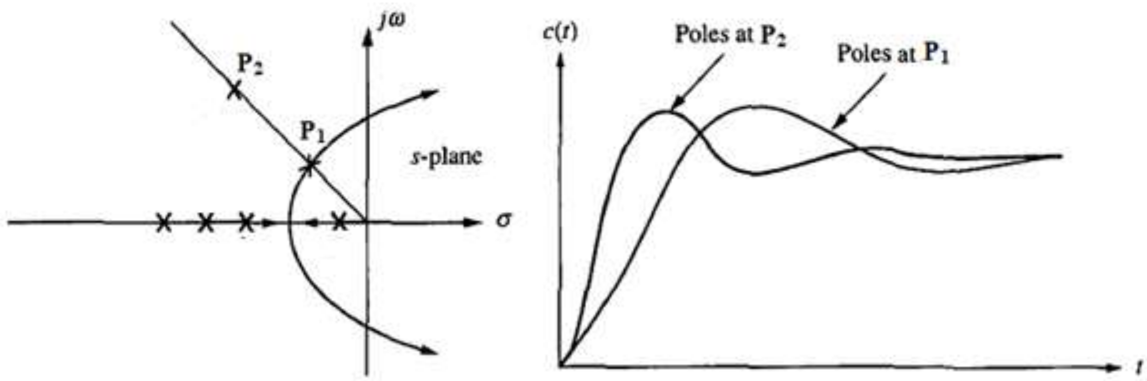
Compensation

Carrying out desired tasks and to maintain stability is the main objective of control system design. Design of control systems start with the understanding of the controlled process. This is accomplished by a controller. The main idea is to design a suitable controller to accomplish desired objective(s). Once understood and the parameters defined, the design of the controller starts with the development of root locus. The design concept start with selecting a desired complex closed-loop pole of the form: $-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$, containing two important system parameters; ζ and ω_n. In order to illustrate the basic design concept for a control system, consider a root locus plot of system shown in Figure 1.



(a) (b)

Figure 1: (a) Root Locus Plot (b) System Transient Response by Pole Shifting

As shown in Figure (1a), point P_1 is on the root locus that gives us a transient performance based on percent overshoot and settling time. If our desired point of operating the plant in a control system is P_1, lying on the root locus, then a simple gain adjustment from the controller is all that is needed for optimization. But, if we desire that transients (disturbances) should settle down quickly in a short period of time without disturbing the percent overshoot, then the operating point of the system should lie leftward on the damping line, supposing point P_2. However, P_2 does not lie on the root locus. Thus optimization of transient response cannot be accomplished by a simple gain adjustment. The design objective is therefore to shift the root locus from point P_1 to P_2, which is located on the same constant damping line (constant zeta), in which case the value of percent overshoot will not be affected while both the rise time and settling time will reduce, thus speeding up the transient response because both rise time and settling time depends on the value of ω_n, which will be greater at P_2 than that at P_1. The process of tending to pass root locus through the desired point is referred to as compensation. This is accomplished by designing a controller that can fulfill the required purpose and is then referred to as compensator. In order to understand the compensation, we will consider two examples in which case a system is given to us and let say what we can do to improve its performance.

Consider the system shown in Figure 2, which is a unity feedback system.

Example 1

The transfer function (T/F) of unity f/b system is:

$$OL\ T/F \rightarrow KG(s)H(s) = \frac{K}{(s+2)(s+4)(s+6)}$$

The followings are the items required to draw the Root Locus of the T/F

(i) No of OL poles : 3
" " " zeros: Nil (3 at ∞). No of loci = 3.

(ii) p: −2, −4 and −6

(iii) Real axis loci: Exist b/w −2 and −4 and −6 and $-\infty$

(iv) There is one break-away point b/w −2 and −4 on real axis

(v) The breakaway point is found by using the angle-mag criteria and taking its derivative and then equating this derivative to zero i.e.

$$KG(s)H(s) = -1.$$

$$K = -(s+2)(s+4)(s+6)$$

$$K = -(s^3+12s^2+44s+48)$$

$$\frac{dk}{ds} = \frac{d}{ds}\left(-(s^3+12s^2+44s+48)\right) = 0$$

which will give us value of s as: s = −2.8 & −5.15
But only s = −2.8 lies b/w −2 and −4 on real axis, so this is the breakaway point. Thus

$$\sigma_b = -2.8$$

(vi) The centre of asymptotes.

$$\delta_c = \frac{\sum p - \sum z}{p - z} = \frac{-2-4-6-0}{3}$$

$$\delta_c = -4.$$

(vii) Angles of asymptotes.

$$\theta_c = \frac{180(2m+1)}{p-z} \qquad m = 0, 1, 2 \ldots$$

$m = 0 \quad \theta_c = \frac{180}{3} = 60°$

$m = 1 \quad \theta_c = \frac{180(3)}{3} = 180°$

$m = 2 \quad \theta_c = \frac{180 \times 5}{3} = 300°$

next $m = 3, \ldots$ etc is repetition.

(viii) $j\omega$-crossing point.

We first obtain the CL T/F of given OL T/F. Since the system is unity feedback

$$\therefore \quad CLT/F = \frac{K}{(s+2)(s+4)(s+6)+k}$$

according to formula $\frac{kG(s)}{1+kG(s)}$.

$$CLT/F = \frac{K}{s^3+12s^2+44s+48+k}$$

The denominator to polynomial is characteristic polynomial i.e.

$$Q(s) = s^3+12s^2+44s+48+k.$$

put $s = j\omega$:

$$\therefore \quad -j\omega^3 - 12\omega^2 + j44\omega + 48 + k = Q(s)$$

$$Q(s) = 0$$

So that:

$$-j\omega^3 - 12\omega^2 + j44\omega + 48 + k = 0 \quad (1)$$

Equating (grouping) imaginary portion of above

ie $-j\omega^3 + j44\omega = 0$.

$$\omega^3 = +44\omega.$$

$$\omega^2 = +44.$$

$$\omega = \pm 6.63$$

or $j\omega = \pm j6.63$.

The gain at this crossing is found by equating the real portion of above eq (1)

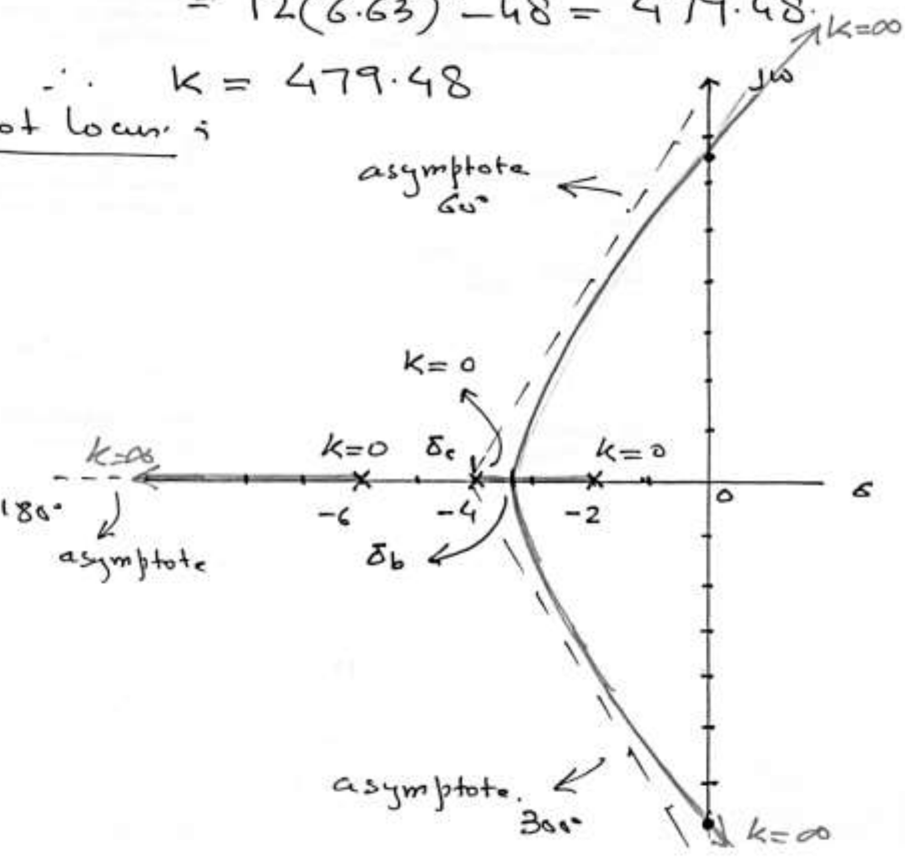
$$-12\omega^2 + 48 + k = 0.$$

or $k = +12\omega^2 - 48$

$$= 12(6.63)^2 - 48 = 479.48.$$

$$\therefore \quad k = 479.48$$

The root locus is



Example 2

Now let us add a simple zero at the origin in the OL T/F so that it is modified as.

$$OLT/F \rightarrow KG(s)H(s) = \frac{sk}{(s+2)(s+4)(s+6)}$$

Proceeding according to steps in example 1

(i) No of OL $p = 3$, No. of OL $z = 1$. (2 at ∞) $\therefore p-z = 3-1 = 2$

(ii) $p: -2, -4, -6$, $z: 0$.

(iii) Real-axis loci: Exist b/w 0 and -2 and b/w -4 and -6.

(iv) There is one breakaway point b/w -4 and -6 according to the rules (see text book)

(v) Breakaway point

$$\frac{dk}{ds} = -\frac{d}{ds}\left[\frac{(s^3+12s^2+44s+48)}{s}\right] = 0$$

or $$\frac{1}{(s+2)} + \frac{1}{(s+4)} + \frac{1}{(s+6)} = \frac{1}{s}.$$

solving it will give:

$$s = -5.06, -2.7, 1.75$$

The breakaway point is -5.06 since it lies b/w -4 and -6 on real axis ~~Thus~~ loci. Thus.

$$\delta b = -5.06$$

(vi) Centre of asymptote.

$$\delta_c = \frac{\Sigma p - \Sigma z}{p-z} = \frac{-2-4-6-0}{2} = -6$$

$$\therefore \delta_c = -6$$

(vii) Angles of asymptotes.

$m = 0$ $\quad \theta_c = \frac{180}{2} = 90°$

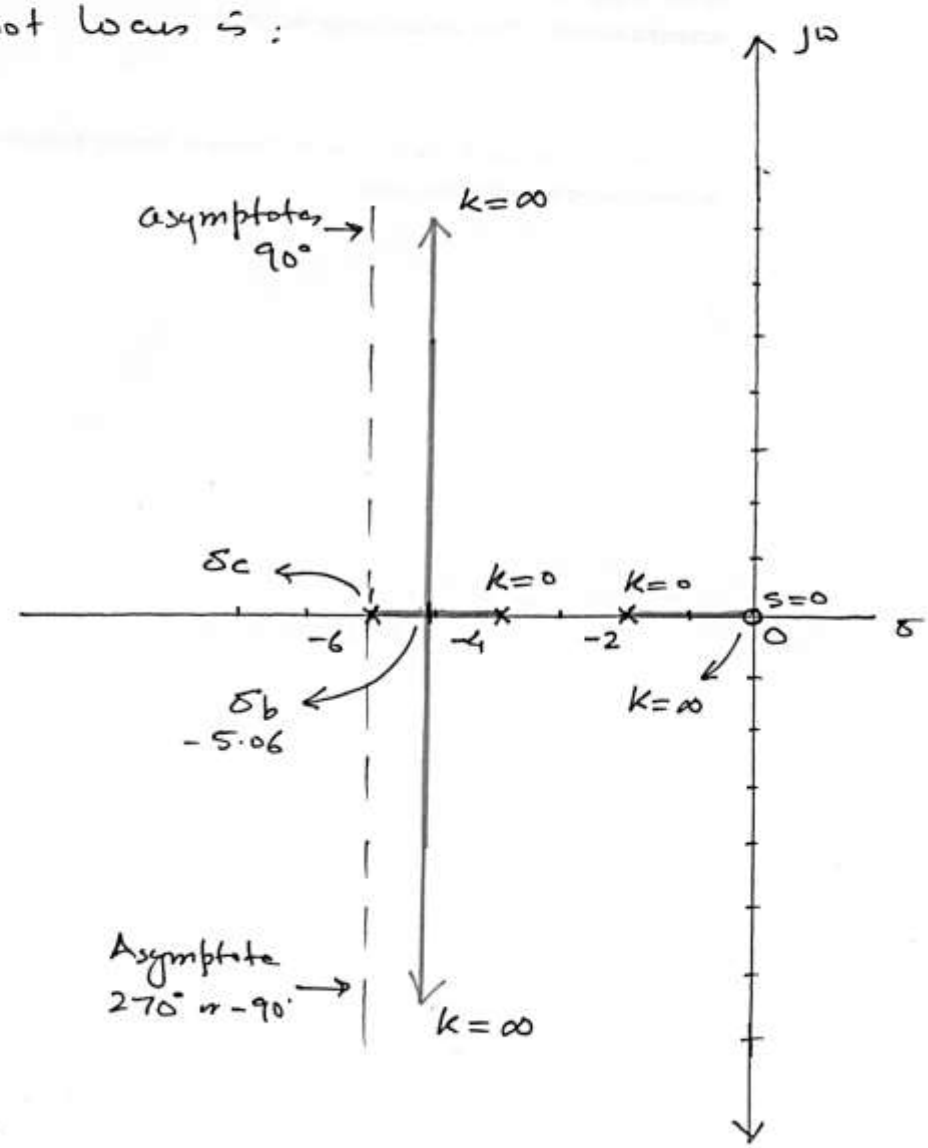
$m = 1$ $\quad \theta_c = \frac{180 \times 3}{2} = 270°$ or $-90°$

next $m = 2, 3 \ldots$ is repetition of same angles

$\therefore \theta_c = 90°$ and $270°$

(viii) There is no $j\omega$-crossing since the Root locus goes up and down vertically and will not cross $j\omega$-axis.

The root locus is:



Example 1 is that of the system shown in Figure 1, which is a unity feedback system with the forward block containing the transfer function.

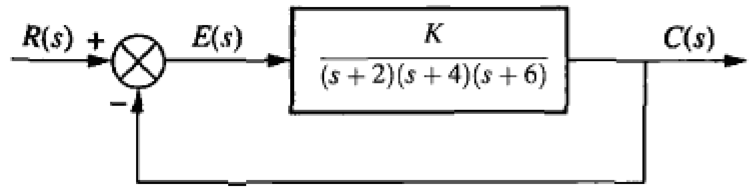


Figure 1

A simple control system can be represented as a unity feedback system with a plant driven by a controller connected in series or in the same forward path as shown in Figure 2.

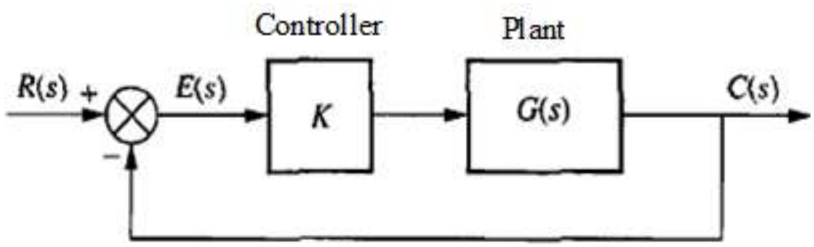


Figure 2

According to the control system configuration of Figure 2, the system of example 1 can therefore be represented as shown in Figure 3.

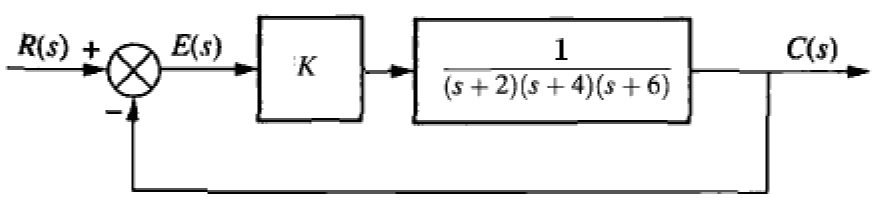


Figure 3

Comparison of the two root locus leads us to the following findings:

1. The root locus for uncompensated system (Figure 2 or 3) is moving from stable half of s-plane (LH-plane) to an unstable half of s-plane (RH-plane) in search of their zeros lying at infinity. On the other hand the root locus for compensated system by including a simple zero (zero at origin) is always stable that is it never leaves the left-half of s-plane to search for zeros lying at infinity. In this particular case it can be concluded that the system becomes stable for all closed-loop poles and thus becomes closer to a minimum phase system (gain margin: infinity).

2. Further it can be noted that the root locus of compensated system is shifted to the left in comparison with that of uncompensated system. This can be ascertained from the break-away point, which in the compensated case is –5.06 and that for uncompensated system is –4. It must be remembered that more the root locus is to the left, faster the transients (disturbances) decay that is the system becomes speedier in getting rid of disturbances and reverting to steady-state. Thus the settling time of transients is less irrespective of percentage overshoot, which means the system is getting better.

What we have done actually, is simply adding a zero at the origin or including a simple single differentiator in the forward path, since: $s = \frac{d}{dt}$. Including a simple zero can be looked upon as including a simple gain differentiator *sK* as a series controller with the plant as shown in Figure 4. Thus it can be noted that providing a derivative controller shifts the root locus to the left side as a general case. However, as a specific case in this example, the system also becomes completely stable.

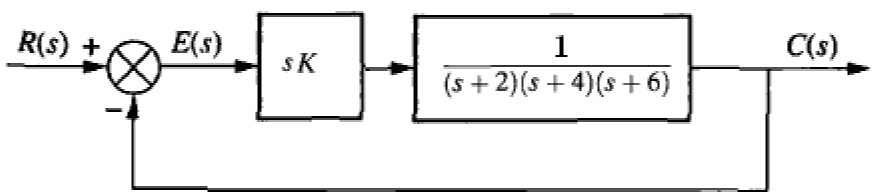


Figure 4

As Example 3, let us now consider a second order, unity feedback system with an open-loop transfer function:

$$KG(s)H(s) = \frac{K}{(s+1)(s+2)}.$$

This system is always stable and is therefore a minimum phase system with its poles lying in the left-half of s-plane. To draw its root locus, it is simple we follow the rules in finding quantities essential for plotting the root locus. These are as follows:

Consider a unity f/b control system with
OL T/F $\longrightarrow KG(s)H(s) = \dfrac{K}{(s+1)(s+3)}$

The root locus is quite easy.

(i) $p: = -1, -3, \quad z = \text{Nil} \quad 2 \text{ at } \infty$

(ii) $p - z = 2 - 0 = 2$ No of loci $= 2$

(iii) Real axis loci lie b/w -1 and -3 only.

(iv) There is this one breakaway point
Using angle-mag criteria

$$KG(s)H(s) = -1$$

$$\frac{K}{(s+1)(s+3)} = -1$$

$$\text{or} \quad K = -(s^2 + 4s + 3)$$

$$\frac{dK}{ds} = -2s - 4 = 0.$$

$$s = -2$$

$$\delta_b = -2$$

(V) Centre of asymptote.

$$\delta_c = \frac{\Sigma p - \Sigma z}{p - z} = \frac{-1-3-0}{2} = -2$$

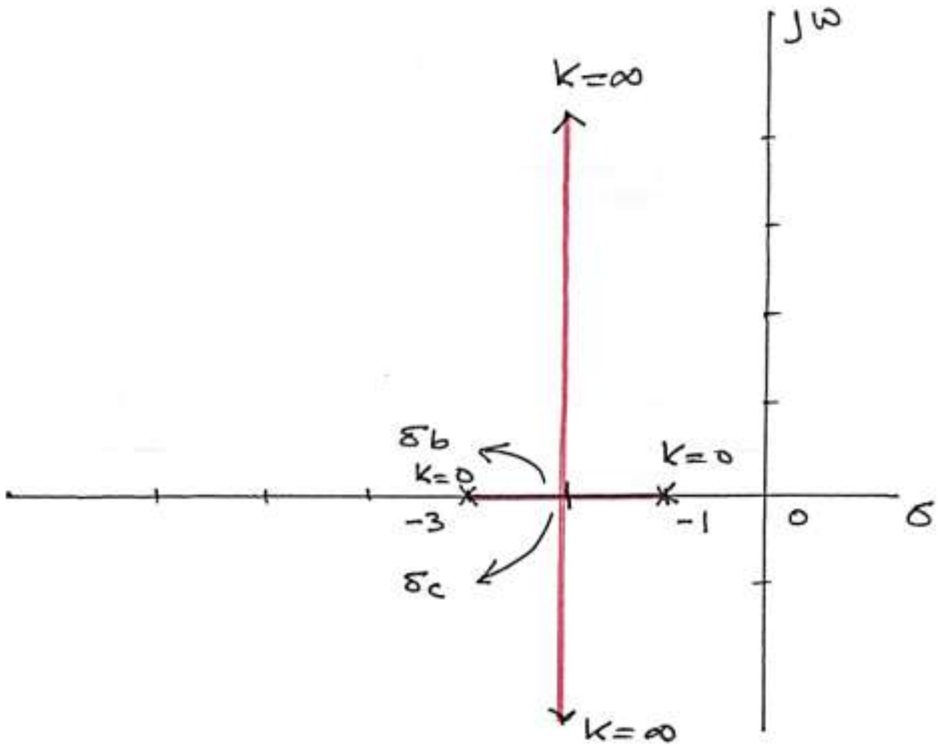
(vi) Angles of asymptote.

$$\theta_c = 90^\circ \text{ and } 270^\circ$$

(vii) There is no $j\omega$-crossing because $\theta_c = 90^\circ, 270^\circ$

Thus the root locus is a st vertical line
drawn at $\delta b = s = -2$

The root locus is shown below:



The root locus is a straight vertical line and therefore does not cross the jw-axis. Thus the system does not move into the unstable region (right-half plane) and is therefore always stable. Thus all the closed-loop poles lie in the left-half plane.

Now let us add a pure gain integrator in the forward path with transfer function 1/s. The open-loop transfer function then takes the form: $KG(s)H(s) = \dfrac{K}{s(s+1)(s+2)}$. This is our Example 4.

To draw the root locus we follow the rules:

Now if we add a pure integrator with T/F of 1/s in the forward path we get a OL T/F of the system as:

$$\text{OLT/F} \longrightarrow KG(s)H(s) = \frac{K}{s(s+1)(s+3)}$$

To draw its root locus we have the following requirements.

(i) $p: 0, -1, -3$. $\quad z:$ Nil $\quad 3$ at ∞

(ii) $p - z = 3 - 0 = 3$ No of loci $= 3$ (equal to No of OL poles).

(iii) Real axis loci lie between 0 and -1 and b/w -3 and $-\infty$

(iv) Thus there is one breakaway point, which can be found using angle-magnitude criteria

ie $\quad KG(s)H(s) = -1$

or
$$K = -\frac{1}{G(s)H(s)} = -s(s+1)(s+3)$$
$$K = -(s^3 + 4s^2 + 3s)$$
$$\frac{dK}{ds} = -\frac{d}{ds}(s^3 + 4s^2 + 3s) = 0$$
$$-(3s^2 + 8s + 3) = 0$$
or
$$3s^2 + 8s + 3 = 0$$

from which $S = -0.45$ and -2.21, in which case only -0.45 lies on the real axis loci

Thus
$$\delta b = -0.45. \qquad (\text{b/w } 0 \text{ and } -1)$$

(v) The centre of asymptote
$$\delta_c = \frac{\Sigma p - \Sigma z}{p - z} = \frac{0 - 1 - 3 - 0}{3} = -1.33$$
$$\therefore \quad \delta c = -1.33$$

(vi) Likewise angles of asymptotes, if worked out will be

$$\theta_c = 60°, 180° \text{ and } 300°$$

(vii) This means root locus will cross jω-axis. The jω-crossing is found by considering Q(s) in CLT/F as:

$$CLT/F = \frac{K}{s(s+1)(s+3)+K}$$

$$= \frac{K}{s^3+4s^2+3s+K}$$

$$Q(s) = s^3+4s^2+3s+k$$

$$s=j\omega \quad Q(j\omega) = -j\omega^3 - 4\omega^2 - j3\omega + K = 0$$

Equating imag part to zero.

$$-j\omega^3 - j3\omega = 0.$$

$$\omega^3 = -3\omega$$

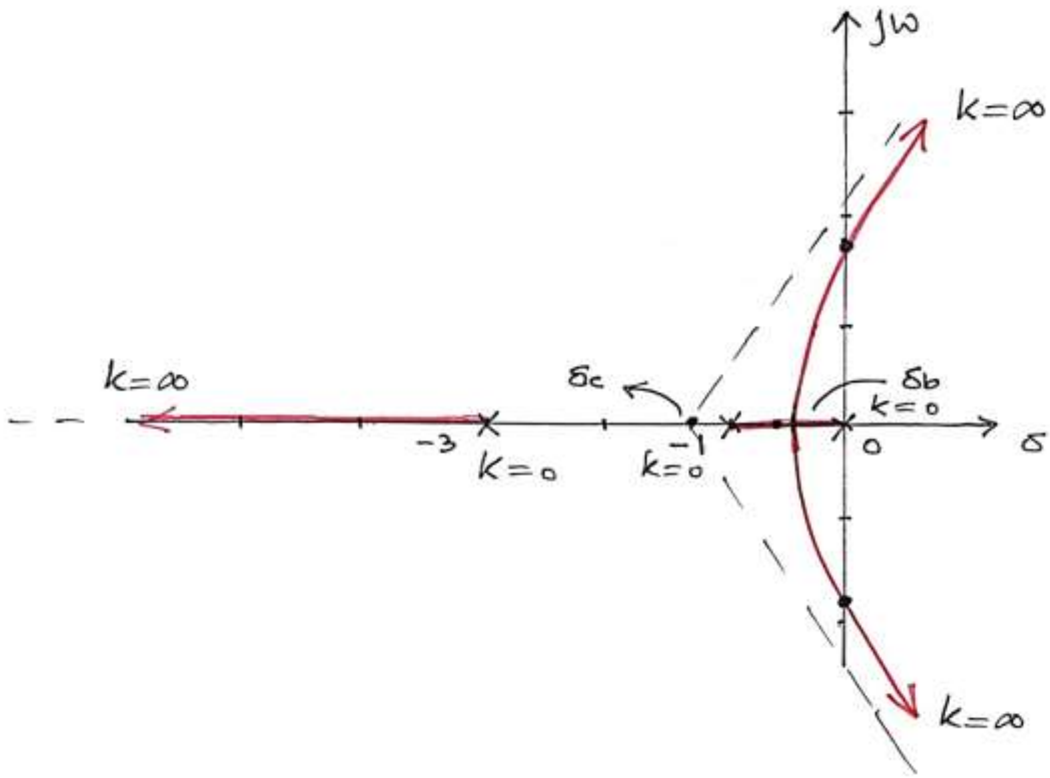
$$\omega^2 = -3$$

$$\omega = \pm j1.732$$

∴ jω-crossing is at $+j1.732$ and $-j1.732$

(viii) Gain at jω-crossing is from equating real portion of $Q(j\omega) = 0$.

The root locus plotted is shown on the next page.



1. We can see that now the root locus is not a vertically straight line but it follows a curve and crosses the jw-axis. Thus adding a pure integrator will make the otherwise stable system to a system that may lead to instability if the operating point is not wisely selected.
2. The root locus, on adding a pure integrator in series with the plant will tend to shift the root locus more towards the right as can be noted from example 3 and 4 with their break-away points shifted to right in case of example 4. This means that the transient response in-terms of settling time will deteriorate. The settling time will increase in case when integrator is included and therefore the transients or disturbance will not die out quickly. This may be a bad news in the case of aero-plane auto pilot system.
3. However, shifting the root locus to the right improves steady-state performance since more gain is added into the system.
4. Adding an integrator in the forward path will also change the system-type. Comparing example 3 and 4 indicate that the system type changes from Type-0 (example 3) to Type-1 (example 4)

Conclusions:

1. Adding a pure gain derivative controller will improve the transient performance by shifting the original root locus to the left. In this cane the settling and rise time becomes less and the disturbances die out quickly (speedy transient response)

2. Adding a pure gain integrator controller will deteriorate the transient performance by shifting the root locus to the right. However, the steady-state performance is improved.
3. Including integrator changes the system type from lower to higher order.

However, practically pure gain derivative and integrator controller does not exist. Instead proportional derivative (PD) and proportional integrator (PI) controllers are used. These controllers are incorporated with an added gain element with their transfer functions as follows:

PD $$G(s) = sK_D + K$$

And

PI $$G(s) = \frac{K_I}{s} + K$$

A combination of the above two is the proportional integral derivative (PID) controllers with transfer function:

PID $$G(s) = \frac{K_I}{s} + sK_D + K$$