RULES FOR PLOTTING ROOT LOCUS

Real axis loci:

1. Given the open-loop transfer function $KG(s)H(s)$, plot all the open-loop poles and zeros in the s-plane.
2. The real axis loci will exist to the left of real axis odd number of pole or zero.
3. The real axis loci must exist between two real axis poles or two real axis zeros or between a real axis pole and a real axis zero.
4. In case a zero is lying at infinity, the loci will exist between a real axis pole and infinity (indicating a zero lying at infinity).

Breakaway and break-in point:

1. A breakaway point exists on the real axis loci formed between two poles.
2. A break-in point exists on the real axis loci formed between two zeros.
3. To find these points:
    a. Equate the open-loop transfer function $KG(s)H(s)$ to –1.
    b. Rearrange and take the derivative: $\frac{d}{ds}(K)$.
    c. Equate the derivative in part b equal to zero for $s = s_b$, forming an equation.
    d. Solve the equation for s_b to yield breakaway and break-in point. Suitable choice is necessary between multiple roots.

Centre and angles of asymptotes:

1. The centre of asymptotes σ_C can be obtained using: $\sigma_C = \frac{\sum p - \sum z}{n}$, where: n is the difference of the number of poles and zeros.
2. Angle of asymptotes can be obtained using: $\theta_C = \frac{\pm 180(2k+1)}{n}$. Repetitions are not considered.

Imaginary axis crossing points:

1. Obtain the closed-loop transfer function from the given open-loop transfer function.
2. Get the characteristic equation from the closed-loop transfer function: $1 + KG(s)H(s) = 0$.
3. Obtain the Fourier transform of the characteristic equation that is: $1 + KG(jw)H(jw) = 0$.
4. Equate separately the imaginary and real components of the Fourier transformed characteristic equation to zero and solve them for w and K. The quantity w gives the $\pm\, jw$ points symmetrically above and below the origin with the value of K for which the system is marginally stable. Alternatively use the Routh table and apply the all-zero row rule to get the auxiliary equation. Solving the auxiliary equation will give w for jw crossing and the condition of equating to zero on the expression containing K at the calculated value of w will give the gain at jw crossing.

Angle of departure from a complex pole:

1. In the given open-loop transfer function, remove the complex pole of the form: $s = -a + jb$ at which the angle of departure is to be calculated.
2. Find the angle of the resultant open-loop transfer function for: $s = -a + jb$. Let this angle be α.
3. The angle of departure θ_D is then: $\theta_D = 180 + \alpha$.

Angle of arrival at a complex zero:

1. In the given open-loop transfer function, remove the complex zero of the form: $s = -c + jd$ at which the angle of arrival is to be calculated.
2. Find the angle of the resultant open-loop transfer function for: $s = -c + jd$. Let this angle be β.
3. The angle of arrival θ_A is then: $\theta_A = 180 - \beta$.

Location of the dominant complex pole:

1. Given the open-loop transfer function, obtain the closed-loop transfer function.
2. Solve the characteristic equation: $1 + KG(s)H(s) = 0$ for a given design value of *K*.
3. In case the design value of *K* is not given, a suitable design value of *K* can be obtained from the range of *K* values for stability from Routh table. Preferably, that value of *K* at which the gain margin should be around 5. The **gain margin** is the ratio of the value of *K* at *jw* crossing point to the design value of *K*.
4. The complex root: $-\zeta w_n + jw_n\sqrt{1-\zeta^2}$ of the characteristic equation in step 2 is then the dominant closed-loop complex pole.

Drawing the damping line:

1. The damping line is the line that extends from the origin that passes through the dominant closed-loop pole.
2. Alternatively if the value of ζ is known, the damping line is drawn at the origin with an angle: $\beta = 180 - \cos^{-1}\zeta$.
3. The dominant closed-pole is then the point of intersection of the damping line on the root locus.

The design value of *K*:

1. Using the magnitude criteria, equate the magnitude of the open-loop transfer function to unity that is: $|KG(s)H(s)| = 1$.
2. Find the design value of *K* from the magnitude criteria above by substituting the dominant complex pole: $-\zeta w_n + jw_n\sqrt{1-\zeta^2}$ for the value of *s*.