Consider a unity f/b control system with
OL T/F $\longrightarrow$ $KG(s)H(s) = \dfrac{K}{(s+1)(s+3)}$

The root locus is quite easy.

(i) p : = −1, −3, z = Nil 2 at ∞

(ii) p − z = 2 − 0 = 2 No of loci = 2

(iii) Real axis loci lie b/w −1 and −3 only.

(iv) There is this one breakaway point
Using angle−mag criteria
$$KG(s)H(s) = -1$$
$$\frac{K}{(s+1)(s+3)} = -1$$
$$\text{or} \quad K = -(s^2+4s+3)$$
$$\frac{dK}{ds} = -2s-4 = 0.$$
$$s = -2$$
$$\sigma_b = -2$$

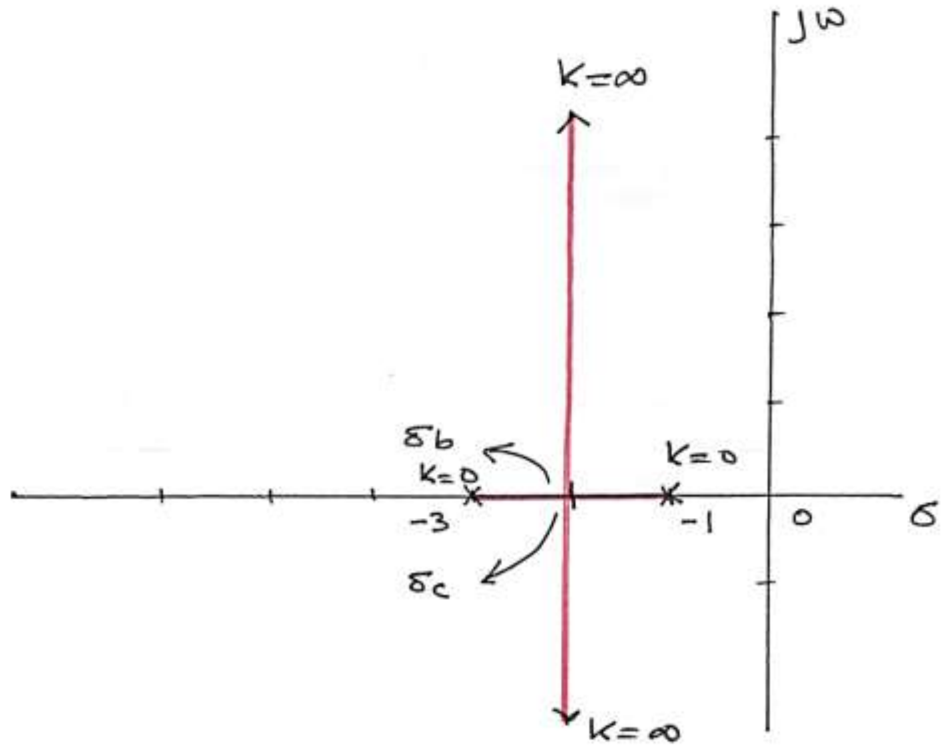
(V) Centre of asymptote.
$$\sigma_c = \frac{\Sigma p - \Sigma z}{p-z} = \frac{-1-3-0}{2} = -2$$

(vi) Angles of asymptote.
$$\theta_c = 90^\circ \text{ and } 270^\circ$$

(vii) There is no jω-crossing because $\theta_c = 90^\circ, 270^\circ$

Thus the root locus is a st vertical line drawn at $\sigma_b = s = -2$

The root locus is shown below:



The root locus is a straight vertical line and therefore does not cross the jw-axis. Thus the system does not move into the unstable region (right-half plane) and is therefore always stable. Thus all the closed-loop poles lie in the left-half plane.

Now let us add a pure gain integrator in the forward path with transfer function 1/s. The open-loop transfer function then takes the form: $KG(s)H(s) = \dfrac{K}{s(s+1)(s+2)}$. This is our Example 4.

To draw the root locus we follow the rules:

Now if we add a pure integrator with T/F of 1/s in the forward path we get a OL T/F of the system as:

$$\text{OLT/F} \longrightarrow KG(s)H(s) = \frac{K}{s(s+1)(s+3)}$$

To draw its root locus we have the following requirements.

(i) p: 0, −1, −3. z: Nil 3 at ∞

(ii) p − z = 3 − 0 = 3 No of loci = 3 (equal to No of OL poles).

(iii) Real axis loci lie between 0 and −1 and b/w −3 and −∞

(iv) Thus there is one breakaway point, which can be found using angle-magnitude criteria

ie $KG(s)H(s) = -1$

or
$$K = -\frac{1}{G(s)H(s)} = -s(s+1)(s+3)$$
$$K = -(s^3+4s^2+3s)$$
$$\frac{dk}{ds} = -\frac{d}{ds}(s^3+4s^2+3s) = 0$$
$$-(3s^2+8s+3) = 0$$
or
$$3s^2+8s+3 = 0$$

from which $S = -0.45$ and -2.21, in which case only -0.45 lies on the real axis loci

Thus
$$\delta b = -0.45. \quad (\text{b/w } 0 \text{ and } -1)$$

(V) The centre of asymptote
$$\delta_c = \frac{\sum p - \sum z}{p-z} = \frac{0-1-3-0}{3} = -1.33$$
$$\therefore \quad \delta c = -1.33$$

(vi) Likewise angles of asymptotes, if worked out will be

$$\theta_c = 60°, 180° \text{ and } 300°$$

(vii) This means root locus will cross jw-axis. The jw-crossing is found by considering Q(s) in CLT/F as

$$CLT/F = \frac{K}{s(s+1)(s+3)+K}$$

$$= \frac{K}{s^3+4s^2+3s+K}$$

$$Q(s) = s^3+4s^2+3s+k$$

$$s=j\omega \quad Q(j\omega) = -j\omega^3 - 4\omega^2 - j3\omega + K = 0$$

Equating imag part to zero.

$$-j\omega^3 - j3\omega = 0.$$

$$\omega^3 = -3\omega$$

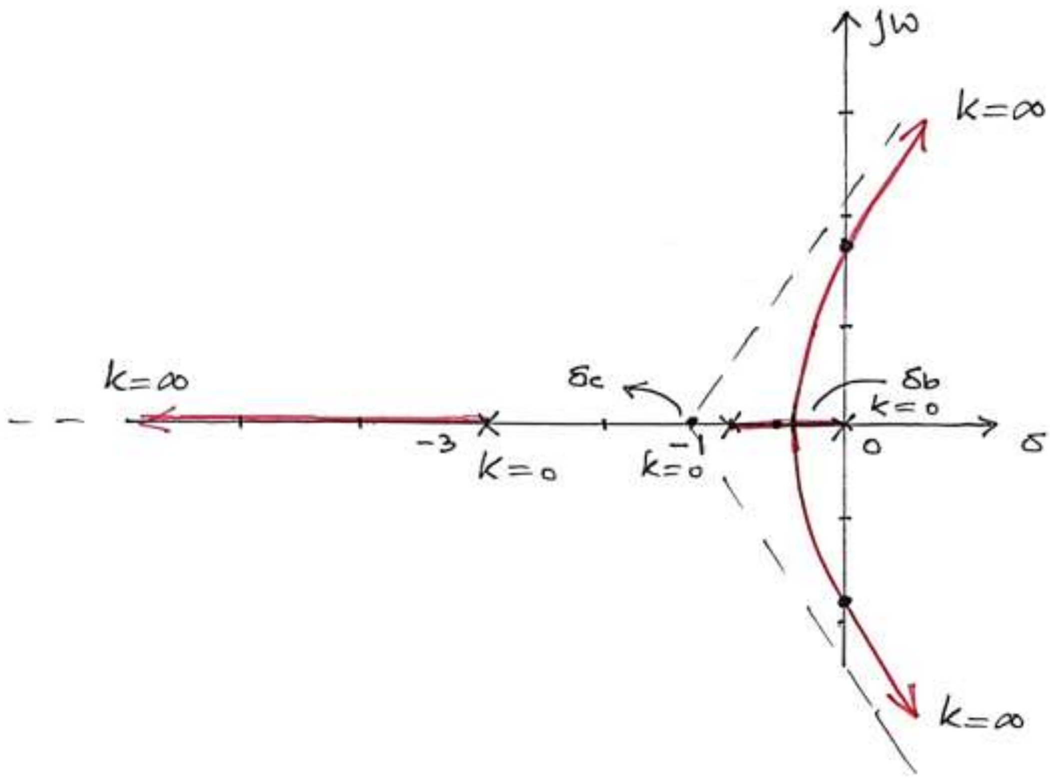
$$\omega^2 = -3$$

$$\omega = \pm j1.732$$

$\therefore$ jw-crossing is at $+j1.732$ and $-j1.732$

(viii) Gain at jw-crossing is from equating real portion of $Q(j\omega) = 0$.

The root locus plotted is shown on the next page.



1. We can see that now the root locus is not a vertically straight line but it follows a curve and crosses the jw-axis. Thus adding a pure integrator will make the otherwise stable system to a system that may lead to instability if the operating point is not wisely selected.
2. The root locus, on adding a pure integrator in series with the plant will tend to shift the root locus more towards the right as can be noted from example 3 and 4 with their break-away points shifted to right in case of example 4. This means that the transient response in-terms of settling time will deteriorate. The settling time will increase in case when integrator is included and therefore the transients or disturbance will not die out quickly. This may be a bad news in the case of aero-plane auto pilot system.
3. However, shifting the root locus to the right improves steady-state performance since more gain is added into the system.
4. Adding an integrator in the forward path will also change the system-type. Comparing example 3 and 4 indicate that the system type changes from Type-0 (example 3) to Type-1 (example 4)

Conclusions:

1. Adding a pure gain derivative controller will improve the transient performance by shifting the original root locus to the left. In this cane the settling and rise time becomes less and the disturbances die out quickly (speedy transient response)

2. Adding a pure gain integrator controller will deteriorate the transient performance by shifting the root locus to the right. However, the steady-state performance is improved.
3. Including integrator changes the system type from lower to higher order.

However, practically pure gain derivative and integrator controller does not exist. Instead proportional derivative (PD) and proportional integrator (PI) controllers are used. These controllers are incorporated with an added gain element with their transfer functions as follows:

PD $$G(s) = sK_D + K$$

And

PI $$G(s) = \frac{K_I}{s} + K$$

A combination of the above two is the proportional integral derivative (PID) controllers with transfer function:

PID $$G(s) = \frac{K_I}{s} + sK_D + K$$