly consists of a single-phase step-up transformer and diodes. The size and rating of the equipment is dependent on the high-voltage step-up transformer and diodes used. A typical commonly used half-wave rectifiers produces outputs up to 30 kV. Due to weight and high cost of components, high-voltage rectifiers are usually limited to the production of DC voltages for up to 100kV. In high voltage laboratories, HVDC is used in testing insulation. The circuit of a simple half-wave rectifier to explain the principle of rectification is shown in Figure (1). A half-wave rectifier essentially uses a single-phase step-up transformer and a single diode *D* with a shunt capacitor *C*, performing as a filter to reduce the harmonics to appreciable level in the output. The capacitance of the test object (insulator) under test can also act as a smoothing capacitor. A half-wave rectifier is a circuit that converts an AC voltage into a pulsating DC voltage using only one (positive) half-cycle of the applied AC voltage.

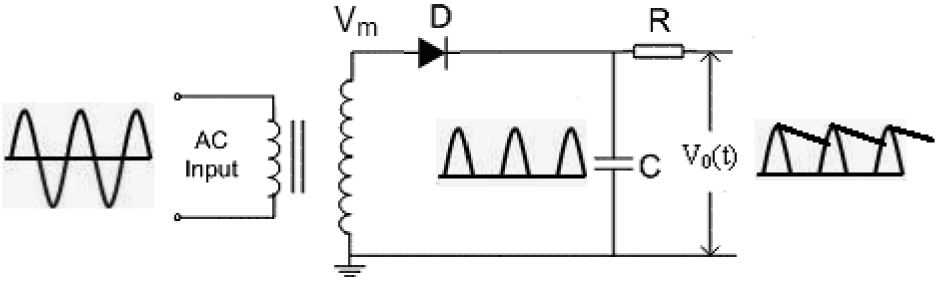

Figure 1: Circuit of a Half-wave Rectifier

Capacitor filters are generally designed to be used under the conditions that output ripples must be limited to 1 to 5%.

If *T* represents the periodic time of oscillation or deviation from the mean value and the periodic frequency *f* of oscillations is given by: $f = \frac{1}{T}$. Since $q = IT$, is the charge for one cycle due to current I during the time T, then $q = I / f$ so that the charge stored in the capacitor C is: $q = 2(\delta V)C$. Therefore:

$$\delta V = \frac{I}{2fC} \qquad 1$$

As can be seen from Eq (1), the ripple in the output voltage can be reduced by increasing the frequency of the AC voltage and also by increasing the capacitance of the smoothing capacitor. The ripple factor of a rectifier is also given by:

$$\varphi = \sqrt{\left(\frac{V_{RMS}}{\overline{V}}\right)^2 - 1} \qquad 2$$

Where $\overline{V}$ is the average value of the output voltage and for a half-wave rectifier is obtained as:

$$\overline{V} = \frac{1}{2\pi}\int_0^{\pi}(V_m \sin \omega t)d(\omega t) = \frac{V_m}{\pi} \qquad 3$$

The RMS value of voltage is:

$$V_{RMS} = \left[\frac{1}{2\pi}\int_0^{\pi} V_m{}^2 \sin^2(\omega t) d(\omega t)\right]^{1/2} = \frac{V_m}{2} \qquad 4$$

The diodes used in rectifier sets must be able to withstand twice the maximum output voltage of the transformer secondary in the reverse bias mode and each capacitor must be capable of charging to twice the maximum voltage of the transformer secondary.

For testing the insulation to its breakdown, an excessively large current flows, which must be taken into consideration for designing the test equipment and a current limiting resistor is generally connected in series with the test object in order to protect the equipment damage.

Full-Wave Rectifier

A full-wave rectifier converts an AC voltage into a pulsating DC voltage using both half-cycles of the applied AC voltage. It uses two diodes of which one conducts during one half-cycle while the other conducts during the other half-cycle of the applied AC voltage. The circuit of a full-wave rectifier and the associated waveform is shown in Figure (2). A full-wave rectifier essentially consists of a step-up, center-tapped transformer and two diodes with a smoothing capacitor. The center-tapped transformer produces two sine waves that are dispersed in opposite direction at the center-tap, so that these waves have a phase-shift of 180°. The average value of the output voltage for a full-wave rectifier is obtained as:

$$\overline{V} = \frac{1}{\pi}\int_0^{\pi}(V_m \sin\omega t)d(\omega t) = \frac{2V_m}{\pi} \qquad 5$$

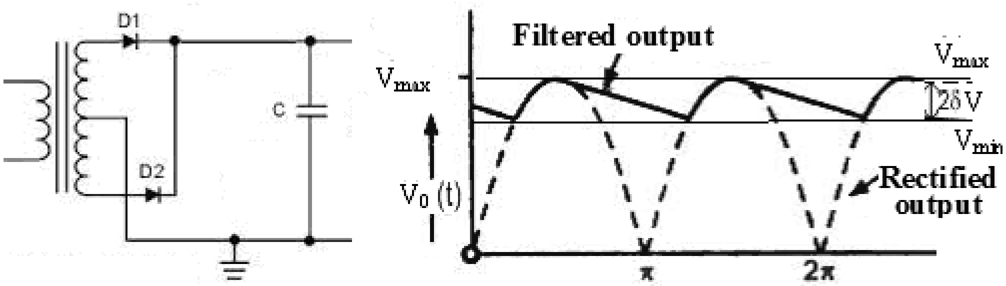

Figure 2: Circuit and Waveform of a Full-wave Rectifier

The RMS value of voltage is:

$$V_{RMS} = \left[\frac{1}{\pi}\int_0^{\pi} V_m{}^2 \sin^2(\omega t)d(\omega t)\right]^{1/2} = \frac{V_m}{\sqrt{2}} \qquad 6$$

Most high-voltage rectifier sets are usually manufactured in small units up to about 100 kV, because of the transformer and diodes. Furthermore, the centre-tapped transformer is expensive. For generating higher DC voltages than 100 kV, other circuits, namely doubler, tripler and quadruple circuits are used.

Example 1: It is required to produce a DC voltage up to 25kV for testing purpose using a half-wave rectifier by using a 1.5kVA, 50Hz, single phase step-up transformer with voltage ratio of 220V/20kV. Assume that that the allowable voltage drop across the diode should not exceed 5% of the output voltage. Design a high voltage rectifier that can limit the ripple to 2.8%.

Solution:

Given that: The transformer is 1.5kVA, 220V/20kV. The peak value of the transformer secondary voltage is $20\sqrt{2} = 28.28$ kV. The allowed voltage drop across the diode is 5% of the of the output voltage, which is: $V_0 = 25$ kV. The voltage drop across the diode should then be:

$$V_d = 0.05 \times 28.28 = \mathbf{1.414kV}$$

The maximum output voltage that can be obtained will be:

$$V_0 = V_m - V_d = 28.28 - 1.414 = \mathbf{26.86kV}$$

Thus the transformer can be used for producing 25kVDC at the output. To limit the ripple to 2.8%, the deviation between maximum and minimum value must be:

$$\delta V = 0.028 \times V_0 = 0.028 \times 26.86 = \mathbf{0.75kV}$$

The full load current of the transformer is: $\frac{1.5}{20} = 75\,\text{mA}$. The capacitor required is:

$$C = \frac{I}{2f\delta V} = \frac{75 \times 10^{-3}}{2 \times 50 \times 750} = \mathbf{1\mu F}$$

Example 2: Repeat Example (1) for a full-wave rectifier using a 1.5kVA, single phase step-up transformer with two identical secondary coils each of voltage ratio 220V/20kV.

Solution:

In this case two similar diodes will be used and the two secondary coils of the transformer should be connected with the correct polarity for providing a centre-tap. At the point of centre-tap a solid ground is connected. Each diode will be contributing independently to give an output voltage pulsating at a frequency of 100Hz. Thus for limiting the ripple to 2.8%, the capacitance of the smoothing capacitor should be:

$$C = \frac{I}{2f\delta V} = \frac{75 \times 10^{-3}}{2 \times 100 \times 750} = \mathbf{0.5\mu F}$$

Voltage Doublers

The name voltage doubler is derived from the fact that the output voltage is double the value of the input voltage (transformer secondary voltage). Voltage doubler circuit reduces the size and cost of the equipment compared to ordinary rectifiers of the same voltage rating. Figure (3) shows a simple voltage doubler circuit.

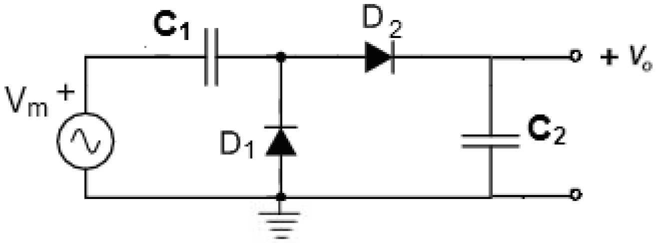

Figure 3: Simple Voltage Doubler

Referring to Figure (3), the voltage doubler is composed of two circuits; a clamper section comprising of capacitor C_1 and diode D_1 and peak detector section (half-wave rectifier) comprising of a diode D_2 and capacitor C_2. The test object is connected across C_2 or conversely, C_2 may represent or comprise the capacitance of the test object. According to its working, the capacitor C_1 charges to the peak secondary voltage of the transformer during the negative half-cycle when the diode D_1 is forward biased. During this period, the diode D_2 remains non-conducting (reverse biased). During the subsequent positive-half cycles, the diode D_2 is forward biased while D_1 is reverse biased. The voltage across C_1 acts in series with the secondary voltage of the transformer and appears across C_2 that may be the capacitance of the test object. Thus, the voltage across the output terminal and ground is $V_0 = 2V_m$. Thus an output of twice the transformer secondary peak voltage will appear across the terminals where the test object is connected.

Cascade Voltage Doubler (Walton Cockroft Generator)

Figure (4) shows a 2-stage voltage doubler or Walton-Cockroft generator. The Walton-Cockcroft design is based on the cascade voltage doublers. Each stage comprises of two diodes and two capacitors with the circuit configuration of a voltage doubler or equal to the number of smoothing capacitors. The advantage is that an expensive high-voltage transformer is not required; at least not as high as the output.

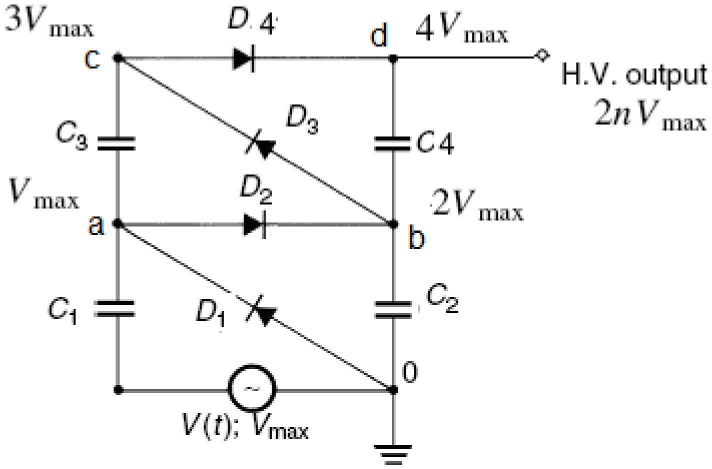

Figure 4: Circuit of 2-stage Walton-Cockroft Generator

The working principle of the generator can be understood by considering open-circuit conditions at the output of the generator, circuit shown in Figure (5), in which case the load current is zero. Based on this condition, the steady-state potentials at all the nodes of the circuit are indicated. Assume that V_{max} is the peak value of the secondary voltage of the high-voltage supply transformer. The charging of the capacitors actually takes place stage-by-stage rather than simultaneously (practically the circuit behaves as if the process takes place simultaneously).

Consider the portion of the circuit containing the diode D1 and the capacitor C1 and the secondary winding of the supply transformer. During the first negative half-cycle of the applied voltage, the capacitor C1 charges-up to full transformer secondary voltage V_{max} through the forward biased diode D1. During the positive half-cycle which follows, the diode D1 is reverse biased, the capacitor C1 will neither discharge instantly nor will charge-up in the other direction, and therefore at the peak of this half-cycle, the point b will be at $2V_{max}$. During the positive half cycle, the diode D2 will forward bias (FB) and D1 will reverse bias (RB), the potential at *a* will swing between 0 and $2V_{max}$, so that point b will be at 2Vm. Thus as the voltage at *b* swing between 0 and $2V_{max}$, the diode *D2* will be forward biased, and the capacitor *C2* would therefore charge to $2V_{max}$.

During the next negative half cycle the process is repeated for D1 and C1, but the diode D3 will also be FB because of voltage 2Vm at point b so that a voltage of 2Vm will swing to point c and together with Vm voltage on C1, the total voltage at point c will be Vm + 2Vm = 3Vm. The diode D2 will be RB at this stage.

During the next positive half cycle, D1 and D3 will be RB, but the diodes D2 and D4 will be FB. This will allow a voltage swing of 3Vm + Vm = 4Vm at point d. Thus after two cycles (two negative half and two positive half) the voltage will be 4Vm.

The HVDC at the output terminal of the generator is thus of the magnitude $2nV_{max}$. For multistage cascade, the peak-to-peak ripple is given by the expression:

$$2\delta V = IT\sum_{i=1}^{n}\frac{i}{C_i} \qquad 7$$

Where *I* is the current and *T* is the period of applied input voltage. Since all the diodes from D'_1 to D'_n allows the same charge to be transferred on to the capacitors in the smoothing section, a cascade voltage doubler has therefore a reduced ripple content as compared to ordinary rectifiers. The total ripple is given by an expression:

$$\delta V = \frac{I}{2f}\left(\frac{1}{C_1}+\frac{2}{C_2}+...+\frac{n}{C_n}\right) \qquad 8$$

All the capacitors, in each stage of the smoothing section (output side) have equal values of capacitance, therefore $C = C_1 = C_2 = ...\ C_n$. The above equation can then be expressed as:

$$\delta V = \frac{I}{fC}\left[\frac{n(n+1)}{4}\right] \qquad 9$$

Once a load is connected at the output terminals of the generator, the output voltage V_0 decreases due to the voltage regulation. If the voltage drop in generator is V_d, then the output voltage is:

$$V_0 = 2nV_{sm} - V_d \qquad 10$$

Where: V_{sm} is the maximum voltage at the secondary of the supply transformer. The voltage drop for an *n*-stage generator is given by an expression:

$$V_d = \frac{I_0}{6fC}(4n^3 + 3n^2 - n) \qquad 11$$

Where I_0 is the load current, f is the supply frequency and C is the stage capacitance. The voltage drop as a result of application of load is then modified as:

$$V_d = \frac{n^2 I_0}{fC} \qquad 12$$

The ripple produced in Walton–Cockroft generators can be calculated in terms of the ripple voltage V_r given by:

$$V_r = \frac{I_0}{2fC}[n(n+1)] \qquad 13$$

Example 3: A 4-stage Walton-Cockroft generator is powered by a 220V/10kV, 50Hz, single phase transformer. Find the output and ripple voltage for a load current of 10mA when the capacitors have value of 2μF.

Solution:

Given that: $f = 50$ Hz, $C = 2$ μF and $I_0 = 10$ mA.

Secondary peak voltage: V_{sm} = 1.414 x 10000 = 14.14 kV. For calculation of voltage drop, using:

$$V_d = \frac{I_0}{6fC}(4n^3 + 3n^2 - n) = \frac{10\times10^{-3}}{6\times50\times2\times10^{-6}}[4(4)^3 + 3(4)^2 - 4] = 5\text{kV}$$

Therefore the output voltage can be calculated from using Eq (9.14):

$$V_0 = 2nV_{sm} - V_d = 2\times4\times14.14\times10^3 - 5000 = \mathbf{108\ kV\ (peak)}$$

Example 4: Calculate the ripple volatge for the generator described in Example 3.

Solution:

Using: $V_r = \frac{I_0}{2fC}[n(n+1)] = \frac{10\times10^{-3}}{2\times50\times2\times10^{-6}}[4(5)] = \mathbf{1\ kV}$

Van de Graff Generator

Van de Graff generator is the most commonly used electrostatic machine for producing very high voltages, but are seldom used for testing purposes. Van de Graff generators are widely used in physical and nuclear sciences laboratories as particle accelerators for bombardment with suitable targets, and as sources for producing high-energy neutrons in the range between 1 and 5 MeV.

Figure (5) illustrates the basic constructional features and simplified principle of working of a Van de Graff generator. It consists of a large hollow metallic dome-shaped electrode A, supported by an insulating pillars or column. An insulated belt runs on two pulleys B and C, the upper and the lower pulley respectively. The upper pulley is idle and the lower pulley is driven by an electric motor, whose speed can be controlled. At the top and bottom of the insulating pillar are charge injector *D* and charge collector *E*, which terminate in row of sharply pointed corona combs. At the charge injector a voltage of around 10 to 30 kV is applied with respect to the ground by using an HVDC source such as a rectifier. At the tip of the corona combs at *D*, the electrical field becomes very high, which ionizes the air thus injecting the positive ions.

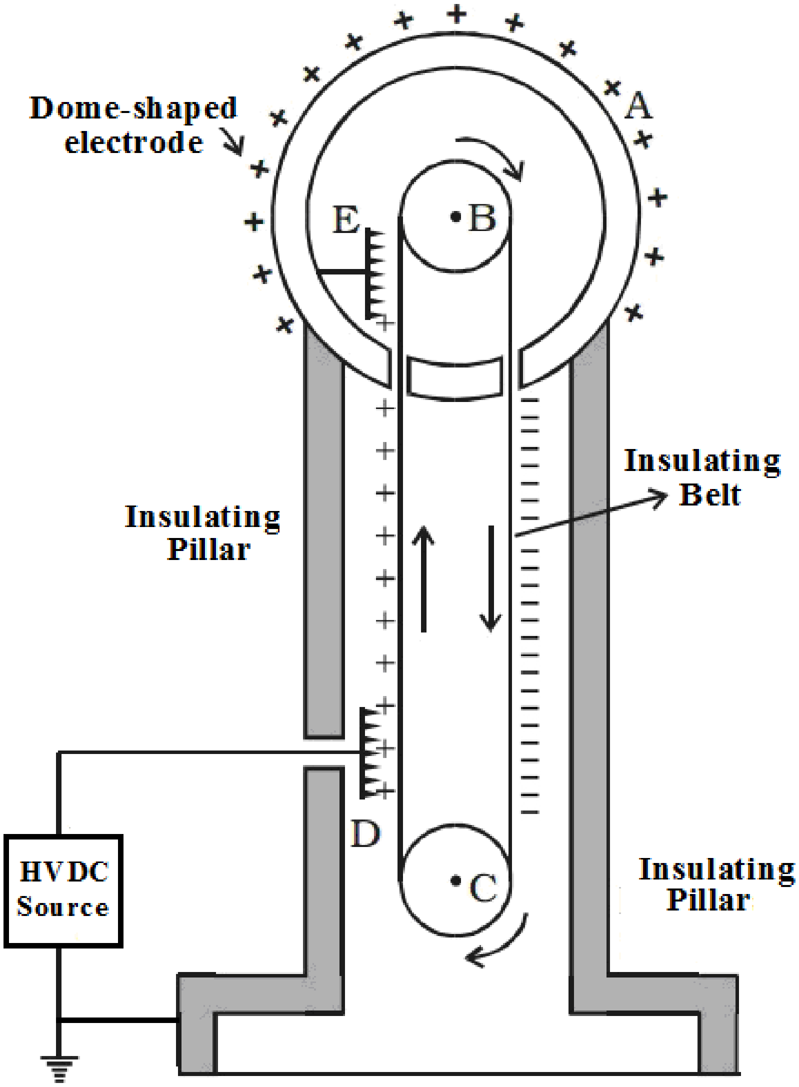

Figure 5: Van de Graff Generator HVDC Source

Positive charge is thus sprayed from a row of corona combs at *D* onto a moving belt, which carries it to the field-free region terminal of electrode *A*. At the electrode *A*, the charge is removed by another row of corona combs or charge collector *E*, allowing it to flow to the outer surface of the electrode terminal. Thus the positive charge carried by the belt, induces negative charge on the combs at *E* and thus positive charge is transferred to the electrode *A*. The high electric field at the points of the electrode *A* ionizes the air and negative charges are repelled onto the belt, discharging it before it passes over the upper pulley. The charge on the electrode A gradual increase in charge magnitude results as the belt goes around the upper and lower pulley. Thus the charge transfer on to the electrode *A* increases its potential with respect to the ground. The potential on the electrode *A* can be estimated from the charge build-up that can be viewed as a point charge system or an isolated charged sphere.

Thus it is expected that as the process of charging continues, a very high potential difference can be created between the electrode *A* and ground, which in the absence of any charge lost will build-up indefinitely. However, the built-up of charge on the electrode *A* is limited by the leakage paths through the supporting columns and air in the vicinity and the potential thus attains a steady-state value. These paths collectively constitute a resistance *R*. The charge transfer depends on the width of the belt *b* and the speed *v* with which the belt is moving. The potential difference *V* between the electrode *A* and the ground is then:

$$V = Rvb\sigma \qquad 14$$

This high potential difference when applied across a spark gap may cause breakdown under favorable conditions. Early Van de Graff generators were operated in air at atmospheric pressure. This required a great deal of space around the terminal and the highest voltages attained were about 1 MV.

Example 5: Find the potential difference that can be created in a typical Van de Graff generator having a belt width of 10cm moving with a constant speed of 10m/s when a charge density of 1μC/m^2 accumulates on the dome-shaped electrode. The total resistance between the electrode and the grounded base through insulating structure of the generator is $10^{13}\Omega$. Also find the continuous leakage current along the insulating structure.

Solution:

Given that: belt width b = 10cm or 0.1m, $\sigma = 10^{-6}$C/m^2, $R = 10^{13}\Omega$ and that the speed of the pulley is maintained constant at 10m/s. Using:

$$V = Rvb\sigma$$

Or

$$V = 10^{13} \times 10 \times 0.1 \times 10^{-6} = \mathbf{10MV}$$

The leakage current is:

$$I = \frac{V}{R} = \frac{10 \times 10^{6}}{10^{13}} = \mathbf{1\mu A}$$

Please report any mistakes in calculations.