THE TRANSMISSION AND DISTRIBUTION OF ELECTRICAL ENERGY

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By

H. COTTON

M.B.E., D.Sc., M.I.E.E.

Emeritus Professor of Electrical Engineering,
University of Nottingham

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PREFACE

The changes introduced into this second edition are threefold, namely: the conversion from the c.g.s. to m.k.s. system of units in the development of formulae, the addition of much new material to chapters dealing with subjects in which there have been great changes in apparatus and in practice, and the extension of many sections dealing with fundamental principles. This third change applies particularly to Chapter I, where a number of important network theorems are explained and numerical examples given; to Chapter XII on insulated cables, where a serious omission in the first edition, the determination of cable temperature rise and cable rating, has been rectified; to Chapter XIV, which is a new chapter dealing with the important mathematical tool of symmetrical components, previously relegated to a section in the chapter on feeder protection; and to Chapters XVIII and XIX on travelling waves and the protection of lines and systems against overvoltages. Again, certain of the earlier material dealing with apparatus now more or less obsolete has been omitted and a large amount of more recent material put in its place.

It may be objected that, in the field of feeder protection, this elimination of old material has not been taken far enough. This now vast subject does not readily lend itself to text-book treatment and it would have been easy to ask manufacturers for technical literature on current methods and apparatus, to make liberal use of scissors and paste, and to call the assembled residue a chapter. This would have resulted in a treatment of the subject completely lacking in coherence and having nothing in the form of a plan devised to give the student a good picture of the whole field. For this reason a certain amount of the older material has been retained.

There has been, over the past few years, considerable development in the design and construction of super voltage cables and, in consequence, the chapter on insulated cables has been largely rewritten and very considerably extended. That on circuit-breakers is entirely new; it has been written for me by Mr. E. O.

Taylor of the Heriot Watt College, and I am glad to express my gratitude to him for undertaking this part of the book.

The final chapters deal with travelling waves and with the apparatus used to protect lines and systems against overvoltages. The original method of treatment, which was very well received, has been retained, but there has been very considerable amplification of the treatment of the effects of changes in line constants, and descriptions given of the construction and characteristics of modern apparatus.

The continued popularity of the book in its original form over a period of twenty-one years has been very gratifying, and it is hoped that, in its new form, it will be equally acceptable.

H. COTTON.

Colwyn Bay

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CHAPTER I

SUPPLY SYSTEMS

Feeders and Distributors

THE conductor system, by means of which electrical energy is conveyed from a power station to a consumer, can, in general, be divided up into two distinct parts, viz. the transmission and distribution systems. These again can be subdivided into primary and secondary transmission, and primary and secondary distribution, and finally, there is the system of supply to the individual consumer. A typical system will thus be somewhat as in Fig. 1.1, although it has to be realised that all existing systems do not include all the parts shown in the figure. For example, some systems may have no secondary transmission, some may be so small that there is, in effect, distribution only and no transmission, and so on. In general, in an A.C. system there will be a change of voltage at each point where the subdivision takes place, this change being effected at a substation, and it therefore follows that there may be several working voltages in the same system. For obvious reasons it has been necessary to standardise voltages, the values in use in this country being as follows:

1. Generating voltages: 6600, 11000, and up to 33000.
2. High-voltage transmission: 275000, 132000, 66000, down to 11000.
3. High-voltage distribution: 11000 and 6600.
4. Low-voltage distribution:
   A.C.: 400 between phases, 230 to neutral.
   D.C.: 3-wire, 2×240 and 2×250.

For A.C. working the standard frequency which has been adopted in this, and in many other, countries is 50 cycles per second. In America a frequency of 60 is adopted for lighting and some of the power load, but 25 cycles is also in use for power. For single-phase traction much lower frequencies are necessary, e.g. 15 or $16\frac{2}{3}$ cycles.

The distribution system, i.e. not including the transmission lines, can be subdivided into feeders, distributors, and service mains. The feeders are the conductors which connect the substations, or in some cases the generating stations, to the areas served by these stations. The distributors are characterised by the numerous tappings which are taken from them for the supply to consumers, and the service mains are the connecting links between distributors and consumers' terminals. The essential difference between feeders and distributors is thus that, whereas the current loading of a feeder is the same along the whole of its length, a distributor has a distributed loading, with consequent variations of current along its length. The functions of these three types of main are illustrated by Fig. 1.2, in which the distributor is shown as a ring main. In some cases smaller feeders are tee'd off from a main feeder, but this does not make any exception to the general rule that tappings are not taken from a feeder to a consumer's premises.

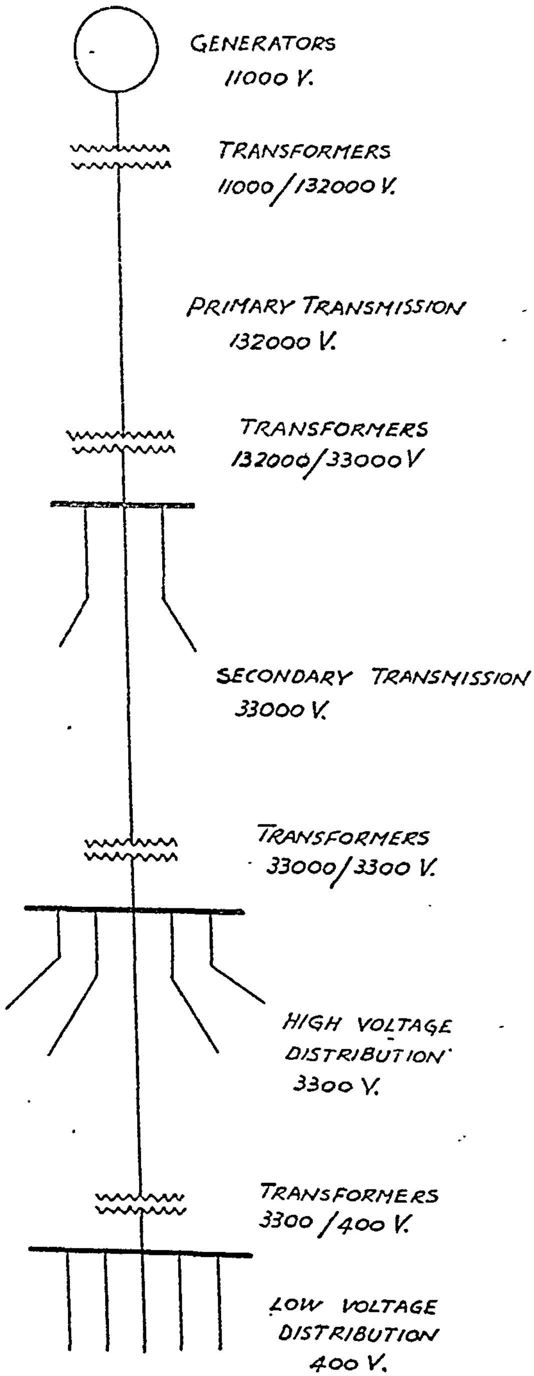


FIG. 1.1.—ELEMENTS OF A TRANSMISSION SYSTEM.

Influence of Working Voltage on Size of Feeders and Distributors

Because of the inevitable drop in voltage along all conductors, it is obvious that it is impossible to keep the potential difference absolutely constant at the terminals of any consumer. On the other hand, large fluctuations in voltage may be very undesirable from the point of view of the consumer, as, for example, in the case of a lighting load in which the luminous output of a lamp is proportional to the voltage raised to some power, which may vary from 3 to 7, depending on the type of lamp in use. The only means of reducing this drop in voltage is to reduce the conductor resistance and therefore to increase the cross-section and cost of the conductor. As in many engineering problems, there are thus two conflicting requirements, these being in the present case low-voltage fluctuation from the consumer's point of view and low cost of conductors from the point of view of the supply company. The inevitable compromise is fixed by a Board of Trade regulation, which states that the voltage at the consumer's terminals

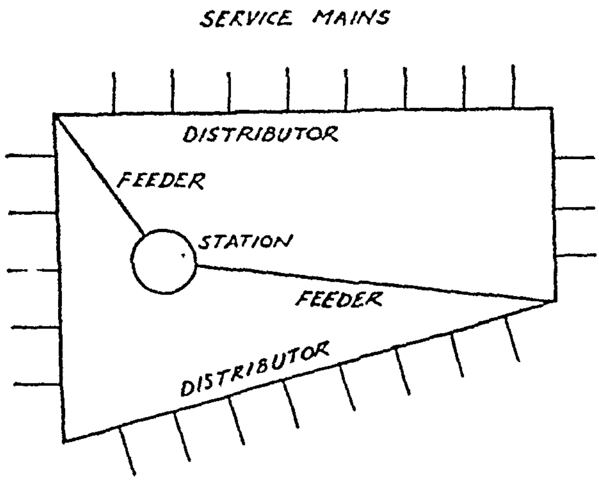


FIG. 1.2.—ELEMENTS OF A DISTRIBUTION SYSTEM.

must not vary by more than plus or minus 6 per cent. of the "declared pressure." In the case of an A.C. supply, there is an additional regulation fixing the variation in frequency at not more than 2½ per cent. above or below the "declared frequency."

By means of over-compounding of the generators, or by other methods of controlling the line voltage, it is possible to keep the voltage variations at the feeding points within very narrow limits, and in consequence most of the permissible drop in volts can take place in the distributors and service mains. Owing to the possibility of automatically compensating for the drops in them feeders can have any desired drop in volts, provided that this is within the range of voltage adjustment made available by compounding or other means, and as a result the fundamental basis of design of a feeder is current-carrying capacity. On the other hand, the drop in volts along any distributor is limited by the above Board of Trade regulation, so that the main basis of design is now the drop in volts. Obviously, the distributor must also be capable of carrying the required current without excessive temperature rise, but it will be found that in practically all cases a distributor will be of ample cross-section if its drop of volts is within the required limits.

Owing to the above differences in feeders and distributors, changes in system voltage will affect the two types differently. Consider first of all the case of a feeder. We have just seen that, owing to the fact that there are no consumers tapped off from it, a feeder is not influenced by the Board of Trade regulation regarding drop in volts, but that it can be designed primarily from the point of view of current-carrying capacity. The allowable current density for a given type of cable laid in a given manner is not a constant, but decreases somewhat as the cable size increases, and this applies also to overhead conductors. The reason for this is that the cooling facilities improve as the cross-section is reduced. As a rough approximation we can, however, assume that the current density is constant. Now, suppose that the voltage of the system is increased n fold, then for a given power delivered the current is reduced to $\frac{1}{n}$th of the previous value, and therefore the cross-section of the feeder is reduced to $\frac{1}{n}$th.

Now take the case of a distributor. Assuming that the voltage at the feeding points can be kept sensibly constant at, say, 4 per cent. above the declared pressure, and allowing, say, 1 per cent. drop along the service mains, the greatest possible drop that can take place in a distributor is 9 per cent. The percentage drop can thus be regarded as a constant. Now consider a section AB of distributor, Fig. 1.3 ; let the current in this section be I and let the drop along it be V. Then its resistance is given by $R = \frac{V}{I}$. Suppose that the voltage is increased n fold, so that, for the same power delivered, the current is reduced to I/n. But the percentage drop is a constant, and therefore the actual drop along

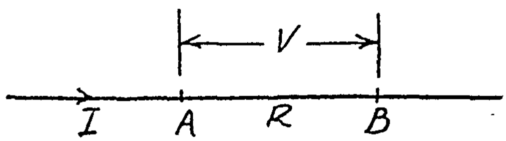


FIG. 1.3.

the section AB can be increased to nV. The resistance of the same length of distributor will thus be:

$$R_1 = \frac{nV}{I/n} = n^2 \frac{V}{I} = n^2R.$$

Now, for a given length, the cross-section of a cable is inversely proportional to its resistance, showing that an increase in the working voltage of n times reduces the cross-section, and therefore the weight of copper, to $\frac{1}{n^2}$ times. The saving in capital cost of copper in distributors is thus much greater than the saving in feeders when the system voltage is increased.

Effect of System Voltage on Transmission Efficiency

Let the power delivered by the line be P watts at a line voltage of V. Let the power factor be cos φ and the system three-phase. Then line current

$$I = \frac{P}{\sqrt{3}\, V \cos\varphi}$$

Let l = length of line in inches.
α = current density in amps. per sq. in.
ρ = specific resistance of conductor.

$$\therefore\ a = \frac{I}{\alpha} = \frac{P}{\sqrt{3}\,\alpha \cos\varphi\, V}$$

Resistance per conductor:

$$R = \frac{\rho l}{a} = \frac{\sqrt{3}\,\alpha\, \rho l \cos\varphi\, V}{P}$$

$$\therefore \text{ Line loss} = 3\, I^2R$$

$$= 3\, \frac{P^2}{3\, V^2 \cos^2\varphi} \times \frac{\sqrt{3}\,\alpha\, \rho l \cos\varphi\, V}{P}$$

$$= \frac{\sqrt{3}\,\alpha\, \rho l\, P}{\cos\varphi\, V} \quad \ldots\ldots\ldots \quad (1)$$

$$\therefore \text{ Line intake} = P + \text{Line loss}$$

$$= P + \frac{\sqrt{3}\,\alpha\, \rho l\, P}{\cos\varphi\, V}$$

$$= P\left(1 + \frac{\sqrt{3}\,\alpha\, \rho l}{\cos\varphi\, V}\right)$$

$$\therefore \text{ Effic. of transmission } = \frac{P}{P\left(1 + \frac{\sqrt{3}\,\alpha\,\rho l}{\cos\varphi\,V}\right)}$$

$$\simeq \left(1 - \frac{\sqrt{3}\,\alpha\,\rho l}{\cos\varphi\,V}\right) \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

$$\text{Resistance drop per line } = RI$$

$$= \frac{\sqrt{3}\,\alpha\,\rho l \cos\varphi\,V}{P} \times \frac{P}{\sqrt{3}\,V\cos\varphi}$$

$$= \alpha\,\rho l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

$$\text{Volume of copper } = 3\,al$$

$$= \frac{3Pl}{\sqrt{3}\,\alpha\cos\varphi\,V}$$

$$= \frac{\sqrt{3}Pl}{\alpha\cos\varphi\,V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

Regarding P, l, α, and ρ as constants, we see, from the above equations, that the supply voltage affects the transmission system as follows:

(1) The line loss is inversely proportional to the supply voltage: it is also inversely proportional to the power factor.

(2) The efficiency of transmission increases as the voltage increases, and also as the power factor increases.

(3) For a given current density the resistance drop per line is a constant, the percentage resistance drop therefore decreasing as the voltage is increased.

(4) The volume of copper required in the transmission line is inversely proportional to the voltage, and also inversely proportional to the power factor.

From the above argument it is clear that for long-distance transmission very high voltages are essential, and with A.C. systems a high power factor is also desirable. Increase in voltage introduces difficulties associated with the insulation of the conductors, whether bare or covered, and also in the case of bare conductors difficulties associated with their mechanical support, and the necessity for very large clearances. For these reasons the voltage is largely fixed by the length of the line. Since there is no hard-and-fast rule, the old rule of 1,000 volts per route mile is still a useful, though very rough, guide.*

* This is considered in detail on p. 101.

Comparison of Conductor Costs—Overhead Systems

We will assume that the power transmitted, length of line, and *maximum* voltage to earth are the same in all cases.

I. *D.C. System.*—Assume that the middle point is at earth potential, so that the line voltage is 2V :

$$\text{Current } I_1 = \frac{P}{2V}$$

$$\text{Line loss } = 2\,I_1^2R_1 = \frac{P^2R_1}{2V^2}$$

where R_1 is the resistance per conductor.

II. *Single-phase System.*—Assume one side earthed as is usual :

$$\therefore V_{RMS} = \frac{V}{\sqrt{2}}$$

$$\therefore \text{Current } I_2 = \frac{P}{V_{RMS}\cos\varphi} = \frac{\sqrt{2}P}{V\cos\varphi}$$

$$\therefore \text{Line loss } = 2I_2^2R_2 = \frac{4P^2R_2}{V^2\cos^2\varphi}$$

To effect a comparison, we will postulate equal line losses, and therefore equal efficiency of transmission :

$$\therefore \frac{4P^2R_2}{V^2\cos^2\varphi} = \frac{P^2R_1}{2V^2}$$

$$\therefore \frac{R_2}{R_1} = \frac{\cos^2\varphi}{8}$$

Hence, since the resistance is inversely proportional to the cross-section for a given length and material, we have :

$$\frac{a_2}{a_1} = \frac{8}{\cos^2\varphi}$$

III. *Three-phase System.*—Neutral point at earth potential :

$$\therefore \text{R.M.S. Phase voltage} = \frac{V}{\sqrt{2}}$$

$$\therefore \text{Line voltage} = \sqrt{3} \times \text{Phase voltage}$$

$$= \frac{\sqrt{3}V}{\sqrt{2}}$$

$$\therefore \text{Current } I_3 = \frac{P}{\sqrt{3}\cdot\frac{\sqrt{3}V}{\sqrt{2}}\cdot\cos\varphi}$$

$$= \frac{\sqrt{2}P}{3V\cos\varphi}$$

$$\therefore \text{Line loss} = 3\, I_3^2 R_3$$

$$= 3 \times \frac{2P^2}{9V^2 \cos^2 \varphi} \times R_3$$

$$= \frac{2P^2 R_3}{3V^2 \cos^2 \varphi}$$

∴ For equal line losses :

$$\frac{2P^2R_3}{3V^2 \cos^2 \varphi} = \frac{P^2R_1}{2V^2}$$

$$\therefore \frac{R_3}{R_1} = \frac{3 \cos^2 \varphi}{4}$$

$$\therefore \frac{a_3}{a_1} = \frac{4}{3 \cos^2 \varphi}$$

$$\therefore a_1 : a_2 : a_3 = 1 : \frac{8}{\cos^2 \varphi} : \frac{4}{3 \cos^2 \varphi}$$

But in Cases I and II there are two conductors per line, while in Case III there are three. Hence, denoting the volume of copper in the line by v, we have :

$$v_1 : v_2 : v_3 = 2 : \frac{16}{\cos^2 \varphi} \cdot \frac{12}{3 \cos^2 \varphi}$$

$$= 1 : \frac{8}{\cos^2 \varphi} : \frac{2}{\cos^2 \varphi}$$

The single-phase system shows up very badly in this comparison, and this is mainly due to the assumption of one side earthed instead of the middle point earthed. For single-phase distribution, as against transmission, it is possible to earth the middle point in some cases, in which case the value of v_2 comes out the same as v_3, namely $\frac{2}{\cos^2 \varphi}$. The above comparison shows that for transmission at high voltages, for which overhead conductors are nearly always used, the D.C. system is much superior to either of the A.C. systems from the point of view of economy copper. For two-phase systems the value of v works out at $\frac{4}{\cos^2 \varphi}$ with a four-wire system and the middle point earthed, and $5{\cdot}82/\cos^2 \varphi$ for a three-wire system with the junction of the two phases earthed. It will form a useful exercise for the reader to work these cases out for himself.

Example.—Estimate the weight of copper required for a three-phase transmission system supplying at 380 kV. between lines a load of 100 MW. at a lagging power factor of 0·9. The length of the line is 150 miles, the efficiency of transmission is 92 per cent., and the neutral point is earthed.

The resistance of a conductor 1 mile long of 1 sq. in. cross-section is 0·046 ohm.; one cubic inch of copper weighs 0·315 lb.

Find also the weight of copper required for the D.C. transmission of the same power with equal losses, working at a voltage between lines double that of the peak voltage to earth of the three-phase supply.

$$\text{Line current } I = 100 \times 10^6/(\sqrt{3} \times 380 \times 10^3 \times 0{\cdot}9) = 169 \text{ amp.}$$

I^2R loss per three conductors

$$= 0{\cdot}08 \times (100 \times 10^6) = 8 \times 10^6 \text{ watts}; = \frac{8}{3} \times 10^6 \text{ watts per conductor}$$

$$\therefore \text{Resistance per conductor} = \frac{8}{3} \times 10^6 \div 169^2 = 93{\cdot}8 \text{ ohm.}$$

$$\therefore \text{Resistance per mile} = 93{\cdot}8/150 = 0{\cdot}625 \text{ ohm.}$$

$$\therefore \text{Conductor area} = 0{\cdot}045/0{\cdot}625 = 0{\cdot}072 \text{ sq. in.}$$

$$\text{Volume per foot run} = 0{\cdot}072 \times 12 = 0{\cdot}865 \text{ cu. in.}$$

$$\text{Weight per foot run} = 0{\cdot}865 \times 0{\cdot}315 = 0{\cdot}273 \text{ lb.}$$

$$\therefore \text{Weight per 150 miles} = 3 \times 150 \times 5280 \times 0{\cdot}273 = 6{\cdot}51 \times 10^5 \text{ lb.}$$

Put V = A.C. volts to neutral and P the power supplied in watts

Then $$I_{d.c.} = P/(2V_2V)$$

$$\therefore \text{Losses } (d.c.) = 2R_{d.c.}[P/(2V_2V)]^2$$

and $$\text{Losses } (a.c.) = 3R_{a.c.}[P/(3V\cos\phi)]^2$$

$$\therefore \frac{R_{d.c.}}{R_{a.c.}} = 4/(3\cos^2\phi)$$

$$\therefore \frac{\text{area } (d.c.)}{\text{area } (a.c.)} = 3\cos^2\phi/4$$

$$\therefore \frac{\text{weight } (d.c.)}{\text{weight } (a.c.)} = \frac{2 \times 3\cos^2\phi}{3 \times 4} = \frac{\cos^2\phi}{2} = \frac{0{\cdot}81}{2}$$

$$\therefore \text{weight } (d.c.) = \frac{6{\cdot}51 \times 10^5 \times 0{\cdot}81}{2} = 2{\cdot}64 \times 10^5 \text{ lb.}$$

Comparison of Conductor Costs—Cable Systems

In the case of insulated cables we take the maximum voltage between conductors as the criterion. Let this be V.

I. *D.C. System.*

$$\text{System voltage} = V$$
$$\therefore I_1 = \frac{P}{V}$$
$$\therefore \text{Total line loss} = 2I_1^2R_1$$
$$= \frac{2P^2R_1}{V^2}$$

II. *Single-phase System.*

The conditions are the same as in the overhead system, so that we have, as before:

$$\text{Total line loss} = \frac{4P^2R_2}{V^2 \cos^2 \varphi}$$
$$\therefore \text{For equal losses } \frac{4P^2R_2}{V^2 \cos^2 \varphi} = \frac{2P^2R_1}{V^2}$$
$$\therefore \frac{R_2}{R_1} = \frac{\cos^2 \varphi}{2}$$
$$\therefore \frac{a_2}{a_1} = \frac{2}{\cos^2 \varphi}$$

III. *Three-phase System,* with neutral point at earth potential, The voltage V is now the maximum voltage between conductors. so that the R.M.S. value of the line voltage is now $V/\sqrt{2}$.

$$\therefore I_3 = \frac{P}{\sqrt{3} \cdot \frac{V}{\sqrt{2}} \cdot \cos \varphi}$$
$$= \frac{\sqrt{2}P}{\sqrt{3}V \cos \varphi}$$
$$\text{Line loss} = 3\, I_3^2R_3$$
$$= 3 \times \frac{2\,P^2}{3\,V^2 \cos^2 \varphi} \times R_3$$
$$= \frac{2\,P^2R_3}{V^2 \cos^2 \varphi}$$
$$\therefore \text{For equal loss } \frac{2\,P^2R_3}{V^2 \cos^2 \varphi} = \frac{2\,P^2R_1}{V^2},$$
$$\therefore \frac{R_3}{R_1} = \frac{\cos^2 \varphi}{1}$$

$$\therefore \frac{a_3}{a_1} = \frac{1}{\cos^2 \varphi}$$

$$\therefore a_1 : a_2 : a_3 = 1 : \frac{2}{\cos^2 \varphi} : \frac{1}{\cos^2 \varphi}$$

$$\therefore v_1 : v_2 : v_3 = 2 : \frac{4}{\cos^2 \varphi} : \frac{3}{\cos^2 \varphi}$$

$$= 1 : \frac{2}{\cos^2 \varphi} : \frac{1 \cdot 5}{\cos^2 \varphi}$$

It will be seen that, with insulated conductors, the D.C. system is not so greatly advantageous as with bare conductors, and since transmission is generally carried out by bare conductors, and low-voltage distribution by cables, it follows that, although D.C. has a very decided advantage for transmission, this advantage is not so marked with distribution. In fact, when other factors are taken into account, the most important probably being the ease with which the voltage in an A.C. system can be changed by means of static transformers, the A.C. system is, on the whole, the most suited to distribution.

In each of the A.C. cases the expression for the volume of copper has $\cos^2 \varphi$ for a denominator, and this shows that reasonable economy in copper can only be achieved if the power factor is as nearly equal to unity as possible. For example, if the power factor is 0·8, then for the equal line loss the volume of copper will be $\frac{1}{(0 \cdot 8)^2} \simeq 1 \cdot 6$ times the volume required at unity power factor.

Distribution Systems

For direct-current distribution the only system of importance is the three-wire system which consists of two "outers" and an earthed middle wire or neutral. This system is used in place of a simple two-wire system because, with consumers connected between the neutral and one of the outers, the system voltage is twice the consumer's voltage, thereby giving the advantages of high-voltage working.

In order to compare the three-wire with the two-wire system from the point of view of copper required assume the same amount of power delivered, the same distance, and the voltage between the outers in the three-wire case twice the voltage in the two-wire case. Also assume, first of all, that the three-wire system is balanced, i.e. the currents in the two outers are equal and therefore the current in the neutral wire is zero.

Let P = power delivered in watts.
V_1 = voltage at feeding end in two-wire system.
V_2 = voltage at load end.

$$\therefore \quad I = \frac{P}{V_2}$$

Hence total drop in volts, reckoning both conductors:

$$2IR_1 = \frac{2PR_1}{V_2}$$

And

$$V_1 = V_2 + \text{drop} = V_2 + \frac{2PR_1}{V_2}$$

Where R_1 is the resistance of one conductor.

$$\therefore \quad \text{Efficiency} = \frac{V_2}{V_2 + \dfrac{2PR_1}{V_2}}$$

In the three-wire system the total voltage is doubled, and therefore with a balanced system, current in each outer is P/2V. Hence if R_2 is the resistance of each outer we have for the efficiency of the three-wire system:

$$\text{Efficiency} = \frac{2V_2}{2V_2 + \dfrac{2PR_2}{2V_2}}$$

Hence, for equal efficiencies we have—

$$\frac{V_2}{V_2 + \dfrac{2PR_1}{V_2}} = \frac{2V_2}{2V_2 + \dfrac{2PR_2}{2V_2}}$$

$$= \frac{V_2}{V_2 + \dfrac{PR_2}{2V_2}}$$

Hence $R_2 = 4R_1$.

Now the cross-section, and therefore the volume of conductor for a given length, is inversely proportional to the resistance. If we represent the volume of copper in the two-wire system by 100, i.e. 50 in each conductor, we have—

$$\text{Volume of each outer in three-wire system} = \frac{50}{4} = 12{\cdot}5.$$

Now, with the D.C. three-wire system, it is usual to make the cross-section of the neutral one-half that of an outer, so that the volume of the neutral is represented by 12·5/2 = 6·25.

∴ Total volume of copper in three-wire system—

$$= 12{\cdot}5 + 12{\cdot}5 + 6{\cdot}25 = 31{\cdot}25$$

$$\therefore \frac{\text{Copper in three-wire system}}{\text{Copper in two-wire system}} = \frac{31{\cdot}25}{100} = \frac{5}{16}$$

Now take the case of an unbalanced three-wire system. Let the voltage between the outers be 2V, the current in one outer I, and that in the other outer mI, where $m < 1$. Then current in the neutral is $(1 - m)$I. If R_2 is the resistance of each outer, as before, then $2R_2$ is the resistance of the neutral, and we have—

$$P = VI + mVI = VI(1 + m)$$
$$\text{Line loss} = I^2R_2 + m^2I^2R_2 + (1 - m)^2I^2 \times 2R_2$$
$$= I^2R_2(3 + 3m^2 - 4m)$$

The current in a two-wire system to convey the same power at voltage V is—

$$\frac{P}{V} = \frac{VI(1 + m)}{V} = I(1 + m)$$

$$\therefore \text{Line loss} = 2(1 + m)^2I^2R_1$$
$$= 2I^2R_1(1 + 2m + m^2)$$

Hence for equal loss—

$$2I^2R_1(1 + 2m + m^2) = I^2R_2(3 - 4m + 3m^2)$$
$$\therefore \frac{R_1}{R_2} = \frac{3 - 4m + 3m^2}{2 + 4m + 2m^2}$$
$$\therefore \frac{a_2}{a_1} = \frac{3 - 4m + 3m^2}{2 + 4m + 2m^2}$$

Hence denoting a_1 by 50 as before—

$$a_2 = 50 \times \frac{3 - 4m + 3m^2}{2 + 4m + 2m^2}$$

For the total cross-section of two outers and the neutral we thus have—

$$2{\cdot}5 \times 50 \times \frac{3 - 4m + 3m^2}{2 + 4m + 2m^2}$$

Hence for the volume of copper in the three-wire system expressed as a percentage of the volume in the two-wire system we have—

$$\frac{2{\cdot}5 \times 50}{100} \times \frac{3 - 4m + 3m^2}{2 + 4m + 2m^2}$$
$$= 1{\cdot}25 \times \frac{3 - 4m + 3m^2}{2 + 4m + 2m^2}$$

Example.—A three-wire feeder whose outers have a cross-section of 0·2 sq. in. each is 1 mile long. It carries 220 amps. in the positive and 200 amps. in the negative outer, and the voltages across the two sides at the feeding point are 200. What size of two-wire feeder will transmit 420 amps. at 200 volts for the same distance with the same total loss? Also compare the amounts of copper in the two feeders.

$$m = \frac{200}{220} = \frac{10}{11}$$

∴ Vol. of copper in three-wire feeder as a percentage of the volume in the two-wire feeder :

$$= 1{\cdot}25 \times \frac{3 - \frac{40}{11} + \frac{300}{121}}{2 + \frac{40}{11} + \frac{200}{121}}$$

$$= \frac{1{\cdot}25 \times 1{\cdot}84}{7{\cdot}29} = 31{\cdot}6 \text{ per cent.}$$

The problem can also be solved without the use of the above equation in the following manner. Taking the specific resistance of copper as $\frac{2}{3}$ microhm. inch, we have for the resistances of the cores of the three-wire feeder :

$$+ \text{ side, } R_2 = \rho\frac{l}{a} = \frac{2}{3 \times 10^6} \times \frac{5{,}280 \times 12}{\cdot 2} = \cdot 21 \text{ ohm.}$$

$$- \text{ side, } R_2 = \cdot 21 \text{ ohm.}$$

$$\text{neutral, } 2R_2 = \cdot 42 \text{ ohm.}$$

Hence total copper loss :

$$= 220^2 \times \cdot 21 + 200^2 \times \cdot 21 + 20^2 \times \cdot 42$$

$$= 18{,}770 \text{ watts.}$$

Let R_1 = resistance of each side of two-wire feeder :

$$\therefore\ 2 \times 420^2 \times R_1 = 18{,}770$$

$$R_1 = \cdot 053 \text{ ohm}$$

Now, the amount of copper is inversely proportional to the resistance for a given length; hence if we call the total copper in the two-wire feeder 100, i.e. 50 per cent. in each side as before, we have for the three-wire feeder :

$$\text{Copper in } + \text{ outer} = \frac{\cdot 053}{\cdot 21} \times 50 = 12{\cdot}63 \text{ per cent.}$$

$$\text{,, ,, } - \text{ outer} = 12{\cdot}63 \text{ ,,}$$

$$\text{Copper in neutral} = \frac{12{\cdot}63}{2} = 6{\cdot}31 \text{ ,,}$$

$$\text{Total} = 31{\cdot}6 \text{ per cent.}$$

Now consider A.C. distribution. For large consumers the supply can be a three-phase one, in fact a polyphase supply will be essential for an important motor load. But with small consumers a single-phase supply is desirable, particularly with private houses and small buildings where the load may be a combined lighting and domestic load. Again, for small powers, up to, say, 5 h.p., it is now possible to obtain single-phase motors whose performance is comparable with that of polyphase motors of the same power and speed, and therefore for small power loads the simplicity of a single-phase service will thus be an advantage. The problem is thus the derivation of single-phase services from a three-phase supply. There are several methods, their diversity being largely due to the fact that in many cases change-overs from D.C. to A.C. supply had to take place, and the old network has been retained, whereas in other cases the complete system has been planned *de novo*. The following are the most common.

Three-phase, Four-wire

(*a*) *With Star-connected Secondary.*—The scheme is illustrated in Fig. 1.4, from which it will be seen that the fourth wire on the

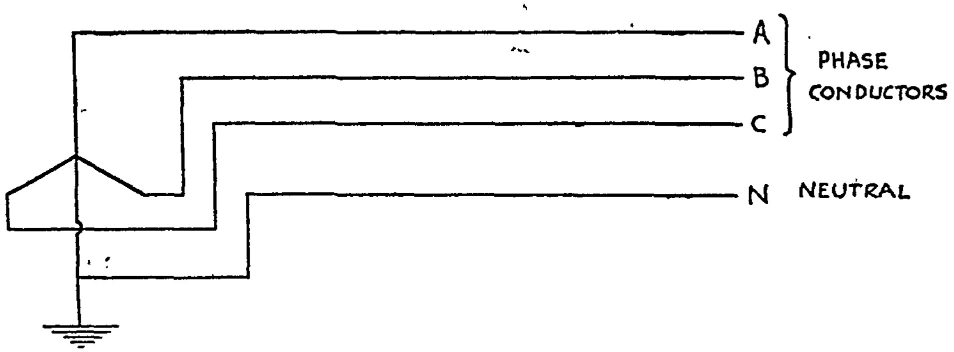


FIG. 1.4.—THREE-PHASE, FOUR-WIRE SYSTEM.

secondary side is derived from the neutral point of the star-connected secondary. The single-phase loading is split up into three sections as nearly equal as possible, while any three-phase load, if existent, is connected to the three-phase conductors. If the single-phase load has been reasonably equally divided between the three phases, then for all practical purposes the system can be regarded as perfectly balanced. It is not necessary to install balancing transformers and, no matter how the system extends, a continuous four-wire distributor can be laid down in the area to be served so as to be available for future new consumers. The disadvantage of the method is that there is no simple ratio between the voltages of the three-phase and the single-phase services, this ratio being 1·732 to 1, as contrasted with the ratio of 2 to 1 in the available voltages of a D.C. or single-phase three-wire system. Actually, standardisation of voltages has largely nullified this disadvantage, the standards in this country being either 400 or 380 volts for the three-phase supply, and either $\frac{400}{\sqrt{3}}$ (= 230) or $\frac{380}{\sqrt{3}}$ (= 220) respectively for the single-phase supply. Since, in general, there will not be a perfect balance in the three single-phase loads there will be a current along the neutral wire.

Example 1.—A three-phase four-wire supply with a voltage of 400/230 is loaded as follows: three single-phase lighting loads of 10, 15, and 20 kW. respectively, and a three-phase power load of 25 kW. at a power factor of 0·8 lagging. Calculate the currents in all four wires.

Let I_1, I_2, and I_3 be the current in the three lighting loads, and I_m the current in the motor load. Then since lighting loads can be assumed to be at unity power factor we have—

$$I_1 = \frac{10,000}{230} = 43{\cdot}5 \text{ amps.}$$

$$I_2 = \frac{15,000}{230} = 65{\cdot}3 \text{ amps.}$$

$$I_3 = \frac{20,000}{230} = 87 \text{ amps.}$$

$$I_m = \frac{25,000}{\sqrt{3} \times 400 \times {\cdot}8} = 45{\cdot}1 \text{ amps.}$$

Also the current I_m in any phase lags behind the corresponding phase voltage by φ = arc cos 0·8 degrees. The vector diagram giving the currents I_A, I_B, and I_C, in the phase conductors, is therefore as shown in Fig. 1.5, and from it we have—

$$I_A^2 = 43{\cdot}5^2 + 45{\cdot}1^2 + 2 \times 43{\cdot}5 \times 45{\cdot}1 \times {\cdot}8$$
$$= 1890 + 2030 + 3140 = 7060$$
$$\therefore I_A = 84 \text{ amps.}$$
$$I_B^2 = 65{\cdot}3^2 + 45{\cdot}1^2 + 2 \times 65{\cdot}3 \times 45{\cdot}1 \times {\cdot}8$$
$$= 4260 + 2030 + 4720 = 11010$$
$$\therefore I_B = 105 \text{ amps.}$$
$$I_C^2 = 87^2 + 45{\cdot}1^2 + 2 \times 87 \times 45{\cdot}1 \times {\cdot}8$$
$$= 7570 + 2030 + 6270 = 15870$$
$$\therefore I_C = 126 \text{ amps.}$$

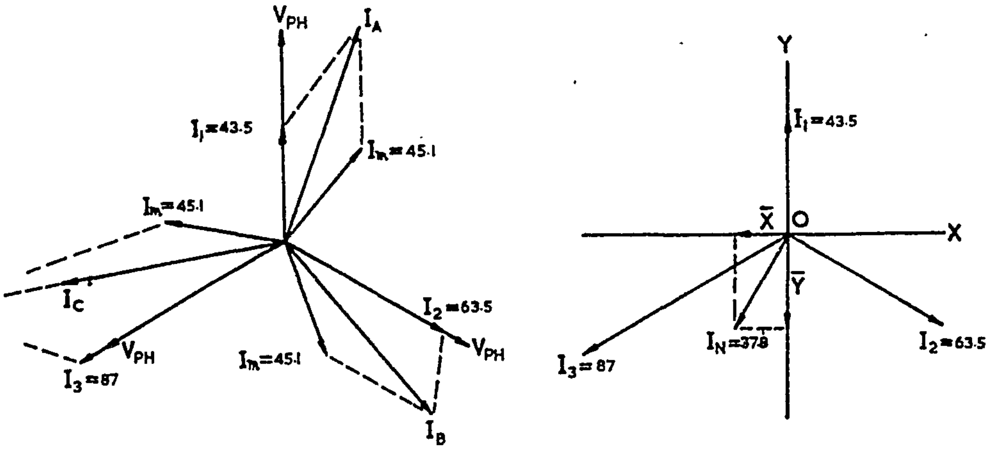


FIG. 1.5.

The current I_N in the neutral wire is the vector sum of the three single-phase currents. Resolving along OX and OY as indicated in the figure we have—

$$\begin{aligned}\overline{X} &= 0 + 65{\cdot}3 \cos 30^\circ - 87 \cos 30^\circ \\ &= -21{\cdot}7 \times {\cdot}866 \\ &= -18{\cdot}8 \text{ amps.} \\ \overline{Y} &= 43{\cdot}5 - 65{\cdot}3 \sin 30^\circ - 87 \sin 30^\circ \\ &= 43{\cdot}5 - 152{\cdot}3 \times 0{\cdot}5 \\ &= -32{\cdot}7 \text{ amps.} \\ \therefore I_N &= \sqrt{18{\cdot}8^2 + 32{\cdot}7^2} \\ &= 37{\cdot}8 \text{ amps.}\end{aligned}$$

Example 2.—Three-phase, four-wire supply with phase voltages at supply end 230 V. All four wires of resistance 2Ω and resistance 1Ω. Loaded as follows: 20A at 1·0 p.f.; 50A at 0·8 p.f. lagging; 80A at 0·7 p.f. lagging; these are loads from line to neutral. In addition, a balanced three-phase load of 30A at 0·9 p.f. lagging. Calculate the line to neutral voltages at the load end, and the three line power factors.

The circuit diagram is given in Fig. 1.6. Using $E_{ph.1}$ as the reference vector, we have—

$$\begin{aligned}&\cos^{-1} 0{\cdot}7 = 45^\circ,\ \sin 45^\circ = 0{\cdot}7,\quad \sin (45 + 60)^\circ = 0{\cdot}99 \\ &\cos^{-1} 0{\cdot}8 = 36^\circ,\ \sin 36^\circ = 0{\cdot}6,\quad \sin (60 - 36)^\circ = 0{\cdot}41 \\ &\cos^{-1} 0{\cdot}9 = 25^\circ,\ \sin 25^\circ = 0{\cdot}43,\ \sin (25 + 60)^\circ = 0{\cdot}89 \\ &\cos 85^\circ = 0{\cdot}09,\ \sin (60 - 25)^\circ = 0{\cdot}58 \\ &\cos 35^\circ = 0{\cdot}82,\ \sin 75^\circ = 0{\cdot}96 \\ &\cos 24^\circ = 0{\cdot}91,\ \cos 75^\circ = 0{\cdot}26\end{aligned}$$

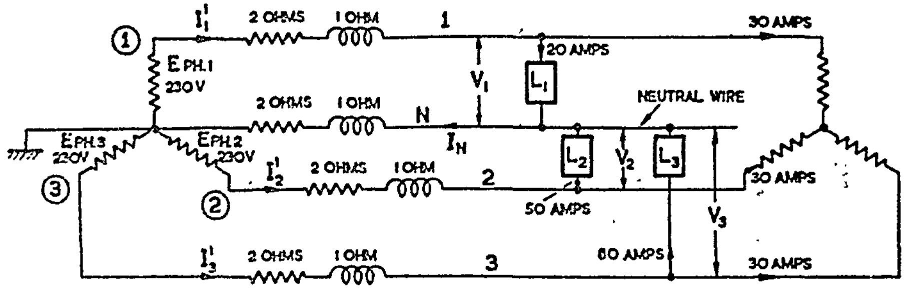


FIG. 1.6.

Current of 20A $= 20 + j\,0$
Current of 50A $= -\,50 \cos 24^\circ - j\,50 \sin 24^\circ$
$= -\,50 \times \cdot 91 - j\,50 \times \cdot 41$
$= -\,45{\cdot}5 - j\,20{\cdot}3$
Current of 80A $= -\,80 \cos 105^\circ + j\,80 \sin 105^\circ$
$= +\,80 \times \cdot 26 + j\,80 \times \cdot 96$
$= 20{\cdot}7 = j\,77{\cdot}0$
Current of 30A $= 30 \times \cdot 9 - j\,30 \times \cdot 43 = 27 - j\,13{\cdot}2$
$= -\,30 \cos 35^\circ - j\,30 \sin 35^\circ = -\,24{\cdot}6 - j\,17{\cdot}1$
$= -\,30 \cos 85^\circ + j\,30 \sin 85^\circ = -\,2{\cdot}7 + j\,29{\cdot}8$
Current $I_1' = (20 + j\,{\cdot}0) + (27 - j\,13{\cdot}2)$
$= 47 - j\,13{\cdot}2$
$= 48{\cdot}8$ A
Current $I_2' = (-\,45{\cdot}5 - j\,20{\cdot}3) + (-\,{\cdot}24{\cdot}6 - j\,17{\cdot}1)$
$= -\,70{\cdot}1 - j\,37{\cdot}4$
$= 80{\cdot}6$ A.
Current $I_3' = (20{\cdot}7 + j\,77{\cdot}0) + (-\,2{\cdot}7 + j\,29{\cdot}9)$
$= 18 + j\,106{\cdot}9$
$= 108$ A.
$I_N = I_1' + I_2' + I_3'$
$= (47 - j\,13{\cdot}2) + (-\,70{\cdot}1 - j\,37{\cdot}4)$
$+ (18 + j\,106{\cdot}9)$
$= -\,5{\cdot}1 + j\,56{\cdot}3$
$= 56{\cdot}5$ A.

Volt drops along the wires:

Drop along wire 1 $= (2 + j\,{\cdot}1)\,(47 - j\,13{\cdot}2)$
$= 107{\cdot}2 + j\,20{\cdot}6$
Drop along wire 2 $= (2 + j\,1)\,(-\,70 - j\,37{\cdot}4)$
$= -\,102{\cdot}8 - j\,144{\cdot}9$
Drop along wire 3 $= (2 + j\,1)\,(18 + j\,106{\cdot}9)$
$= -\,70{\cdot}9 + j\,231{\cdot}8$
Drop along N $= (2 + j\,1)\,(-\,5{\cdot}1 + j\,56{\cdot}3$
$= -\,66{\cdot}5 + j\,107{\cdot}5$

The phase voltages at the load end.

Along the three paths to earth from the supply there are, in each case, three volt drops, viz. volt drop along the wire, the volt drop across the load, the volt drop along the neutral.

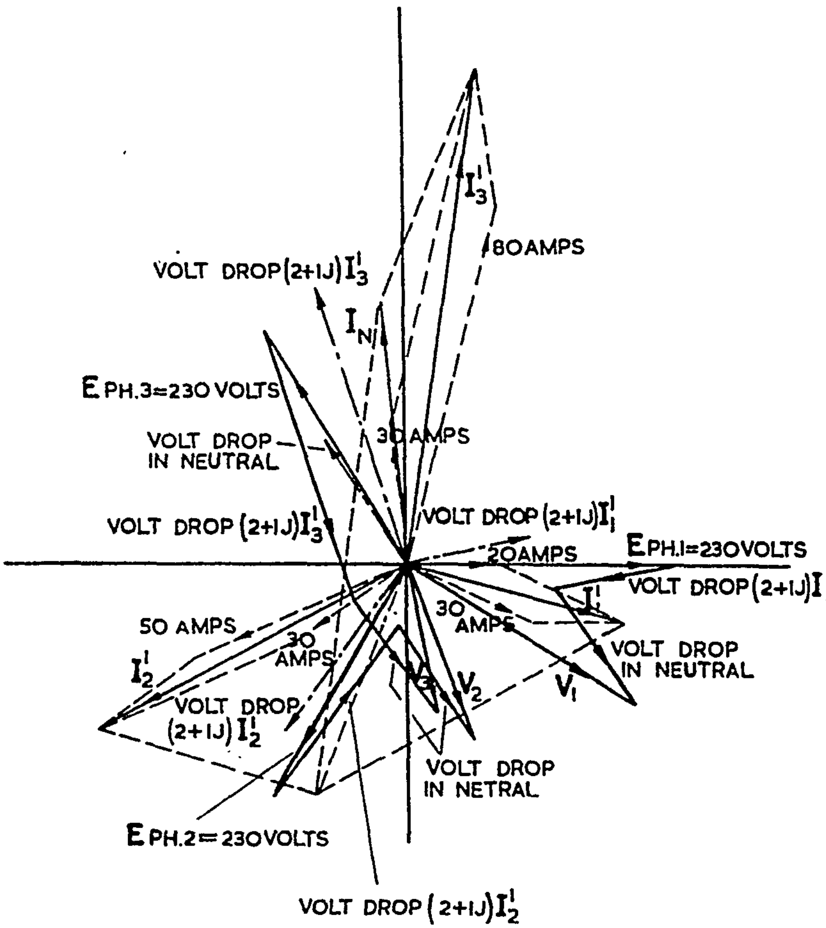


FIG. 1.7.

$$\therefore\ 230 + j\,0 = (107{\cdot}2 + j\,20{\cdot}6) + V_1 + (-\,66{\cdot}5 + j\,107{\cdot}5)$$
$$= 40{\cdot}6 + j\,128{\cdot}1 + V_1$$
$$V_1 = 189{\cdot}4 - j\,128{\cdot}1$$
$$= 229\text{ V.}$$
$$-\,230\cos 60^\circ - j\,230\sin 60^\circ = (-\,102{\cdot}8 - j\,144{\cdot}9) + V_2 + (-\,66{\cdot}5 + j\,107{\cdot}5)$$
$$-\,115 - j\,199 = -\,169{\cdot}3 - j\,37{\cdot}4 + V_2$$
$$V_2 = 54{\cdot}3 - j\,161{\cdot}6$$
$$= 170\text{ V.}$$

of 90 degrees, and therefore if these two are interconnected it is possible to use two-phase motors for the power load.

An advantage of the Scott method is that if an existing D.C.

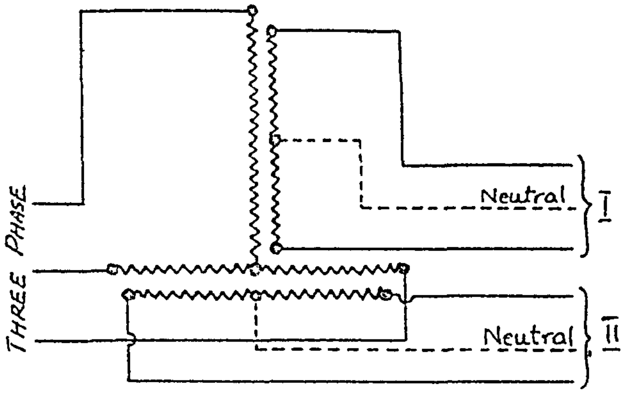


FIG. 1.11.—TWO-PHASE SIX-WIRE SUPPLY FROM SCOTT-CONNECTED TRANSFORMERS.

three-wire network is taken over, the network need only be divided into two halves, and three-wire services can be maintained by connecting the neutral wire to the mid-point of the appropriate secondary. Such a scheme is illustrated in Fig. 1.11. In this case the potentials of the two single-phase lines are fixed with respect to one another, and, provided that the lines are adjacent to one another, a two-phase service for power can be provided. Such a two-phase service is indicated in the figure.

Balancers for Three-phase Four-wire Systems

Even with the star-connected secondary it is not always possible to obtain a satisfactory balance with the three single-phase loads, and in such cases it is necessary to install balancing apparatus. Since any lack of balance manifests itself by a current in the neutral wire, the restoration of balance can be effected by diverting this current to the other wires. One method is to employ a three-phase 1/1 ratio transformer with interconnected star windings on the secondary side, the star point of this winding being connected to the neutral wire, as shown in Fig. 1.12. In addition, a choke coil is included in the neutral, to ensure that out of balance current flowing from the load shall be forced to pass through the secondary windings of the balancing transformer. The balancer does not divert the whole of the out-of-balance current into the three-phase conductors, the amount of current so diverted being proportional to the inverse ratio of the impedances of the direct path for the current, via the line, load, and neutral, to the second path, line, load, balancer, neutral point, and balancer. For satisfactory results it therefore follows that the impedance of the balancer must be kept as low as possible, while the impedance of

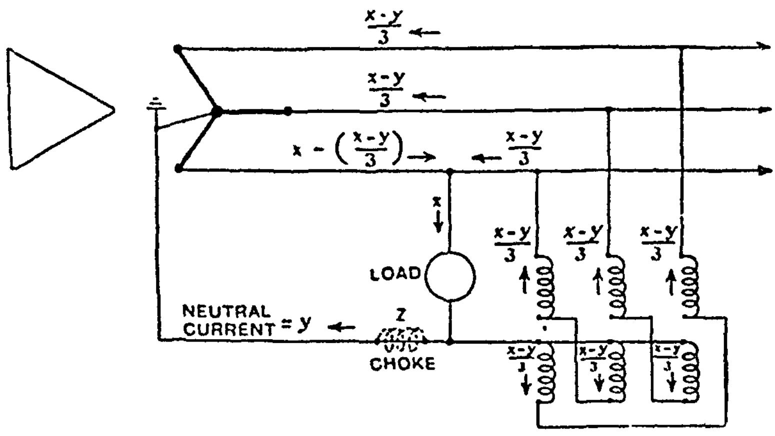


FIG. 1.12.—CONNECTIONS OF BALANCING TRANSFORMER.

the fourth wire to the flow of out-of-balance current must be as high as possible. In some cases the required high impedance of the fourth wire has been achieved by making a break in this wire, but this has the disadvantage that even though the fourth wire

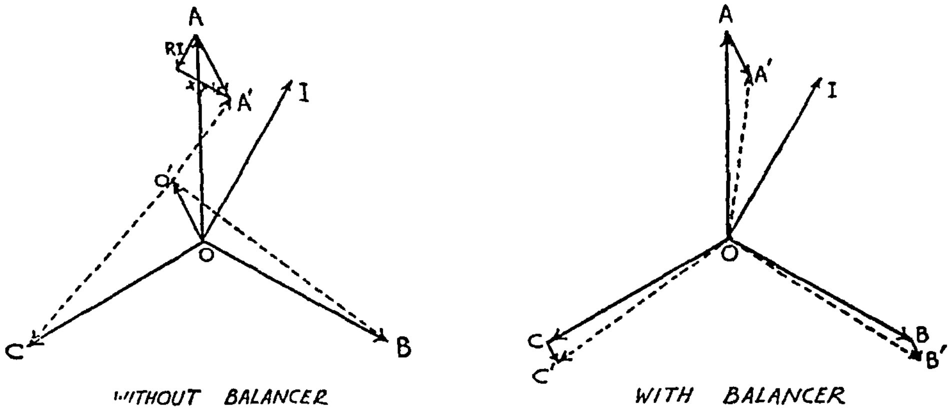


FIG. 1.13.—TO ILLUSTRATE THE EFFECT OF THE BALANCING TRANSFORMER ON THE VOLTAGE BALANCE.

may be earthed at the supply point, beyond the break there is the possibility of the line wires assuming too high a voltage with respect to earth. The best method of obtaining the high impedance is thus to include the choke coil, as shown in the figure.

In the figure, the worst possible case is assumed, i.e. a load on

$$-230\cos 60^\circ + j\,230\sin 60^\circ = (-70{\cdot}9 + j\,231{\cdot}8) + V_3 + (-66{\cdot}5 + j\,107{\cdot}5)$$
$$-115 + j\,199 = -137{\cdot}4 + j\,339{\cdot}3 + V_3$$
$$V_3 = 22{\cdot}4 - j\,140{\cdot}3$$
$$= 145{\cdot}5 \text{ V.}$$

The complete vector diagram is given in Fig. 1.7.

(*b*) *With Delta-connected Secondary.*—This is a less commonly used method which has been devised to give a ratio of 2 to 1

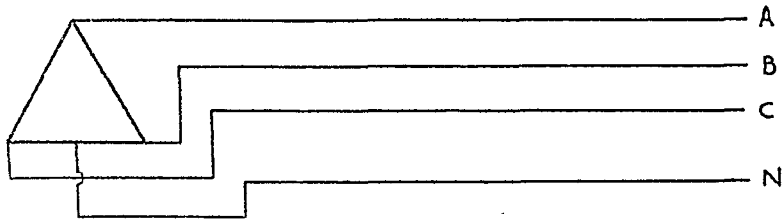


FIG. 1.8.—THREE-PHASE, FOUR-WIRE SYSTEM DERIVED FROM DELTA-CONNECTED TRANSFORMER.

between the three-phase and single-phase voltages. The circuit diagram is shown in Fig. 1.8, from which it will be seen that the whole of the single-phase load is taken by one phase only, the fourth wire being derived from a mid-point tapping on that phase. Owing to the position of the tapping the voltage of the single-phase service is one-half that of the line voltage of the three-phase

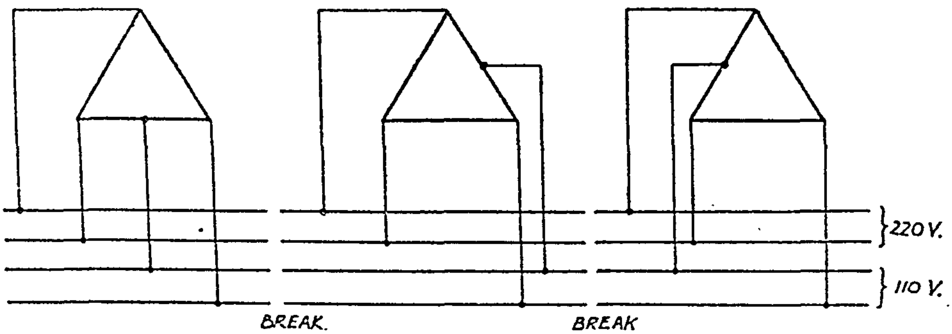


FIG. 1.9.—SUBDIVISION OF FOUR-WIRE DISTRIBUTOR FED FROM A DELTA-CONNECTED TRANSFORMER.

service. This is the only advantage of the method. The obvious disadvantage is that the load is completely unbalanced, and it is therefore necessary to divide the district served into a number of sections which are quite independent so far as the secondary sides of the transformers are concerned, each section being fed by its own transformer. Also to obtain reasonable balance in the primary circuit it is desirable to have three sections, the three mid-point tappings being taken from each of the three phases in turn. Fig. 1.9 will make this clear. It is obvious that with this system it is not possible to lay a continuous four-wire distributor through the area to be served.

In connection with the four-wire systems it is necessary to note that a three-wire lead-sheathed D.C. cable cannot be used for this purpose whenever a change-over from D.C. to A.C. takes place, although at first sight it appears that the lead sheath could be used in place of the fourth wire. Apart from the limited current-carrying capacity imposed by the smallest core, there is a regulation to the effect that A.C. circuits shall be connected to earth at one point only, viz. where energy is delivered to the circuit. As the lead-sheath is earthed throughout its length (at any rate, it is not efficiently insulated from earth by the outer covering of the cable), its use as a fourth wire would be a contravention of this regulation.

Two-phase System

The Scott method provides two single-phase supplies from a three-phase service, the necessary plant comprising two single-phase transformers arranged as follows. Both transformers have

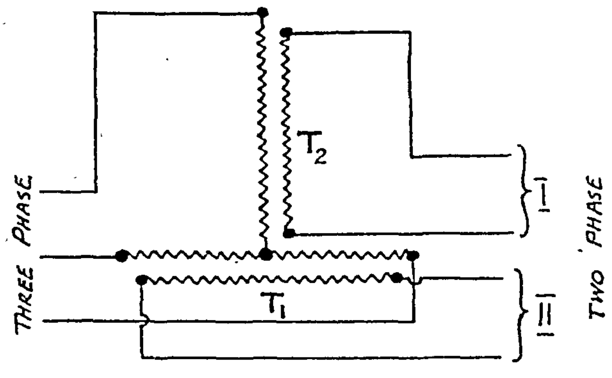


FIG. 1.10.—SCOTT-CONNECTED TRANSFORMERS.

identical secondaries wound for the voltage of the single-phase supplies—transformer T_1, Fig. 1.10, has a centre tapping to its primary, while transformer T_2 has 0·866 times as many primary turns as T_1. The theory of the method is given in textbooks on Electrical Technology and need not be considered here. The output consists of two single-phase supplies with a phase difference one phase only, but the vector diagrams of Fig. 1.13 show that the balancer gives a much improved performance from the point of view of voltage balance.

Network Theorems

The circuit parameters in the case of transmission and distribution circuits energised at a frequency of 50 cycles per second can be regarded as (*a*) linear, and (*b*) bilateral. A linear parameter is one in which the current produced by an applied P.D. is proportional to that P.D. There is no case of a parameter being rigorously linear, since a change in current will have some effect on the magnitude of the parameter. Thus increased current will, because of increased temperature, cause an increase in the resistance of normal conducting materials. An increased current will cause an increase in the electromagnetic forces acting between parallel conductors, thereby producing possible deformation, with change in inductance, and so on. Iron-cored reactors may experience very appreciable changes with change in current. For practical purposes the linearity of circuit parameters can be assumed, the resistance being that corresponding to a known, or assumed, temperature.

A bilateral parameter is one whose impedance is independent of the direction of current flow.

The Reciprocity Theorem

If an e.m.f. E acting in any branch p of a network consisting of linear and bilateral impedances results in a current I in another branch q, then the same e.m.f. E acting in the branch q will result in the flow of the same current I in the branch p.

The ratio of the voltage in branch p to resulting current in branch q is called the transfer impedance Z_{pq}. Thus the reciprocity theorem states that $Z_{pq} = Z_{qp}$.

Example.—The network of Fig. 1.14.

Let an E.M.F. $E = 100 + j\,0$ act in branch AB

$$Z_{AB} = (3 + j\,4) + \frac{5(4 - j\,3)}{5 + (4 - j\,3)} = 5{\cdot}5 + j\,3{\cdot}17$$

$$I_{AB} = \frac{100}{5{\cdot}5 + j\,3{\cdot}17} = 11{\cdot}47 - j\,8{\cdot}5$$

$$\therefore\ I_{EF} = (11{\cdot}47 - j\,8{\cdot}5) \times \frac{5}{9 - j\,3} = 7{\cdot}16 - j\,2{\cdot}32$$

Now let the E.M.F. $E = 100 + j\,0$ be transferred from branch AB to branch EF.

$$Z_{EF} = (4 - j\,3) + \frac{5\,(3 + j\,4)}{5 + (3 + j\,4)} = 6{\cdot}5 - j\,1{\cdot}74$$

$$I_{EF} = \frac{100}{6{\cdot}5 - j\,1{\cdot}74} = 14{\cdot}34 - j\,3{\cdot}9$$

$$\therefore\ I_{AB}' = (14{\cdot}34 - j\,3{\cdot}9) \times \frac{5}{8 + j\,4} = 7{\cdot}16 - j\,2{\cdot}32$$

Thus $$Z_{pq} = Z_{qp} = \frac{100}{7{\cdot}16 - j\,2{\cdot}32} = 12{\cdot}7 + j\,4{\cdot}12$$

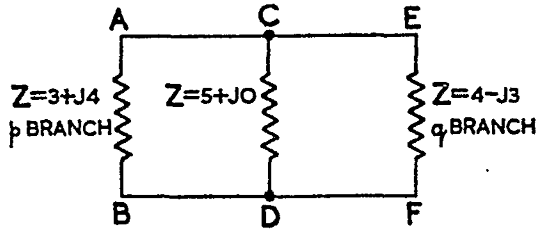


FIG. 1.14

The Superposition Theorem

If several E.M.F.s of the same frequency act in a network composed of linear impedances, then the current in any branch is the sum of the current which would flow if the various E.M.F.s

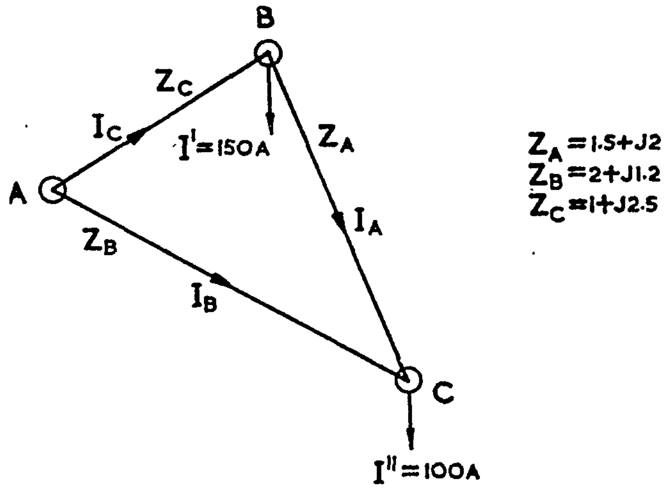


FIG. 1.15

acted independently and all the other sources acted as impedances.

Example.—A three-phase generating station at A with a line voltage of 6,600 volts supplies two substations at B and C through two independent feeders; the substations are also interconnected. The impedances of the feeders per phase are: A to B, $(1 + j\,2{\cdot}5)$ ohms, A to C, $(2 + j\,1{\cdot}2)$ ohms, and the interconnector $(1{\cdot}5 + j\,2)$ ohms. The load at B is 150 amps. at a power-factor of 0·7 lagging, and at C, 100 amps. at 0·9 lagging, both power-factors being referred to the voltage at A. Calculate the current flowing in each feeder. Find also the voltage difference between the two substations if the interconnector is removed, the loads being maintained at the above values.

First of all we will solve by the use of Kirchoff's Laws. Take the voltage to neutral at A as the reference vector; fig. 1.15.

When $\cos\phi = 0{\cdot}7$, $\sin\phi = 0{\cdot}714$; when $\cos\phi = 0{\cdot}9$, $\sin\phi = 0{\cdot}436$

$$\therefore I_C = I_A + 150(0{\cdot}7 - j\,0{\cdot}714)$$

$$I_B = 100\,(0{\cdot}9 - j\,0{\cdot}436) - I_A$$

$$\Sigma\; IZ = 0$$

$$\therefore 0 = I_A(1{\cdot}5 + j\,2) - (90 - j\,43{\cdot}6 - I_A)\,(2 + j\,1{\cdot}2) + (I_A + 105 - j\,107)\,(1 + j\,2{\cdot}5)$$

$$\therefore I_A\,(1{\cdot}5 + j\,2 + 2 + j\,1{\cdot}2 + 1 + j\,2{\cdot}3) - (90 - j\,43{\cdot}6)\,(2 + j\,1{\cdot}2) + (105 - j\,107)\,(1 + j\,2{\cdot}5) = 0$$

$$I_A = -\frac{141 + j\,132}{4{\cdot}5 + j\,5{\cdot}7}$$

$$= -\,26{\cdot}5 + j\,3{\cdot}8 = 26{\cdot}7\text{ A}$$

$$I_B = 90 - j\,43{\cdot}6 - (-\,26{\cdot}5 + j\,3{\cdot}8)$$

$$= 116 - j\,47{\cdot}4 = 125\text{ A}$$

$$I_C = -\,26{\cdot}5 + j\,3{\cdot}8 + 105 - j\,107$$

$$= 79 - j\,103 = 130\text{ A}$$

When the interconnector BC is removed—

$$V_B = V_A - 150\,(0{\cdot}7 - j\,0{\cdot}714)\,Z_C$$

$$= 6600/\sqrt{3} - (105 - j\,107)\,(1 + j\,2{\cdot}5)$$

$$= 3810 - (373 + j\,155)$$

$$= 3437 - j\,155 = 3440\text{ V}$$

$$\therefore V_B = \sqrt{3} \times 3440 = 5960\text{ V, line to line}$$

$$V_C = V_A - 100\,(0{\cdot}9 - j\,0{\cdot}436)\,Z_B$$

$$= 3810 - (90 - j\,43{\cdot}6)\,(2 + j\,1{\cdot}2)$$

$$= 3578 - j\,21 = 3578\text{ V}$$

$$\therefore V_C = \sqrt{3} \times 3578 = 6190\text{ V, line to line}$$

Thévenins Theorem (Helmholtz)

Any system of linear impedances with two accessible terminals may be replaced by an E.M.F. acting in series with an impedance. The e.m.f. is that acting between the terminals when they are unconnected externally, and the impedance is that presented by the system when all the E.M.F.s in the system are zero, the sources being represented by their internal impedances only.

This theorem, in a modified form, is also called the redundant impedance theorem. The method of applying it is to remove members until the network is such that each load is supplied by one path only. The p.d. across the point from which a member was removed is calculated. Then the impedance of the reduced network between these two points is calculated and to it is added the impedance of the member itself. The quotient of the calculated p.d. and impedance then give the current in that particular number.

In the previous example, imagine the interconnector BC to be disconnected. We have already calculated the corresponding values of V_B and V_C, namely—

$$V_B = 3437 - j\,155;\ V_C = 3578 - j\,21$$

The potential of C with respect to B is thus

$$(3437 - j\,155) - (3578 - j\,21) = -\,141 - j\,134$$

Total impedance

$$= (1{\cdot}5 + j{\cdot}2) + (2 + j\,1{\cdot}2) + (1 + j\,2{\cdot}5) = 4{\cdot}5 + j\,5{\cdot}7$$

Hence current in interconnector

$$= \frac{-\,141 - j\,134}{4{\cdot}5 + j\,5{\cdot}7}, \text{ as before}$$

Solution by Superposition Method

For the solution of the network under discussion a variant of the theorem, as normally stated, is again required. It is as follows: if a number of currents of the same frequency are tapped off from points in a network of linear conductors, then the current in any particular member is the vector sum of the various currents it would carry if the currents from the network were tapped off one at a time and all the others were zero.

Denote the load taken off at B as I' and that taken off at C as I'', then—

$$I' = 150(0{\cdot}7 - j\,0{\cdot}714);\ I'' = 100(0{\cdot}9 - j\,0{\cdot}436)$$

Let I' only be tapped off, I'' therefore being zero.

$$I_C = I' \times \frac{Z_A + Z_B}{Z_A + Z_B + Z_C}$$

$$= 150\,(0{\cdot}7 - j\,0{\cdot}714) \times \frac{3{\cdot}5 + j\,3{\cdot}2}{4{\cdot}5 + j\,5{\cdot}7}$$

$$= 56{\cdot}8 - j\,79{\cdot}9$$

transformation of admittances are similar to those in the delta-star transformation of impedances.

Millman's Theorem

This theorem, claimed by Millman, is very valuable for the solution of unbalanced star-connected loads, particularly the four-wire case. Fig. 1.18 shows a three-phase, four-wire system, the load admittances Y_1, Y_2, and Y_3 being, in each case, the total

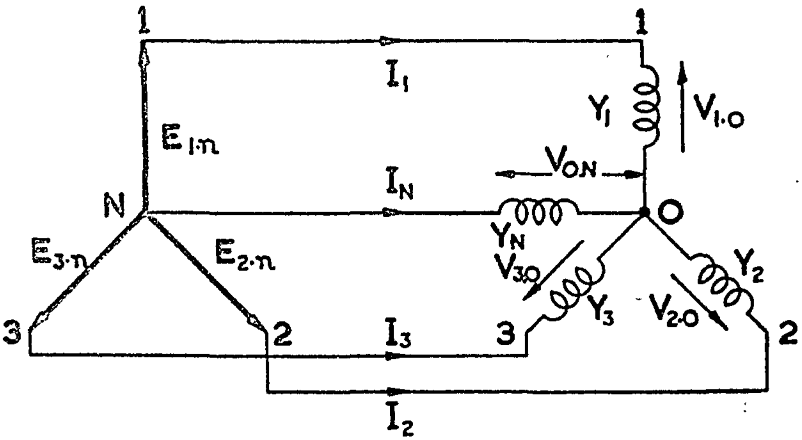


FIG. 1.18.—Pertaining to Millman's Theorem.

admittance for load, phase, and line. Y_N is the admittance of the neutral wire. The relationships between supply E.M.F.s and load voltages are—

$$E_{1.2} = E_{1.n} - E_{2.n} = V_{1.o} - V_{2.o}$$
$$E_{2.3} = E_{2.n} - E_{3.n} = V_{2.o} - V_{3.o}$$
$$E_{3.1} = E_{3.n} - E_{1.n} = V_{3.o} - V_{1.o}$$

For the voltage of the neutral at the load end, we have—

$$\begin{aligned} V_{0.n} &= E_{1.n} - V_{1.o} \\ &= E_{2.n} - V_{2.o} \\ &= E_{3.n} - V_{3.o} \end{aligned}$$

The current relationship is—

$$I_1 + I_2 + I_3 + I_n = 0$$
$$V_{1.o}Y_1 + V_{2.o}Y_2 + V_{3.o}Y_3 - V_{o.n}Y_n = 0$$
$$\therefore (E_{1.n} - V_{o.n})Y_1 + (E_{2.n} - V_{o.n})Y_2 + (E_{3.n} - V_{o.n})Y_3 - V_{o.n}Y_n = 0$$
$$\therefore V_{o.n} = (E_{1.n}Y_1 + E_{2.n}Y_2 + E_{3.n}Y_3)/(Y_1 + Y_2 + Y_3 + Y_n)$$

This can be generalised as—

$$V_{o.n} = \Sigma EY/\Sigma Y$$

In the three-phase, four-wire system the summation includes all four admittances, but only three voltages as there is no applied voltage in the neutral circuit. The equation also applies to the three-wire case since we then put $Y_n = 0$, the summations ΣEY and EY both involving three terms.

Example.—Three-phase, 230-volt supply; phase impedances, line and load $5 + j\,0$, $3 + j\,0{\cdot}5$, and $5 + j\,5$; neutral line impedance $2 + j\,1$.

Using $E_{1.n}$ as the reference quantity we have—

$$\begin{aligned} E_{1.n} &= 230 + j\,0 \\ E_{2.n} &= -\,115 - j\,199 \\ E_{3.n} &= -\,115 + j\,199 \end{aligned}$$

Load and neutral admittances—

$$\begin{aligned} Z_1 &= 5 + j\,0 & \therefore\ Y_1 &= 0{\cdot}2 - j\,0 \\ Z_2 &= 3 + j\,0{\cdot}5 & Y_2 &= 0{\cdot}324 - j\,0{\cdot}054 \\ Z_3 &= 5 + j\,5 & Y_3 &= 0{\cdot}100 - j\,0{\cdot}100 \\ Z_n &= 2 + j\,1 & Y_n &= 0{\cdot}400 - j\,0{\cdot}200 \\ & & \Sigma(Y) &= 1{\cdot}024 - j\,0{\cdot}354 \end{aligned}$$

$$\begin{aligned} E_{1.n}Y_1 &= 230\,(0{\cdot}2 - j\,0) &&= 46 + j\,0 \\ E_{2.n}Y_2 &= (-\,115 - j\,199)(0{\cdot}324 - j\,0{\cdot}054) &&= 47{\cdot}95 - j\,58{\cdot}29 \\ E_{3.n}Y_3 &= (-\,116 + j\,199)(0{\cdot}100 - j\,0{\cdot}100) &&= 8{\cdot}4 + j\,31{\cdot}4 \\ & & \Sigma(EY) &= 6{\cdot}45 - j\,26{\cdot}89 \end{aligned}$$

$\therefore$ Potential of 0 with respect to N

$$\begin{aligned} V_{o.n} = \Sigma(EY)/\Sigma(Y) &= \frac{6{\cdot}45 - j\,26{\cdot}89}{1{\cdot}024 - j\,0{\cdot}354} \\ &= 13{\cdot}73 - j\,21{\cdot}5 \end{aligned}$$

$$\begin{aligned} V_{1.o} &= E_{1.n} - V_{o.n} = (230 + j\,0) - (13{\cdot}73 - j\,21{\cdot}5) \\ &= 216{\cdot}3 + j\,21{\cdot}5 \\ V_{2.o} &= E_{2.n} - V_{o.n} = (-\,115 - j\,179) - (13{\cdot}73 - j\,21{\cdot}5) \\ &= -\,128{\cdot}7 - j\,177{\cdot}5 \\ V_{3.o} &= E_{3.n} - V_{o.n} = (-\,115 + j\,199) - (13{\cdot}73 - j\,21{\cdot}5) \\ &= -\,128{\cdot}7 + j\,220{\cdot}5 \end{aligned}$$

The currents are

$$\begin{aligned} I_1 &= (216{\cdot}3 + j\,21{\cdot}5)\,(0{\cdot}2 - j\,0) &&= 43{\cdot}26 + j\,4{\cdot}30 \\ I_2 &= (-\,128{\cdot}7 - j\,177{\cdot}5)\,(0{\cdot}324 - j\,0{\cdot}054) &&= 51{\cdot}28 - j\,50{\cdot}55 \\ I_3 &= (-\,128{\cdot}7 + j\,220{\cdot}5)\,(0{\cdot}1 - j\,0{\cdot}1) &&= 9{\cdot}18 + j\,34{\cdot}92 \\ I_n &= (13{\cdot}73 - j\,21{\cdot}5)\,(0{\cdot}4 - j\,0{\cdot}2) &&= 1{\cdot}20 - j\,11{\cdot}35 \end{aligned}$$

The vector sum of I_1, I_2, and I_3 is $1{\cdot}16 - j\,11{\cdot}33$

It should, of course, be the same as I_n—the slight difference being the result of cumulative slide-rule errors.

$$I_B = -I_A = I' \times \frac{Z_C}{Z_A + Z_B + Z_C}$$
$$= 150\,(0{\cdot}7 - j\,0{\cdot}714) \times \frac{1 + j\,2{\cdot}5}{4{\cdot}5 + j\,5{\cdot}7}$$
$$= 48{\cdot}5 - j\,26{\cdot}9$$

Now let I″ only be tapped off, I′ therefore being zero.

$$I_C = I_A = I'' \times \frac{Z_B}{Z_A + Z_B + Z_C}$$
$$= 100\,(0{\cdot}9 - j\,0{\cdot}436) \times \frac{2 + j\,1{\cdot}2}{4{\cdot}5 + j\,5{\cdot}7}$$
$$= -j\,22{\cdot}1 - j\,23{\cdot}1$$
$$I_B = I'' \times \frac{Z_A + Z_C}{Z_A + Z_B + Z_C}$$
$$= 100\,(0{\cdot}9 - j\,0{\cdot}436) \times \frac{2{\cdot}5 + j\,4{\cdot}5}{4{\cdot}5 + j\,5{\cdot}7}$$
$$= 67{\cdot}5 + j\,20{\cdot}4$$

The total currents are therefore

I_A	I_B	I_C
$-48{\cdot}5 + j\,26{\cdot}9$ $22{\cdot}1 - j\,23{\cdot}1$	$48{\cdot}5 - j\,26{\cdot}9$ $67{\cdot}5 - j\,20{\cdot}4$	$56{\cdot}8 - j\,79{\cdot}9$ $22{\cdot}1 - j\,23{\cdot}1$
Totals $-26{\cdot}4 + j\;3{\cdot}8$	$116\quad - j\,47{\cdot}3$	$78{\cdot}9 - j\,103$

Star-delta and Delta-star Transformations

The impedances are, in general, complex and the equations are therefore vector equations. Consider first of all the trans-

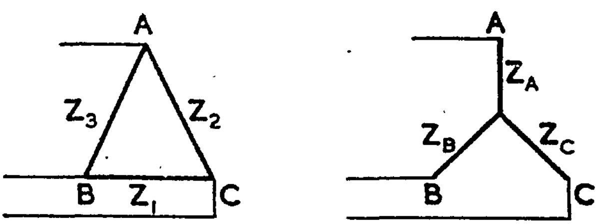


FIG. 1.16

formation from delta to star, Fig 1.16. The two systems are said to be equivalent if the corresponding line currents are identical in magnitude and phase when corresponding terminals are connected to identical supply systems or networks. This requires that the impedances, and therefore the admittances between corresponding terminals must be the same in each system. For equal impedances between terminals A, B, and C

$$Z_B + Z_C = Z_1(Z_2 + Z_3)/(Z_1 + Z_2 + Z_3) \quad . \quad . \quad (1)$$
$$Z_C + Z_A = Z_2(Z_3 + Z_1)/(Z_1 + Z_2 + Z_3) \quad . \quad . \quad (2)$$
$$Z_A + Z_B = Z_3(Z_1 + Z_2)/(Z_1 + Z_2 + Z_3) \quad . \quad . \quad (3)$$

adding (1) and (2) and subtracting (3) we have

$$2Z_C = (Z_1Z_2 + Z_1Z_3 + Z_2Z_3 + Z_2Z_1 - Z_3Z_1 - Z_3Z_2)/(Z_1 + Z_2 + Z_3)$$

$$\therefore Z_C = Z_1Z_2/(Z_1 + Z_2 + Z_3)$$

Similarly $\quad Z_A = Z_2Z_3/(Z_1 + Z_2 + Z_3)$

and $\quad Z_B = Z_3Z_1/(Z_1 + Z_2 + Z_3)$

Now consider the star-delta transformation, Fig. 1.17. It is convenient to make the transformations in terms of admittance. For equal admittances between terminals A and B—

$$Y_AY_B/(Y_A + Y_B) = Y_3 + Y_1Y_2/(Y_1 + Y_2)$$

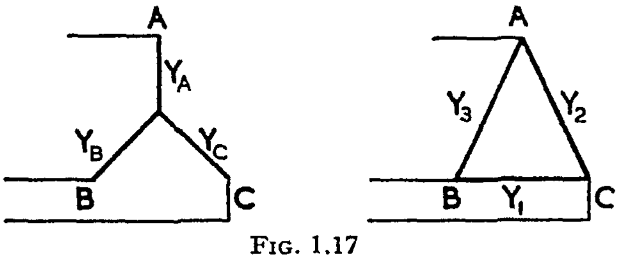


FIG. 1.17

The solution of three equations of this form is cumbersome and we obtain a simpler solution by imagining that equal impedances are connected between corresponding terminals. As long as they are equal, they can be given any value we like, and we will therefore give them the value zero. Thus, suppose that terminals B and C are connected together in each case: equating the admittances between terminal A and the common junction of B and C gives—

$$Y_A(Y_B + Y_C)/(Y_A + Y_B + Y_C) = Y_2 + Y_3 \quad . \quad (1)$$

Similarly $\quad Y_B(Y_C + Y_A)/(Y_A + Y_B + Y_C) = Y_3 + Y_1 \quad . \quad (2)$

and $\quad Y_C(Y_A + Y_B)/(Y_A + Y_B + Y_C) = Y_1 + Y_2 \quad . \quad (3)$

Adding (1) and (2) and subtracting (3), as before, gives

$$Y_1 = Y_BY_C/(Y_A + Y_B + Y_C)$$
$$Y_2 = Y_CY_A/(Y_A + Y_B + Y_C)$$
$$Y_3 = Y_AY_B/(Y_A + Y_B + Y_C)$$

This shows that the forms of the equations in the star-delta

Kouwenhoven's Short-Circuit Theorem

If a number of generators in parallel are supplying a load of given impedance, the common terminal P.D. is equal to the product of the sum of the generator short-circuit currents and the total network impedance.

Let there be n generators in parallel supplying a load of total impedance Z_L.

Let I_{1s} = short-circuit current of generator 1 at open-circuit E.M.F. E_1

Let I_{2s} = short-circuit current of generator 2 at open-circuit E.M.F. E_2

Let I_{ns} = short-circuit current of generator n at open-circuit E.M.F. E_n

Then $I_{1s} = E_1/Z_1$; $I_{2s} = E_2/Z_2$; . . . . . $I_{ns} = E_n/Z_n$

also

$$\left.\begin{array}{l} E_1 - I_1Z_1 = V \\ \vdots \\ E_n - I_nZ_n = V \end{array}\right\}$$

$$\therefore \left.\begin{array}{l} I_{1s}Z_1 - I_1Z_1 = V \\ \vdots \\ I_{ns}Z_n - I_nZ_n = V \end{array}\right\}$$

$$\therefore \left.\begin{array}{l} I_{1s} - I_1 = V/Z_1 \\ \vdots \\ I_{ns} - I_n = V/Z_n \end{array}\right\}$$

$$\therefore \quad \Sigma I_s - \Sigma I = \Sigma(V/Z)$$

$$\begin{aligned} \therefore \quad \Sigma I_s &= I_L + \Sigma(V/Z) \\ &= V/Z_L + V\Sigma(1/Z) \\ &= V[1/Z_L + \Sigma(1/Z)] \\ &= V/Z_T \text{ where } Z_T = \text{total impedance} \end{aligned}$$

$$\therefore \quad V = (\Sigma I_s) \cdot Z_T$$

which proves the theorem.

CHAPTER II

DISTRIBUTORS

D.C. Distributor fed at One End

We have seen that the main consideration in the design of a distributor is the drop of volts along it. This drop depends upon the nature of the current loading of the distributor, and also on whether the distributor is fed at one end only or at both ends. Consider first of all a distributor with concentrated loads, and fed from one end. In the D.C. case we are concerned only with the resistance of the distributor and not with its reactance. Let i_1,

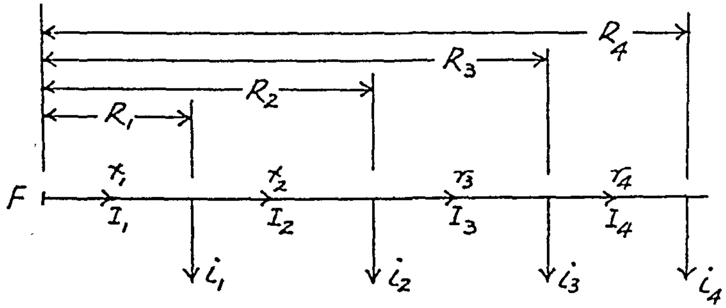


Fig. 2.1.—Distributor fed at One End.

i_2, i_3, etc., be the currents tapped off, and I_1, I_2, I_3, etc., the currents in the various sections of the distributor. Let r_1, r_2, r_3, etc., be the resistances of these various sections, and R_1, R_2, R_3, etc., the total resistances from the feeding point F to the successive tapping points (Fig. 2.1). Then total drop

$$\begin{aligned} &= r_1 I_1 + r_2 I_2 + r_3 I_3 + \dots \\ &= r_1 (i_1 + i_2 + i_3 + \dots) \\ &\qquad + r_2 (i_2 + i_3 + \dots) \\ &\qquad\qquad + r_3 (i_3 + i_4 + \dots) \\ &= i_1 r_1 + i_2 (r_1 + r_2) + i_3 (r_1 + r_2 + r_3) + \dots \\ &= i_1 R_1 + i_2 R_2 + i_3 R_3 + \dots \\ &= \Sigma (iR) \end{aligned}$$

Thus the drop *at the far end* of a distributor fed at one end is given by the sum of the moment of the various currents tapped off, about the feeding point. The drop at any intermediate point is equal to the sum of the moments of the currents up to that point, plus the moment of all the currents beyond that point assumed to be acting at that point. Thus the drop at the third tapping point will be:

$$i_1 R_1 + i_2 R_2 + i_3 R_3 + (i_4 + i_5 + i_6 + \ldots) R_3$$
$$= i_1 R_1 + i_2 R_2 + I_3 R_3$$

and similarly for any other intermediate point. The total drop,

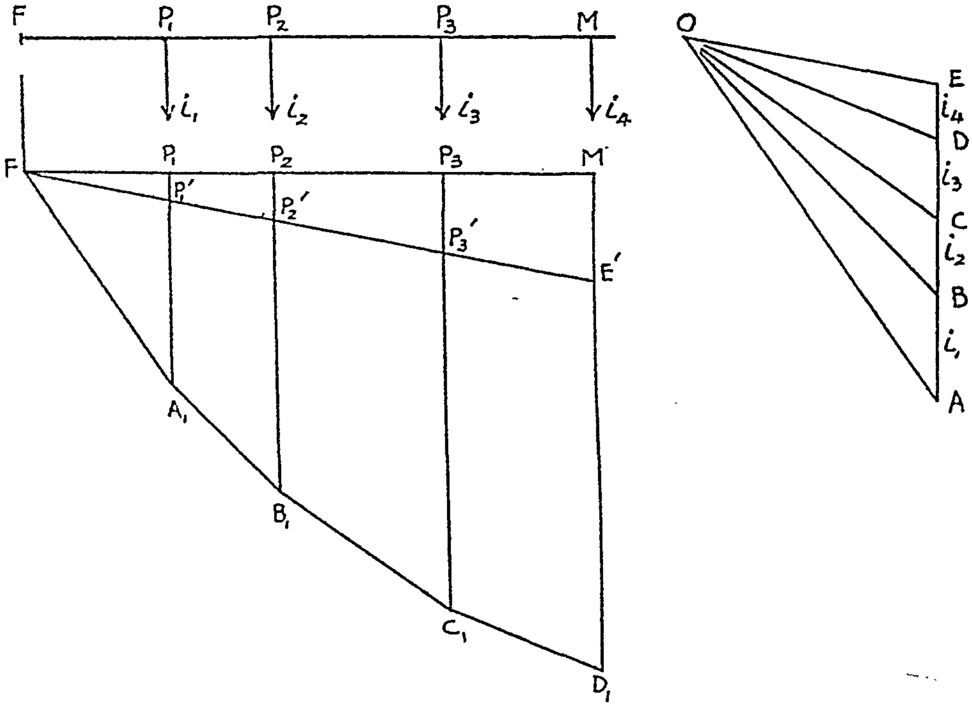


FIG. 2.2.—GRAPHICAL CONSTRUCTION FOR DETERMINING THE DROP.

reckoning both conductors, will, of course, be twice the above value.

The drop may also be determined by a simple graphical method. Since the drop is given by a sum of moments, the currents tapped off being regarded as forces and the resistances as distances, it follows that the problem can be solved by the methods of graphic statics. Draw FM (Fig. 2.2) to represent the length of the distributor to scale, and mark off the positions P_1, P_2, etc., of the various tapping points. Next draw any vertical line AY and mark off lengths AB, BC, CD, etc., to represent to the currents i_1, i_2, etc., to scale. Then from any convenient point O join OA, OB, OC, etc., and finally draw FA_1 parallel to OA, A_1B_1 parallel to OB, etc., FE′ parallel to OE.

Now, the length FM will represent a certain resistance, namely the resistance of the length FM of distributor, and therefore to the same scale the perpendicular from O will also represent a certain resistance: let this resistance be R. Then by similar triangles:

$$\frac{P_1'A_1}{FP_1'} = \frac{EA}{OE}$$

$$\therefore\ P_1'A_1 = \frac{FP_1' \times EA}{OE}$$

$$\text{Also}\ \frac{FP_1'}{OE} = \frac{FP_1}{R}$$

$$\therefore\ FP_1' = \frac{OE \times FP_1}{R}$$

$$\therefore\ P_1'A_1 = \frac{OE \times FP_1}{R} \times \frac{EA}{OE}$$

$$= \frac{FP_1 \times EA}{R}$$

$$= \frac{r_1 \times (i_1 + i_2 + i_3 + \dots)}{R}$$

$$\text{Or}\ P_1'A_1 \times R = r_1 I_1$$

$$= \text{Drop from F to } P_1$$

Hence the length P_1A_1 multiplied by the resistance R gives the drop from F to P_1. Similarly the length $P_2'B_1$ gives the drop from F to P_2; the length P C_1 the drop to P_3, and so on, from which we see that the potentials of the various tapping points are given by the vertical distances of the points A_1, B_1, C_1, etc., below the line FE_1. If the point O is chosen to be on the same horizontal level as E (i.e. the last point obtained on setting off the currents along AY), then the line FE′ will coincide with FM, and the potentials of the various tapping points will be given by the vertical distances of the points A_1, B_1, C_1, etc., below FM.

Three-wire Distributor fed at One End

All the currents tapped off from the positive outer are of the same sign, and so are all the currents flowing into the negative outer, with the result that each outer can be considered separately and treated as in the above section. The only difference is that each of these drops must no longer be multiplied by two as in the case of a two-wire distributor. In the case of the middle wire the currents are of two kinds, viz. those flowing to it from the positive outer which are reckoned positive, and those flowing from it to the negative outer, which are reckoned negative. If the resultant drop in the neutral comes out positive it is added to the drop in the positive outer and deducted from the drop in the negative outer to give the total drops on the two sides. If negative, it is deducted from the drop in the positive outer and added to the drop in the negative outer. An example will make this clear.

Example.—A three-wire distributor 300 yards long is fed at one end, the P.D. on each side at the feeding point being 230 volts. It is loaded as follows:

Yards from feeding point + side .	30	100	140	160	210	240
Load, amps.	20	40	50	10	25	30
Yards from feeding point — side .	60	80	120	180	260	—
Load, amps.	30	25	35	60	25	—

The outers each have a resistance of 0·15 ohm, and the neutral has a cross-section of one-half of either outer.

$$\text{Resistance per yard of outer} = \frac{15}{300} = 5 \times 10^{-4} \text{ ohm}$$

$$\therefore \text{Resistance per yard of neutral} = 10 \times 10^{-4} \text{ ohm}$$

Drop in positive outer:

$$\begin{aligned}\Sigma(iR) &= (20 \times 30 + 40 \times 100 + \ldots. + 30 \times 240) \times (5 \times 10^{-4})\\ &= 25650 \times (5 \times 10^{-4})\\ &= 12{\cdot}82 \text{ volts}\end{aligned}$$

Drop in negative outer:

$$\begin{aligned}\Sigma(iR) &= (30 \times 60 + 25 \times 80 + \ldots. + 25 \times 260) \times (5 \times 10^{-4})\\ &= 25300 \times (5 \times 10^{-4})\\ &= 12{\cdot}65 \text{ volts}\end{aligned}$$

Drop in neutral:

$$\begin{aligned}&= (20 \times 30 + 40 \times 100 + \ldots. + 30 \times 240) \times (10 \times 10^{-4})\\ &\quad - (30 \times 60 + 25 \times 80 + \ldots. + 25 \times 260) \times (10 \times 10^{-4})\\ &= (25650 - 25300) \times (10 \times 10^{-4})\\ &= + {\cdot}35 \text{ volt}\end{aligned}$$

$$\begin{aligned}\therefore \text{Total drop on positive side} &= 12{\cdot}82 + {\cdot}35\\ &= 13{\cdot}2 \text{ volts}\\ \therefore \text{P.D. at far end on positive side} &= 230 - 13{\cdot}2 = 216{\cdot}8 \text{ volts}\\ \text{Total drop on negative side} &= 12{\cdot}65 - {\cdot}35\\ &= 12{\cdot}3 \text{ volts}\\ \therefore \text{P.D. at far end on negative side} &= 230 - 12{\cdot}3 = 217{\cdot}7 \text{ volts}\end{aligned}$$

A problem of this nature can also be solved by the graphical method, the two outers and the neutral all requiring a separate diagram. In constructing the diagram for the neutral it is important to note that when setting off the currents on the vertical AY, the currents from the positive outer are set off upwards, while those flowing from the neutral to the negative outer are set off downwards. As an illustration the diagram for the neutral wire in the above example is drawn to scale in Fig. 2.3.

Distributor fed at Both Ends

Whenever possible it is desirable that a long distributor should be fed at both ends instead of at one end only, since in this way

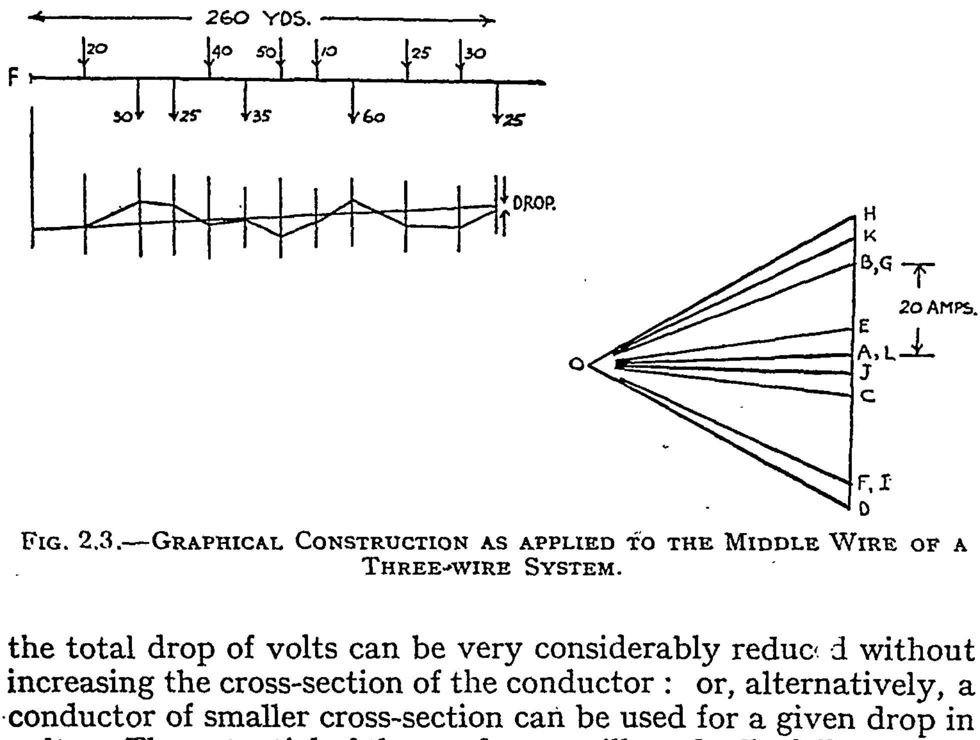


Fig. 2.3.—Graphical Construction as applied to the Middle Wire of a Three-wire System.

the total drop of volts can be very considerably reduced without increasing the cross-section of the conductor : or, alternatively, a conductor of smaller cross-section can be used for a given drop in volts. The potential of the conductor will gradually fall from one feeding point, say F_1, Fig. 2.4, reach a minimum at one of the tapping points, and will then rise again as the other feeding point F_2 is approached. There is thus some tapping point M, at which the potential is a minimum. Between F_1 and M all the currents tapped off will be supplied from the end F_1, while between F_2 and M all the currents tapped off will be supplied from F_2. With regard to the point M itself the current tapped off there will, in general, come partly from F_1 and partly from F_2, and we will denote these two parts by x and y respectively. If the distributor were cut in two at M, thereby giving two separate distributors F_1M_1 and F_2M_2, each fed from one end only, the currents tapped off at M_1 and M_2 would be x and y respectively, and the potential distribution along these two distributors would be identical with that along the original distributor F_1F_2. Hence by means of this

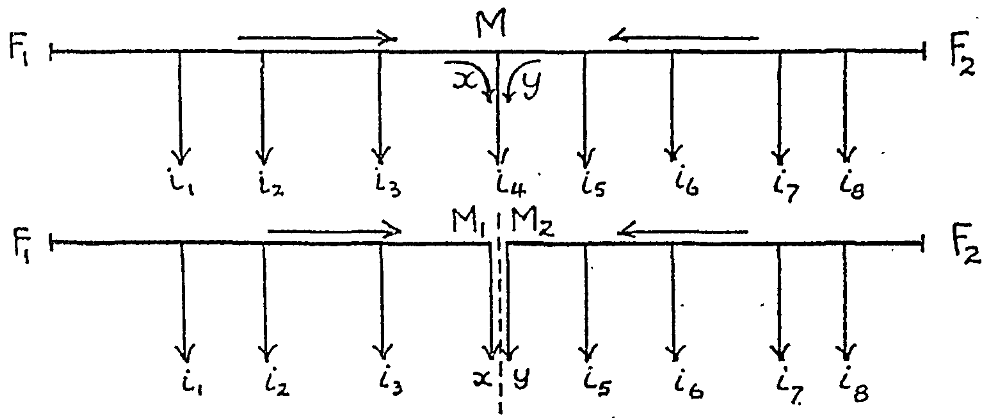


FIG. 2.4.—DISTRIBUTOR FED AT BOTH ENDS.

artifice we can make use of the method pertaining to a distributor fed at one end only. In order to calculate the drop we thus require first of all to locate the point M, after which we calculate the currents x and y. This is done by means of a pair of equations:

$$\left.\begin{array}{r}x + y = i_M \\ \text{Drop from } F_1 \text{ to } M_1 \ldots\ldots = \text{Drop from } F_2 \text{ to } M_2\end{array}\right\}$$

The best method of locating the point of minimum potential is to take moments about the two ends, and by comparing the sum

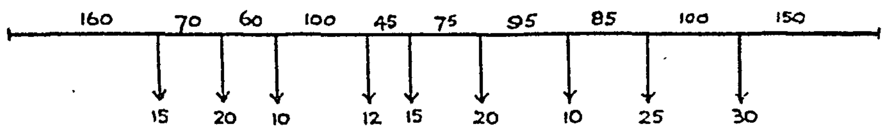


FIG. 2.5.

of these as the calculation proceeds, to make a guess at the point. The following example will illustrate the method.

A two-core distributor half a mile long is fed at both ends, these two ends having the same potential. The cross-section of each core is 0·05 sq. in. The loads tapped off are as shown in Fig. 2.5, the numbers adjacent to each distributor section indicating yards. Take the specific resistance of copper as 0·7 microhm inch.

In order to locate M the moments about the two ends are tabulated as below:

Moments about F_1 in ampere-yards.	Sums.	Moments about F_2 in ampere-yards.	Sums.
15 × 160 = 2,400 . .	2,400	30 × 150 = 4,500 . .	4,500
20 × 230 = 4,600 . .	7,000	25 × 250 = 6,250 . .	10,750
10 × 290 = 2,900 . .	9,900	10 × 335 = 3,350 . .	14,100
12 × 390 = 4,680 . .	14,580	20 × 430 = 8,600 . .	22,700
15 × 435 = 6,525 . .	21,105		

The table indicates that the sixth tapping point from F_1 is the required point. We then have for the two equations for x and y:

$$x + y = 20$$
$$21105 + 510\,x = 14100 + 430\,y$$
$$\therefore x = 1{\cdot}7 \text{ amps. and } y = 18{\cdot}32 \text{ amps.}$$

$\therefore$ Drop at M per conductor

$$= 21105 + 510 \times 1{\cdot}7$$
$$= 21973 \text{ ampere-yards.}$$

$$\text{Resistance per yard} = \frac{\rho l}{a}$$
$$= \frac{0{\cdot}7 \times 10^{-6} \times 36}{{\cdot}05}$$
$$= 50{\cdot}4 \times 10^{-5} \text{ ohm.}$$
$$\text{Hence, drop per conductor} = 2{\cdot}1973 \times 10^{4} \times 50{\cdot}4 \times 10^{-5}$$
$$= 11{\cdot}1 \text{ volts.}$$

Hence, total drop, reckoning both conductors, will be 22·2 volts.

An alternative method is to denote the total current fed in at

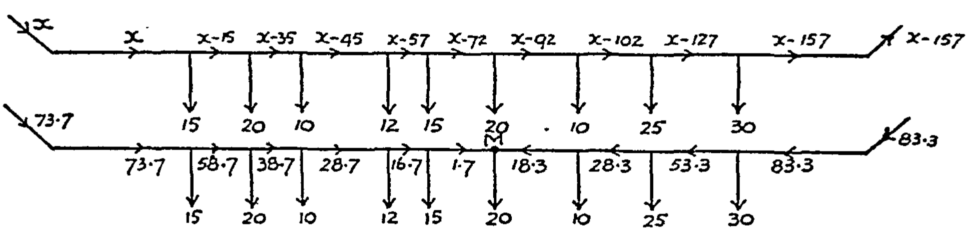


FIG. 2.6.

one end, say F, by x, set out the current distribution and calculate the total drop along the whole distributor as given by the sum of the moments about F_1. This drop is, of course, zero for a distributor in which the potentials of F_1 and F_2 are the same, and in this way an equation for x is obtained. The currents are set out in Fig. 2.6, and we have:

$$\text{Resistance per yard, single} = 5{\cdot}04 \times 10^{-4} \text{ ohm.}$$
$$\therefore \text{Resistance per yard, double} = 10{\cdot}08 \times 10^{-4} \text{ ohm.}$$

$$\therefore \frac{10{\cdot}08}{10^4}\left\{\begin{array}{l} 160x+70(x-15)+60(x-35)+100(x-45)+45(x-57) \\ \quad +75(x-72)+95(x-92)+85(x-102) \\ \quad\quad +100(x-127)+150(x-157) \end{array}\right\}=0$$

which reduces to

$$940x = 69275$$
$$\therefore x = 73{\cdot}7 \text{ amps.}$$

This gives the current distribution in the second diagram of Fig. 2.6, and indicates that the point M has the lowest potential.

$\therefore$ Drop at M (reckoning both conductors)

$$= \frac{10{\cdot}08}{10^4}(150 \times 83{\cdot}3 + 100 \times 53{\cdot}3 + 85 \times 23{\cdot}3 + 95 \times 18{\cdot}3)$$
$$= 22{\cdot}2 \text{ volts, as before.}$$

The graphical construction for a distributor fed at both ends is similar to that for a distributor fed at one end only, but it is a convenience to take the point O roughly opposite the middle of the vertical AE. The drop polygon is drawn as before by drawing a series of parallels to OA, OB, OC, etc., as in Fig. 2.2, and the polygon is then closed by joining FD_1. The point on the polygon which has the greatest perpendicular distance from FD_1 corresponds to the point M of minimum potential. Finally, if a line OX is drawn parallel to FD_1 the point X will divide the current I_M in the ratio of x to y. Fig. 2.7 shows a method of solution by means of a current-loading diagram.

F_1 and F_2 not at the Same Potential

In practice it is very probable that the points F_1 and F_2 will not be of the same potential, and in such a case the whole potential distribution along the distributor will be altered. Suppose there is a difference of v volts between F_1 and F_2, F_1 being at the higher potential. Then there will be a current of v/R flowing from F_1 to F_2, where R is the resistance of each core, and this current will be superposed on the distributor currents due to the loads tapped off. Suppose that this current is less than y, then the point M will still remain the point of minimum potential, but the current flowing to M from the left-hand side will now be $\left(x + \frac{v}{R}\right)$, while the current flowing to M from the right-hand side will be $\left(y - \frac{v}{R}\right)$.

Thus, in the previous example, suppose that v is 5 volts. R, per conductor, is 0·443 ohm, so that the additional current is

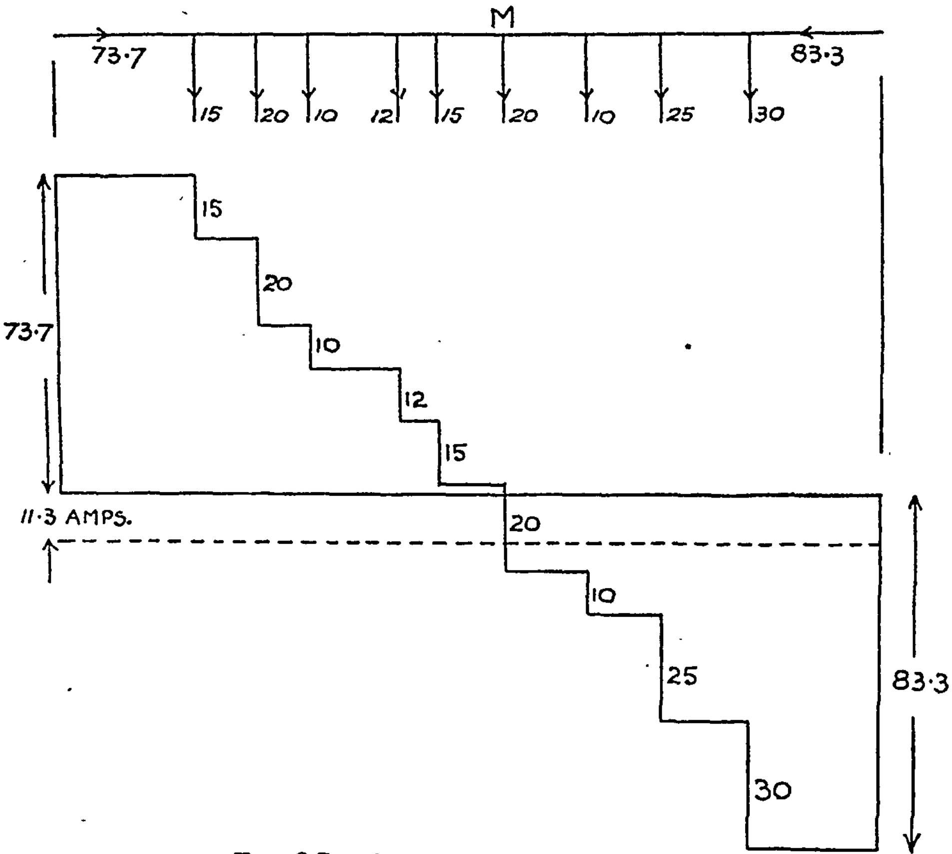


FIG. 2.7.—CURRENT-LOADING DIAGRAM.

5/0·443 = 11·3 amps. This is less than y, so that the point of minimum potential is still at M, but the new value of x is 1·7 + 11·3 = 13 amps., and the new value of y is 18·3 − 11·3 = 7 amps. Hence, the new drop from F_1 to M *is*

$$
\begin{aligned}
&21105 + 510 \times 13 \\
&= 27735 \text{ ampere-yards} \\
&= 27735 \times 5{\cdot}04 \times 10^{-4} \\
&= 14 \text{ volts.}
\end{aligned}
$$

Alternatively, assuming the point of minimum potential unchanged, superposed current

$$i = (V_1 - V_2)/R = v/R \text{ say}$$

Drop (single) from left-hand end to M

$$= [\Sigma(il) + il_M] \, R/L$$
$$= (R/L) \, \Sigma(il) + (l_M/L) \, v$$
$$\Sigma(il) = 21{,}105 + 1{\cdot}7 \times 510 = 21{,}972$$
$$il_M = 11{\cdot}3 \times 510 = 5{,}760$$
$$\text{Sum} = \underline{\underline{27{,}732}}$$

Hence, as before, drop from left-hand end to M is equal to 27,732 ampere-yards or 14 volts.

In order to take account in the graphical method of this difference of potential between F_1 and F_2 we lower the point D_1 to D_1' by an amount $F_2'F_2''$ corresponding to the potential difference v. The points A_1, B_1, C_1, etc., are lowered by amounts proportional to the distances of the corresponding tapping points from F_1, and a new vector polygon thus drawn. The point which is at the greatest vertical distance from the new closing line FD_1' gives the point of minimum potential. The necessary modification to the current-loading diagram is indicated by the dotted construction in Fig. 2.7.

Uniformly loaded Distributor

Let i = current tapped off per unit length
r = resistance per unit length (Fig. 2.8)
l = total length of distributor.

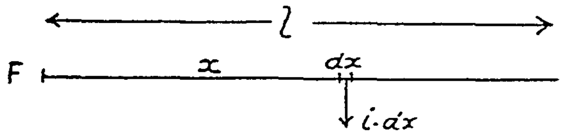


FIG. 2.8.—UNIFORMLY LOADED DISTRIBUTOR FED AT ONE END.

Then for a distributor fed at one end only we have for the drop at a distance x from F

$$\begin{pmatrix}\text{Sum of moments}\\ \text{up to } x\end{pmatrix} + \begin{pmatrix}\text{moment of whole load beyond}\\ x\text{, assumed acting at } x\end{pmatrix}$$
$$= \int_0^x rx \,.\, idx + i(l - x)\, rx$$
$$= \tfrac{1}{2}irx^2 + irlx - irx^2$$
$$= irlx - \tfrac{1}{2}irx^2$$

At the far end $x = l$

$$\therefore \text{Drop} = \tfrac{1}{2}irl^2$$
$$= \tfrac{1}{2}(il) \times (rl)$$
$$= \tfrac{1}{2}\text{IR}$$

where I is the total current fed in at F, and R is the total resistance. Thus a uniformly loaded distributor fed at one end gives a total drop equal to that produced by the whole of the load assumed concentrated at the middle point.

Now let the distributor be fed at both ends. Then with both ends at the same potential the point of minimum potential is obviously the middle point. We can thus imagine the distributor cut into two at the middle point, thus giving two uniformly loaded distributors, each fed at one end. The resistance of each is R/2 and the total current fed into each is I/2.

∴ Drop at middle point

$$= \frac{1}{2} \times \frac{\text{I}}{2} \times \frac{\text{R}}{2}$$
$$= \tfrac{1}{8}\text{IR}$$

This shows the great advantage of feeding a distributor at both ends, a uniform load giving only one-quarter of the drop produced by feeding at one end only.

The equation

$$\text{Drop at } x = irlx - \tfrac{1}{2}irx^2$$

shows that the drop diagram for a uniformly loaded distributor fed at one end is a parabola.

Example 1.—A D.C. two-wire distributor AB, 500 yards long, is supplied from both ends, the voltage at A being 210 and at B 220. The cross-section of each of the conductors is 0·4 sq. in. If a uniform load of 2 amp. per yard is taken from the half of the cable adjacent to A, and concentrated loads of 200 amps. at a distance of 100 yards, and 250 amps. at a distance of 200 yards from B, calculate the voltage drop to the middle point, and the current supplied at each end. The resistance of a cable 100 yards long and 0·1 sq. in. cross-sectional area is 0·024 ohm.

The loading diagram is given in Fig. 2.9. Starting from end B, the voltage drop up to the far end, in ampere-yards, is—

$$100\ \text{I}_\text{B} + 100\ (\text{I}_\text{B} - 200) + 50\ (\text{I}_\text{B} - 450) + \int_0^{250} [(\text{I}_\text{B} - 450) - 2x]_{dx}$$
$$= 500\ \text{I}_\text{B} - 217{,}500 \text{ amp.-yards}$$
$$1 \text{ yard} = 1{\cdot}2 \times 10^{-4} \text{ ohm.}$$
$$\therefore \text{voltage drop} = 1{\cdot}2 \times 10^{-4} \times (500\ \text{I}_\text{B} - 217{,}500)$$

This must be equal to the P.D. between A and B—namely, 10 volts.

$$\therefore 10 = 1{\cdot}2 \times 10^{-4} (500\ I_B - 2{\cdot}175 \times 10^5)$$
$$\therefore I_B = 300{,}900/500$$
$$= 602 \text{ amps.}$$
$$\therefore I_A = 500 + 250 + 200 - 602$$
$$= 348 \text{ amps.}$$

Voltage drop to middle point

$$= (250\ I_B - 42{,}500) \times 1{\cdot}2 \times 10^{-4}$$
$$= (250 \times 602 - 42{,}500) \times 1{\cdot}2 \times 10$$
$$= 12{\cdot}96 \text{ volts}$$

Example 2.—A D.C. two-wire distributor, AB, 300 yards long, is fed from both ends and supplies a uniformly distributed load of 0·15 amp. per yard, together with the following concentrated

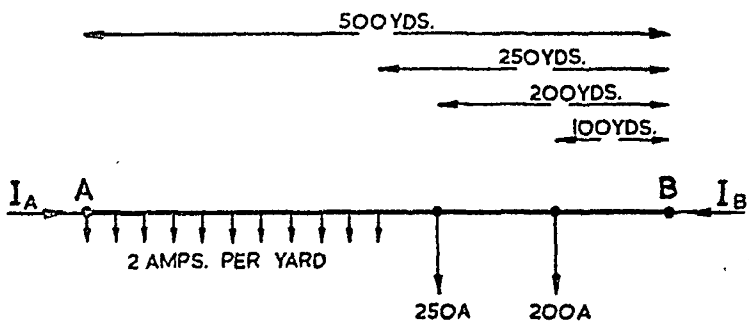


Fig. 2.9.—Distributed and Concentrated Loads.

loads: 50 amps. at C, 60 amps. at D, 40 amps. at E. The supply voltages at A and B are 206 and 200 respectively, and the resistances of each core is 0·00015 ohm per yard. If AC = 75, CD = 100, DE = 50 yards, calculate the current supplied at each end.

Resistance per two cores = 0·0003 ohm per yard.

Let current fed in at A be I amp. and denote the distributed load by i amp. per yard; then voltage drop over any length l, Fig. 2.10.

$$= \int_o^l (I - ix)\ rdx$$
$$= rl\ (I - \tfrac{1}{2}il)$$

Hence, reckoning both distributed and concentrated loads—
Drop over section AC

$$= r \times 75\ (I - \tfrac{1}{2} \times 0{\cdot}15 \times 75)$$
$$= r \times 75\ (I - 5{\cdot}6)$$

and distributed current tapped off $= 0{\cdot}15 \times 75 = 11{\cdot}3$ amps.
Drop over section CD.

$$\text{Current fed in, } I' = I - (50 + 11{\cdot}3) = I - 61{\cdot}3 \text{ amps.}$$
$$\therefore \text{Volt drop} = r \times 100\,[(I - 61{\cdot}3) - \tfrac{1}{2} \times 0{\cdot}15 \times 100]$$
$$= r \times 100\,(I - 68{\cdot}8)$$

and the distributed current tapped off $= 0{\cdot}15 \times 100 = 15{\cdot}0$ amps.
Drop over section DE.

$$\text{Current fed in, } I'' = I - (61{\cdot}3 + 60 + 15) = I - 136{\cdot}3 \text{ amps.}$$
$$\therefore \text{Volt drop} = r \times 50\,[(I - 136{\cdot}3) - \tfrac{1}{2} \times 0{\cdot}15 \times 50]$$
$$= r \times 50\,(I - 140)$$

and the distributed current tapped off $= 0{\cdot}15 \times 50 = 7{\cdot}5$ amps.
Drop over section EB.

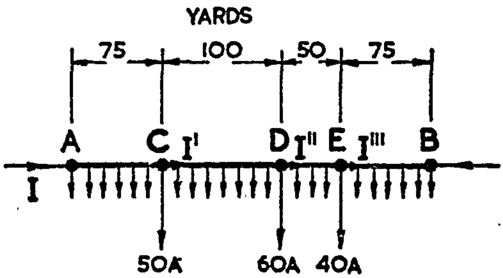


FIG. 2.10.—DISTRIBUTED AND CONCENTRATED LOADS.

$$\text{Current fed in, } I''' = I - (136{\cdot}3 + 7{\cdot}5 + 40)$$
$$= I - 183{\cdot}8 \text{ amps.}$$
$$\therefore \text{Volt drop} = r \times 75\,[(I - 183{\cdot}8) - \tfrac{1}{2} \times 0{\cdot}15 \times 75]$$
$$= r \times 75\,(I - 189{\cdot}4)$$

$\therefore$ Total volt drop
$$= r \times 75\,(I - 5{\cdot}6) + r \times 100\,(I - 68{\cdot}8) + r \times 50\,(I - 140) + r \times 75\,(I - 189{\cdot}4)$$

Putting $r = 0{\cdot}0003$, and equating the expression to the available drop of 6 volts, we have—

$$I = 161 \text{ amps.}$$

The total current is 195 amps. and therefore the current fed in an end B is

$$195 - 161 = 34 \text{ amps.}$$

Ring Distributor

As is indicated by the name, a ring distributor is a distributor which is arranged to form a closed circuit, and it can have one or several feeding points. It is quite clear that for the purpose of calculating the voltage distribution the distributor can be considered as consisting of a series of open distributors each fed at both ends. The chief interest in the ring distributor therefore lies in the economy in copper which can be effected by its use and by the proper choice in the number of feeding points. The following treatment of the problem is due to Russell,* and in order that it may be amenable to mathematical treatment it is assumed that the distributor is in the form of a circle, that the generating station, G, is in the centre, and that the feeding points, F, are arranged uniformly round the distributor. Fig. 2.11.

Let I be the total current output of the station, and n the number of feeding points. Then the current in each feeder will

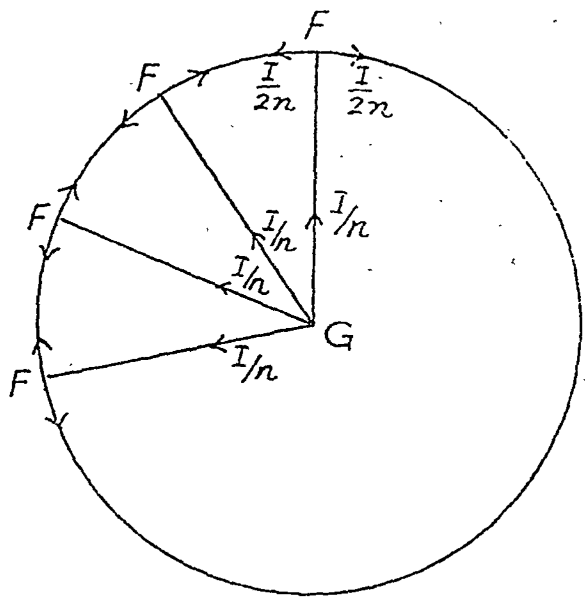


FIG. 2.11.—RING MAIN.

be I/n and the current adjacent to each feeding point in each distributor section $I/2n$.

Let v_1 = volt drop along each feeder
a_f = cross-section of feeder
r = radius of circle.

$$\therefore v_1 = \frac{I}{n} \times \frac{2\rho r}{a_f} \text{ (reckoning both sides)}$$

$$\therefore a_f = \frac{I}{n} \times \frac{2\rho r}{v_1}$$

$$= \frac{2\rho r I}{v_1 n}$$

* *The Theory of Electric Cables and Networks.*

If v is the total allowable drop for the whole system, then $(v - v_1)$ will be the drop in each distributor. Hence $(v - v_1)$

$$= \frac{1}{8} \text{ current} \times \text{resistance}$$

where current = sum of currents tapped off over one section

$$= \frac{I}{n}$$

and resistance $= \frac{\rho}{a_d} \times \frac{2\pi r}{n}$

where a_d = cross-section of a distributor

$$\therefore v - v_1 = \frac{1}{8} \times \frac{I}{n} \times \frac{2 \times 2\pi\rho r}{a_d\, n} \text{ (reckoning both sides)}$$

$$= \frac{2\pi\rho r I}{4n^2\, a_d}$$

$$\therefore a_d = \frac{2\pi\rho r I}{4n^2(v - v_1)}$$

Volume of copper in radial feeders

$$= 2nra_f$$

$$= 2nr \times \frac{2\rho r I}{v_1 n}$$

$$= \frac{4Ir^2\rho}{v_1}$$

Volume of copper in ring main

$$= 2 \times 2\pi r \times a_d$$

$$= 2 \times 2\pi r \times \frac{2\pi\rho r I}{4n^2(v - v_1)}$$

$$= \frac{4\pi^2 r^2 \rho I}{2n^2(v - v_1)}$$

$\therefore$ Total volume of copper :

$$\text{Vol.} = \frac{4Ir^2\rho}{v_1} + \frac{4\pi^2 r^2 \rho I}{2n^2(v - v_1)}$$

$$\therefore \frac{d.\text{Vol.}}{dv_1} = -\frac{4Ir^2\rho}{v_1^2} + \frac{4\pi^2 r^2 I\rho}{2n^2(v - v_1)^2}$$

This is zero and corresponds to a minimum volume

when
$$\frac{4Ir^2\rho}{v_1^2} = \frac{4\pi^2 r^2 \rho I}{2n^2(v - v_1)^2}$$

which reduces to

$$v_1 = \frac{v \times n\sqrt{2}}{\pi + n\sqrt{2}}$$

Substituting this in the equation for the volume we have for the minimum volume corresponding to the given total drop v:

$$\frac{4Ir^2\rho(\pi + n\sqrt{2})}{v\,n\sqrt{2}} + \frac{4\pi^2 r^2 I\rho}{n^2\left(v - \dfrac{vn\sqrt{2}}{\pi + n\sqrt{2}}\right)}$$

which reduces to

$$\frac{4Ir^2\rho}{v}\left(1 + \frac{\pi}{n\sqrt{2}}\right)^2$$

If n were infinite this would reduce to

$$\text{Vol}_\infty = \frac{4Ir^2\rho}{v}$$

$$\therefore \frac{\text{Vol}}{\text{Vol}_\infty} = \left(1 + \frac{\pi}{n\sqrt{2}}\right)^2$$

The dependence of this ratio on the number n of feeding points is indicated by the following table:

n.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	100.
$\frac{\text{Vol}}{\text{Vol}_\infty}$.	10·4	4·46	3·03	2·43	2·09	1·88	1·73	1·63	1·56	1·49	1·014

We see at once how a saving in copper in such a system can be effected by using a large number of radial feeders.

In some cases two distant points of a ring distributor may be

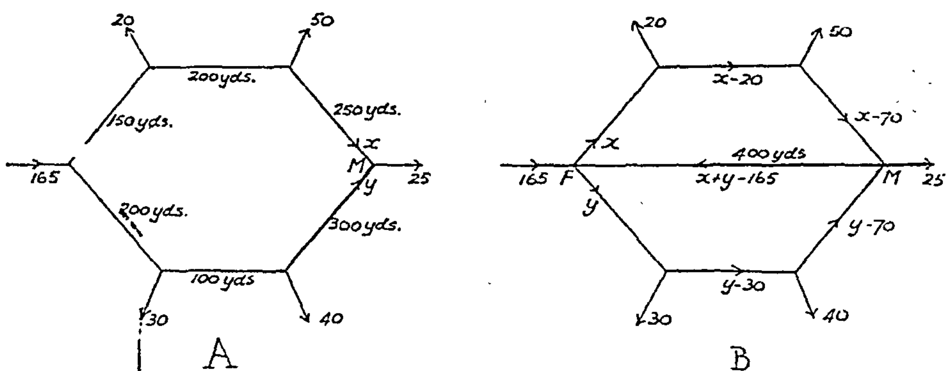


FIG. 2.12.—RING DISTRIBUTOR WITH INTERCONNECTOR.

joined by an interconnector, the function of this interconnector being to reduce the drop of volts between these points. In such a case the method of considering each section individually is no longer applicable, and it is necessary to make use of Kirchhoff's laws. An example of this type is illustrated in Fig. 2.12 in which Fig. A shows a simple ring main fed at F, and Fig. B the same main with an interconnector. Consider case A first of all, and let r be the resistance per yard of double run. The ring is obviously similar to a straight distributor fed at both ends, and both ends at the same potential. It is also obvious that M will be the point of minimum potential. Hence applying the previous methods we have:

$$x + y = 25$$
$$20 \times 150 + 50 \times 350 + x \times 600 = 30 \times 200 + 40 \times 300 + v \times 600$$
$$\text{or} \quad y - x = 4{\cdot}2.$$

Hence $x = 10{\cdot}4$ amps.; $y = 14{\cdot}6$ amps.

The total drop at M is thus:

$$(20 \times 150 + 50 \times 350 + 10{\cdot}4 \times 600)\, r$$
$$= 26740\, r \text{ volts}$$

Assume that this drop can be 20 volts

$$\therefore r = \frac{20}{26740} \text{ ohm per yard of double run}$$
$$= {\cdot}75 \times 10^{-3} \text{ ohm, which fixes the size of conductor.}$$

Now consider case B. Denoting by x and y the currents flowing to left and right from the point F, the current flowing *towards* F along the interconnector will be $x + y - 165$, and the current distribution will be as shown in Fig. 2.12. It will be seen that the circulation of current in the top loop is completely clockwise, and in the bottom loop completely counter-clockwise, so that in each case we can equate the sums of the drops to zero. Let the resistance per yard of double run of interconnector be r'.

$$\therefore 150xr + 200(x - 20)r + 250(x - 70)r + 400(x + y - 165)r' = 0$$
$$\text{and } 200yr + 100(y - 30)r + 300(y - 70)r + 400(x + y - 165)r' = 0$$

Dividing throughout by r and simplifying, we have—

$$600x - 21500 + (400x + 400y - 66000)\frac{r'}{r} = 0$$

$$600y - 24000 + (400x + 400y - 66000)\frac{r'}{r} = 0$$

It is instructive to take a series of values for the ratio r'/r, but for the present purpose will take a ratio of unity, giving

$$10x + 4y = 875$$
$$4x + 10y = 900$$

the solution of which is $x = 61{\cdot}5$ amps., $y = 65{\cdot}4$ amps.

The current along the interconnector is thus:

$$165 - (61{\cdot}5 + 65{\cdot}4) = 38{\cdot}1$$

Hence total drop to point M = drop along interconnector

$$= 38{\cdot}1 \times 400 \times {\cdot}75 \times 10^{-3}$$
$$= 11{\cdot}4 \text{ volts.}$$

Thus the use of the interconnector has cut down the voltage drop to nearly one-half: alternatively the current loading can be increased, provided, of course, that the cable section is sufficient to allow for this.*

Example.—The dimensions, currents, and voltages of a ring distributor are given in Figs. 2.13, 2.14, and 2.15. Compare the voltages of the load points A to J and the efficiencies of trans-

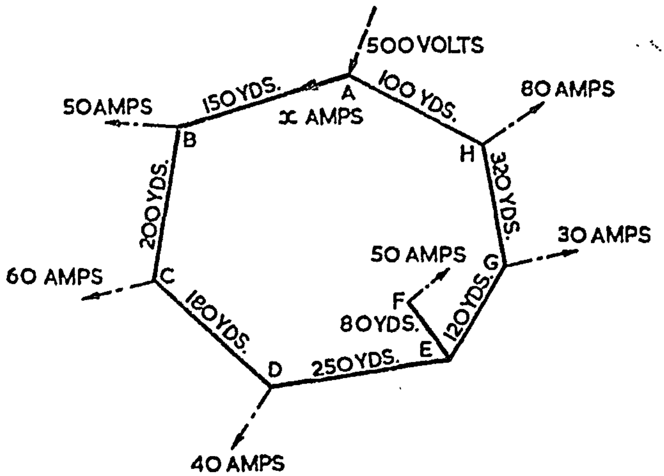


FIG. 2.13.

mission when (*a*) the distributor is fed at A only; (*b*) when it is fed at A and also at the mid-point of the 1,200-yard section; (*c*) when it is fed at the same two points and there is an interconnector, also loaded, between C and F. The resistance per yard of double conductor is 0·0008 ohm.

Case (*a*) (Fig. 2.13)

In this case a current of x amps. is assumed to flow in the section AB. All the currents are then stated in terms of x and the volt drop round the ring equated to zero.

$$\therefore 0 = 0{\cdot}12x + 0{\cdot}16\,(x - 50) + 0{\cdot}144\,(x - 50 - 60) + 0{\cdot}20\,(x - 50 - 60 - 40) + 0{\cdot}096\,(x - 50 - 60 - 40 - 50) + 0{\cdot}256\,(x - 50 - 60 - 40 - 50 - 30) + 0{\cdot}08\,(x - 50 - 60 - 40 - 50 - 30 - 80)$$
$$= 0{\cdot}264x - 39{\cdot}16$$

* In addition the point of minimum potential is no longer at M. Along the top route this point is now at the 50-amp. tapping, and along the bottom route at the 40-amp. tapping.

$\therefore x = 39{\cdot}16/0{\cdot}264 = 148$ amps.

Efficiency

$$\eta = \frac{\text{input} - \text{losses}}{\text{input}}$$

$$= \frac{500\ (148 + 162) - 8620}{500\ (148 + 162)} \times 100$$

$= 94{\cdot}6$ per cent.

The volt drops are tabulated later.

Case (*b*) (Fig. 2.14)

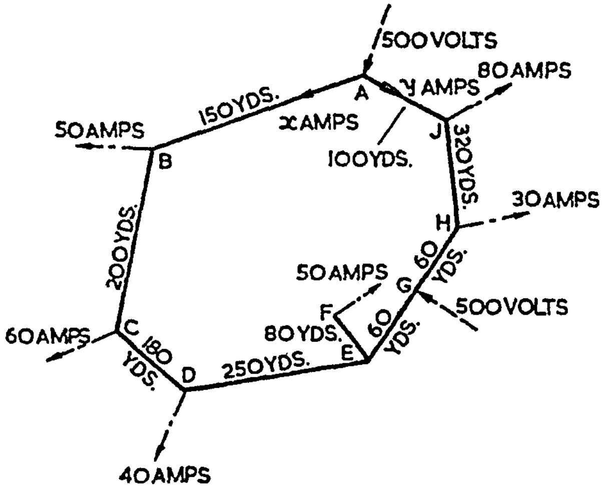


FIG. 2.14.

A current of x amps. is assumed to flow in the section AB and y amps. in the section AJ. Currents in the various sections are then calculated in the sections between A and G, travelling in both directions.

In terms of x—

$$0 = 0{\cdot}12x + 0{\cdot}16\ (x - 50) + 0{\cdot}144\ (x - 50 - 60) + 0{\cdot}2\ (x - 50 - 60 - 40) + 0{\cdot}048\ (x - 50 - 60 - 40 - 50)$$

$0{\cdot}672x = 63{\cdot}44$

$x = 94{\cdot}5$ amps.

In terms of y—

$$0 = 0{\cdot}08y + 0{\cdot}256\ (y - 80) + 0{\cdot}048\ (y - 80 - 30)$$

$0{\cdot}384y = 25{\cdot}76$

$y = 67{\cdot}00$ amps.

$$\eta = \frac{500\ (94{\cdot}5 + 67{\cdot}0 + 105{\cdot}5 + 43{\cdot}0) - 3220}{500\ (94{\cdot}5 + 67{\cdot}0 + 105{\cdot}5 + 43{\cdot}0)} \times 100$$

$= 97{\cdot}9$ per cent.

Case (*c*) (Fig. 2.15)

In this case it is necessary to assume three unknown currents, viz.: x amps. in AB, y amps. in CG, and z amps. in AH. These three currents are found by taking voltage drops round the network ABCGFH, and between the point of intake A and E.

Volt drop round the network—

$$0 = 0{\cdot}12x + 0{\cdot}16\,(x - 50) + 0{\cdot}24y + 0{\cdot}16\,(y - 50) - 0{\cdot}08z - 0{\cdot}256\,(z - 80)$$

$$\therefore 0 = 0{\cdot}28x + 0{\cdot}40y - 0{\cdot}336z + 4{\cdot}48 \quad . \quad . \quad . \quad . \quad (1)$$

Volt drop between A and H. First of all, in a counter-clockwise direction—

$$0 = 0{\cdot}12x + 0{\cdot}16\,(x - 50) + 0{\cdot}144\,(x - 60 - y - 50)$$

$$\therefore x = 0{\cdot}58y + 91{\cdot}0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Next, in a clockwise direction—

$$0 = 0{\cdot}08z + 0{\cdot}256\,(z - 80) + 0{\cdot}048/z - 80 - 30 + y - 50)$$

$$\therefore z = -\,0{\cdot}125y + 73{\cdot}5 \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The solution of these three equations is

$$x = 85{\cdot}94 \text{ amps.}; \; y = -\,8{\cdot}75 \text{ amps.}; \; z = 74{\cdot}59 \text{ amps.}$$

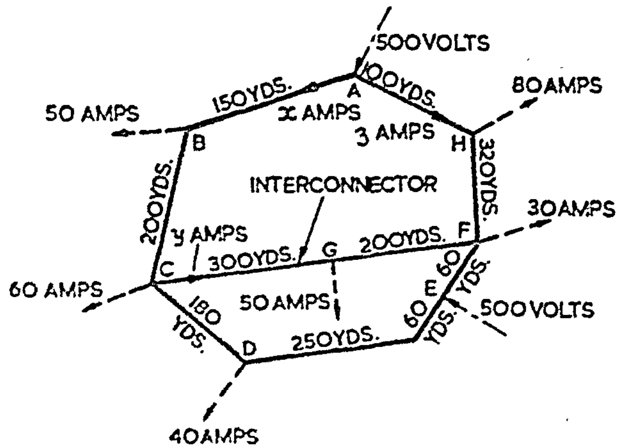


Fig. 2.15.

The negative sign for y means that this current flows in the opposite direction to that indicated in Fig. 2.14.

$$\eta = \frac{500\,(85{\cdot}94 + 74{\cdot}59 + 94{\cdot}16 + 55{\cdot}31) - 3339}{500\,(85{\cdot}94 + 74{\cdot}59 + 94{\cdot}16 + 55{\cdot}31} \times 100$$
$$= 97{\cdot}9 \text{ per cent.}$$

The volt drops at the various points for all three cases are tabulated below, and are plotted in Fig. 2.16. With a regulation which requires that the maximum allowable volt drop is 6 per cent. of the supply voltage, and therefore 30 volts in the example, we see that case (*a*) is not permissible.

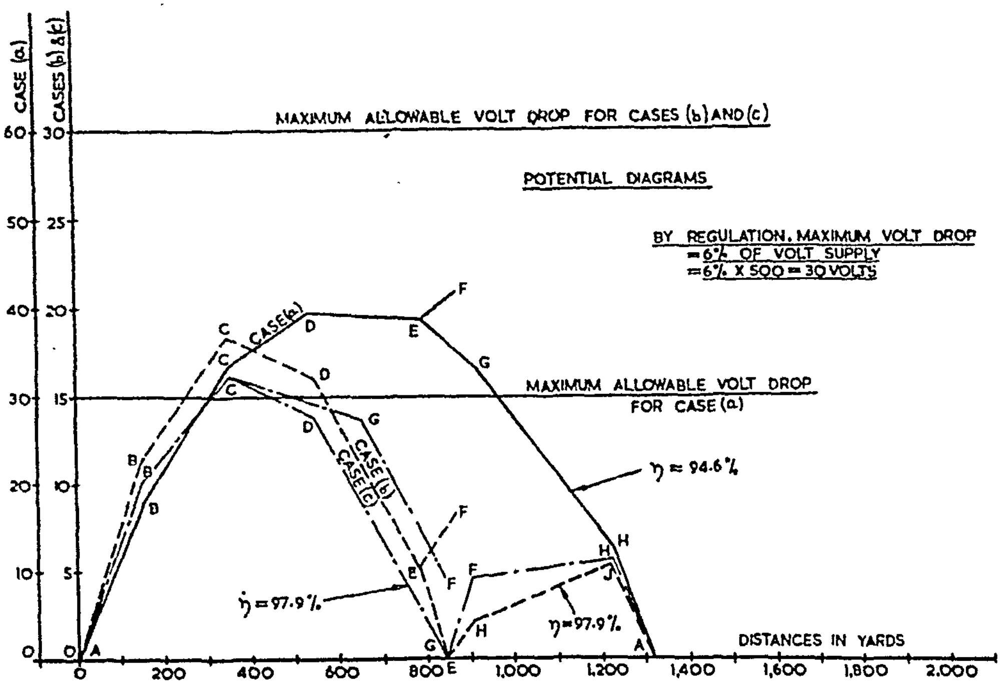


FIG. 2.16.—VOLTAGE DISTRIBUTION DIAGRAMS FOR THE RING MAINS IN THE EXAMPLE.

CASE (*a*)

Position	Length in yds.	Resistance in ohms.	Current in amps.	Volt Drop in volts.	Power Loss in watts.	Volts at:
AB	150	0·120	148	17·76	2630	B:—482·24
BC	200	0·160	98	15·68	1530	C:—466·56
CD	180	0·144	38	5·60	212	D:—460·96
DE	250	0·200	– 2	– 0·40	0·8	E:—461·36
EF	80	0·064	+ 50	+ 3·20	160	F:—458·16
EG	120	0·096	– 52	– 4·96	258	G:—466·32
GH	320	0·256	– 82	– 21·04	1730	H:—487·36
HA	100	0·080	– 162	– 12·76	2100	A:—500·12

Total Power Loss = 8620 watts.

CASE (*b*).

Position.	Length in yds.	Resistance in ohms.	Current in amps.	Volt Drop in volts.	Power Loss in watts.	Volts at:
AB	150	0·120	94·5	11·34	1070	B:—488·66
BC	200	0·160	44·5	7·11	316	C:—481·55
CD	180	0·144	– 15·5	– 2·23	35	D:—483·78
DE	250	0·200	– 55·5	– 11·10	616	E:—494·88
EF	80	0·064	50·0	3·20	160	F:—491·60
EG	60	0·048	– 105·5	– 5·06	534	G:—499·94
AJ	100	0·080	67·0	5·36	359	J:—494·64
JH	320	0·256	– 13·0	– 3·33	43	H:—497·97
HG	60	0·048	– 43·0	– 2·03	87	G:—500·00

Total Power Loss = 3220 watts.

CASE (*c*).

Position.	Length in yds.	Resistance in ohms.	Current in amps.	Volt Drop in volts.	Power Loss in watts.	Volts at:
AB	150	0·120	85·94	10·30	885	B:—489·70
BC	200	0·016	35·94	5·76	207	C:—483·94
CD	180	0·144	– 15·31	– 2·21	34	D:—486·15
DE	310	0·248	– 55·31	– 13·80	764	E:—499·95
AH	100	0·080	74·59	5·96	444	H:—494·04
HF	320	0·256	– 5·41	– 1·38	7	F:—495·42
FE	60	0·048	– 94·16	– 4·53	427	E:—499·96
CG	300	0·240	8·75	2·10	18	G:—493·32
GF	200	0·160	58·75	9·40	553	C:—483·92

Total Loss in Watts = 3339.

Calculation of Sub-mains

In an extensive system it is commonly necessary to tee off a distributor at some point on a main distributor, and in such a case some economy in copper may result in the correct choice of cross-sections for both main and sub-distributor. There are, of course, very many possible arrangements, but one only will be considered to illustrate the method. This is a distributor fed at both ends, with a loaded sub-distributor tee'd off, and a distributed load between one feeding point and the T. The scheme is illustrated in Fig. 2.17. Now for the calculation of the drop of

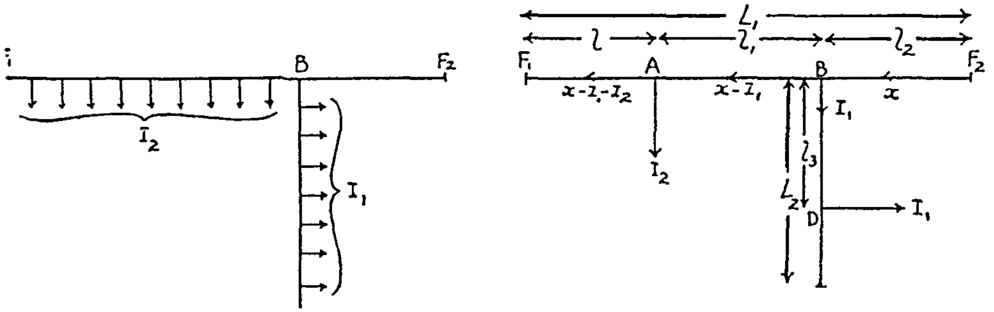


FIG. 2.17.—EXAMPLE OF A SUB-MAIN.

volts any system of distributed loads can be replaced by a concentrated load of the same total magnitude, $\Sigma(i)$, assumed acting at such a point that the moment of this current is equal to the sum of the moments of the actual currents. For example, in the case of the uniformly loaded distributor fed at one end we saw that the drop was equivalent to that due to the total current assumed concentrated at the middle point. Adopting this method the system is simplified to the second diagram of Fig. 2.17, A and D being the points at which the concentrated currents I_2 and I_1 respectively give the same total drop of volts as in the actual system. Denoting the current fed in at F_2 by x, the current distribution will be as illustrated. Denoting the resistance per unit length, single, by r, and equating the total drop along $F_1 F_2$ to zero, we have

$$2\{xrl_2 + (x - I_1)rl_1 + (x - I_1 - I_2)rl\} = 0$$
$$\therefore\ xl_2 + (x - I_1)l_1 + (x - I_1 - I_2)l = 0$$

which reduces to

$$x = \frac{I_1(l + l_1) + I_2 l}{l + l_1 + l_2}$$
$$= \frac{I_1(l + l_1) + I_2 l}{L_1}$$

Let v' = volt drop at B.
v = volt drop at far end of sub-main.
l_3 = distance of D from B.
a_1 = cross-section of main distributor.
a_2 = cross-section of sub-main.

∴ Volt drop in sub-main :

$$v - v' = 2I_1 r l_3$$
$$= 2I_1 \frac{\rho l_3}{a_2}$$
$$\therefore a_2 = \frac{2I_1 \rho l_3}{v - v'}$$

Again $$v' = 2xrl_2$$
$$= 2x \frac{\rho l_2}{a_1}$$
$$\therefore a_1 = \frac{2x\rho l_2}{v'}$$

Total volume of copper in main and sub-distributors :

$$\text{Vol} = 2(L_1 a_1 + L_2 a_2)$$
$$= 2L_1 \times \frac{2x\rho l_2}{v'} + 2L_2 \times \frac{2I_1 \rho l_3}{v - v'}$$
$$\therefore \frac{d\,\text{Vol}}{dv'} = -\frac{4L_1 l_2 \rho x}{(v')^2} + \frac{4L_2 l_3 \rho I_1}{(v - v')^2}$$

Equating this to zero, we have for the condition which gives the minimum volume of copper for the given allowable total drop of v :

$$v' = \frac{v}{\sqrt{\frac{L_2 l_3 I_1}{L_1 l_2 x}} + 1}$$

In this way v' and $(v - v')$ can be calculated, and therefore the cross-sections a_1 and a_2 can be calculated. It is to be remembered that this method of design ensures only that the volt drops shall be correct : it is also necessary to ensure that the calculated cross-sections a_1 and a_2 shall be sufficient to carry the appropriate currents without overheating. If the calculated cross-sections are sufficient for the current to be carried, then they can be adopted in practice. If too small, then obviously the cables must be chosen from the point of view of heating alone, since this will, under such circumstances, give a total drop less than the allowable value v.

Stepped or Tapered Mains

When a series of currents is tapped off from any conductor it is obvious that the current will vary along the length of the conductor. For example, a uniformly loaded distributor fed at one end carries a current which varies progressively from a maximum at the feeding point to zero at the far end. If the conductor is of uniform cross-section throughout, it therefore follows that, from the point of view of current-carrying capacity, there is a great deal too much copper in use, and that an appreciable saving can be effected by choosing the cross-section in accordance with the current to be carried; in other words, in stepping or tapering the conductor. At first sight it might appear that it would be sufficient to make the cross-section proportional to the current to be carried (neglecting the small changes of current density with cross-section), but the criterion is a fixed voltage drop and not fixed current density.

Consider the distributor shown in Fig. 2.18, with loads of I_1 and

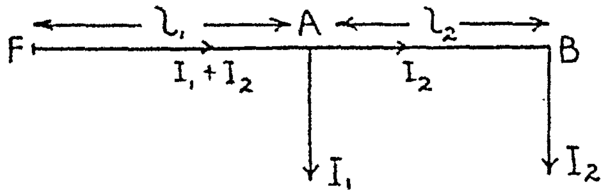


FIG. 2.18.

I_2 tapped off at A and B respectively. Let the lengths and cross-sections be l_1, a_1 for FA and l_2, a_2 for AB. Then:

Resist. from F to A, reckoning both sides $= \dfrac{2\rho l_1}{a_1}$.

Drop from F to A, $$v' = \frac{2\rho l_1(I_1 + I_2)}{a_1}$$

$$\therefore a_1 = \frac{2\rho l_1(I_1 + I_2)}{v'}$$

Drop from A to B, $$v - v' = \frac{2\rho l_2 I_2}{a_2}$$

$$a_2 = \frac{2\rho l_2 I_2}{v - v'}$$

Volume of copper in the distributor, reckoning both sides:

$$\text{Vol} = 2l_1a_1 + 2l_2a_2$$

$$= \frac{4\rho l_1^2(I_1 + I_2)}{v'} + \frac{4\rho l_2^2 I_2}{v - v'}$$

$$\therefore \frac{d.\text{Vol}}{dv'} = -\frac{4\rho l_1^2(I_1 + I_2)}{(v')^2} + \frac{4\rho l_2^2 I_2}{(v - v')^2}$$

$$= 4\rho\left\{\frac{l_2^2 I_2}{(v - v')^2} - \frac{l_1^2(I_1 + I_2)}{(v')^2}\right\}$$

Substituting the previous values of v' and $(v - v')$, we have for the condition for the minimum volume of copper :

$$\frac{l_2^2 I_2 a_2^2}{4\rho^2 l_2^2 I_2^2} = \frac{l_1^2 (I_1 + I_2) a_1^2}{4\rho^2 l_1^2 (I_1 + I_2)^2}$$

$$\frac{a_2^2}{I_2} = \frac{a_1^2}{I_1 + I_2}$$

$$\therefore \frac{a_1}{a_2} = \sqrt{\frac{I_1 + I_2}{I_2}}$$

In other words, the cross-sections of the various portions of a stepped conductor should be proportional to the square-roots of the appropriate currents. Since the current density is equal to the quotient of current and cross-section, it also follows that in such a stepped distributor the current density will not be uniform.

As an example of a stepped distributor consider that shown in Fig. 2.19. The voltage of the system is 400 and the total drop is

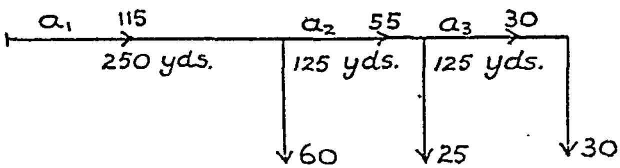


FIG. 2.19.

not to exceed 3 per cent. of this, i.e. 12 volts. Taking the distributor sections in pairs and commencing with the last pair, we have—

$$\frac{a_2}{a_3} = \sqrt{\frac{I_2 + I_3}{I_3}}$$

$$\therefore a_2 = a_3 \times \sqrt{\frac{25 + 30}{30}} = 1{\cdot}354\ a_3.$$

Next considering the first pair we have—

$$\frac{a_1}{a_3} = \sqrt{\frac{I_1 + I_2 + I_3}{I_3}}$$

$$\therefore a_1 = a_3 \times \sqrt{\frac{60 + 25 + 30}{30}} = 1{\cdot}958\ a_3$$

Again taking $\rho = \frac{2}{3} \times 10^{-6}$ ohm per inch cube :

$$R_1 = \frac{2 \times 2 \times 10^{-6} \times 250 \times 36}{3a_1} = \frac{1{\cdot}2 \times 10^{-2}}{a_1} \text{ ohms}$$

$$R_2 = \frac{2 \times 2 \times 10^{-6} \times 125 \times 36}{3a_2} = \frac{\cdot 6 \times 10^{-2}}{a_2} \text{ ohms}$$

$$\text{and } R_3 = \frac{2 \times 2 \times 10^{-6} \times 125 \times 36}{3a_3} = \frac{\cdot 6 \times 10^{-2}}{a_3} \text{ ohms,}$$

the above values taking into account both positive and negative sides. Hence for the total drop of volts we have—

$$R_1(I_1 + I_2 + I_3) + R_2(I_2 + I_3) + R_3I_3$$

$$= \frac{1{\cdot}2 \times 10^{-2} \times 115}{1{\cdot}958\, a_3} + \frac{\cdot 6 \times 10^{-2} \times 55}{1{\cdot}354\, a_3} + \frac{\cdot 6 \times 10^{-2} \times 30}{a_3}$$

$$= \frac{\cdot 705}{a_3} + \frac{\cdot 245}{a_3} + \frac{\cdot 180}{a_3} = \frac{1{\cdot}13}{a_3}$$

$$\therefore \frac{1{\cdot}13}{a_3} = 12$$

$$\therefore a_3 = \frac{1{\cdot}13}{12} = \cdot 0943 \text{ sq. in.}$$

$$\therefore a_2 = 1{\cdot}354 \times \cdot 0943 = \cdot 128 \text{ sq. in.}$$

$$\text{and } a_1 = 1{\cdot}958 \times \cdot 0943 = \cdot 185.$$

The nearest standard cross-sections are 0·1, 0·12, and 0·2 sq. in. respectively, and taking these values we have for the total volume of copper in the distributor—

$$2(\cdot 2 \times 250 + \cdot 12 \times 125 + \cdot 1 \times 125) \times 36$$
$$= 72 \times 77{\cdot}5.$$

If the distributor had been of uniform cross-section of ·2 sq. in. throughout, the volume of copper in it would have been—

$$2 \times \cdot 2 \times 500 \times 36 = 72 \times 100.$$

$$\therefore \frac{\text{Copper in stepped distributor}}{\text{Copper in uniform distributor}} = \frac{77{\cdot}5}{100} = \cdot 775.$$

It is to be noted that the practicability of laying tapered distributors is very debatable for the following reasons. Standard cross-sections are of necessity few, and it may thus be impossible to choose such sections without departing considerably from the economy aimed at. The scheme will entail more jointing than a scheme with uniform section, and it is very desirable that the number of joints should be kept to a minimum. But probably the most important objection is that future additions to any system may entirely alter the distribution of current in the various sections of any given main and may therefore render a stepped conductor useless. The only case in which there is real justification for a stepped conductor is that of a long distributor fed from one end and terminating at the other end at the boundary of the undertaking.

Alternating Current Distribution

Since, in general, the various loads tapped off from a given distributor will be at different power factors, the currents in the distributor sections will be given by the vector sums of the load currents and not by the arithmetic sums. In addition, the drop of volts will not be due to resistance only as in a D.C. distributor, but to the combined effects of resistance and reactance. Now, if a current I flows along a conductor of resistance R and inductive reactance X (neglecting the effect due to capacitance), the drop of volts will be given by

$$(RI \cos \varphi + XI \sin \varphi)$$

Hence, if the various currents are resolved into working components and wattless components by multiplying them by these appropriate $\cos \varphi$ and $\sin \varphi$ respectively, the distributions of these two kinds of component can be calculated separately. In the case of the working components the drop will be due to resistance only, and in the case of the wattless components due to wattless components only. In calculating the drop due to the working components the resistances only are used, and in calculating the drop due to the wattless components the inductive reactances only are used, and in this way the problem is solved in two halves, the calculation for each being identical with that for a D.C. distributor.

As an example consider the distributor shown in Fig. 2.20; the cable used is a 0·2 sq. in. two-core paper insulated cable, for which the resistance and reactance are 0·25 and 0·125 ohm per 1,000 yards of double run respectively. The actual loadings are shown in Fig. A, and for the working components we have—

$$100 \times \cdot 707 = 70\cdot 7\ ;\ \ 120 \times 1 = 120\ ;\ \ 80 \times \cdot 8 = 64$$

These are shown in Fig. B. For the wattless components we have—

$$100 \times \cdot 707 = 70\cdot 7\ ;\ \ 120 \times 0 = 0\ ;\ \ 80 \times \cdot 6 = 48$$

and these are shown in Fig. C. The resistances and reactances are also indicated in Figs. B and C, and taking moments we have—

Drop due to working components

$$\begin{aligned} &= 70\cdot 7 \times \cdot 025 + 120 \times \cdot 0625 + 64 \times \cdot 1 \\ &= 1\cdot 77 + 7\cdot 5 + 6\cdot 4 \\ &= 15\cdot 67 \text{ volts} \end{aligned}$$

Drop due to wattless components $= 70{\cdot}7 \times {\cdot}0125 + 48 \times {\cdot}05$
$= {\cdot}88 + 2{\cdot}4$
$= 3{\cdot}28$

Hence total drop $= 15{\cdot}67 + 3{\cdot}28 = 18{\cdot}95$ volts.

With a large number of loads there will be considerable labour attached to such a method, and provided that an approximate

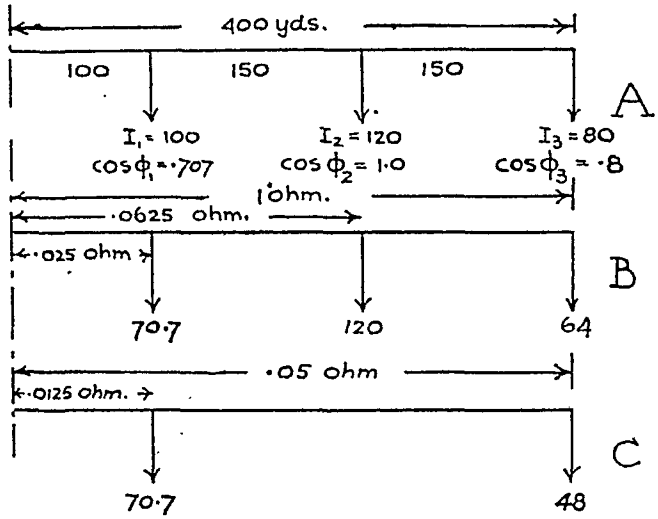


FIG. 2.20.—DISTRIBUTOR WITH A.C. LOADING.

solution is sufficient a certain amount of calculation can be saved by the following method.

The "distributing centre" or centre of gravity of the load is situated the following distance from F :

$$\frac{100 \times 100 + 120 \times 250 + 80 \times 400}{300}$$
$$= 240 \text{ yds.}$$

for which the resistance and reactance per double run are :

$$R = \frac{{\cdot}25 \times 240}{1000} = {\cdot}06 \text{ ohm}$$
$$X = \frac{{\cdot}125 \times 240}{1000} = {\cdot}03 \text{ ohm}$$

The average power factor is :

$$\text{Cos } \varphi_{av} = \frac{100 \times {\cdot}707 + 120 \times 1 + 80 \times {\cdot}8}{300}$$
$$= 0{\cdot}85$$

$$\therefore \text{ Sin } \varphi_{av} = \sqrt{1 - (\cdot 85)^2} = \cdot 525$$

$$\therefore \text{ Drop } \simeq \{\cdot 06 \times \cdot 85 + \cdot 03 \times \cdot 525\} \times 300$$

$$\simeq 20 \text{ volts}$$

as against 19 volts by the more exact method.

Division of Load between Lines in Parallel

In order to maintain continuity of supply it is common practice to work two or more cables or overhead lines in parallel in the case of a supply to an important district, so that in the event of a fault developing on one line the other, or others, will be able to carry the load until the fault has been rectified. If all the lines follow the same route and are composed of the same type of cable or the same type of overhead conductor, then they will normally divide the total current equally between themselves. If the routes are different, and/or the line constants, e.g. resistance and reactance per mile are different, then there will be no longer equality of divisions of the total current. It is sufficient to take the case of two lines in parallel, in which case the division of current is determined exactly as for the division of current between two impedances in parallel.

Let the impedances of the two lines be Z_1 and Z_2, then for their combined impedance we have—

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Let I = total current delivered to both lines

$$\therefore I \times \frac{Z_1 Z_2}{Z_1 + Z_2} = \text{total drop of volts.}$$

Now let I_1 and I_2 be the currents flowing in the two lines, then :

$$I_1 = \frac{\text{Drop}}{Z_1}$$

$$= \frac{IZ_2}{Z_1 + Z_2}$$

and

$$I_2 = \frac{IZ_1}{Z_1 + Z_2}$$

In this calculation it is convenient to take the voltage vector as the reference axis; then if the power factor of the total current is cos φ, we have—

$$E = E + j0$$

$$I = I \cos \varphi - j\, I \sin \varphi$$

The following example, taken from a London University paper on Electrical Power, will illustrate the method:

Example 1.—A total load of 12,000 kW. at a power factor of 0·8 lagging is transmitted to a substation by two overhead three-phase lines connected in parallel. One line has a conductor resistance of 2 ohms per conductor and a reactance (line to neutral) of 1·5 ohms, the corresponding values for the other line being 1·5 and 1·2 respectively. Calculate by an exact method the power transmitted by each overhead line.

Let A and B refer to the two lines respectively. Then

$$Z_A = R_A + jX_A = 2 + j \times 1{\cdot}5$$
$$Z_B = R_B + jX_B = 1{\cdot}5 + j \times 1{\cdot}2$$

Total load current

$$I = \frac{12000 \times 1000}{\sqrt{3} \times {\cdot}8 \times 10^6} = 8{\cdot}66, \text{ assuming a line voltage of } 10^6 \text{ volts}$$

for convenience.

$$\therefore I = 8{\cdot}66({\cdot}8 - j \times {\cdot}6)$$

$$\therefore I_A = \frac{IZ_B}{Z_A + Z_B}$$

$$= \frac{8{\cdot}66({\cdot}8 - j \times {\cdot}6)\,(1{\cdot}5 + j \times 1{\cdot}2)}{(2 + j \times 1{\cdot}5) + (1{\cdot}5 + j \times 1{\cdot}2)}$$

which reduces to

$$I_A = \frac{8{\cdot}66}{19{\cdot}52}(6{\cdot}882 - j \times 5{\cdot}39)$$

∴ Power transmitted by A

$$= 10^6 \times \frac{8{\cdot}66 \times 6{\cdot}882}{19{\cdot}52} \times \frac{\sqrt{3}}{1000}$$
$$= 5280 \text{ kW.}$$

Similarly $$I_B = \frac{8{\cdot}66({\cdot}8 - j \times {\cdot}6)\,(2 + j \times 1{\cdot}5)}{(2 + j \times 1{\cdot}5) + (1{\cdot}5 + j \times 1{\cdot}2)}$$

which reduces to

$$I_B = \frac{8{\cdot}66}{19{\cdot}52}(8{\cdot}75 - j \times 6{\cdot}75)$$

∴ Power transmitted by B

$$= 10^6 \times \frac{8{\cdot}66 \times 8{\cdot}75}{19{\cdot}52} \times \frac{\sqrt{3}}{1000}$$
$$= 6720 \text{ kW.}$$

This gives a total power transmitted of

$$5280 + 6720 = 12000 \text{ kW.}$$

Example 2.—A 25,000-kW. three-phase load is to be supplied at 33 kV. between lines, over a distance of 20 miles from a substation. The load and the substation are connected by a three-conductor overhead line, and a three-core cable in parallel with the line. If the overhead line has an impedance of $0{\cdot}15 + j\,0{\cdot}25$

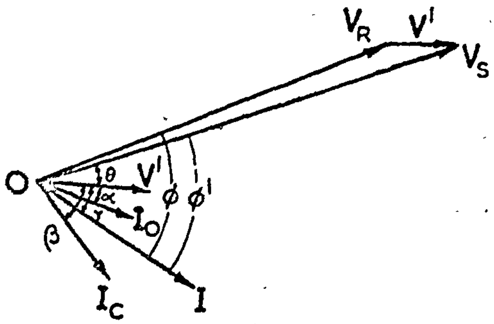


Fig. 2.21.

ohm per mile and the cable $0{\cdot}2 + j\,0{\cdot}05$ ohm per mile, calculate the magnitude and phase of the currents in the overhead line and in the cable, with respect to the supply voltage. Assume that the load power factor is 0·8 lagging. What is the power supplied by the substation?

Current delivered to load $= 25{,}000 \times 10^3/(\sqrt{3} \times 33 \times 10^3 \times 0{\cdot}8)$
$= 546$ amps.

Taking the voltage drop V′ from substations to load as the reference: Fig. 2.21.

Impedance of overhead line $= 20(0{\cdot}15 + j\,0{\cdot}25) = 3 + j\,5$
Impedance of cable $= 20(0{\cdot}2 + j\,0{\cdot}05) = 4 + j\,1$
Current in overhead line $I_o = V'/(3 + j\,5)$
$= V'(0{\cdot}0884 - j\,0{\cdot}147)$
Current in cable $I_c = V'/(4 + j\,1)$
$= V'(0{\cdot}235 - j\,0{\cdot}0588)$

Hence

$$546^2 = (V')^2[(0{\cdot}0884 + 0{\cdot}235)^2 + (0{\cdot}147 + 0{\cdot}0588)^2]$$

giving $V' = 1430 \text{ volts} = 1{\cdot}43 \text{ kV.}$

$$\therefore I_o = (0{\cdot}0884^2 + 0{\cdot}147^2)^{\frac{1}{2}} \times 1430$$
$$= 245 \text{ amps.}$$
$$I_c = (0{\cdot}235^2 + 0{\cdot}0588^2)^{\frac{1}{2}} \times 1430$$
$$= 346 \text{ amps.}$$

Angle of lag of I_o with respect to V′, $\alpha = \tan^{-1}(0{\cdot}147/0{\cdot}0884)$
$= 59°$ (Fig. 2.21.)

Angle of lag of I_c with respect to V', $\beta = \tan^{-1} (0{\cdot}0588/0{\cdot}235)$
$= 14° 21'$
Angle of lag of I with respect to V', $\gamma = \tan^{-1} (0{\cdot}205/0{\cdot}323)$
$= 32° 27'$
Angle of lag of I with respect to receiving end voltage V_R
$\phi = \cos^{-1} 0{\cdot}8$
$= 36° 52'$
Phase difference between V_R and V' $= 36° 52' - 32° 27'$
$= 4° 25'$

$$V_R = 33/\sqrt{3} = 19 \text{ kV}.$$

Hence for the sending-end voltage we have

$$V_S^2 = (19 \cos 4° 25' + 1{\cdot}43)^2 + (19 \sin 4° 25')^2$$
$$= 20{\cdot}33^2 + 1{\cdot}46^2$$
$$V_S = 20{\cdot}38 \text{ kV}.$$

Phase difference between V_S and V', $\theta = \tan^{-1} (1{\cdot}46/20{\cdot}33)$
$= 4° 7'$
∴ Phase difference between V_S and I, $\phi' = 32° 27' + 4° 7'$
$= 36° 34'$
∴ $\cos \phi' = 0{\cdot}803$
∴ Power supplied by substation $= 3 \times 20{\cdot}38 \times 546 \times 0{\cdot}803$
$= 26{\cdot}700$ kW.
Phase difference between V_S and $I_o = 59° + 4° 7' = 63° 7'$
$= \cos^{-1} 0{\cdot}45$
Phase difference between V_S and $I_C = 14° 2' + 4° 7' = 18° 9'$
$= \cos^{-1} 0{\cdot}95$
∴ $I_o = 245$ amps. at a power factor of 0·45 lagging
$I_C = 346$ amps. at a power factor of 0·95 lagging
both with respect to the sending-end voltage.

Line Loss

So far we have dealt with the drop of volts in a distributor, but not with the loss of power taking place in it: in this section we will consider only the I^2R loss due to the conductor resistance. For a distributor with concentrated loads the currents in the various sections are determined, and then the total I^2R loss is the sum of the losses for the various sections. Now consider a uniformly loaded distributor fed at one end. Using the same notation as before, we have—

Total current in distributor at a point distant x from the feeding point $= li - xi$.

∴ I^2R loss in a length dx, reckoning both sides:

$$= 2(li - xi)^2 dx \times r$$
$$= 2i^2 r(l^2 - 2lx + x^2)dx$$

The loss for any length x is therefore given by:

$$\int_0 2i^2r(l^2 - 2lx + x^2)dx = 2i^2r\left(l^2x - lx^2 + \frac{1}{3}x^3\right)$$

At the far end, where $x = l$, this reduces to:

$$2i^2r\left(l^3 - l^3 + \frac{1}{3}l^3\right) = \frac{2}{3}i^2rl^3$$

$$= \frac{2}{3}(li)^2 \times (lr)$$

$$= \frac{2}{3}I^2R$$

where I is the total current and R the resistance per single run. The loss per side is thus $\frac{1}{3}I^2R$, showing that the distribution centre for the computation of power loss is one-third the distance from the feeding point, and not one-half. It will thus be clear that if a system of distributed loads is reduced to the equivalent system of concentrated loads for the purpose of calculating the voltage regulation, this equivalent system cannot be used for the calculation of the line loss.

Best Position for Substation

In the early days of electricity supply there was very little long-distance transmission, and each undertaking was more in the nature of a comprehensive distributing system. Consequently it was fairly common practice to locate the power station near the centre of gravity of the load, in order to economise in the cost of cables. This procedure led to the choice of many unfortunate sites for power stations, but within recent years the majority of such stations have been converted to substations, for which such a basis of calculation is justifiable.

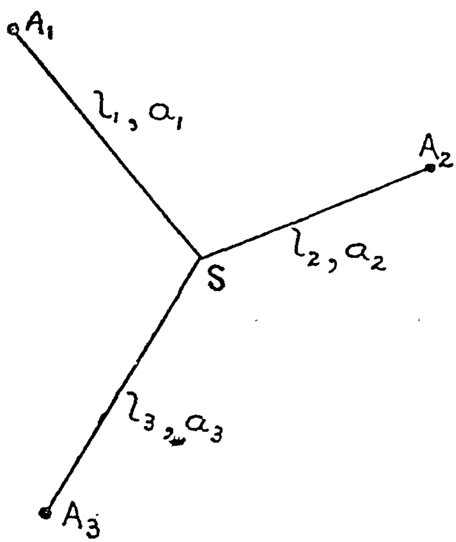


FIG. 2.22.

Consider a number of load centres A_1, A_2, A_3, etc., Fig. 2.22, supplied from a substation S. Let the lengths and cross-sections of the feeders be l_1, a_1; l_2, a_2; l_3, a_3, etc.; and let I_1, I_2, I_3, etc., be the currents. Assuming the same allowable volt drop, v, in all cases, we have:

$$a_1 = \frac{2I_1l_1\rho}{v}$$

$$\therefore \text{Vol}_1 = 2l_1a_1 = \frac{4I_1l_1^2\rho}{v}$$

Hence for the total volume of copper we have—

$$\text{Vol} = \frac{4I_1l_1^2\rho}{v} + \frac{4I_2l_2^2\rho}{v} + \ldots\ldots$$

$$= \frac{4\rho}{v}(I_1l_1^2 + I_2l_2^2 + \ldots\ldots)$$

Now, if we have a series of masses m_1, m_2, m_3, etc., situated distances l_1, l_2, l_3, etc., from a point, the quantity $\Sigma(ml^2)$ is a minimum when this point coincides with the centre of gravity of the system of masses. By analogy, the quantity (Il^2), and therefore the total volume of copper, will be a minimum when the substation S is situated at the centre of gravity of the system, the currents I_1, I_2, etc., being treated as masses.

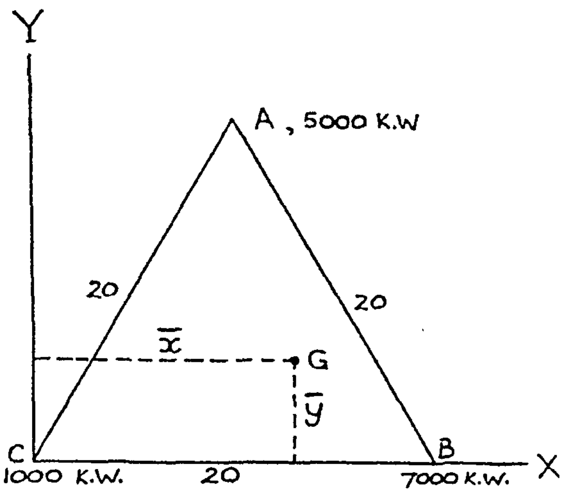


FIG. 2.23.

Example.—Three towns, A, B, and C, are situated at the corners of an equilateral triangle of 20-mile side. It is proposed to supply power to the extent of 5,000, 7,000, and 1,000 kW respectively to A, B, and C, from one substation. Where should the substation be located? (London Univ.)

Assuming an A.C. system and the same power factor for all three loads, the loads themselves can be treated as masses (Fig. 2.23). Then—

$$\bar{x} \times 13000 = 5000 \times 10 + 7000 \times 20 + 0$$
$$= 190000$$
$$\therefore \bar{x} = \frac{190000}{13000} = 14{\cdot}6 \text{ miles}$$
$$\bar{y} \times 13000 = 5000 \times \frac{20\sqrt{3}}{2} + 0 + 0$$
$$y = \frac{5000 \times 20 \times 1{\cdot}732}{13000 \times 2} = 6{\cdot}66 \text{ miles}$$

CHAPTER III

INSULATION RESISTANCE OF A SYSTEM

Fault Resistance

THE leakage of electricity which takes place under working conditions in a distribution system consists partly of leakage across the insulation of the cables, but mainly of surface leakage across apparatus and fittings. These leakages will cause currents to flow even if all loads are disconnected. Imagine the system to be perfect and resistances connected between the various bus-bars and earth, the values of these resistances being adjusted until the currents flowing through them are equal, both in magnitude and direction, to the actual values of the earth currents due to leakages. Then the values of these resistances are called the fault resistances of the system. The following are some of the methods which have been developed to determine the fault resistances during operation, i.e. without the necessity for interrupting the supply.

Two-wire System

A polarised voltmeter is connected, firstly between the positive main and earth and secondly between the negative main and earth, and readings taken in each case. If the system is working with one pole earthed, the earth connection is removed temporarily, while the readings are taken. Let V_1 and V_2 be the two readings, and let V be the system voltage. Also let f_1 and f_2 be the fault resistances of the positive and negative sides respectively, and r the resistance of the voltmeter. The currents and potentials in the two cases are represented in the diagrams of Fig. 3.1, from which we have—

$$\frac{V_1}{f_1} + \frac{V_1}{r} = \frac{V - V_1}{f_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\frac{V - V_2}{f_1} = \frac{V_2}{f_2} + \frac{V_2}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\therefore \frac{V_1}{f_1} + \frac{V_1}{f_2} + \frac{V_1}{r} - \frac{V}{f_2} = 0 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{and } \frac{V_2}{f_1} + \frac{V_2}{f_2} + \frac{V_2}{r} - \frac{V}{f_1} = 0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The above equations will give f_1 and f_2 individually if they are required. Generally the total fault resistance only is required, and in a two-wire system this is given by—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

Hence, adding equations 3 and 4 and making the above substitution, we have—

$$\frac{V_1 + V_2}{F} + \frac{V_1 + V_2}{r} = \frac{V}{F}$$

$$\therefore\ F = r\left(\frac{V}{V_1 + V_2} - 1\right)$$

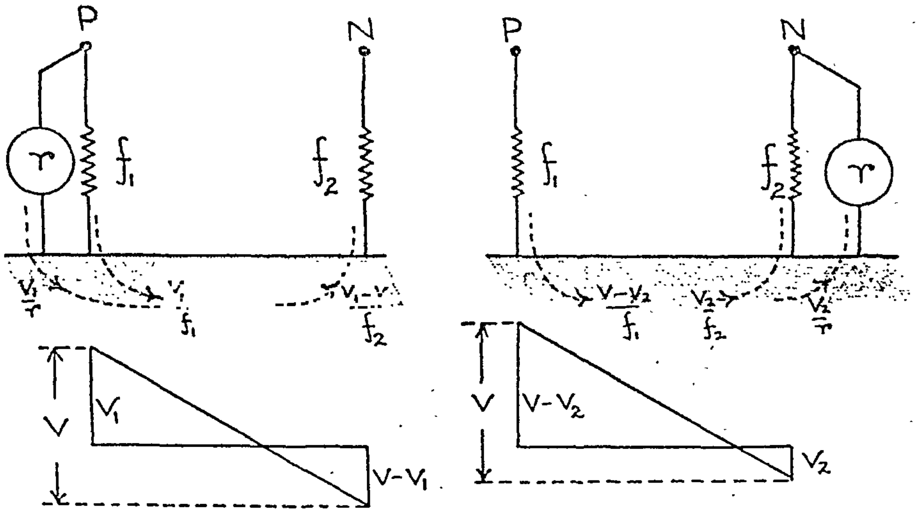


FIG. 3.1.—FAULT RESISTANCE OF A TWO-WIRE SYSTEM.

If an ammeter instead of a voltmeter is used, and I_1 and I_2 are the readings, we have from the above—

$$F = \left(\frac{V}{\frac{V_1}{r} + \frac{V_2}{r}} - r\right)$$

$$= \frac{V}{I_1 + I_2} - r$$

Example.—A voltmeter of resistance 9,000 ohms gave the following readings on a 220-volt, two-wire system : between positive main and earth 130 volts, between negative main and earth 45 volts. Calculate the individual and total fault resistances. If the negative main is earthed through an ammeter what will be the reading?

$$F = 9000\left(\frac{220}{130+45} - 1\right)$$
$$= 9000\left(\frac{220}{175} - 1\right)$$
$$= 2340 \text{ ohms}$$

again from equation 3:

$$\frac{V_1}{F} + \frac{V_1}{r} = \frac{V}{f_2}$$
$$\therefore \frac{130}{2320} + \frac{130}{9000} = \frac{220}{f_2}$$

giving $f_2 = 3220$ ohms

and from equation 4:

$$\frac{V_2}{F} + \frac{V_2}{r} = \frac{V}{f_1}$$
$$\therefore \frac{45}{2320} + \frac{45}{9000} = \frac{220}{f_1}$$
$$\therefore f_1 = 9020$$

If the negative main is earthed the resistance f_2 is short-circuited, thereby bringing the potential of this main to that of the earth. Consequently the positive main will be at a potential of $V = 220$ volts above earth, and the earth current flowing will be—

$$\frac{V}{f_1} = \frac{220}{9020} = \cdot 0244 \text{ ampere}$$

Three-wire System

Many methods have been devised for the determination of the fault resistance of a three-wire system while working, and three will be described. The first two necessitate the temporary removal of the earth connections of the middle wire, but the third method dispenses with this.

Method 1

A polarised voltmeter is connected, first between the positive main and earth, and second between the neutral and earth, the earth connection of the middle wire being temporarily removed. Let V_1 and V_2 be the readings, and 2V be the system voltage

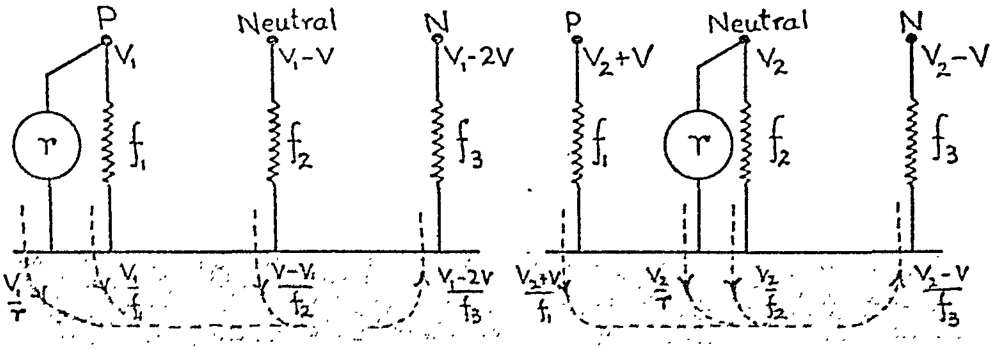


FIG. 3.2.—FAULT RESISTANCE OF A THREE-WIRE SYSTEM.

maintained between the outers. In each test the algebraic sum of the earth currents must be zero, and therefore by reference to Fig. 3.2 we have—

$$\frac{V_1}{f_1} + \frac{V_1}{r} + \frac{V_1 - V}{f_2} + \frac{V_1 - 2V}{f_3} = 0 \quad . \quad . \quad . \quad (1)$$

and

$$\frac{V_2 + V}{f_1} + \frac{V_2}{f_2} + \frac{V_2}{r} + \frac{V_2 - V}{f_3} = 0 \quad . \quad . \quad . \quad (2)$$

substituting

$$\frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{F}$$

we have

$$\frac{V_1}{F} + \frac{V_1}{r} - \frac{V}{f_2} - \frac{2V}{f_3} = 0 \quad . \quad . \quad . \quad (3)$$

and

$$\frac{V_2}{F} + \frac{V}{f_1} + \frac{V_2}{r} - \frac{V}{f_3} = 0 \quad . \quad . \quad . \quad (4)$$

subtracting 4 from 3:

$$\frac{V_1 - V_2}{F} + \frac{V_1 - V_2}{r} - \frac{V}{F} = 0$$

$$\text{giving } F = r\left(\frac{V}{V_1 - V_2} - 1\right)$$

If an ammeter with series resistance is used in place of the voltmeter and its readings are I_1 and I_2, then

$$V_1 = I_1 r \text{ and } V_2 = I_2 r$$

$$\text{giving } F = \left(\frac{V}{I_1 - I_2} - r\right)$$

Example.—The middle wire of a three-wire D.C. system is earthed through a resistance of 10 ohms in series with an ammeter.

On the positive side there is a fault to earth of 20 ohms, and on the negative side of 18. If 200 volts is maintained between the middle wire and each of the outers, calculate the potentials of the positive, negative, and middle wires, and the ammeter reading (C. and G.).

$$f_1 = 20\,;\; f_2 = \infty\,;\; f_3 = 18\,;\; r = 10$$

$$\therefore \quad \frac{V_2 + 200}{20} + \frac{V_2}{10} + \frac{V_2 - 200}{18} = 0$$

which reduces to $V_2 = 5{\cdot}4$ volts.

$\therefore$ Voltage of + outer above earth $= 205{\cdot}4$ volts.

Voltage of − outer below earth $= 205{\cdot}4 - 2 \times 200$

$= -\ 194{\cdot}6$ volts.

ammeter current $= \dfrac{V_2}{10}$

$= {\cdot}54$ amp.

Method 2 (Russell's Test)

The earth connection to the middle wire is temporarily removed as before, but measurements are only made on the middle wire.

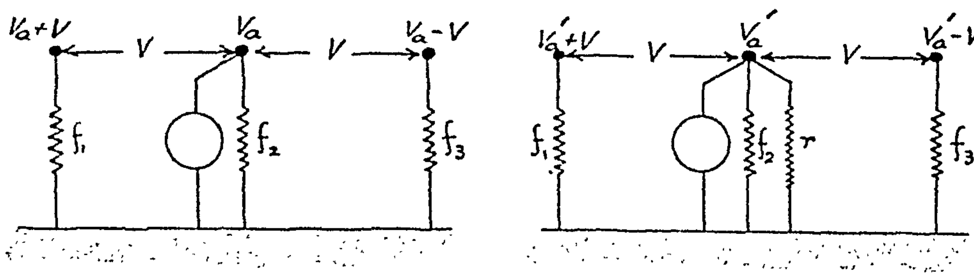


FIG. 3.3.—FAULT RESISTANCE OF A THREE-WIRE SYSTEM. RUSSELL'S TEST.

Firstly, an electrostatic voltmeter is connected between the middle wire and earth and the reading taken: let it be V_a. Secondly, a resistance r is connected between the middle wire and earth. This changes the potential distribution of the whole system with respect to earth, and the voltmeter will give a different reading V'. The potentials of the system during the two tests are as shown in Fig. 3.3, and equating the algebraic sum of the earth currents to zero, as before, we have—

$$\frac{V_a + V}{f_1} + \frac{V_a}{f_2} + \frac{V_a - V}{f_3} = 0 \quad . \; . \; . \quad (1)$$

$$\frac{V'_a + V}{f_1} + \frac{V'_a}{f_2} + \frac{V'_a}{r} + \frac{V'_a - V}{f_3} = 0 \quad . \; . \; . \quad (2)$$

$$\frac{V_a}{F} + \frac{V}{f_1} - \frac{V}{f_3} = 0 \quad . \; . \; . \quad (3)$$

$$\frac{V'_a}{F} + \frac{V}{f_1} - \frac{V}{f_3} + \frac{V'_a}{r} = 0 \quad . \; . \; . \quad (4)$$

Subtracting equation (4) from (3)

$$\frac{V_a - V'_a}{F} - \frac{V'_a}{r} = 0$$

which reduces to

$$F = r\left(\frac{V_a}{V_a} - 1\right)$$

Note that the electrostatic voltmeter takes no current and therefore has no influence on the potentials in the system.

Method 3

The previously described tests are carried out to determine the total insulation resistance of the system. If the determination is made systematically and a gradual lowering of the insulation resistance is indicated, one method of localising the fault is to disconnect different parts of the system in turn and repeat the test until the faulty section is found. The Kapp-Coales test largely obviates this as it gives the values of the individual fault resistances. The P.D.s on the positive side and on the negative side are alternately raised and depressed keeping the total voltage constant. Thus if the normal system voltage is 2 × 220, the amended values for the first test will be, say, 230 on the positive and 210 on the negative, and for the second test 210 and 230 respectively. The middle wire is earthed through a resistance r.

Let the system voltage between outers be

$E_1 + E_2$ in the first test
and $E'_1 + E'_2$ in the second test.

Let e and e' be the P.D.s of the middle wire with respect to earth in the two tests, then working in terms of conductances instead of resistances we can write:

$$g_1 = \frac{1}{f_1}; \; g_2 = \frac{1}{f_2}; \; g_0 = \frac{1}{f_0}$$

the suffixes 1, 2, and 0 now referring to the two outers and the middle wire respectively. Hence for the whole system :

$$g = g_0 + g_1 + g_2$$

also let $$y = \frac{1}{r} \text{ and } k = g + y.$$

$$\therefore (E_1 + e)g_1 + e(g_0 + y) - (E_2 - e)g_2 = 0$$

$$\therefore \quad E_1 g_1 + e(g_0 + g_1 + g_2 + y) - E_2 g_2 = 0$$

$$\therefore \quad E_1 g_1 - E_2 g_2 + ek = 0 \quad \ldots \quad (1)$$

similarly $$E_1' g_1 - E_2' g_2 - e'k = 0 \quad \ldots \quad (2)$$

subtracting,

$$g_1(E_1 - E_1) - g_2(E_2 - E_2') + k(e - e') = 0$$

But $$E_2 - E_2' = E_1' - E_1$$

$$\therefore g_1 + g_2 = \frac{e - e'}{E_1' - E_1} \cdot k \qquad (3)$$

Now $g_0 = g - (g_1 + g_2)$

and $k = g + y$

$$\therefore g_0 = g - \frac{e - e'}{E_1' - E_1}(g + y) \quad \ldots \quad (4)$$

But g is known from the Russell or a similar test and hence g_0 can be calculated from equation (4).

Also, combining equations (1) and (3)

$$g_1 = (g + y)\left(\frac{E_2}{2E} \cdot \frac{e - e'}{E_1' - E_1} - \frac{e}{2E}\right)$$

$$g_2 = (g + y)\left(\frac{E_1}{2E} \cdot \frac{e - e'}{E_1' - E_1} + \frac{e}{2E}\right)$$

where $$2E = E_1 + E_2 = E_1' + E_2'$$

Polarity of a Fault

If the leakage on any side becomes so great as to constitute a fault, the side on which the fault exists, i.e. the polarity of the fault, can be determined by what is called a flash test. This is carried out as follows, Fig. 3.4. A selector switch S_1 is arranged so that connection can be made to any of the three bus-bars, and a knife-switch S_2 can connect the bar so chosen to earth through an adjustable resistance and a fuse. The usual earth connection of the middle wire is temporarily removed, and the three bus-bars are flashed in turn by closing S_2 for a moment and then opening it, at the same time watching the ammeter A in series with the

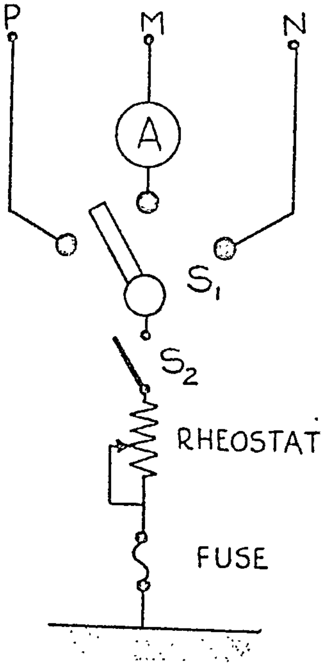


FIG. 3.4.—CONNECTIONS FOR "FLASH" TEST.

middle wire. Suppose the positive is flashed and there is a change in the current, then the fault will be on the middle wire. If the positive is flashed and there is no change in the current, then the fault will be on the positive side, since the closing of S_2 will make no difference if the potential of this side is already down to earth or thereabouts. Similarly for the negative side. In some cases, depending on the nature of the fault, the current may be very unsteady, in which case a period of reasonably steady current has to be waited for and the test repeated several times.

Localisation of Faults

The faults most likely to occur in cable installations are breakdowns of the insulation, leading to:

(*a*) a fault to earth or to the protective metallic sheathing,
(*b*) a cross or short-circuit between two cores.
or (*c*) an open circuit.

Murray Loop Test

This is perhaps the best-known test for locating a fault to earth. The faulty cable is "looped" to a sound cable by means of a connection made at the far end. The two ends A and B at which the test is made are then joined by the two ratio arms of a Wheatstone's bridge, and a galvanometer with the usual key is also connected across AB. The arrangement is shown in Fig. 3.5. The junction C of the ratio arms P and Q is connected to a battery via a switch, and the other pole of the battery is connected to earth, thus being in electrical contact with the fault to earth at F. The equivalent bridge network is given in the figure and when a balance is obtained:

$$\frac{\text{Resist. of cable between A and F}}{\text{Resist. of cables AD and DF}} = \frac{P}{Q}$$

First assume that the sound cable looped to the faulty cable is of the same cross-section, so that each has the same resistance r

per unit length, then if x is the distance of the fault from A and l is the length of each cable, we have:

$$\frac{rx}{r(2l - x)} = \frac{P}{Q}$$

$$\therefore \frac{x}{2l - x} = \frac{P}{Q}$$

$$\therefore \frac{x}{2l} = \frac{P}{P + Q}$$

Hence, knowing l the distance x can be calculated. If the sound cable has a cross-section a' and the faulty cable a cross section a, we have—

$$\frac{rx}{r(l - x) + r'l} = \frac{P}{Q}$$

or

$$\frac{x}{l - x + \frac{r'}{r} \cdot l} = \frac{P}{Q}$$

$$\text{But } \frac{r'}{r} = \frac{a}{a'}$$

$$\therefore \frac{x}{l - x + \frac{a}{a'} l} = \frac{P}{Q}$$

or

$$\frac{x}{l\left(1 + \frac{a}{a'}\right) - x} = \frac{P}{Q}$$

$$\therefore \frac{x}{l\left(1 + \frac{a}{a'}\right)} = \frac{P}{P + Q}$$

Example.—A faulty cable 600 yards long has 7/15 cores. It is looped to a sound 19/16 cable of the same length and the ratio P/Q is equal to 1·5. Calculate the position of the fault.

Resistance per yard of faulty cable $r = \cdot 872 \times 10^{-3}$ ohm
Resistance per yard of sound cable $r' = \cdot 407 \times 10^{-3}$ ohm

$\therefore$ Equivalent length of sound cable $l' = \frac{\cdot 407}{\cdot 872} \times l$
$= \cdot 468 l$

$$\therefore \frac{x}{l - x + l'} = \frac{P}{Q}$$

$$\frac{x}{l + l'} = \frac{P}{P + Q}$$

$$\therefore \quad \frac{x}{600 \times 1{\cdot}468} = \frac{1}{2{\cdot}5}$$

$$x = \frac{600 \times 1{\cdot}468}{2{\cdot}5}$$

$$= 354 \text{ yards.}$$

It will be seen from the connection diagram that the resistance of the fault is in series with the battery, and therefore the only effect of such resistance (provided it is not too high) is to reduce the sensitiveness of the test. This can be improved by adding more cells. If the resistance of the fault is so high as to be comparable with the insulation resistance of the cable, the leakage current through the insulation will be comparable with that through the fault, and the distance x as calculated from the ratio P/Q for balance will not give the correct location of the fault. Actually, in such a case, it is generally possible to leave matters until the fault develops.

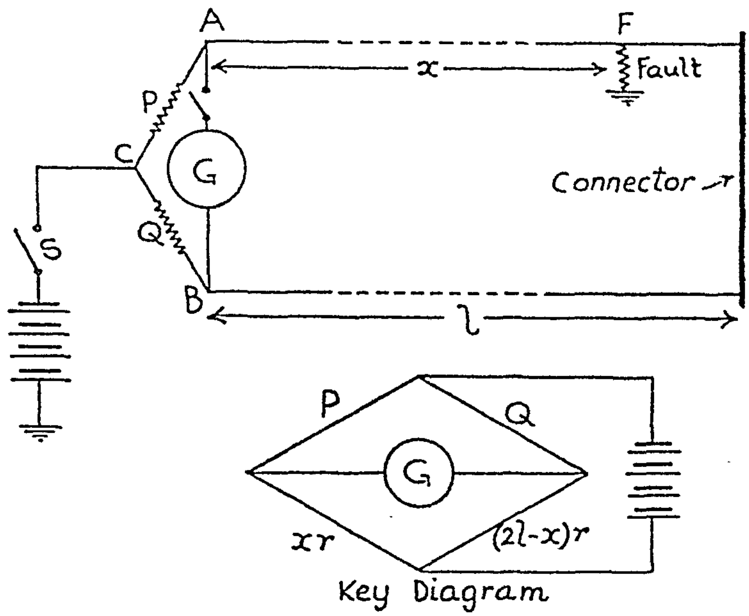


FIG. 3.5.—MURRAY LOOP TEST.

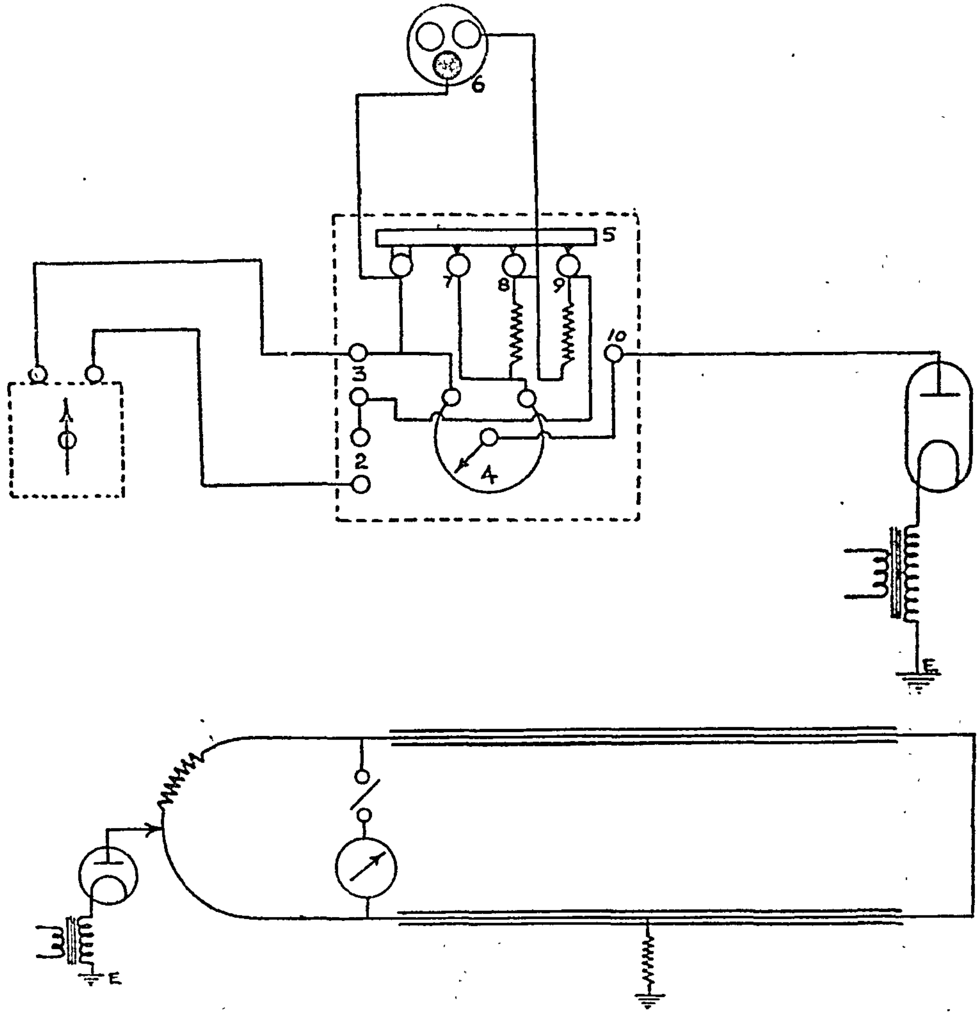
The Murray loop test can be modified so as to locate a fault occurring between two cores in the same cable. One of the faulty cores is now made to take the place of the earth return in the previous test, and the other is looped to a sound cable. The theory of the method is the same as for a fault to earth. The resistance of the fault does not affect the method, as before, unless it is very high.

With high-voltage cables a fault which is too serious to be left to develop may still be of too high a resistance for location by the original loop test. Again, the large amount of oil present in the insulation of such cables may cause a fault to seal up when the working voltage is removed, the resistance of the fault when the cable is dead being thus very much greater than the resistance when the fault is energised. Hence, when locating the fault it is necessary to apply a high voltage to the bridge network, this voltage being usually derived from a high-voltage transformer and rectifying valve (Kenotron). The connections for such a test are shown in Fig. 3.6, from which it will be seen that there is no change in the method except the provision of the high voltage in place of the low voltage from a battery. The spark-gap is used to protect the galvanometer. The voltage applied is read by the voltmeter, and the current through the fault by the milliammeter. The voltage is controlled so that the current is not too great. The procedure, as given by Urmston, is as follows: "Pressure is . . . applied, and when the breakdown occurs the fault current is limited to 3 or 4 ma., and a preliminary balance made. The current is then adjusted to 40 or 50 ma., and the balance readjusted."

When the fault is broken down a comparatively low voltage will pass the required current, but if the fault clears itself there may be a sudden dangerous rise of voltage due to the voltage regulation of the apparatus. For this reason the ratio arms, which may be in the form of a slide wire, must be highly insulated in order to safeguard the operator. When the fault breaks down the cable suddenly discharges, and the spark-gap, which is set to about 0·01 inch, is provided to take this discharge. Without this precaution spark-over might occur somewhere in the galvanometer circuit, owing to its high inductance.

Fault-localising Bridges

Such bridges have been devised to avoid the necessity for any calculation, that is to say, they are direct reading. The best known is the Raphael Bridge, illustrated in Fig. 3.7. It consists of two slide wires each two feet long, and two sliders. One of these, P, makes contact with only one wire, but the other, S, makes contact with both. The slider P is the usual type of contact maker used on slide-wire bridges, but S is a connector which can be clamped in any desired position, its function being to make the length of wire actually utilised a convenient multiple of the length of the loop. The slider P is then moved until a balance is



1. Galvanometer.
2. Contact Key.
3. Resistance Bridge.
4. Slide-wire Resistance.
5. Spark-gaps
6. Faulty Core of Cable.
7. 100 Scale.
8. 200 Scale.
9. 500 Scale.
10. D.C. High-tension Supply.

FIG. 3.6.—LOOP TEST AS APPLIED TO A HIGH-VOLTAGE CABLE.

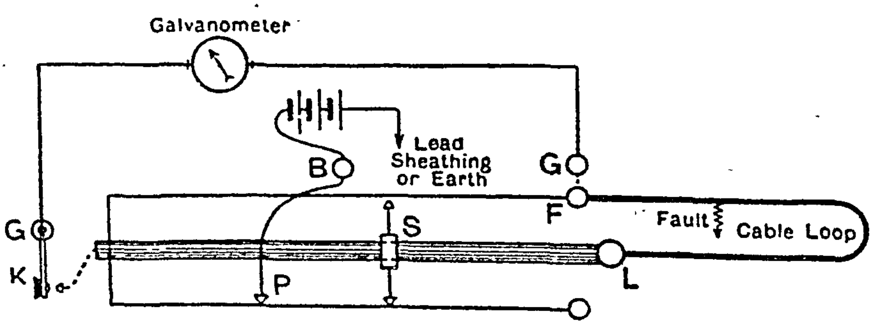


FIG. 3.7.—RAPHAEL FAULT-LOCALISING BRIDGE (MUIRHEAD).

obtained, and the scale reading corresponding to the position of P gives directly the distance of the fault from the testing end. The relationship at balance is the same as for the ordinary loop test, viz.

$$\frac{x}{2l - x} = \frac{P}{Q}$$

$$\therefore \quad \frac{x}{2l} = \frac{P}{P + Q}$$

But (P + Q) is the length of the slide wire, while $2l$ is the length of the loop, so that if the length (P + Q) in scale divisions is made

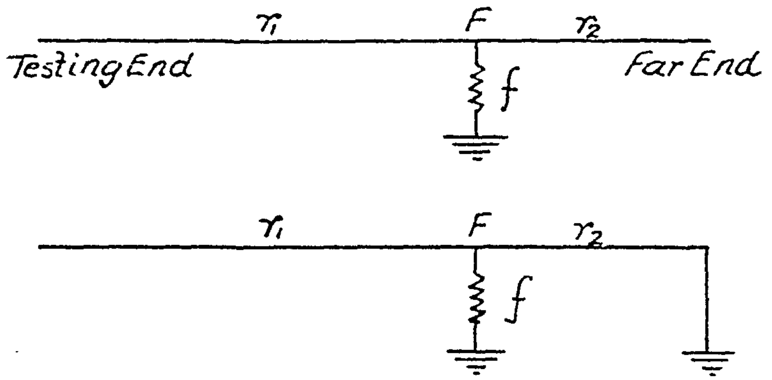


FIG. 3.8.

equal to the $2l$ in yards, the position of P on the scale will give the distance x in yards.

Blavier and Earth Overlap Tests

It is not always possible to find a sound cable with which to complete a loop, and in such a case an entirely different method must be adopted. In the Blavier test the resistance between line and earth is measured (*a*) with the far end insulated, (*b*) with the far end connected to earth. The conditions in the two tests are illustrated in Fig. 3.8. Denoting the measured values of the resistance by R_1 and R_2 we have—

$$R_1 = r_1 + f \quad \ldots \text{(1)}$$

$$R_2 = r_1 + \frac{r_2 f}{r_2 + f} \quad \ldots \text{(2)}$$

Substituting $r_2 = r - r_1$

and eliminating f by the relationship

$$f = R_1 - r_1$$

we have, from equation (2),

$$R_2 = r_1 + \frac{(r - r_1)(R_1 - r_1)}{r - r_1 + R_1 - r_1}$$

$$\therefore \quad R_2 = r_1 + \frac{rR_1 - rr_1 - R_1r_1 + r_1^2}{r + R_1 - 2r_1}$$

which reduces to

$$r_1^2 - 2R_2r_1 + (R_2r + R_2R_1 - rR_1) = 0$$

$$\therefore \quad r_1 = \frac{2R_2 \pm \sqrt{4R_2^2 - 4R_2r - 4R_2R_1 + 4rR_1}}{2}$$

$$= R_2 \pm \sqrt{(R_2 - R_1)(R_2 - r)}$$

Since r_1 is obviously less than R_2 the negative sign is taken giving—

$$r_1 = R_2 - \sqrt{(R_2 - R_1)(R_2 - r)}$$

In the "earth overlap" test two measurements are made: (*a*) as (*b*) in the Blavier test; (*b*) as (*a*) but with the measurement made at the far end and the previous testing end earthed. The value of r_1 is then given by—

$$r_1 = R_1\left(\frac{r - R_2}{R_1 - R_2}\right)\left\{1 - \sqrt{\frac{R_2(r - R_1)}{R_1(r - R_2)}}\right\}$$

Voltage-drop Tests

These tests, like the loop tests, require a sound cable running parallel to the faulty cable, but the function is now to provide a return path for a steady current of sufficient magnitude to cause

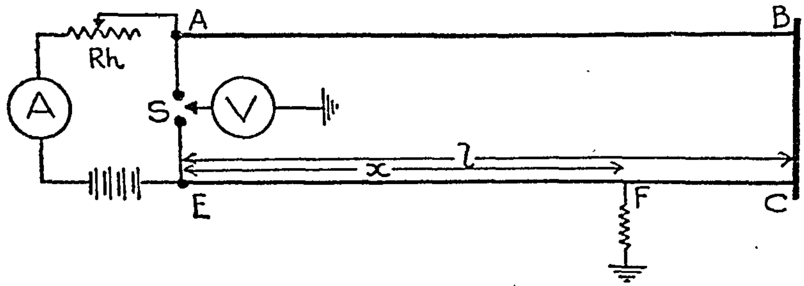


FIG. 3.9.—VOLTAGE-DROP TEST.

a measurable voltage drop along the faulty cable. Consider the circuit shown in Fig. 3.9; by means of the switch S, the voltmeter V can be connected between E and earth or A and earth. The current is adjusted by means of the rheostat R*h* to give suitable voltmeter readings, the magnitude of the current being kept constant while the two readings are made. Let the two readings be V_1 and V_2: then since, for a given current, the drop of volts along a circuit is proportional to the resistance, we have—

$$V_1 \propto \text{Resistance of EF}$$
$$V_2 \propto \text{Resistance of AB} + \text{CF}$$

$$\therefore \frac{V_1}{V_2} = \frac{x}{2l - x}$$

or

$$\frac{V_1}{V_1 + V_2} = \frac{x}{2l}$$

If the sound cable is of different cross-section from the faulty one, or one or both of the cables is stepped, then a correction must

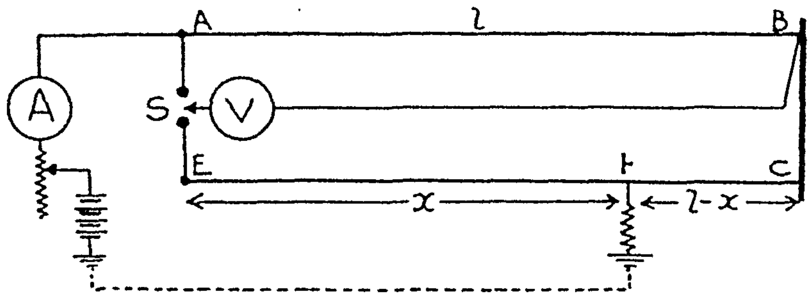


FIG. 3.10.—VOLTAGE-DROP TEST.

be made as for the loop test. In this test it is necessary that a high-resistance voltmeter should be used, since the fault resistance is in series with the voltmeter resistance. Also a high-resistance voltmeter will not take any appreciable current, an essential condition in all "drop-of-volts" tests.

The modification indicated in Fig. 3.10 will give better results; and if V_1 and V_2 are the readings taken with the switch making contact with E and A respectively—

$$\frac{V_1}{V_2} = \frac{\text{Drop along FC}}{\text{Drop along AB}}$$
$$= \frac{l - x}{l}$$

In the first test the length EF of faulty cable merely forms one of the leads to the voltmeter.

With any form of drop-of-volts test it is essential to avoid overheating of the cable by the passage of too heavy a current, otherwise the resistance per unit length of the sound and faulty cables may not be known with any degree of accuracy.

Test for an Open Circuit

The usual method of determining the position of a complete disconnection is to measure the electrostatic capacity of the cable, since this capacity is proportional to the length of cable between the testing end and the fault. If the capacity per unit length of

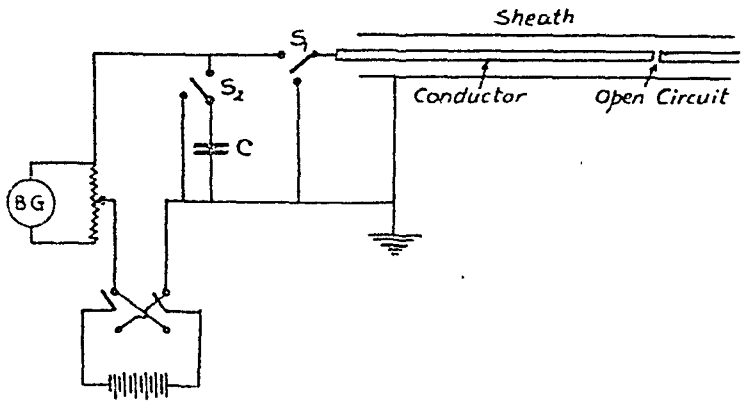


FIG. 3.11.—TEST FOR AN OPEN-CIRCUIT FAULT.

the cable is already known, then the distance is given by the quotient of the measured capacity and the capacity per unit length. If the capacity per unit length is not known, then the capacity must be measured at each end. Fig. 3.11 shows the method of measurement by means of a ballistic galvanometer, in which B.G. is the galvanometer with its shunt, and C a standard condenser. First the cable is charged by closing S_1, S_2 being open, and the galvanometer throw noted: let it be θ_1. Then S_1 is opened and the standard condenser charged by closing S_2: let the throw now be θ_2. Then—

$$\frac{\text{Capacity of cable}}{\text{Capacity of standard}} = \frac{\theta_1}{\theta_2}$$

If the capacity is measured at each end, C_1 and C_2 being the observed values and l the length of cable, then the distance of the fault from the end giving the reading C_1 is—

$$\frac{C_1}{C_1 + C_2} \times l$$

This method is not applicable when the broken core has a comparatively low fault resistance to earth. The following statement due to Urmston is of importance: "Under the conditions that the normal insulation of the core is high compared with the fault resistance, that the length of cable is short, and that the resistance of the conductor to earth is more than 500 ohms, the broken conductor acts practically as a condenser shunted by a resistance. In such a case an A.C. impedance bridge can be used directly for

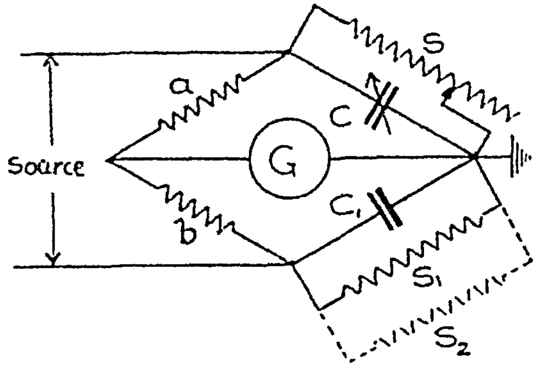


FIG. 3.12.—WIEN BRIDGE CONNECTIONS.

measuring the capacitances C_1 and C_2 (above). The connections generally used are similar to those of the Wien bridge and are shown in Fig. 3.12, where *a* and *b* are non-inductive ratio arms of resistance *a* and *b*, *c* and c_1 are condensers, S is a resistance connected across the terminals of condenser C, S_1 is a resistance connected across the terminals of C_1, and G is a detector. The condition for balance of the bridge is :

$$\frac{a}{b}=\frac{C_1}{C}=\frac{S}{S_1}$$

In applying the bridge in practice the testing current can be taken from A.C. mains or from a thermionic valve oscillator ; *a* and *b* are preferably of the same resistance, C is a variable condenser, the capacitance of which can be varied up to an amount equal to that of the cable to be tested, S is a variable 10,000-ohm non-inductive resistance, C_1 is the capacitance of the faulty conductor, S_1 is the fault resistance, and T is either a vibration galvanometer or a telephone receiver.

As will be seen, by adjustment of C and S perfect balance can be obtained when the fault resistance S_1 is not greater than 10,000 ohms. If the fault resistance S_1 is greater than 10,000 ohms, a second 10,000-ohm resistance S_2 can be connected in parallel with the cable capacitance C_1. This resistance S_2 is shown dotted in the figure.

Induction Method

This method is used for the location of faults to earth, and can only be applied when the cable has no metallic sheathing or

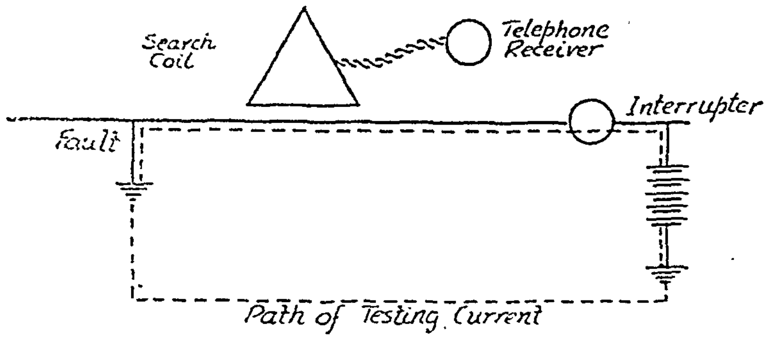


FIG. 3.13.—INDUCTION METHOD OF LOCATING A GROUND FAULT.

armouring. Its application at the present time is thus decidedly limited, but when it can be employed it has the great advantage of indicating the exact position of the fault without the necessity for any calculation, or any assumption as to cable resistance. The cable is supplied with an intermittent current derived from a battery and interrupter, Fig. 3.13, and the route of the cable is explored with a search coil connected to a telephone receiver. This search coil is triangular in form, and Raphael suggests a triangle

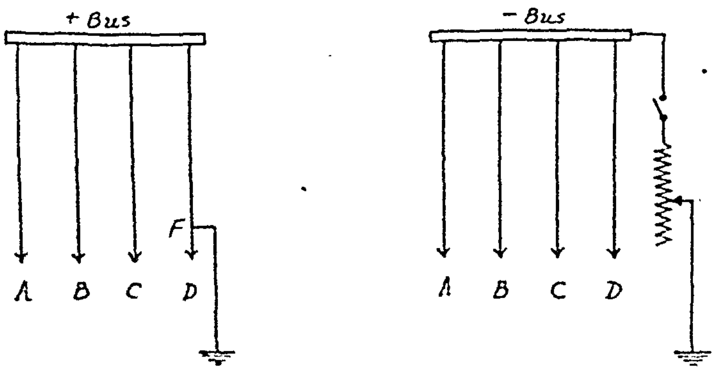


FIG. 3.14.—LOCATION OF FAULTY CABLE BY INDUCTION METHOD.

of 3 ft. side with about two hundred turns of 26 or 30 S.W.G. wire. The coil is held near to the ground with its plane parallel to the run of the cable, as shown in the figure. Until the fault is reached the cable will carry current, and the intermittent nature of the magnetic field set up will induce an intermittent E.M.F. in the coil, which will produce a note in the telephones. As soon as the fault is passed the cable will carry no current and the note will cease.

This method can also be used to locate a faulty cable at the station. Consider, for example, a D.C. two-wire system which is known to have a fault on one of the positive feeders. The search coil is held to each of the positive cables in turn while the negative bus-bar is connected to earth through a bank of lamps: see Fig. 3.14. When the faulty cable is reached a click will be heard in the telephones since the additional current to earth on the closing of the switch will flow through this cable, the sudden increase in current through it therefore inducing a momentary E.M.F. in the search coil.

Board of Trade Regulations

The Board of Trade requires that the earth current of D.C. systems shall be continuously recorded, and for this reason special panels, called Board of Trade panels, have to be included on the switchboard. In the case of a D.C. three-wire system the regulations require that the middle wire shall be earthed if the pressure between middle and either outer exceeds 125 volts. This earth connection has to be made at one point only, viz. a generating station or substation, and the current in the earth circuit must not exceed one-thousandth of the maximum output current of the system.

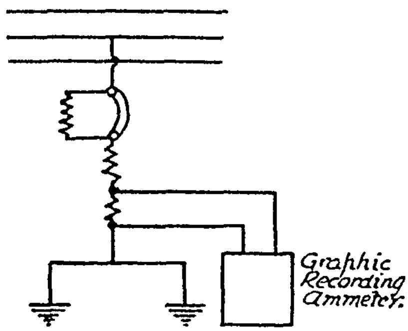


FIG. 3.15.—CONNECTIONS OF BOARD OF TRADE PANEL.

The apparatus on the necessary Board of Trade panel consists of the following: a single-pole circuit-breaker with overload release operating at the maximum allowable earth current; an auxiliary switch arranged to sound an alarm whenever the circuit-breaker opens; a limiting resistance across the contacts of the circuit-breaker of such value that with half voltage impressed across it the earth current is still kept below the maximum limit; a graphic recording ammeter. The connections are shown in Fig. 3.15, from which it will be seen that two earth plates are provided. This is to enable the contact resistances to be checked.

With traction systems the arrangement shown in Fig. 3.16 is adopted. Normally the lower double-throw knife switch is kept in the down position, thus enabling the recording ammeter RA to keep a constant record of the earth current, the maximum allowable value of which is 2 amps. per mile of track, or 5 per cent. of the normal system load, whichever may be the less. A second requirement is that the earth contacts shall be maintained in such

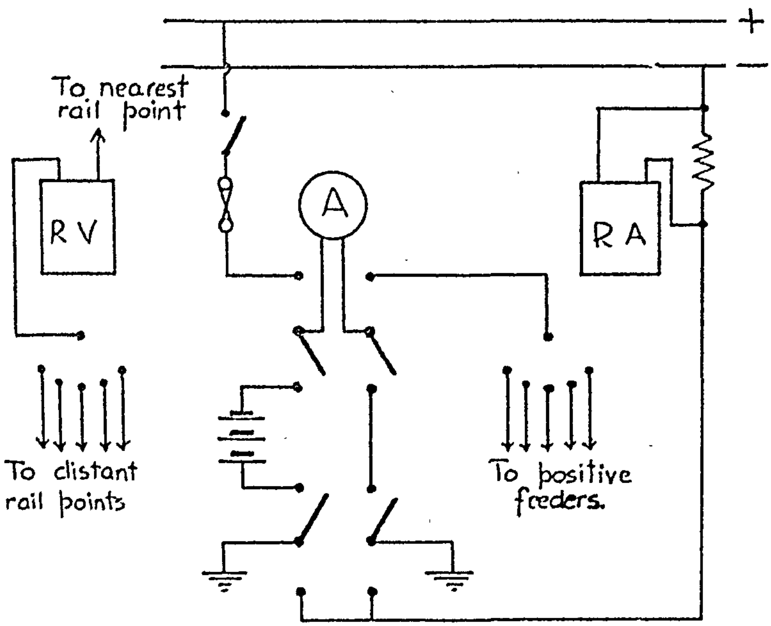


FIG. 3.16.—CONNECTIONS OF BOARD OF TRADE PANEL (TRACTION SYSTEM).

a condition that with 4 volts across them at least 2 amps. shall pass between the plates. This is checked by throwing the upper knife switch to the bottom position and the lower switch to the top position.

It is also specified that the leakage current from any insulated feeder shall not exceed 0·01 amp. per mile of single track. To test this the selector switch on the right is connected to each positive feeder in turn. By opening a feeder circuit-breaker, putting the selector switch on the corresponding contact and closing the upper knife-switch on the top contacts the indicating ammeter A gives a direct reading of the current leaking from the positive bus-bar, via that feeder and back to the earthed negative bus-bar.

Finally it is required that the conductivity of the rail return shall be such that the drop between any two rail points shall not exceed 7 volts. The recording voltmeter RV is connected between a feeding point on the track through the left-hand selector switch to pilot wires run back to the station from the extreme ends of every line track.

Fault Resistance of an A.C. System

A neat method of determining experimentally the fault resistance of a three-phase, four-wire system has recently been devised by Carr and Shackleton. Denoting the fault resistances of the line conductors by f_1, f_2, and f_3, and of the neutral by f_N, we have for the total fault resistance—

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \frac{1}{f_N}$$

Let the *phase* voltages be E_1, E_2, E_3, with angular displacements of zero, φ_2, and φ_3 respectively, and let the capacitances of the four conductors be C_1, C_2, C_3, and C_N respectively. "In an insulated system the voltage to earth of the several conductors is determined by the currents flowing through the leakage resistances and capacitances. If the neutral point is now solidly earthed (i.e. through a negligibly low impedance) the magnitude I and the phase φ_N of the current returning through the earth connection are similarly dependent on these characteristics. The component of this current in phase with E_1 is—

$$\begin{aligned} I \cos \varphi_N = {} & \frac{E_1}{f_1} + \frac{E_2}{f_2} \cos \varphi_2 + \frac{E_3}{f_3} \cos \varphi_3 \\ & + E_1\omega C_1 \sin 0^\circ + E_2\omega C_2 \sin \varphi_2 + E_3\omega C_3 \sin \varphi_3 \end{aligned}$$

If now the voltage of the neutral point be raised above earth by an amount E in phase with E_1, the in-phase component of the modified earth return current I′ will be given by :—

$$\begin{aligned} I' \cos \varphi'_N = {} & \frac{E_1}{f_1} + \frac{E_2}{f_2} \cos \varphi_2 + \frac{E_3}{f_3} \cos \varphi_3 + E_1\omega C_1 \sin 0^\circ \\ & + E_2\omega C_2 \sin \varphi_2 + E_3\omega C_3 \sin \varphi_3 \\ & + \frac{E}{F} + E\omega(C_1 + C_2 + C_3 + C_N) \sin 0^\circ \end{aligned}$$

Since $\sin 0^\circ = 0$, it follows that—

$$I' \cos \varphi'_N - I \cos \varphi_N = \frac{E}{F}$$

$$\text{or } F = \frac{E}{I' \cos \varphi'_N - I \cos \varphi_N}$$

The in-phase current of the neutral earth current may be simply determined by means of a wattmeter having the voltage coil energised from a supply in phase with, and proportional to, E_1. If the voltage conditions of the system are determined, the exact phase and magnitude of I may be ascertained by resolving in a different direction (for example, in phase with E_3). As a check the magnitudes of I and I′ may be determined by means of an ammeter in the earth connection."

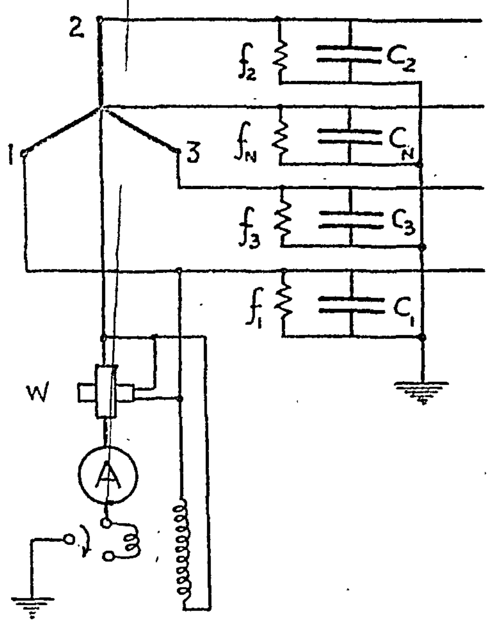


FIG. 3.17.—FAULT RESISTANCE OF AN A.C. SYSTEM.

The connection diagram and circuit conditions are given in Fig. 3.17.

As tests on A.C. systems are more involved than those on D.C. systems owing to the presence of capacitance, the following simple test is often carried out in practice on networks with insulated neutral, or where it is possible temporarily to remove the earth connection. An electrostatic voltmeter is connected between one main and earth, and periodically a resistance is connected between that main and earth, and the value of the resistance adjusted to give some definite lowering of the normal voltage to earth, say 10 per cent. A high resistance indicates that that main has a high fault resistance, and if the resistance became progressively lower it means that the fault resistance is gradually diminishing. The test is made on each phase in succession.

CHAPTER IV

FEEDERS

Kelvin's Law

We have seen that volt drop is not such a vital factor in the design of a feeder as with a distributor, and in consequence a feeder can be designed on the basis of current-carrying capacity and, where practicable, of minimum financial loss.

The financial loss per annum occasioned by conductors of electricity is made up of (*a*) interest on the capital cost of the conductors, plus an allowance for depreciation, (*b*) the cost of the energy wasted in virtue of the ohmic resistance of the conductor, losses in the metallic sheaths of cables, and, in the case of high-voltage insulated cables, losses in the insulating material. For a route of given length the weight, and therefore the cost of the copper, is proportional to the cross-section: hence the annual value of the combined interest and depreciation is also proportional to the cross-section, and can be written as £Pa,

where P = a constant,
a = cross-section of conductor.

The ohmic resistance is proportional to $1/a$, and therefore for a given curve of demand for current throughout the year, i.e. for a given annual load curve, the energy lost in the conductor will be proportional to the resistance, and therefore proportional to $1/a$. The annual value of this energy will, therefore, be proportional to $1/a$ and can be written as £Q/a, where Q is another constant. For a variable load the current used in the calculation of the line I^2R loss will be the R.M.S. value of the current reckoned throughout the year. Neglecting for the moment losses other than ohmic, we have

$$\text{Total annual financial loss} = £\left(Pa + \frac{Q}{a}\right).$$

If plotted against cross-section, a, the graph of the loss Pa is a straight line through the origin, while the graph of the loss Q/a is a rectangular hyperbola, as shown by the curves 1 and 2 in Fig. 4.1. The graph of total loss is given by curve 3, and it exhibits a minimum at that value of a corresponding to the intersection of the two component curves. The above argument has not been concerned with any insulating covering in the case of cables, or of insulators in the case of bare conductors, from which we see that for bare conductors the most economical cross-section is that which makes the annual value of the interest and depreciation on the conductor equal to the annual value of the electrical energy wasted due to the resistance of the conductor. This is known as Kelvin's Law.

It is to be noticed that a feeder cross-section, as calculated from this law, may not always be a practicable one, because it may be too small for the safe carrying of the necessary current.

Example.—If the cost of an overhead line is £600a (where

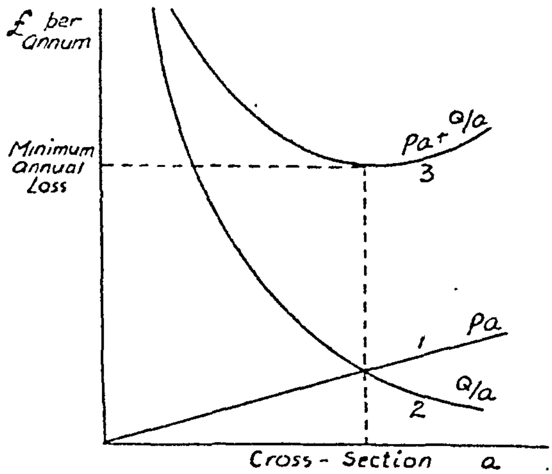


FIG. 4.1.—ILLUSTRATING KELVIN'S LAW.

a is the sectional area in square inches), and if the interest and depreciation charges on the line are 8 per cent., estimate the most economical current-density to use for a transmission requiring full-load current for 60 per cent. of the year. The cost of generating electrical energy is 0·5*d.* per unit. The resistance of a conductor 1 mile long and 1 sq. in. cross-section is 0·043 ohm. (Lond. Univ.)

Consider one mile, and let the full-load current be I, then in a two-conductor line in which each conductor is of resistance R, power lost at full current

$$= 2I^2R$$

$$= 2I^2 \times \cdot 043 \times \frac{1}{a} \text{ watts}$$

$$= \frac{\cdot 086I^2}{a} \times 10^{-3} \text{ kW.}$$

The annual loss of energy is equal to that produced by the above loss taking place continually for 60 per cent. of the year, i.e. for $(\cdot 6 \times 365 \times 24)$ hours. Hence, at £1/480 per unit, annual value of energy lost

$$= £\left(\frac{\cdot 086\mathrm{I}^2}{a} \times 10^{-3} \times \cdot 6 \times 365 \times 24 \times \frac{1}{480}\right)$$

$$= £\cdot 00094\,\frac{\mathrm{I}^2}{a}$$

The annual value of interest and depreciation

$$= 8\,\% \text{ of } 600a = £48a$$

Hence, for minimum total annual loss

$$48a = \cdot 00094\,\frac{\mathrm{I}^2}{a}$$

$$\therefore \text{ Current density } = \frac{\mathrm{I}}{a} = \sqrt{\frac{48}{\cdot 00094}}$$

$$= 226 \text{ amps. per sq. in.}$$

Effect of Insulation

We will now consider the influence of the insulation in the case of a covered cable. For a given type of cable, e.g. with vulcanised indiarubber, or impregnated paper insulation, for a given type of armouring, and for a given working voltage, the cost of the covering does not vary very much with the cross-section of the cores, with the result that as a rough approximation we can regard the covering as adding a constant term to the cost of the cable. The annual value of the interest and depreciation can, therefore, be written £$(\mathrm{P}a + \mathrm{R})$, where R is another constant representing the cost of the insulation. The effect of this on the graph of interest and depreciation is to raise the graph of £Pa vertically through a distance equal to £R. Similarly, the graph of total annual loss, namely, £$(\mathrm{P}a + \mathrm{Q}/a + \mathrm{R})$ is merely the graph of £$(\mathrm{P}a + \mathrm{Q}/a)$ raised vertically through a distance £R, without any horizontal displacement of the point of minimum cost. This is illustrated in Fig. 4.2. It follows from this that the insulation does not have any effect on the value of the cross-section which gives the minimum annual loss, so that for an insulated cable the law can be stated as follows. The most economical cross-section is that which makes the annual value of the interest and depreciation due to the conductor in the cable equal to the annual value of the energy lost.

Example.—A 500-volt, 2-core feeder, two miles long, supplies a maximum current of 200 amps., and the demand is such that the copper loss per annum is such as would be produced by the full current flowing for six months. The resistance of a conductor one mile long of 1 sq. in. cross-section is 0·046 ohm. The cost of the cable including installation is £(6a + 1·2) per yard, and interest and depreciation charges 10 per cent. The cost of energy is 0·5d.

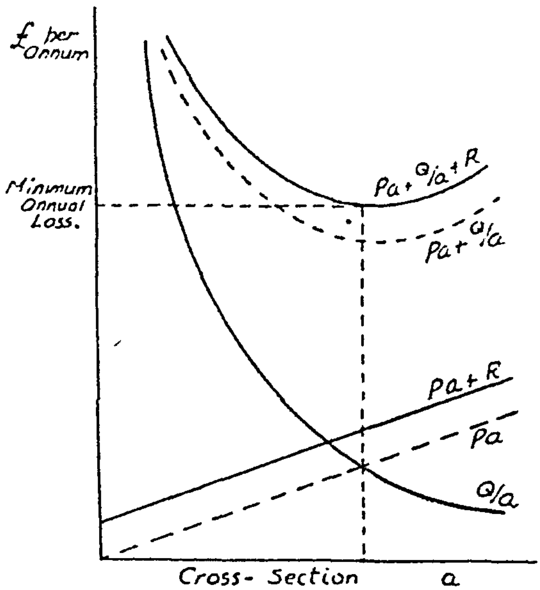


FIG. 4.2.—ILLUSTRATING KELVIN'S LAW.

per unit. Find the most economical cross-section and plot curve of component and total annual losses.

For one mile, cost of conductors in cable

$$= £6a \times 1760$$
$$= £10560a$$

∴ Interest and depreciation

$$= £1056a \text{ per annum.}$$

Power lost per two cores per mile, with maximum current flowing

$$= 2I^2R$$
$$= 2 \times (200)^2 \times \frac{\cdot 046}{a} \text{ watts}$$

∴ Annual value of energy lost per mile of cable

$$= £\left\{2 \times (200)^2 \times \frac{\cdot 046}{a} \times 10^{-3} \times \frac{365 \times 24}{2} \times \frac{1}{480}\right\}$$

$$= £\frac{33\cdot 58}{a}$$

Hence for the most economical cross-section :

$$1056a = \frac{33\cdot 58}{a}$$

$$a = \sqrt{\frac{33\cdot 58}{1056}} = \cdot 178 \text{ sq. in.}$$

From the above figures we have for the values of the three constants P = 1056, Q = 33·58, and R = 211·2 for each mile of cable, so that the curves are as shown in Fig. 4.3.

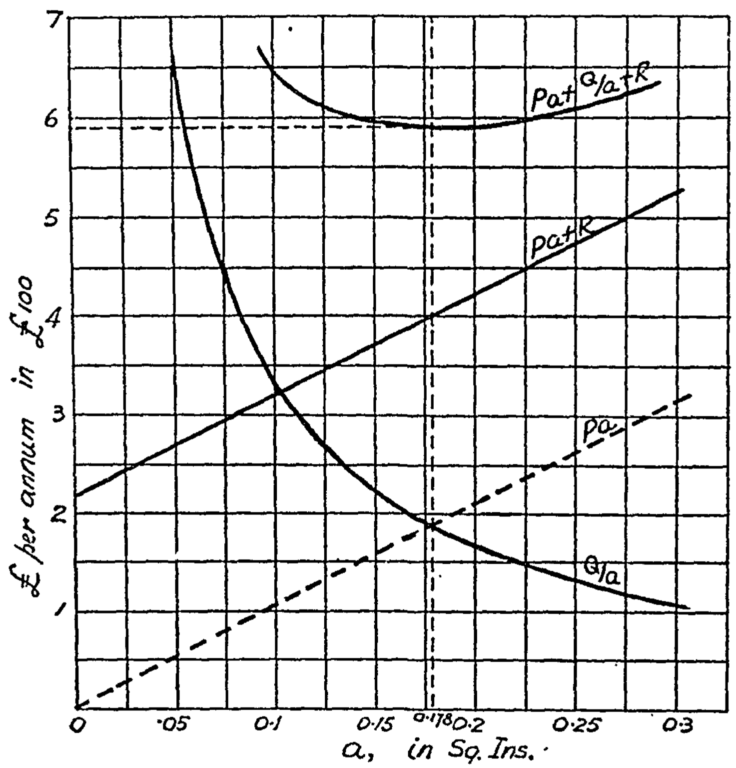


FIG. 4.3.—NUMERICAL EXAMPLE.

The determination of the R.M.S. current may present some difficulty and in practice it is usually estimated as follows. Let I_{max} be the maximum value of the load current and f_2 the annual load factor, then the average value of the current is given by the product $f_2 I_{max}$. In order to convert this to the R.M.S. current it is necessary to multiply by another constant f_f called the "form factor," not to be confused with the form factor concerned with the wave shape of an alternating quantity. This form factor is related to the load factor in the manner represented by the graph of Fig. 4.4. Alternatively the graph of load factor of losses *v.* system load factor can be used, the load factor of the losses being the factor by which the maximum line loss must be multiplied in order to obtain the average loss.

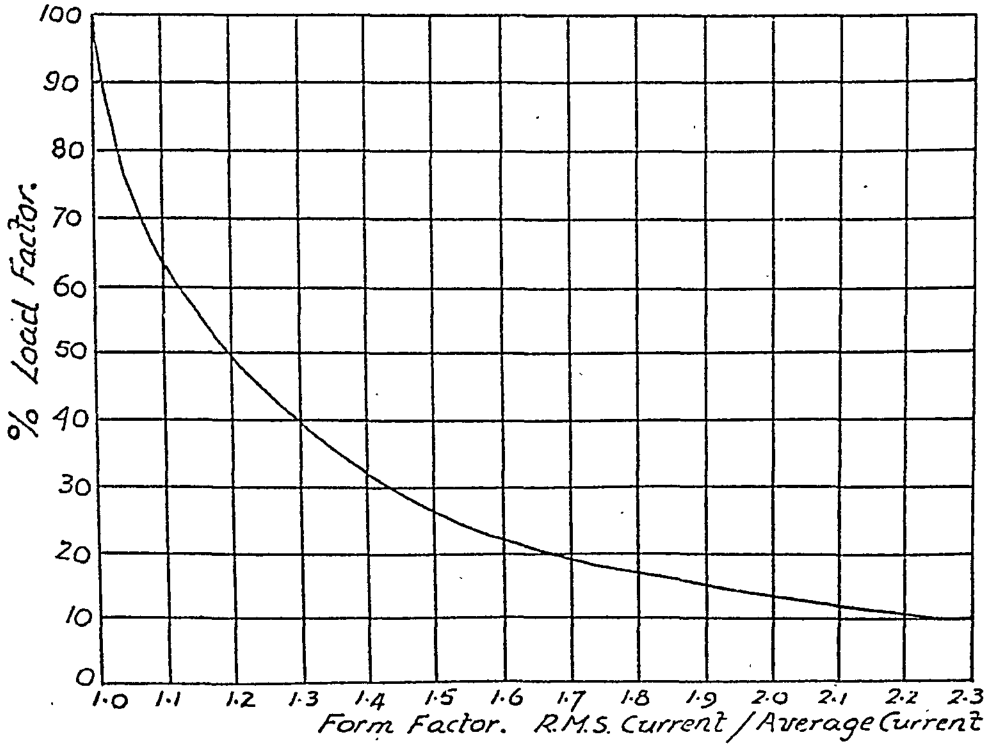


FIG. 4.4.—GRAPH FOR THE DETERMINATION OF LOAD FACTOR OF LOSSES.

The following alternative method is convenient: the empirical law is based on experiments carried out on the British Grid system. If f is the load factor, then the empirical law is—

$$\text{Loss load factor} = 0{\cdot}2f + 0{\cdot}8f^2$$

The loss load factor is also given by—

$$\text{Loss load factor} = \int I^2 R\,dt / (I_m^2 R \times 8760)$$
$$= \text{Total loss per annum} / (I_m^2 R \times 8760)$$
$$\therefore \text{Total losses per annum} = I_m^2 R \times 8760 \times \text{loss load factor} \quad \text{Wh.}$$
$$= 8{\cdot}76\, I_m^2 R \times \text{loss load factor} \quad \text{kWh.}$$

I_m is the maximum designed current for the line in question and 8760 is, of course, the number of hours per annum.

Example.—Assuming the empirical relation between the load factor and the loss load factor to be such that loss load factor $= 0{\cdot}2f + 0{\cdot}8f^2$, where f is the load factor, estimate the most economical cross-sectional area of a feeder required to supply a three-phase load which has a maximum demand of 750 kW.; the total energy required to be supplied per annum is $2{\cdot}4 \times 10^6$ kWh. at unity power factor. The three-core, 11,000-volt cable may be assumed to cost £$(400 + 6000a)$ per 1,000 yards, where a is the area of each core in square inches. The cost of energy wasted is 0·8*d.* per kWh.; interest and depreciation is 6 per cent. per annum. The specific resistance of copper at the working temperature is 0·7 microhm-inch.

$$\text{Load factor } f = 2{\cdot}4 \times 10^6/(750 \times 8760) = 0{\cdot}36$$
$$\therefore \text{Loss load factor} = 0{\cdot}2 \times 0{\cdot}36 + 0{\cdot}8 \times 0{\cdot}36^2$$
$$= 0{\cdot}18$$
$$\therefore \int I^2R\,dt \text{ over one year} = 0{\cdot}18 \times 8{\cdot}76 \times I_{max}{}^2R$$
$$I_{max} = 750{,}000/(\sqrt{3} \times 11{,}000) = 39{\cdot}4 \text{ amps.}$$
$$R = 0{\cdot}7 \times 10^{-6} \times 36 \times 10^3/a$$
$$= 25{\cdot}2 \times 10^{-3}/a$$
$$\therefore \text{kWh. loss per 1,000 yds.} = 0{\cdot}18 \times (25{\cdot}2 \times 10^{-3}/a) \times 39{\cdot}4^2 \times 8{\cdot}76$$
$$= 61{\cdot}3/a = \text{kWh. per annum per core.}$$
$$\text{Cost of cable per 1,000 yds.} = £(400 + 6000a)$$
$$\text{Capital charge per annum} = 0{\cdot}06\,(400 + 6000a)$$
$$= £(24 + 360a)$$

Total cost of losses, reckoning all three cores

$$= 3 \times (61{\cdot}3/a) \times (0{\cdot}8/240)$$
$$= £0{\cdot}613/a$$

∴ Total cost per annum per 1,000 yds.

$$= £(360a + 24 \times 0{\cdot}613/a)$$

Differentiating with respect to a and equating to zero we have, for the most economical section—

$$0 = 360 - 0{\cdot}613a^{-2}$$
$$a^2 = 0{\cdot}613/360$$
$$= 0{\cdot}17 \times 10^{-2}$$
$$\therefore a = 0{\cdot}41 \text{ sq. in.}$$

The probable area would thus be 0·5 sq. in. for each core.

Limitations to Application of Kelvin's Law

In practice Kelvin's Law has to be applied with considerable discretion because it does not always give a conductor size which is suitable. This is inevitable when it is realised that energy costs and interest and depreciation charges have no relation to the purely physical aspects of the problem such as resistance, volt drop, temperature rise, and so on, the method therefore being essentially artificial. Thus if there could be two identical systems having identical demands it follows that from the engineering point of view they should require identical treatment. But if energy costs and interest and depreciation charges were different in the two cases the application of Kelvin's Law would require entirely different cable sizes.

For cable systems the cost of copper is not the only variable, and in addition the current to be carried by any size will be limited by the permissible temperature rise. In fact, with cable systems it is very common for the current density corresponding to the most economical section to be too high. Again, in the case of cables there are sheath losses, and with high voltages, say 20,000 or more, dielectric hysteresis losses, these latter having a load factor of 100 per cent., since they take place continually and independently of load variations. Since the cost per unit decreases as the load factor increases, the cost of generating hysteresis loss is less than the cost of generating the same amount of I^2R loss, and, in fact, it is almost impossible to derive any expression for conductor cross-section when such losses are taken into account. In the case of high-voltage cable systems it is therefore preferable to take a series of cross-sections and work out the total annual loss, due to all causes, for each. A check on the safe current loading and on the voltage drop will then enable the most suitable cross-section to be chosen.

In the case of overhead lines the question of temperature rise is of less importance, and as there are no losses additional to the I^2R loss, and obeying different laws, it follows that the cross-sections derived from Kelvin's Law have more chance of being acceptable. Actually, for a three-phase, 20,000-volt overhead system Kelvin's Law is directly applicable, giving reasonable current densities and voltage drops.

Another limiting factor is the cost per kilowatt-hour of energy lost. This cost is not the same as the cost of generating one kilowatt-hour at the station because generating costs are dependent on the load factor, and the load factor of the I^2R losses is different from the load factor of the load itself. We have also seen that with dielectric loss the load factor of the losses is greater than that of the load.

In the elementary treatment of the application of the law to insulated cables it was assumed that for a given type and voltage, the cost of insulation was independent of the cross-section. This, of course, is only an approximation and for a rigorous solution it would be necessary to take into consideration the capital costs of all portions which vary with the cross-section.

In conclusion it can thus be said that, if there be no special limiting conditions of voltage drop, Kelvin's Law can be applied to overhead lines up to, say, 30,000 volts. In the case of cable systems it is advisable to examine the available cross-sections above and below the calculated section and to ensure that the temperature rise will not be excessive.

In order to illustrate how the differences in the load factor of the various losses can be taken into account a solution is given of the following problem taken from a paper set by the City and Guilds of London Institute. Calculate the cost of transmitting 10,000 kW. by three-phase current at unity power factor for 15 miles (*a*) at 33,000 volts, (*b*) at 44,000 volts, including cost of transforming from a station voltage of 11,000 volts, given: Capital cost of switchgear and transformers, £2 per kVA.; cost per mile of 33,000-volt cable, £0·7 per kVA.; cost per mile of 44,000-volt cable, £0·55 per kVA.; interest and depreciation, 10 per cent.; load factor of supply, 50 per cent.; load factor of copper losses, 37 per cent.; full-load copper loss in transformers, 1 per cent.; no-load loss in transformers, 1 per cent.; maximum current density in cables, 1,000 amps: per sq. in.; specific resistance of conductor, 0·7 microhm per inch cube. Cost of energy, 0·5*d*. per unit at 37 per cent. load factor, and 0·3*d*. at 100 per cent. load factor.

Cost of transformers and switchgear	= £20,000
Cost of 33,000-volt cable = 15 × ·7 × 10,000	= £105,000
Total	= £125,000

∴ Interest and depreciation per annum	= £12,500
No-load losses, 1 per cent.	= 100 kW.

Annual cost of this at 100 per cent. load factor

$$= £100 \times \frac{\cdot 3}{240} \times 365 \times 24 = \quad £1,095$$

Annual cost of transformer losses, at 37 per cent. load factor

$$= £100 \times \frac{\cdot 5}{240} \times 365 \times 24 \times \cdot 37 = £675$$

at 33,000 volts, and unity power factor

$$\text{The max current, I} = \frac{W}{\sqrt{3}E} = \frac{10^7}{\sqrt{3} \times 33000} = 175 \text{ amps.}$$

$$\therefore \text{ at 1,000 amps. per sq. in. } a = \frac{175}{1000} = \cdot 175 \text{ sq. in.}$$

∴ Resist. per mile of single core

$$= \frac{\cdot 7 \times 5280 \times 12 \times 10^{-6}}{\cdot 175} = \cdot 254 \text{ ohm.}$$

∴ Resist. per core—15 miles $= 15 \times \cdot 254 = 3{\cdot}8$ ohms.

∴ Full load copper loss, reckoning all three cores

$$= 3 \times 175^2 \times 3{\cdot}8 \times 10^{-3} = 349 \text{ kW.}$$

At 37 per cent. load factor the annual cost of this loss

$$= £349 \times 365 \times 24 \times \cdot 37 \times \frac{\cdot 5}{240} = £2{,}360.$$

∴ Total cost of copper losses $= 2360 + 675 = £3{,}035$.

Since the load factor of the supply is 50 per cent., the average load is 5,000 kW. and therefore the total number of units supplied per annum

$$= 5000 \times 24 \times 365$$

Taking a straight line law for the relationship between cost per unit and load factor, we have if y is the cost per unit and x the load factor :

$$y = mx + c$$

$$\therefore \cdot 5 = \cdot 37m + c$$

$$\cdot 3 = m + c$$

$$\therefore \cdot 2 = - \cdot 63m$$

$$\therefore m = - \frac{\cdot 2}{\cdot 63} = - \cdot 318$$

$$\therefore c = \cdot 3 - m = \cdot 618$$

Hence when $x = \cdot 5$

$$y = - \cdot 318 \times \cdot 5 + \cdot 618 = \cdot 46$$

∴ Cost of load per annum

$$= £5000 \times 24 \times 365 \times \frac{\cdot 46}{240} = £84{,}000$$

Hence, total annual cost at 33,000 volts is as follows:

Interest and depreciation	12,500
No-load loss	1,095
Total copper losses	3,035
Supply	84,000
Total	£100,630

and similarly for 44,000 volts.

Working Voltage

The working voltage depends, of course, on the distance of transmission. We have already stated the rough rule of 1,000 volts per mile, but it is to be noted that in practice the voltage per mile varies from about 600 to 1,500. The following table forms an approximate guide in the case of overhead transmissions:

Distance.	Kilovolts per mile.
Up to 10 miles	1·5 to 1·2
10 to 15 miles	1·2 to 1·0
50 to 75 miles	1·0 to 0·9
75 to 100 miles	0·9 to 0·8
100 to 150 miles	0·8 to 0·7

The volts per mile increase with the amount of power to be transmitted in order that the line loss and line drop may be kept within reasonable limits. The above values can be taken as applying to a base load of 1,000 kVA., and 1 per cent. can be added to the kilovolts per mile for each additional 1,000 kVA.

For overhead lines more than 20 miles long the *Cable Research Handbook* gives the following formula:

$$\text{Approx. economical voltage (kV.)} = 5{\cdot}5\sqrt{\text{miles} + \frac{\text{kVA.}}{150}}$$

For an important undertaking the only satisfactory method of arriving at the most economical voltage is to take two probable standard voltages and work out for each voltage the total annual cost (transmission loss, plus interest and depreciation), as indicated in the previous example, for cables, transformers, and switchgear.

Cables v. Overhead Lines

The primary consideration in the comparison of competitive systems is that of cost, and overhead lines score heavily over insulated cables in this respect. The following figures of capital cost per kW per mile give a general idea of the relative costs, but the ratios naturally vary in individual cases. The basic

kW	200	500	1,250	10,000 to 15,000	30,000
Voltage, kV	6 to 10	6 to 10	10 to 12	50 to 100	100
Overhead	3,500 to 2,000	2,000 to 1,000	1,200 to 600	300 to 150	100
Underground	7,500 to 5,000	4,000 to 2,500	2,200 to 1,500	350 to 200	150

figure of 100 arbitrary units is taken for overhead transmission of 30,000 kW. at 100 kV. It will be seen that with both systems the cost per kW. per mile decreases enormously as the voltage and amount of power are increased, but that in all cases the figures are in favour of overhead transmission, which system shows up to particular advantage (viz. a ratio of about 1 : 2) for low voltages and powers. It is largely for these reasons that the overhead system is used not only for high-voltage transmission of bulk supply, but also low-voltage distribution to small towns and in rural districts. In large cities overhead lines are not suitable, and buried cables are the rule, in spite of their much greater cost.

An additional advantage of the overhead system for distributors is that tappings can be made at any time without any disturbance, and, as a result, the cost of the service mains is very small. This is of great importance in an area which is developing rapidly. For feeders, which are not likely to be disturbed for tapping purposes, buried cables are advantageous in that they are less liable to breakdown through lightning, or storms, or even wilful damage, and they thus offer a safer guarantee of continuity of supply. To offset this, any repairs which have to be made necessitate trenching and expensive jointing.

Where voltage regulation in A.C. systems is of importance the cable scores over the overhead line, since the very small spacing of the conductors gives a very low inductive drop. With low- or medium-voltage systems the greater charging current of the cable is of no consequence.

CHAPTER V

LINE CONSTANTS

Conductors

FOR overhead transmission by bare conductors the only metals used for short or moderate spans are hard-drawn copper and aluminium. For long spans copper-clad steel or steel-cored aluminium are used, the justification for these constructions being that they enable the use of long spans without increase in tower height because of their high breaking loads.

So far as the combined properties of conductivity and weight are concerned, aluminium has an advantage over copper, as is shown by the following table, but the chief objection to aluminium is its low tensile strength and high coefficient of expansion. In the smaller sizes aluminium presents a comparatively large surface, so that wind pressure is of greater importance than with copper. This together with the low tensile strength requires that sags shall be calculated very carefully, so as to avoid danger of over-straining. The great advantage of hard-drawn copper is its high mechanical strength: it should have a minimum ultimate tensile strength of 55,000 lb. per sq. in., which is more than twice the value for aluminium.

Aluminium is light compared with copper, its specific gravity being 2·71 as against 8·89, and it has an electrical conductivity of 60 per cent. that of copper. Hence for equal conductance an aluminium conductor has 1·66 times the cross-section of copper, but its weight is only one-half that of the copper conductor.

The comparison between the two materials is summarised in the table on page 104, in which the values in the copper column refer to hard-drawn copper, i.e. copper which will give before fracture an elongation of 2 per cent. in a length of 10 inches.

Resistance

The ohmic resistance of a uniform conductor of length l and cross-section a is given by the expression

$$R = \frac{\rho l}{a}$$

Property.	Aluminium.	Copper.
Electrical conductivity (silver = 100)	58·5	97·5
Specific gravity	2·71	8·89
Tensile strength, lb. per sq. in.	26,000–33,000	55,700–62,500
Elastic limit, per cent. of tensile strength	70 per cent.	75 per cent.
Resistance of a conductor 1,000 yds. long, 1 sq. in. cross-section at 60° F.	Soft ·03932 ohm Hard ·04070 ohm	·02398 ohm ·02442 ohm
Temp. coeff. at 0° C.	·00434	·00427
Wt. per 1,000 yds., 1 sq. in. cross-section	3,520 lb.	11,700
Ratio of conductivities for equal area	·6	1·0
Ratio of areas for equal resistance	1·66	1·0
Ratio of diameters for equal resistance	1·29	1·0
Ratio of weights for equal area	1·0	3·3
Ratio of weights for equal resistance	1·0	2·0
Coeff. linear expansion per ° C.	·0000245	·0000170
Coeff. linear expansion per ° F.	·0000136	·0000095

where the value of ρ depends upon the temperature of the conductor. If ρ_0 and ρ_1 are the values of ρ corresponding to two different temperatures θ_0 and θ_1 then

$$\rho_1 = \rho_0\{1 + \alpha(\theta_1 - \theta_0)\}$$

where α is the temperature coefficient of the material. The value of α is not a constant but depends upon the initial temperature θ_0. Thus at 0°C. the values of α for copper and aluminium are ·00427 and ·00434 respectively. If these values of α are used in resistance calculations, then the resistance must first of all be referred to zero temperature; for example, if R_0, R_1, and R_2 are the resistances of a given conductor at 0°C., θ_1°C. and θ_2°C. respectively, then

$$R_1 = R_0(1 + \alpha_0\theta_1)$$
$$R_2 = R_0(1 + \alpha_0\theta_2)$$
$$\therefore \quad R_2 = R_1 \times \frac{1 + \alpha_0\theta_2}{1 + \alpha_0\theta_1}$$

To avoid this an expression of the type

$$R_2 = R_1\{1 + \alpha_1(\theta_2 - \theta_1)\}$$

can be used provided that the temperature coefficient α_1 has the value corresponding to the initial temperature θ_1. This value is derived from α_0 as follows:

Comparing the above equations for R_2 we see that

$$1 + \alpha_1(\theta_2 - \theta_1) = \frac{1 + \alpha_0\theta_2}{1 + \alpha_0\theta_1}$$

which reduces to

$$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0\theta_1}$$

For example, since $\alpha_0 = \cdot 00427$ for copper, we have for the value of α_1 at an initial temperature of 40°C,

$$\alpha_{40} = \frac{\cdot 00427}{1 + 40 \times \cdot 00427} = \cdot 00364$$

and similarly for any other initial temperature.

Skin Effect

When a conductor is carrying a steady direct current this current will distribute itself uniformly over the whole cross-section. With alternating currents the distribution is not uniform since the current density increases from the middle towards the outer layers. At low frequencies the effect is very small, in fact it is only of importance with high frequencies or with solid conductors of large cross-section. The increase in current density in the outer layers causes a greater I^2R loss for a given current, the effect therefore being similar to that of an increase in ohmic resistance. For frequencies of 50 or less the increase in effective resistance is inappreciable for solid copper conductors up to 0·5 in. diameter, about 2·5 per cent. for 0·75 in. diameter, and 8 per cent. for 1 in. diameter. In an aluminium cable the effect is the same as in a copper cable of equal conductivity. Thus since the resistivity of copper is 0·6 times that of aluminium, the increased resistance due to skin effect on an aluminium cable of *a* sq. in. cross-section will be the same percentage as on a 0·6*a* sq. in. copper cable.

The skin effect is much smaller with stranded than with solid conductors. It increases with the permeability and cross-section of the conductor, and also with the frequency.

Inductance

Consider first of all a single-phase overhead line consisting of two parallel conductors whose spacing *d*, in metres, is large compared with the conductor radius *r*, also in metres. Each conductor, if it carried current alone, would set up a magnetic field whose lines of force would be concentric circles having their centre at the centre of the conductor, and lying in planes perpendicular to the length of the conductor. Some of the lines of force would be inside the conductor, and the others outside.

Consider first of all the flux inside the conductor. The current inside a line of force of radius x (Fig. 5.1) is $I \times x^2/r^2$, where I is the

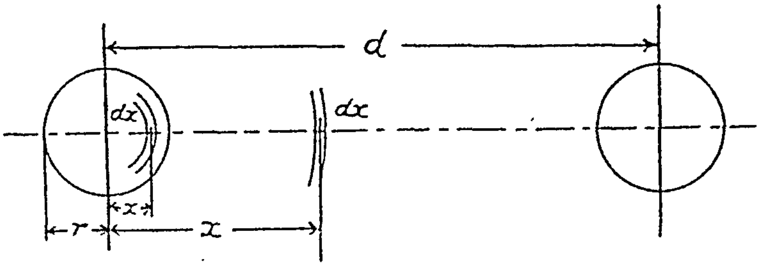


FIG. 5.1.—INDUCTANCE OF AN OVERHEAD LINE.

total current in amperes. Hence the field strength inside the conductor at a distance x from the centre is:

$$H_x = \frac{\text{current}}{2\pi \times \text{distance}} = I \times \frac{x^2}{r^2} \times \frac{1}{2\pi x}$$

∴ Flux density

$$B_x = \mu_0 H_x = \mu_0 Ix/2\pi r^2 = \frac{Ix}{2\pi r^2}$$

Hence the flux through a cylindrical shell of radial thickness dx and axial length 1 metre is:

$$\mu_0 Ix/2\pi r^2 \times 1 \times dx = (\mu_0 I/2\pi r^2)x dx.$$

But this flux links with only x^2/r^2 of the conductor, so that the linkage of the shell is:

$$(\mu_0 I/2\pi r^2) \times (x^2/r^2) \times x dx = (\mu_0 I/2\pi r^4)x^3 dx$$

∴ Total linkage inside the conductor

$$(\mu_0 I/2\pi r^4)\int_0^r x^3 dx = \mu_0 I/8\pi$$

Now consider the external flux; we need only consider that portion lying between the two conductors since none of the other lines of force link with the loop formed by the two conductors and do not, in consequence, contribute to the self-induced E.M.F. At any distance x, the field strength due to one conductor alone is:

$$H_x = I/2\pi x$$
$$\therefore B_x = \mu_0 I/2\pi x$$

Hence the flux through a cylindrical shell of thickness dx and axial length 1 metre is $(\mu_0 I/2\pi)dx/x$. Hence linkage

$$= \mu_0 I/2\pi \int_r^d dx/x = (\mu_0 I/2\pi) \log_e d/r$$

The total linkage per conductor, reckoning both flux inside and flux outside the conductor, is thus given by:

$$\mu_o I/8\pi + (\mu_o I/2\pi)\log_e\frac{d}{r} = (\mu_o I/4\pi)(\tfrac{1}{2} + 2\log_e d/r)$$

The total linkage for both conductors is therefore:

$$\mu_o I/4\pi(1 + 4\log_e d/r)$$

Hence the linkage per ampere is:

$$(\mu_o/4\pi) \times (1 + 4\log_e d/r)$$

$$\text{Putting } \mu_o = 4\pi \times 10^{-7}$$

$$\text{Linkage per amp.} = (1 + 4\log_e d/r) \times 10^{-7}$$

Now for any circuit the coefficient of self-induction is given by the expression:

$$L = (\text{linkage per ampere}) \quad \text{henries}$$

In the case under consideration we therefore have:

$$L = (1 + 4\log_e d/r) \times 10^{-7} \text{ henries per metre}$$
$$= 4\log_e d/r \times 10^{-7} \text{ approx. for wide spacings.}$$

If the log is reduced to the base 10, and the inductance expressed in the more practicable form of henries per mile, we have, finally:

$$L = 14{\cdot}8 \times 10^{-4}\log_{10} d/r$$

Three-phase Line with Symmetrically Spaced Conductors

Imagine the line with three equilaterally spaced conductors to be replaced by a six-conductor line, the six conductors being arranged in three pairs, as shown in Fig. 5.2. Then each individual phase will have two conductors, as indicated, which can be regarded as forming a loop. Let the currents in two of these loops be I_1 and I_2, the actual current I_a flowing in conductor A being the vector difference of I_1 and I_2. Again, the third loop is directly opposite to the conductor A, so that the flux produced by it will not link with A, and will therefore induce no voltage in A. It is thus unnecessary to take the third loop into consideration when calculating the inductance of A, so that the E.M.F. induced in A is due to the currents I_1 in B and I_2 in C. The E.M.F. induced in both conductors of loop 1 is $L\frac{dI_1}{dt}$, where L is the inductance of a pair of conductors. Hence the E.M.F. induced in one conductor only of loop 1, say conductor A, is $\frac{1}{2}L\frac{dI_1}{dt}$. Similarly the E.M F. induced

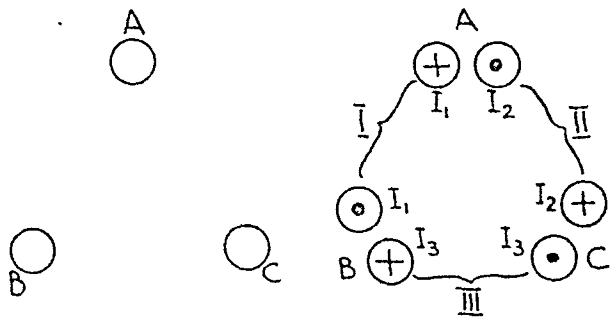


FIG. 5.2.—INDUCTANCE OF A THREE-PHASE LINE.

in conductor A by the current I_2 in loop 2 is $\frac{1}{2}L\frac{dI_2}{dt}$.

∴ Total E.M.F. induced in conductor A

$$= \tfrac{1}{2}L\left(\frac{di_1}{dt} - \frac{di_2}{dt}\right)$$

Now $i_A = i_1 - i_2$,

$$\therefore \frac{di_1}{dt} - \frac{di_2}{dt} = \frac{di_A}{dt}$$

∴ Total E.M.F. induced in conductor A

$$= \tfrac{1}{2}L\frac{di_A}{dt}$$

Hence the equivalent inductance of one conductor of a three-conductor line with equilateral spacing is equal to one-half the inductance of any pair of the conductors taken together.

Three-phase Line with Unsymmetrical Spacing

Let A, B, and C be the conductors and a, b, and c the spacings as shown in Fig. 5.3. Consider the conductor A. The linkage per ampere per metre length of A external to it and due to its own current is:

$$\mu_0/2\pi\int_r^\infty dx/x$$

$$= 2 \times 10^{-7}\int_r^\infty dx/x$$

Linkage with A due to each ampere in conductor B, that is mutually inductive effect of B on A, is:

$$2 \times 10^{-7}\int_c^\infty \frac{dx}{x}$$

since, as shown by the dotted lines, which represent the lines of force set up round B by the current in B, that portion of the flux

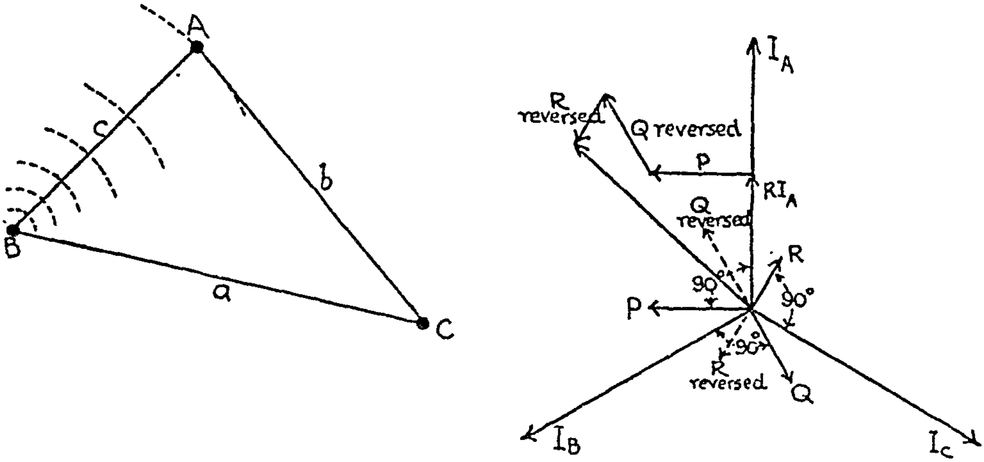


FIG. 5.3.—OVERHEAD LINE WITH UNSYMMETRICAL SPACING.

due to B which crosses the space between the conductors A and B does not link with A.

Similarly, the linkage per metre length of A due to each ampere in conductor C is:

$$2 \times 10^{-7} \int_b^\infty \frac{dx}{x}$$

The total equivalent inductance of A is the sum of its own self-inductance and the mutually inductive effects of its neighbours. Hence, if we denote the instantaneous currents in the three conductors by i_A, i_B, and i_C, we have for the total linkage per metre length of conductor A due to all causes:

$$\left[\left(\tfrac{1}{2} + 2\int_r^\infty \frac{dx}{x}\right) i_A + 2\ i_B \int_c^\infty \frac{dx}{x} + 2\ i_C \int_b^\infty \frac{dx}{x}\right] \times 10^{-7}$$
$$= [(\tfrac{1}{2} - 2 \log_e r)\, i_A - (2 \log_e c) \times i_B$$
$$- (2 \log_e b) \times i_C + 2 \log_e \infty\, (i_A + i_B + i_C)] \times 10^{-7}$$

Now the sum of the instantaneous currents in the three conductors is at every instant zero, whether the system has a current balance or not, and therefore the last term must be zero.

The physical meaning of this is that the total flux surrounding the three conductors is zero.

Hence, considering the other terms only, and remembering that the self and mutually induced E.M.F.s in any conductor are proportional to the rate of change of linkage with respect to time, and therefore to the rate of change of current with respect to time, we have

Voltage induced in each metre length of conductor A

$$=\left\{(\tfrac{1}{2}-2\log_e r)\frac{di_A}{dt}-2\log_e c\cdot\frac{di_B}{dt}-2\log_e b\cdot\frac{di_C}{dt}\right\}\times 10^{-7}$$

The term:

$$\tfrac{1}{2}-2\log_e r\frac{di_A}{dt}$$

leads the vector of the current i_A by 90°; it is represented in Fig. 51 by the vector P. The voltage expressed by the term

$$2\log_e c\cdot\frac{di_B}{dt}$$

leads the current i_B by 90°. It is represented on the diagram by the vector Q, and to take the negative sign into account it is necessary to reverse Q, as shown. Finally, the voltage expressed by

$$2\log_e b\cdot\frac{di_C}{dt}$$

leads the current i_C by 90°, and is represented by the vector R. This vector also has to be reversed. Any of the three currents can be written in the form:

$$i = I_{max}\sin\omega t$$

$$\therefore \frac{di}{dt} = I_{max}\,\omega\cos\omega t$$

$$\therefore \left(\frac{di}{dt}\right)_{max} = I_{max}\,\omega$$

Hence, if I_A, I_B, and I_C are the R.M.S. values of the three currents, the numerical values of the various inductive drops in conductor A will be:

$$(\tfrac{1}{2}-2\log_e r)\times\omega I_A\times 10^{-7}$$

$$2\log_e c\times\omega I_B\times 10^{-7}$$

$$\text{and } 2\log_e b\times\omega I_C\times 10^{-7}$$

In order to illustrate the application of the method we will consider three special cases:

(*a*) *Equilateral spacing*

Then $a = b = c = d$

∴ Voltage induced in each metre length of conductor A

$$e_A=\left\{(\tfrac{1}{2}-2\log_e r)\frac{di_A}{dt} - 2\left(\log_e d.\frac{di_B}{dt}+2\log_e d\cdot\frac{di_C}{dt}\right)\right\}\times 10^{-7}$$

$$=\left\{(\tfrac{1}{2}-2\log_e r)\frac{di_A}{dt}-2\log_e d\left(\frac{di_B}{dt}+\frac{di_C}{dt}\right)\right\}\times 10^{-7}$$

But we have seen that

$$\frac{d}{dt}\left(i_B + i_C\right) = -\frac{di_A}{dt}$$

Hence,

$$e_A = (\tfrac{1}{2} - 2\log_e r + 2\log_e d)\frac{di_A}{dt} \times 10^{-7}$$
$$= \left(\frac{1}{2} + 2\log_e\frac{d}{r}\right) \times 10^{-7} \times \frac{di_A}{dt}$$

Hence, equivalent self-inductance per conductor

$$= \left(\frac{1}{2} + 2\log_e\frac{d}{r}\right) \times 10^{-7}$$
$$= \frac{1}{2}\left(1 + 4\log_e\frac{d}{r}\right) \times 10^{-7} \text{ henries per metre}$$

It is thus one-half the self-inductance of the loop formed by two of the conductors, as proved previously.

(*b*) *The three conductors in the same plane*

This arrangement of conductors is in very common use. Let e_A, e_B, and e_C stand for the E.M.F., induced in each metre length of each of the conductors A, B, and C. Then, with conductor B in the middle and with phase sequence A, B, C, we have—

$$e_A = \left\{(\tfrac{1}{2} - 2\log_e r)\frac{di_A}{dt} - 2\left(\log_e d\cdot\right)\frac{di_B}{dt} - 2\left(\log_e 2d\cdot\right)\frac{di_C}{dt}\right\} \times 10^{-7}$$
$$= \left\{(\tfrac{1}{2} - 2\log_e r)\frac{di_A}{dt} - 2\left(\log_e d\right)\left(\frac{di_B}{dt} + \frac{di_C}{dt}\right) - 2\left(\log_e 2\cdot\right)\frac{di_C}{dt}\right\} \times 10^{-7}$$
$$= \left\{\left(\frac{1}{2} + 2\log_e\frac{d}{r}\right)\frac{di_A}{dt} - 2\log_e 2\cdot\frac{di_C}{dt}\right\} \times 10^{-7}$$
$$= \left\{\left(\frac{1}{2} + 2\log_e\frac{d}{r}\right)\frac{di_A}{dt} - 1{\cdot}39\,\frac{di_C}{dt}\right\} \times 10^{-7}$$

The R.M.S. values of the drops represented by the two terms of this induced voltage are:

$$\left(\frac{1}{2} + 2\log_e\frac{d}{r}\right)I_A\omega \times 10^{-7} \text{ leading } I_A \text{ by } 90^\circ$$

and $1{\cdot}39\ I_C\omega \times 10^{-8}$ lagging 90° behind I_C.

Similarly for conductor c.

$$e_C = \left\{\left(\frac{1}{2} + 2\log_e \frac{d}{r}\right)\frac{di_C}{dt} - 1{\cdot}39\,\frac{di_A}{dt}\right\} \times 10^{-7}$$

the drops due to the two terms being

$$\left(\frac{1}{2} + 2\log_e \frac{d}{r}\right) I_C\omega \times 10^{-7} \text{ leading } I_C \text{ by } 90^\circ$$

and $1{\cdot}39\ I_A\omega \times 10^{-8}$ lagging 90° behind I_A.

Considering now conductor B, we have—

$$e_B = \left\{ (\tfrac{1}{2} - 2\log_e r)\,\frac{di_B}{dt} - 2\log_e d\cdot\frac{di_A}{dt} - 2\log_e d\cdot\frac{di_C}{dt}\right\} \times 10^{-7}$$

$$= \left(\frac{1}{2} + 2\log_e \frac{d}{r}\right)\frac{di_B}{dt} \times 10^{-7} \text{ volts per cm.}$$

The R.M.S. value of the drop due to it is

$$\left(\frac{1}{2} + 2\log_e \frac{d}{r}\right) I_B\omega \times 10^{-7} \text{ leading } I_B \text{ by } 90^\circ.$$

The vector diagrams for these drops in volts are given in Fig. 5.4, the diagrams for each phase being drawn separately so as to avoid confusion. It will be seen that the total drops are not equal,

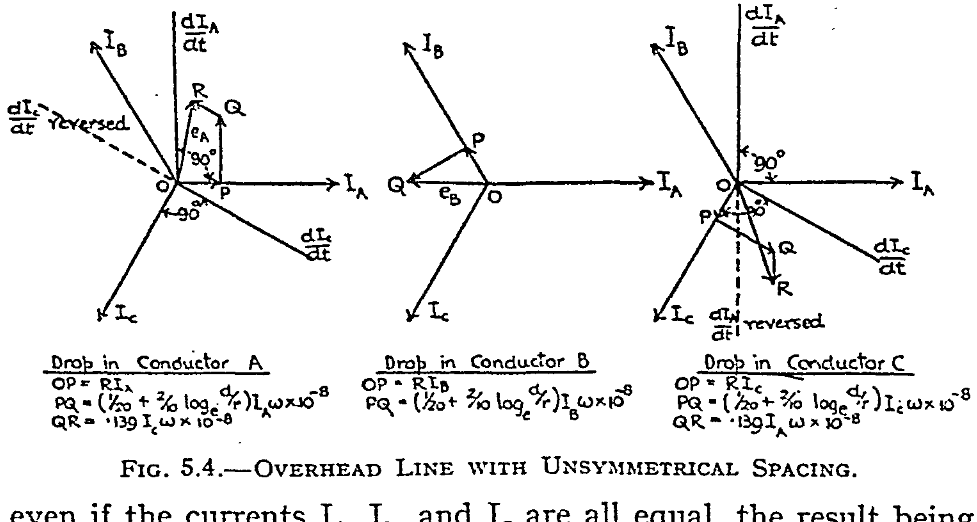


FIG. 5.4.—OVERHEAD LINE WITH UNSYMMETRICAL SPACING.

even if the currents I_A, I_B, and I_C are all equal, the result being that the line voltages at the receiving end will not be equal unless the conductors are frequently transposed. Also the magnitudes and phases of the drops depend upon the sequence of the phases in the three conductors, i.e. whether A, B, C, or A, C, B. The reason for this is that an alteration in phase sequence alters the phase relations of the various components of any of the drops.

(*c*) *Conductors at the corners of a right-angled triangle*

If conductor B is at the right angle we have, using the same notation as before:

$$e_A = \left\{ (\tfrac{1}{2} - 2\log_e r)\frac{di_A}{dt} - 2\log_e d \cdot \frac{di_B}{dt} - 2\log_e \sqrt{2}\,d \cdot \frac{di_C}{dt} \right\} \times 10^{-7}$$

$$= \left\{ \left(\frac{1}{2} + 2\log_e\frac{d}{r}\right)\frac{di_A}{dt} - 2\log_e \sqrt{2}\,\frac{di_C}{dt} \right\} \times 10^{-7}$$

$$= \left\{ \left(\frac{1}{2} + 2\log_e\frac{d}{r}\right)\frac{di_A}{dt} - 6{\cdot}95\,\frac{di_C}{dt} \right\} \times 10^{-7}$$

$$e_B = \left\{ \left(\frac{1}{2} + 2\log_e\frac{d}{r}\right)\frac{di_B}{dt} \right\} \times 10^{-7}$$

$$e_C = \left\{ \left(\frac{1}{2} + 2\log_e\frac{d}{r}\right)\frac{di_C}{dt} - 6{\cdot}95\,\frac{di_A}{dt} \right\} \times 10^{-7}$$

The vector diagrams are therefore similar to those for a line with all three conductors in the same plane, except that the unbalancing of the voltages at the far end of an untransposed line will be smaller.

Equalisation of Reactance Drops by Transposition

Suppose that a line with unsymmetrical spacing is efficiently transposed in the manner indicated in Fig. 5.5: we require the equivalent equilateral spacing which gives the same inductance as the conductor inductances of the transposed line. Let d be the

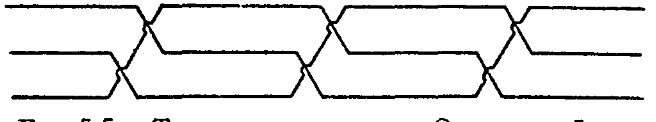


FIG. 5·5.—TRANSPOSITION OF AN OVERHEAD LINE.

spacing for the equivalent line, then for a length of l miles, a more convenient unit of length than the mile for practical purposes

$$L = \tfrac{1}{2} \times 14{\cdot}8 \times 10^{-4} \log_{10}\frac{d}{r} \times l \text{ henries.}$$

But since any given configuration will exist for one-third of the length of the line we also have

$$L = \tfrac{1}{2} \times 14{\cdot}8 \times 10^{-4} \log_{10}\frac{d_1}{r} \times \frac{l}{3} + \tfrac{1}{2} \times 14{\cdot}8 \times 10^{-4} \log_{10}\frac{d_2}{r} \times \frac{l}{3} + \tfrac{1}{2} \times 14{\cdot}8 \times 10^{-4} \log_{10}\frac{d_3}{r} \times \frac{l}{3}$$

$$= \tfrac{1}{2} \times 14{\cdot}8 \times 10^{-4} \times \frac{l}{3}\left(\log_{10}\frac{d_1}{r} + \log_{10}\frac{d_2}{r} + \log_{10}\frac{d_3}{r}\right)$$

Hence, equating the two expressions for L, we have

$$3 \log_{10}\frac{d}{r} = \log_{10}\frac{d_1}{r} + \log_{10}\frac{d_2}{r} + \log_{10}\frac{d_3}{r}$$

$$\log_{10}\left(\frac{d}{r}\right)^3 = \log_{10}\left(\frac{d_1}{r} \times \frac{d_2}{r} \times \frac{d_3}{r}\right)$$

$$\therefore\ d = \sqrt[3]{d_1\, d_2\, d_3}$$

This shows that provided an overhead line be transposed in such a manner that each of the three possible conductor arrangements exists for one-third of the total length of the line, the inductive drop of volts will be equal to that in a line of the same length, having conductors of the same size, and an equilateral spacing of $\sqrt[3]{d_1\, d_2\, d_3}$.

Capacitance

Consider first of all a single-phase line with two parallel conductors. Since the spacing is large compared with the conductor diameter, the surface density of the electrostatic charge on either conductor will be practically unaffected by the charge on the neighbouring conductor, so that the charges will act as though concentrated at the axes.

Let the conductor A, Fig. 5.6, have a charge of $+Q$ farads

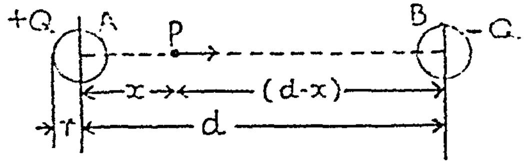


FIG. 5.6.—CAPACITANCE OF AN OVERHEAD LINE.

per metre length, then the conductor B will have a charge of $-Q$ farads per metre length. Hence, Intensity at any point P due to A $= Q/2\pi\varepsilon_0 x$ in the direction of the arrow.

Intensity at same point P due to B $= Q/2\pi\varepsilon_0(d - x)$ in same direction.

Hence total intensity at point P

$$E_x = Q/2\pi\varepsilon_0 x + Q/2\pi\varepsilon_0(d - x)$$

$$\therefore E_x = Q/2\pi\varepsilon_0 \left(\frac{1}{x} + \frac{1}{d - x}\right)$$

Now, the potential difference between the conductors is equal to the work done in moving one coulomb from one conductor to the other against the electrostatic force. If one coulomb is moved a distance dx the work done is obviously $E_x dx$ joules.

$\therefore$ Potential difference :

$$V = \int_r^{d-r} E_x \, dx$$

$$= Q/2\pi\varepsilon_0 \int_r^{d-r} \left(\frac{1}{x} + \frac{1}{d - x}\right) dx$$

$$= Q/2\pi\varepsilon_0 \Big[\log_e x - \log_e (d - x)\Big]_r^{d-r}$$

$$= Q/\pi\varepsilon_0 \log_e \frac{d - r}{r} \text{ volts}$$

Now the electrostatic capacity between the cylinders per metre axial length is given by the equation

$$C = \frac{Q}{V}$$

$$\therefore C = \frac{Q}{(Q/\pi\varepsilon_0) \log_e \frac{d - r}{r}}$$

$$= \pi\varepsilon_0/\log_{10} \frac{d - r}{r} \text{ farads per metre}$$

$$= \pi\varepsilon_0/2{\cdot}303 \log_{10} \frac{d - r}{r}$$

1 mile $= (5280 \times 12 \times 2{\cdot}54) \times 10^{-2}$ metres

$\varepsilon_0 = 8{\cdot}845 \times 10^{-12}$

and these substitutions give

$$C = \frac{1{\cdot}94 \times 10^{-8}}{\log_{10} \frac{d - r}{r}} \text{ farads per mile}$$

Since r is small compared with d it is usually justifiable to replace $(d - r)$ by d in the above expression giving

$$C = \frac{1{\cdot}94 \times 10^{-8}}{\log_{10} \frac{d}{r}} \text{ farads per mile.}$$

Capacitance of Three-phase Line

(*a*) *Equilateral spacing*

Consider two of the conductors, say A and B, in Fig. 5.7. The

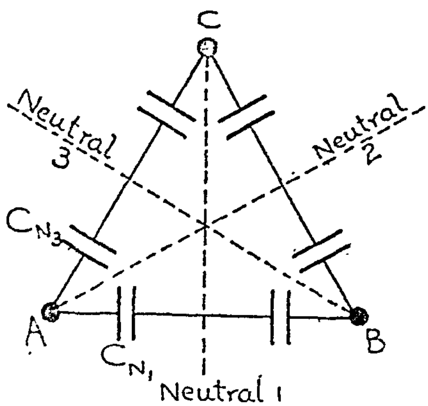


FIG. 5.7.—CAPACITANCE OF A THREE-PHASE LINE.

line-to-line capacity is given by

$$C \doteq \frac{1{\cdot}94 \times 10^{-8}}{\log_{10} \frac{d}{r}}$$

Now this capacity can be regarded as consisting of two line-to-neutral capacities in series, so that for the line-to-neutral capacity of A we have—

$$C_{N1} = \frac{3{\cdot}88 \times 10^{-8}}{\log_{10} \frac{d}{r}} \text{farads per mile.}$$

Similarly, the capacity of conductor A with respect to the neutral plane 3 is given by

$$C_{N3} = \frac{3{\cdot}88 \times 10^{-8}}{\log_{10} \frac{d}{r}} \text{farads per mile.}$$

These two capacities must be added vectorially, the phase angle being 120°, that between the E.M.F.s which act across them. But the resultant of two numerically equal vectors inclined at 120° is numerically equal to either of the vectors, with the result that the line-to-neutral capacitance of any one of the lines is equal to

$$\frac{3{\cdot}88 \times 10^{-8}}{\log_{10}\frac{d}{r}} \text{ farads per mile.}$$

(*b*) *Unsymmetrical spacing*

Using the same notation as for the inductance of a three-phase line with unsymmetrical spacing we have—

$$C = \frac{3{\cdot}88 \times 10^{-8}}{\log_{10}\frac{d}{r}} \times l \text{ farads}$$

$$\text{But } C = 3{\cdot}88 \times 10^{-8} \times \frac{l}{3}\left(\frac{1}{\log_{10}\frac{d_1}{r}} + \frac{1}{\log_{10}\frac{d_2}{r}} + \frac{1}{\log_{10}\frac{d_3}{r}}\right)$$

$$\therefore \frac{1}{\log_{10}\frac{d}{r}} = \frac{1}{3\log_{10}\frac{d_1}{r}} + \frac{1}{3\log_{10}\frac{d_2}{r}} + \frac{1}{3\log_{10}\frac{d_3}{r}}$$

$$\therefore \log_{10}\frac{d}{r} = \frac{3\log_{10}\frac{d_1}{r}\log_{10}\frac{d_2}{r}\log_{10}\frac{d_3}{r}}{\log_{10}\frac{d_1}{r}\cdot\log_{10}\frac{d_2}{r} + \log_{10}\frac{d_2}{r}\cdot\log_{10}\frac{d_3}{r} + \log_{10}\frac{d_3}{r}\cdot\log_{10}\frac{d_1}{r}}$$

$$\simeq \tfrac{1}{3}(\log_{10}d_1 + \log_{10}d_2 + \log_{10}d_3) - \log_{10}r.$$

For equilateral spacing $\log_{10}\frac{d}{r}$ is equal to the above expression,

$$\therefore \log_{10}d \simeq \tfrac{1}{3}(\log_{10}d_1 + \log_{10}d_2 + \log_{10}d_3)$$

$$\therefore d \simeq \sqrt[3]{d_1\, d_2\, d_3}$$

Thus the spacing to be used in the calculation of line capacitance is the same as that for the calculation of line inductance.

Spacing

The spacing of the conductors is decided not only by the electrical consideration of working voltage, but also by such mechanical considerations as the length of the span, weight, and diameter of the conductor (since these decide the amplitude of the swing which may be set up by winds), the strength and direction of prevalent winds, and the amount of sag under the worst conditions. Owing to these varied factors which, naturally, are different for different lines and localities, it is very difficult to give any rule which can be applied without judgment of individual requirements, but the following have been used:

(i) For low-voltage lines using pin-type insulators:

Up to 6,000 volts	2 ft.
6,000 to 10,000 volts	2 ft. 6 in.
10,000 to 20,000 volts	3 ft.

(ii) Spacing (feet) $= \{1 + (\cdot 1 \times \text{line voltage in kV.})\}$

(iii) Spacing (inches) $= \cdot 048a + 4\sqrt{V}$

where a = span length in feet

V = working pressure in kV.

(iv) Spacing (inches) $= 20 + \frac{V}{1\cdot 4} + \left(\frac{a}{120}\right)^2$

a and V having the same meaning as in (iii).

(v) Spacing (metres) $= c\sqrt{d} + \frac{V^2}{20000}$ where d is the sag in metres, and c is taken as 0·75 for copper and steel-cored aluminium and 1·0 for aluminium.§

Usual Values of Inductance and Capacitance

Consider first a low-voltage line with heavy conductors, say a spacing of 2 feet (which is about a minimum) and a conductor diameter of 1 inch:

$$\therefore \frac{d}{r} = \frac{24}{\frac{1}{2}} = 48$$

$$\log_{10}\frac{d}{r} = 1\cdot 6812.$$

$$\therefore L = 14\cdot 8 \times 10^{-4} \times 1\cdot 6812$$

$$= 2\cdot 49 \times 10^{-3} \text{ henries per mile}$$

$$\text{and } C = \frac{1\cdot 94 \times 10^{-8}}{1\cdot 6812}$$

$$= 1\cdot 16 \times 10^{-8} \text{ farads per mile}$$

Now consider a high-voltage line, e.g. a 132 kV. British Grid line with a spacing of 12 feet and a conductor diameter of 0·77 inches:

$$\therefore \frac{d}{r} = \frac{144}{\cdot 385} = 374$$

$$\therefore \log_{10}\frac{d}{r} = 2\cdot 5729$$

$$\therefore L = 14\cdot 8 \times 10^{-4} \times 2\cdot 5729$$

$$= 3\cdot 8 \times 10^{-3} \text{ henries per mile}$$

$$\text{and } C = \frac{1{\cdot}94 \times 10^{-8}}{2{\cdot}5729}$$
$$= {\cdot}755 \times 10^{-8} \text{ farads per mile}$$

It will be seen that in spite of the great difference in the spacings the inductances and capacitances are of the same order in the two cases.

Again, still using the approximate expressions for L and C, we have for the product

$$LC = 1{\cdot}48 \times 10^{-3} \log_{10}\frac{d}{r} \times \frac{1{\cdot}94}{\log_{10}\frac{d}{r}} \times 10^{-8}$$
$$= 2{\cdot}89 \times 10^{-11}$$

This is a constant for every overhead line, no matter what the spacing and conductor diameter may be. The significance of this quantity is explained in Chapter XVIII.

Importance of Capacitance in Overhead Lines

The capacitance of the line may be regarded as due to a uniform series of condensers connected between the conductors, as shown

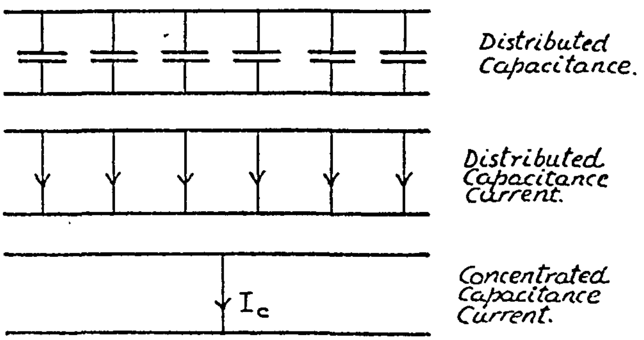


FIG. 5.8.—CAPACITANCE OF AN OVERHEAD LINE.

in Fig. 5.8. Neglecting the progressive drop of volts in the line due to load current, it will be seen that the effect of capacitance is to produce a loading similar to that of a uniformly loaded distributor. The total current I_c due to capacitance may thus be regarded as concentrated at the middle point of the line.

Take the case of a single-phase line with two conductors each of resistance R, then the total drop of volts, E_c, due to I_c is $2RI_c$. Let

$$E = 1000 \text{ volts, 50 cycles}$$
$$d = 24'', \; r = \tfrac{1}{2}''$$
$$l = 1 \text{ mile}$$

$$\text{Then } C = 1{\cdot}16 \times 10^{-8} \text{ farads}$$
$$I_c = EC\omega$$
$$= 1000 \times 1{\cdot}16 \times 10^{-8} \times 314$$
$$= {\cdot}00364$$
$$a = {\cdot}7854 \times (2r)^2 = {\cdot}7854 \text{ sq. in.}$$
$$\therefore R = \frac{{\cdot}046}{a} \times \text{miles}$$
$$= \frac{{\cdot}046}{{\cdot}7854} \times 1 = {\cdot}0586 \text{ ohm}$$
$$\therefore E_c = 2 \times {\cdot}00364 \times {\cdot}0586 = {\cdot}0004 \text{ volt, which is}$$

absolutely negligible. We see from that the effect of capacitance in short lines can be ignored.

Now let

$$E = 132000 \text{ volts, 50 cycles}$$
$$d = 144'', r = {\cdot}385$$
$$l = 100 \text{ miles}$$
$$\text{Then } C = ({\cdot}755 \times 10^{-8}) \times 100$$
$$= {\cdot}755 \times 10^{-6} \text{ farads}$$
$$\therefore I_c = 132000 \times {\cdot}755 \times 10^{-6} \times 314$$
$$= 31{\cdot}4 \text{ amps.}$$
$$a = {\cdot}7854 \times ({\cdot}77)^2 = {\cdot}465 \text{ sq. in.}$$
$$\therefore R = \frac{{\cdot}046}{{\cdot}465} \times 100 = 9{\cdot}9 \text{ ohms.}$$
$$\therefore E_c = 2 \times 31{\cdot}4 \times 9{\cdot}9 = 622 \text{ volts.}$$

This also is small in comparison with E, so that even in a very long high-voltage line the capacitance is not important so far as the numerical value of the line drop is concerned (this is considered in detail in Chapter VII). But suppose that the line is open-circuited at the far end, then in order to maintain the voltage at that end there must be available in the generating station plant of capacity

$$EI_c \times 10^{-3} = 132000 \times 31{\cdot}4 \times 10^{-3}$$
$$= 4150 \text{ kVA.}$$

which will be loaded at zero power factor, leading. On the other hand the power factor of the fully loaded line will most certainly be lagging, and we thus see that owing to the line capacitance there will be large changes in power factor at the generating station with changes in load. The significance of this is discussed in Chapter XIII.

CHAPTER VI

SHORT AND MODERATELY LONG TRANSMISSION LINES

Short Single-phase Line

In the case of a short transmission line the line constants can be regarded as " lumped " instead of distributed and, as we have seen, capacitance can be neglected.

Let V_0 = line voltage at sending end
V = line voltage at receiving end
I = line current
$\cos\varphi$ = power factor
R = resistance per conductor
X = reactance of the loop formed by the two conductors
$= L\omega$

where $L = 14{\cdot}8 \times 10^{-4} \log_{10} \frac{d}{r}$

The equivalent circuit diagram and vector diagram are as shown in Fig. 6.1. Resolving along OX and OY, we have

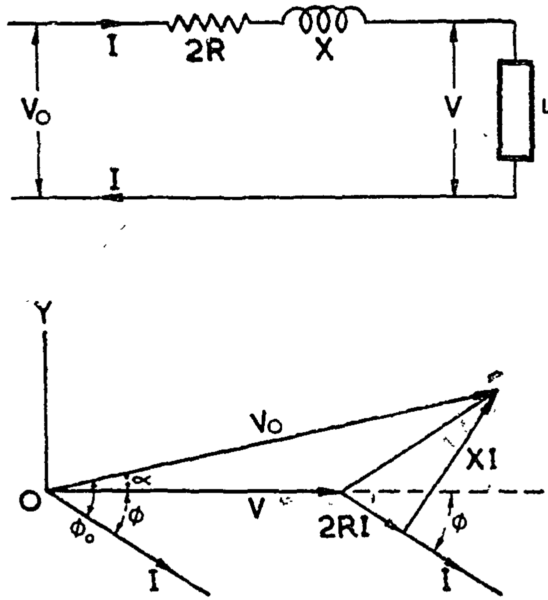


Fig 6.1.—Voltage Regulation of a Short Line.

$$(V_0)_X = V + 2RI\cos\varphi + XI\sin\varphi$$
$$(V_0)_Y = XI\cos\varphi - 2RI\sin\varphi$$
$$\therefore V_0 = [(V + 2RI\cos\varphi + XI\sin\varphi)^2 + (XI\cos\varphi - 2RI\sin\varphi)^2]^{\frac{1}{2}}$$

As a rule the drops 2RI and XI are small in comparison with V and V_0, and therefore $(V_0)_Y$ will be small. Hence, since $(V_0)_Y$ is perpendicular to $(V_0)_X$ we have—

$$V_0 \simeq (V_0)_X \simeq V + 2RI\cos\varphi + XI\sin\varphi$$

Example.—Find the size of bare wire for a single-phase overhead line required to transmit 500 h.p. for 10,000 yds. with a loss of 10 per cent. of the power delivered. Receiving pressure, 5,000 volts; frequency, 50; power factor of load, 0·8. Also determine the distance between the wires so that the reactance voltage of the line shall be 600. Hence find the voltage and power factor at the sending end.

$$\text{Power delivered} = 500 \times 746 = 373000 \text{ watts.}$$
$$\therefore \text{Line loss} = 37300 \text{ watts}$$
$$I = \frac{P}{V\cos\varphi} = \frac{373000}{5000 \times 0{\cdot}8}$$
$$= 93{\cdot}4 \text{ amps.}$$
$$\therefore 2 \times (93{\cdot}4)^2 \times R = 37300$$
$$\therefore R = \frac{37300}{2 \times (93{\cdot}4)^2} = 2{\cdot}14 \text{ ohms per conductor}$$
$$\therefore 2{\cdot}14 = \frac{{\cdot}046}{a} \times \frac{10000}{1760}$$
$$a = \frac{{\cdot}046 \times 10000}{2{\cdot}14 \times 1760}$$
$$= {\cdot}122 \text{ sq. in.}$$
$$\therefore r = {\cdot}2 \text{ inch}$$
$$XI = L\omega I = 600 \text{ volts}$$
$$\therefore L = \frac{600}{93{\cdot}4 \times 314}$$
$$\text{But } L = 14{\cdot}8 \times 10^{-4} \log_{10}\frac{d}{r} \times \frac{10000}{1760}$$
$$\therefore \log_{10}\frac{d}{r} = \frac{600}{93{\cdot}4 \times 314} \times \frac{1760}{14{\cdot}8 \times 10^{-4} \times 10000}$$
$$= 2{\cdot}43$$
$$\therefore \frac{d}{r} = 269{\cdot}2$$
$$\therefore d = 269{\cdot}2 \times {\cdot}2 = 53{\cdot}84, \text{ say, } 54 \text{ inches}$$

$$(V_0)_x = V + 2RI\cos\varphi + XI\sin\varphi$$
$$= 5000 + 2 \times 2{\cdot}14 \times 93{\cdot}4 \times {\cdot}8 + 600 \times {\cdot}6$$
$$= 5680$$
$$(V_0)_Y = XI\cos\varphi - 2RI\sin\varphi$$
$$= 600 \times {\cdot}8 - 2 \times 2{\cdot}14 \times 93{\cdot}4 \times {\cdot}6$$
$$= 240 \text{ volts.}$$
$$\therefore V_0 = \{(5680)^2 + (240)^2\}^{\frac{1}{2}} \simeq 5680 \text{ volts.}$$
$$\tan\alpha = \frac{(V_0)_Y}{(V_0)_X} \text{ (see Fig. 6.1)}$$
$$= \frac{240}{5680} = {\cdot}0423$$
$$\therefore \alpha = 2^\circ\, 25'$$
$$\text{again } \varphi = \text{arc}\cos({\cdot}8) = 36^\circ\, 52'$$
$$\therefore \varphi_0 = \varphi + \alpha = 39^\circ\, 17'$$
$$\therefore \cos\varphi_0 = {\cdot}774$$

Short Three-phase Line

We have seen that the inductance per phase is equal to one-half of the inductance of the loop formed by two conductors, and therefore if we use this value for the inductance and the resistance per single conductor for the resistance, and at the same time work in phase voltages instead of line voltages, then the calculation for a three-phase line is identical to that for a single-phase line.

Example.—A three-phase line delivers 3,000 kW. at 11,000 volts a distance of 10 miles. Line loss 10 per cent. of power delivered, power factor of load 0·8, frequency 50, spacing of conductors 5 feet, equilateral. Calculate the voltage at the sending end.

$$\text{Phase voltage at load end } V_{ph} = \frac{V}{\sqrt{3}} = \frac{11000}{\sqrt{3}} = 6350$$
$$\text{Line current } I = \frac{P}{\sqrt{3}V\cos\varphi} = \frac{3000000}{\sqrt{3} \times 11000 \times {\cdot}8}$$
$$= 197 \text{ amps.}$$
$$\text{Line loss } 3I^2R = 10\% \text{ of } 3000000 \text{ watts.}$$
$$\therefore R = \frac{300000}{3 \times (197)^2} = 2{\cdot}58 \text{ ohms.}$$

This gives a cross-section of 0·164 sq. in. and therefore a radius of 0·228 inch.

$$\therefore \frac{d}{r} = \frac{5 \times 12}{\cdot 228} = 263$$

$$\therefore \log_{10}\frac{d}{r} = 2{\cdot}42$$

$$\begin{aligned}
\therefore L &= 14{\cdot}8 \times 10^{-4} \times 2{\cdot}42 \times 10 \\
&= \cdot 0358 \text{ henry per loop} \\
\therefore L_{ph} &= \cdot 0179 \text{ henry per phase} \\
\therefore X_{ph} &= \cdot 0179 \times 314 = 5{\cdot}63 \text{ ohms.} \\
\therefore V_{0ph} &= E_{ph} + (R_{ph}I \cos \varphi + X_{ph}I \sin \varphi) \\
&= 6350 + (2{\cdot}58 \times \cdot 8 + 5{\cdot}63 \times \cdot 6)197 \\
&= 7{,}430 \text{ volts} \\
\therefore V_0 &= \sqrt{3} \times 7430 \\
&= 12860 \text{ volts}
\end{aligned}$$

Instead of referring all quantities to one phase, as in the above example, we can, alternatively, use the single-phase line which has the same drop of volts. The particulars of this equivalent single-phase line are deduced as follows:

In a single-phase line formed by two conductors, and carrying a current I, the two components of the total drop are

$$\begin{aligned}
\text{Resistance drop} &= 2I_1R \\
\text{Reactance drop} &= I_1L\omega \\
\therefore \text{Total drop} &= I_1\{4R^2 + (L\omega)^2\}^{\frac{1}{2}}
\end{aligned}$$

In a three-phase line of the same length, with the same spacing (equilateral) and the same conductor diameter, the resistance and reactance per phase are R and $\frac{1}{2}L\omega$ respectively, so that the drop per phase with a current I_2 is

$$I_2\{R^2 + \tfrac{1}{4}(L\omega)^2\}^{\frac{1}{2}}$$

$\therefore$ Drop in potential difference between two conductors of the three-phase line carrying current I_2, i.e. the drop in line voltage with current I_2, is

$$\begin{aligned}
&\sqrt{3} \times \text{Drop per phase} \\
&= \sqrt{3}I_2\{R^2 + \tfrac{1}{4}(L\omega)^2\}^{\frac{1}{2}}
\end{aligned}$$

If the drops in the single-phase and three-phase lines are equal, then

$$I_1 \times \{4R^2 + (L\omega)^2\}^{\frac{1}{2}} = \sqrt{3}I_2 \times \{R^2 + \tfrac{1}{4}(L\omega)^2\}^{\frac{1}{2}}$$

$$\therefore \frac{I_1}{I_2} = \frac{\sqrt{3}}{2}$$

If the line pressures and the power factors are the same in the single- and three-phase lines, say V and $\cos\varphi$ respectively, then

$$\text{Power conveyed by single-phase line } P_1 = VI_1 \cos\varphi$$

$$\text{and power conveyed by three-phase line } P_2 = \sqrt{3}VI_2 \cos\varphi$$

$$\therefore \text{Ratio of powers } \frac{P_1}{P_2} = \frac{I_1}{\sqrt{3}I_2} = \frac{1}{2}$$

Hence we have the following rule: The total drop of volts in a balanced three-phase overhead line with symmetrically arranged conductors is the same as that in a single-phase overhead line having conductors of the same spacing and diameter, operating at the same line voltage and the same load power factor, but transmitting only one-half of the power transmitted by the three-phase line.

In order to illustrate the application of this method take the 11,000-volt line of the previous example.

Power conveyed by equivalent single-phase line

$$P_1 = \frac{3000}{2} = 1500 \text{ kW}$$

$$\therefore I_1 = \frac{P_1}{V\cos\varphi} = \frac{1500000}{11000 \times \cdot 8} = 170{\cdot}5 \text{ amps.}$$

$$X = \text{Reactance per loop}$$
$$= 2 \times 5{\cdot}63 = 11{\cdot}26 \text{ ohms}$$

$$\therefore V_0 = V + (2R\cos\varphi + X\sin\varphi)I_1$$
$$= 11000 + (2 \times 2{\cdot}58 \times \cdot 8 + 11{\cdot}26 \times \cdot 6) \times 170{\cdot}5$$
$$= 12860 \text{ volts, as before.}$$

Moderately Long Three-phase Line

For voltages up to, say, 20,000, we have seen that the line capacitance can be ignored, but as the voltage and length of the line increase, capacitance becomes of gradually increasing importance. In the same way the leakage of current across insulators becomes of increasing importance as the voltage increases. Thus the exact calculation of a very long high-voltage line is an elaborate process; in fact, the amount of elaboration depends upon the voltage. For voltages up to, say, 100,000, satisfactory solutions can be obtained from what are called the τ and π methods. In the τ, or middle-condenser, method the whole of the line capacity is assumed to be concentrated at the middle point of the line, and half the line resistance and reactance to be lumped on either side, as shown in Fig. 6.2(A). The leakage conductance or leakance, G, can also be assumed as concentrated at the middle point of the line and represented by the non-inductive conductance G shunting the capacitance C.

In the π method the capacitance and leakance are split into two halves, which are situated at either end of the line, as in Fig. 6.2(B). Obviously the capacitance and leakance at the generator end have no effect on the line drop, but they affect the kVA. and the power factor at that end. A more accurate

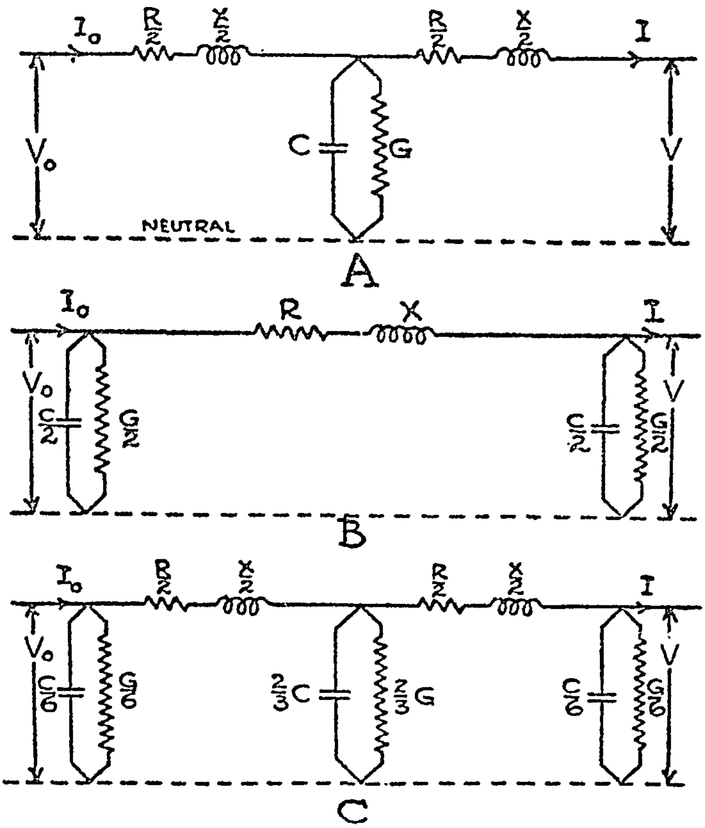


FIG. 6.2.—τ and π METHODS OF SOLUTION FOR A MODERATELY LONG LINE.

method is to combine the τ and π methods by assuming that two-thirds of the capacitance and leakance are concentrated at the middle of the line and one-sixth at either end, as in Fig. 6.2(C). Still another method which gives very good results is to divide the line into halves and use a τ connection for each: this assumes one-half of the capacitance and one-half of the leakance assumed concentrated at one-quarter the line length from both supply and load ends.

As an example of the τ method consider the following case. 5,000 kW. at power factor 0·9 are to be transmitted 50 miles, pressure at load end 60,000 volts, frequency 50. Line loss 12 per cent. of power delivered.

$$\text{Line current at far end} \quad I = \frac{5000000}{\sqrt{3} \times 60000 \times \cdot 9}$$
$$= 53{\cdot}6 \text{ amps.}$$

∴ Line loss (assuming that this is the current throughout the whole length of the line)

$$= 3I^2R = 12\% \text{ of } 5000000 = 600000 \text{ watts}$$

$$\therefore R = \frac{600000}{3 \times (53{\cdot}6)^2} = 69{\cdot}5 \text{ ohms}$$

$$\therefore \text{Resistance per mile} = \frac{69{\cdot}5}{50} = 1{\cdot}39 \text{ ohms}$$

$$\therefore \text{Cross-section } a = \frac{\cdot 046}{1{\cdot}39} = \cdot 0331 \text{ sq. in.}$$

$$\therefore r = \cdot 103 \text{ inch}$$
$$\text{Let } d = 6 \text{ ft.} = 72 \text{ inches}$$

$$\therefore \log_{10} \frac{d}{r} = \log_{10} \frac{72}{\cdot 103} = 2{\cdot}8445$$

$$\therefore \text{Inductance per phase } L_{ph} = \tfrac{1}{2} \times 14{\cdot}8 \times 10^{-4} \times 2{\cdot}8445 \times 50$$
$$= \cdot 105 \text{ henry}$$

∴ Inductive reactance per phase at 50 cycles

$$X_{ph} = \cdot 105 \times 314$$
$$= 33 \text{ ohms}$$

$$\text{Capacitance per phase } C_{ph} = \frac{3{\cdot}9}{10^8 \times 2{\cdot}8445} \times 50$$
$$= \cdot 67 \times 10^{-6} \text{ farads}$$

$$\text{Phase voltage at load end } V_{ph} = \frac{60000}{\sqrt{3}} = 34700 \text{ volts.}$$

The circuit for the τ method has therefore the constants shown in Fig. 6.3. Taking the voltage V_{ph} as the reference vector we have

$$V_{ph} = 34700 + j{\cdot}0$$
$$\cos \varphi = \cdot 9 \text{ and } \sin \varphi = \cdot 4357$$
$$\therefore \dot{I} = I \cos \varphi - jI \sin \varphi$$
$$= 48{\cdot}2 - j23{\cdot}3$$
$$\dot{Z}_{AB} = 34{\cdot}8 + j16{\cdot}5$$

∴ Drop per phase between B and A:

$$\dot{I}\,\dot{Z}_{AB} = (48{\cdot}2 - j23{\cdot}3)\,(34{\cdot}8 + j16{\cdot}5)$$
$$= 2064 - j15$$
$$\therefore \dot{V}'_{ph} = \dot{E}_{ph} + \dot{I}\,\dot{Z}_{AB}$$
$$= (34700 + j{\cdot}0) + (2064 - j15)$$
$$= 36764 - j15$$

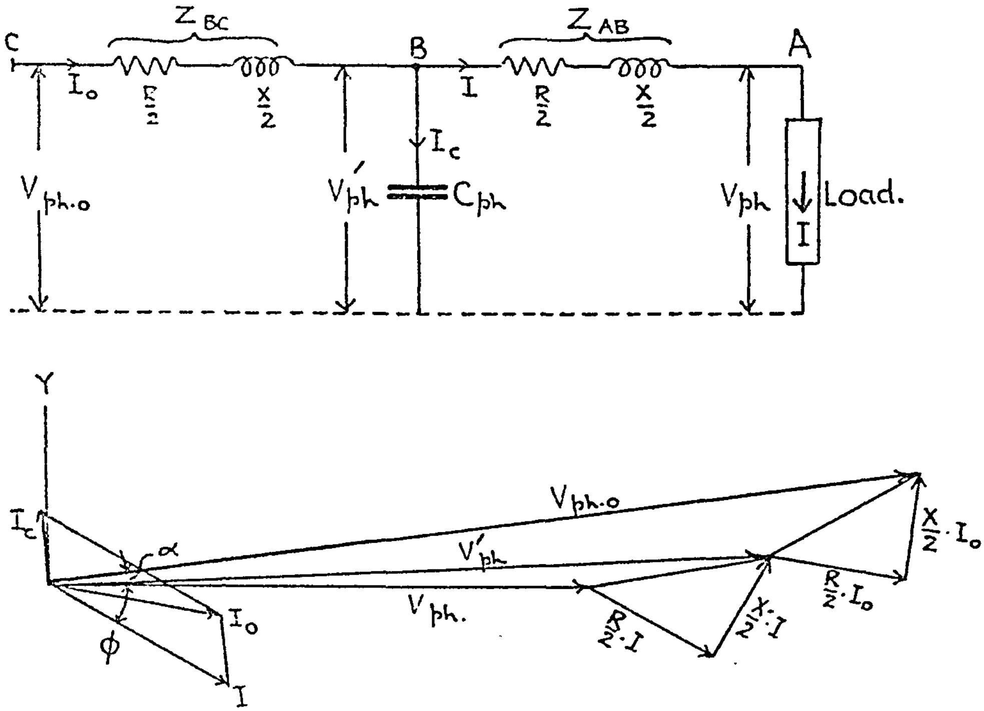


FIG. 6.3.—REGULATION DIAGRAM FOR THE τ METHOD.

For the capacitance we have

$$\dot{Z}_c = 0 - j\frac{10^6}{314 \times {\cdot}67} = -j4750$$
$$\therefore \dot{I}_c = \frac{\dot{V}'_{ph}}{\dot{Z}_c} = \frac{36764 - j15}{-j4750}$$
$$= \frac{j36764 + 15}{4750}$$
$$= {\cdot}00316 + j7{\cdot}74$$
$$\therefore \dot{I}_o = \dot{I} + \dot{I}_c$$
$$= 48{\cdot}2 - j23{\cdot}3 + {\cdot}00316 + j7{\cdot}74$$
$$= 48{\cdot}2 - j15{\cdot}6$$

∴ Drop per phase between C and B :

$$I_0 Z_{BC} = (48{\cdot}2 - j15{\cdot}6)\,(34{\cdot}8 + j16{\cdot}5)$$
$$= 1937 + j254$$
$$\therefore V_{ph.0} = E'_{ph} + I_0 Z_{BC}$$
$$= 36764 - j15 + 1937 + j254$$
$$= 38701 + j239$$
$$\therefore V_{ph.0} = (38701^2 + 239^2)^{\frac{1}{2}}$$
$$= 38701 \text{ volts}$$
$$\therefore V_0 = \sqrt{3} \times 38701$$
$$= 67000 \text{ volts}$$
$$\therefore \% \text{ line drop} = \frac{67000 - 60000}{67000} \times 100 = 10{\cdot}45$$

Now consider the nominal π method of solution: take the case of a three-phase, fifty-cycle line 75 miles long, supplying a load of 40,000 kVA. at a receiving-end voltage of 110 kW. and a lagging power factor of 0·8. Line constants as follows:

Resistance, 11 ohms.

Inductive reactance, line to neutral, 38 ohms.

Capacitive susceptance, line to neutral, 3×10^{-4} mhos.

Determine the sending-end voltage and power factor, and the efficiency of transmission.

Line constants
$$R = 11 \text{ ohms}$$
$$X = 38 \text{ ohms}$$
$$Y = 3 \times 10^{-4} \underline{|90^\circ} \text{ mhos}$$
$$Z = 11 + j38 = 39{\cdot}58 \underline{|73^\circ\,54'}$$

The nominal π circuit is given in Fig. 6.4.

at the load end—

$$V_L = 110{,}000/\sqrt{3} = 63{,}500 \text{ V to neutral}$$
$$\cos\phi_L = 0{\cdot}8,\ \sin\phi_L = 0{\cdot}6$$
$$I_L = 40000/(3 \times 63{\cdot}5) = 210 \text{ amps. at a lagging p.f. of } 0{\cdot}8$$
$$\therefore I_L = 210 \times {\cdot}8 - j210 \times {\cdot}6$$
$$= 168 - j126$$
$$I_{C.L} = 63500 \underline{|0^\circ} \times 1{\cdot}5 \times 10^{-4} \underline{|90^\circ}$$
$$= 0 + j9{\cdot}52$$
$$\therefore I_{line} = (168 - j126) + (0 + j9{\cdot}52)$$
$$= 168 - j116{\cdot}48$$
$$= 204{\cdot}4 \underline{|- 34^\circ\,42'}$$

$$ZI_{\text{line}} = 39{\cdot}58 \angle 73^\circ 54' \times 204{\cdot}4 \angle -34^\circ 42'$$
$$= 8090 \angle 39^\circ 12'$$
$$= 6260 + j5100$$

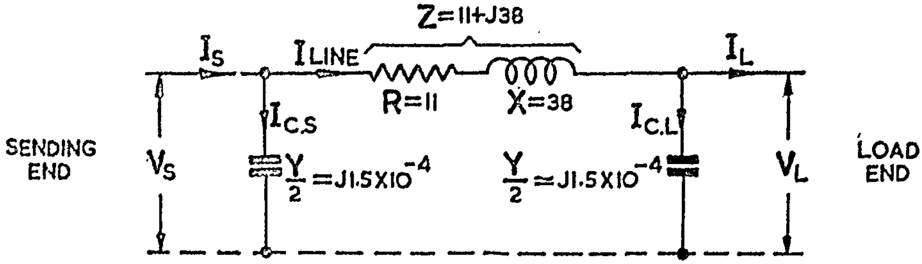


FIG. 6.4—CIRCUIT FOR NOMINAL π SOLUTION.

∴ at the supply end—

$$V_S = 63500 + 6260 + j5100 \text{ to neutral}$$
$$= 69760 + j5100$$
$$= 69900 \angle 4^\circ 10'$$
$$I_{C.S} = 69900 \angle 4^\circ 10' \times 1{\cdot}5 \times 10^{-4} \angle 90^\circ$$
$$= 10{\cdot}48 \angle 94^\circ 10'$$
$$= -0{\cdot}762 + j10{\cdot}4$$
$$I_S = I_L + I_{C.S}$$
$$= (168 - j116{\cdot}48) + (-0{\cdot}762 + j10{\cdot}4)$$
$$= 167{\cdot}24 - j106{\cdot}08$$
$$= 197{\cdot}9 \angle -32^\circ 22'$$
$$\phi_S = 30^\circ 22' + 4^\circ 10'$$
$$= 36^\circ 32'$$
$$\therefore \cos\phi_S = 0{\cdot}8052$$
$$\text{Line intake} = 3\, V_S\, I_S \cos\phi_S$$
$$= 3 \times 69{,}900 \times 197{\cdot}9 \times 0{\cdot}8052$$
$$= 33400 \text{ kW.}$$
$$\text{Load intake} = 40000 \times 0{\cdot}8 = 32000 \text{ kW.}$$
$$\therefore \text{Line loss} = 1400 \text{ kW.}$$
$$\eta = \frac{32000}{33400} \times 100$$
$$= 95{\cdot}8 \text{ per cent.}$$

In this example we have not taken any account of leakance: a complete solution for a long line taking all factors into account is given in the following chapter.

CHAPTER VII

LONG TRANSMISSION LINES

For a complete solution of the problem it is, of course, necessary to take into account the distributed nature of the various line constants, as shown in Fig. 7.1, and to discard the approxima-

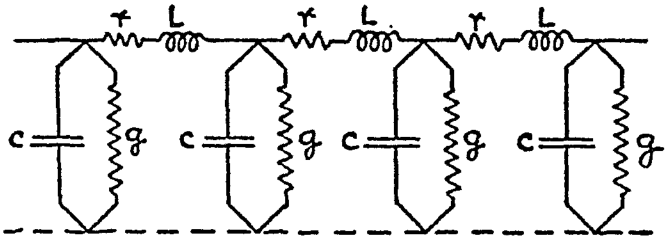


Fig. 7.1.—Distributed Constants of a Long Line.

tions brought about by the simplification of lumped constants.

Consider an element of length δl of the line, let the currents at the boundary planes be

$$i \text{ and } i + \frac{\partial i}{\partial l}\cdot\delta l$$

and let the potential between the conductor and the neutral be v. Let c be the capacitance per unit length, then the electrostatic charge on the element is

$$cv\delta l$$

Hence, neglecting for a moment the leakage current, we have

$$\frac{\partial}{\partial t}(cv\delta l) = \text{rate of accumulation of electricity within the element of length}$$

$$= -\frac{\partial i}{\partial l}\cdot\delta l$$

$$\therefore -\frac{\partial i}{\partial l} = c\cdot\frac{\partial v}{\partial t}$$

If now g is the leakage conductance or leakance per unit length, then obviously the leakage current per unit length is vg, so adding this to the right-hand side of the above equation we have

$$-\frac{\partial i}{\partial l} = vg + c\cdot\frac{\partial v}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now let L stand for the self-induction *per unit length* of conductor, and r the resistance per unit length, then

$$\mathrm{L}\,\delta l \cdot \frac{\partial i}{\partial t} + r\,\delta l i = -\frac{\partial v}{\partial l}\cdot \delta l$$

$$\therefore -\frac{\partial v}{\partial l} = ri + \mathrm{L}\frac{\partial i}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We can rewrite equations (1) and (2) as follows:

$$-\frac{\partial i}{\partial l} = \left(g + c\frac{\partial}{\partial t}\right)v \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{and } -\frac{v}{\partial l} = \left(r + \mathrm{L}\frac{\partial}{\partial t}\right)i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Differentiating equation (4) with respect to l we have

$$-\frac{\partial^2 v}{\partial l^2} = \left(r + \mathrm{L}\frac{\partial}{\partial t}\right)\cdot\frac{\partial i}{\partial l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and substituting the value of $\partial i/\partial l$ given by equation (3) we have

$$\frac{\partial^2 v}{\partial l^2} = \left(r + \mathrm{L}\frac{\partial}{\partial t}\right)\left(g + c\frac{\partial}{\partial t}\right)v$$

It is well known that when dealing with sinusoidal functions the operator $\partial/\partial t$ can be replaced by $j\omega$, where

$$\omega = 2\pi f$$

$$\therefore \frac{\partial^2 v}{\partial l^2} = (r + j\omega\mathrm{L})(g + j\omega c)v$$

Now put $\mathrm{L}\omega = x$ the inductive reactance per unit length
$c\omega = h$ the capacity susceptance per unit length

$$\therefore \frac{\partial^2 v}{\partial l^2} = (r + jx)(g + jh)v$$

$$= m^2 v, \text{ say} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

We have now eliminated i from the equations and we can write down the solution for v:

$$v = \mathrm{A}e^{ml} + \mathrm{B}e^{-ml} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

To find the current i we make use of the original equation (4), writing $j\omega$ for $\partial/\partial t$. We then have:

$$-\frac{\partial v}{\partial l} = (r + jx)i$$

But we have, by differentiating equation (7):

$$\frac{\partial v}{\partial l} = m(\mathrm{A}e^{ml} - \mathrm{B}e^{ml})$$

$$\therefore i = \frac{m}{r + jx}(-\mathrm{A}e^{ml} + \mathrm{B}e^{-ml})$$

Now write:

$$n^2 = \frac{r + jx}{g + jh}$$

$$\therefore\ ni = -Ae^{ml} + Be^{-m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

We have now deduced expressions for v and i, and by means of them the potential difference and current can be calculated at any point of the line. The only places for which the calculation need be made are the two ends. At the generating end we have:

$$l = 0$$

$$V_{ph.0} = \text{line voltage (R.M.S.) per phase}$$

$$I_0 = \text{line current}$$

Therefore from equations (7) and (8):

$$\left.\begin{aligned} V_{ph.0} &= A + B \\ nI_0 &= -A + B \end{aligned}\right\}$$

hence

$$A = \frac{V_0 - nI_0}{2}$$

$$B = \frac{V_0 + nI_0}{2}$$

Substituting these values in the same equations we have for the voltage and current at any point distant l from the generating end, l here standing for any distance:

$$V_{ph} = \tfrac{1}{2}V_0(e^{ml} + e^{-ml}) - \tfrac{1}{2}nI_0(e^{ml} - e^{-ml})$$

and

$$I = \tfrac{1}{2}I_0(e^{ml} + e^{-ml}) - \tfrac{1}{2}\cdot\frac{V_0}{n}(e^{m} - e^{-ml})$$

Now the expression:

$$(e^{ml} \pm Ie^{-ml})$$

can be expressed in terms of the hyperbolic sines and cosines as follows:

$$\sinh ml = \frac{e^{ml} - e^{-ml}}{2}$$

$$\cosh ml = \frac{e^{ml} + e^{-m}}{2}$$

$$\therefore\ \left.\begin{aligned} V_{ph} &= V_{ph.0}\cosh ml - nI_0 \sinh ml \\ I &= I_0 \cosh ml - \frac{V_0}{n}\sinh ml \end{aligned}\right\} \quad . \quad (9)$$

As stated before, a knowledge of the terminal conditions is, as a rule, all that we require, and therefore if we now let l stand for the total length of the line, and V_{ph} and I for the values of phase voltage and current at the load end, the equations (9) give the voltage and current at the load end in terms of the voltage and current at the generating end. They are therefore used to determine the line regulation if the conditions at the generator end are fixed. On the other hand, if the current at the load end is to be kept constant, and the necessary conditions at the generating end to give these specified conditions at the load end are required, a new pair of equations is necessary. These equations are obtained by using the values V_{ph}, I and l when calculating the constants A and B, thus giving:

$$A = \frac{V - nI}{2}$$

$$B = \frac{V + nI}{2}$$

Now regard the load end as the origin, the generating end therefore being at a distance of $-l$. Substituting the new constants, and substituting $-l$ for $+l$, we then have:

$$V_{ph.0} = \tfrac{1}{2}V(e^{ml} + e^{-ml}) + \tfrac{1}{2}nI(e^{ml} - e^{-ml})$$

$$I_0 = \tfrac{1}{2}I(e^{ml} + e^{-ml}) + \tfrac{1}{2}\frac{V}{n}(e^{ml} - e^{-ml})$$

Expressing these in terms of hyperbolic functions we finally have:

$$\left.\begin{aligned} V_{ph.0} &= V\cosh ml + nI\sinh ml \\ I_0 &= I\cosh ml + \frac{V}{n}\sinh ml \end{aligned}\right\} \quad . \quad . \quad . \quad (10)$$

When the line is on open circuit at the load end $I = 0$, and the necessary voltage at the generating end to maintain the specified voltage V_{ph} at the load end is

$$V'_{ph.0} = V_{ph}\cosh ml$$

The current at the generating end will then be:

$$I'_0 = \frac{V}{n}\sinh ml$$

This current is the complete charging current of the line, taking into account the leakage as well as the capacity current.

The hyperbolic sines and cosines can be written in the forms of infinite series as follows:

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots$$

three terms being, as a rule, sufficient.*

* They can, of course, be obtained directly from tables.

As an example take the following case. A three-phase line with symmetrically arranged conductors delivers 10,000 kW. at a voltage of 120,000 at the load end. Power factor of the load 0·8, frequency 50. The conductors are of diameter 0·4 in. and are spaced 150 in. apart. Length of line 200 miles.

As this is to be an exact solution, it is preferable to use the more accurate expression for the inductance, viz.:

$$L = \tfrac{1}{2}\left(1 + 4 \log_e \frac{d}{r}\right) \times 10^{-9} \text{ henries per cm. for a three-phase line}$$

which, reduced to more practicable units, becomes

$$L = \left(8 + 74{\cdot}1 \log_{10} \frac{d}{r}\right) \times 10^{-5} \text{ henries per mile.}$$

Collecting the line constants we have:

$$\begin{aligned}
r &= {\cdot}45 \text{ ohm per mile} \\
L &= 2{\cdot}21 \times 10^{-3} \text{ henries per mile} \\
\omega &= 2\pi f = 314 \\
x &= L\omega = {\cdot}694 \text{ ohm per mile} \\
c &= \frac{3{\cdot}878 \times 10^{-8}}{\log_{10} \dfrac{d}{r}} \\
&= 1{\cdot}385 \times 10^{-8} \text{ farads per mile} \\
\therefore\ h &= c\omega = 4{\cdot}35 \times 10^{-6} \text{ mhos per mile}
\end{aligned}$$

It is difficult to give a figure for the leakance, but $g = 5 \times 10^{-9}$ mhos per mile is a fairly representative value.

We now calculate the constants m and n:

$$\begin{aligned}
m &= \sqrt{(r + jx)(g + jh)} \\
&= \sqrt{({\cdot}45 + j \times {\cdot}694)(5 \times 10^{-9} + j \times 4{\cdot}35 \times 10^{-6})} \\
&= \sqrt{-3{\cdot}0166 + j \times 1{\cdot}961} \times 10^{-3} \\
&= {\cdot}000540 + j \times {\cdot}00182\ * \\
n &= \sqrt{\frac{r + jx}{g + jh}} \\
&= \sqrt{\frac{{\cdot}45 + j \times {\cdot}694}{5 \times 10^{-9} + j \times 4{\cdot}35 \times 10^{-6}}} \\
&= \sqrt{{\cdot}155 - j \times {\cdot}103} \times 10^{3}\ \dagger
\end{aligned}$$

* If $\sqrt{a + jb} = x + jy$, then $x = \pm \left\{\dfrac{\sqrt{a^2 + b^2} + a}{2}\right\}^{\frac{1}{2}}$

$y = \pm \left\{\dfrac{\sqrt{a^2 + b^2} - a}{2}\right\}^{\frac{1}{2}}$

† $\dfrac{r + jx}{g + jh} = \dfrac{rg + hx}{g^2 + h^2} + j \times \dfrac{gx - rh}{g^2 + h^2}$

$$= 413 - j \times 124{\cdot}5$$

$$ml = 200({\cdot}00054 + j \times {\cdot}00182)$$

$$= {\cdot}108 + j \times {\cdot}364$$

$$\cosh\ ml = \cosh\ ({\cdot}108 + j \times {\cdot}364)$$

$$= \cosh {\cdot}108 \times \cos {\cdot}364 + j \times \sinh {\cdot}108 \times \sin {\cdot}364$$

$$\cosh {\cdot}108 = 1{\cdot}0059\ ;\ \ \sinh {\cdot}108 = {\cdot}1082$$

$$\cos {\cdot}364 = \cos \left(\frac{{\cdot}364 \times 180}{\pi}\right)^{\circ} = {\cdot}9348$$

$$\sin {\cdot}364 = \sin \left(\frac{{\cdot}364 \times 180}{\pi}\right)^{\circ} = {\cdot}3551$$

$$\therefore \cosh\ ml = 1{\cdot}0059 \times {\cdot}9348 + j \times {\cdot}1082 \times {\cdot}3551$$

$$= 940 + j \times {\cdot}0385$$

$$\sinh\ ml = \sinh\ ({\cdot}108 + j \times {\cdot}364)$$

$$= \sinh {\cdot}108 \times \cos {\cdot}364 + j \times \cosh {\cdot}108 \times \sin {\cdot}364$$

$$= {\cdot}1082 \times {\cdot}9348 + j \times 1{\cdot}0059 \times {\cdot}3551$$

$$= {\cdot}1011 + j \times {\cdot}3575$$

$$n \sinh\ ml = (413 - j \times 124{\cdot}5)({\cdot}1011 + j \times {\cdot}3575)$$

$$= 86{\cdot}26 + j \times 135{\cdot}06$$

$$\frac{\sinh\ ml}{n} = \frac{{\cdot}1011 + j \times {\cdot}3575}{413 - j \times 124{\cdot}5}$$

$$= (-\ 1{\cdot}475 + j \times 86{\cdot}2) \times 10^{-5}$$

We are now in a position to commence the calculations of voltage and current. At the load end the voltage to neutral is given by:

$$V_{ph} = \frac{120000}{\sqrt{3}} = 69400$$

The power factor of the load is $\cos \varphi = 0{\cdot}8$, and therefore expressing the voltage symbolically, the direction of the vector of load current I being taken as the axis of reference, we have:

$$V_{ph} = 69400 \cos \varphi + j \times 69400 \sin \varphi$$

$$= 69400 \times {\cdot}8 + j \times 69400 \times {\cdot}6$$

$$= 55520 + j \times 41640$$

The phase relationships are shown in Fig. 7.2.

Power per phase at load end:

$$P = \frac{10000}{3} = 3333 \text{ kW.}$$

$$= 3{\cdot}333 \times 10^{6} \text{ watts}$$

$$I = \frac{3{\cdot}333 \times 10^{6}}{69400 \times {\cdot}8}$$

$$= 60 \text{ amperes}$$

The necessary terminal conditions at the generating end are:

$$\left.\begin{aligned} V_{ph.0} &= V\cosh ml + nI\sinh ml \\ I_0 &= I\cosh ml + \frac{V}{n}\sinh ml \end{aligned}\right\}$$

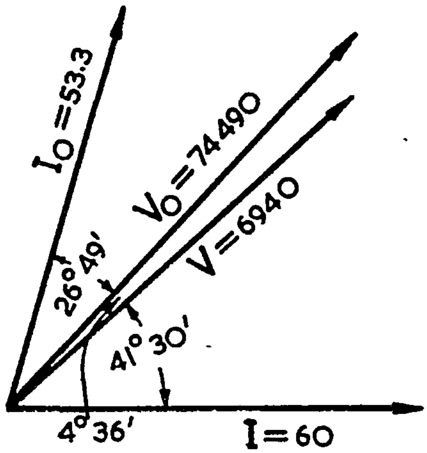


FIG. 7.2.

Calculating the various terms separately, we have:

$$\begin{aligned} V\cosh ml &= (55520 + j \times 41640)(\cdot 940 + j \times \cdot 0385) \\ &= 50590 + j \times 41280 \\ nI\sinh ml &= 60(86\cdot 26 + j \times 135\cdot 06) \\ &= 5176 + j \times 8104 \\ \therefore\ V_{ph.0} &= (50590 + j \times 41280) + (5176 + j \times 8104) \\ &= 55770 + j \times 49380 \\ \therefore\ V_{ph.0} &= (55770^2 + 49380^2)^{\frac{1}{2}} \\ &= 74490 \text{ volts to neutral} \end{aligned}$$

The phase of this voltage with respect to the load current (not the generated current) is given by the angle ψ, where

$$\cos\psi = \frac{55770}{74490} = \cdot 749$$

$$\therefore\ \psi = 41^\circ 30'$$

whereas $\varphi = 36^\circ 54'$

For the current at the generating end:

$$\begin{aligned} I\cosh ml &= 60(\cdot 940 + j \times \cdot 0385) \\ &= 56\cdot 4 + j \times 2\cdot 31 \\ \frac{V}{n}\sinh ml &= (55520 + j \times 41640)(-1\cdot 475 + j \times 86\cdot 2) \times 10^{-5} \\ &= -36\cdot 7 + j \times 47\cdot 2 \end{aligned}$$

$$\therefore \dot{I}_0 = (56{\cdot}4 + j \times 2{\cdot}31) + (-36{\cdot}7 + j \times 47{\cdot}2)$$
$$= 19{\cdot}7 + j \times 49{\cdot}5$$
$$\therefore I_0 = (19{\cdot}7^2 + 49{\cdot}5^2)^{\frac{1}{2}}$$
$$= 53{\cdot}3 \text{ amps.}$$

which, it will be seen, is appreciably smaller than the load current, owing to the magnitude and phase of the charging current. The phase angle θ of this current with respect to the load current I is given by

$$\cos\theta = \frac{19{\cdot}7}{53{\cdot}3} = {\cdot}37$$
$$\therefore \theta = 68^\circ\ 19'$$

Thus the total generated current I_0 *leads* $E_{ph.0}$ by

$$\varphi_0 = 68^\circ\ 19' - 41^\circ\ 31'$$
$$= 26^\circ\ 49'$$

The power per phase at the generating end is obtained by adding the product of the real parts of $V_{ph.0}$ and I_0 to the product of the imaginary parts of $V_{ph.0}$ and I_0

$$\therefore P_0 = (55770 \times 19{\cdot}7 + 49380 \times 49{\cdot}5) \times 10^{-3}$$
$$= 3543 \text{ kW. per phase.}$$

This same result can, of course, be obtained from the expression:

$$P_0 = V_{ph.0}\, I_0 \cos\varphi_0$$
$$= 74490 \times 53{\cdot}3 \times {\cdot}8398 \times 10^{-3}$$
$$= 3543 \text{ kW. per phase}$$

The efficiency of the line is therefore

$$\frac{3333}{3543} \times 100 = 93{\cdot}9 \text{ per cent.}$$

The voltage per phase at the generating end to give the same voltage V at the load end when the line is open-circuited is

$$\dot{V}'_{ph.0} = V_{ph} \cosh ml$$
$$= (55520 + j \times 41640)({\cdot}940 + j \times {\cdot}0385)$$
$$= 50586 + j \times 41279$$
$$\therefore V_{ph.0} = (50586^2 + 41279^2)^{\frac{1}{2}}$$
$$= 65300$$

On the other hand, if the voltage at the station is maintained at $V_{ph.0} = 74490$ volt per phase, and the load is suddenly thrown off the line, the voltage per phase at the load end will be T'_{ph}, where

$$V_{ph.0} = V' \cosh ml$$

$$\therefore V'_{ph} = \frac{V_{ph.0}}{\cosh ml}$$

$$= \frac{55770 + j \times 49380}{\cdot 940 + j \times \cdot 0385}$$

$$= \frac{55770 \times \cdot 940 + 49380 \times \cdot 0385}{\cdot 940^2 + \cdot 0385^2}$$

$$+ j \times \frac{\cdot 940 \times 49380 - \cdot 0385 \times 55770}{\cdot 940^2 + \cdot 0385^2}$$

$$= 61380 + j \times 50014$$

$$= 79160 \text{ volts.}$$

This final calculation illustrates the difference between line drop in volts and line regulation. The percentage voltage drop is given by

$$\frac{V_0 - V}{V} \times 100$$

and this is also the value of the line regulation when the effects of capacity and leakage can be neglected. On the other hand, the

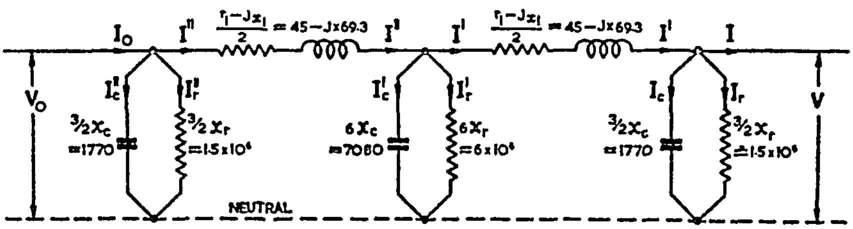


FIG. 7.3.—APPROXIMATE SOLUTION FOR THE LONG LINE.

regulation is defined as the change in pressure at the load end when the load is thrown off, the voltage at the generating end remaining constant. Thus in the case of a long high-voltage line the percentage regulation will be greater than the percentage line drop, as is shown by the calculations above.

It is instructive to make the calculations for the same line by one of the approximate methods, and we will choose the combined τ and π methods. The circuit diagram is given in Fig. 7.3, and the data are as follows:

$$r = \cdot 45 \text{ ohm per mile}$$

$$L = 2 \cdot 21 \times 10^{-3} \text{ henries per mile}$$

$$C = 1{\cdot}385 \times 10^{-8} \text{ farads per mile}$$
$$r_1 = 90 \text{ ohms per conductor}$$
$$x_1 = 2{\cdot}21 \times 10^{-3} \times 314 \times 200$$
$$= 138{\cdot}6 \text{ ohms per conductor}$$
$$x_c = 10^8/1{\cdot}385 \times 200 \times 314$$
$$= 1180 \text{ ohms per conductor}$$
$$x_r = 1/200 \times 5 \times 10^{-9}$$
$$= 10^6 \text{ ohms per conductor}$$
$$= \text{leakage resistance}$$
$$\therefore 6x_c = 7080 \text{ ohms}$$
$$\tfrac{3}{2}x_c = 1770 \text{ ohms}$$
$$6x_r = 6 \times 10^6 \text{ ohms}$$
$$\tfrac{3}{2}x_r = 1{\cdot}5 \times 10^6 \text{ ohms}$$

It will be convenient to refer the phases of the various voltages and currents to that of the voltage V_{ph}. At a power factor of 0·8 the load current is 60 amperes, whereas, at unity power factor, to give the same power, the current would be 48 amperes.

$$\therefore \underset{\cdot}{V}_{ph} = V_{ph} = 69400 + j.0$$
$$\underset{\cdot}{I} = 48 + j \times 35{\cdot}9$$
$$\underset{\cdot}{Z}_1 = R_1 - jX_1$$
$$= 90 - j \times 138{\cdot}6$$
$$\underset{\cdot}{I}_c = \underset{\cdot}{V}_{ph}/j \times 6x_c$$
$$= \frac{69400}{j \times 7080}$$
$$= -j \times 9{\cdot}8$$
$$\underset{\cdot}{I}_r = \underset{\cdot}{V}_{ph}/6x_r$$
$$= \frac{69400}{6 \times 10^6}$$
$$= {\cdot}01 + j \times 0$$
$$\underset{\cdot}{I}' = \underset{\cdot}{I} + \underset{\cdot}{I}_c + \underset{\cdot}{I}_r$$
$$= (48 + j \times 35{\cdot}9) - j \times 9{\cdot}8 + {\cdot}01$$
$$= 48 + j \times 26{\cdot}1$$
$$\underset{\cdot}{V}'_{ph} = \underset{\cdot}{V} + \tfrac{1}{2}\underset{\cdot}{I}'\underset{\cdot}{Z}_1$$
$$= 69400 + \tfrac{1}{2}(48 + j \times 26{\cdot}1)(90 - j \times 138{\cdot}6)$$
$$= 73380 - j \times 2150$$
$$\underset{\cdot}{I}'_c = \underset{\cdot}{V}'_{ph}/j \times \tfrac{3}{2}x_c$$
$$= \frac{73380 - j \times 2150}{j \times 1770}$$
$$= -j \times 41{\cdot}5 - 1{\cdot}22$$
$$\underset{\cdot}{I}' = \underset{\cdot}{V}'_{ph}/j \times \tfrac{3}{2}x_r$$
$$= \frac{73380 - j \times 2150}{1{\cdot}5 \times 10^6}$$

$$= \cdot 05 - j \times \cdot 001$$

$$\dot{I}'' = \dot{I}' + \dot{I}'_c + \dot{I}'_r$$

$$= (48 + j \times 26{\cdot}1) - (1{\cdot}22 + j \times 41{\cdot}5) + (\cdot 05 - j \times \cdot 001)$$

$$= 46{\cdot}8 - j \times 15{\cdot}4$$

$$\dot{V}_{ph.0} = \dot{V}'_{ph} + \tfrac{1}{2}\dot{I}''Z_1$$

$$= (73380 - j \times 2150) + \tfrac{1}{2}(46{\cdot}8 - j \times 15{\cdot}4)(90 - j \times 138{\cdot}6)$$

$$= 74430 - j \times 6077$$

$$\dot{I}''_c = \frac{\dot{V}_{ph.0}}{j \times 6x_c}$$

$$= \frac{74430 - j \times 6077}{j \times 7080}$$

$$= -j \times 10{\cdot}5 - 0{\cdot}9$$

$$\dot{I}''_r = \frac{\dot{V}_{ph.0}}{6x_r}$$

$$= \frac{74430 - j \times 6077}{6 \times 10^6}$$

$$= \cdot 01 - j \times \cdot 001$$

$$\dot{I}_0 = \dot{I}'' + \dot{I}''_c + \dot{I}''_r$$

$$= (46{\cdot}8 - j \times 15{\cdot}4) - (0{\cdot}9 + j \times 10{\cdot}5) + (\cdot 01 - j \times \cdot 001)$$

$$= 45{\cdot}9 - j \times 25{\cdot}9$$

$$\therefore V_0 = (74430^2 + 6077^2)^{\frac{1}{2}}$$

$$= 74500 \text{ volts}$$

$$\text{and } I_0 = (45{\cdot}9^2 + 25{\cdot}9^2)^{\frac{1}{2}}$$

$$= 52{\cdot}6 \text{ amps.}$$

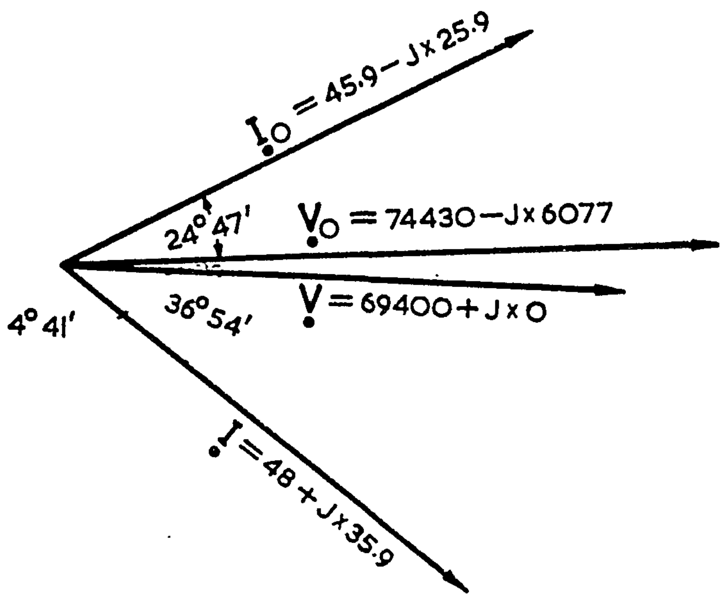


FIG. 7.4.

On comparing these results with those obtained by the use of hyperbolic functions it will be seen that the two calculated values of the voltage drops are almost identical, but that there is a small difference in the two values for the current I_0. The example chosen therefore represents about the limit to which the more approximate method can be applied with any degree of confidence. For still higher working voltages it is very desirable to use the hyperbolic method.

The vector diagram as determined by the split-condenser method is given in Fig. 7.4. In order to determine the various phase angles it is convenient to calculate their tangents. Thus the voltage E_0 leads the voltage V by

$$\begin{aligned}\text{arc tan}\,\frac{6077}{74430} &= \text{arc tan}\ {\cdot}0818\\ &= 4°\ 41'\end{aligned}$$

as compared with 4° 36′ as given by the hyperbolic method.

$$\begin{aligned}\varphi_o &= \text{arc tan}\,\frac{25{\cdot}9}{45{\cdot}9} - 4°\ 41'\\ &= \text{arc tan}\ {\cdot}565 - 4°\ 41'\\ &= 29°\ 28' - 4°\ 41'\\ &= 24°\ 47'\end{aligned}$$

as compared with 26° 49′ as given by the hyperbolic method.

It is of interest to note that the τ method will give a correct value for the voltage drop if the values of line impedance Z and line admittance a per mile are replaced by the adjusted value

$$z' = z \times \frac{\tanh\left(\dfrac{l\sqrt{az}}{2}\right)}{\dfrac{l\sqrt{az}}{2}}$$

$$a' = a \times \frac{\sinh\,(l\sqrt{az})}{l\sqrt{az}}$$

For the π method the adjusted values are

$$z' = z \times \frac{\sinh\,(l\sqrt{az})}{l\sqrt{az}}$$

$$a' = a \times \frac{\tanh\left(\dfrac{l\sqrt{az}}{2}\right)}{\dfrac{l\sqrt{az}}{2}}$$

CHAPTER VIII

CORONA

If an alternating potential difference is applied to two wires whose spacing is large in comparison with the diameter, and the potential difference is gradually increased, a point will be reached when a faint luminous glow of violet colour will make its appearance, and at the same time a hissing noise will be heard. This phenomenon is accompanied by the formation of ozone, as is indicated by the characteristic odour of this gas. If the potential difference is raised still farther, the glow and the noise will increase in intensity until eventually a spark-over will take place. If the wires are perfectly uniform and smooth the glow will be uniform along their length, but if there are any roughnesses these will be picked out by brighter illumination. With wires only a short distance apart in comparison with their diameter, spark-over may take place before there is any luminous glow.

If the applied potential difference is direct instead of alternating the two wires no longer have the same appearance, the positive wire having a uniform glow while the negative wire has a more patchy glow, often accompanied by streamers if there are any rough places. It thus appears at first sight that the phenomenon is different for the two types of applied potential difference, but if with an alternating potential difference the wires are examined stroboscopically it will be found that the appearance is now similar to that with a direct potential difference.

The above phenomena are termed corona, and an important point in connection with corona is that it is accompanied by a loss of power, and this means that there is a flow of current to the wire. The current wave form is not sinusoidal, but has the characteristic shape shown in Fig. 8.1. In practice this non-sinusoidal current and the non-sinusoidal drop of volts caused by it may be more important than the loss in power.

Critical Voltage

If a long uniform round wire of radius r is charged to q coulombs per metre length there will be an electrostatic field in the neighbourhood of the wire, the field intensity at a distance x from the centre being

$$H_x = q / (2\pi\varepsilon_0 x)$$

The variation of potential gradient across the space between two conductors of 0·77 in. diameter with a spacing of 12 ft. is shown in Fig. 8.2.

For air under ordinary conditions near sea-level and without impurities, the value of the potential gradient at which ionisation

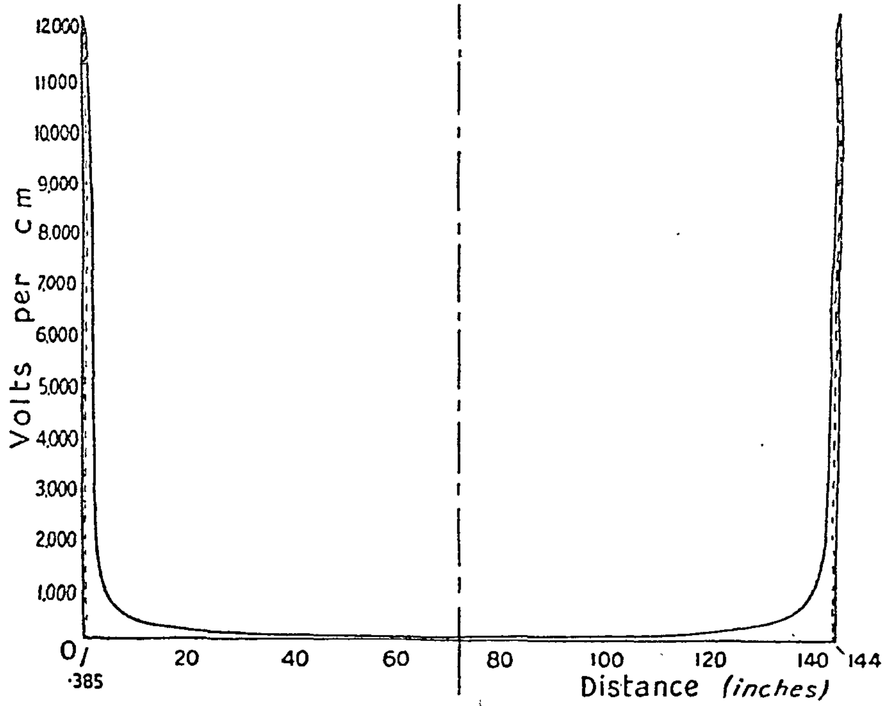


FIG. 8.2.—VARIATION OF POTENTIAL GRADIENT BETWEEN TWO 0·77-IN. CONDUCTORS, SPACED 12 FT. TOTAL P.D. 132,000 VOLTS.

takes place can be taken as 30,000 volts per cm., giving as the condition at which corona will begin to take place:

$$30000 = \frac{V'}{r \log_e \frac{d}{r}}$$

$$\therefore V' = 30000\, r \log_e \frac{d}{r}$$

$$= 30000 \times 2{\cdot}3\, r \log_{10} \frac{d}{r}$$

Thus let $r = {\cdot}77/2 = {\cdot}385$ in.

$d = 12$ ft. $= 144$ in.

Then $V' = 30000 \times 2{\cdot}3 \times ({\cdot}385 \times 2{\cdot}54) \times \log_{10} \frac{144}{{\cdot}385}$

$= 174000$ volts.

This gives the *maximum volts to neutral* at which corona will commence, so that for a single-phase two-conductor line the R.M.S. voltage to which the line must be raised to start corona will be:

$$\frac{2 \times 174000}{\sqrt{2}} = 246000 \text{ volts.}$$

For a three-phase line the voltage to neutral is $1/\sqrt{3}$ of the line voltage, so that the line voltage to start corona with the same diameter and spacing is:

$$\frac{\sqrt{3} \times 174000}{\sqrt{2}} = 210000 \text{ volts.}$$

Before dealing with the factors which modify this value in practice, it is necessary to appreciate several other points in connection with corona. In the first place, if the line voltage is raised to the critical value, called the disruptive critical voltage, ionisation of the immediately adjacent air will commence, but there will be no *visual* corona. The impact to produce a visual glow requires a greater velocity and therefore a greater line voltage than that given by the above equation. It can be shown that the air must be broken down at a distance of $(r + 0{\cdot}3\sqrt{r})$ from the centre of the conductor, the maximum voltage to neutral for visual corona at normal temperature and pressure thus being:

$$V' = 30000(r + {\cdot}3\sqrt{r}) \log_e \frac{d}{r}$$

Thus for a grid line at N.T.P. we have for the necessary R.M.S. line voltage:

$$\frac{\sqrt{3}}{\sqrt{2}} \times 30000 \times 2{\cdot}3 \times ({\cdot}385 + {\cdot}3\sqrt{{\cdot}385}) \times 2{\cdot}54 \times \log_{10} \frac{144}{{\cdot}385}$$
$$= 315{,}500$$

The second point is that the phenomenon is confined to a sheath of air round each conductor. This sheath of air is itself conducting, with the result that there is a virtual increase in the diameter of the wire, so far as the distribution of potential gradient in the electrostatic field is concerned. Suppose that the radius of this sheath is r', then since $r' > r$ the maximum value of the potential gradient is reduced from

$$\frac{V'}{r \log_e \frac{d}{r}} \quad \text{to} \quad \frac{V'}{r' \log_e \frac{d}{r'}}$$

A charged particle situated in the field will be acted on by a force of

$$E_x = H_x q' \text{ newtons}$$

where q' is the charge on the particle. The acceleration produced by this force will be inversely proportional to its mass, but under

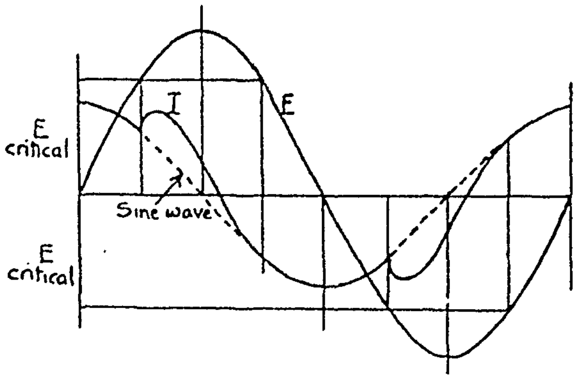


FIG. 8.1.—WAVE FORM OF CORONA CURRENT.

this acceleration, velocity will be acquired. In the atmosphere there are always a number of such charged particles, or ions, present and, in consequence, there will be movement of these particles whenever electrostatic fields are present, the average distance moved without collision being the mean free path. Frequent collisions occur with the uncharged molecules of air, and, provided that the impact is sufficiently violent, electrons will be dislodged from these molecules, thereby increasing the number of ions. The effect is thus seen to be cumulative, the final result being that the air will be rendered conducting. For a given condition of the atmosphere the velocity acquired by any ion depends upon the potential gradient, and, since a definite kinetic energy of an ion of given mass will be required to dislodge an electron from a molecule of air, it follows that this dislodgment is also dependent on the potential gradient. Hence, if the potential of the wire is gradually increased from zero there will be no change in the conducting state of the atmosphere until the potential gradient has become sufficiently great to give this velocity.

From the expression for H_x it is obvious that the potential gradient has its maximum value at the surface of the wire and that it decreases rapidly as the distance increases. Hence, when the gradient has been raised to the necessary value, it is the layer of air immediately surrounding the wire which will become conducting, and not the whole body of air acted upon by the electrostatic field.

Now consider a two-conductor transmission line, and let the centre plane be regarded as the neutral plane. Then if V′ is the voltage of one conductor with respect to this plane we have, using the previous notation :

$$V' = \int_r^{\frac{d}{2}} \left(\frac{q}{x} + \frac{q}{d-x}\right) dx/2\pi\varepsilon_0$$

$$= \left(\frac{q}{2\pi\varepsilon_0}\right) \log_e \frac{d-r}{r}$$

At any point distant x from a conductor, the potential with respect to the neutral plane is

$$V'_x = \int_x^{\frac{d}{2}} \left(\frac{q}{x} + \frac{q}{d-x}\right) dx/2\pi\varepsilon_0$$

$$= \left(\frac{q}{2\pi\varepsilon_0}\right) \log_e \frac{d-x}{x}.$$

To obtain the potential gradient dV'_x/dx, we have to differentiate with respect to x, and we have

$$\frac{dV'_x}{dx} = -\left(\frac{q}{2\pi\varepsilon_0}\right)\frac{d}{x(d-x)}$$

But
$$q = \frac{V'}{\log_e \frac{d-r}{r}} \times 2\pi\varepsilon_0$$

$$\therefore \frac{dV'_x}{dx} = V' \times \frac{d}{x(d-x)} \times \frac{1}{\log_e \frac{d-r}{r}}$$

The gradient is a maximum at the surface of the conductor, where $x = r$, and since d is large compared with r we can write d in place of $(d - r)$ with negligible error. Hence, in general :

$$\frac{dV'_x}{dx} = \frac{V'}{x} \times \frac{1}{\log_e \frac{d-r}{r}}$$

and at the surface :

$$\left(\frac{dV'_x}{dx}\right)_{x=r} = \frac{V'}{r} \times \frac{1}{\log_e \frac{d-r}{r}}$$

$$\simeq \frac{V'}{r \log_e \frac{d}{r}}$$

Consider again the two-conductor line with $d = 144$ in. and $r = 0{\cdot}385$ in. Suppose that the line is operated at a voltage of 200,000 volts, thereby giving a potential gradient, if there is no corona, of

$$\frac{200000}{2{\cdot}3 \times {\cdot}385 \times 2{\cdot}54 \times \log_{10} \frac{144}{{\cdot}385}} = 34600$$

Now suppose that this gradient is sufficient to cause corona, and that the conditions are such that the corona sheath is double the diameter of the wire, then the maximum potential gradient will be reduced to

$$\frac{200000}{2{\cdot}3 \times {\cdot}77 \times 2{\cdot}54 \times \log_{10} \frac{144}{{\cdot}77}} = 19600$$

This value is well below the critical value of 30,000, showing that, in the case considered, the further spread of the corona into the space between the two wires is impossible.

The effect of corona can thus be considered as an easing of the electrostatic stress, and it ensures that a flash-over between two conductors is quite impossible. This is not necessarily the case between two parallel cylinders whose spacing is not very large in comparison with the diameter. In such a case the ratio of the maximum to minimum potential gradient is much smaller than for an overhead line, the electrostatic field between the two cylinders is much more uniform, and it is possible that as the potential difference between the cylinders is gradually increased, a flash-over may take place without the preliminary formation of corona. The condition for such a spark-over is the same as for the formation of corona, namely, that the voltage gradient at the surface of the conductor shall be greater than the critical value :

$$\frac{V'}{r \log_e \frac{d}{r}} > 30000$$

Now, if the ratio d/r is equal to e, then if r is increased, the ratio d/r decreases more rapidly than r increases, so that if such an increase in r were due to an initial formation of corona, the corona would spread at once and result in flash-over. In other words, for a ratio of d/r less than e, a virtual increase in r through corona will cause an increase, instead of a decrease, in the potential gradient.

Hence, theoretically, flash-over takes place without corona if $\frac{d}{r} < e$, while corona takes place first of all if $\frac{d}{r} > e$. In practice a flash-over may take place with no, or very little, preliminary corona, if d/r is less than about 15, but as this ratio is always exceeded with overhead lines, a flash-over can be regarded as impossible.

If the p.d. is gradually raised above that at which visual corona first appears the luminous glow will spread out further and further from the conductors until, finally, the insulation of the air is broken down completely and an arc passes between the conductors. This voltage is called the sparkover voltage.

If the spacing is only a few times the diameter of the wires, sparkover may take place without prior corona formation. The approximate rule is $\frac{d}{r} < \frac{3}{1}$

Conditions affecting Corona

We have seen that corona is due to the bombardment of air molecules, with subsequent dislodging of electrons, by ionised particles. Corona will thus be affected by the physical state of the atmosphere, as well as by conditions of the line. Considering first the atmospheric conditions the factors are :

i. Number of ions.
ii. Size and charge per ion.
iii. Mean free path.

The first two will be determined by the electrical state of the atmosphere and also by atmospheric pollution, while the third will be controlled mainly by the density of the air. The density is a deciding factor in another respect, viz. that the voltage gradient for the breakdown of the air is proportional to the density. In stormy weather the number of ions may be very considerably greater than normal, and corona may then form at a voltage much less than the voltage required in fair weather. It is impossible to give any figure for this reduced voltage, but 80 per cent. may be taken as a rough guide.

Considering next the physical conditions of the line the factors are :

i. Voltage of the line.
ii. Ratio d/r.
iii. Contour of the surface.
iv. State of the surface.

For stranded conductors, as almost invariably used, the shape of the cross-section is a series of arcs of circles each of much smaller diameter than the conductor as a whole. The potential gradient for such a conductor will, in consequence, be greater than for the equivalent smooth conductor, so that the breakdown voltage for a stranded conductor will be somewhat less than for a smooth conductor, the ratio being 0·88 to unity. The effect of dirt on the surface is to increase the irregularity and thereby cause a still further decrease in the breakdown value, an average value for the ratio when both stranding and dirt are taken into account being about 0·85 to unity.

Power Loss due to Corona

The formation of corona is associated with a loss of power, which will have some effect on the efficiency of the line, but will not be of sufficient importance to have any appreciable effect on the voltage regulation. As in the case of the critical voltage for corona, the power loss is affected both by atmospheric and line conditions. The loss is calculated by means of Peek's formula. It is first necessary to calculate the critical voltage to neutral, and this is given by the following expression:

$$V_c' = 2302\, m_0 g_0 \delta r \log_{10} \frac{d}{r}$$

where m_0 = irregularity factor

g_0 = disruptive critical voltage gradient for air in kilovolts per inch, effective value, at a temperature of 77° F., and a pressure of 29·9 in. It is taken as 53·6 kilovolts per inch.

δ = density factor

The value of δ is given by:

$$\delta = \frac{17{\cdot}9\, p}{459 + t}$$

where p = barometric pressure in inches

t = air temperature in ° F.

The value of m_0 depends upon the shape of the cross-section of the wire, and on the state of the surface. For an absolutely smooth wire of one strand of circular section, m_0 is equal to unity, but if the wire is roughened owing to weathering, m_0 will be slightly less than unity. With stranded cables the departure of the cross-section from the circular form causes m_0 to be less than unity even when the cable is in perfect condition. Suitable values for m_0 can be found from the following table:

Irregularity Factor

Polished wires	1·0
Roughened or weathered wires	0·98 to 0·93
Seven-strand cables	0·87 to 0·83
Large cables with more than seven strands	0·90 approx.

For the calculation of the disruptive critical voltage the value of the irregularity factor m_d may be slightly different from m_0 except in the case of a polished conductor.

Knowing V_c' the corona loss in kW is calculated from the following empirical formula, in which the voltages are in kV.

$$P_c = 390 \times \frac{(f+25)l}{\delta} \times \sqrt{\frac{r}{d}} \times (V_{ph} - V_c')^2 \times 10^{-5}\,\text{kW}.$$

Example 1.—Take the case of the line considered in Chapter VII. We will assume the following conditions:

$$m_0 = \cdot 85$$
$$p = 29\cdot 5 \text{ in.}$$
$$t = 32^\circ \text{ F.}$$
$$\therefore \quad \delta = 1\cdot 07$$
$$\therefore \; V_c' = 2302 \times \cdot 85 \times 53\cdot 6 \times 1\cdot 07 \times \cdot 2 \times 2\cdot 875$$
$$= 64600 \text{ volts, or } 64\cdot 6 \text{ kV.}$$

For V_{ph} we can take the mean phase voltage of the line, viz.:

$$\frac{(69400 + 74490)}{2} = 72000 \text{ volts, or } 72 \text{ kV.}$$

$$\therefore \; P_c = 390 \times \frac{50+25}{1\cdot 07} \times 200 \times \sqrt{\frac{\cdot 2}{150}} \times (72 - 64\cdot 6)^2 \times 10^{-5}$$
$$= 110 \text{ kW. per phase}$$
or 330 kW. altogether.

Making the necessary alteration to the power intake at the generating end, we have:

$$P_0 = 3543 + 110$$
$$= 3653 \text{ kW.}$$

$\therefore$ Line efficiency:

$$= \frac{3336}{3653} \times 100$$
$$= 91\cdot 3 \text{ per cent.}$$

The inclusion of the density factor in the equation for the critical voltage shows that for the lower atmospheric pressures associated with high altitudes the critical voltage will be very materially reduced. A long line running over both plains and mountains will thus have different critical voltages at different points along the route.

Example 2.—Find the corona characteristics of a three-phase line 100 miles long, consisting of three 0·10-in. stranded copper conductors equally spaced, 8-ft. delta spacing, air temperature 80° F., altitude 8,000 ft., corresponding to an approximate barometric pressure of 28·8 in. Line voltage 110 kV., frequency 50 cycles per sec.

$$\text{Conductor diameter} = 0{\cdot}408 \text{ in.}$$

$$r = 0{\cdot}204 \times 2{\cdot}54 = 0{\cdot}518 \text{ cm.}$$

$$d/r = 96/0{\cdot}204 = 470$$

$$\log_{12} d/r = 2{\cdot}673$$

$$\sqrt{\frac{r}{d}} = \frac{1}{\sqrt{470}} = 0{\cdot}461$$

$$\delta = \frac{17{\cdot}9 \times 28{\cdot}8}{459 + 80} = 0{\cdot}956$$

$$1 + 0{\cdot}3/\sqrt{\delta r} = 1 + 0{\cdot}3/(0{\cdot}956 \times 0{\cdot}518)^{\frac{1}{2}}$$

$$= 1{\cdot}428$$

$$\text{Take } m_d = 0{\cdot}85$$

$$m_0 = 0{\cdot}72$$

$$\text{Disruptive critical voltage } V_d = 30/\sqrt{2} m_d \delta r \log_e \frac{d}{r} \text{ r.m.s. kV. to neutral}$$

$$= 30/\sqrt{2} \times 0{\cdot}85 \times 0{\cdot}956 \times 0{\cdot}518 \times 2{\cdot}3 \times 2{\cdot}673$$

$$= 54{\cdot}5 \text{ kV. to neutral}$$

$$\text{Visual critical voltage } V_v = 30/\sqrt{2}\, m_0 r(1 + 0{\cdot}3/\sqrt{\delta r}) \log_e \frac{d}{r}$$

$$= 30/\sqrt{2} \times 0{\cdot}72 \times 0{\cdot}956 \times 0{\cdot}518 \times 1{\cdot}428 \times 2{\cdot}3 \times 2{\cdot}673$$

$$= 66 \text{ kV. to neutral}$$

$$\text{Fair weather loss} = 390 \times \frac{(f + 25)\, l}{\delta} \times \sqrt{\frac{r}{d}} \times (V_{pb} - V'_c)^2 \times 10^{-5}$$

$$= 390 \times \frac{75 \times 100}{0{\cdot}956} \times 0{\cdot}461$$

$$\times (110/\sqrt{3} - 54{\cdot}5)^2 \times 10^{-5}$$

$$= 114 \text{ kW. per conductor}$$

$$= 342 \text{ kW. per three conductors}$$

Approx. loss during storm conditions. The disrupted critical value is taken as 0·8 of the value for fair weather, viz.— $0{\cdot}8 \times 54{\cdot}5 = 43{\cdot}6$ kV.

$$\therefore \text{Loss} = 390 \times \frac{75 \times 100}{0{\cdot}956} \times 0{\cdot}461 \times (110/\sqrt{3} - 43{\cdot}6)^2 \times 10^{-5}$$

$$= 560 \text{ kW. per conductor}$$

$$= 1680 \text{ kW. per three conductors}$$

Practical Importance of Corona

The above example shows that under normal conditions the loss of power due to corona is of no great importance, and consequently corona calculations do not enter directly into transmission line design. The basis of such design is essentially financial, the most economical line being the most acceptable. If such a line has a critical voltage under normal weather conditions of about 10 per cent. above the operating conditions, then it will be quite satisfactory, although Woodruff* suggests that "in a region where there are only a few storms each year on the average, it might be wise to allow considerable corona loss on these rare occasions, so as to enjoy the continuous benefit of the higher voltage without increasing too much the cost of line construction." If a line designed on economical considerations has an operating voltage near to, or greater than, the critical voltage for corona, then, either the size of the conductors must be increased, or the spacing must be increased, this entailing heavier towers. Either remedy will, of course, increase the cost of the line. Alternatively the line as designed can be operated at a lower voltage than anticipated, but the small choice of voltage due to standardisation may make this impossible. If such a reduced voltage is adopted the kVA. capacity will be reduced in proportion. Hence, as a rule, one of the other remedies will be preferable. With important transmission lines in which a large conductor is required, a stranded conductor of aluminium, possibly with a steel core, will be of such diameter that corona loss will be avoided, except possibly under storm conditions.

* *Electric Power Transmission and Distribution*, p. 109.

An advantage of corona is that it reduces transients, since charges induced on the line by lightning or other causes will be partially dissipated as a corona loss. In this way it acts as a safety valve, and in one or two cases lines have been purposely designed to have an operating voltage near to the critical voltage in order to do away with the necessity for, and expense of, lightning arrester gear. An objection to this scheme is that the critical voltage is not fixed for a given line, but may vary considerably with changes in the weather.

The disadvantages of corona are firstly, there is a definite loss of power, although this is not important except under abnormal weather conditions; and secondly, the non-sinusoidal corona current causes a non-sinusoidal drop of volts, and these may cause some interference with neighbouring communication circuits due to electromagnetic and electrostatic induction. The shape of the wave of corona current is such that there may be a large third harmonic. This is considered in detail in Chapter VIII.

CHAPTER IX

INDUCTIVE INTERFERENCE WITH NEIGHBOURING COMMUNICATION CIRCUITS

It is very common for a telephone line to be run along the same route as a power line, possibly for a few miles only or, in a few cases, for many miles. In the case of a communication line which is the property of a power company this line may be run on the same towers as the power line. Interference with such communication circuits may be due to both electromagnetic and electrostatic action, the former producing currents which are superposed on the true speech currents, thereby setting up distortion, and the latter raising the potential of the communication circuit as a whole. In extreme cases this raising of the potential above that of the ground may be sufficiently high to render the handling of the telephone receiver extremely dangerous, and in such cases elaborate precautions have to be taken to avoid this danger.

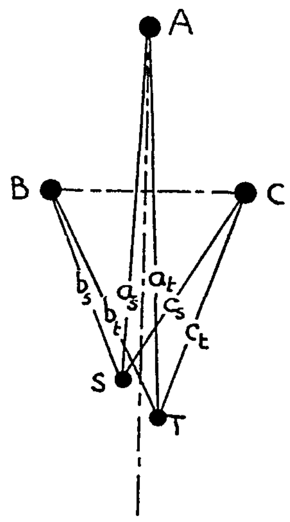


FIG. 9.1.—NEIGHBOURING POWER AND TELEPHONE LINES.

Electromagnetic Effect

Let A, B, and C, Fig. 9.1, be the power conductors and S and T the telephone conductors.

Let a_s = distance between A and S
a_t = distance between A and T
b_s = distance between B and S
b_t = distance between B and T
c_s = distance between C and S
c_t = distance between C and T
r = radius of each power conductor

Consider the loop formed by the conductors A and S. The self-induction of this is given by one half of the expression for the self-induction of a pair of parallel conductors, because only the conductor A is contributing to the E.M.F. induced in S. Denoting this self-induction by $L_{A.S.}$ we have

$$L_{A.S.} = 7{\cdot}4 \log_{10} \frac{a_s}{r} \times 10^{-4} \text{ henries per mile}$$

Similarly $L_{A.T.} = 7{\cdot}4 \log_{10} \frac{a_t}{r} \times 10^{-4}$ henries per mile

Hence, for the mutual induction between conductor A and the loop ST we have

$$M_A = L_{A.T.} - L_{A.S.}$$
$$= 7{\cdot}4 \left(\log_{10} \frac{a_t}{r} - \log_{10} \frac{a_s}{r}\right) \times 10^{-4}$$
$$= 7{\cdot}4 \times 10^{-4} \log_{10} \frac{a_t}{a_s} \text{ henries per mile}$$

Similarly for the mutual inductions M_B, between B and the loop ST, and M_C, that between C and the loop ST. Now these three mutual inductions are the result of fluxes which have the three-phase phase displacement of 120°, so that for the mutually inductive effect of all three power conductors on the loop ST, we have

$$\vec{M} = \vec{M}_A + \vec{M}_B + \vec{M}_C$$

The E.M.F. induced in the loop ST is then given by

$E_m = 2\pi f M I$ volts per mile, where I is the current in the power line.

It will be seen that since M is the vector sum of M_A, M_B, and M_C, there is a partial cancellation of the induced voltages due to the fundamental of the power current, and the greater the distance of the telephone circuit from the power circuit, the greater will be this cancellation, because M_A, M_B, and M_C will become more nearly equal in magnitude. This cancellation does not take place with harmonics which are a multiple of three, and consequently these frequencies, if present, may be very troublesome. The total mutual induction is, for these harmonics, the arithmetic sum of the three components, thus giving a total induced voltage of about three times the induced voltage per phase. These harmonics are troublesome in two other ways: firstly, because the induced E.M.F. is proportional to the frequency, and secondly, because the higher frequencies come within the audible range. Their effects could, of course, be eliminated by using a three-phase system with insulated neutral and without a fourth wire, but such a system is the exception and not the rule. It is also to be noted that certain types of load, such as rectifiers, have a tendency to introduce appreciable harmonics, and so also has the presence of corona.

If the distance between the power line and the communication circuit is not constant, or if branch lines are fed by the main transmission line, then it is necessary to consider each section of the line separately when calculating the induced E.M.F.

Electrostatic Effect

When a conductor such as A, Fig. 9.2, is run parallel to an infinite plane, the potential distribution between A and this plane is exactly similar to that between its image and the plane. Hence, if there is a quantity of electricity, q coulombs, per unit length of A, the potential of A with respect to the earth is given by

$$V_A = \frac{1}{2\pi\varepsilon_0}\int_r^a \left(\frac{q}{x} + \frac{q}{2a-x}\right) dx$$

$$= \frac{1}{2\pi\varepsilon_0}\, q \log_e \frac{2a-r}{r}$$

The potential of S above the earth due to the conductor A is given by a similar expression except that the limits of integration are now from a_s to a. Hence, denoting the potential S due to A by $V_{S.A.}$, we have

$$V_{S.A.} = \frac{1}{2\pi\varepsilon_0}\int_{a_s}^a \left(\frac{q}{x} + \frac{q}{2a-x}\right) dx$$

$$= \frac{1}{2\pi\varepsilon_0} \times q \log_e \frac{2a-a_s}{a_s}$$

$$\therefore V_{S.A.} = V_A \times \frac{\log_e \dfrac{2a-a_s}{a_s}}{\log_e \dfrac{2a-r}{r}}$$

$$= V_A \times \frac{\log_{10} \dfrac{2a-a_s}{a_s}}{\log_{10} \dfrac{2a-r}{r}}$$

Similarly, we can calculate the potentials above earth of S due to conductors B and C, and denoting these by $V_{S.B.}$ and $V_{S.C.}$ respectively, we have finally for the resultant potential of S with respect to earth

$$\vec{V}_S = \vec{V}_{S.A.} + \vec{V}_{S.B.} + \vec{V}_{S.C.}$$

Similarly for the potential of conductor T.

Numerical Example

30,000 kW. delivered 100 miles at 130,000 volts, power factor 0·9, frequency 50. Equilateral spacing of 11 ft., conductors of radius 0·216 in. Lowest conductor 50 feet from the ground.

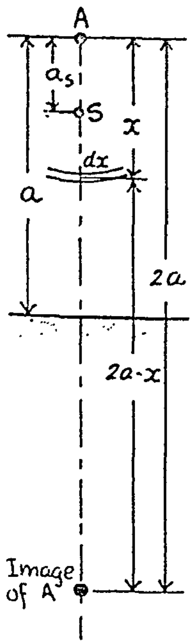


FIG. 9.2.

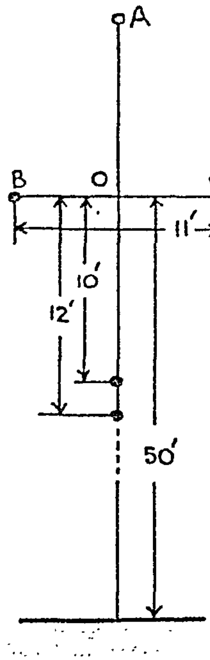


FIG. 9.3.

The telephone line is run on the same supporting towers, the conductors being 10 feet and 12 feet lower.

From Fig. 9.3 we have

$$AO = \frac{11\sqrt{3}}{2} = 9{\cdot}5 \text{ ft.}$$

$$\therefore a_s = 9{\cdot}5 + 10 = 19{\cdot}5 \text{ ft.}$$

$$a_t = 21{\cdot}5 \text{ ft.}$$

$$b_s = \sqrt{5{\cdot}5^2 + 10^2} = 11{\cdot}4 \text{ ft.}$$

$$c_s = 11{\cdot}4 \text{ ft.}$$

$$b_t = \sqrt{5{\cdot}5^2 + 12^2} = 13{\cdot}2 \text{ ft.}$$

$$c_t = 13{\cdot}2 \text{ ft.}$$

$$\therefore M_A = 7{\cdot}4 \times 10^{-4} \log_{10} \frac{21{\cdot}5}{19{\cdot}5}$$

$$= {\cdot}335 \times 10^{-4} \text{ henries per mile}$$

$$M_B = 7{\cdot}4 \times 10^{-4} \log_{10} \frac{13{\cdot}2}{11{\cdot}4}$$

$$= \cdot 468 \times 10^{-4} \text{ henries per mile}$$

$$M_C = M_B$$

$$= \cdot 468 \times 10^{-4} \text{ henries per mile}$$

$$\therefore M = (4{\cdot}68 - 3{\cdot}35) \times 10^{-5}$$

$$= 1{\cdot}33 \times 10^{-5} \text{ henries per mile}$$

$$I = \frac{30000 \times 1000}{\sqrt{3} \times 130000 \times {\cdot}9}$$

$$= 148{\cdot}4 \text{ amps.}$$

$$\therefore E_m = 2\pi f MI$$

$$= 2\pi \times 50 \times 1{\cdot}33 \times 10^{-5} \times 148{\cdot}4$$

$$= {\cdot}62 \text{ volt per mile.}$$

Thus for the whole route of 100 miles the induced voltage at fundamental frequency in the telephone circuit will be 62 volts.

For the electrostatic effect we have for conductor S:

$$V_{S.A.} = V_A \times \frac{\log_{10} \dfrac{2a - a}{a_s}}{\log_{10} \dfrac{2a - r}{r}}$$

$$= \frac{130000}{\sqrt{3}} \times \frac{\log_{10} \dfrac{119 - 19{\cdot}5}{19{\cdot}5}}{\log_{10} \dfrac{119 \times 12}{{\cdot}216}}$$

$$= 13800 \text{ volts}$$

$$V_{S.B.} = V_B \times \frac{\log_{10} \dfrac{2b - b_s}{b_s}}{\log_{10} \dfrac{2b - r}{r}}$$

$$= \frac{130000}{\sqrt{3}} \times \frac{\log_{10} \dfrac{100 - 11{\cdot}4}{11{\cdot}4}}{\log_{10} \dfrac{100 \times 12}{{\cdot}216}}$$

$$= 19000 \text{ volts}$$

$$V_{S.C.} = V_{S.B.} = 19000 \text{ volts.}$$

Hence, remembering that the total potential V_S is the vector sum of the above three components, we have

$$V_S = 19000 - 13800$$

$$= 5200 \text{ volts.}$$

Similarly for the potential of conductor T above earth.

This is, of course, an extreme case, but it illustrates the magnitude of the quantities which may be involved under the worst conditions.

Transposition

The numerical example above shows that in some cases the electromagnetically induced current in the communication circuit may be so great as to render speech impossible. The disturbance can be kept down by means of a thorough transposition of the conductors of both the power line and the telephone line. This transposition has the effect of splitting the induced E.M.F. into a series of mutually opposing E.M.F.s, the principle being identical

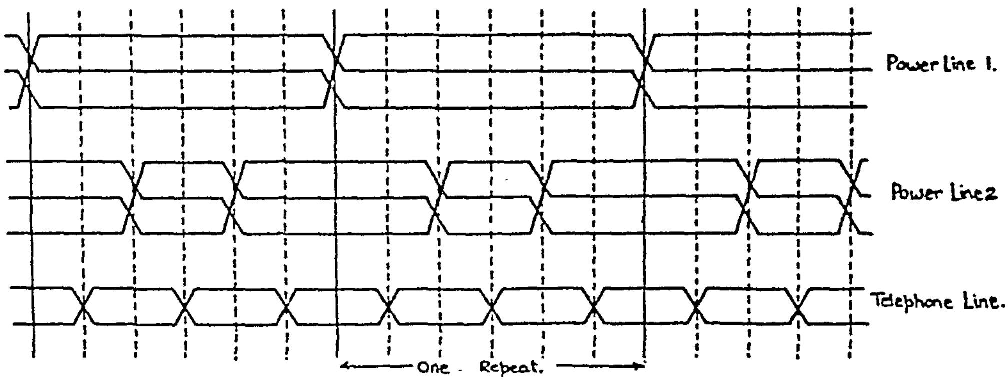


FIG. 9.4.—TRANSPOSITION SCHEME FOR TWO POWER LINES AND A TELEPHONE LINE.

with that underlying the transposition of heavy laminated conductors in large alternators and transformers. In the case of a telephone line running parallel to a single-circuit power line, if the power line has no branch lines, i.e. the current is constant throughout its length, and the spacings and distances between the two circuits remain constant, then a single transposition of the conductors of the telephone line is theoretically sufficient, but with both circuits run on the same towers it may be necessary to transpose the power conductors every three or four miles, and the telephone conductors about every 500 feet. The number of transpositions necessary is governed largely by the sensitiveness of the receiving apparatus. In the case of a telephone line running parallel to a double-circuit power line the problem is much more difficult, and it is necessary to transpose the conductors of both power lines in addition to those of the telephone line. A possible scheme is shown in Fig. 9.4, from which it will be seen that the scheme of transposition is a regular one for each individual circuit, and that it is arranged that not more than one transposition will take place at any one point in the line.

Each transposition of a telephone line consists of a complete

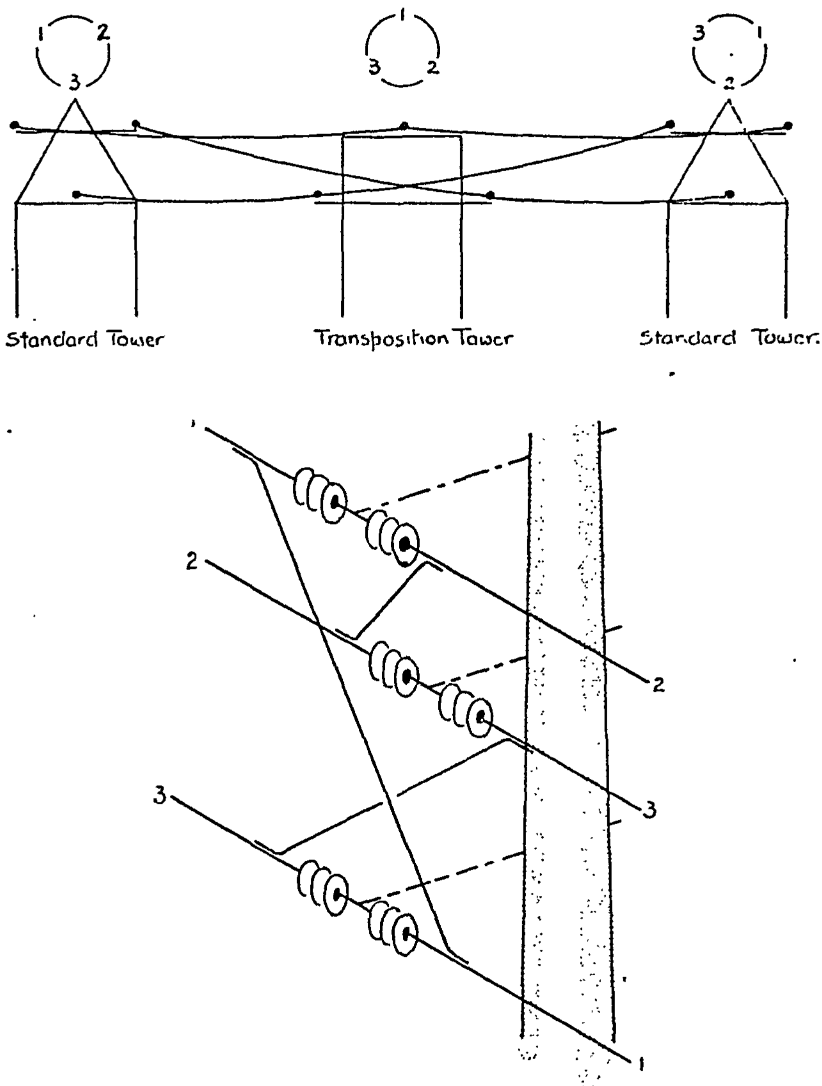


FIG. 9.5.—TWO METHODS OF TRANSPOSITION.

cross-over of the two conductors, while each transposition of a three-phase line consists of a twist, in a plane at right angles to the run of the line, of one-third of a revolution. Thus three transpositions are then necessary to bring the phases back to their original positions. Various methods of carrying out the trans-

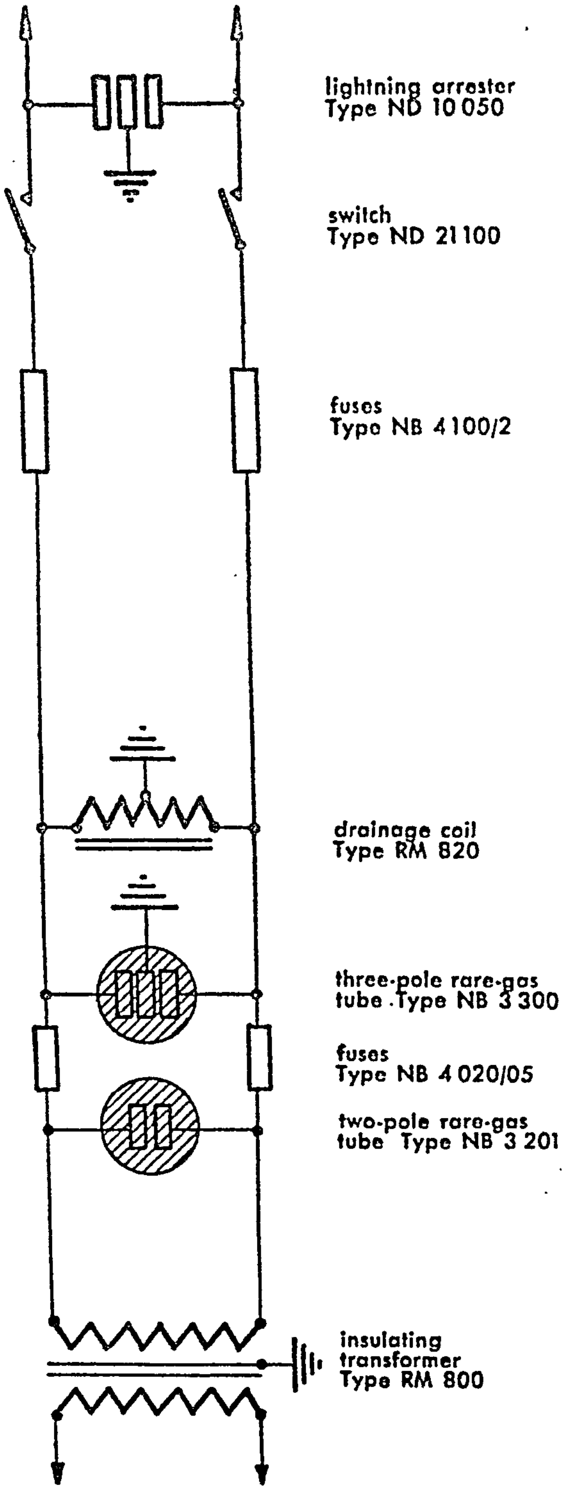


FIG. 9.6.—PROTECTIVE SYSTEM FOR A COMMUNICATION CIRCUIT. (*Ericsson Telephones, Ltd.*)

positions on both telephone and power lines are shown in Fig. 9.5.

It is to be noted that the electrostatic charging of the telephone line will also result in the flow of current, and this current also will tend to interfere with the clarity of the speech, an effect which cannot be eliminated entirely by transposition.

In extreme cases of electrostatic charging, as in single-phase electric railway systems with overhead trolley wires, it may be necessary completely to isolate the telephone apparatus from the telephone line by means of highly insulated transformers, and also to ensure the dissipation of the induced charges by means of such devices as earthed "drainage" coils and lightning arresters. One such scheme due to the Ericsson Co. is illustrated in Fig. 9.6. Beginning from the line it consists of a switch with an air-gap arrester, high-tension fuses, drainage coil, three-pole rare-gas tube with the centre pole earthed, further fuses, a two-pole rare-gas tube, and finally, an isolating transformer which completely isolates the receiving apparatus from the line. The three arresters are inserted so as to relieve the system of any excess voltages occurring between the line conductors, while the drainage coil, which is simply a choke with centre point connected to earth, prevents any accumulation of static charge. The rare-gas tubes are very sensitive to high-frequency disturbances such as might be produced by harmonics in the power line: they discharge freely as soon as the voltage reaches the neighbourhood of 300.

CHAPTER X

INSULATORS FOR OVERHEAD LINES

Materials and Types

THE insulators used in connection with overhead systems employing bare conductors are composed almost invariably of glazed porcelain, although some moulded materials are used for low voltages, and glass has been used on the European Continent and in America for medium voltages. The British Standard Specification gives particulars of porcelain only, and requires that "the porcelain shall be ivory white, sound, free from defects, and thoroughly vitrified so that the glaze is not depended upon for insulation." This thorough vitrification of the porcelain is of paramount importance, since the presence of pores or other air-spaces will lower the dielectric strength. Any sealed-in impurities will also decrease the dielectric strength, and it therefore follows that porcelain for electrical purposes must be both thoroughly air-free and impervious to the entrance of gases and liquids. Apart from the above requirement, electrical porcelain is practically identical with the pottery which has come down to us through the ages, since the obvious requirement of high dielectric strength is inherent in all porcelain which is homogeneous, and free from impurities. The dielectric strength of mechanically sound porcelain is of the order of 15,000 to 17,000 volts for every one-tenth inch thickness. Actually, it is very difficult to manufacture perfectly homogeneous porcelain of a thickness required for certain types of insulator, and it is then necessary to adopt a two- or three-piece construction, the various pieces being fixed and glazed separately and then cemented together.

The ultimate strength of electrical porcelain, as measured on the standard ceramic test piece, is 100,000 lbs./sq. in. in compression and 7,000 lbs./sq. in. in tension. The figures realised on the larger sections used in insulators are lower than these and may be taken as 40,000, and 2,000 to 3,000, for compression and tension respectively.

Very sound insulators are made from glass, its advantages being the very high dielectric strength of 35,000 volts per one-tenth inch thickness, and the possibility of adopting a one-piece design, no matter how large the insulator may be. It has also a lower coefficient of thermal expansion which minimises the strains due to temperature changes, is transparent to heat rays, thereby heating up but slightly when exposed to sunlight, and is mechanically stronger than porcelain when under compression. In tension it has about the same strength as porcelain. The disadvantages of glass are that moisture more readily condenses on the surface, and that in large sizes the great mass of material, combined with the irregular shape, may result in internal strains after cooling. Under ordinary atmospheric conditions toughened glass is therefore limited to about 30,000 volts, while in dry climates it can be used up to 50,000 volts.

Although B.S. 137 does not include glass other than toughened, forms of untoughened glass are in current use. These are the ordinary lime-soda glass and the boro-silicate glasses. Untoughened glass is used on the Continent and in America for low and medium voltages, say up to 20 kV. Its resistance to continuous mechanical loads and temperature changes is poor compared with porcelain and toughened glass.

Types

There are three types used in connection with overhead lines, viz.:

1. Pin-type.
2. Suspension-type.
3. Strain-type.

As the name suggests, the pin-type insulator is attached to a steel bolt or pin which is secured to a cross-arm on the transmission pole. The above-mentioned British Standards Specification requires that the porcelain shall not engage directly with a hard metal screw. B.S. 137 recognises two methods :—

(1) The provision of a taper thread cut on the head of the pin which screws into a threaded soft metal thimble cemented into the insulator.

(2) The provisions of a cast lead thread on the steel spindle which screws directly into a thread formed in the porcelain ; on the continent the pin, which has a plain top, is still sometimes wrapped with hemp and the threaded porcelain screwed on.

For operating voltages up to about 25,000 with ordinary designs of insulator a one-piece construction can be adopted, up to about 45,000 volts a two-piece, up to 66,000 volts a three-piece, and beyond this a four-piece insulator. Recent progress in design and manufacture has enabled much thicker sections to be adopted, with the result that for working voltages up to 33,000 a single-piece construction is possible, and not more than two parts even in the largest sizes. Actually, the tendency is to use pin-type insulators for voltages up to 50,000 only, since they become uneconomical for higher voltages. This is because their cost increases much more rapidly than the voltage, the law being

$$\text{Cost} \propto E^x \text{ where } x > 2$$

According to Taylor, the ratio of average initial cost per mile of pin-type to suspension-type is about three to four or five, but

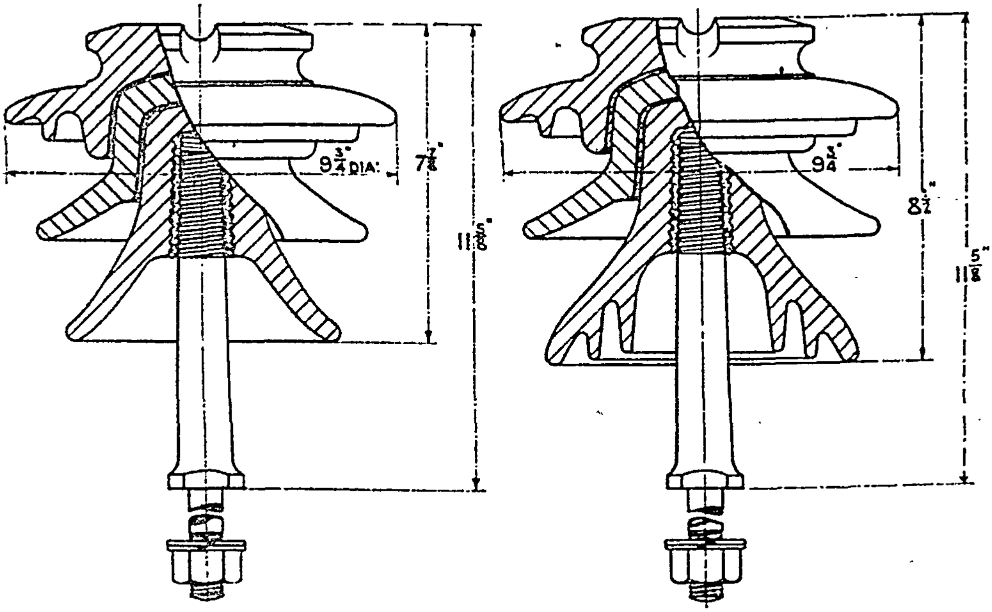


FIG. 10.1.—PIN-TYPE INSULATORS.

the cost of replacement for suspension insulators is usually much lower than for pin-type.

Two typical porcelain pin-type insulators are illustrated in Fig. 10.1. The characteristics of this type are as follows:

Type	Usual Working Voltage (kV.)	Average Puncture Voltage (kV.)	Height (in.)	Max. Diameter (in.)	Seepage Distance (in.)	Net Weight of Porcelain (lb.)
11004	3·3	85	3⅝	3⅝	5¼	1 3/16
5063	6·6	120	4¼	4 1/16	6¾	1¾
5065	11·0	140	6⅝	5¾	11¾	3 15/16
11253	33·0	250	9	10¾	29	23⅝

The usual working voltage refers to insulators for use in an industrial atmosphere in this country. Smaller insulators are used in clean conditions.

The significance of these figures will be appreciated from the following notes. There should be sufficient thickness of porcelain between the line conductor and the insulator pin (or other metal work) to give a factor of safety of up to 10 against puncture, but the insulator should be designed so that it will spark-over before it will puncture. The ratio of the spark-over voltage to the working voltage is called the safety factor, and for pin-type insulators this factor is much higher for low voltages than it is for

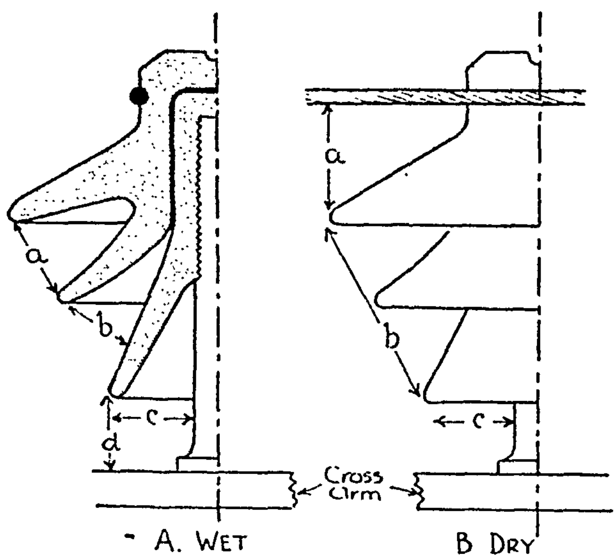


Fig. 10.2.—Arcing Distances, Wet and Dry.

high. The present tendency is to use pin-type insulators for low voltages only, say up to 11 kV., for which the factors of safety are 8·3 dry and 5 wet.

With a wet insulator the surfaces of the various pieces, or " sheds " as they are sometimes called, have no insulating value, so that the total arcing distance is the sum of the shortest distances from the edge of one shed to the nearest point on the next lower shed, plus the distance from the edge of the next lowest shed to the pin. With a wet insulator the arcing distance is thus the sum $a + b + c$ in Fig. 10.2(A). With a clean, dry insulator the surfaces of the sheds have their proper insulating value and the sparking distance is, therefore, the shortest distance from conductor to pin which avoids the insulator, e.g. $a + b + c$ in Fig.

10.2 (B). The pin must be long enough to make the vertical distance d in Fig. 10.2(A) greater than c, otherwise, in the case of an arc-over, the discharge will take place to the cross-arm instead of to the pin, thereby necessitating the renewal of the cross-arm. It should also be noticed that in the case of a steel cross-arm on a wooden pole, this should be connected to the earth wire of the pole, and in the case of a wooden cross-arm the earthing wire should be brought to a metal strip laid along the top of the arm so that the pins of the insulators pass through it.

Porcelain insulators of ordinary design arc over at somewhere about 10,000 volts per inch, so that if the sum of the distances $a + b + c$, dry, were 10 in., the insulator would arc over at about 100,000 volts.

B.S. 137 specifies the following factors under clean area conditions :—

Working Voltage (kV.)	Voltage to Earth (kV.)	Dry S.O.V. / Volts to Earth	Wet S.O.V. / Volts to Earth
11	6·3	8·3	5
66	38	4·2	3·5
275	150	3·7	3·3

In this table the higher values naturally refer to strings of suspension-type insulators.

The insulator and its pin, or other support, should be sufficiently strong mechanically to withstand the resultant force due to the combined effects of wind pressure and weight of span (and ice load if any). At terminal poles there is, in addition, the almost horizontal pull due to the tension of the conductor. This, in particular, causes such a great bending moment at the bottom of the pin, with pin-type insulators, this being transmitted to the cross-arm, that for a line insulated with pin-type insulators it is desirable to use some type of strain insulator at all terminal or dead-ending poles. In connection with the mechanical strength it is to be noted that the insulator is stronger than the pin. In fact, the pin should be designed as a cantilever and the elastic limit of the steel should be just reached at the load for which the pin is designed.

Recent researches on dielectrics has shown that thickness of insulation and length of sparking distance are not the only factors which decide the electrical performance of an insulator, but that the distribution of electrostatic tubes of force and equipotential surfaces is also of considerable importance in deciding corona formation and flash-over voltage. Researches carried out by Gilchrist and Klinefer have shown that the various sheds should be shaped so as to conform as closely as possible to the equipotential surfaces, that the body should be shaped so as to conform to the electrostatic tubes of force, that the leakage resistance of the various sheds should be approximately equal, and that the capacitance of the various sheds should be approximately equal. These requirements necessitate an insulator of entirely different shape from that usually adopted, but, as it happens, they are only of real importance when the voltage is so high that the suspension-type insulator would be adopted. Fig. 10.3 shows that modern insulators do largely conform to these requirements.

Suspension Insulators

We have seen that the cost of a pin-type insulator increases very rapidly as the working voltage is increased. For high voltages this type is therefore uneconomical, and there is the further disadvantage that replacements are expensive. For these reasons high-voltage lines are insulated by means of suspension insulators in which, as their name indicates, the line conductor is suspended below the point of support by means of the insulator or insulators. Several important advantages follow from this system.

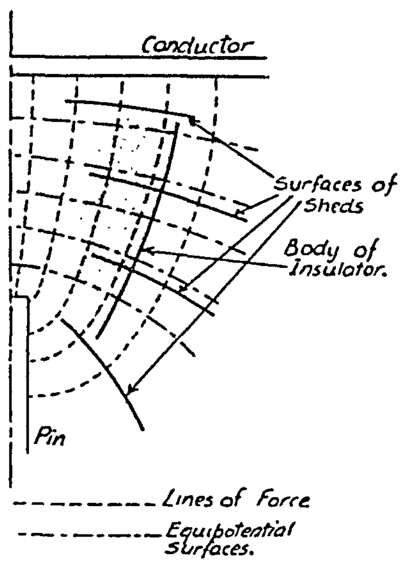


FIG. 10.3.—ELECTROSTATIC FIELD OF A PIN-TYPE INSULATOR.

(i) Each insulator is designed for a comparatively low working voltage, usually about 11,000 volts, and the insulation for any required line voltage can be obtained by using a "string" of a suitable number of such insulators.

(ii) In the event of failure of an insulator, one unit, instead of the whole string, has to be replaced.

(iii) The mechanical stresses are reduced, since the line is suspended flexibly; with pin-type insulators the rigid nature of the attachment results in fatigue and ultimate brittleness of the wire, due to the alternating nature of the stress. Also since the string is free to swing there is an equalisation of the tensions in the conductors of successive spans.

(iv) In the event of an increase in the operating voltage of the line, this can be met by adding the requisite number of units to each string, instead of replacing all insulators, as would be necessary with the pin-type.

Owing to the free suspension, the amplitude of swing of the conductors may be large compared with that on a pin-type insulated line and the spacings should therefore be increased.

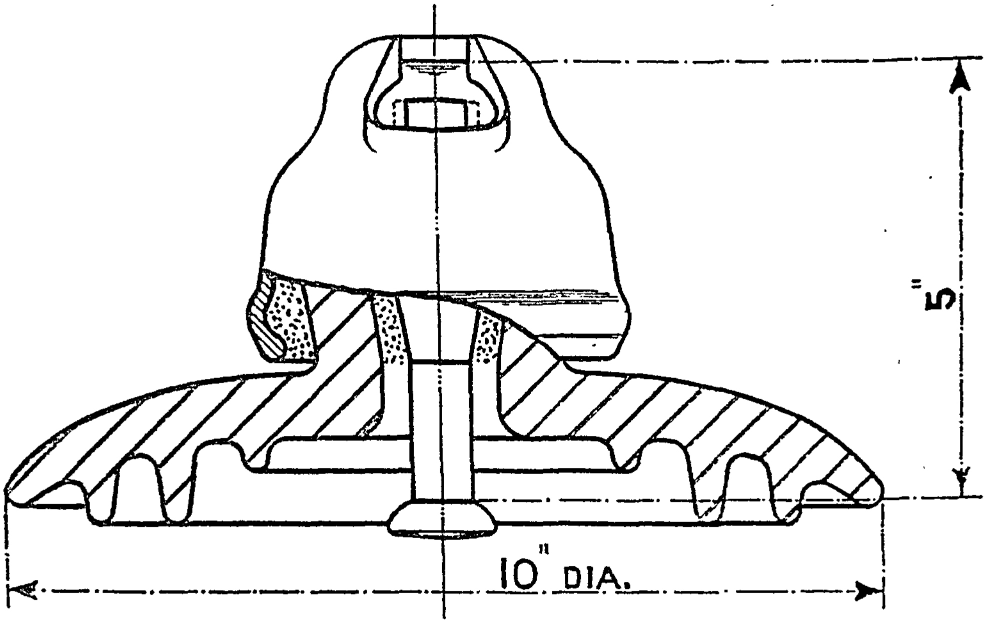


FIG. 10.4.—SUSPENSION INSULATOR.
(*Messrs. Buller's, Ltd.*)

There are several types of suspension insulator, that illustrated in Fig. 10.4 being most frequently used in this country, having been adopted for the insulation of the Grid lines. It will be seen that it consists of a single disc-shaped piece of porcelain grooved on the under surface to increase the surface leakage path, and to a metal cap at the top, and to a metal pin underneath. The cap is recessed so as to take the pin of another unit, and in this way a string of any required number of units can be built up. The cap is secured to the insulator by means of cement. Various means of securing the pin have been tried, but all have been abandoned in favour of cementing. Mechanical methods of fixing have proved unsatisfactory since they caused concentrations of mechanical stress which led to failure in service. On the other hand, cement acts as a good distributor of mechanical stress and cemented insulators of good mechanical design have an excellent service record.

The usual diameter of this type of insulator is ten inches, since it has been found that this size gives a suitable ratio of spark-over to puncture voltage. Increasing the diameter increases the spark-over voltage, of course, but it lowers the above ratio and this is undesirable.

Another type of suspension insulator, the "Hewlett" insulator, is shown in Fig. 10.5. This has ten-inch discs, each disc having two curved tunnels which lie in planes at right angles to one another. Lead-covered steel U links are threaded through these tunnels and are fastened to similar links on adjacent upper and lower units. Thus no cementing or special fastening is required, and the design is very simple. This ten-inch pattern has been found very suitable for lines up to, and including, 33,000 volts, where the conditions of mechanical loading allow of its use. Since the ultimate mechanical strength is decided by the steel links, and not by the porcelain, this pattern is very strong and has the peculiar advantage that the breaking of a porcelain disc will not allow the line to fall, or, in fact, interrupt the service if a string of several units is used. Its disadvantage is that this construction is, of necessity, associated with high electrostatic stresses in the porcelain immediately between the links, so that the liability to puncture is greater than with other types.

The Hewlett insulator is also used as a strain insulator (see below), particularly on pin-type insulator lines up to 33 kV. It is supplied in various disc diameters from 6 in. to 10 in. and for mechanical working loads from 2,000 to 8,000 lbs.

The performance characteristics for single units of the ten-inch and six-inch discs are as follows:

Type.	Spark-over Voltage, Dry (kV.).	Spark-over Voltage, Wet (kV.).
10-in. disc	75	48
6-in. disc	55	27

Strain Insulators

These insulators are used to take the tension of the conductors at line terminals and at points where the line is dead-ended, as for

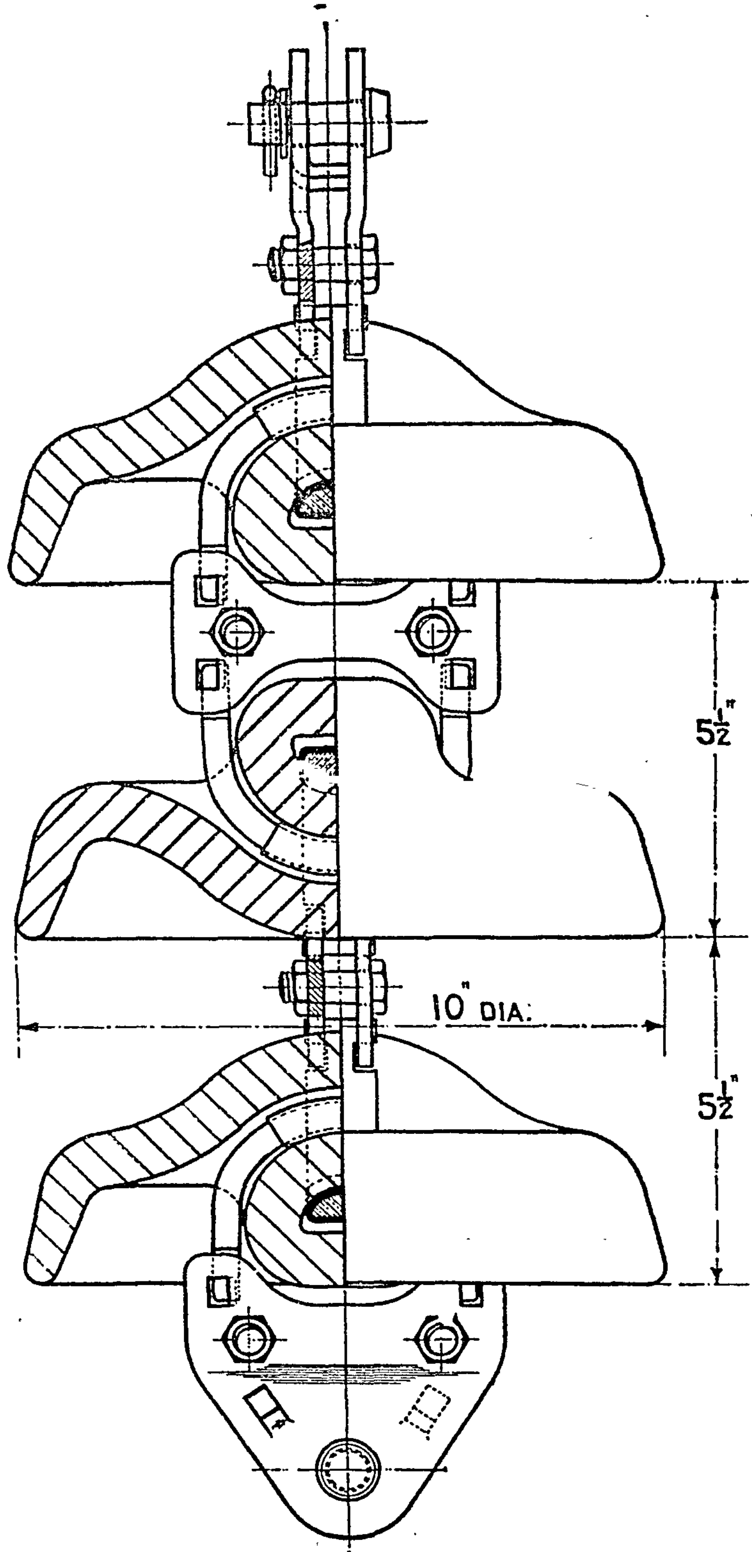


FIG. 10.5.—HEWLETT INSULATORS. (*Messrs. Buller's, Ltd.*)

example some road-crossings, junctions of overhead lines with cables, river crossings, at angle towers where there is a change in direction of the line, and so on. For light low-voltage lines, say, up to 11,000 volts, the shackle insulator is suitable, but for higher voltages a string of suspension-type insulators is necessary. Where the tension is exceedingly high, as at long river spans, two, three, or even four strings of insulators in parallel have been used. However, the present practice in this country is to avoid the use of multiple strings where heavy mechanical loads are required, and to use units of higher mechanical rating. Standard units are made for 4,000, 8,000, and 12,000 lbs. maximum working load, which suffice for all normal construction on lines up to 275 kV. For river crossings it is necessary to use multiple strings.

It will be realised that when used as strain insulators the discs are in a vertical instead of a horizontal plane. This may make some difference to the spark-over voltage, wet, the value for the standard ten-inch insulator being 55,000, and for the standard six-inch Hewlett insulator 33,000 volts.

Potential Distribution over a String of Suspension Insulators

If a string of similar suspension insulators could be situated so far from neighbouring metal work that the capacitances between this metal work and the metal of the insulators would be negligibly small in comparison with the capacitance of each unit, then the potential difference applied to the whole string would be divided equally between the various units. The capacitance of each unit is sometimes called the "mutual capacity." In practice this condition is not fulfilled because of the nearness of the tower, the cross-arm, and the line, and we shall see that these additional capacitances have an important effect in proportioning the potential difference between the units. Denoting the spark-over voltage by S.O.V. we have for a string of n insulators

$$\text{String efficiency} = \frac{\text{S.O.V. for } n \text{ insulators}}{n \times \text{S.O.V. for one insulator}}$$

This efficiency is generally higher for wet than for dry flash-over, except for a small number of units in the string ; it also depends on the ratio

$$\frac{\text{Capacity per insulator}}{\text{Capacity to earth}}$$

The following results of an actual test on ten-inch suspension insulators show how the string efficiency depends upon the number of units in the string, and also on the condition, whether dry or wet.

No. in series.	S.O.V., Dry (kV.).	String Effic., Dry.	S.O.V., Wet (kV.).	String Effic., Wet.
		Per cent.		*Per cent.*
1	75	100	48	100
2	140	93·4	90	92
3	195	86·7	128	89
4	245	81·8	166	86·5
5	295	78·8	205	85·5
6	345	76·7	245	85·1
7	395	75·4	280	83·4
8	445	74·2	320	83·4
9	490	72·8	355	82·2
10	535	71·4	385	80·3

Fig. 10.6 shows a string of suspension insulators with the circuit diagram of capacitances. It will be seen that the mutual capacitances of the various units are in series between line and

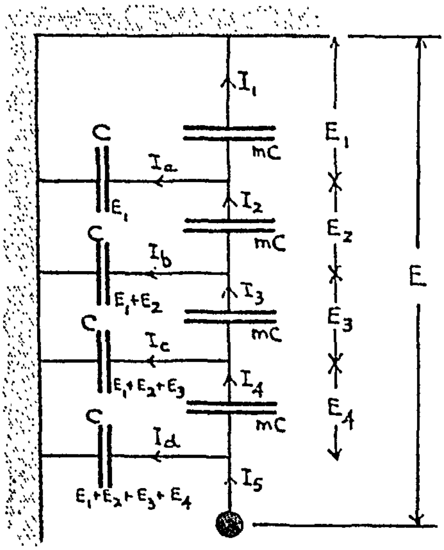


FIG. 10.6.—CAPACITANCE OF A STRING OF SUSPENSION INSULATORS.

ground, while the capacities to ground act in the manner of a series of shunts to ground. The performance is thus dependent on the ratio

$$m = \frac{\text{mutual capacitance}}{\text{capacitance to ground}}$$

Let C = capacitance to ground

$\therefore mC$ = mutual capacitance

With the notation indicated in the figure we have

$$I_1 = \omega C m E_1$$
$$I_a = \omega C E_1$$
$$\therefore I_2 = I_1 + I_a$$
$$= \omega C E_1 (1 + m)$$
$$\therefore E_2 = \frac{I_2}{\omega C m}$$
$$= E_1 \cdot \frac{m + 1}{m} = E_1\left(1 + \frac{1}{m}\right) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The voltage producing the current I_b

$$E_1 + E_2 = E_1\left(1 + \frac{m + 1}{m}\right)$$
$$= E_1 \cdot \frac{2m + 1}{m}$$
$$\therefore I_b = \omega C E_1 \cdot \frac{2m + 1}{m}$$

also $I_3 = I_b + I_2$

$$= \omega C E_1 \left(\frac{2m + 1}{m} + 1 + m\right)$$
$$= \omega C E_1 \left(\frac{m^2 + 3m + 1}{m}\right)$$

But $I_3 = \omega m C E_3$

$$\therefore E_3 = E_1 \cdot \frac{m^2 + 3m + 1}{m^2} = E_1\left(1 + \frac{3}{m} + \frac{1}{m^2}\right) \quad . \quad (2)$$

The voltage producing the current I_c

$$E_1 + E_2 + E_3 = E_1 \left(1 + \frac{m + 1}{m} + \frac{m^2 + 3m + 1}{m^2}\right)$$
$$\therefore I_c = \omega C E_1 \left(1 + \frac{m + 1}{m} + \frac{m^2 + 3m + 1}{m^2}\right)$$
$$= \omega C E_1 \left(\frac{3m^2 + 4m + 1}{m^2}\right)$$
$$\therefore I_4 = I_3 + I_c$$
$$= \omega C E_1 \left(\frac{m^2 + 3m + 1}{m} + \frac{3m^2 + 4m + 1}{m^2}\right)$$

But $I_4 = \omega C m E_4$

$$\therefore E_4 = E_1 \left(\frac{m^2 + 3m + 1}{m^2} + \frac{3m^2 + 4m + 1}{m^3}\right)$$

$$= E_1\left(\frac{m^2+3m+1}{m^2}+\frac{3m^2+4m+1}{m^3}\right)$$

$$= E_1\left(1+\frac{6}{m}+\frac{5}{m^2}+\frac{1}{m^3}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For the fifth insulator from the top, similar reasoning gives

$$E_5 = E_1\left(1+\frac{10}{m}+\frac{15}{m^2}+\frac{7}{m^3}+\frac{1}{m^4}\right)$$

and so on for the complete string. Finally we have

$$E = E_1 + E_2 + E_3 + \quad . \quad . \quad .$$

and thus by substituting the above values of E_2, E_3, etc., in terms of E_1 we have an equation for E_1 in terms of E. The voltages E_2, E_3, etc., can then be calculated.

As a numerical example take the following problem. Find the potential difference across each unit of an overhead line suspension insulator consisting of four similar units. The pressure between the line conductor and earth is 60 kilovolts and the ratio of the capacity of each unit insulator to the capacity relative to earth, of each intermediate section of the connecting metalwork, is five to one. It is assumed that no leakage takes place (City and Guilds).

$$m = 5$$

$$\therefore E_2 = E_1\left(1+\frac{1}{5}\right) = 1{\cdot}2E_1$$

$$E_3 = E_1\left(1+\frac{3}{5}+\frac{1}{25}\right) = 1{\cdot}64E_1$$

$$E_4 = E_1\left(1+\frac{6}{5}+\frac{5}{25}+\frac{1}{125}\right) = 2{\cdot}408E_1$$

$$\therefore E = E_1 + E_2 + E_3 + E_4$$

$$\therefore 60 = E_1(1 + 1{\cdot}2 + 1{\cdot}64 + 2{\cdot}408) = 6{\cdot}248E_1$$

$$\therefore E_1 = \frac{60}{6{\cdot}248} = 9{\cdot}6 \text{ kV.}$$

$$E_2 = 1{\cdot}2E_1 = 11{\cdot}5 \text{ kV.}$$

$$E_3 = 1{\cdot}64E_1 = 15{\cdot}8 \text{ kV.}$$

$$E_4 = 2{\cdot}408E_1 = 23{\cdot}1 \text{ kV.}$$

Total 60·0 kV.

This example shows very clearly that the units far removed from the line conductor are stressed very much below their normal value, and that only the unit adjacent to the line is highly stressed. The string efficiency is also given by

$$\frac{\text{Voltage across string}}{n \times \text{voltage across unit adjacent to line}}$$

where n is the number of units in the string. Thus, when $n = 4$, as in the numerical example, and $m = 5$

$$\text{String efficiency} = \frac{E}{4 \times E_4}$$
$$= \frac{60}{4 \times 23{\cdot}5}$$
$$= {\cdot}638 \text{ or } 63{\cdot}8\,\%$$

It will be seen that the performance of the string is dependent on the value of m, and that as m is increased the division of voltage becomes more equalised. Thus

when $m = 1$, $E_2 = 2E_1$; $E_3 = 5E_1$; $E_4 = 13E_1$; and so on
when $m = 5$, $E_2 = 1{\cdot}2E_1$; $E_3 = 1{\cdot}64E_1$; $E_4 = 2{\cdot}48E_1$; and so on
when $m = 10$, $E_2 = 1{\cdot}1E_1$; $E_3 = 1{\cdot}31E_1$; $E_4 = 1{\cdot}65E_1$; and so on
when $m = \infty$, $E_2 = E_1$; $E_3 = E_1$; $E_4 = E_1$; and so on

Thus, when m is small the top units are performing very little work and adding further units has very little effect on the voltage across the unit adjacent to the line conductor. For high line voltages, say, over 100,000 volts, it is thus imperative that m shall be large, otherwise an impossibly large number of units per string will be required.

In the above discussion the effects of leakage and corona have been neglected; they are to equalise somewhat the voltage distribution. The capacitances between the various connectors and the line conductor have also been neglected. These also tend to equalise the distribution and, with certain ratios between the various capacitances, may give the minimum voltage, not on the top unit, but on one lower down. In every case the maximum voltage is on the bottom unit.

Methods of Equalising the Potential

1. *Elimination of m*

Since an increase in m improves the performance of the string the obvious method is to make m as large as possible, and therefore to make the capacitances to ground relative to the mutual capacitances as small as possible. Something can be done by using a long cross-arm, but obviously this method is limited by the strength and cost of the towers; $m = 10$ is about the limit that can be achieved by this method. Piernet has made m very high, and eliminated the capacitances to earth by using an insulating material for the connections between units, instead of metal, but the method has not been generally adopted.

2. *Grading of the Units*

From the previous calculations it will be apparent that if the mutual capacitance of the lowest unit can be increased, and that of the other units progressively decreased, the top unit having the lowest capacitance; then, since the voltage for a given current is

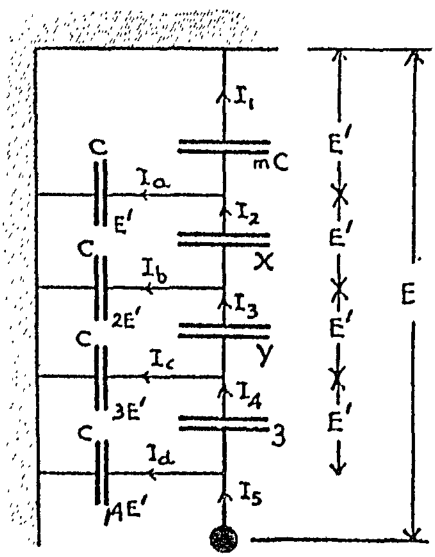


FIG. 10.7.—CAPACITANCE GRADING.

inversely proportional to the capacitance, it follows that the effect will be to reduce the voltage across the lower units and to increase that across the higher units. By correct grading of the capacitances complete equality of voltages can be obtained. In the following discussion the capacitances between metal work and line conductor are again neglected. Referring to Fig. 10.7, and putting

C = each capacitance to earth
mC = mutual capacitance of top unit
x = mutual capacitance of second unit
y = mutual capacitance of third unit, and so on
$I_1 = \omega C m E'$
$I_a = \omega C E'$

$$\therefore I_2 = I_1 + I_a$$
$$= \omega CE'(1 + m)$$

But $E' = \dfrac{I_2}{\omega x}$

$$\therefore E' = \frac{\omega CE'(1 + m)}{\omega x}$$

$$\therefore \quad x = C(1 + m) = mC + C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Voltage producing $I_b = 2E'$

$$\therefore I_b = 2\omega CE'$$

also $I_3 = I_b + I_2$

$$= 2\omega CE' + \omega CE' (1 + m)$$
$$= \omega CE'(3 + m)$$

But $I_3 = E'\omega y$

$$\therefore y = C(3 + m)$$
$$= C(1 + 2 + m)$$
$$= mC + (1 + 2)C$$

and so on, giving the capacity for the nth unit

$$C_n = mC + (1 + 2 + 3 + \ldots + \overline{n - 1})C$$

For example, taking $m = 5$ as before, the capacities of the various units, in terms of C, are as follows :

$$C_1 = 5C$$
$$C_2 = 5C + C = 6C$$
$$C_3 = 5C + (1 + 2)C = 8C$$
$$C_4 = 5C + (1 + 2 + 3)C = 11C, \text{ and so on.}$$

It will be obvious that to carry out the capacitance grading to this extent will be quite impossible in practice, since it will require that all the insulators in the string will be different from one another. Good results have been obtained by using standard insulators for most of the string and larger units for that adjacent to the line, and possibly the next insulator above. Also with comparatively light lines it is possible to use the smaller Hewlett units for most of the string and two or three standard 10-in. units at the bottom. In this way the total number of units required, and, therefore, the cost of the string, can be reduced, but there is still the operating disadvantage that stocks of different-sized insulators must be carried. Alternatively the capacitances of the bottom units can be increased by fitting metal caps, or even by painting a portion of the top surface with a conducting paint. In practice, the method of capacitance grading is only suitable for very high-voltage lines, say, 200,000 volts or over.

3. *Static Shielding*

The voltage distribution is controlled in this method by the employment of a grading or guard ring, which usually takes the form of a large metal ring surrounding the bottom unit and connected to the metalwork at the bottom of this unit, and therefore to the line. This ring, or shield, has the effect of increasing the capacitances between metal work and line, which capacitances we

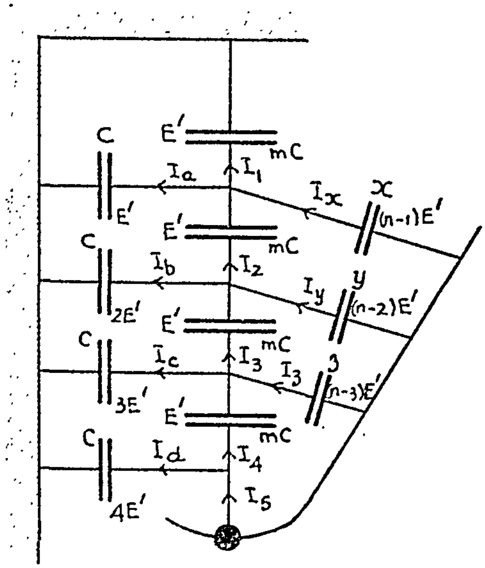


FIG. 10.8.—STATIC SHIELDING.

have neglected in the previous discussions. Using the same notation as before, assuming similar units, and denoting the capacities to the shield by x, y, z . . ., we have, from Fig. 10.8,

$$I_2 = I_1 + I_a - I_x$$
$$I_3 = I_2 + I_b - I_y \text{ etc.}$$

But if the voltage is E′ across every unit and all n units are identical, the currents I_1, I_2, I_3, etc., must be equal, from which we have:

$$I_x = I_a\,;\; I_y = I_b\,;\; I_z = I_c$$
$$\therefore\; E'C\omega = (n-1)E'x\omega$$
$$2E'C\omega = (n-2)E'y\omega$$
$$3E'C\omega = (n-3)E'z\omega$$
$$\therefore\; x = \frac{C}{n-1}$$
$$y = \frac{2C}{n-2}$$
$$z = \frac{3C}{n-3}$$

Hence in general, the capacitance from the shield to the pth link from the top (the link being the connector between two consecutive units, and not the metalwork joining the whole string to the cross-arm), is given by:

$$C_p = \frac{pC}{n - p}$$

With this method also it is impossible to obtain in practice an equal distribution of voltage, but considerable improvements are possible nevertheless. For example, tests on a certain 14-unit

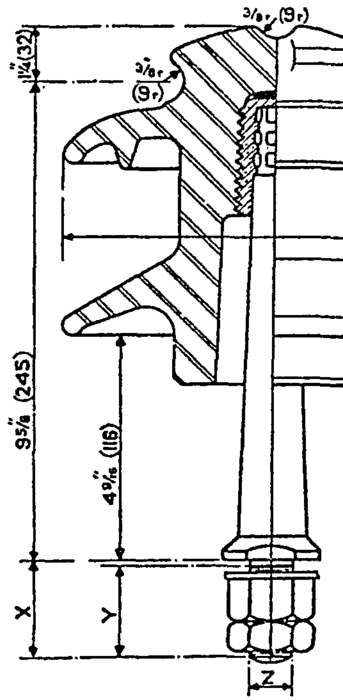


FIG. 10.9.—PIN-TYPE, ANTI-DIRT INSULATOR.
(Messrs. Steatite and Porcelain Products, Ltd.)

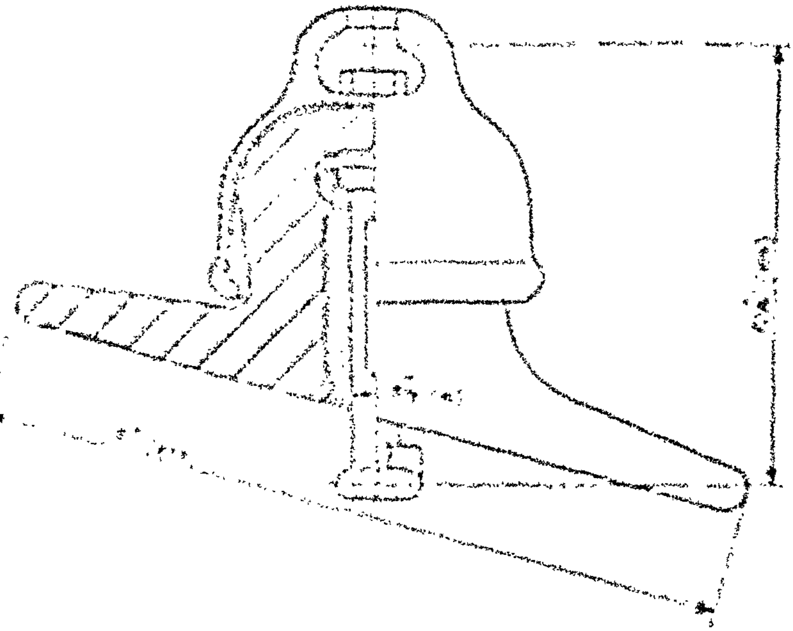
string gave 18·3 per cent. of the total voltage on the bottom unit unshielded, and 11·8 per cent. when shielded.

Incidentally, the grading shield serves the purpose of an arcing shield when used in conjunction with an arcing horn fixed at the top end of the string. In the event of a power arc following a flash-over due to some type of over-voltage, the arc will usually take the path between horn and shield and stay clear of the insulator string.

Special Types of Insulator

Where the conditions of service depart from normal, for example, where there are smoke, chemical, or salt deposits, it may be [illegible] design of insulator, some insulators of [illegible] satisfactorily. Also in districts [illegible] are frequent, or where there may be over- [illegible] steep-fronted travelling waves (see Chapter XVIII), [illegible] set up are in the nature of [illegible] it may be found that insulators which operate with [illegible] over-voltages of line frequency may punc- [illegible] steep-fronted waves. Hence, in such situa- [illegible] special design are required.

The requirements of anti-fog insulators are firstly, a long creep- [illegible] profile which derives the maximum



[illegible]

[illegible]

[illegible]

type gave good service in some localities, it was unsatisfactory for the following reason : Imagine the insulator clean, its leakage resistance thus being a maximum ; during a fine spell there will be a gradual accumulation of deposit, which will have little effect so long as the atmosphere is dry. Prior to a spell of wet weather there is usually a considerable increase in atmospheric humidity before the fall of any rain, and consequently the surface insulating property will break down with possible spill-over of the insulator *before* the rain can wash the deposit away. This has been over-

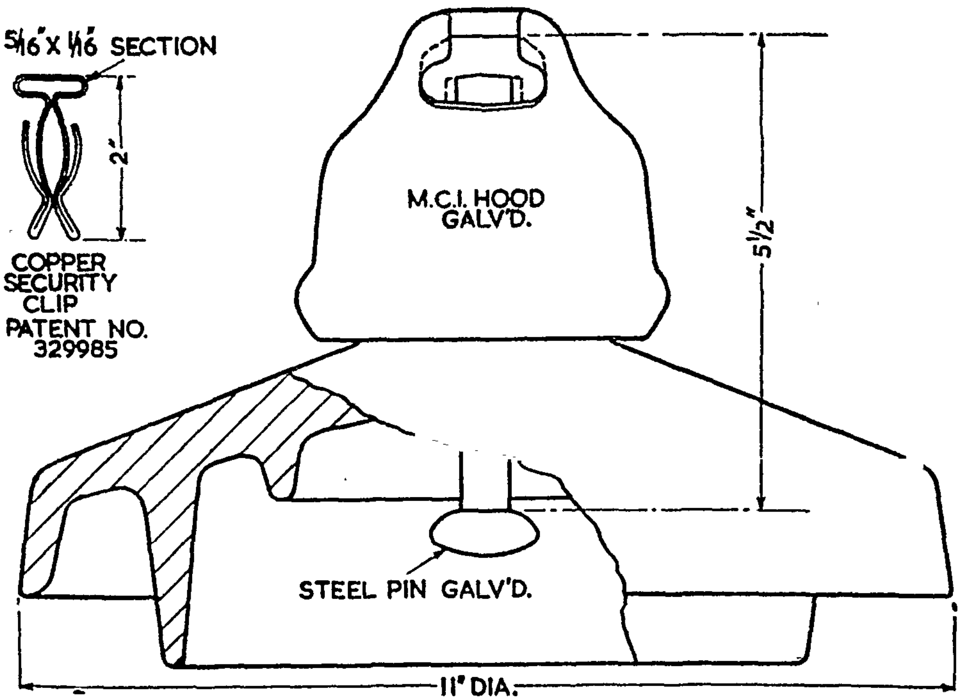


FIG. 10.11.—DEEP-SHED INSULATOR.

(Messrs. Buller's, Ltd.)

come by the provision of a long leakage path, one method being to design the insulator with a deeper shed than usual. An insulator of this type is illustrated in Fig. 10.11.

In order to avoid puncture under the action of steep-fronted voltage waves it is essential that the intense electrostatic stresses in the neighbourhood of the pin shall be avoided. This is accomplished by doing away with the pin altogether and using a solid-core insulator whose base is secured to a shank of suitable shape. A pin-type insulator of this type for a working voltage of 33 kV. is shown in Fig. 10.12.

It has been pointed out that the ideal design of insulator is one in which the surface coincides with an equipotential surface. With porcelain this ideal can only be approximately attained, but Dr. H. B. Smith developed a suspension type in which the insulation was achieved by a wooden rod, and the shape of the electro-

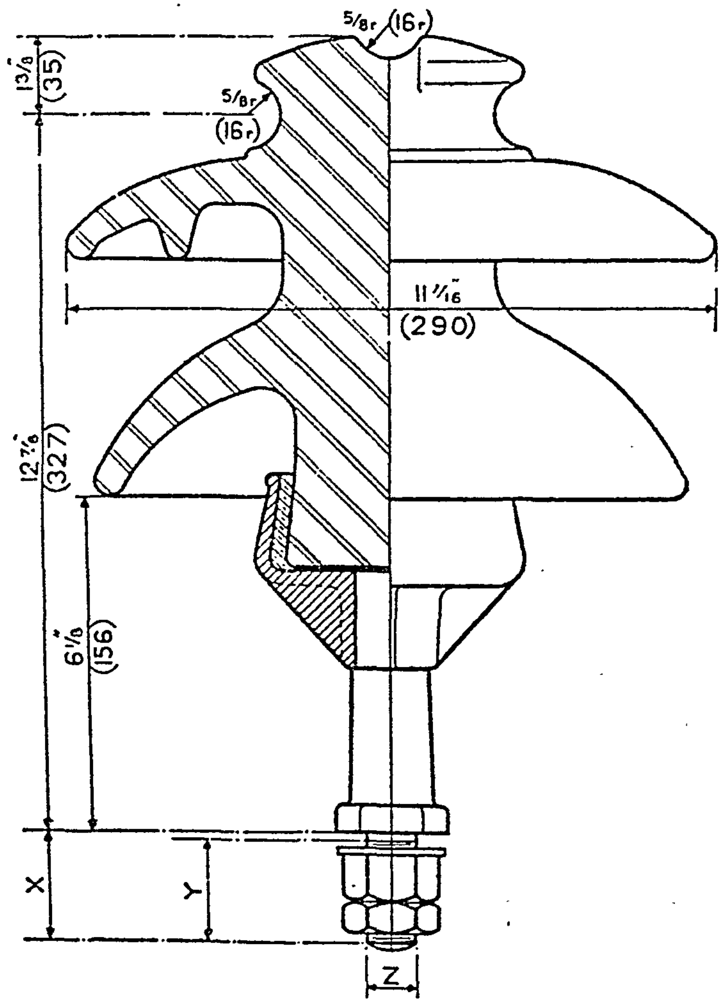


FIG. 10.12.—SOLID-CORE INSULATOR.
(*Messrs. Steatite and Porcelain Products, Ltd.*)

static field controlled by means of an upper and lower metal shield. The design of a 110-kV. unit is shown in Fig. 10.13, from which it will be seen that the insulator proper is a wooden rod 2 in. in diameter. The upper metal shield is dished, and is 45 in. in diameter, while the lower metal shield is a spun copper ring formed by a 6-in. tube and having an outside diameter of 17 in.

The overall length of the complete unit is 38½ in. The tests results of this insulator are 280 kV. for dry and 200 kV. for wet flash-over. Corona formation is absent until the applied voltage reaches about 95 per cent. of the flash-over voltage. Owing to its central position the insulating rod is situated in the region of minimum field strength, so that when electrical breakdown does occur, it is somewhere in the surrounding field and not along the rod. The mechanical strength is higher than that of a string of suspension units for the same voltage.

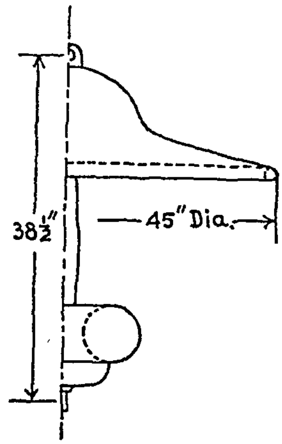


FIG. 10.13.— SMITH'S SUSPENSION INSULATOR FOR 110 kV.

This insulator is described in order to show one of the ways in which the problem of the very high-voltage insulator has been tackled. It has not been adopted for two purely practical reasons. (*a*) Neither wood nor any other organic material has been found suitable for out-door insulators ; (*b*) any design which depends on a uniform electric field is easily disturbed by dirt and moisture deposits.

Testing of Insulators

B.S. 137 groups the tests made on insulators into three categories : flash-over tests, sample tests, and routine tests. In each category there is a group of individual tests. Flash-over tests are a design test taken to three insulators only to prove the correctness of the design ; sample tests are to prove the quality of manufacture and are taken on ½ per cent. of the insulators supplied ; routine tests are carried out on all insulators.

I. Flashover Tests :

(*a*) 50 per cent. dry impulse flash-over test.
(*b*) Dry flash-over and dry one-minute test.
(*c*) Wet flash-over and one-minute rain-test.

II. Sample Tests :

(*d*) Temperature-cycle test.
(*e*) Mechanical test.
(*f*) Electro-mechanical test.
(*g*) Puncture test.
(*h*) Porosity test.

III. Routine Tests :

(*i*) Electrical routine test.
(*j*) Mechanical routine test.

The following is a brief description of the above tests :

(*a*) The test is made on a clean insulator mounted as far as possible in the normal manner. The impulse generator delivers a positive 1/50 microsecond impulse wave * of amplitude such that about half of the impulses applied cause flash-over of the insulator. The polarity is then reversed, a negative 1/50 impulse being applied. There must be at least 20 applications of the impulse in each case, and the insulator must not be damaged.

The relationships between the test voltage for the above, and all other, tests to the insulator rating number, which is the voltage, expressed in kV., used in the one minute rain-test, are as follows :

STANDARD INSULATOR RATING NUMBERS AND TEST-VOLTAGES
C.R.M.S. Values ; 50 cycles per second

1	2	3	4	5	6	7
					Puncture test-voltage	
Insulator rating member	Minimum dry flash-over voltage	Minimum wet flash-over voltage	One-minute dry test-voltage	One-minute rain test-voltage	Pin insulator or line post insulator	String insulator unit
	kV.	kV.	kV.	kV.	kV.	kV.
16	38	17	36	16	68	
22	44	23	42	22	80	
30	53	32	50	30	95	
50	74	53	70	50	130	1·3 times actual dry flash-over voltage of unit.
70	95	74	90	70	170	
90	116	95	110	90	210	
110	137	116	130	110	250	
130	160	137	152	130	290	
170	210	180	200	170		
210	255	220	240	210		
250	300	265	285	250		
310	370	325	350	310		
410	475	430	450	410		
500	590	525	560	500		
600	710	630	670	600		
700	820	735	780	700		

(*b*) A voltage of power frequency is applied to a clean insulator mounted as far as possible in the normal manner, and the voltage gradually increased up to the specified value. This voltage is maintained for one minute.

The voltage is then increased gradually until flash-over occurs. The insulator is then flashed-over at least four more times, the voltage being raised gradually to reach flash-over in about 10

* See B.S. 923, also p. 506.

seconds. The mean of at least five consecutive flash-over voltages must not be less than the value specified in column 2.

(*c*) The insulator is sprayed with water of resistance 9,000–11,000 ohm-cm., drawn from a source of supply at a temperature within 10° C. of the ambient temperature in the neighbourhood of the insulator under test, and directed at an angle of 45 degrees, the volume of water being equivalent to a precipitation of 0·12 in. per minute. The insulator must withstand the test-voltage specified in column 5 for one minute.

The insulator with 50 per cent. of the one-minute rain test-voltage applied to it is then sprayed for two minutes, the voltage raised to the one-minute test-voltage in approximately 10 seconds and maintained there for one minute.

The voltage is then gradually raised until flash-over occurs, and the insulator is then flashed at least four more times, the time taken to reach the flash-over voltage being, in each case, about 10 seconds. The flash-over voltage must not be less than the value specified in column 3.

(*d*) In the temperature-cycle test the insulator is subjected three times to the following temperature cycle : immersed for T minutes in a water-bath at not less than 70° C. higher than that of the main water, taken out and immersed as quickly as possible in a main water bath and left in this bath for T minutes.

$T = (15 + W/3)$ where $W =$ wt. of insulator in lbs. The insulator must withstand this series of tests without damage to the porcelain or glaze.

(*e*) Applied to pin insulators and line post insulators. The test is a bending test in which a load of three times the specified maximum working load (twice for a post insulator) is applied for one minute. There must be no damage to the insulator, and, in the case of the post insulator, the permanent set must be less than 1 per cent. In the case of the port insulator the load is then raised to three times, and there must be no damage to the insulator or its pin (or pins).

A permanent set-test is also made on the pin type. A load of twice the maximum, applied for one minute, must not produce a permanent set of more than 1 per cent.

(*f*) In this test, which is applied to suspension or tension units only, the insulator is mechanically stressed to a tension of 2½ times the specified maximum working load, this being maintained for one minute. Simultaneously, 75 per cent. of the dry spark-over voltage is applied.

(*g*) In the case of pin or post insulators the voltage is applied between the pin and lead foil bound over the top and side grooves. In the case of suspension units between the metal fittings, the insulator is completely immersed in insulating oil at room temperature and the voltage raised as rapidly as is consistent with correct measurement, to the value given in columns 6 and 7. The insulator must not be punctured.

Alternatively, an impulse over-voltage test may be made, in this case in air with the insulator arranged as for flash-over tests, a negative polarity 1/50 microsecond wave is applied of twice the amplitude of the negative 50 per cent. impulse flash-over voltage as determined in test (*a*) above (i.e. a prospective voltage of twice the 50 per cent. flash-over voltage). Twenty such impulses shall then be applied.

The procedure to be repeated at prospective voltages of 2·5, 3, 3·5 . . . times the 50 per cent. flash-over voltage, and continued until the insulator is punctured or the limit of the generator reached. The insulator, to pass the test, must not be punctured by prospective voltages of three times, or less, the flash-over voltage.

(*h*) Pieces freshly broken from a complete finished insulator to show no signs of impregnation (when further broken) after being immersed for 24 hours in a 0·5 per cent. alcohol solution of fuchsin under a pressure of 2,000 lbs. per sq. in.

(*i*) Pin insulators are inverted and immersed in water sufficiently deep to cover the attachment to the neck groove ; the spindle hole is also filled with water. The test is commenced at a low voltage, which is increased rapidly until a flash-over occurs every few seconds. The voltage is maintained at this value for at least five minutes, or, if failures occur, for five minutes after the last punctured piece has been removed. At the conclusion, the voltage is reduced to about one-third of the test-voltage before switching off. With suspension units there is, of course, no necessity for water immersion.

(*j*) After assembly every string insulator unit is suspended in a horizontal or vertical position, and a tensile load 20 per cent. in excess of the maximum specified working load applied for one minute.

Impulse Ratio

With an insulator the spark-over voltage where an impulse is applied is greater than the power-frequency voltage necessary for spark-over. The reason is that, with an impulse, the elapsed time between application and attainment of peak voltage is so exceedingly short that there is no time for preparation in the way of reducing the break-down voltage of the surrounding medium. With a power-frequency alternating voltage there is sufficient time for collision processes resulting in ionisation, thus having the effect of lowering appreciably the break-down voltage. The ratio of the two spark-over voltages is called the impulse ratio. Thus :

$$\text{Impulse ratio} = \frac{\text{Impulse spark-over voltage}}{\text{Power-frequency spark-over voltage}}$$

The impulse ratio has a value round about 1·4 for pin-type and 1·3 for suspension-type insulators.

For insulators destined for service in districts where there is some form of atmospheric pollution, salt deposit, and so on, the special types designed for such districts are tested under conditions which approach, as far as possible, the actual working conditions. Thus, insulators can be tested in fog, with various kinds of deposit, or under salt spray.

CHAPTER XI

MECHANICAL CHARACTERISTICS

AN overhead line must be designed from the mechanical point of view to withstand the worst probable, but not the worst possible, conditions. This is because the cost of a line which would withstand a severe hurricane would be excessively high, and therefore from the financial point of view it is justifiable to run the risk of failure under such conditions of extreme severity. In still air the conductor will be acted on by its own weight only, and if at the same time the temperature is high, the amount of the resulting sag will give a low tension. This combination of still air with high temperature thus gives the easiest conditions. The worst conditions are a combination of low temperature, which reduces the

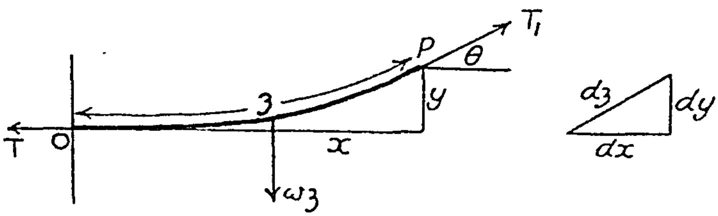


FIG. 11.1.

sag; snow or ice coating, which increase the weight per unit length and also increase the projected area per unit length; and a high cross wind.

Sag or Dip

Let O (Fig. 11.1) be the lowest point of the wire in still air. Consider a length OP of curved length z, and let

$$w = \text{weight per unit length}$$
$$T = \text{tension at O}$$
$$T_1 = \text{tension at P}$$

Then $$T_1 \sin\theta = wz$$

and $$T_1 \cos\theta = T$$

Let $$T = cw, \text{ where } c \text{ is a constant}$$

Then, dividing the first by the second of the above equations,

$$\tan\theta = \frac{wz}{T}$$
$$= \frac{z}{T/w}$$
$$= \frac{z}{c}$$

Again, for an element of length dz

$$(dz)^2 = (dy)^2 + (dx)^2$$
$$\therefore \left(\frac{dz}{dy}\right)^2 = 1 + \left(\frac{dx}{dy}\right)^2$$
$$\text{Now, } \tan\theta = \frac{dy}{dx} = \frac{z}{c}$$
$$\therefore \left(\frac{dz}{dy}\right)^2 = 1 + \left(\frac{c}{z}\right)^2$$
$$= \frac{c^2 + z^2}{z^2}$$
$$\therefore \frac{dz}{dy} = \frac{\sqrt{c^2 + z^2}}{z}$$
$$\therefore \frac{zdz}{\sqrt{c^2 + z^2}} = dy$$
$$\therefore \int \frac{zdz}{\sqrt{c^2 + z^2}} = \int dy$$
$$\therefore \sqrt{c^2 + z^2} = y + A$$

If the base line from which y is measured is taken at a distance c from the bottom point 0, then

$$\text{when } z = 0 \quad y = c \quad \text{and} \quad \therefore A = 0$$

thus giving

$$y = \sqrt{c^2 + z^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This gives the relationship between the vertical displacement y and the curved distance z along the wire. To determine the relationship between x and z we proceed as follows.

$$y^2 = c^2 + z^2$$

Hence, differentiating with respect to x,

$$2y\frac{dy}{dx} = 2z\frac{dz}{dx}$$

$$\therefore \sqrt{c^2 + z^2} \cdot \frac{dy}{dx} = z \cdot \frac{dz}{dx}$$

$$\text{But } \frac{dy}{dx} = \frac{z}{c}$$

$$\therefore \sqrt{c^2 + z^2} \cdot \frac{z}{c} = z \cdot \frac{dz}{dx}$$

$$\therefore dx = \frac{cdz}{\sqrt{c^2 + z^2}}$$

Integrating, we have

$$x + \mathrm{B} = \int \frac{c\,dz}{\sqrt{c^2 + z^2}}$$

$$= c \operatorname{logh} (z + \sqrt{c^2 + z^2})$$

When $x = 0$, $z = 0$, $\therefore \mathrm{B} = c \operatorname{logh} c$

$$\therefore x = c[\operatorname{logh} (z + \sqrt{c^2 + z^2}) - \operatorname{logh} c]$$

$$\text{or } x = c \operatorname{logh} \frac{z + \sqrt{c^2 + z^2}}{c}$$

$$\therefore \frac{x}{c} = \operatorname{logh} \frac{z + \sqrt{c^2 + z^2}}{c}$$

$$\text{or } \frac{z + \sqrt{c^2 + z^2}}{c} = e^{\frac{x}{c}}$$

$$z + \sqrt{c^2 + z^2} = ce^{\frac{x}{c}} \quad \ldots \ldots \ldots \quad (2)$$

$$\therefore \frac{1}{z + \sqrt{c^2 + z^2}} = \frac{1}{c} \cdot e^{-\frac{x}{c}}$$

which, on rationalising, reduces to

$$\sqrt{c^2 + z^2} - z = ce^{-\frac{x}{c}} \quad \ldots \ldots \ldots \quad (3)$$

Adding equations (2) and (3), we have

$$2\sqrt{c^2 + z^2} = ce^{\frac{x}{c}} + ce^{-\frac{x}{c}}$$

$$\sqrt{c^2 + z^2} = \frac{c}{2}(e^{\frac{x}{c}} + e^{-\frac{x}{c}})$$

$$\therefore y = c \cosh \frac{x}{c} \quad \ldots \ldots \quad (4)$$

Subtracting equation (3) from (2), we have

$$2z = ce^{\frac{x}{c}} - ce^{-\frac{x}{c}}$$

$$z = \frac{c}{2}(c^{\frac{x}{c}} - e^{-\frac{x}{c}})$$

$$\therefore z = c \sinh \frac{x}{c} \quad \ldots \ldots \quad (5)$$

Expanding the cosh h term in equation (4), we have

$$y = \frac{c}{2}\left\{\left(1 + \frac{x/c}{1!} + \frac{(x/c)^2}{2!} + \ . \ . \ .\right) + \left(1 + \frac{-x/c}{1!} + \frac{(-x/c)^2}{2!} + \ . \ . \ .\right)\right\}$$

$$= \frac{c}{2}\left\{2 + \frac{2(x/c)^2}{2!} + \frac{2(x/c)^4}{4!} + \ . \ . \ .\right\}$$

Neglecting all terms after the second, we have

$$y = c\left(1 + \frac{x^2}{2c^2}\right) = c + \frac{x^2}{2c}$$

$$\therefore y - c = \frac{x^2}{2c}$$

Denoting $(y - c)$ by y', we have

$$y' = \frac{x^2}{2c}$$

which is the equation to a parabola.

If we denote the span by l and the half-length of the span by l_1, then when $x = l_1$, $y' = d$, the maximum sag.

$$\therefore d = \frac{l_1^2}{2c}$$

$$\text{or } d = \frac{l_1^2 w}{2T} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (6)$$

This gives the maximum sag under quiet conditions in terms of the span, the weight per unit length, and the tension at O.

It is instructive to note that this approximate equation can be determined very simply as follows. Referring again to Fig. 11.1, and assuming that the curvature is so small that the curved length can be taken as equal to its horizontal projection, we have for the two external forces acting on the length OP of the wire (i) the tension T, and (ii) the weight wx, which acts at a distance of $x/2$ from P. Hence, equating the moments of these two forces about P, we have

$$Ty = wx \times \tfrac{1}{2}x$$

$$\therefore y = \frac{wx^2}{2T}$$

But at O, $y = d$ and $x = l_1$

$$\therefore d = \frac{wl_1^2}{2T}$$

Total Length of Wire

$$z = c \sin h \frac{x}{c}$$

$$= \frac{c}{2}\left(e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right)$$

$$= \frac{c}{2}\left[\left\{1 + \frac{(x/c)}{1} + \frac{(x/c)^2}{2\,!} + \ldots\right\} - \left\{1 + \frac{(-x/c)}{1} + \frac{(-x/c)^2}{2\,!} + \ldots\right\}\right]$$

$$= c\left\{\frac{(x/c}{1} + \frac{(x/c)^3}{3\,!} + \ldots\right\}$$

Taking the first two terms only, we have approximately

$$z = c\left(\frac{x}{c} + \frac{x^3}{6c^3}\right)$$

For the half length we substitute $x = l_1$, giving

$$z_{\frac{1}{2}} = c\left(\frac{l_1}{c} + \frac{l_1^3}{6c^3}\right) = \left(l_1 + \frac{l_1^3}{6c^2}\right)$$

Hence, for the total length z_t

$$z_t = 2l_1 + \frac{l_1^3}{3c^2}$$

But $\frac{l_1^2}{c} = 2d$, so that $\frac{l_1^3}{3c^2} = \frac{4}{3} \cdot \frac{d^2}{l_1}$

$$\therefore z_t = 2l_1 + \frac{4}{3} \cdot \frac{d^2}{l_1} \qquad (7)$$

Tension at the Ends

At any point, $T_1 \cos\theta = T \quad \therefore T_1^2 \cos^2\theta = T^2$

$T_1 \sin\theta = wz \quad \therefore T_1^2 \sin^2\theta = w^2z^2$

$\therefore$ adding, $T_1^2 = T^2 + w^2z^2$

Now, $y^2 = c^2 + z^2$

$\therefore w^2y^2 = w^2c^2 + w^2z^2$

and $c = T/w$

$\therefore w^2y^2 = T^2 + w^2z^2$

$= T_1^2$

$\therefore T_1 = wy$

Again $y = y' + c$

$\therefore T_1 = wy' + wc$

at the end, $y' = d$ and $T = T_s$, the tension at the end

$\therefore T_s = wd + wc$

or $T_s = T + wd \qquad (8)$

For very small sags we can neglect wd in comparison with T, showing that when d is small the tension is approximately uniform throughout the wire.

Effects of Wind and of Ice Covering

The severest conditions are those due to a combination of wind and ice loading. Under such conditions the forces acting on each unit length of wire are (1) the weight w acting vertically downwards, (2) the weight per unit length of the maximum coating of ice likely to form round the conductor, this also acting vertically downwards, and (3) the wind pressure per unit length, this pressure being assumed to act horizontally. Denoting the ice loading per foot run by w_i lb., and the wind pressure per foot run by p lb., each foot of wire is acted on by a total vertical force of $(w + w_i)$ lb., and a horizontal force of p lb., the resultant thus being

$$W = \sqrt{(w + w_i)^2 + p^2}$$

Under such conditions the necessary equations are obtained by substituting W for w.

Since the conditions of wind and ice loading vary from time to time the Electricity Commissioners have specified the worst conditions for which the line shall be calculated. These are:

1. For voltages exceeding 650 D.C. or 325 A.C. a wind pressure of 8 lb. per sq. ft. on the projected area of the ice-covered conductor; a radial thickness of ice of $\frac{3}{8}$ in., the ice assumed to have a weight of 57 lb. per cubic ft.; and a temperature of 22° F. Under these conditions the factor of safety of the line shall be 2.

2. For voltages not exceeding 650 D.C. or 325 A.C. the same regulations apply, but with the exception that the ice load is taken as $\frac{3}{16}$ in. radial thickness.

There is not, as yet, entire agreement as to the relationship between wind velocity and wind pressure on a surface. The *Cable Research Handbook* gives the following expressions:

p = wind pressure per sq. ft. of plane surface.
V_1 = wind velocity in *indicated* m.p.h.
V_a = actual wind velocity in m.p.h.
$p = \cdot 0032\ V_1^2$ . . . (N.P.L. and Eiffel Tower).
$p = \cdot 0052\ V_1^2$ . . . (Grashof's Experiments).
$p = \cdot 0042\ V_a^2$ . . . (Coombs, America).

The difference between indicated and actual velocities is due to the fact that anemometers do not give the actual value.

The above regulation does away with the uncertainty regarding the effective wind area of a circular wire, since it specifies the projected area.

Let r'' = conductor radius, bare.
i'' = radial thickness of ice coating.

Then projected area per foot run

$$= \frac{2(r + i)}{12} \text{ sq. ft.}$$

the wind pressure thus being

$$8 \times \frac{2(r + i)}{12} = \frac{4}{3}(r + i) \text{ lb. per foot run.}$$

When the wind pressure is expressed in lb. per sq. ft. of plane surface, then the projected area has to be multiplied by a factor which has the following values:

Smooth wire	·666
7 strand	·890
19 strand	·935

Actually it is usual to take an all-round figure of 0·6 for all types of wire.

Example 1.—An overhead line is erected across a span of 500 ft. on level supports. The conductor, which has an effective diameter of 0·814 in., a cross-sectional area of 0·475 sq. in., and weighs 1·54 lb. per ft. run, is subjected to a wind pressure of 8 lb. per sq. ft. of projected area. Assuming a maximum working stress of 15,000 lb. per sq. in., estimate approximately the sag under the given load conditions.

$$d = l_1^2 w/2T$$

Weight per ft. run = 1·54 lb.

Wind force per ft. run = $8 \times 1 \times (0{\cdot}814/12)$
= 0·542 lb.

∴ Effective weight
per ft. run $w = (1{\cdot}54^2 + 0{\cdot}542^2)^{\frac{1}{2}}$
= 1·631 lb.

T = effective. cross-sectional area (sq. in.) × allowable stress
= $0{\cdot}475 \times 15{,}000$
= 7,130 lb.

l_1 = half span = 250 ft.

$$\therefore d = \frac{250^2 \times 1{\cdot}63}{2 \times 7130}$$
$$= 7{\cdot}16 \text{ ft.}$$

Example 2.—A 37/14 copper conductor having a radius of 0·28 in. and a weight per foot run of $w = 0{\cdot}731$ lb. Span 70 yards. Line voltage 11,000.

Under the conditions of wind and ice specified in the above Regulations we have:

$$i = \tfrac{3}{8}'' = {\cdot}375$$
$$\therefore (r + i) = ({\cdot}28 + {\cdot}375) = {\cdot}655 \text{ in.}$$
$$\therefore p = \tfrac{4}{3} \times {\cdot}655 = {\cdot}87 \text{ lb. per ft. run}$$

The ice is in the form of a cylinder of internal diameter ·56 in., and external diameter 1·31 in. Hence the volume of 1-ft. run of ice covering is:

$$\frac{\pi}{4}(1{\cdot}31^2 - {\cdot}56^2) \times 12 = 13{\cdot}3 \text{ cub. in.}$$
$$\therefore w_i = \frac{13{\cdot}3 \times 57}{12^3} = {\cdot}44 \text{ lb.}$$
$$\therefore W = \sqrt{(w + w_i)^2 + p^2}$$
$$= \sqrt{({\cdot}731 + {\cdot}44)^2 + {\cdot}87^2} = 1{\cdot}46 \text{ lb.}$$

The breaking load of copper is taken as 24 tons per sq. in., or 53,800 lb. per sq. in. The cross-section of the conductor being 0·182 sq. in., and the factor of safety 2, the maximum allowable tension under the stated conditions is:

$$T = 53800 \times {\cdot}182 \div 2 = 4880 \text{ lb.}$$
$$\text{also } l_1 = \tfrac{1}{2} \text{ of } 70 \times 3 = 105 \text{ ft. (half-span)}$$

Hence, from the equation:

$$d = \frac{Wl_1^2}{2T}$$
$$d = \frac{1{\cdot}46 \times 105^2}{2 \times 4880}$$
$$= 1{\cdot}65 \text{ ft.}$$

Conditions at Erection

The above example shows how the sag can be calculated for any stipulated conditions. In practice a line will be erected under a set of conditions which will be very different from the wind and ice conditions specified in the Regulations, and the problem is therefore to determine the sag under the erection conditions which will ensure the factor of safety of 2 under the prescribed wind and ice conditions. There is, in addition, a second condition to be fulfilled, viz. that when the sag is a maximum, the minimum height of the conductor shall not be less than a certain specified distance above the ground. The British Regulations specify the following minimum ground clearances at 122° F.:

Working Pressure in kV.	Minimum Ground Clearance.
Not exceeding 66	20 ft.
66 to 110	21 ft.
110 to 165	22 ft.
Exceeding 165	23 ft.

For voltages not exceeding 650 D.C. or 325 A.C. the minimum ground clearances of any line conductor (other than a service line), earth wire, or auxiliary conductor at any point of the span are as follows:

19 ft. across a public road.
17 ft. in other positions.
15 ft. in positions inaccessible to vehicular traffic for service lines.
19 ft. across a carriageway.
17 ft. along a carriageway.

The starting-point with all calculations is the wind and ice condition, since for this condition only is the factor of safety specified. For the length of wire in the span under wind and ice conditions we have:

$$z_1 = 2l_1 + \frac{4}{3} \cdot \frac{d_1^2}{l_1} = l + \frac{8}{3} \cdot \frac{d_1^2}{l} \quad . \quad . \quad . \text{ from equation (7)}$$

The suffix 1 refers to these conditions and the suffix 2 to the conditions at erection temperature, and with no wind or ice.

If the conductor is slackened off, the tension T_1 will be removed, and in consequence there will be an elastic contraction whose magnitude will be governed by Young's Modulus E. The amount of this contraction will be given by:

$$\frac{z_1 T_1}{aE}$$

where a is the cross-section.

So that for the unstressed length of the conductor, *at the same temperature*, we have:

$$z' = z_1 - \frac{z_1 T_1}{aE} = z_1\left(1 - \frac{T_1}{aE}\right)$$

If the line is erected at some temperature higher than the 22° F. specified in the wind and ice conditions, say, t° F. higher, then the unstressed length of conductor in the span will be :

$$z'' = z'(1 + \alpha t)$$

where α is the coefficient of linear expansion per degree F. Under tension, and at the temperature corresponding to the rise of t° F., i.e. under erection conditions, the length of conductor in the span must thus be :

$$z_2 = z''\left(1 + \frac{T_2}{aE}\right)$$

But $$T_2 = \frac{l_1^2 w}{2d_2} = \frac{l^2 w}{8d_2}$$

$$\therefore\ z_2 = z''\left(1 + \frac{l^2 w}{8aEd_2}\right)$$

Again $$z_2 = \left(2l_1 + \frac{4}{3}\cdot\frac{d_2^2}{l_1}\right) = \left(l + \frac{8}{3}\cdot\frac{d_2^2}{l}\right)$$

$$\therefore\ l + \frac{8}{3}\cdot\frac{d_2^2}{l} = z''\left(1 + \frac{l^2 w}{8aEd_2}\right)$$

This is an equation in d_2 which reduces to :

$$d_2^3 - \tfrac{3}{8}l(z'' - l)d_2 - \frac{3z''l^3 w}{64aE} = 0 \quad . \quad . \quad . \quad . \quad (9)$$

The quantity z'' is given by :

$$z'' = z_1\left(1 - \frac{T_1}{aE}\right)(1 + \alpha t)$$

$$= z_1\left(1 + \alpha t - \frac{T_1}{aE}\right)$$

$$= l\left(1 + \frac{W^2 l^2}{24T_1^2}\right)\left(1 + \alpha t - \frac{T_1}{aE}\right)$$

Morecombe gives an equation for d_2 in terms of d_1, the derivation being as follows : the method is based on the conception of the critical temperature which can be defined as that temperature which, under conditions of no wind and no ice, gives the same sag as under the basic wind and ice loading at 22° F. We have used the suffix 1 to refer to these basic conditions and we will use the suffix c for the conditions at the critical temperature.

Obviously
$$d_c = d_1$$
$$\therefore \frac{wl^2}{8T_c} = \frac{Wl^2}{8T_1}$$
$$\therefore T_c = \frac{w}{W} \cdot T_1$$

Elongation due to temperature rise of $(t_c - 22)°$ F.
$$= l\alpha(t_c - 22)$$

Contraction due to reduction in tension from T_1 to T_c
$$= \frac{l}{aE}(T_1 - T_c)$$

These two quantities are equal, since the sag is unchanged
$$\therefore \frac{l}{aE}(T_1 - T_c) = l\alpha(t_c - 22)$$
$$\therefore t_c - 22 = \frac{1}{aE\alpha}(T_1 - T_c)$$
$$= \frac{T_1}{aE\alpha}\left(1 - \frac{T_c}{T_1}\right)$$
$$= \frac{T_1}{aE\alpha}\left(1 - \frac{w}{W}\right)$$

Assuming that the line is erected at some other temperature, $t°$ F., and under no wind or ice loading, the calculations of the sag at erection thus involves changes in temperature and tension only, but not in w. Assuming that t is less than t_c, the decrease in length due to temperature change is
$$z_c\alpha(t_c - t)$$

Increase in length due to increased tension is
$$\frac{z_c}{aE}(T - T_c) = \frac{z_cT_c}{aE}\left(\frac{T}{T_c} - T_1\right)$$
$$\text{But } Td = \frac{wl^2}{8} = T_cd_c$$
$$\therefore \text{Elastic increase} = \frac{z_cT_c}{aE}\left(\frac{d_c}{d} - 1\right)$$

The change in length is thus :
$$z_c\alpha(t_c - t) - \frac{z_cT_c}{aE}\left(\frac{d_c}{d} - 1\right)$$

But $\quad z_c = l + \frac{8d_c^{\,2}}{3l}$ and $z = l + \frac{8d^2}{3l}$

the change in length thus being also given by:

$$\frac{8}{3l}(d_c^2 - d^2)$$

Hence we have:

$$\frac{8}{3l}(d_c^2 - d^2) = z_c\left\{\alpha(t_c - t) - \frac{T_c}{aE}\left(\frac{d_c}{d} - 1\right)\right\}$$

As an approximation we can substitute l for z_c, giving:

$$\frac{8}{3l^2}(d_c^2 - d^2) = \alpha(t_c - t) - \frac{T_c}{aE}\left(\frac{d_c}{d} - 1\right)$$

Alternatively, a cubic equation for the tension T_2 can be derived in a similar manner, this method being used by Painton.

The unstressed length of line at erection is:

$$z'' = l\left(1 + \frac{W^2l^2}{24T_1^2}\right)\left(1 + \alpha t - \frac{T_1}{aE}\right)$$

The stressed length under tension T_2 can therefore be written in the form:

$$z_2 = z''\left(1 + \frac{T_2}{aE}\right)$$

$$= l\left(1 + \frac{W^2l^2}{24T_1^2}\right)\left(1 + \alpha t - \frac{T_1 - T_2}{aE}\right) \quad . \quad . \quad (10)$$

But $$z_2 = l + \frac{8}{3}\cdot\frac{d_2^2}{l}$$

and $$d_2 = \frac{wl^2}{8T_2}$$

$$\therefore \frac{d_2^2}{l} = \frac{w^2l^3}{64T_2^2}$$

giving $$z_2 = l\left(1 + \frac{w^2l^2}{24T_2^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Equating (10) and (11) we have:

$$\left(1 + \frac{W^2l^2}{24T_1^2}\right)\left(1 + \alpha t - \frac{T_1 - T_2}{aE}\right) = 1 + \frac{w^2l^2}{24T_2^2}$$

Since all the quantities involved are small in comparison with unity, we can write:

$$1 + \frac{W^2l^2}{24T_1^2} + \alpha t - \frac{T_1 - T_2}{aE} = 1 + \frac{w^2l^2}{24T_2^2}$$

which reduces to:

$$T_2^3 + \left\{aE\left(\frac{W^2l^2}{24T_1^2} + \alpha t\right) - T_1\right\}T_2^2 - \frac{aEw^2l^2}{24} = 0$$

There is little to choose between the two methods so far as the amount of calculation is concerned, except that with the second method the sag, d_2, has to be calculated when the tension T_2 has been evaluated.

Example.—The British Grid lines have a normal span of 900 ft., and the conductors are steel-cored aluminium consisting of a central core of 7 strands of galvanised steel wire, each of 0·11-in. diameter, with 30 strands of aluminium wire of the same diameter of 0·11 in. The particulars are as follows: The line is being erected in still air at 122° F., so that $t = 100°$ F.

Overall diameter, 0·77 in.

Diameter with ice loading, 1·52 in. (this assumes an ice thickness of $\frac{3}{8}$ in.).

Corresponding wind loading per foot run $p = 1·01$ lb.

Weight per foot run $w =$ ·571 lb.

Wt. of ice per foot run $w_i =$ ·535 lb.

$$\therefore \quad W = \sqrt{(w + w_i)^2 + p^2} = \sqrt{(·571 + ·535)^2 + (1·01)^2} = 1·49$$

Breaking load $= 17720$ lb.

$$\therefore T_1 = \frac{\text{Breaking load}}{\text{Factor of safety}} = \frac{17720}{2} = 8860$$

$$E = 13·31 \times 10^6$$

$$\alpha = 10·24 \times 10^{-6}$$

Cross-section $a =$ ·3515 sq. in.

Substituting these values in the equation for T_2, we have:

$$T_2^3 + \left\{·3515 \times 13·31 \times 10^6\left(\frac{1·49^2 \times 900^2}{24 \times 8860^2} + 10·24 \times 10^{-6} \times 100\right) - 8860\right\}T_2^2 - \frac{·3515 \times 13·31 \times 10^6 \times ·571^2 \times 900^2}{24} = 0$$

This reduces to: $T_2^3 + 457T_2^2 - 515 \times 10^8 = 0$

the solution of which is $T_2 = 3{,}570$ lb.

Hence the sag under erection conditions must be:

$$\frac{l_1^2w}{2T_2} = \frac{(450)^2 \times ·571}{2 \times 3570}$$

$$= 16·2 \text{ ft.}$$

Supports not on the Same Level

Let the distance between the two supports be $2l_1$ as before, and let the lowest point O of the conductor be x from the support at

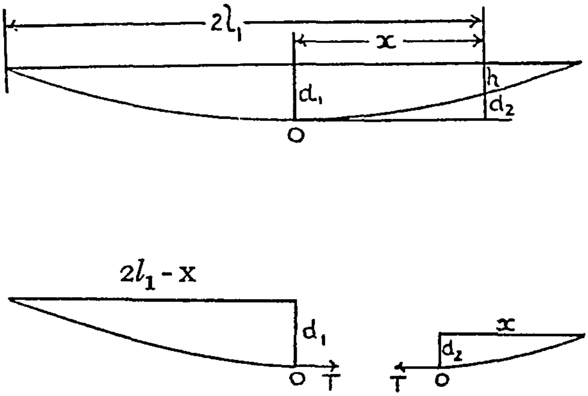


FIG. 11.2.

the lower level (Fig. 86). Then the distance of O from the support at the higher level is $(2l_1 - x)$. This modifies the equation for the sag to :

$$d_1 = \frac{(2l_1 - x)^2 w}{2T}$$

$$\text{and } d_2 = \frac{x^2 w}{2T}$$

$$\therefore\ h = (d_1 - d_2) = \frac{w}{2T}(4l^2 - 4lx + x^2 - x^2)$$

$$= \frac{w}{2T}(4l^2 - 4lx)$$

which gives $$l - x = \frac{2Th}{4lw}$$

$$\therefore\ x = l - \frac{Th}{2lw}$$

By considering the portion of the span either to the right or to the left of O the calculation can be made exactly as in the previous case. An important aspect of the problem when the supports are at different levels is the inclination of the conductor at the lower support. Under certain circumstances the conductor at this point may be almost horizontal or may even be upwards instead of downwards, with the result that with suspension-type insulators very considerable difficulties will be experienced. The inclination at this point will be zero if x is zero, and the inclination will be negative, i.e. upwards instead of downwards, if x is negative. Painton suggests the suspending of weights from the insulator strings as one way out of this difficulty.

SUGGESTIONS FOR FURTHER READING

LEWIS, W. W., *Transmission Line Engineering* (McGraw Hill).
LOWE, E. A., *Electrical Power Transmission* (McGraw Hill).
MORECOMBE, W., *Overhead Power Lines* (Chapman & Hall).
TAYLOR, W. T., *Distribution of Electricity by Overhead Power Lines* (Griffen).
WATSON, G. G., *Sags and Tensions in Overhead Power Lines* (Pitman).
WOODRUFF, L. F., *Principles of Electric Power Transmissions and Distribution* (Wiley).

CHAPTER XII

INSULATED CABLES

Construction of Low-voltage Cables

FOR all cables the conductor was, at one time, almost universally stranded copper, but aluminium is now being used to a considerable extent. The stranding is necessary to secure flexibility, and the conductor numbers are 3, 7, 19, 37, and so on, all numbers except 3 having a centrally disposed conductor with all the others round it. Thus a 7-strand has a central conductor with 6 round it; the 19-strand is a 7-strand with a ring of 12 additional wires round it; the 37-strand, a 19-strand with 18 additional wires round it; and so on. The cable size can therefore be denoted by the total cross-sectional area of the core, or by a designation

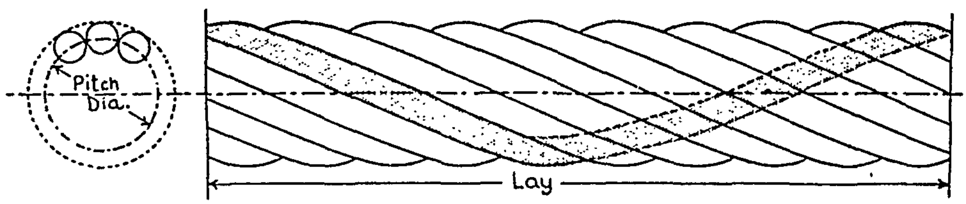


FIG. 12.1.—TO ILLUSTRATE THE "LAY" OF A CABLE.

giving number of strands and gauge number of each strand. Thus a cable made up of 19 strands each of gauge 11 conductor of diameter 0·116 in. will have a total cross-section of 0·20 sq. in., and it can therefore be referred to as a 0·20 sq. in. cable, a 19/11s, or a 19/·116 cable.

The various conductors are spiralled round the central conductor, and when there is more than one layer alternate layers are spiralled in opposite directions. This is to prevent "bird-caging" when the conductor is bent. The distance measured along the axis of the cable corresponding to one complete turn of the spiral is called the lay, and this is illustrated in Fig. 12.1. The lay adopted as standard by the Cable Makers' Association is twenty times the pitch diameter. This spiralling of the conductors obviously increases the resistance, since the length of each spiralled conductor is greater than the central strand, and the flow of current is along the various conductors. It is therefore necessary to be particular in the designation of the cross-section.

The nominal cross-section is the area of the cross-section of one conductor in a plane perpendicular to its length, multiplied by the number of conductors.

The actual cross-section is the area of the oblique cross-section due to the cutting of the stranded cable by a plane perpendicular to the core of the cable, multiplied by the number of conductors.

The equivalent cross-section is the cross-section of a solid conductor of the same length as the cable and having the same resistance at the same temperature.

The actual cross-section is greater than the nominal cross-section, but the equivalent cross-section is less because of the increase in resistance due to spiralling.

If l is the lay, then for an axial length of l, a point travelling along the spiral will make one complete revolution. Thus, if d is the pitch diameter for the particular layer concerned, the circumference of this revolution will be πd. The length of conductor along the spiral is thus the hypotenuse of a right-angled triangle whose base is l and perpendicular πd. Hence, denoting the length along the spiral by l',

$$l' = \{l^2 + (\pi d)^2\}^{\frac{1}{2}}$$

Now let the lay be $n : 1$; thus, n will be 20 for a lay of 20 : 1, then

$$l = nd \text{ or } d = l/n$$

$$\therefore\ l' = \left(l^2 + \frac{\pi^2 l^2}{n^2}\right)^{\frac{1}{2}}$$

$$= l\left(1 + \frac{\pi^2}{n^2}\right)^{\frac{1}{2}}$$

Now let m be the number of strands, one of these being a central strand, then length of central strand will be l, but that of the other $(m - 1)$ strands will be l'. The sum of all the lengths is therefore

$$l + (m - 1)\, l' = l\{1 + (m - 1)\,(1 + \pi^2/n^2)^{\frac{1}{2}}\}$$

But if it were not for the spiralling the sum of these lengths would be ml, and therefore the weight of copper in the cable is increased

$$\frac{1 + (m - 1)\,(1 + \pi^2/n^2)^{\frac{1}{2}}}{m} \text{ times}$$

Thus, consider a 19-strand cable with a lay of 20 : 1, then the multiplier giving the increased weight is

$$\frac{1 + 18\,(1 + \pi^2/400)^{\frac{1}{2}}}{19} = \frac{19{\cdot}221}{19} = 1{\cdot}0116$$

The effect of stranding on the resistance of the cable is determined as follows. There are $(m - 1)$ wires of length l' and one of length l in parallel, and therefore, since they are all of the same cross-section, we have for the total resistance

$$\frac{1}{R'} \propto \frac{1}{l} + \frac{m-1}{l'}$$

If all the wires were straight and of length l, the resistance would be given by

$$\frac{1}{R} \propto \frac{m}{l}$$

The magnitude of the multiplying ratio for the increase in resistance due to stranding is thus given by

$$\frac{R'}{R} = \frac{m/l}{\dfrac{1}{l} + \dfrac{m-1}{l'}}$$

$$= \frac{m/l}{\dfrac{1}{l} + \dfrac{m-1}{l(1 + \pi^2/n^2)^{\frac{1}{2}}}}$$

$$= \frac{m(1 + \pi^2/n^2)^{\frac{1}{2}}}{(1 + \pi^2/n^2)^{\frac{1}{2}} + m - 1}$$

For the cable for which $m = 19$ and $n = 20$ this has the value

$$\frac{19(1 + \pi^2/400)^{\frac{1}{2}}}{(1 + \pi^2/400)^{\frac{1}{2}} + m - 1} = 1{\cdot}011$$

Insulating Materials

The main points regarding the properties of the dielectrics used for cable insulation are as follows:

1. High insulation resistance, but only in so far as the treatment required to attain this does not reduce the life and flexibility of the cable. This applies particularly to impregnated paper cables.
2. High dielectric strength.
3. Good mechanical properties, i.e. tenacity and elasticity.

4. Immune to attacks by acids and alkalis, over a range of temperature of about 0° F to 200° F.

5. Must be non-hygroscopic, or, if hygroscopic, must be enclosed in a watertight covering.

6. Not too costly, and easily manipulated from the point of view of manufacture.

Impregnated Paper

With impregnated paper insulated cables the main insulation consists of paper tape impregnated with compound. In the early cables of this type pure manila paper was used, but it is now almost entirely superseded by a chemically pulped paper made from wood chippings; it should be long fibred, strong, free from deleterious substances, and without pinholes or blemishes.

With the insulation thicknesses necessitated by ever increasing voltages, wrinkling and creasing of the paper may be caused if cables of the very highest voltages are bent over the normal "haul-off" wheel of the lapping machine. To overcome this it is necessary that the various layers shall be able to slip over one another when the cable is bent. In addition the papers must be applied with the greatest possible tension so that as much paper as possible is included in a given thickness of insulation. The 300-kV. oil-filled cable described on p. 242 was hauled off the lapping machine by a "caterpillar," thus avoiding the initial bending.

It is extremely important that the paper shall be free from any traces of ligneous fibres and from metallic or other conducting spots. The compound with which the paper is impregnated should be of such a consistency that it is plastic at ordinary temperatures, has no tendency to drain away from the cable, or to alter its distribution in the dielectric under the conditions of temperature and electrostatic stress that arise in operation. The dielectric strength of impregnated paper is 20,000 to 30,000 volts per mm. and the dielectric constant is about 3·6.

Compounds vary from one manufacturer to another, but they are all based either on paraffinic or on naphthenic mineral oil. Resin frequently is added to lower viscosity and to improve the impregnating qualities of the compound.

Where cables lie on gradients or run vertically for more than a few yards, mass-impregnation by impregnating compound containing a micro-crystalline wax can be adopted. This compound has the property of being fluid under the temperatures used during the impregnating process, but sufficiently solid at operating temperatures to obviate the danger of gravitating down slopes. Oil compounds, not having this property, would tend to leave the insulation unduly dry at all the high points of the route.

Vulcanised Indiarubber (V.I.R.)

V.I.R. cables are not, as a rule, of large size, since paper has largely superseded rubber for power mains, but they are very commonly employed for comparatively small wiring, such as lighting and power installations. Since sulphur is used in the vulcanising process it is necessary to protect the copper from the sulphur, and this was effected at one time by first tinning each conductor and then covering each core with a thin layer of pure rubber. At the present time the control of the sulphur in the vulcanising compound is such that the pure rubber layer is now omitted. The thickness of vulcanised rubber requisite for the working voltage and size of cable is then applied. A waterproof cotton tape is then added and further protection against the ingress of moisture given by a braiding of jute soaked in compound. For use in mines V.I.R. cables are sometimes sheathed with bitumen because of the excellent damp-resisting properties of this material.

The dielectric strength of indiarubber is from 10,000 to 20,000 volts per mm. and the dielectric constant (S.I.C.) about 2·5, but by loading the rubber with certain materials such as talc, zinc oxide, and sulphur, the value can be increased to 6·1 without appreciably weakening the dielectric strength.

The great advantage of paper over vulcanised indiarubber is its superior heat conductivity and its ability to withstand higher temperatures without deterioration, the result of which is that a cable of given size can be worked at a higher current density than a V.I.R. cable. Thus the maximum permissible temperature for a paper cable is about 50° C. to 80° C., according to voltage and method of installation, against about 48° C. for V.I.R. The superior current-carrying capacity of paper cables is illustrated by the table on page 210.

Ratings for cables in air are lower on small sizes and higher on large sizes as compared with cables laid direct.

It is apparent from the table that paper cables have a much greater current-carrying capacity than either V.I.R. or bitumen insulated cables. The figures also show that the old-fashioned rule of 1,000 amperes per square inch is very inaccurate, since the permissible current density depends not only on the class of insulation, but also on the cross-section of the core, being greater for small than for large cross-sections.

Paper cables have also the advantages of marked durability, a high dielectric strength, low cost, and low electrostatic capacity, this last being of importance with extra-high voltages only.

CURRENT RATINGS OF V.R. AND PAPER-INSULATED CABLES

Nominal Area of Conductor.	Number and Diameter of Wires.	D.C.		A.C. Single-phase.		A.C. Three-phase.	
		Lead-covered V.R.	Lead-covered Paper.	Lead-covered V.R.	Lead-covered Paper.	Lead-covered V.R.	Lead-covered Paper.
Sq. in.	Inch.	Amps.	Amps.	Amps.	Amps.	Amps.	Amps.
0·007	7/·036	29	33	29	33	29	33
0·01	7/·044	38	50	38	50	38	50
0·0145	7/·052	45	67	45	67	45	67
0·0225	7/·064	56	89	56	89	56	89
0·03	19/·044	65	103	65	103	65	103
0·04	19/·052	78	123	78	123	78	123
0·06	19/·064	102	160	102	160	102	160
0·1	19/·083	160	262	160	262	157	261
0·15	37/·072	207	340	206	339	201	336
0·2	37/·083	251	412	249	410	242	405
0·3	37/·103	320	530	314	523	304	513
0·4	61/·093	402	630	377	613	355	583
0·5	61/·103	458	739	421	690	390	655
0·75	91/·103	580	952	510	856	455	788
1·0	127/·103	701	1188	583	1005	512	898

These figures are taken from the *I.E.E. Regulations*, 15th Edition, 1950, and refer to lead-covered cables run open on cleats with spacing and clearance as follows:

Normal Cross-sectional Area of Conductor.	Approximate Vertical Distance between Cable Centres.	Approximate Horizontal Distances of Cable Centres from Wall.
sq. in.		
0·1 to 0·3 inclusive	Twice the dia. of the finished	1¼ inches
0·4 to 1·0 inclusive	cable 3½ inches	2¼ inches

Since paper is hygroscopic it is essential that all paper cables should be protected from the possible ingress of moisture, this being accomplished by enclosing the cores and insulation in a moisture-proof sheath. For the great majority of installations this sheath is lead or aluminium. For mining work a sheath has occasionally been of vulcanised bitumen. At the present time this material is very rarely, if ever, used in cable manufacture.

Varnished Cambric

This is a cotton cloth impregnated and coated with varnish, the finished surface being very smooth. The varnish is pure linseed oil with petroleum bitumen added for the purpose of neutralising any acid which may be present. The cambric is lapped on to the conductor in the form of a tape, as in the case of paper-insulated cables, and, to allow for the sliding of one turn over another as the cable is bent, the surfaces, besides being finished as smooth as possible, are lubricated with a petroleum-jelly compound. As the insulating material is hygroscopic, it is usual to lead-sheath such cables. The dielectric strength is about 4,000 volts per mm. and the dielectric constant 2·5 to 3·8.

With this insulation the conductors of twin- and multi-core cables are, with a few exceptions, shaped.

Polyvinyl Chloride (P.V.C.)

This is a synthetic material obtained from acetylene, the grade depending on the polymerisation process used. It is in the form of a powder, and for the purpose of cable insulation it has to be compounded with a plasticiser, the function of which is to form a gell and so render the material plastic over the desired range of temperature. The mechanical properties—i.e. elasticity and recovery from stretching—are not so good as those of rubber; the insulation resistance is also lower. P.V.C. is inert to oxygen and almost inert to oils and to many alkalis and acids, and there are then extreme environmental conditions for which it may be preferable to rubber. So far, the specifications of the British Standards Institute lists cables of this type up to a maximum of 0·06 sq. in. conductor. The I.E.E. regulations on current rating give the same values as for rubber cables.

Silicone Rubber

The usual organic materials used for electrical insulation have, in their molecular architecture, chains of carbon atoms: the silicones have chains of alternate silicon and oxygen atoms. The molecules are very large and their structure leads to considerably greater thermal stability than for organic compounds. It is this high stability which renders these materials important. Thus silicone varnish applied to woven glass cloth has withstood a temperature of 175° C. for years without deterioration.

It is the material silicone rubber which is of interest in cable manufacture, and its thermal properties are such that, not only is stability retained, but flexibility also over the great range of – 90° C. to + 250° C. If decomposition takes place as the result of fire, a non-conducting silica ash is formed thereby eliminating the immediate risk of electrical breakdown provided the ash can remain *in situ.* This is possible when a silicone-rubber-insulated conductor has a glass braiding. Mechanically, its tensile strength is low, 300–1,000 lb./sq. in. as against rubber, which should not be less than 2,000 lb./sq. in. Its resistance to oils, to ozone, and to water are fairly good, but to strong acids and alkalis it is poor. The dielectric strength is 1,200–3,000 volts per mm. and the dielectric constant at power frequency 3·6–9·0.

Mechanical Protection

Since all the insulating materials used in the construction of cables are mechanically weak, some form of protection against mechanical injury is necessary. For power supply with cables laid direct in the ground, this usually takes the form of an armouring of steel tape wound on in two layers so that the upper layer covers the joint in the lower layer. For surface installations this is very satisfactory, but it is very inflexible, and therefore not suited to installations where the bending of the cable cannot be avoided, e.g. in mines; also, owing to their lack of longitudinal strength, steel-tape armoured cables are not considered suitable for installation on supports above ground. In addition there is a Home Office regulation to the effect that the armouring of a colliery cable shall have an electrical conductivity of at least 50 per cent. that of the largest conductor enclosed within the armouring. This prohibits the use of steel tape owing to its very small lay; and it is therefore necessary to use an armouring of steel wire. For very small cables a single layer of such wire is suitable, but for large cables a double layer is preferable, in spite of the fact that the necessary conductivity can be obtained by a single layer of larger-sized wire. This is because the double wire armouring renders the cable easier to handle and gives greater flexibility for the same conductivity. Double wire armouring is used chiefly for cables run vertically as in the mine shaft. The cable may be held in heavy cleats every few yards, or suspended from the upper end by means of a cone clamp which grips the armour wires. With regard to ordinary main cables, an aluminium sheathing has been recently introduced. This does not need armouring, and is usually finished with watertight protection such as rubber sandwich or a covering of P.V.C.

All cables, whether armoured or not, have an external covering, or serving of impregnated braiding or tape as an additional protection against damage and moisture.

British Standard Specifications B.S. 7, 480 Pts I and II, 608, 708, 760, 1557 and 2004

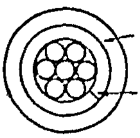
In the previous sections the various components of the insulated cable have been described. The dimensions of these components and the manner in which they are assembled in the completed cable are governed, in certain voltage ranges, by British Specifications, of which the more important are:

Title.	B.S.
1. Rubber-insulated Cables and Flexible Cords for Electric Power and Lighting. (This covers voltages up to 11 kV. inclusive)	7/1953
2. Impregnated-paper-insulated Cables for Electricity Supply. Part I. Lead or Lead-alloy sheathed Cables for working voltages up to and including 33 kV. .	480. Part I/1954
3. Impregnated-paper-insulated Cables for Electricity Supply. Part II. Aluminium-sheathed Cables for working voltages up to and including 22 kV. . .	480. Part II/1954
4. Varnished-cambric-insulated Cables for Electricity Supply. (This covers voltages up to 11 kV. inclusive) .	608/1943
5. Rubber-insulated Trailing Cables for use in mines. (Up to 660 volts inclusive)	708/1950
6. Paper-insulated Cables for use in Mines. (This covers voltages up to 6·6 kV. inclusive)	760/1943
7. Polythene-insulated Cables sheathed with P.V.C. (Up to 250 volts inclusive)	1557/1954
8. P.V.C.-insulated Cables and Flexible Cords. (Up to 250 volts inclusive)	2004/1953

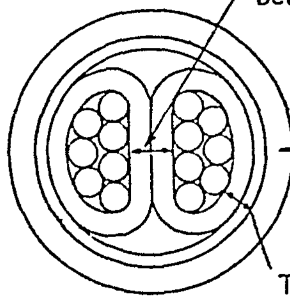
For working pressure above 33 kV., cables are styled " super voltage " and are subject to specifications issued by purchasing authorities or their consultants. All " oil-filled " and " gas-pressure " cables come under this heading.

Typical Low-voltage Cables

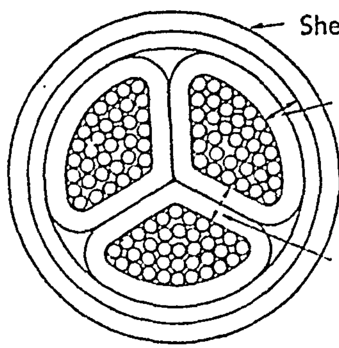
For voltages up to about 6,600, the electrostatic stresses are so small that the thermal conductivity is quite as important as the distribution of the electrostatic field, and no special constructions have to be adopted to control this field. Single-core cables have cores of circular section, and the layers of insulation and protection are concentric, so that there is nothing of special interest from the point of view of cable geometry. The multi-core cables



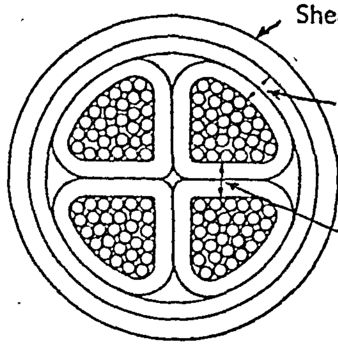
Single-core cable



Twin-belted cable



Three-core belted cable



Four-core belted cable

Fig. 12.2.—SECTIONS OF TYPICAL SINGLE AND MULTI-CORE CABLES.

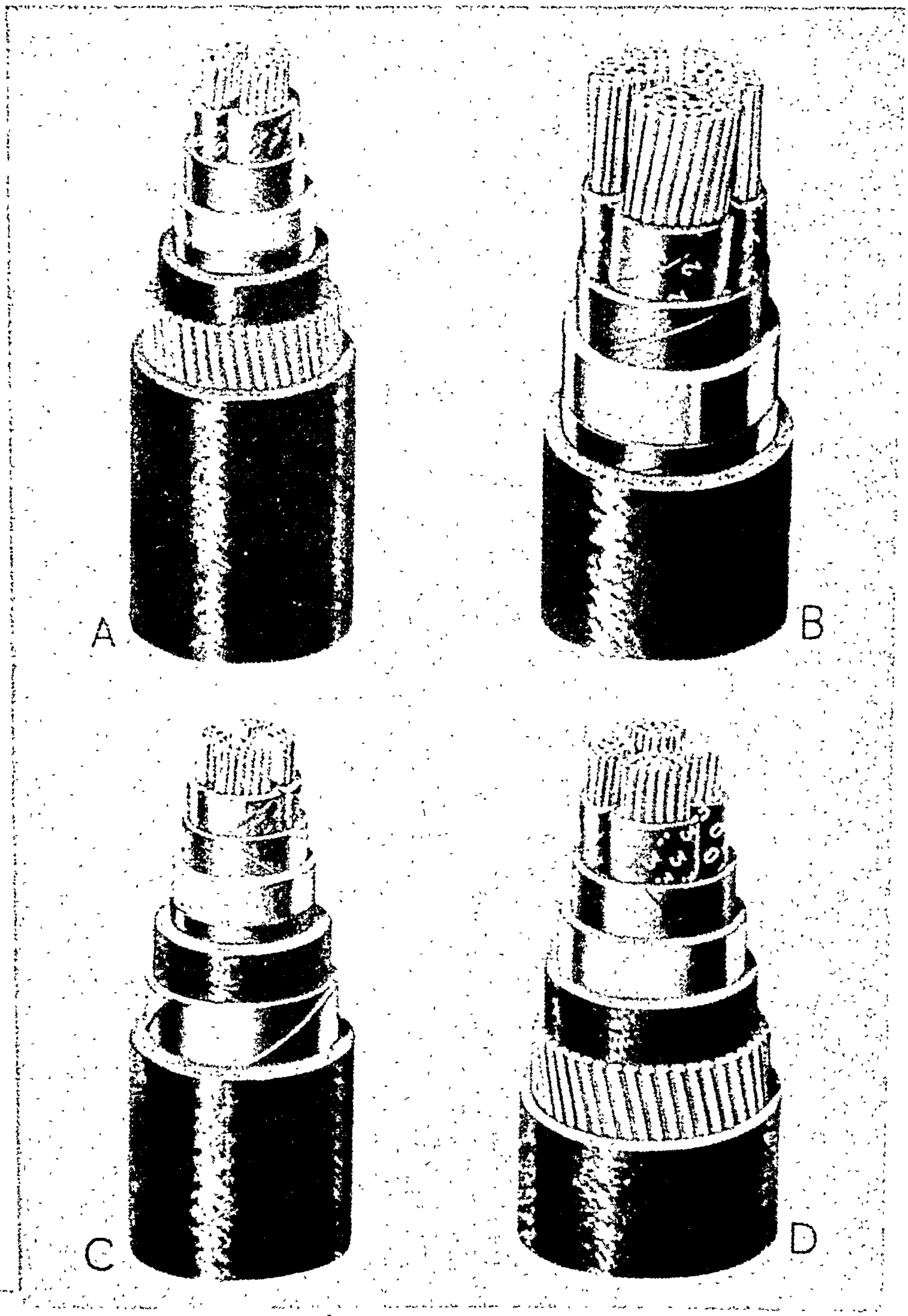


FIG. 12.3.—TELESCOPED VIEWS OF TYPICAL LOW-VOLTAGE CABLES.

A. Twin core, belted, lead sheathed and steel wire armoured.
B. Three core, belted, and lead sheathed.
C. Four core, belted, lead sheathed and steel tape armoured.
D. Four core, belted, lead sheathed and steel wire armoured: three main conductors of equal area and reduced neutral conductor.

(*W. T. Henley's Telegraph Works Co. Ltd.*)

can have cores which are either circular or shaped, the latter having a much better copper space factor—namely, copper area ÷ overall cable area—and therefore a smaller overall diameter for a given copper area. Where there is no difficulty in the way of high electrostatic flux density, the shaped conductor thus offers appreciable advantage, and it is used for voltages as high as 11 kV.

Multi-core cables are of the belted type, the cores are separately insulated, "wormed" together and the interstices filled with a packing or filling of fibrous material, so as to give a circular section. An overall circumferential belt of insulation is then applied. With paper insulation a lead sheath is extruded on to the belt; next, there is a bedding layer consisting of compounded fibrous materials, the armouring, whether steel tape or steel wire, and finally an outer covering, or serving, of layers of fibrous compounded material with additional compound applied under and over each layer.

The sections of single-core and multi-core belted cables are illustrated by the diagrams of Fig. 12.2.* Views of a few typical low-voltage cables are given in Fig. 12.3.

Extra-high-voltage Cables

For voltages greater than, say, 11,000, it is necessary to control the electrostatic field in order that a cable of reasonable overall diameter, and with good heat-dissipating properties, may be produced. With low-voltage cables we have seen that the size is determined by considerations of voltage drop in the case of distributors, and of economics in the case of feeders. Very high-voltage cables are, of course, only used as feeders, but their cost is so high and their construction so different from that of the low-voltage cable that laws of economy, such as Kelvin's Law, no longer apply. In fact, it can be said that the size of a high-voltage cable is governed solely by the current it has to carry and the manner in which it is laid, since it must be worked at its maximum safe temperature owing to its very high cost. Hence thermal considerations become as important as purely electrical considerations.

Electrostatic Stresses in Single-core Cables

Let r = radius of conductor.
R = internal radius of sheath.
ϵ = relative permittivity of the dielectric.
q = charge per metre axial length.

* Reproduced by permission of the British Standards Institution.

E = potential of conductor relative to sheath.
g = potential gradient at any radius x.

$$g = \frac{q}{2\pi\epsilon_0\epsilon}$$

and
$$E = \int_r^R \frac{q}{2\pi\epsilon_0\epsilon} \cdot dx = \frac{q}{2\pi\epsilon_0\epsilon}\,[\text{logh}\ x]_r^R$$
$$= \frac{q}{2\pi\epsilon_0\epsilon} \cdot \text{logh}\ \frac{R}{r}$$
$$\therefore q = \frac{2\pi\epsilon_0\epsilon E}{\text{logh}\left(\frac{R}{r}\right)}$$

Hence by substitution
$$g = \frac{E}{x\ \text{logh}\ \frac{R}{r}}$$

At the surface of the conductor, where $x = r$, we therefore have the maximum potential gradient of
$$g_{max} = \frac{E}{r\ \text{logh}\ \frac{R}{r}}$$

Again, $$g_{min} = \frac{E}{R\ \text{logh}\ \frac{R}{r}}$$
$$\therefore \frac{g_{max}}{g_{min}} = \frac{R}{r}\,*$$

The permissible value of the stress in the cable dielectric depends upon the margin of safety desired and upon the degree of dielectric heating allowed. In high-voltage cables the value of g is taken to the highest possible limit in order to keep down the overall diameter, but since the dielectric losses increase rapidly with potential gradient, the maximum allowable value of g is only about one-fifth of the breakdown value, viz. 40 to 50 kV. per cm.

For a given value of E and of R there is a certain value of the conductor radius r which gives a minimum potential gradient at the conductor surface. The expression for g_{max} has a minimum value when
$$r\ \text{logh}\ \frac{R}{r} = \text{a maximum}$$

* The irregular surface of the conductor due to stranding increases the value of g_{max} by about 20 per cent. This can be avoided in practice by covering the conductor with a thin smooth metal sheath.

$$\text{when} \quad \text{logh}\,\frac{R}{r} = 1$$

$$\text{when} \quad \frac{R}{r} = e = 2{\cdot}71882$$

Hence for a minimum g_{max} we have the condition

$$r = \frac{R}{2{\cdot}71828}$$

$$\text{Let}\ \frac{R}{r} > e$$

$$\text{say} \quad \frac{R}{r} = me, \text{ where } m > 1$$

$$\therefore\ g_{max} = \frac{E}{r\ \text{logh}\,\frac{R}{r}} = \frac{E}{\frac{R}{me}\ \text{logh}\ me}$$

$$= \frac{Eme}{R(1 + \text{logh}\ m)}$$

Suppose the layer of insulation next to the conductor breaks down, then it will become conducting and will have the effect of an increase in r which will decrease m and reduce g_{max} until, when $m = 1$ (i.e. $\frac{R}{r} = e$), g_{max} is a minimum. If breakdown proceeds farther, say through charring owing to too high a temperature, then g_{max} begins to increase again and breakdown occurs. Stubbings comments as follows on this theory: "The fact that so striking a result has never been produced experimentally with certainty has led to a belief, in many minds, that the maximum electric stress in a dielectric is not the dominating factor which causes breakdown of the insulation. . . . It must be remembered, however, that the assumption of homogeneity of the dielectric, assumed in the stress calculations, is far from true where paper-insulated cables are concerned, and the apparent contradictions of the maximum stress theory may be due rather to lack of homogeneity of the dielectric than to any inherent inaccuracy in the elementary theory itself."

It must not be assumed, however, that the theory is of no value; it provides a useful guide to the calculation of dielectric thickness so long as a reasonable factor of safety is assumed, as in so many engineering calculations. For high-voltage cables, designs based on this theory give a very high value for r, much too large from the point of view of current-carrying capacity, and it is therefore necessary to increase the conductor radius. This can be accomplished in three ways:

(i) Using aluminium instead of copper.
(ii) Using copper wires stranded round a central core of hemp.
(iii) Using a central lead tube instead of hemp. Very successful cables have been constructed on this principle.

Grading of Cables*

By the grading of a cable is meant the subdivision of the dielectric in such a way that the difference between g_{max} and g_{min} is reduced, with the result that less dielectric is required and the overall diameter reduced. There are two methods:

(i) By using layers of dielectric having different permittivities ϵ. This is called "capacitance grading."

(ii) By using metal intersheaths to fix the potentials at certain radii. This is called "intersheath grading."

Capacitance Grading

We have seen that

$$g = \frac{q}{2\pi\epsilon_0 \epsilon x}$$

and therefore, since the dielectric must withstand the maximum potential gradient of $q/2\pi\epsilon_0 \epsilon x$, a homogeneous dielectric is uneconomically used at all radii greater than r. Suppose we could vary the permittivity at different radii x in such a way that

$$\epsilon \propto \frac{1}{x} = \frac{m}{x} \text{ say}$$

then for any value of x

$$g = \frac{q}{2\pi\epsilon_0 \left(\frac{m}{x}\right) x} = \frac{q}{2\pi\epsilon_0 m} = \text{a constant}$$

An infinite graduation in ϵ is, of course, impossible, but a compromise can be effected by using two or three dielectrics with different values of ϵ. Suppose, for example, there are three layers of dielectric of outer radii r_1, r_2, and R and of dielectric con-

* The method of grading is no longer adopted, having been superseded by more practicable methods. This section is retained because it illustrates important fundamental principles.

stants ϵ_1, ϵ_2, and ϵ_3, as in Fig. 12.4. Then the distribution of potential gradient, as compared with that for a homogeneous dielectric, will be modified and the maximum values of the potential gradient will be

$$\frac{q}{2\pi\epsilon_0\epsilon_1 r} \text{ at } x = r;\quad \frac{q}{2\pi\epsilon_0\epsilon_1 r_1} \text{ at } x = r_1.$$

$$\frac{q}{2\pi\epsilon_0\epsilon_2 r_1} \text{ at } x = r_1;\quad \frac{q}{2\pi\epsilon_0\epsilon_2 r_2} \text{ at } x = r_2$$

$$\text{and } \frac{q}{2\pi\epsilon_0\epsilon_3 r_2} \text{ at } x = r_2;\quad \frac{q}{2\pi\epsilon_0\epsilon_3 R} \text{ at } x = R$$

Now let G_1, G_2, and G_3 be the dielectric strengths of the three

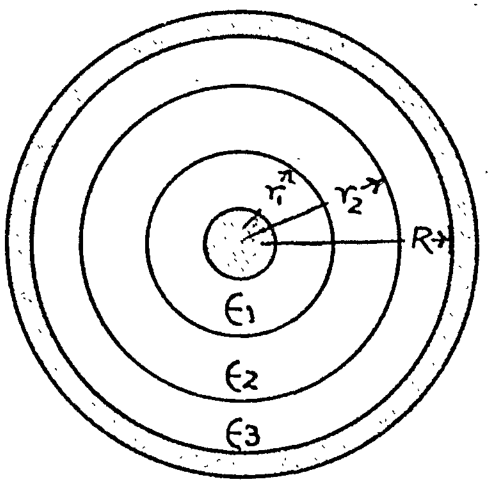


FIG. 12.4.—TO ILLUSTRATE CAPACITANCE GRADING.

materials ; then assuming that all three have the same factor of safety, f, we have

$$\frac{q}{2\pi\epsilon_0\epsilon_1 r} = \frac{G_1}{f};\quad \frac{q}{2\pi\epsilon_0\epsilon_2 r_1} = \frac{G_2}{f};\quad \frac{q}{2\pi\epsilon_0\epsilon_3 r_2} = \frac{G_3}{f}$$

$$\therefore\ \epsilon_1 r G_1 = \epsilon_2 r_1 G_2 = \epsilon_3 r_2 G_3$$

$$\text{Now, } r < r_1 < r_2$$

$$\text{Hence, } \epsilon_1 G_1 > \epsilon_2 G_2 > \epsilon_3 G_3$$

The position of the dielectric is thus decided by the product ϵG and not on G, although Russell states that "when the main object we have in view is to make, at all costs, the factor of safety of the cable as high as possible it is, in general, advisable to put the insulating material having the greatest dielectric strength next to the core, and, if possible, grade the dielectric by using outer layers having smaller dielectric coefficients."

If all three materials are worked at the same maximum potential gradient we have

$$\frac{1}{\epsilon_1 r} = \frac{1}{\epsilon_2 r_1} = \frac{1}{\epsilon_3 r_2}$$

$$\therefore \epsilon_1 r = \epsilon_2 r_1 = \epsilon_3 r_2$$

For the potential difference across the inner layer we have

$$E_1 = \int_r^{r_1} g \, . \, dx = \int_r^{r_1} \frac{q}{2\pi\epsilon_0\epsilon_1 x} \, . \, dx$$

$$= \frac{q}{2\pi\epsilon_0\epsilon_1} \operatorname{logh} \frac{r_1}{r}$$

$$= \frac{q}{2\pi\epsilon_0\epsilon_1 r} \cdot r \operatorname{logh} \frac{r_1}{r}$$

$$= g_{max} \, r \operatorname{logh} \frac{r_1}{r}$$

$$\text{similarly } E_2 = g_{max} \, r_1 \operatorname{logh} \frac{r_2}{r_1}$$

$$\text{and } E_3 = g_{max} \, r_2 \operatorname{logh} \frac{R}{r_2}$$

and for the total potential difference between core and earthed sheath

$$E = E_1 + E_2 + E_3$$

$$= g_{max.} \left(r \operatorname{logh} \frac{r_1}{r} + r_1 \operatorname{logh} \frac{r_2}{r_1} + r_2 \operatorname{logh} \frac{R}{r_2} \right)$$

As a numerical example let $r = 1{\cdot}0$ cm.

$$\epsilon_1 = 5, \ \epsilon_2 = 4, \ \epsilon_3 = 3$$

$$\therefore 5 \times 1 = 4 \times r_1 = 3 \times r_2$$

giving $r_1 = 1{\cdot}25$ and $r_2 = 1{\cdot}67$

Assume also that the radius R is 2·5 cm. and that $g_{max} = 40{,}000$ volts per cm.

$$\therefore E = 40000 \left(1 \times \operatorname{logh} \frac{1{\cdot}25}{1} + 1{\cdot}25 \operatorname{logh} \frac{1{\cdot}67}{1{\cdot}25} + 1{\cdot}67 \operatorname{logh} \frac{2{\cdot}5}{1{\cdot}67} \right)$$

$$= 40000 \times 2{\cdot}3 \, (\log_{10} 1{\cdot}25 + 1{\cdot}25 \log_{10} 1{\cdot}33 + 1{\cdot}67 \log_{10} 1{\cdot}5)$$

$$= 50170 \text{ volts}$$

Now suppose that the cable had a homogeneous dielectric but that the values of r, R, and g_{max} were the same, then for the maximum permissible potential difference between core and sheath we should have

$$\begin{aligned} E' &= g_{max}\, r \,\text{logh}\, \frac{R}{r} \\ &= 40000 \times 1 \times \text{logh}\, \frac{2{\cdot}5}{1} \\ &= 40000 \times 2{\cdot}3 \times \log_{10} 2{\cdot}5 \\ &= 36600 \text{ volts} \end{aligned}$$

Thus for the same conductor diameter and the same overall dimensions the graded cable can be operated at a voltage of 13,900 higher than the homogeneous cable, an increase of 38 per cent. The division of voltage between the various layers of the graded cable is given by

$$\begin{aligned} E_1 &= 2{\cdot}3 \times 40000 \times \log_{10} 1{\cdot}25 = 7200 \text{ volts} \\ E_2 &= 2{\cdot}3 \times 40000 \times 1{\cdot}2 \log_{10} 1{\cdot}33 = 15900 \text{ volts} \\ E_3 &= 2{\cdot}3 \times 40000 \times 1{\cdot}67 \log_{10} 1{\cdot}5 = 27100 \text{ volts} \end{aligned}$$

It must be remembered that the permissible values of g_{max} are peak values, and therefore all the voltages in the above calculation are peak, and not R.M.S., values.

Intersheath Grading

With this method a homogeneous dielectric is used, but it is divided into layers by the interposition of metallic intersheaths which are held at a definite potential between that of the conductor and earth. There is thus a definite potential difference between the inner and outer radii of each sheath, so that each sheath can be treated like a homogeneous single-core cable. Calling the various radii r, r_1, r_2, . . . R, as before, we have

$$g_{max1} = \frac{E_1}{r \,\text{logh}\, \frac{r_1}{r}}$$

$$g_{max2} = \frac{E_2}{r_1 \,\text{logh}\, \frac{r_2}{r_1}} \text{ and so on.}$$

But for a homogeneous dielectric we obviously require the condition

$$g_{max1} = g_{max2} = g_{max3} \;.\;.\;.$$

$$\therefore \frac{E_1}{r \,\text{logh}\, \frac{r_1}{r}} = \frac{E_2}{r_1 \,\text{logh}\, \frac{r_2}{r_1}}$$

Again, since all these potential differences are in phase, since the cable is, in effect, a series of condensers in series, we also have the condition

$$E = E_1 + E_2 + E_3 + \ldots$$

If all the n layers have the same thickness d, we have

$$r_1 = r + d\,;\; r_2 = r + 2d\,;\; r_3 = r + 3d$$

$$\therefore \frac{E_1}{r \operatorname{logh} \frac{r+d}{r}} = \frac{E_2}{(r+d) \operatorname{logh} \frac{r+2d}{r+d}} = \ldots = \frac{E}{M}$$

$$\text{where } M = r \operatorname{logh} \frac{r+d}{r} + (r+d) \operatorname{logh} \frac{r+2d}{r+d} + \ldots$$

$$+ \{ r + (n-1)d \} \operatorname{logh} \frac{r+nd}{r+(n-1)d}$$

$$\therefore E_m = \frac{E}{M}\left\{ r + (m-1)d \right\} \operatorname{logh} \frac{r+md}{r+(m-1)d}$$

Putting $m = 1, 2, 3 \ldots n$, we have for the peak values of the voltages across the various layers to give a uniform value for g_{max}

$$E_1 = \frac{E}{M} r \operatorname{logh} \frac{r+d}{r}\,;\; E_2 = \frac{E}{M}(r+d) \operatorname{logh} \frac{r+2d}{r+d}\,;\; \text{etc.}$$

Also, if E is given in R.M.S. volts, then obviously E_1, E_2, etc., will also be in R.M.S. volts.

In practice there is considerable difficulty in arranging for many intersheaths, this difficulty being mainly associated with the provision of the different voltages for the intersheaths, and as a result it is usual to design a cable of this type with only one intersheath even for voltages as high as 100,000. This very considerably simplifies the design of the cable. Let r and R be the radius of core and outer radius of dielectric, as before, and let r_1 be the radius of the intersheath. Then for $g_{1.max}$ to be equal to $g_{2.max}$ we have

$$g_{max} = \frac{E_1}{r \operatorname{logh} \frac{r_1}{r}} = \frac{E_2}{r_1 \operatorname{logh} \frac{R}{r_1}}$$

Again, $\qquad E_2 = E - E_1$

$$\therefore g_{max}\, r_1 \operatorname{logh} \frac{R}{r_1} = E - g_{max}\, r \operatorname{logh} \frac{r_1}{r}$$

giving

$$g_{max} = \frac{E}{r_1 \operatorname{logh} \frac{R}{r_1} + r \operatorname{logh} \frac{r_1}{r}}$$

Now, r is a constant, and if, in order to make a comparison with the most economical cable with homogeneous core and no inter-sheath, we adopt for R the previously obtained value of

$$R = er$$

R is also a constant, and the independent variable in the above equation is r_1. Hence, differentiating with respect to r_1, we have

$$\frac{dg_{max}}{dr_1} = \frac{d}{dr_1} \cdot \frac{E}{r_1 \text{ logh } \frac{er}{r_1} + r \text{ logh } \frac{r_1}{r}}$$

This is equal to zero and corresponds to a minimum when

$$\frac{r_1}{r} \text{ logh } \frac{r_1}{r} = 1$$

The solution of this is

$$r_1 = 1{\cdot}76r$$

and therefore

$$g_{max} = \frac{E}{1{\cdot}76\, r \text{ logh } \frac{er}{1{\cdot}76r} + r \text{ logh } \frac{1{\cdot}76r}{r}}$$
$$= \frac{E}{1{\cdot}33r}$$

But for the non-intersheath cable we have

$$g'_{max} = \frac{E}{r \text{ logh } \frac{R}{r}} = \frac{E}{r \text{ logh } e}$$
$$= \frac{E}{r}$$

Hence for the same dimensions the use of one intersheath has raised the maximum voltage the cable can withstand in the ratio

$$\frac{g'_{max}}{g_{max}} = \frac{E/r}{E/1{\cdot}33r} = 1{\cdot}33$$

an increase in 33 per cent.

The following particulars refer to an actual design of single-core cable with single intersheath. The conductor is a single layer of 15 S.W.G. copper wire laid round a central lead tube and surrounded by an outer lead tube, the function of which is to give a smooth surface in contact with the dielectric.

	Radial thickness (in.).	Radius (in.).
Inner lead tube	·060	·195
Conductor 19 copper wires S.W.G. 15 . . .	·072	·267
Outer lead tube	·050	·317
First paper layer	·545	·862
Lead intersheath	·050	·912
Second paper layer	·565	1·477
Outer lead sheath	·160	1·637

This cable is one of three designed for a three-phase supply, using separate single-core cables, with a pressure of 173 kV. between phases. It will be seen that the intersheath is of finite, though small, thickness, and this has a small effect on the magnitude of the voltage gradient.

Grading in Practice

Grading is only applicable to very high-voltage working for which the ratio R/r is large. This is very well summed up by Russell as follows : " Grading is only useful when the inner radius R of the outer conductor is large compared with the outer radius r of the inner conductor. For example, in Jonas's method * if R/r is 5, we can double the applied pressure without affecting the factor of safety by grading the insulating material into four layers. But if R/r is 2 the permissible increase is only 30 per cent. In exactly the same way the conducting layer method (intersheath) is only valuable when R/r is large."

With capacitance grading there is the practical difficulty of obtaining materials whose permittivities can be given the required values. The permittivity for impregnated paper can be varied between 2·8 and 4·0 and that for rubber between 4·2 and 6·1. The necessary difference can thus be obtained by the use of paper and rubber in the same cable, but rubber is very expensive, and for this reason alone has been largely superseded by other materials for cable construction. A second difficulty is the possibility of change of the permittivity with time. Such a change would completely alter the distribution of potential gradient and might even lead to breakdown at the normal working voltage.

With the intersheath method, the main objection is the possibility of damage to the intersheath during the laying of the cable,

* A stranded central core surrounded by a closely fitting lead tube, and with four layers of dielectric, with dielectric constants varying from 6·1 at the centre to 4·0 outside.

since the intersheath must, of necessity, be thin. The intersheath has also to carry a charging current which, in a very long line, may be sufficient to cause overheating.

For the above reasons the modern tendency is to avoid grading

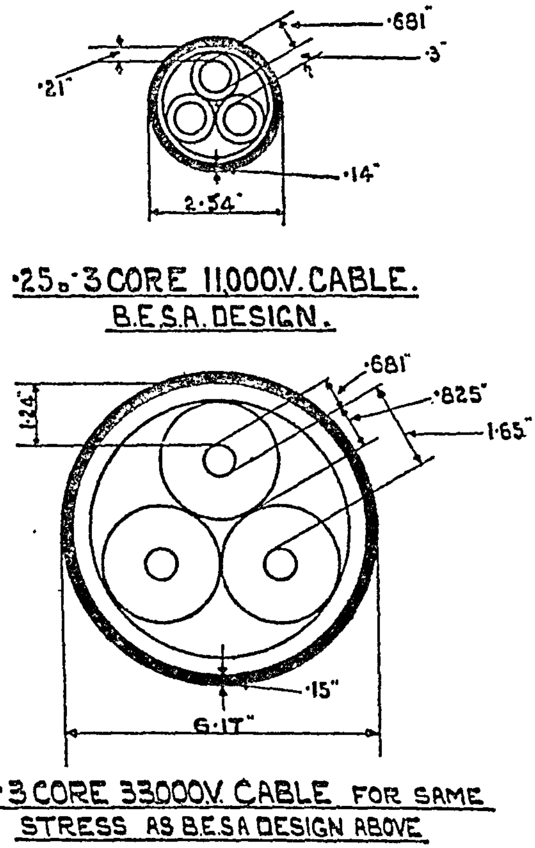


FIG. 12.5.—COMPARATIVE SIZES OF 11-KV. AND 33-KV. CABLES DESIGNED FOR EQUAL RADIAL STRESSES.
(*Electrical Power Engineers' Association.*)

in favour of the oil-filled and gas-cushion principles described later.

Three-core Cables

The design of a high-voltage three-phase cable is different from that of the previously described low- and medium-voltage three-phase cables for two reasons: firstly, if the stresses are the same for all voltages then the overall diameter of the high-voltage cable becomes so great that the cost of the cable will outweigh the advantage of reduced copper section given by the high voltage; and secondly, the electrostatic stress is not a pure radial stress, and at high voltages the tangential component of this stress assumes almost fundamental importance. The first of these points is well illustrated by Fig. 12.5 which shows the dimensions of an 11-kV. cable with insulation as specified by the British Standards Institution, and a 33-kV. cable designed to have the same radial stress, and having cores of the same cross-section. An overall diameter of 6·17 in. is quite impossible, and it is therefore necessary to increase the radial stress in order to obtain acceptable dimensions.

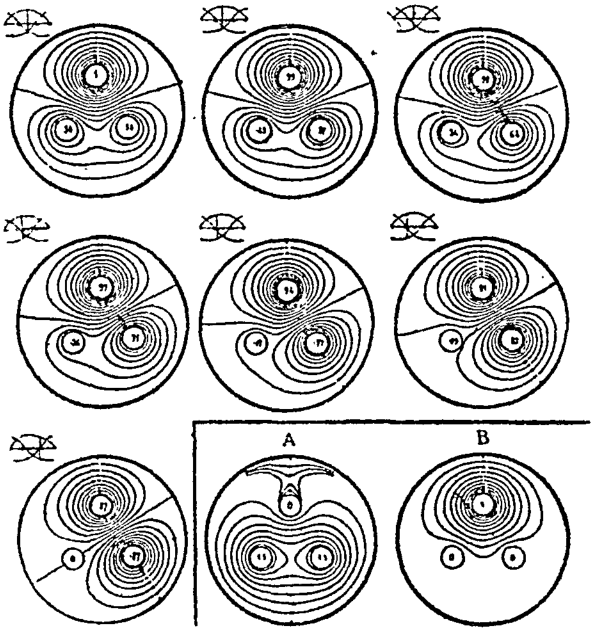


FIG. 12.6.—VARIATIONS IN THE SHAPE OF THE EQUIPOTENTIAL SURFACES OF A THREE-PHASE BELTED CABLE DURING THE CYCLE OF VOLTAGE CHANGES.

The existence of a tangential stress arises from the fact that there is a rotation of voltage in the three cores, with the result that since the centres of the cores lie on a circle, the electrostatic field is a rotating and not an alternating one. The distribution of equipotential surfaces at various points in the cycle is illustrated in Fig. 12.6* from which it will be seen that at many points in the field the lines of force, being perpendicular to the equipotential surfaces, are not radial. Consequently, the potential gradient is not radial, and therefore has a tangential component

* The first seven figures are from experimental data due to Emanuelli and Atkinson. See "Ricerca Sperimentale della Distribuzione del Gradiente di Potenziale nei Cavi Trifasi," *l'Elettrotecnica*, vol. 8, p. 573. Also *High Voltage Cables* (Chapman & Hall). The figures A and B are calculated. The stresses can be calculated by Russell's formula, *Alternating Currents*, vol. i.

which acts along, instead of across, the layers of paper. Now, the electrical resistance of laminated paper is much greater across the layers than along them, and also the dielectric strength is much greater across the layers. Owing to this tangential stress there is a leakance current, as distinct from a capacitance current, along the layers of paper adjacent to the centre filling of the cable, and the loss of power sets up local heating, with the consequent risk of breakdown. That this actually takes place in a belted cable is illustrated by Fig. 12.7, which is a reproduction of an actual breakdown test on a three-core belted cable. It will be seen that charring occurred at those places where the stress was tangential to the layers of paper. Even if there is no ionisation or vacuous space initially, the tangential leakance current will char the paper and oxidise the compound, thereby setting up a conducting path across which a breakdown current can pass.

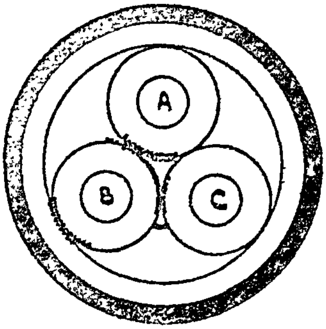


Fig. 12.7.—Breakdown of a Three-phase Belted Cable.

The elimination of the tangential stress can be accomplished only by screening each core separately, so that the cable becomes, in effect, three separate single-core cables laid up within the same protective covering. The "H"-type cable, invented by Hochstadter, consists of three paper-insulated cores, and over the insulation of each is wound a layer of perforated metallised paper. These cores are laid up in the ordinary way with the metallised screens in contact with one another, and also with an additional screen wrapped round all three cores, this outer screen being of cotton tape with fine copper wires interwoven. The cable has no belt insulation, but is lead covered and armoured. The perforations in the metallised paper sheaths assist in the complete impregnation of the cable which, in fact, becomes practically a homogeneous mass in which there is little likelihood of air or gas pockets. All four screens and the lead sheath are at earth potential, with the result that the electrical stresses are purely radial and the dielectric losses are therefore reduced. In addition, there are no sheath losses and the metallised screens assist in the dissipation of heat. All of these combine to give a greater current-carrying capacity than the equivalent belted cable. Thus a 0·194 sq. in. cable with radial thickness of paper of 0·236 in. will carry 261 amps. if of ordinary belted construction, and 298 amps. if of " H " construction, a gain of 14·2 per cent. According to the size of core and thickness of insulation the gain varies from

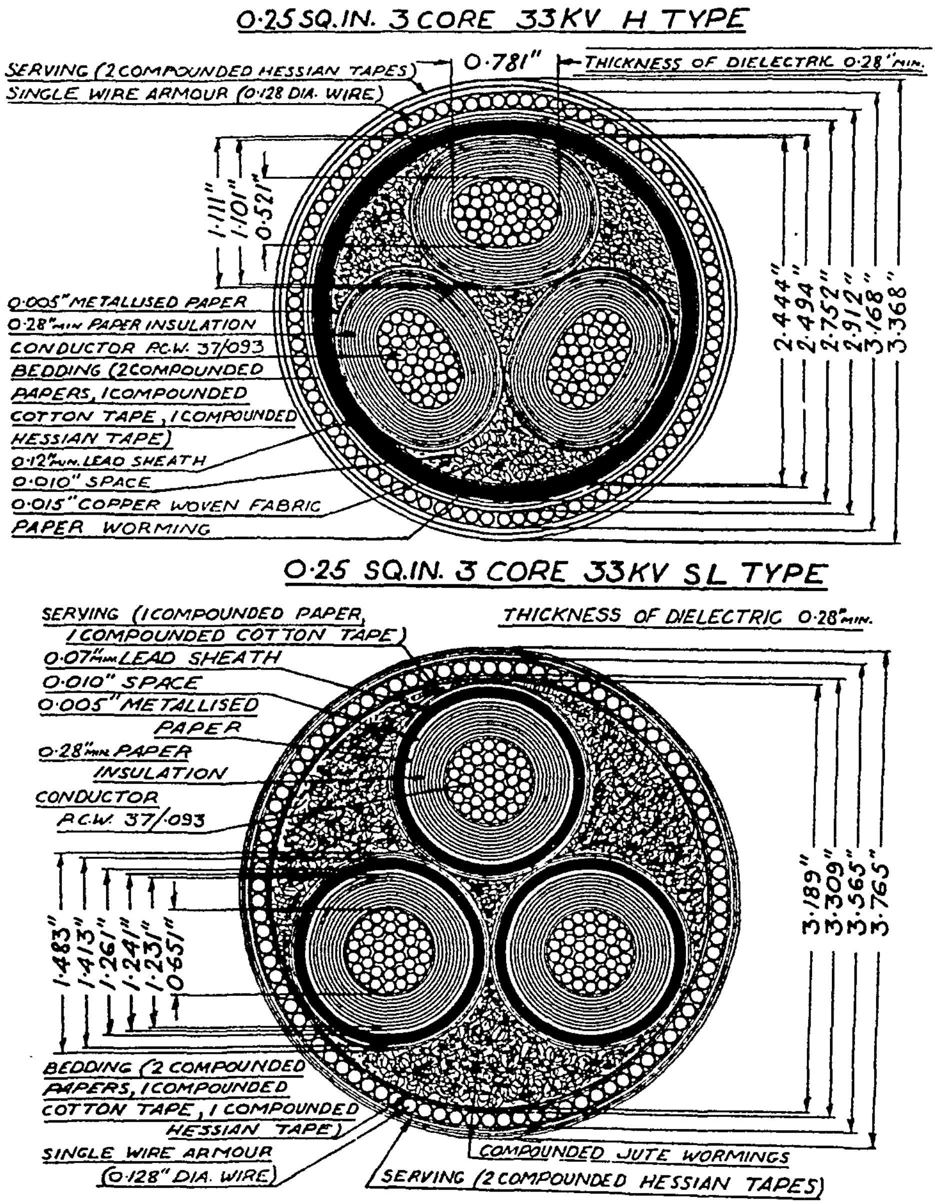


Fig. 12.8.—Cross-sections of " H "-type and " SL "-type Cables.
(*Johnson & Pillips, Ltd.*)

14 to 21·2 per cent. The " H "-type cable has been used for pressures up to 66 kV.

In the " S.L." cable the three cores can be regarded as three

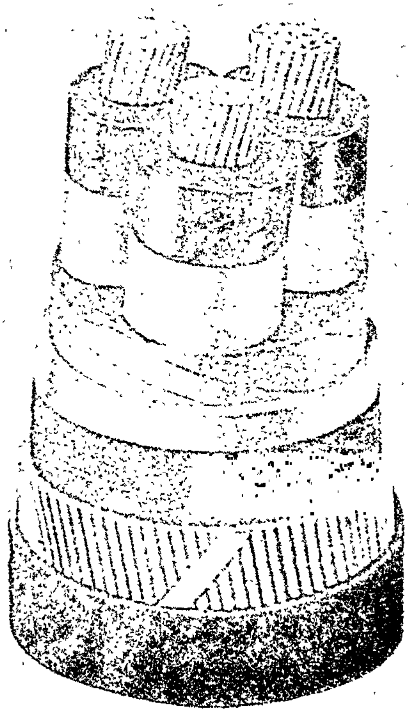


FIG. 12.9.—TELESCOPED VIEW 33,000 V OF " H "-TYPE CABLE. (*W. H. Henley's Telegraph Works Co. Ltd.*)

separate single-core cables since each has its own lead sheath. The three cables are laid up with fillers in the ordinary way, armoured and served overall with impregnated hessian, but there is no lead covering additional to the sheaths on the individual cores. The " S.L." type is also constructed with a metallised paper tape immediately underneath, and in contact with, each lead screen. In another special form of " S.L." cable, steel reinforcing strands are laid up in the spaces between the cores in order to give longitudinal strength without the sacrifice of flexibility. The various types of screened-core cable are illustrated in cross-section in Fig. 12.8 and in construction by Fig. 12.9. The " S.L." construction is useful on short runs because the terminating equipment is simplified.

Dunsheath summarises the advantages of cables with metal-sheathed covers over plain three-core cables as follows:

1. Long-time failures on plain three-core cables originate in core-to-core and not core-to-earth faults. The metal-sheathed core cable has a greater core-to-core thickness for a given overall diameter than a plain cable.

2. Direct core-to-core faults are prevented by the metal sheaths, and failure from any cause results in breakdown to earth, which is an advantage from the operating point of view.

3. Electric stress uniformly radial in all parts of the dielectric.

4. The dielectric subjected to stress is homogeneous, since no wormings or packings are in the electric field.

5. Mechanical distortion due to any cause cannot introduce voids in the dielectric. Voids in the packings are of no consequence because of advantage 4.

6. The metal sheaths facilitate the dissipation of heat, thereby improving the current-carrying capacity as explained above.

With regard to the relative advantages of the "H" and "S.L." types, it is to be noted that the lead tubes in the latter are of small diameter, so that bending results in minimum distortion of the lead. The wormings of the "H" type are full of oil, while those of the "S.L." type are not, the latter thus having a smaller oil content. This is an advantage in hilly country, where the draining of the oil may result in considerable pressure and consequent risk of distortion. The three sheaths of the "S.L." cable are much thinner than the single sheath of the "H" type and therefore call for great care in manufacture. Finally, the separate sheaths of the "S.L." cable are the seats of induced currents, but the resulting losses are small, and appear to be of no practical significance.

Super-voltage Power Cables

The previous discussions of the operation of insulated cables and the designs based on them have been concerned almost solely with the distribution of electrostatic stress in a medium assumed to be homogeneous, and with the control of this stress. With this assumption it is necessary to limit the maximum stress to about 4 or 5 kV. per mm., and the maximum operating temperature to 50° or 60° C. in order to ensure freedom from breakdown. When it is remembered that the instantaneous breakdown value of the impregnated paper used for the conductor insulation is in the neighbourhood of 40 kV. per mm., and the long-time breakdown value about 16 kV. per mm., it is clear that there is a very inefficient utilisation of the material, and that the stress must be increased to about 7 kV. per mm., giving a factor of safety of 2½ to 3, if very high voltage transmission by insulated cables is to compete economically with overhead transmission.

The above limits, although low, are necessary because it is now realised that the dielectric is far from homogeneous, but that with normal manufacturing methods it is impossible to avoid the existence of gas-filled or vacuous spaces in the layers. Again, even if a new cable could be constructed without any of these spaces, or voids, it would be impossible to avoid their formation under the influence of movements of the compound and expansion and contraction of the paper during normal operation. When the dimensions of these voids reach a certain value, depending on the strength of the field in the neighbourhood, there will be local breakdown with resulting ionisation. If the ionisation is sufficiently intense, chemical action will follow and that particular part of the cable will be gradually destroyed. All such voids are liable to breakdown because, due to their lower dielectric constant, they are exposed to a relatively high potential gradient, and in addition their breakdown strength is low, only about one-tenth of that of impregnated paper, *when they are at atmospheric pressure.* It is only possible to make better utilisation of the dielectric by ensuring that the dielectric is homogeneous, or that any voids, if present, will not break down. So far there have been two methods of solution to this problem, both necessitating that the impregnating material shall be under considerable pressure.

Before describing the various pressure cables it is necessary to consider several factors which have influenced the design of these cables. These are as follows:

i. Instantaneous and long-time breakdown strength.
ii. Dielectric losses.
iii. Stability.
iv. Deterioration.

(1) Instantaneous and Long-time Breakdown Strength

The time-voltage breakdown test is a test of the quality of the material of which the insulation of the cable is composed. The

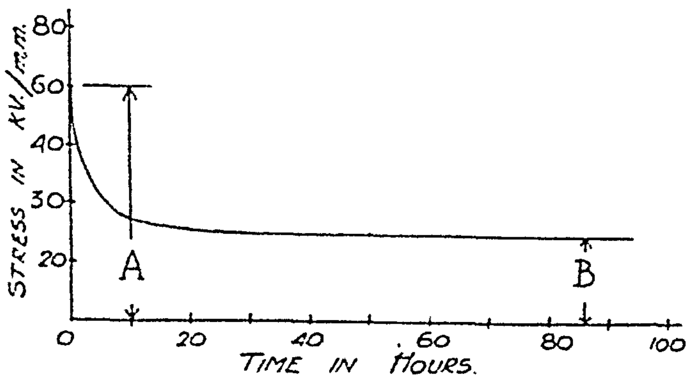


FIG. 12.10.

shape of the curve is somewhat as indicated in Fig. 12.10, the terminal values being determined by the nature of the insulation, and also by the conditions of the test. The instantaneous breakdown strength is given by the ordinate A, while the long-time breakdown strength is given by B. Bowden and Main make the following observations on the time-voltage characteristic: "This time-voltage breakdown is neither a purely/thermal nor a purely electrical one, but . . . the explanation of it must lie in the heterogeneity of the cable dielectric. The insulation of a cable is a comparatively coarse and heterogeneous body, and it is therefore to be expected that there are present in the dielectric heterogeneous places of different intensity, extent, and number which, under high electrical stress, are subject in the course of time to gradual disintegration. The time necessary for this effect is greater the lower the stress and the less intensive and extensive the heterogeneity. When a certain voltage is reached, discharges are set up and pressure breakdown occurs in the largest and most extensive of these places. As the voltage increases, more such places of lesser intensity and extent are involved, and the intensity of the disintegration process at places already ionised increases. Conversely, as the voltage falls, more and larger heterogeneous places become free from pressure breakdowns until, ultimately, ionisation ceases even in the regions of most pronounced heterogeneity.

"If the cable be subjected to excessive temperature variations the time-voltage curve is displaced downwards, the displacement depending on the temperature and the number of heat cycles which the cable has undergone.

"It must therefore be assumed that the asymptotic value of the time-voltage curve may be considerably lowered under operating conditions, and having regard to the necessary factor of safety of $2\frac{1}{2}$ to 3, the usual value of the maximum stress of 4 to 5 kV. per mm. (for impregnated paper) seems reasonable, although the material will stand, momentarily, ten times this stress.

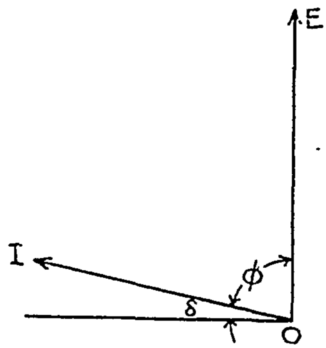


FIG. 12.11.

"It will be seen that better utilisation of the material would be possible only if by some means the heterogeneity of the dielectric, or at least its effect, could be reduced and in consequence the time-voltage curve raised."

(2) Dielectric Losses

The breakdown of a cable is not necessarily electrical: it may be thermal or mechanical. Thermal breakdown is due to rise in temperature resulting from all the losses taking place in the cable. One of these losses is the dielectric loss due to dielectric hysteresis. The current taken by an unloaded cable does not lead by 90 degrees, but by an angle φ, which is less than 90 degrees, Fig. 12.11.

$$\text{Let } \varphi + \delta = 90°$$

$$\text{Then dielectric loss} = EI \cos \varphi$$
$$= EI \sin \delta = EI\delta, \text{ since } \delta \text{ is small.}$$

Regarding the unloaded cable as a condenser of capacitance C, we have

$$I = EC\omega$$

$$\therefore \text{ Dielectric loss} = E^2\omega C\delta$$

This shows that the dielectric loss is proportional to the square of the voltage, with the result that, although it is of little importance in low-voltage working, it is of very great importance with high voltages, say, over 22 kV.

We have already seen the effect of a heating/cooling cycle on the voltage-time/breakdown curve; such a cycle is also of importance in connection with the cable power factor. Curve A, Fig. 12.12, shows the power factor/voltage curve for a typical newly manufactured cable. Heating to 70° C. gives the curve B, and the performance under both conditions A and B is satisfactory. After cooling from the conditions of B, the characteristic becomes similar to that of curve C, the rapid rise at just over half normal voltage indicating the commencement of ionisation at this voltage. This indicates an important change in the cable structure, resulting from the temperature cycle, and if such an increase in loss is localised it may represent considerable damage which may ultimately lead to breakdown. This in spite of the fact that the power factor of 0·03 is not excessive.

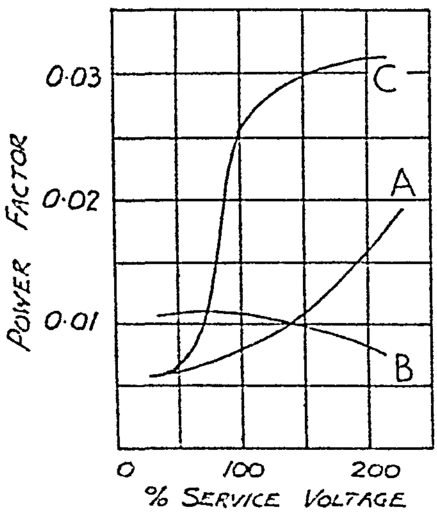


FIG. 12.12.—EFFECT OF HEATING/COOLING CYCLES ON POWER FACTOR.

(3) Ionisation

Curve C of Fig. 12.12 shows that the dielectric loss rises very rapidly when ionisation occurs, whereas without ionisation the "pure" loss is proportional to the square of the voltage. Ionisation can thus mean the physical action of gaseous ionisation inside a void space within the dielectric, or it can refer to the rising power-factor characteristic with voltage.

If the body of the dielectric is uniform it appears that ionisation cannot take place, but if voids are present, say, due to imperfect impregnation or as the result of successive expansions and contractions, then discharges can take place. The magnitude of the discharge will then depend upon the gas pressure within the void and also on the potential gradient; the lower the pressure the lower the voltage required to produce a discharge. The voltage gradient acting across a void is inversely proportional to the permittivity, with the result that with an impregnated paper dielectric the gradient across the void may be three or four times that acting on the dielectric in the immediate neighbourhood of the void. The effect of ionisation is the gradual breakdown of the dielectric due to chemical action.

(4) **Stability**

The term stability is used to denote the characteristic by which a cable will retain its freedom from ionisation during service or under conditions of load designed to represent service conditions. The successive expansions and contractions due to load cycles may set up conditions which encourage the formation of voids even if the cable is initially free from voids. As the temperature is raised the compound expands, with the resulting flow of compound between the layers of paper. On subsequent cooling the forces tending to restore the compound to its original position are less, and, in addition, the temperature variations across the cross-section are responsible for variations in viscosity. Thus it is practically impossible for the final state of the cable to be identical with the initial state, and when the changes have been repeated many times voids are almost inevitable.

Thus, in order to test a cable for stability it is necessary to subject the cable to heating cycles while under the impressed voltage, and to measure the power factor over a range of voltage after each cycle. Assuming identical initial performances for a number of cables the best cable will be that whose final power factor/voltage curve shows the least departure from the original curve.

The three factors involved in a stability test are :

i. Duration (length and number of cycles).
ii. Temperature attained.
iii. Voltage applied.

One suggestion for item (i) is a short time test of about three heating and cooling cycles extending over a fortnight, the idea being to obtain the results in a reasonable time. A second suggestion is to extend the test over a year, since, owing to the nature of the problem, nothing like finality can be secured in a short period. Dunsheath considers that under suitable conditions the 14-day test will give a satisfactory idea of the stability of a cable, and that a 12-month test is an expensive luxury.

Item (ii), the maximum temperature, is governed by the probable temperature under working conditions, a reasonable maximum being 65° C. The effect of the maximum temperature on the power factor is indicated by Fig. 12.13, in which Fig. B is for a

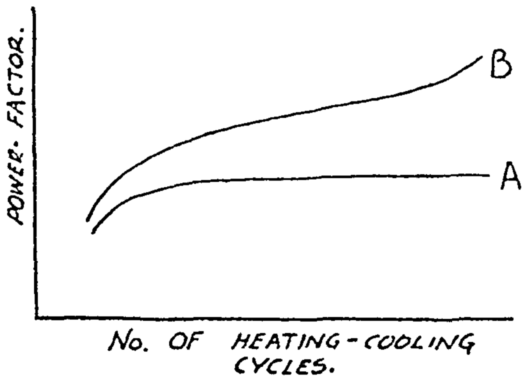


FIG. 12.13.—EFFECT OF HEATING/COOLING CYCLES ON POWER FACTOR.

higher temperature than A. A curve such as B indicates that the stability limit has been exceeded.*

Item (iii) is also controversial. " If a cable is perfectly stable with 110 per cent. of working voltage applied during three cycles up to a maximum temperature of 65° C., it is very unlikely to fail in normal service for many years through ionisation deterioration at 100 per cent. working voltage. On the other hand, if a greater percentage of overvoltage during a stability test is found to produce no ionisation, then obviously this second cable is giving a greater factor of safety." Dunsheath points out that a 14-day test with a 10 per cent. overvoltage would have doubled the power factor of a cable made a few years ago, but with a modern cable there would be very little change in the slope of the curve.

(5) Deterioration

It will be clear that the presence of voids, with consequent ionisation, is one of the chief causes of the deterioration of a cable dielectric. This deterioration is accompanied by very complex chemical changes, one of which is the formation of a hard yellow wax called an " x " product. At the same time hydrogen gas is released, Hirshfield having obtained as much as 400 c.c. from less

* See also p. 260.

than 2 grm. of compound in 25 hours. Comparing the relative amounts of "x" and hydrogen in experimental cables, and making allowance for all the possible destinations of the hydrogen, it was found that a large proportion of this gas was not thus accounted for. The most obvious inference was that water had been formed, and on examining lengths of cable after test as much as 6·5 grm. of water per 10 ft. were found. Further experiment showed that the oxygen required for the formation of water must have been derived from the paper, thus proving that both paper and compound suffer deterioration. The gas spaces between the layers of paper commonly have a dentritic appearance "as though the gas were spreading and feeling its way between the layers." These spaces are sometimes accompanied by blackening of the compound, also in dentritic formation, and in extreme cases by scoring and puncture of the paper.

If the applied voltage is below that required to set up ionisation, then the long-time breakdown voltage will be independent of the time of application for all voltages below that value. At voltages sufficiently high to produce ionisation the quality of both paper and compound is of great importance, and with high-grade materials a slight amount of ionisation has no practical importance.

The desirable qualities of a cable, oil or compound, are summarised by Say as follows:

(*a*) Low coefficient of expansion.
(*b*) Low viscosity at impregnation temperature.
(*c*) High viscosity at working temperature (solid cables only).
(*d*) Solidifying point below service temperatures.
(*e*) Some lubricating property.
(*f*) Low permittivity and temperature coefficient, and high resistivity.
(*g*) High dielectric strength.
(*h*) Chemically stable and free from occluded gases.

Summarising, it can be said that two essential requirements must be fulfilled in the design of a super-voltage cable: (1) the suppression of ionisation by the inhibition of void formation under all possible operating conditions; (2) the most efficient use of the insulation by the adoption of the highest possible electric stress. It will also be obvious that for very high voltages, e.g. above 132 kV., the problem will be simplified by the use of three single-core cables: (1) because of radial symmetry, geometrical, electrical, and thermal; (2) because of the very large diameter and consequent loss of flexibility of a three-core cable. However, a three-core impregnated pressure cable has been made for this voltage, its overall diameter being 4·84 in.

Types of Super-voltage Cable

In all cases the insulation consists of oil-impregnated paper tape. There are two methods of void control:

1. The oil filled cable;
2. The gas pressure, or compression, cable.

There are two methods of maintaining the gas pressure: (*a*) external pressure, (*b*) internal pressure.

There are two sub-divisions of the external pressure cable, viz. Pipe-line type and Self-contained type. They are also both known as compression cables.

There are three sub-divisions of the internal pressure cable, viz. High-Pressure Gas-filled, Gas cushion, and Impregnated Pressure.

Oil-filled Cables

We have seen that void formation is of such importance in connection with cable breakdown that any means adopted to prevent such formation must result in greatly increased reliability in service, and also increase the voltage at which an insulated cable can be worked. The advantages of the oil-filled cable over the solid type are:

(*a*) More perfect impregnation.
(*b*) No ionisation, oxidation, or void formation.
(*c*) Increased temperature range in service possible.
(*d*) The maximum allowable dielectric stress can be increased.
(*e*) Smaller-sized cable for given voltage and kVA. rating.
(*f*) Cable can be impregnated after sheathing.
(*g*) A defect in the lead sheathing will be indicated by an oil leakage.

The disadvantages are, greater cost and greater complication.

As the name indicates, the feature of the oil-filled cable is that it is kept constantly supplied with oil by means of reservoirs, with the result that expansions or contractions merely produce changes in level in reservoirs and consequently do not result in voids. The direct result is that, in comparison with a solid cable of the same instantaneous breakdown strength, the long-time breakdown strength is about doubled.

There are three main types of oil-filled cable, viz.:

(*a*) Single-core, conductor channel.
(*b*) Single-core, sheath channel.
(*c*) Three-core, filler-space channels.

These are illustrated in Fig. 12.14. Type (*a*) has a hollow conductor which acts as an oil channel, and is the simplest from the point of view of the cable itself. Its disadvantage is that this channel is at the middle of the cable and is at full voltage with respect to earth, so that a very complicated system of joints is

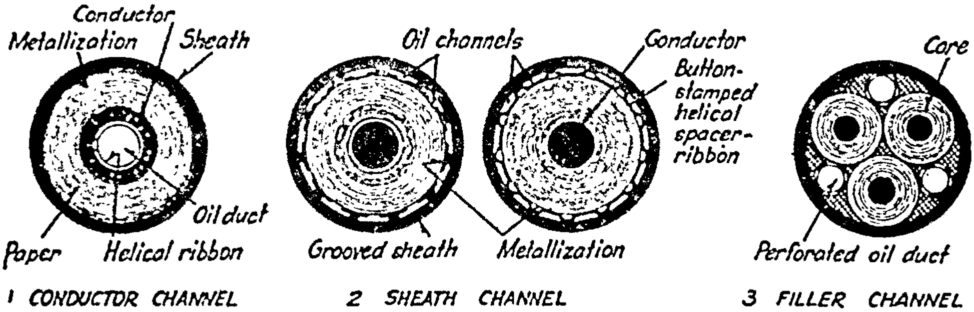


FIG. 12.14.—DIFFERENT TYPES OF OIL-FILLED CABLE.

necessary. As explained previously, the large diameter of conductor, consequent on its hollow construction, is an advantage from the point of view of potential gradient. The copper strands of which the conductor is composed are laid up over a helical metal ribbon.

In type (*b*) the oil channels are produced either by grooving the sheath or by arranging spacers between sheath and insulation, the second being more sound from the mechanical point of view. The resistance to oil-flow of type (*b*) is 6 to 8 times that of type (*a*), so that a large number of feeding points is necessary. As an offset to this, the channels are at earth potential, so that the joints and installation are simpler.

Type (*c*) is a three-phase cable with the oil-channels located in the filler spaces. These channels are composed of perforated metal-ribbon tubing, and like those of type (*b*) are at earth potential.

The type of joint used in connection with oil-filled cables is fixed largely by the route taken by the cable. The standard joint acts merely as a connector between consecutive cable lengths, exactly as with solid cables, and it is arranged to offer no restriction to the flow of oil. The semi-stop joint offers a certain amount of restriction, while the stop joint completely seals off one section from the next.

The oil tanks whose function it is to deal with the variations in volume of oil in the cable caused by expansion or contraction are kept at practically constant pressure, irrespective of the amount of oil they may contain. This is accomplished by constructing the tank in the form of a stack of flexibly walled disc-like cells. There are two types. In the gravity-feed type the oil is at atmospheric pressure, and the necessary hydrostatic pressure is obtained by raising the tank to the appropriate height above the cable level. In the balanced-pressure type the tank is sealed and

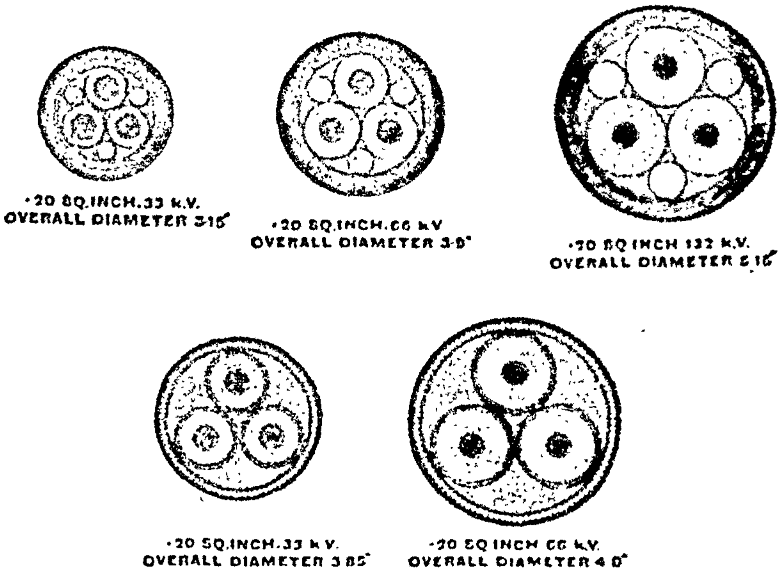


FIG. 12.15.—COMPARISON OF OIL-FILLED AND NORMAL CABLES.

contains an inert gas at the required pressure. It is imperative that the oil shall not be in contact with the atmosphere, this condition being secured in the first type by means of a breather.

The great reduction in size obtained by the oil-filled construction is clearly shown by the comparative drawings of Fig. 12.15, which show, to scale, both normal and oil-filled cables. The construction of a three-phase cable of this type is well illustrated by the dimensional drawing of Fig. 12.16, and the make-up view of Fig. 12.17.

Since the early days of the oil-filled cable the maximum dielectric stress has been appreciably increased: thus in the case of 132-kV. single-core cables the increase is from 75 to 90 kV. per cm. In the case of a 300-kV. cable the stress is designed for

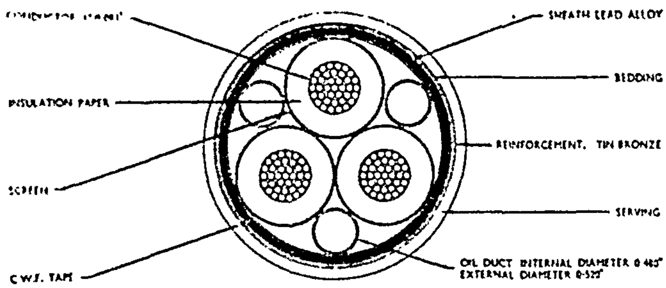


Fig. 12.16.—Cross-section of 3-core Oil-filled 66 kV. Cable for use in Systems with Neutral Earthed. Overall Cable Diameter 3·045″ Normal Maximum Stress in Dielectric at Conductor Surface at Normal Operating Voltage, 80 kV /cm.
(*British Insulated Callender's Cables Ltd.*)

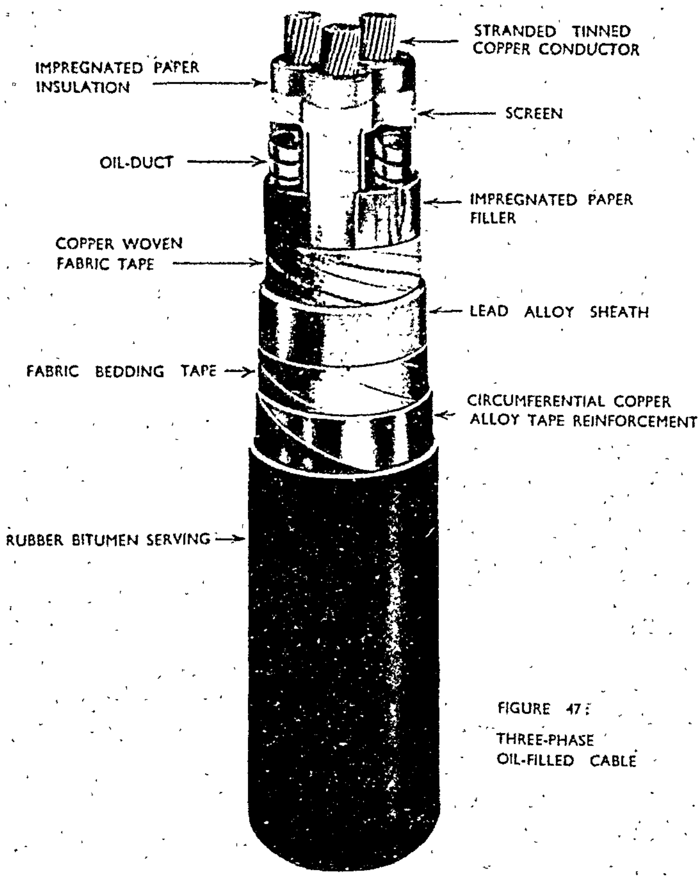
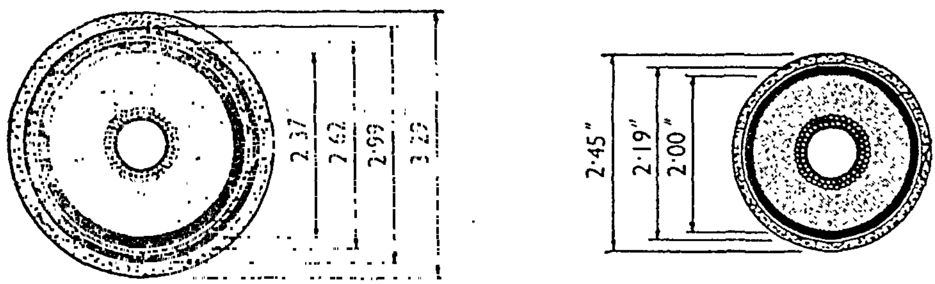


Fig. 12.17.—Make-up view of 3-core Oil-Filled Cable.

120 kV. per cm. The reduced thickness of dielectric reduces the overall cable size and, a very important operating condition, reduces the volume of oil and therefore the size of the oil-feeding tanks. The outer protection has also been simplified, only one lead sheath being used, and the pressure-reinforcing tapes being protected by rubber or similar material which prevents any corrosion due to the soil in which the cable is laid. Fig. 12.18 gives a comparison of 132 kV., single-core, oil-filled cables manufactured in 1930 and at the present day.

Another recent development is the use of aluminium sheathing: in the case of a three-core cable the strength and rigidity of the



CABLE MANUFACTURED IN 1930.

PRESENT-DAY CABLE

FIG. 12.18.—COMPANIES OF 1930 SINGLE-CORE 132 KV. OIL-FILLED CABLE WITH SIMILAR CABLE OF PRESENT-DAY MANUFACTURE.
(*G.E.C.*)

aluminium as compared with lead results in considerable economy because the external reinforcing tapes are not necessary, and the spiral oil ducts can be eliminated.

As far as oil pressure is concerned, the minimum permissible pressure is 2–3 lb./sq. in., the maximum static pressure 30–75 lb./sq. in., and the pressure during transient conditions 100 lb./sq. in. or more. The static pressure depends on the profile of the cable route and the normal electrical loading. The transient pressures are due to (*a*) absorption or emission of oil by the dielectric because of temperature changes; (*b*) to the frictional resistance to oil flow; this, in turn, depending on the oil viscosity, the duct section, and the length of cable. The maximum transient pressure occurs if full load is switched on to a dead cold cable.

The following particulars refer to a 300-kV. single-core, oil-filled cable, installed as a termination for a 300-kV. overhead line in

Canada.* The cable is 670 yds. long and is laid on racks at the side of a tunnel, there being a difference in level between the ends of 36 ft. The end of the run where the cable leaves the tunnel to connect to the overhead line is subjected, in winter, to exceedingly low temperatures, a minimum working temperature of — 20° C. being specified. This necessitated an oil having only about one-third of the viscosity of the oil normally used.

The detailed make-up of the cable is as follows:

Diameter of oil duct	0·75 in.
Size of conductor	0·4 sq. in. (66/·090 in.).
Conductor screen	3 × 0·005 in. carbon black paper tapes.
Insulation	Minimum thickness 0·95 in. made up of 144 paper tapes graded from 0·0035 in. at conductor to 0·0075 in. in outer layers. Direction of lapping reversed every eight papers.
Outer screen	3 × 0·005 in. carbon black paper tapes, the third being interlocked with metallised paper tape.
Diameter over outer screen	3·12 in.
Lead alloy sheath	0·2 per cent. tin, 0·7 per cent. cadmium, remainder lead (applied on continuous screw press).
Impregnation	Low viscosity mineral oil.
Reinforcement	Two silicon bronze reinforcing tapes 0·008 in. thickness.
Protective finish	Proofed cloth tape. Two compounded hessian tapes. (All the above applied with waterproof compound at this stage.)
Overall diameter	3·83 in.

The use of screening by semi-conducting carbon paper is of interest. It is to ensure a high degree of dryness prior to lead sheathing, the carbon paper presenting a less impenetrable barrier to water vapour than metal or metallised paper tape.

External Pressure Cables

The voltage required to set up ionisation inside a void increases as the pressure is increased, and consequently if the pressure to which a cable of ordinary construction is subjected can be raised sufficiently, no ionisation will take place. At the same time the radial compression due to this increased pressure will tend to close any voids. This is the fundamental principle of the pressure cable, and power-factor measurements taken during stability tests show that the subjecting of a cable to such a pressure gives a marked improvement in performance. Thus, tests made by Höchstadter showed that with a certain type of three-phase cable, ionisation disappeared completely at a pressure of 12 atmospheres. In addition, the long-time breakdown stress was

* By permission of the Pirelli-General Cable Works, Ltd.

raised from 180 kV. per cm. at atmospheric pressure to 400 kV. per cm. at 15 atmospheres.

The pressure cable originally due to Hockstadter, Vogel, and Bowden is illustrated in Fig. 12.19, from which it will be seen that it is triangular instead of circular in section. It is of the pipe-line type. The triangular section reduces the weight and gives a lower thermal resistance, but the main reason for the triangular shape is that the lead sheath acts as a pressure membrane, for which purpose it is only about three-quarters the normal thickness. The sheath is protected by a thin metal tape. The thickness of

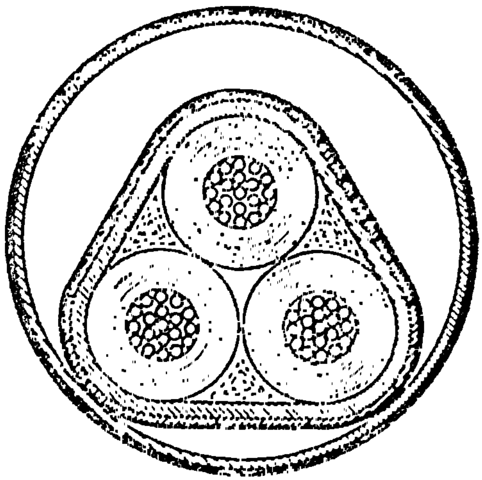


FIG. 12.19.—H.S.O. PRESSURE CABLE.
(*Royal Society of Arts.*)

the insulation for a working voltage of 66,000 and conductor sections 0·15 sq. in. is 0·306 in. The figure shows cores of circular section, but modern practice is to use an oval section as this gives an improved copper space factor.

The cable is installed in a steel pipe of somewhat larger section, this pipe being filled with nitrogen at a pressure of from 12 to 15 atmospheres. In comparison with a normal cable this construction gives twice the working voltage and about one and a half times the working current. With a potential gradient of 100 kV. per cm. the dielectric power factor at 15° C. is 0·6 per cent. The somewhat increased cost is more than offset by the greatly increased operating voltage and current, and the steel pipe forms an ideal mechanical protection for the cable.

In the self-contained type an additional reinforced-lead sheath is used, otherwise the principle is the same as that of the pipe-line type. The pipe is filled with nitrogen at a pressure of 200 lb./sq. in.

Internal Pressure Cables

The control of the dielectric power-factor can be effected by means of internally applied pressure, as is illustrated by the characteristics of Fig. 12.20 which gives the results of tests made on a 132-kV. single-core cable (*a*) without the application of internal pressure, (*b*) with the application of a pressure of 160 lb./sq. in. The attraction of the internal application of pressure

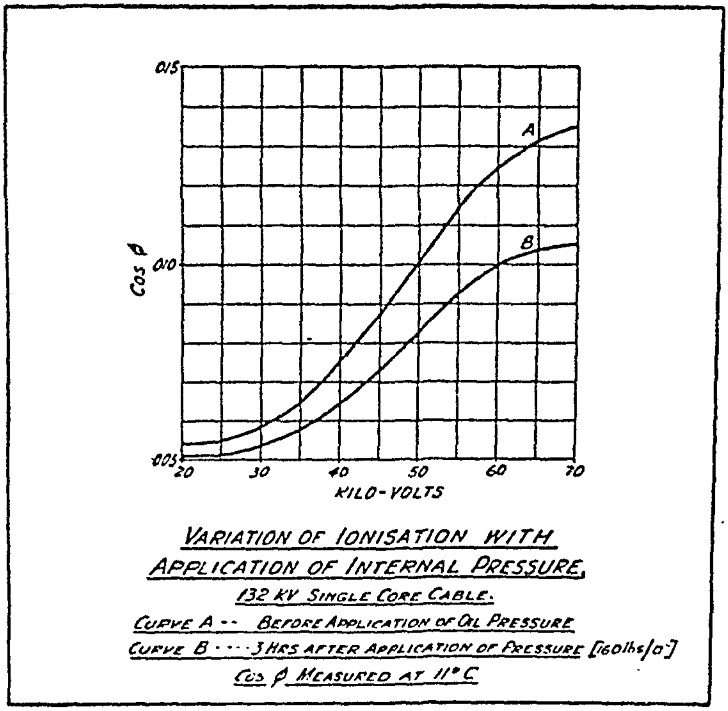


FIG. 12.20.
(*Electrical Power Engineers' Association.*)

is that external accessories are eliminated, and with certain designs the cable can be run vertically without any fear of drainage.

1. The High-pressure Gas-filled Cable

In this cable spaces are provided in the dielectric itself, these spaces being in the form of gaps between the edges of adjacent turns of the spirally lapped paper strip, and between the outer and inner faces of adjacent layers. The stranded conductor is insulated with pre-impregnated paper strip from which surplus compound has been removed, applied to the conductor in air.

The interstices provided by the above construction are filled with gas as a component part of the dielectric, the pressure being of the order of 200 lb./sq. in. for super-voltage cables, and about 100 lb./sq. in. for 33 kV. Pressure is retained by means of a lead sheath which, in the case of single-core cables, has a diametral clearance of about 0·025 in. This facilitates the axial flow of gas, which also passes along the unimpregnated strand. In the case of multi-core cables this clearance is not necessary, the filler spaces and strands providing a sufficiently low-resistance path for the flow of gas.

Whatever the type it will be clear that the very highest voltages, e.g. 100 kV. and above, necessitate the single-core construction

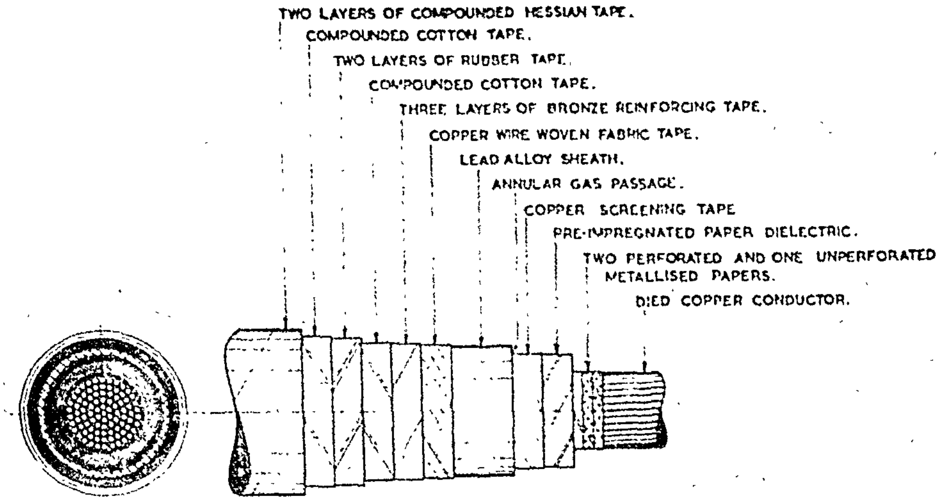


Fig. 12.21.—Make-up Drawing of Gas-filled Cable.
(*W. T. Glover & Co. Ltd.*)

and Figs. 12.21 and 12.22 show the manner in which the cable is made up. It is to be noted that an outer lead sheath is no longer used for moisture proofing. Corrosion prevention is secured by means of a special anti-corrosion serving containing a layer of vulcanised rubber, moprene, or P.V.C., since this provides a layer of material impervious to water. It will also be noted that the layers of paper are not all of the same thickness but that the thickness gradually increases with the radius. In this way a considerable degree of control of the potential gradient is achieved.

The justification for the use of gas in place of oil is made by Beaver and Davis as follows: " . . . in the oil-filled system it is necessary to have oil reservoirs situated along the cable route;

in the case of the compression cable reservoirs have to be installed to deal with the compound expansion occurring within valving ends and terminals; while in the case of the Oilostatic cable, oil-pressure reservoirs have to be provided at the terminations of the cable.

To obviate these disadvantages, a type of cable is clearly desirable in which the compensating arrangements are self-contained within the confines of the lead sheath, i.e. in which expansion due to temperature rise is accommodated at constant-

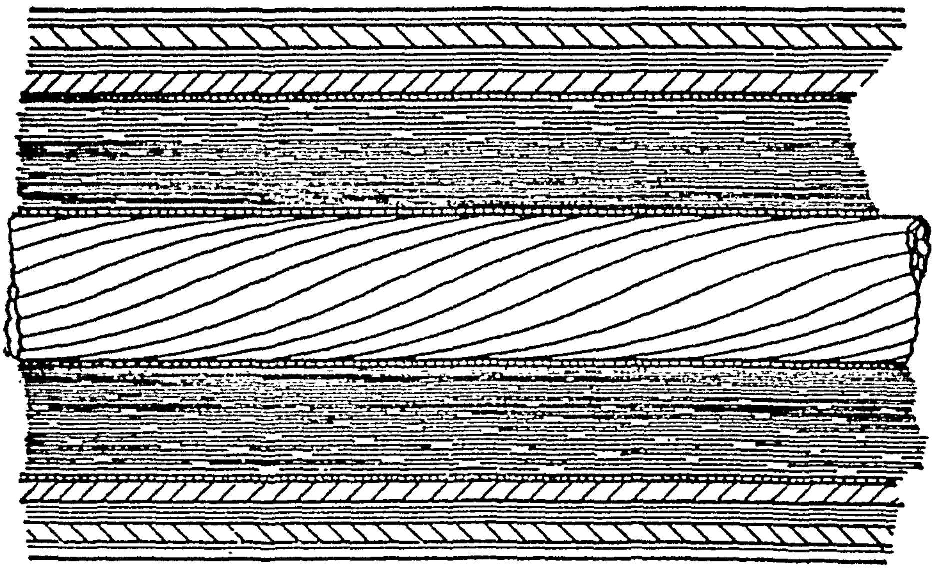


FIG. 12.22.—CROSS-SECTION OF GAS-FILLED CABLE.
(*W. T. Glover & Co. Ltd.*)

volume, thereby avoiding the necessity for extraneous apparatus as well as any consideration of gradient, and at the same time suppressing ionisation to a predesigned extent."

2. The Gas-cushion Cable

In this a screened space is provided between the lead sheath and the dielectric, this space providing at all points along the length of the cable accommodation for the storage of inert gas under pressure. This storage is maintained by the subdivision of the screened space into a series of gas cushions by means of barriers, with the result that the cable may be cut for jointing without losing gas from more than a short length. Also, should the cable be damaged the loss of gas will be local. The special feature of the cable is thus that it is not necessary to arrange for the transmission of pressure to the cable from outside, so that the cable is a complete unit with its own armouring, requiring no external pipe protection whatever.

The gas cushions are formed by means of spiral spacers wound over the metallised paper-screened core, the spacers being made up of layers of metallised paper retained in position by a narrow metal strip. This metal strip is coated with a film of low melting-point alloy before application to the cable, so that it may subsequently adhere to the lead sheath and provide a perfect barrier to prevent escape of gas.

FIG. 12.23.—66 kV. Three-core Gas-cushion Cable with Aluminium Sheath and Anti-corrosive Serving. The Gas Transference Pipe is seen in one of the Worming Spaces.
(*W. T. Henley's Telegraph Works Co. Ltd.*)

The gas-cushion construction of the present time is a simplification of the above. The stranded conductor is paper-insulated, screened, lead-sheathed, metallic-reinforced, and finally protected by a second lead sheath or waterproof covering having rubber in its composition. The feature of a large number of separate gas pockets, each acting as a separate gas pressure cable section, has been abandoned in favour of a continuous gas space throughout the length of the cable. The construction of a modern gas-cushion cable is illustrated in Fig. 12.23.

3. The Impregnated-pressure Cable

In initial manufacture this type of cable is similar to the solid type, except that provision is made for longitudinal gas flow, and the impregnating compounds used are suitable for the higher dielectric stresses necessary for super-voltages. In single-core cables the sheath clearance is about 0·07 in., and in three-core cables about 0·03 in. With the three-core cable there is, in addition, a lead gas-channel pipe of about 0·25 in. bore which occupies part of the space normally occupied by the filler. The object of this pipe is to provide a low resistance path between joints.

The cable has a mass-impregnated paper dielectric and this is maintained under a pressure of 200 lb./sq. in. by means of nitrogen. Special reinforcement is provided to cater for the large hoop and longitudinal stresses set up. This reinforcement consists of longitudinal and circumferential metallic tapes. In the case of a single-core cable, non-ferrous reinforcement is used. The sheath alloy (0·1 per cent. tin) was chosen because of its excellent slow creep characteristics and its ability to withstand slow distention without failure. A rubber-bitumen sandwich is applied to protect sheath and reinforcement from corrosion, this serving being graphited to facilitate installation work.

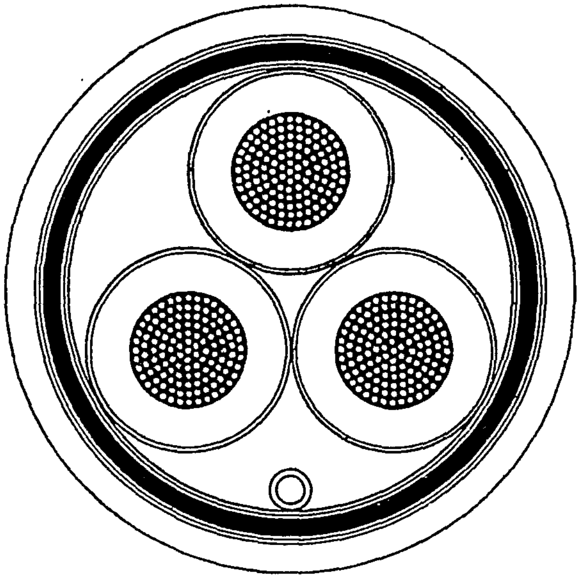


FIG. 12.24.—BARKING-ILFORD 132-KV. IMPREGNATED-PRESSURE CABLE.
(*British Insulated and Callender's Cables Ltd.*)

Fig. 12.24 is a cross-section of the Barking-Ilford three-core 132-kV. cable of this type. Fig. 12.25 is a telescoped view of the cable with all the components labelled.

Design Considerations

Although there are many factors which must be taken into account, there is one factor which can be regarded as critical. This critical design factor is decided by the voltage range as follows:

up to 33 kV. ionisation
33 to 132 kV. impulse strength
above 132 kV. thermal stability (see p. 235).

Up to 33 kV. cables are of the solid type and the voltage is such that there is not sufficient thickness of dielectric to produce

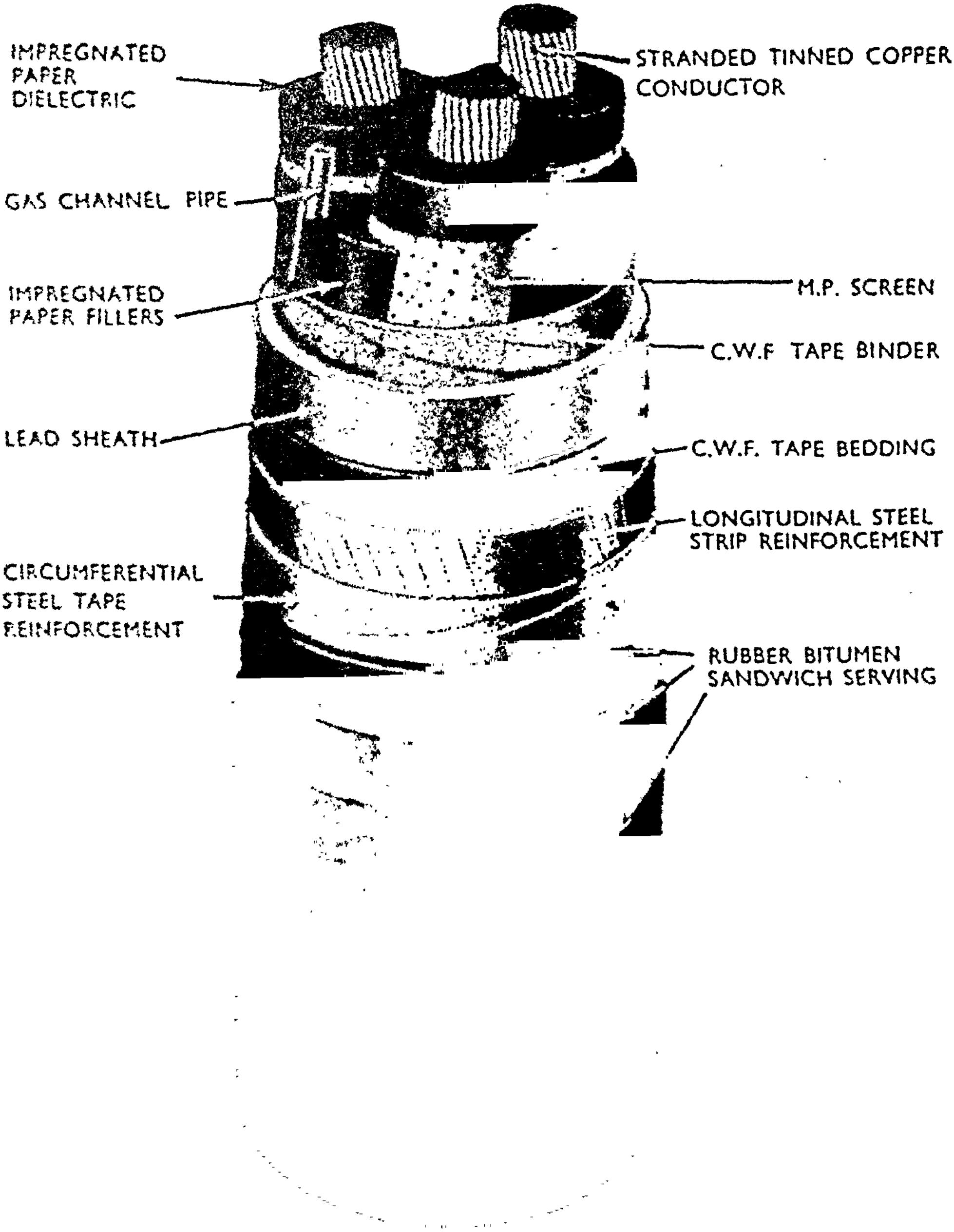


FIG. 12.25.—MAKE-UP VIEW OF BARKING-ILFORD 132-KV. IMPREGNATED-PRESSURE CABLE.

(*British Insulated and Callender's Cables Ltd.*)

any danger of thermal instability. There is no special means adopted to maintain the insulation in an unvariable state, and consequently there is the possibility of ionisation after a time, with resulting dielectric deterioration.

Between 33 and 132 kV. there is again little chance of thermal instability, and in virtue of the special construction the state of the dielectric is unlikely to change with time. Failure in this range is almost exclusively due to corrosion or other non-electrical breakdown of the sheath. The design, from the electrical point of view, is consequently based on something other than ionisation or thermal stability, namely, impulse voltage requirements.

Above 132 kV. The impulse voltage which can be applied on test is not proportional to the voltage, for in this range such a requirement would render economical design impossible. The criterion is thermal instability since there is now sufficient thickness of insulation to make this possible. As with the middle range a breakdown due to purely electrical causes is very improbable.

Values of Insulation Thickness and Dielectric Stress

For cables not within the super-voltage class the minimum insulation thicknesses are given in the various British Specifications listed on p. 213. In the case of all classes of pressurised cable the thickness is based on values of maximum dielectric stress which have been proved acceptable by laboratory tests and operational experience. The table on page 252 gives values of maximum dielectric stress in kV./cm. which are representative of present-day practice.

The Heating of Cables

The temperature rise of a cable under operating conditions depends on five factors:

1. The production of heat within the external periphery of the cable.
2. The conveyance of heat as far as the periphery—that is, up to the boundary of the surrounding medium.
3. The conveyance of heat through this medium, and therefore away from the cable.
4. The current rating of the cable, or cables, singly and collectively, where installed under the varying conditions encountered in practice.
5. The nature of the load, i.e. whether continuous or intermittent: not infrequently the rating under short-circuit conditions has to be taken into account.

Type.	Conductor Section. sq. in.	33 kV.	66 kV.	132 kV.
Solid-type Single-core	0·1 0·2 0·3 0·4	38 35 35 35		
Solid-type Three-core Screened	0·1 0·2 0·3 0·4	39 37 36 35		
Solid-type Single-core	0·2 0·3 0·4		44 41 38	
Pressurised	Type:			
	Oil-filled	Minimum of 0·13 in.	80	90
	Compression	—	93·5	110
	Gas-filled (Internal Pressure)	75	85	90
	Gas-cushion (Internal Pressure)	75	85	85
	Impregnated Pressure (Internal Pressure)	85	93·5	100

Heat Production

Within the cable there are three sources of heat—namely, I^2R losses in the conductors, losses in the dielectric, and losses in the metallic sheathings and armourings.

In calculating I^2R losses it is usual to start with the resistances given in standard resistance tables, which are for a temperature of 60° F. or 15·6° C. Three corrections are then made:

1. An operating temperature of 65° C. is assumed, the tabulated resistance therefore being multiplied by—

$$1 + \alpha\,(65 - 15{\cdot}6) \doteq 1 + 49{\cdot}4\alpha$$

where α is the temperature coefficient of copper referred to the Centigrade system and for an initial temperature of 15·6° C. The value of α is 40×10^{-4}; i.e. 0·004.

2. To allow for stranding, the resistance of a single-core cable is multiplied by the factor 1·02.

3. To allow for the lay of the whole conductor in a multi-core cable, the resistance is multiplied by an additional factor of 1·02.

Dielectric loss is of importance only in super-voltage cables:

it is not capable of direct calculation although, for what it is worth, the following formula can be used:

$$\text{Dielectric loss} = V^2 C\omega \tan \delta \times 10^{-6} \text{ watts/metre}$$

where V = voltage between phases

C = capacitances to neutral μF per metre length

δ = loss angle of dielectric

Sheath and armouring losses, while not absent in three-core cables, are dominating factors in single-core cables carrying alternating currents. In Vol. II of the *Cable Research Handbook* it is stated " that even with large three-core cables the sheath loss should be less than 5 per cent. of the copper loss . . . and the heating effect of this at the conductor may be ignored."

Now consider the case of single-core cables. Let—

R = conductor resistance in ohms/mile

R_S = sheath resistance in ohms/mile

d = distance between cable centres

x = mean radius of lead sheath

I = conductor current

Consider the case of two cables side by side as shown in cross-section in Fig. 12.26. If the sheaths are bonded together at the two ends, the arrangement is that of one air-transformer with primary and secondary windings having one turn each. The resulting sheath currents will give rise to sheath losses. If the sheaths are not bonded, then open-circuit sheath voltages will be set up and these may result in sparking and pitting in case of accidental contact, and possibly in A.C. electrolysis.

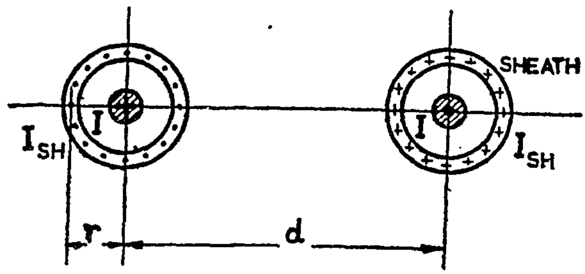


FIG. 12.26.—PERTAINING TO SHEATH LOSSES IN SINGLE-CORE CABLES.

The mutual inductance, M, of the two sheaths with respect to the two cores is given by the flux per ampere, wb./amp., over the range r to d, the argument being analogous to that pertaining to the inductance of a pair of parallel conductors. Hence—

$$M = 4 \log_e(d/r) \times 10^{-7} \text{ henry/metre.}$$

The induced E.M.F. along both sheaths in series is therefore—

$$2E_S = M\omega I$$
$$= 4 \log_e(d/r)\omega I \times 10^{-7} \text{ volts/metre}$$
$$\therefore E_S = 2 \log_e(d/r)\omega I \times 10^{-7} \text{ volts/metre}$$
$$= 4{\cdot}6 \log_{10}(d/r)\omega I \times 10^{-7} \text{ volts/metre}$$

If the sheaths are bonded at one end, the voltage between them is $2E_S$ at the far end. With three-phase transmission by three separate sheathed single-core cables with equilateral spacing of d, the voltage between adjacent sheaths at the end remote from a bond at one end is $\sqrt{3}\,E_S$.

Example.—Three single-core lead-sheathed cables carry three-phase current of 300 A. The nominal conductor area of the cores is 0·2 sq. in., the sheath thickness is 0·06 in. and the diameter over the sheath 0·90 in. They are supported in equilateral formation with a distance between cable centres of 2 in. Calculate the induced E.M.F. in each sheath, given that the cables are 1 mile long.

$$r = (0{\cdot}9 - 0{\cdot}06)/2 \quad \text{mean} = 0{\cdot}42 \text{ in.}$$
$$d/r = 2/0{\cdot}42$$
$$= 4{\cdot}76$$
$$\log_{10}(d/r) = 0{\cdot}6776$$
$$\therefore \quad l = 1760 \times 36 \times 2{\cdot}54/100$$
$$= 1609 \text{ metres}$$
$$\therefore E_S = 4{\cdot}6 \times 0{\cdot}6776 \times 314 \times 300 \times 1609 \times 10^{-7}$$
$$= 46{\cdot}8 \text{ volts per sheath.}$$

Sheath Currents

If the sheaths are bonded, the induced sheath E.M.F.s give rise to sheath currents: the circuit parameters are R, M, and R_S. The specific resistance of lead at 20° C. is 22·0 microhm cm. For the sheath in the above example—

$$l = 1{\cdot}609 \times 10^5 \text{ cm.}$$
$$a = {\cdot}7854\,(0{\cdot}9^2 - 0{\cdot}78^2) \times 2{\cdot}54^2 \text{ sq. cm.}$$
$$= 1{\cdot}03$$
$$\therefore R_S = 22 \times 10^{-6} \times 1{\cdot}609 \times 10^5/1{\cdot}03$$
$$= 3{\cdot}43 \text{ ohms per sheath.}$$

Owing to the very loose magnetic coupling between the cores and the sheaths it is not possible to determine the sheath currents in terms of the equality of primary and secondary M.M.F.s: the best way is to make use of the equivalent circuit of the air-cored transformer. This equivalent circuit is given in Fig. 12.27. Fig. (a) is the general transformer circuit, while Fig. (b) is the same

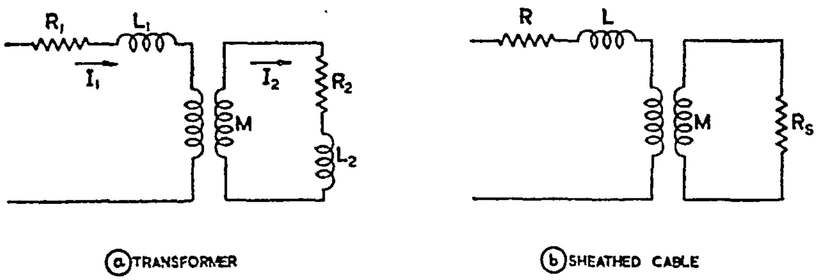


FIG. 12.27.—EQUIVALENT CIRCUIT OF SHEATHED CABLE.

circuit with parameters modified to suit this particular problem. Thus the sheath impedance can be assumed to be due to the resistance R_S only. In terms of the circuit of Fig. (a) the equivalent resistance and inductance of the primary winding are*—

$$R' = R_1 + R_2 \frac{M^2\omega^2}{R_2^2 + L_2^2\omega^2}$$

$$L' = L_1 - L_2 \frac{M^2\omega^2}{R_2^2 + L_2^2\omega^2}$$

In the circuit of Fig. (b), L_2 is neglected and the suffixes are dropped, giving—

$$R' = R + R_S \frac{M^2\omega^2}{R_S^2 + M^2\omega^2}$$

$$L' = L - M \frac{M^2\omega^2}{R_S^2 + M^2\omega^2}$$

The I^2R loss per core alone is I^2R, while the combined core loss and sheath loss is I^2R'. Hence the sheath loss per cable is

$$I^2(R' - R) = I^2R_SM^2\omega^2/(R_S^2 + M^2\omega^2)$$

Hence

$$\frac{\text{Sheath loss}}{\text{Conductor loss}} = \frac{I^2R_SM^2\omega^2/(R_S^2 + M^2\omega^2)}{I^2R}$$

$$= R_SM^2\omega^2/R(R^2 + M^2\omega^2)$$

When considering grouped cables it is convenient to regard

* See *Electrical Technology*, H. Cotton; or *Vectors for Electrical Engineers*, E. Mallett.

each single-core cable as a separate phase, so that for the parameters L and M we now use—

$$L = 2 \log_e(d/r_c) \times 10^{-7} = 4{\cdot}6 \log_{10}(d/r_c) \times 10^{-7} \text{ henry/metre}$$
$$M = 2 \log_e(d/r) \times 10^{-7} = 4{\cdot}6 \log_{10}(d/r) \times 10^{-7} \text{ henry/metre}$$

when r_c is the radius of the core.

Consider again the three 0·2-sq. in. cables of the previous example. The resistance per 1,000 yards of 0·2-sq. in. stranded conductor at 60° F. is 0·1223 ohm; this allows for the stranding of the individual wires. Hence, for a one-mile length at 65° C. we have—

$$R = 0{\cdot}1223\,(1 + 49{\cdot}4 \times 0{\cdot}004) \times 1{\cdot}76$$
$$= 0{\cdot}26 \text{ ohm}$$
$$d_c = 0{\cdot}581 \text{ in.} \therefore r_c = 0{\cdot}291 \text{ in.}$$
$$d/r_c = 2/0{\cdot}291 = 6{\cdot}87$$
$$\log_{10}(d/r_c) = 0{\cdot}8370$$
$$\therefore L = 4{\cdot}6 \times 0{\cdot}837 \times 10^{-7} \times 1609$$
$$= 5{\cdot}08 \times 10^{-4} \text{ henry}$$
$$R_S = 3{\cdot}43 \text{ ohms.}$$
$$M = 4{\cdot}6 \times \log_{10}(d/r) \times 10^{-7} \times 1609$$
$$= 4{\cdot}6 \times 0{\cdot}6776 \times 10^{-7} \times 1609$$
$$= 5{\cdot}04 \times 10^{-4} \text{ henry}$$
$$M^2\omega^2 = (5{\cdot}04 \times 10^{-4} \times 314)^2 = 0{\cdot}025$$
$$\therefore \frac{\text{Sheath loss}}{\text{Conductor loss}} = 3{\cdot}43 \times 0{\cdot}025/[0{\cdot}26\,(3{\cdot}93^2 + 0{\cdot}025)]$$
$$= 0{\cdot}0294$$

Thus in this particular case the sheath loss is approximately 3 per cent. of the conductor loss. It will be obvious that the governing factor is the value of R_S, and for low values of R_S the sheath losses may be very high. Thus Dunsheath cites a case in which, with R_S equal to 0·3 ohm per mile, the sheath loss is 172 per cent. of the core loss. These losses would be eliminated if the cables were installed with insulating joints in the lead sheaths, but this would be accompanied by the presence of the full open-current sheath voltages. A successful method is a transposition of the sheaths similar to that carried out with overhead conductors. "The maximum reduction in sheath loss, accompanied by a maximum reduction in voltage between sheaths and earth, is obtained by a combination of reactance and cross-bonding."

The usual practice is to group the cables in trefoil in contact with one another and to link the sheaths together at the ends of the run with bonds of large cross-section. The close proximity of the cables keeps the currents down to a minimum by reducing M to its lowest possible value, and the bonds allow the currents to circulate freely.

Where cables fan out to spacings of several feet on being led to terminal sealing ends, the ends are insulated from earth and from each other, it being preferable to deal with the standing voltages, which usually are limited to about 7 volts, than the very large circulating currents that would flow if the sealing-end bases were bonded together. In transforming stations and such-like localities two or more single-core cables are often run in parallel for each phase of the supply. When this is done, each group of cables should embrace all three phases, as otherwise the circulating current losses would exercise a most detrimental effect on the capacities of the cables.

Armouring losses are sufficient to preclude the use of steel tape armouring on single-core cables and to render non-magnetic wire armouring essential if large currents are to be carried. They also preclude anything but wire armouring on super-voltage three-core cables.

In addition to the above circulating currents, there are also sheath losses due to eddy currents. As we have seen, the circulating current losses can be controlled: the eddy current losses cannot.

As an example of bonding in practice consider the example illustrated in Fig. 12.27(a) which shows a super-voltage installation consisting of three single-core oil-filled cables. A, B, C and D are cross-bonds which permit the free circulation of sheath currents. It is to be noted that there are no bonds below the sealing ends as it is usually preferable to allow limited standing voltages where single-core cables are spaced widely apart. For short trefoil runs, only one bond at the break of the trefoil formation is necessary.

Factors affecting the Carrying Capacity

Impregnated paper cables provide the highest current ratings. At the lower voltages, V.I.R. and plastic-insulated cables are on equal terms, although not necessarily having the same range of usages. Outer coverings exercise a considerable influence on the carrying capacity of a cable: the addition of armouring and serving to a L.V. plain lead-covered three- or four-core cable

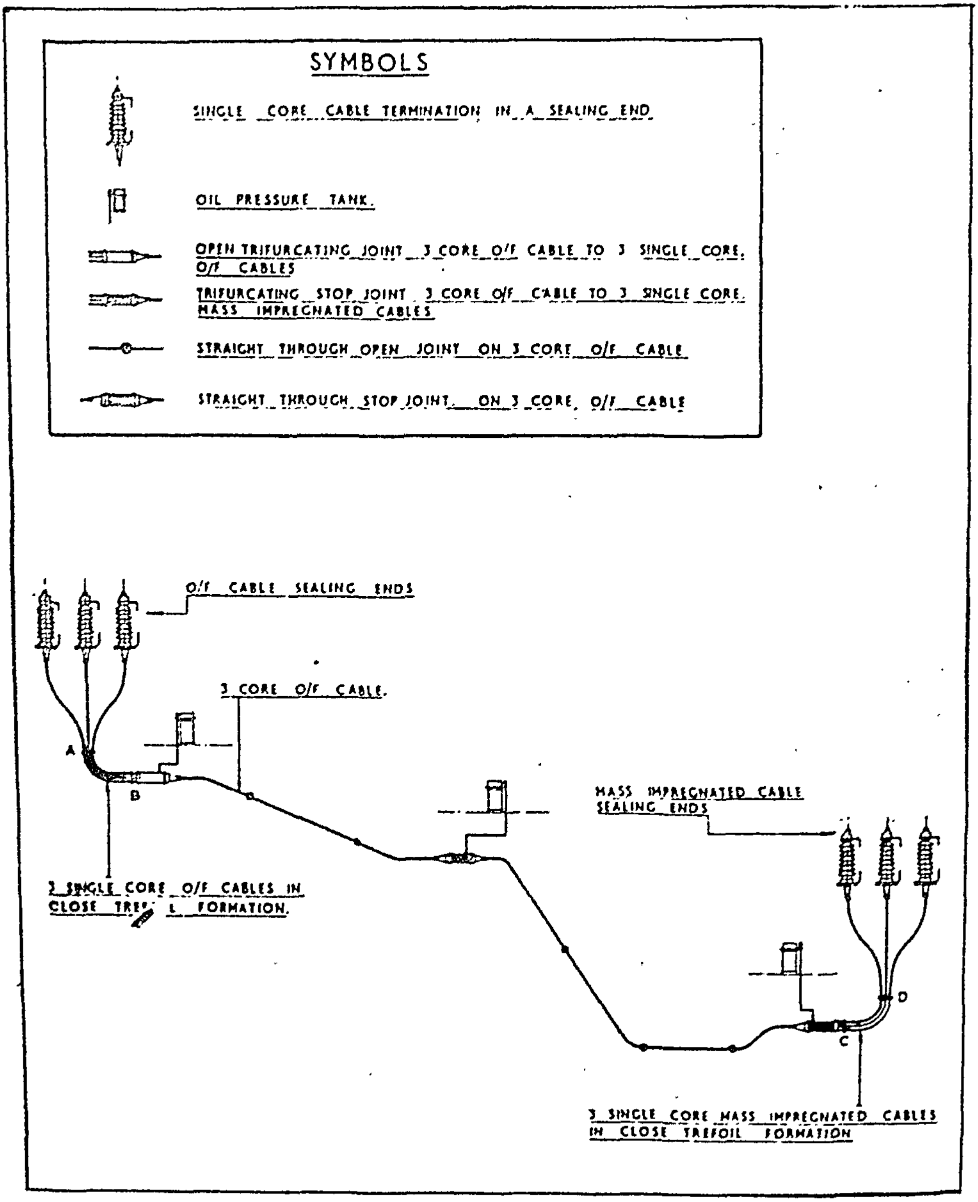


FIG. 12.27A.—INSTALLATION OF THREE SINGLE-CORE OIL-FILLED SUPER-VOLTAGE CABLES.
(*W. T. Henley's Telegraph Works Co. Ltd.*)

destined for drawing into ducts increases its current rating by approximately 20 per cent. by providing greater radiating surface.

The greatest variety of circumstances is met with when cables are laid direct in the ground. Heat, having reached the outer surface of the cable, has to be conveyed away from it. The method of calculating the heat flow is the same as that used for the determination of the internal thermal condition of the cable, and is explained below. The thermal resistivity, the "g," of the soils along the different parts of the cable route have to be determined, and are then used to compute the cable ratings at the different points. If certain parts are sufficiently heat-insulating by nature, the conductor cross-section may have to be increased in such places in order that there may be no loss of current-carrying capacity. This is particularly important in the case of the costly super-voltage cables, and it shows that with such cables a careful survey of the proposed route is a necessity.

Waterlogged ground provides the best conditions for the cable when laid, although it may be troublesome to deal with while laying operations are in progress. Dry dust, with air circulation practically non-existent, represent the worst conditions.

As well as thermal resistivity, the natural temperature of the soil has to be taken into account. This is usually subject to a slow annual variation, and for general purposes a figure of 15° C. is assumed unless there is more definite information.

Grouping of Cables

The proximity of a number of cables to one another has an appreciable effect on the rating. The amount of heat to be dissipated is proportional to the number of cables, but the number of free paths by which heat can escape into the general body of the earth or atmosphere is reduced. In the case of cables laid direct or drawn into ducts and kept continuously on load, the whole locality immediately surrounding the cables attains a steady temperature somewhat above the general level. Consequently the ratings of cables loaded in this manner have to be adjusted with this eventuality in view.

Changes in circumstances, such as the augmentation of circuits, or some variation in local conditions, may necessitate the derating of a cable to a reduced capacity.

Where cables are run outdoors they should be shielded from the direct rays of the sun, but without restricted ventilation. If this is not possible, a higher ambient temperature should be assumed, the recommended values being:

Cables less than 1½ in. overall diameter . .	15° C.
Cables 1½ in. or greater overall diameter . .	20° C.

Where conductor sizes are determined solely by consideration of voltage drop there is less necessity to weigh the above factors closely, but where the criterion is current loading, every one must be taken into account.

Grouping factors which, in certain circumstances, may be as low as 0·48 for cables laid direct in the ground, are given in *E.R.A. Report F/T* 128.

Maximum Allowable Conductor Temperatures

The limiting factor in current rating is the temperature to which the insulation nearest the conductor can be raised without suffering deterioration. The allowable values are:

V.I.R.- and P.V.C.-insulated cables . .	60° C.
Impregnated-paper-insulated cables	
1. Oil-filled and gas-pressure cables . .	85° C.
2. 33-kV. solid-type cables, armoured . .	65° C.
3. 22-kV. screened cables, armoured . .	65° C.
4. 22-kV. belted cables, armoured . . .	55° C.
5. 11-kV. screened cables, armoured . .	70° C.
6. 11-kV. belted cables, armoured . . .	65° C.
7. 6·6-, 3·3-, and 1·1-kV. cables, armoured .	80° C.

It is of vital importance that the allowable temperature rise should not be exceeded, since the dielectric loss rises very rapidly beyond a certain temperature whose value depends on the dielectric and the type of cable. Thus in the case of an oil-filled paper-insulated cable, the power factor will be a minimum of 0·002 at about 45° C. As the temperature is lowered, the power factor increases somewhat, becoming 0·003 at 15° C. This is of no importance because the temperature will only diminish with reduction in loading and therefore of total cable loss. When the temperature is increased, the power factor again increases, becoming 0·004 at about 90° C. As this corresponds to increased I^2R loss, it means that a condition may be reached at which any further increase in temperature will result in more additional heat generation than can be effectively dissipated. The temperature of the cable will then continue to rise until eventual breakdown occurs. This phenomenon is known as thermal instability.

As an example, the power-factor/temperature characteristic of the 132-kV. three-core, impregnated-pressure cable illustrated in Fig. 12.24 is given in Fig. 12.28.

Determination of Cable Rating

The method of computation is based on Ohms' Law in thermal, instead of electrical, units. The formula used gives the trans-

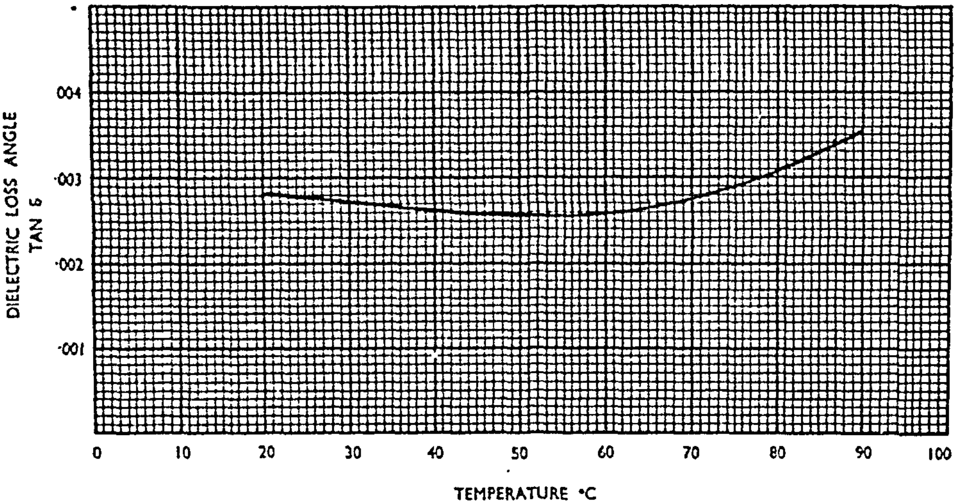


FIG. 12.28.—POWER-FACTOR V. TEMPERATURE CHARACTERISTIC OF 132-KV. IMPREGNATED-PRESSURE CABLE.
(British Insulated and Callender's Cables Ltd.)

ference of heat across a layer between whose opposite faces there exists a difference in temperature: it is—

$$\text{Heat flow in thermal watts} = \frac{\text{Temperature difference, }^\circ\text{C.}}{\text{Thermal resistance in thermal ohms}}$$

A thermal ohm is the difference in degrees C. between opposite faces of a 1-cm. cube produced by the flow of 1 watt of heat; it is expressed in units of °C./watt/cm. In addition to inherent resistivity, heat flow is affected by the geometrical shape of the material, and in practice this may necessitate the use of factors obtained by indirect methods. Thus the thermal resistance of a three-core cable with shaped conductors cannot be stated accurately, owing to the absence of any standardisation of conductor section. With circular cores the lines of heat flow and the isothermal lines are by no means simple, as is shown by Fig. 12.29.

Metals are regarded as possessing zero resistivity, so that metal sheathings and armourings are left out of the calculations except in so far as they are sources of heat.

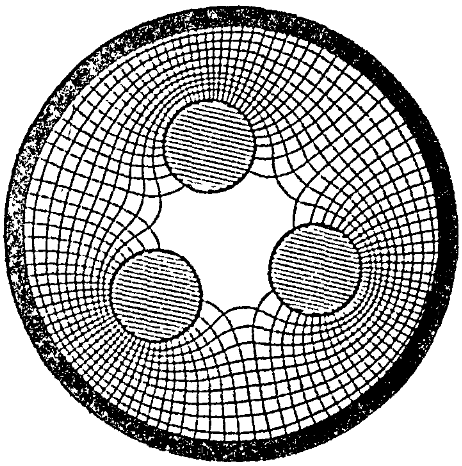


FIG. 12.29.—HEAT-FLOW LINES AND EQUIPOTENTIALS IN 3-PHASE CABLE WITH HOMOGENEOUS DIELECTRIC.
(*Cable Research Handbook.*)

1. *Single-core Cable*

Let r = radius of core
r_1 = radius of dielectric (Fig. 12.30)
$\varkappa$ = thermal resistivity of the dielectric

Then thermal resistance of an annulus of thickness dx at radius x

$$= \frac{\varkappa dx}{2\pi x} \text{ thermal ohms per cm. length}$$

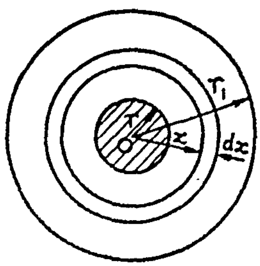


FIG. 12.30.

Hence, total thermal resistance per cm. length

$$S_1 = \int_r^{r_1} \frac{\varkappa}{2\pi} \cdot \frac{dx}{x} = \frac{\varkappa}{2\pi} \log_e (r_1/r) \text{ thermal ohms/cm.}$$

The usual values for $\varkappa$ are

Solid-type cable below 2·2 kV. .	750 thermal ohms/cm.
Oil-filled cable	450 thermal ohms/cm.
Impregnated-pressure cable . .	550 thermal ohms/cm.

2. *Three-core Cable*

For a three-core cable the calculation is more complex. For the belted-type cable a formula due to Simon—" Cable Geometry and the Calculation of Current Carrying Capacity ": *Trans.*

A.I.E.E., Vol. 42, p. 600—derived from experimental work, gives fairly accurate results: it is—

$$S_1 = \frac{\varkappa}{6\pi}\left(0{\cdot}85 + \frac{0{\cdot}2t}{T}\right)\log_e\left[\left(4{\cdot}15 - \frac{1{\cdot}1t}{T}\right)\left(\frac{T+t}{r}\right) + 1\right]$$

where t = thickness of belt insulation

T = thickness of conductor insulation

r = conductor radius, as before

Another expression, said to be accurate to within 10 per cent., is

$$S_1 = \frac{\varkappa}{6\pi}\log_e\frac{r_2^6 - a^6}{3r_2^3a^2r}$$

where r has the same meaning as before, a is the radius of the circle at which the centres of the conductor cross-sections lie, and r_2 is the outer dielectric radius.

3. *Protective Covering*

As this is in the form of a cylinder, the expression is of the same form as that for a single-core cable: it is—

$$S_2 = \frac{\varkappa_2}{2\pi}\log_e\frac{r_3 - A/2}{r_2 + A/2}\text{ thermal ohms/cm.}$$

where r_3 = radius over the outer covering of the cable

r_2 = radius over the lead sheath

A = thickness of armouring

$\varkappa_2$ = thermal resistivity of bedding and serving: usually taken as 500

4. *Thermal Resistance of the Ground*

The thermal resistivity of the soil, the "g" of the soil, depends upon the type of soil and also upon the amount of moisture present. This is illustrated by the curves of Fig. 12.31. The thermal resistance, on the assumption that the surface of the ground is a plane isothermal and that the ground is homogeneous, is derived in a manner analogous to the calculation of a horizontal overhead conductor with respect to ground. It is—

$$G = \frac{g}{2\pi}\log_e\frac{2L}{r_3}$$

where L = depth of cable axis below ground.

It is found that, in practice, the values of g determined in the laboratory must be multiplied by a correction factor of 2/3 in order to correspond with the results of experiments on buried cables. The amended formula is thus—

$$G = \frac{g}{3\pi} \log_e \frac{2l}{r_3}$$

A representative value of g for a soil of average moisture content is 180. If the multiplying factor of $g/2\pi$ is used, then the value of g must be amended to 120, and this is sometimes done.

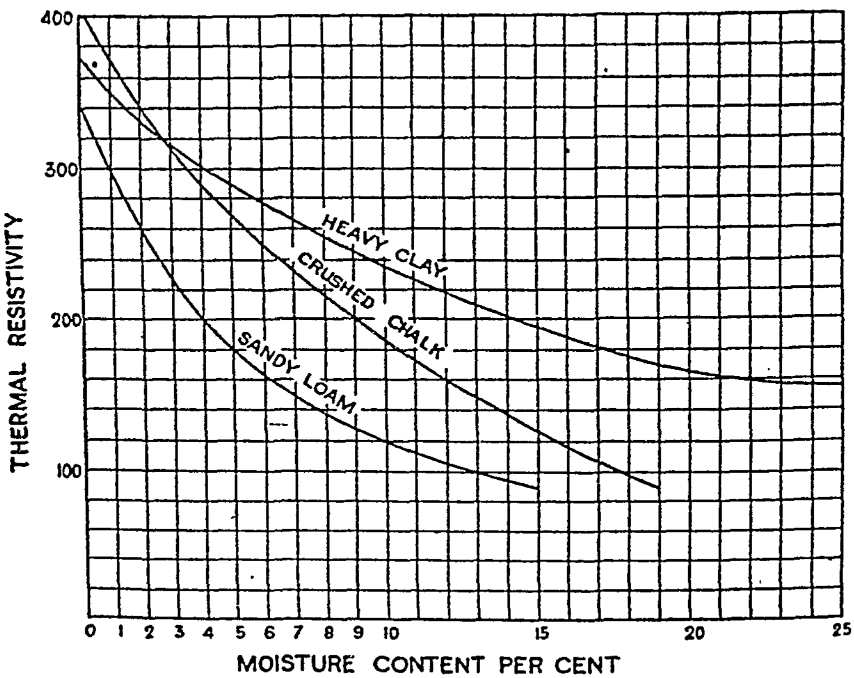


FIG. 12.31.—SOIL THERMAL RESISTANCE V. MOISTURE CONTENT. (*Cable Research Handbook.*)

When the g value is different from this, the *E.R.A. Report F/T* 128 gives correction factors by which the rating based on the assumed value of 180 (or 120) must be multiplied. They are, based on g = 120—

g	80	100	120	180	240
Rating Correction Factor . .	1·12	1·05	1·00	0·88	0·80

5. *Cables exposed to the Air*

The heat dissipated by a surface in direct contact with the air and with unrestricted ventilation is equal to—

$$2\pi r_2 k(\theta_s - \theta_a)^{1\cdot 25} \text{ watts/cm. length}$$

where r_2 = external radius of cable, usually the lead sheath.

θ_s = temperature of cable surface.

θ_a = ambient temperature.

k = an emissivity constant whose value varies with the external radius of the cable: thus

when $r_2 = 0{\cdot}2$ in., $k = 0{\cdot}00178$
when $r_2 = 2{\cdot}0$ in., $k = 0{\cdot}00115$

For a given value of θ, the maximum permissible core temperature, the corresponding value of θ_s can be determined by equating the heat flow through the cable to the heat emitted from its surface. To take sheath losses into account, denote the ratio of the sheath losses to the copper losses by λ; then we have—

$$(1 + \lambda)\left(\frac{\theta - \theta_s}{S}\right) = 2\pi r_2 k(\theta_s - \theta_a)^{1\cdot 25}$$

where S is the thermal resistance of the dielectric determined as above.

Having determined θ_s, the equivalent thermal resistance of the cable surface can be calculated from—

$$\frac{\theta_s - \theta_a}{2\pi r_2 k(\theta_s - \theta_a)^{1\cdot 25}} \text{ or } \frac{1}{2\pi r_2 k(\theta_s - \theta_a)^{0\cdot 25}} \text{ thermal ohms/cm.}$$

Alternatively, if the temperature difference $(\theta_s - \theta_a)$ is small, as, for example, along a mine roadway, Newton's law of cooling can be assumed provided an appropriate value of k is used. The heat dissipation of the surface is then—

$$2\pi r_2 k(\theta_s - \theta_a) \text{ watts/cm. length}$$

6. *Calculation of Current Rating*

When the thermal resistance is known, it is possible to calculate the value of the current I which will raise the conductor temperature to the maximum allowable value. This is the cable rating under the specified conditions and under continuous loading, or loading sufficiently prolonged to enable steady thermal conditions to be attained. Neglecting, for the moment, the dielectric losses, the insulation has to pass heat due to the core losses, while bedding and serving have to pass the heat due to sheath losses in addition.

With n cores, copper losses $= nI^2R$ watts/cm. length.

With a core temperature of $\theta°$ C., the sheath temperature θ_s is given by

$$nI^2R = \frac{\theta - \theta_s}{S_1}$$

Total losses flowing from sheath $= (1 + \lambda)I^2R$, so that for an ambient temperature of θ_a,

$$(1 + \lambda)nI^2R = \frac{\theta_s - \theta_a}{S_2 + G}, \text{ R is now the resistance per cm.}$$

$$\therefore nI^2R\,S_1 + n(1 + \lambda)I^2R(S_2 + G) = \theta - \theta_a$$

$$\therefore I = \sqrt{\frac{\theta - \theta_a}{nR[S_1 + (1 + \lambda)(S_2 + G)]}} \text{ amps.}$$

7. *Effect of Dielectric Losses*

This effect cannot be ignored in the cases of voltages of 66 kV. and upwards. It is independent of the load current except in so far as the current causes an increase in the temperature of the dielectric. The method is to compute the dielectric loss at the maximum permissible temperature, assume that it is all concentrated at the core, and then calculate the temperature rise θ_d it would produce if it were the only source of heat. This is subtracted from the maximum permissible temperature rise θ, and the difference $(\theta - \theta_d)$ used in place of θ in the above expression for I. Thus—

$$I = \sqrt{\frac{\theta - \theta_d - \theta_a}{nR[S_1 + (1 + \lambda)(S_2 + G)]}} \text{ amps.}$$

In order that thermal stability may be attained, it is essential that the additional heat due to increasing dielectric loss shall not rise with temperature more rapidly than the heat dissipation. Thus, stability depends largely on the external thermal resistance, and with super-voltage cables it is essential that the value of G should be determined experimentally along the route.

Numerical Examples

Example 1.—Calculation of rating—cable in air.

Three-core, 0·05-sq. in., jute-insulated, vulcanised-bitumen sheathed, double-wire armoured, low-tension cable, installed along a mine roadway having an ambient temperature of 70° F. Allowable maximum temperature of jute 145° F.

$$\theta = 145° \text{ F.} = 62{\cdot}8° \text{ C.}$$
$$\theta_a = 70° \text{ F.} = 21{\cdot}1° \text{ C.}$$
$$\therefore \theta - \theta_a = 41{\cdot}7° \text{ C.}$$

(*a*) *Jute Insulation*

Thickness of conductor insulation T = 0·04 in.
Thickness of belt insulation t = 0·04 in.
Conductor radius r = 0·145 in.
Thermal resistivity of jute $\varkappa$ = 750

∴ Thermal resistance of jute

$$S_1 = \frac{750}{6\pi}\left(0{\cdot}85 + \frac{0{\cdot}2 \times 0{\cdot}04}{0{\cdot}04}\right)\log_e\left[\left(4{\cdot}15 - \frac{1{\cdot}1 \times 0{\cdot}04}{0{\cdot}04}\right) \times \left(\frac{0{\cdot}04 + 0{\cdot}04}{0{\cdot}145}\right) + 1\right]$$

= 41 thermal ohms/cm.

(*b*) *Vulcanised Bitumen Sheath*

Outer radius r_2 = 0·6 in.
Inner radius r_1 = 0·44 in.
$\varkappa_1$ for V.B. = 500

∴ Thermal resistance of V.B. sheath

$$S_2 = \frac{500}{2\pi}\log_e\frac{0{\cdot}6}{0{\cdot}44}$$

= 24·7 thermal ohms/cm.

(*c*) *Double Wire Armouring and Tape over V.B. Sheath*

Outer radius r_3 = 1·095 in.
Inner radius r_2 = 0·6 in.
$\varkappa$ = 300 (for combined tape and armouring)

∴ Thermal resistivity of armouring and covering

$$S_3 = \frac{300}{2\pi}\log_e\frac{1{\cdot}095}{0{\cdot}6}$$
$$\doteq 28{\cdot}7$$

(*d*) *Air*

Thermal resistance of air

$$S_4 = 1/2\pi k r_3$$

$$\text{where } k = \text{emissivity} = 0{\cdot}0009$$

$$r_3 = 1{\cdot}095 \text{ in.}$$

$$\therefore S_4 = 1/(2\pi \times 0{\cdot}0009 \times 2{\cdot}54 \times 1{\cdot}095)$$

$$= 63{\cdot}5$$

(*e*) *Calculation of Rating*

Total thermal resistance

$$S = 41 + 24{\cdot}7 + 28{\cdot}7 + 63{\cdot}5$$

$$= 157{\cdot}9 \text{ thermal ohms/cm.}$$

$$n = 3$$

$$R = 6{\cdot}6 \times 10^{-6} \text{ ohms/cm. at } 145^\circ \text{ F.}$$

$$\therefore I = \sqrt{\frac{41{\cdot}7}{3 \times 6{\cdot}6 \times 10^{-6} \times 157{\cdot}9}}$$

$$= 116 \text{ amps.}$$

Since the various portions of the cable all pass the same amount of heat, the various thermal resistances can be regarded as being in series, in the sense that a number of resistors all carrying the same current are in series (Fig. 12.32). The tempera-

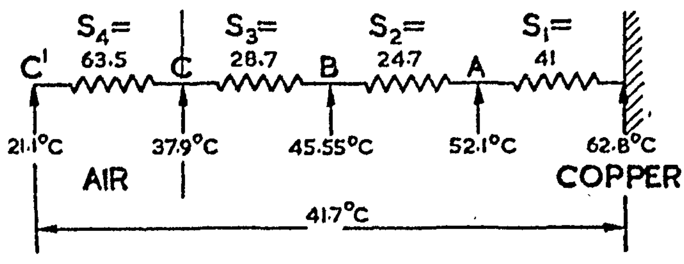


FIG. 12.32.—TEMPERATURE DISTRIBUTION WITHIN CABLE.

ture drop in any one part is thus proportional to the thermal resistance of that part. The various temperature drops—

$$\frac{S_1}{S} \times 41{\cdot}7 = \frac{41}{157{\cdot}9} \times 41{\cdot}7 = 10{\cdot}7^\circ$$

∴ Temperature at outer surface of jute insulation, point A

$$= 62{\cdot}8 - 10{\cdot}7 = 52{\cdot}1^\circ \text{ C.}$$

$$\frac{S_2}{S} \times 41{\cdot}7 = \frac{24{\cdot}7}{157{\cdot}9} \times 41{\cdot}7 = 6{\cdot}55^\circ$$

∴ Temperature at outer surface of V.B. sheath, point B

$$= 52{\cdot}1 - 6{\cdot}55 = 45{\cdot}55° \text{ C.}$$

$$\frac{S_3}{S} \times 41{\cdot}7 = \frac{28{\cdot}7}{157{\cdot}9} \times 41{\cdot}7 = 7{\cdot}65°$$

∴ Temperature at outer surface of cable, point C

$$= 45{\cdot}55 - 7{\cdot}65 = 37{\cdot}9° \text{ C.}$$

$$\frac{S_4}{S} \times 41{\cdot}7 = \frac{63{\cdot}5}{157{\cdot}9} \times 41{\cdot}7 = 16{\cdot}8°.$$

= Temperature drop from cable surface to air

∴ Temperature of air = 37·9 − 16·8 = 21·1° C, as specified.

Example 2.—Calculation of rating—cable laid in the ground.

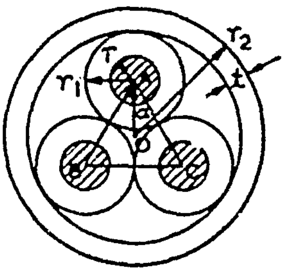


FIG. 12.33.—DIMENSIONS OF CABLE IN EXAMPLE 2.

The *C.R. Handbook*, Vol. II, gives the following data for an 11-kV., 0·25-sq. in., three-core armoured cable, paper insulated. It is buried to a depth of 3 ft.

Core radius r = 0·325 in.

Insulation thickness = 0·15 in.

∴ Insulation radius r_1 = 0·325 + 0·15 = 0·435 in.

Belt insulation thickness = 0·06 in.

∴ Outer radius of belt insulation

r_2 = 0·997 in.

a = 0·502 in.

These dimensions are indicated in the cross-section of Fig. 12.33

$\varkappa_1$ for insulation = 550

$$\therefore S_1 = \frac{\varkappa}{6\pi} \times \log_e \frac{r_2^6 - a^6}{3r_2^3a^2r}$$

$$= \frac{550}{6\pi} \times 2{\cdot}3 \log_{10}\left(\frac{0{\cdot}997^6 - 0{\cdot}502^6}{3 \times 0{\cdot}997^3 \times 0{\cdot}502^2 \times 0{\cdot}325}\right)$$

$$= 30{\cdot}3 \text{ thermal ohms/cm.}$$

The radii of serving and armouring are 1·222 and 1·542, and the thermal resistivity 300.

$$\therefore S_2 = \frac{300}{2\pi} \log_e \frac{1{\cdot}542}{1{\cdot}222}$$

$$= 11{\cdot}1 \text{ thermal ohms/cm.}$$

with soil of $g = 180$

$$G = \frac{180}{3\pi} \log_e \frac{72}{1{\cdot}542} = 73{\cdot}4 \text{ thermal ohms/cm.}$$

$n = 3$

$R = 1{\cdot}329 \times 10^{-6}$ ohms/cm. at the maximum permissible temperature of 65° C., and allowing 2 per cent. for stranding and 2 per cent. for warning. Assuming $\theta_s = 15°$ C.

$$I = \sqrt{\frac{\theta - \theta_s}{nR(S_1 + S_2 + G)}}$$

$$= \sqrt{\frac{65 - 15}{3 \times 1{\cdot}329 \times 10^{-6} \times 114{\cdot}8}}$$

$$= 330 \text{ amps.}$$

If the Simon formula is used to determine S_1 the rating comes out at 326 amps., showing that, for the example chosen, there is no appreciable difference.

Example 3.—Effect of grouping. Two cables like those of example 2 buried 3 ft. and spaced horizontally 1 ft. between centres.

The G of one cable for a cylinder of earth of radius equal to the cable spacing *d*, on the assumption that the isothermals due to one cable alone are cylinders with their axes at the cable axis, is—

$$G_d = \frac{180}{3\pi} \log_e \frac{2l}{d}$$

$$= \frac{180}{3\pi} \log_e \frac{72}{12} = 34{\cdot}2$$

The temperature rise of one cable due to its own losses is proportional to $(S_1 + S_2 + G)$. Its temperature rise due to its own losses plus the heat conducted to it from the other cable, assumed equally loaded, is proportional to $(S_1 + S_2 + G + G_d)$. Hence, since the heat generation is proportional to the square of the current, we have for the new current I_1 in terms of the current I in an isolated cable for the same temperature rise of the copper—

$$I_1^2(S_1 + S_2 + G + G_d) = I^2(S_1 + S_2 + G)$$

$$\therefore I_1 = I\sqrt{\frac{S_1 + S_2 + G}{S_1 + S_2 + G + G_d}}$$

$$= I\sqrt{\frac{30{\cdot}3 + 11{\cdot}1 + 73{\cdot}4}{30{\cdot}3 + 11{\cdot}1 + 73{\cdot}4 + 34{\cdot}2}}$$

$$= 291 \text{ amps.}$$

Cable Short-circuits

In the event of a short-circuit of short duration there will be so little time for the transfer of heat from the cores to the dielectric that it can be assumed that the whole of the heat generated is utilised in raising the temperature of the copper. Provided the duration is small, there appears to be no danger of deterioration of the cable dielectric wire if the conductor temperature reaches 300° C. The criterion is that this temperature shall not be maintained, since, under such conditions, there is no margin for contingencies, and a small delay in tripping time might seriously endanger the insulation. The temperature rise of the lead sheath can impose severe restrictions on the permissible value of the short-circuit current.

Under conditions of heavy short-circuit, severe mechanical strains are set up due to (1) the rapid expansion and contraction of the conductors, (2) repulsive forces due to the intense magnetic field in the immediate neighbourhood of the cores. This combination of thermal and magnetic stresses is imposed on the conductor joints and cable terminations and these also have a limiting effect on the allowable short-circuit current.

The coefficient of expansion of copper is 0·000017 per °C. For a temperature rise of 150° C. this corresponds to an elongation of 0·255 per cent. Thus the free expansion of one mile of cable would be—

$$0{\cdot}225 \times 5280/100 = 11{\cdot}85 \text{ ft.}$$

In order fully to restrain this expansion—i.e. assuming no expansion allowed to take place—the necessary force would be that required to produce this amount of elongation. For the above values this is 8·54 tons/sq. in. This is, of course, an extreme case not realised in practice. With rapid expansion the tendency is to cause snaking of the cable and extension at the bends, and this relieves the stress. Subsequent cooling takes place much more slowly and the cable may offer greater resistance to deformation, thereby throwing much of the strain on the joints.

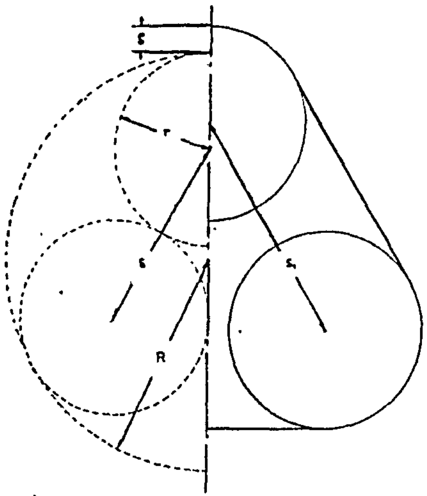
The stresses due to magnetic attraction are of importance only in multi-core cables where the small distance between core centres may result in considerable deformation since there is only the inherent strength of the internal structure to offer resistances. The nature of the distortion in the three cases of round-, sector- and oval-shaped cores is illustrated in Fig. 12.34.* An illustration of the magnitude of the quantities involved is provided by the following data for 0·1-sq. in. cables with round conductors. The rupturing force is in lb./in. run, and the current is that value which will produce this force.

Voltage.	Rupturing Force.	Current.
L.T.	149	33,400
11 kV.	372	66,000
22 kV.	820	98,000
22 kV. (screened)	122	40,900
33 kV. (screened)	122	44,800

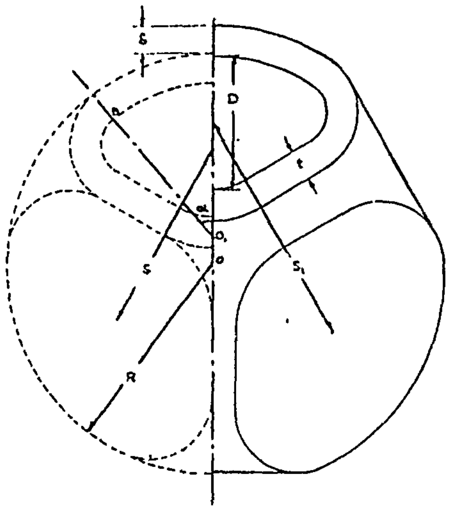
Example.—A control board is situated at the load end of a 570-yard length of 440-volt cable, conductors 0·1 sq. in. This cable is supplied by a 6,600/440 volt transformer whose primary is fed by a 2,370-yard length of 0·06-sq. in. H.T. cable connected to bus-bars of large capacity. One of the cables fed by the control board is a 113-yard length of 0·05 cable. A dead short-circuit occurs at the end of this cable. Calculate the temperature rise of the cores of the 0·05-sq. in. cable given:

Resistance per mile of 0·05-sq. in. cable	0·894 ohm
Reactance per mile of 0·05-sq. in. cable	0·125 ohm
Resistance per mile of 0·10-sq. in. cable	0·429 ohm
Reactance per mile of 0·10-sq. in. cable	0·118 ohm
Resistance per mile of H.T. cable	0·724 ohm
Reactance per mile of H.T. cable	0·142 ohm

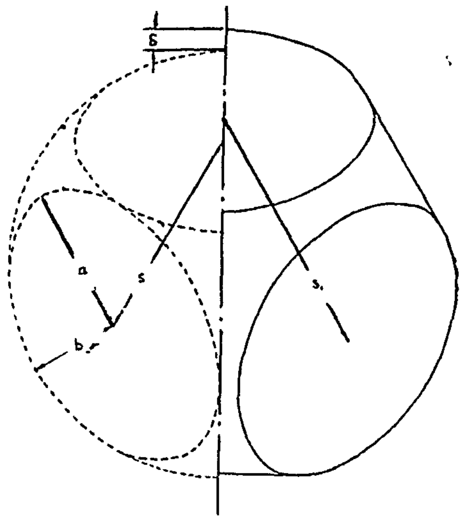
* Taken from Siemens *Engineering Bulletin*, June/Sept. 1952.



ORIGINAL POSITION POSITION AFTER DISTORTION
Fig. a. Three-core (round) Cable.



ORIGINAL POSITION POSITION AFTER DISTORTION
Fig. b. Three-core (sector) Cable.



ORIGINAL POSITION POSITION AFTER DISTORTION
Fig. c. Three-core (oval) Cable.

The degree to which a 3-core cable can distort without actual disruption is limited to the position where the belt or the copper-reinforced tape becomes tangential as the cores move apart. The three possible cases for round, sector and oval conductors are illustrated in *Figs. a, b and c.* It is possible to calculate the movement that will bring about such a position from the following formulae, which can be taken as being sufficiently accurate for all practical purposes.

A. 3-Core (Round) Cable.

Let r = radius of one insulated core.
R = radius of the three laid-up cores.
S = original axial spacing of the cores.
S_1 = axial spacing of the cores after distortion.
δ = radial movement of the cores when distortion occurs

Then :— $S = 2r$
$S_1 = 2 \cdot 420\, r$
$\delta = 0 \cdot 2425\, r$

B. 3-Core (Sector) Cable.

Let R = radius of laid-up cores.
r = radius of curvature of *conductor* sector $= O_1B$.
t = thickness of insulation over conductor.
α = angle as shown in *Fig. b* ≃ 40° for all common conductor sizes.
O = centre of cable.
O_1 = centre of curvature of *conductor* sector.
S = original axial spacing of the cores.
S_1 = axial spacing of the cores after distortion.
D = depth of *conductor* sector
δ = radial movement of the cores when distortion occurs

Then :—
$S = 1 \cdot 022\, D + 2 \cdot t$
$S_1 = S + \sqrt{3}\,\delta$
$\delta = 0 \cdot 210R + 1 \cdot 156(r+t)[0 \cdot 866 - \sin(60-\alpha) - 0 \cdot 0174\alpha]$.

C. 3-Core (Oval) Cable. (22 and 33 kV.)

Let a = major semi-axis of insulated core.
b = minor semi-axis of insulated core.
S = original axial spacing of the cores.
S_1 = axial spacing of the cores after distortion.
δ = radial movement of the cores when distortion occurs.

Then
$S = \sqrt{3b^2 + a^2}$
$S_1 = a + \frac{2\pi}{3}\left(b + \sqrt{b^2 + \frac{a^2}{3}}\right) - \pi\sqrt{\frac{a^2+b^2}{2}}$
$\delta = \frac{1}{\sqrt{3}}(S_1 - S)$.

FIG. 12.34.—DEFORMATION UNDER SHORT-CIRCUIT CONDITIONS OF 3-PHASE CABLES WITH VARIOUS CONDUCTOR SHAPES. (*Siemen's Engineering Bulletin.*)

Resistance per phase of transformer referred to
L.T. side . . . . 0·027 ohm
Reactance . . . . 0·078 ohm
Reactance per phase of transformer referred to
L.T. side . . . . 0·078 ohm
Operating time of overload trip 2·5/(overload above setting)

Using the symbols of Fig. 12.35, the various resistances and reactances are—

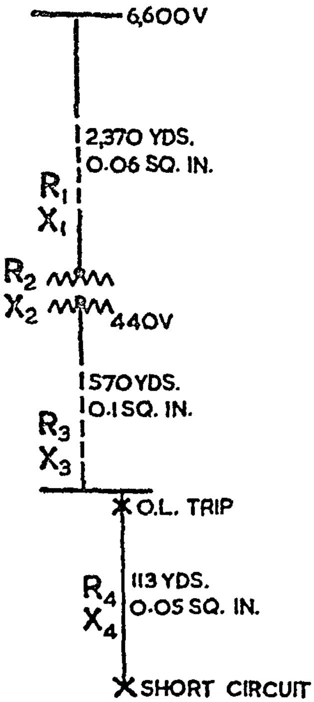


FIG. 12.35.

$$R_1 = 0{\cdot}724 \times 2370/1760 = 0{\cdot}98 \text{ ohm}$$
$$X_1 = 0{\cdot}142 \times 2370/1760 = 0{\cdot}19 \text{ ohm}$$

Where referred to the L.T. side, these become—

$$R_1 = 0{\cdot}98 \times (440/6600)^2 = 0{\cdot}0044 \text{ ohm}$$
$$X_1 = 0{\cdot}19 \times (440/6600)^2 = 0{\cdot}00084 \text{ ohm}$$
$$R_2 = 0{\cdot}027 \text{ ohm}$$
$$X_2 = 0{\cdot}078 \text{ ohm}$$
$$R_3 = 0{\cdot}429 \times (570/1760) = 0{\cdot}139 \text{ ohm}$$
$$X_3 = 0{\cdot}118 \times (570/1760) = 0{\cdot}038 \text{ ohm}$$
$$R_4 = 0{\cdot}894 \times (113/1760) = 0{\cdot}058 \text{ ohm}$$
$$X_4 = 0{\cdot}125 \times (113/1760) = 0{\cdot}008 \text{ ohm}$$

Total resistance and reactance referred to L.T. side—

$$R = 0{\cdot}228;\ X = 0{\cdot}125$$
$$\therefore Z = (0{\cdot}228^2 + 0{\cdot}125^2)^{\frac{1}{2}} = 0{\cdot}259 \text{ ohm}$$
$$V_{ph} = 440/\sqrt{3} = 254$$
$$\therefore I_{s.c} = 254/0{\cdot}259 \simeq 1000 \text{ amps. assumed symmetrical.}$$

Assuming that this value is attained instantaneously, and that the full-load current in the 0·05-sq. in. cable is 100 amps.—

overload = 900 per cent.

Assuming an overload setting of 100 per cent.—

Overload above setting = 800 per cent.

$\therefore$ Operating time = 2·5/8 = 0·316 sec.

Resistance of 0·05-sq. in. cable per cm. run = $5{\cdot}54 \times 10^{-6}$ ohm

$$\therefore \text{Calories per cm.} = 1000^2 \times 5{\cdot}54 \times 10^{-6} \times 0{\cdot}316/4{\cdot}2 = 0{\cdot}417$$

$$\text{Vol. of 1 cm. run} = 0{\cdot}05 \times (2{\cdot}54)^2 \times 1 = 0{\cdot}323 \text{ cm.}^3$$

$$\therefore \text{Mass of 1 cm. run} = 2{\cdot}7 \text{ gm.}$$
$$\text{Sp. ht} = 0{\cdot}1$$
$$\therefore 2{\cdot}7 \times 0{\cdot}1 \times \theta = 0{\cdot}417$$
$$\theta = \frac{0{\cdot}417}{0{\cdot}27}$$
$$= 1{\cdot}54^\circ \text{ C.}$$

This example shows that where fault current is limited by the impedance of a long cable run and clearance times are short, the thermal effect of a short-circuit can be very small.

Current Ratings of Super-Voltage Cables

These are determined solely in terms of conductor temperature rise. The following values refer only to cables laid direct in the ground, this being the normal practice in this country for such cables. The values are the maximum continuous loadings that can be carried without exceeding the allowable temperature rise. Values for solid-type cables are given for comparison, the best criterion, in this respect, being perhaps the current density. It will be seen that for pressurised cables (and this applies to all types) the current density is appreciably greater than for the solid-type.

In the case of solid-type cables, the allowable operating temperature is dependent on the risk of the formation of voids. Another limiting factor is the risk of movement and ultimate sheath fracture in the case of unarmoured cables installed in ducts. These two considerations do not apply to pressurised cables.

Conductor Section sq. in.	Solid Type, 33 kV., 3-core.		Solid Type, 66 kV., Single-core, Circular Conductor.		Pressurised Cables 33 kV. to 132 kV.	
	Amps.	Amps/sq. in.	Amps.	Amps/sq. in.	Amps.	Amps/sq. in.
0·2 . .	290	1,480	280	1,420	350	1,740
0·4 . .	420	1,030	410	1,030	505	1,290
0·6 . .	505	840	490	820	610	1,030

Capacitance of Insulated Cables

The capacitance of a cable system is of much greater importance than that of an overhead line of the same length owing to the nearness of the conductors to one another and to the earthed sheath, and to their separation by a dielectric of permittivity greater than that of air.

Assuming a perfectly uniform dielectric it is possible to calculate the capacitance, but in practice it is usual to measure this quantity on the finished length of a cable, since the dielectric is far from uniform and the calculated value is, at the best, only approximate.

Consider the case of a three-phase cable. Since there is a potential difference between pairs of conductors and between each conductor and sheath there will be a system of electrostatic fields in the cross-section of the cable somewhat as indicated in Fig. 12.36A. This figure gives the average distribution of electrostatic tubes of force ; actually, the distribution is continually changing because

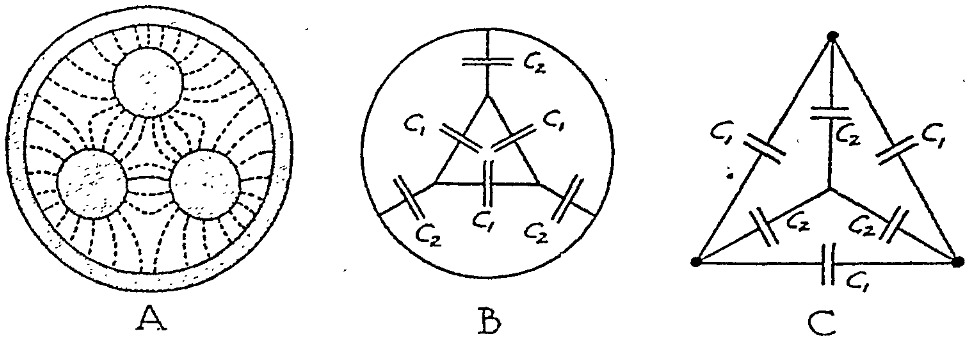


FIG. 12.36.—ELECTROSTATIC FIELD OF A THREE-CORE BELTED CABLE.

of the changing P.D.s between pairs of conductors, and between conductors and earth. The existence of a system of tubes of force between two metal bodies indicates that there exists electrostatic capacitance between these bodies, and we thus see that there are six such capacitances with a three-phase cable, as indicated in Fig. 12.36B. Three of these are mesh-connected, and three star-connected, the lead sheath forming the star point. The arrangement of 12.36B is thus equivalent to the system of six condensers in 12.36C.

It is convenient to reduce this system to the equivalent star-connected system. Let the three mesh-connected condensers each of c_1 be equivalent to three star-connected condensers each of c_3, Fig. 12.37 A and B. Let the line voltage be E. Thus phase current due to the actual mesh-connected condensers

$$= Ec_1\omega$$

$$\therefore \text{Line current} = \sqrt{3}\, Ec_1\omega$$

Phase voltage in the equivalent star

$$= E/\sqrt{3}$$

∴ Phase current in the equivalent star

$$= \text{Line current in the equivalent star}$$
$$= Ec_3\omega/\sqrt{3}$$
$$\therefore\ E/\sqrt{3} \cdot C_3\omega = \sqrt{3}Ec_1\omega$$
$$\therefore\ c_3 = 3c_1$$

Hence, the whole cable is equivalent to three star-connected condensers each of capacitance $(3c_1 + c_2)$, as shown in Fig. 12.22 C.

The capacitances c_1 and c_2 are determined by measurement as follows: first the capacitance between two cores is measured, the third core being either insulated or connected to the sheath.

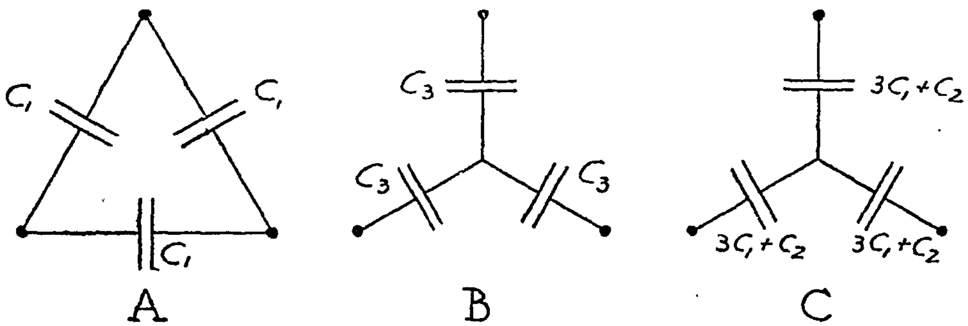


FIG. 12.37.—EQUIVALENT CAPACITANCE OF A THREE-CORE CABLE.

This eliminates one of the condensers c_2, so that if C_1 is the measured capacity

$$C_1 = c_1 + \frac{c_1}{2} + \frac{c_2}{2}$$
$$= \tfrac{1}{2}(3c_1 + c_2)$$

Next, the three cores are bunched together and the capacitance measured between them and the sheath. The bunching eliminates all three condensers c, leaving the three condensers c_2 in parallel. Hence, if C_2 is the measured capacitance

$$C_2 = 3c_2$$

Paper-insulated concentric cables of about 0·2 sq. in. cross-section have a capacitance between conductors of the order of 0·2 to 0·6 mfd. per mile, while with rubber or bitumen dielectric the capacitance may be somewhat greater. Between the outer conductor and the sheath of a concentric cable of the same size the capacitance is of the order of 1 mfd. per mile. With three-core cables up to about 0·1 sq. in. the capacitances are of the order: (*a*) between the three conductors joined and the sheath, 0·3 mfd. per mile; (*b*) between any two conductors, 0·15 mfd. per mile.

As a numerical example take the case of a 6,600 volt three-core cable having the above capacitances. Then per mile length of cable we have

$$C_1 = \cdot 15 \text{ and } C_2 = \cdot 3$$

$$\therefore \left.\begin{aligned} \tfrac{1}{2}(3c_1 + c_2) &= \cdot 15 \\ 3c_2 &= \cdot 3 \end{aligned}\right\}$$

The solution of which is

$$c_1 = \cdot 07 \text{ mfd. and } c_2 = \cdot 1 \text{ mfd.}$$

Hence, the equivalent star total capacitance per mile of cable is

$$3c_1 + c_2 = \cdot 2 + \cdot 1 = \cdot 3 \text{ mfd.}$$

$$\text{Phase voltage} = 6600/\sqrt{3} = 3810 \text{ volts}$$

$\therefore$ Charging current at 50 cycles

$$= 3810 \times \cdot 3 \times 10^{-6} \times 314 = \cdot 36 \text{ amp.}$$

Tests on Cables

For cables not in the super-voltage class the tests to be carried out are laid down in the appropriate British Specifications. Thus for Impregnated-paper-insulated Cables with Lead or Lead-alloy Sheaths, B.S. 480; Pt. I; 1954, the tests, purely electrical, are as follows:

Acceptance Tests at Works

(*a*) *Conductor Resistance.*

(*b*) *Voltage Test.*—The applied voltage must be of approximately sinusoidal shape and of any frequency between 25 and 100 c/s. It must be increased gradually to the full value and maintained continuously for 15 minutes between conductors and between each conductor and sheath. The required values of the test voltages are tabulated in the Specification and, as one illustration of the magnitude relative to the normal voltages, the figures for 11,000-V. cables for earthed systems are given.

Voltage Designation.	Belted Cables.				Single-core, S.L., and Screened Cables.	
	Between Conductors.		Between any Conductor and Sheath.		Between any Conductor and Sheath.	
	Cable as manufactured.	After bending test.	Cable as manufactured.	After bending test.	Cable as manufactured.	After bending test.
11,000	24,000	36,000	14,000	21,000	15,000	22,000

It will be seen that a voltage test is made before and after a bending test. In this the cable has to be bent around a cylinder of specified diameter to make one complete turn: it is then unwound and the process repeated in the opposite direction. This cycle of operations has to be carried out three times.

(c) *Dielectric Power-factor/voltage Test, for 33-kV. cables only.*—Each core of every drum of completed cable is tested for dielectric power-factor at room temperature at the following A.C. single-phase 50-cycle voltages:

9·5, 19, 28·5, 38·0 kV.

The measured power-factor at normal working voltage shall not exceed the value declared by the manufacturer and shall in no case exceed 0·01.

The ionisation—i.e. the difference in power-factor between half normal working voltage and twice normal working voltage—shall not exceed the value declared by the manufacturer and shall in no case exceed 0·0006 for three-core screened cable or 0·001 for single-core and screened S.L.-type cable. The manufacturer can also be asked to produce evidence to show that the power-factor at normal working voltage does not exceed 0·01 at a series of temperatures ranging from 15° C. to 65° C.

Sample Tests at Works

These include the bending test above and a dripping or drainage test for cables which have to be installed vertically.

Tests when Installed

A voltage test similar to the above is carried out in the same manner but with somewhat reduced voltages. Thus the value of 24 kV., 14 kV., and 11 kV. for belted cables as manufactured and the value 15 kV. for single-core, S.L., and screened cables, become 20 kV., 11·5 kV., and 12 kV. respectively.

*Tests on Pressurised Cables**

At one time a high-voltage A.C. test was required, 3·46 times the working voltage being applied for 15 minutes. This kind of test is quite unrepresentative of working conditions and may impose unnecessary limitations on the dielectric stress for which the cable is designed. It has been replaced by an impulse test.

Type approval tests, as distinct from type tests which could be demanded for each separate installation, are stipulated for

* Schedule F of Specification Central Electricity Board—c/(1947).

each design of cable and accessory. These tests are carried out on the maximum and the minimum conductor sizes for each design and voltage rating, and, if successful, no further type tests are required, except in the case of changes in the design. The dielectric thermal resistance test included in the schedule is applied only to the minimum conductor sizes. The tests are as follows:

(*a*) *Loading Cycle Tests.*—A test loop, comprising cable and each type of accessory to be subjected to 20-load cycles to a minimum conductor temperature 5° C. in excess of the design value, with the cable energised to 1·5 times the working voltage. The cable to be tested at a stipulated minimum internal pressure.

(*b*) *Thermal Stability Test* (132-kV. cables only).—After test (*a*) the cable to be energised at 1·5 times working voltage and the loading current adjusted to give a maximum temperature 5° C. in excess of the design value. The current to be maintained at this value for a period of six hours, with other test conditions unaltered, to prove that the cable is thermally stable.

For 275-kV. cables 1·33 × working voltage is proposed.

(*c*) *Impulse Test.*—A test loop, comprising cable and each type is accessory to be subjected to 10 positive and 10 negative impulses at the voltage values given below.

Working Voltage. kV.	Impulse Test Voltage. kV.	Impulse Test: Peak Working Voltage Ratio.
33	194	7·2
66	342	6·4
132	640	6·0
275	1050	4·7

Brazier, Hollingsworth, and Williams have this to say about the above requirements: "Most design features are independent of operating voltage, the essential difference brought about by this voltage being the maximum electric stress at the conductor at which the dielectric is operated. This may be increased, within limits, as the system voltage increases, because the impulse-withstand requirement, which is generally the deciding factor, becomes progressively less onerous. Up to 132 kV., the British Electricity Authority employs the empirical formula 4·5 (E + 10) for the impulse requirement, where E is the nominal system voltage between phases. This corresponds to factors of 10, 9, and 8·4 times the working voltages at 33 kV., 66 kV., and

132 kV. respectively. At 275 kV. the level has been set at 1050 kV., corresponding to a factor of 6·6."

(*d*) *Cold Power-factor/Voltage Test.*—The power factor of a 100-yard length of cable to be measured at 0·5, 1·0, 1·5, and 2 times working voltage with the cable at the stipulated minimum internal pressure. The values not to exceed the makers' guaranteed values.

(*e*) *Dielectric Thermal Resistance Test.*—The thermal resistance of the cable to be measured.

(*f*) *Mechanical Test of Metallic Reinforcement.*—A sample of cable to withstand twice the maximum specified interval pressure for a period of seven days.

(*g*) *Binding Test.*—The cable to be subjected to three binding cycles round a drum of diameter 20 times the diameter of the pressure retaining sheath. The sample then to withstand the routine voltage test carried out on all production lengths of cable.

Suggestions for Further Reading

(*a*) *Books*

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CHAPTER XIII

VOLTAGE CONTROL

OWING to the inevitable resistance and reactance of all conducting systems, whether overhead or underground, there will be a drop of volts along the system when current is flowing, and furthermore this drop will vary with the current, and in the case of an A.C. system with the power factor also. If transformations of voltage take place at one or more points, then the drops in the transformers will be superposed on the line drops. For a short transmission line it is possible to design an economical system in which all these drops will not exceed, say, 20 per cent. of the station bus-bar voltage, this drop being within the capacity of the automatic voltage regulators which control the generator terminal voltage. Where the current must flow along a transmission line before reaching the distribution system, these automatic voltage regulators can be compounded so as to give the alternators a rising voltage characteristic similar to that of an over-compounded D.C. generator, and in this way the voltage variation at the junction between the transmission and distribution systems can be kept within close limits.

At the present time distribution systems are much more extensive than the early D.C. distribution systems for which the previously quoted B.O.T. Regulations regarding allowable voltage variation were devised, and, furthermore, the majority of the modern systems employ alternating, instead of direct, currents. It therefore follows that in addition to the compounding of the voltage regulators at the central station it is necessary to boost up the voltage at various points in the distribution system itself. There are various methods of effecting this.

With very long transmission lines the line capacitance becomes of importance, this having the effect not only of adding its own component to the total line drop, but also of causing wide variations of power factor with changes of load. This, combined with the fact that the total line drop with such a system cannot, for economical reasons, be kept at the low percentage value possible with short lines, necessitates that the control of the receiving-end voltage shall be effected by an entirely different method. The method is to connect an idle-running synchronous motor, sometimes called a synchronous condenser, across the line at the receiving end, and to arrange automatic regulation of its excitation in such a way that with full load on the line the excitation is a maximum, while with no load on the line the excitation is a minimum. Hence, when the line is fully loaded the motor delivers the maximum leading kVA. to it, thereby compensating for the lagging current taken by the load, while when the line is unloaded the motor draws wattless lagging kVA. from it, thus preventing the rise in voltage caused by the line capacitance when the line is open ended. In this way it is possible to maintain the voltage at the receiving end constant while the voltage at the sending end is held constant. The method is considered in further detail on p. 306.

With the exception of the synchronous condenser method the problem of the control of the voltage of a distribution system is essentially the same as that of a transmission system, except that in the latter it may be necessary to transmit power at will in either direction. The following methods in which the control can be carried out on the H.T. or the L.T. side are in use:

1. Tap-changing transformers.
2. Induction regulators.
3. Moving-coil regulators.
4. Boosters.

In addition, there are special methods applicable to the L.T. side.

Tap-changing Transformers

Since the voltage ratio of a transformer is dependent on the turns ratio (being exactly equal to it at no load) it follows that if the turns ratio can be altered then the voltage ratio will be altered by a corresponding amount. With this method it is usual to vary the turns ratio by altering the number of turns in use on the H.T. side, because the current in this side is smaller and the current-carrying parts of the necessary gear therefore smaller. It is obvious that the number of turns in use must be capable of variation whilst the transformer is on load, and for this reason the equipment is termed "on-load" tap-changing equipment. It is clear that if a section of the winding is to be cut in or out when carrying current, without breaking the circuit or temporarily short-circuiting this section, then auxiliary gear of some kind must be employed in order to provide a parallel path for the current. Four systems have been advocated, viz.:

1. Split parallel windings, with duplicated tappings connected to two "off-load" tap changers, plus two oil switches for breaking the current.

2. Single winding with tappings connected to contactors or switches, and a reactor to enable the transfer to be made from tapping to tapping.

3. As method 2, but with a centre-tapped reactor.

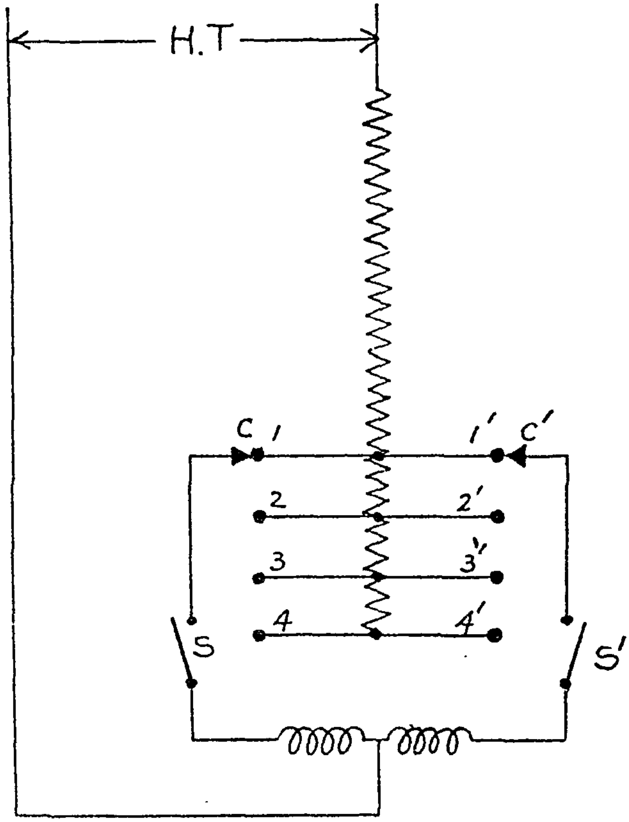


FIG. 13.1.—PRINCIPLES OF ON-LOAD TAP CHANGING.

4. Similar to 2 and 3, except that an induction regulator is employed in place of the reactor.

Although all these methods have been tried, method 3 is the only one which has survived the requirements of cost and maintenance, and details of the others will therefore not be given. The principle of the method can be followed very easily by reference to Fig. 13.1, in which only one phase is shown for convenience. Suppose that the tapping 1,1′ is in use and it is desired to change to the tapping 2, 2′; then the switches S and S′ will both be closed. First the switch S is opened and then the contact C is moved from tapping 1 to 2. It is now clear that contact C′ must not be moved with switch S still open or the H.T. circuit will be interrupted, and consequently the next step is to close S. As switch S′ is still closed this means that the reactor is now across the tapping 1, 2′ and will, in consequence, carry a current equal to the voltage between these points divided by its impedance, in addition to carrying one half of the load current. The next step is to open S′ and to move the contact C′ from point 1′ to 2′, and during this adjustment the left-hand half of the reactor carries the whole of the load current. Finally, the switch S′ is closed and the transformer is then in normal operation but with the tapping 2, 2′ in service instead of the tapping 1, 1′.

From the above descriptions it is clear that the following equipment is required:

1. Transformer with the required tappings on the H.T. side.
2. Centre-tapped reactor.
3. Tapping switches.
4. Two circuit-breakers or contactors.
5. Operating mechanism and timing system for correlating the various adjustments.

It will be obvious that if both switches S and S′ are closed while contact C is at 1 and C′ at 2′, then the voltage obtained will be intermediate between the voltages at 1, 1′ and 2, 2′. By employing a continuously rated reactor this intermediate position can be utilised, thereby doubling the number of voltage steps for a given number of tappings. The above method necessitates heavy mechanical gear for the movements of the contacts, these movements being correlated with the opening and closing of the switches S and S′. This mechanism can be dispensed with and all the operations rendered electrical by providing a separate switch to each tapping point, as illustrated in Fig. 13.2, in which only two tapping points are shown for simplicity. The three possible voltages for the two tappings are given by the three schemes of connection shown, and it will be seen that except for the voltage midway between those of the two tappings, the positions involve one half of the reactor being left in circuit. The reactor must therefore be rated to allow for its carrying the full line current continuously. In addition, the drop due to the reactor will be deducted vectorially from the line voltage, although as this drop is in quadrature with the current its effect will be negligible unless the power factor is lower than about 0·6.

A further modification is illustrated diagrammatically in Fig. 13.3, in which it will be seen that there are three transition positions between each permanent position. This method

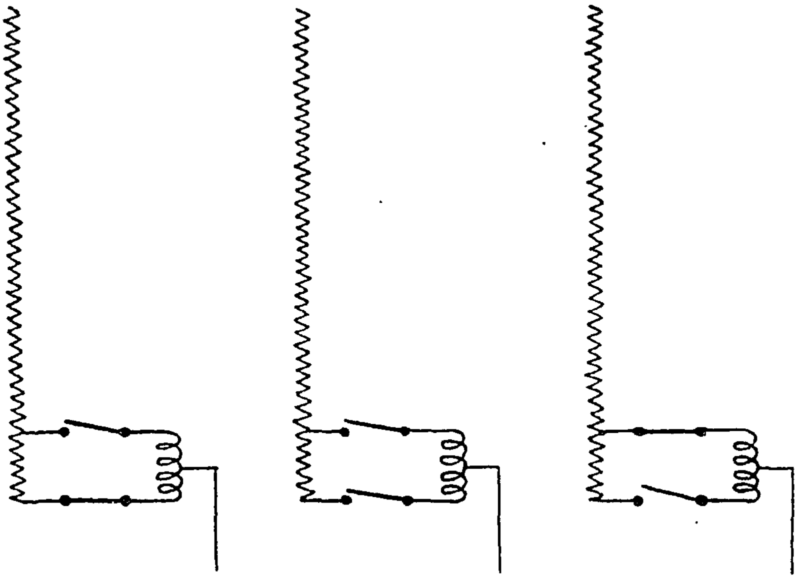


Fig. 13.2.

eliminates the reactor drop in a permanent position since, in position 2, the M.M.F.s are in opposition. The tappings on the main transformer winding are selected by means of a drum-type rotary switch on the smaller units, and a dial-type rotary switch

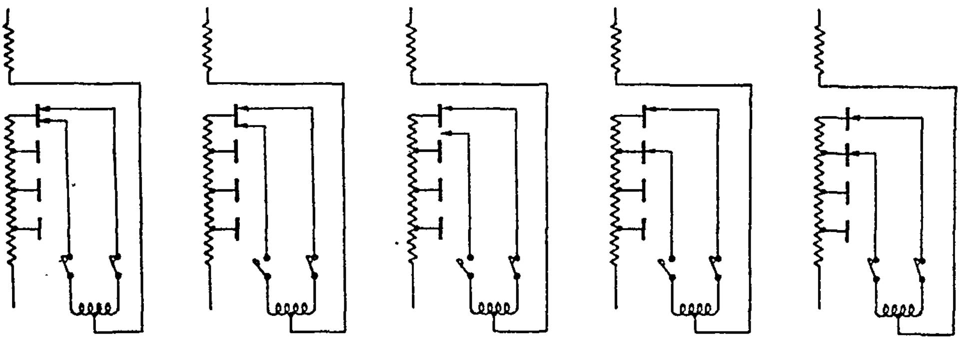


Fig. 13.3.

on the larger. As will be seen from the key diagram of Fig. 13.3, the circuits are not made or broken on the selector switch but by the diverter switch, which is of contactor type. The two diverter-contactors to each phase are operated through a cam and toggle mechanism. Since the circuit-opening during the changing of a tap is performed by the diverter-contactors, these are mounted in a separate chamber. The selector-switch chamber is open to the oil in the main transformer tank, and the tap-leads can be brought through the opening between the two chambers without the necessity for the provision of multiple bushings. Fig. 13.4 shows a developed diagram of connections of a drum-type selector switch.

For larger units the system of tap-changing is as above, but the selector switches are of the dial-type and the diverter-contactors are mounted in pairs in three separate tanks, one per phase, instead of all in one chamber as in the smaller units. One pair of diverter-contactors for a 66-kV. tap-changer is shown in Fig. 13.5, its tank being lowered. The connection diagram for the dial-type selector switch is given in Fig. 13.6. The two separate movable members—shown overlapping on contact 11—correspond with the contacts marked with the arrows on the schematic diagram of Fig. 13.3.

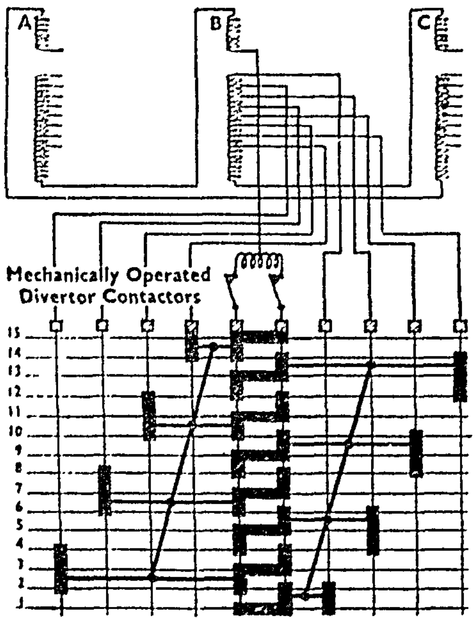


FIG. 13.4.—DEVELOPED DIAGRAM OF CORRECTIONS OF DRUM-TYPE TAP-CHANGER.
(*English Electric Co. Ltd.*)

Another system, suitable for voltages up to 275 kV., and currents up to 800 amps., is illustrated diagrammatically in Fig. 13.7. It comprises twin selector switches for each phase, associated diverter switches, and transition resistors instead of reactors. The figure shows the sequence of operations in the case of a transformer with a star-connected winding. Fig. (a) is for voltage steps up to 14, and Figs. (b) and (c) for steps up to 18. In the "coarse-fine" arrangement of Fig. (b) the section 11–12 contains ten times as many turns as each tapping section such as 5–6. The tappings 1–9 and 11 are connected to the selector switches, which make two revolutions to cover the tapping range, one with the larger section in circuit and the other with it out. The arrangement of Fig. (c) has a tapping section which can be added to, or subtracted from, the main winding according to the position of the reversing switch. The selector switches make two revolutions to cover the tapping range, one with the reversing switch connected to 11, the other with it connected to 1.

To change from tapping 10 to tapping 9 in Fig. A, the main contactor M_1 is opened and the auxiliary contactor A_2 is closed, thereby bridging the tapping 9–10 with the transition resistors R_1 and R_2. The opening of A_1, followed by the closing of M_2, completes the tap change. Before the operating mechanism comes to rest, the selector switch not carrying current moves to tapping 8, in readiness for the next tap change in the same direction. To make a tap change from 10 to 11, the selector switch moves from tapping 9 to 11 before the sequential opening of the diverter switches A1, A2, M1, and M2—that is, while not carrying any current.

FIG. 13.5.—DIVERTOR-CONTACTORS FOR ONE PHASE OF A 66 kV. TAP-CHANGER. (*English Electric Co. Ltd.*)

Continuity of circuit is maintained during a tap change, and all current is broken by the diverter switches which are housed in a separate compartment with separate oil. The use of transition resistors ensures that the current broken by the diverter switches is of high power factor, easing the switching duty and minimising arcing at the contacts.

Fig. 13.8 shows a much simpler scheme for lower voltage and current ratings. The tap changer consists of (*a*) the fixed contacts, which are mounted radially on both sides of a bakelite panel—one panel per phase, and (*b*) moving contacts comprising two arms per phase, one on either side of the fixed contact panel, and permanently offset with respect to one another. In the tapping positions both arms are in contact with the same fixed contact. During a tapping change the leading arm registers with

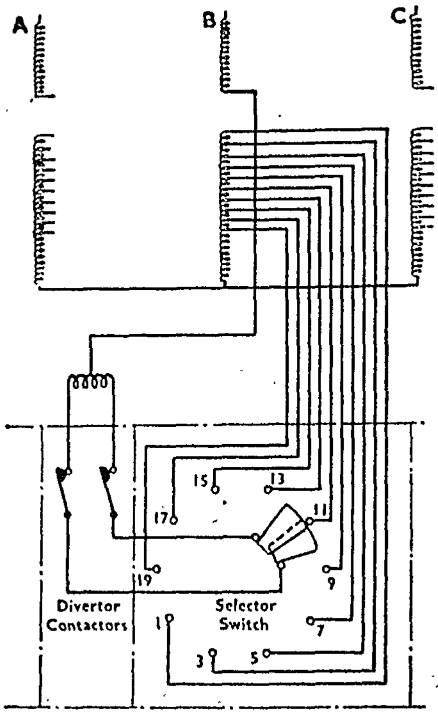


FIG. 13.6.—DEVELOPED DIAGRAM OF CONNECTIONS OF DIAL-TYPE TAP-CHANGER.
(*English Electric Co. Ltd.*)

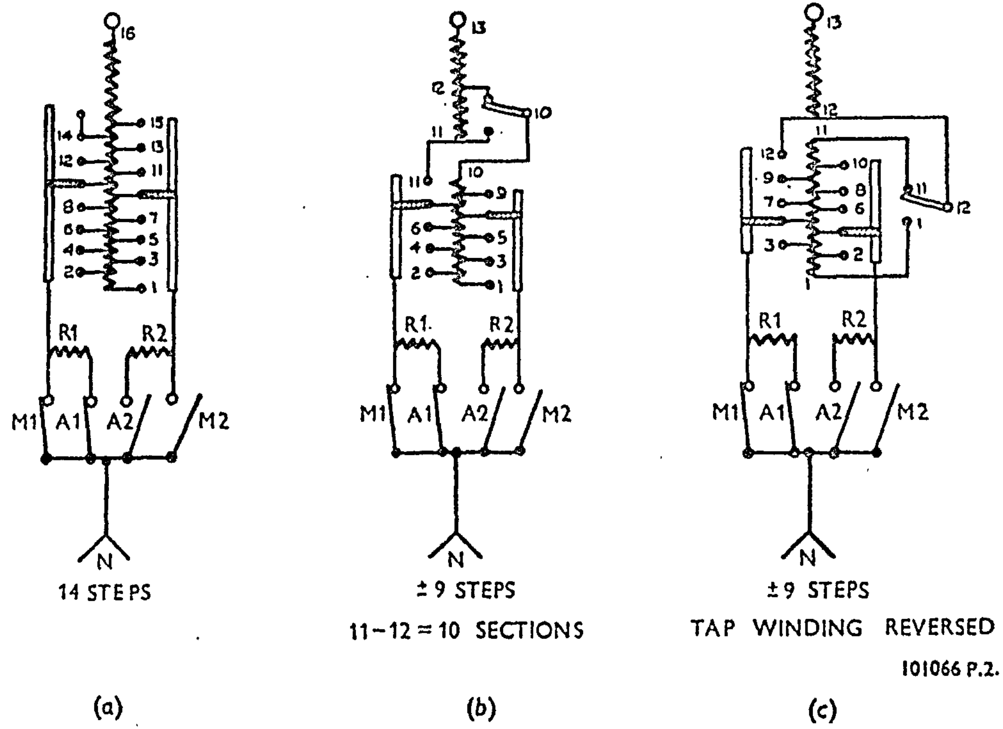
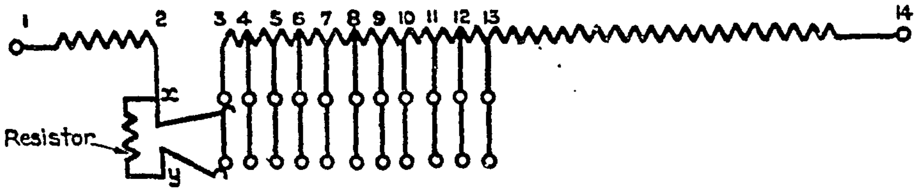


FIG. 13.7.—TAP-CHANGING SYSTEM USING TRANSITION RESISTORS.
(*Metropolitan Vickers Electrical Co. Ltd.*)

the next fixed contact before the lagging arm leaves the last fixed contact. During the change the resistor momentarily bridges the two fixed contacts.



Tapping Position No.		1		2		3		and	10		11
Moving	x to tap	13	12	12	11	11	10	continued	4	3	3
Contacts	y to tap	13	13	12	12	11	11	to	4	4	3

FIG. 13.8.—CONNECTIONS OF TAP-CHANGING GEAR SUITABLE FOR LOW VOLTAGE AND CURRENT RATINGS.
(*Metropolitan Vickers Electrical Co. Ltd.*)

Location of the Tap-changing Gear

Fig. 13.9 shows the cost of the tapping switch and its associated automatic gear against the transformer kVA. output, and it is

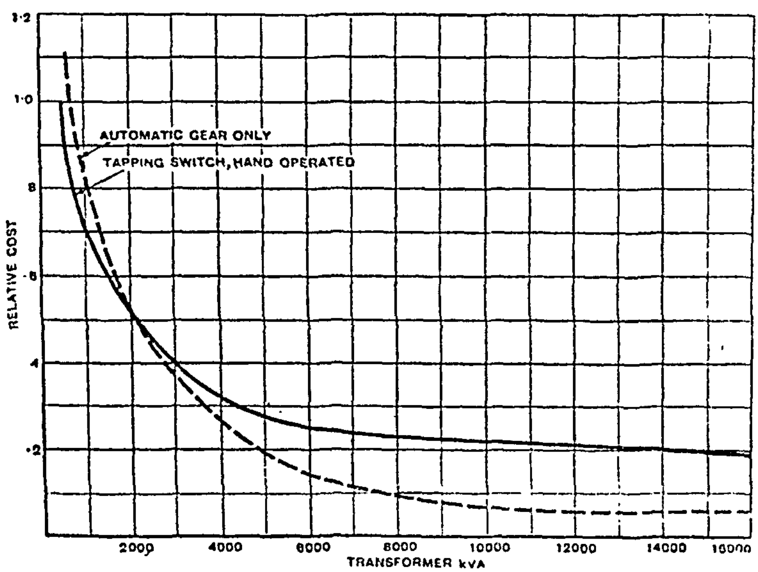


FIG. 13.9.—COST OF TAP-CHANGING GEAR.

at once clear that the relative cost of both increases very rapidly as the kVA. is reduced. From the point of view of first cost it is therefore very desirable to install such apparatus as near to the generating station as possible. Consider, for example, a distribution system employing a number of 500-kVA. transformers, with the bulk supply to the whole system passing through a 5,000-kVA. transformer. Voltage regulation could be accomplished at the bulk supply transformer at a relative cost of 0·27 for the tapping switch and 0·19 for the automatic gear, while with correction at the 500-kVA. transformers the relative costs would be increased to about 1·0 and 1·2 respectively. It is to be noted, however, that where the drop in the lower-voltage lines is very large it may be necessary to regulate at the smaller transformers in addition to, or instead of, regulation at the main transformers.

With regard to the question as to whether the tappings should be provided on the H.T. or L.T. side of the transformer it is necessary to realise that in any typical installation the following regulation requirements may be required :

(*a*) A constant secondary voltage/load characteristic with a varying primary voltage.

(*b*) A rising secondary voltage/load characteristic with a constant primary voltage.

(*c*) A rising secondary voltage/load characteristic with a varying primary voltage.

If the applied voltage of a transformer without voltage regulating apparatus is not equal to the normal voltage, then the flux density will be different from normal : an applied voltage above normal will raise the flux density with the result that the core-loss will be increased, harmonics due to the non-sinusoidal nature of the magnetising current will be increased, and, if there is any hum from the transformer, this also will be increased. If, on the other hand, the applied voltage is reduced, the magnetic characteristics will be improved, but, if the kVA. output is up to the full value, the current, and therefore the copper losses, will be increased.

The following very instructive table is taken from a paper by Nunn.

Transformer Characteristic.	Constant Secondary Voltage with Varying Primary Voltage.				To vary Secondary Voltage with Constant Applied Primary Voltage.			
	Switch on Primary.		Switch on Secondary.		Switch on Primary.		Switch on Secondary.	
	Volts above normal.	Volts below normal.	Volts above normal.	Volts below normal.	Volts above normal.	Volts below normal.	Volts above normal.	Volts below normal.
Flux density, Harmonics, Iron Loss	Constant	Constant	Increases	Decreases	Increases	Decreases	Constant	Constant
Primary Current	Decreases	Increases	Decreases	Increases	Constant	Constant	Constant	Constant
Secondary Current	Constant	Constant	Constant	Constant	Decreases	Increases	Decreases	Increases

The table shows that a tap-changer connected to the primary winding to correct for primary supply voltage variations will only alter the primary current, whereas if connected to the secondary winding two characteristics will alter. If the tap-changer is used for varying the secondary voltage with a constant applied primary voltage, then it will only alter the secondary current if connected to the secondary winding, whereas if connected to the primary two characteristics will again vary. Similarly, the requirement of a rising voltage/load characteristic with a varying primary voltage will give varying transformer characteristics proportional to the sum of the variations of the separate voltage variation requirements.

The above considerations indicate that, from a purely technical point of view, the tap-changer should be connected to the primary winding if correction is required for a varying primary supply voltage, and to the secondary side when a rising voltage/load characteristic is desired with constant primary voltage. In the case of a rising voltage/load characteristic with varying supply voltage, all the characteristics will be affected, whichever winding is connected to the tap changer, and the choice will therefore depend on the magnitudes of the separate variations.

The above discussion has not taken into account the difference in cost of switches of equal rating but different voltages, and from the point of view of cost, instead of technical perfection, it may be the best to connect the switch to the theoretically incorrect winding.

Induction Regulators

In construction the induction regulator is similar to the induction motor, except that the rotor is held stationary in a position which can be adjusted, either manually or by power. The primary winding is connected across the supply, while the secondary winding is connected in series with the line whose voltage is to be controlled. From the electrical point of view it is immaterial whether the rotor or the stator carries the line current. For small sizes the stator is usually made the primary and the line current is brought to the rotor by flexible leads. For large sizes the magnitude of the line current requires that the secondary should be the fixed winding, i.e. the stator, and consequently the rotor becomes the primary.

In a three-phase regulator the primary winding generates a rotating field of constant magnitude, and consequently the induced voltage in the secondary will be constant in magnitude. On the other hand, the phase of this voltage can be varied by varying the relative position of rotor to stator. After leaving the secondary the line has a total voltage equal to the vector sum of its incoming voltage and the induced secondary voltage, and it is clear from Fig. 13.10 how this total voltage can be controlled from full buck to full boost by varying the rotor position. In the figure OA is the incoming line voltage, AB the secondary induced voltage, and OB the resultant line voltage. With a single regulator it will be clear that the phase of the total line voltage will, in general, be different

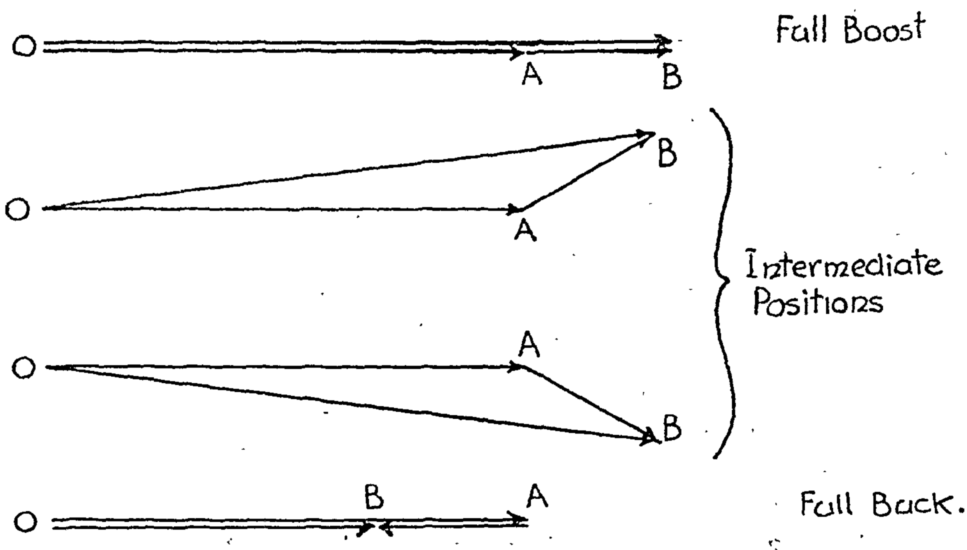


FIG. 13.10.—PRINCIPLE OF THE INDUCTION REGULATOR.

from that of the incoming line voltage. With a single feeder this phase shift is of no consequence, but where parallel lines are in operation, or in the case of ring mains or of interconnectors, it is not permissible, and it is then necessary to employ a double induction regulator consisting of two units, each of which contributes one half of the voltage added to each phase. These are so connected that the total voltage is always in phase with the incoming voltage, as shown in Fig. 13.11.

An alternative method of neutralising this phase shift is to employ three single-phase regulators. Another application of a bank of single-phase regulators is to the voltage control of a four-wire distribution network where considerable out-of-balance load is experienced, and it is therefore desirable to regulate the voltage between each phase and the neutral separately. The single-phase regulator has a wound stator and rotor like the three-phase regulator, but, in addition, it has a short-circuited winding on the rotor, the function of which is to limit the leakage reactance of the stator winding, which would otherwise reach a prohibitive value as the rotor reached the mid, or neutral, position. In other

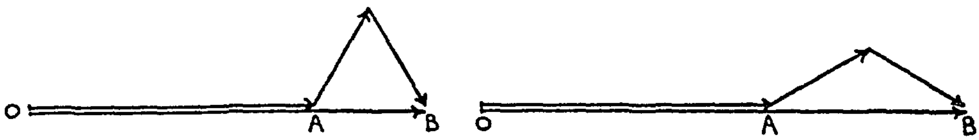


FIG. 13.11.

words, it prevents the apparatus from behaving like a choke when the rotor is in this position. The construction of the single-phase regulator will be followed from Fig. 13.12.

An advantage of the induction regulator is that the voltage variation is gradual, instead of in steps.* Also there are no switches or moving contacts of any type, the maintenance of the

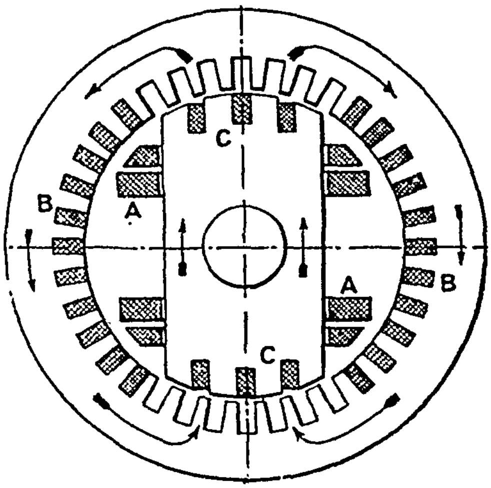


FIG. 13.12.—ELECTRIC AND MAGNETIC CIRCUITS OF S. & C. SINGLE-PHASE REGULATOR.

regulator itself thus being practically nil. For these reasons induction regulators are sometimes preferred to tap-changing methods, in spite of a higher first cost. The rotor of the single-phase regulator is liable to vibrate in the alternating field, and care has to be taken to minimise noise and reduce the deterioration of the insulation due to vibration.

* It should be noted that the relays used to control automatic induction regulator equipment require a voltage variation of 1 to 1½ per cent. to make them operate, so that the infinite gradation of voltage is not attained in practice.

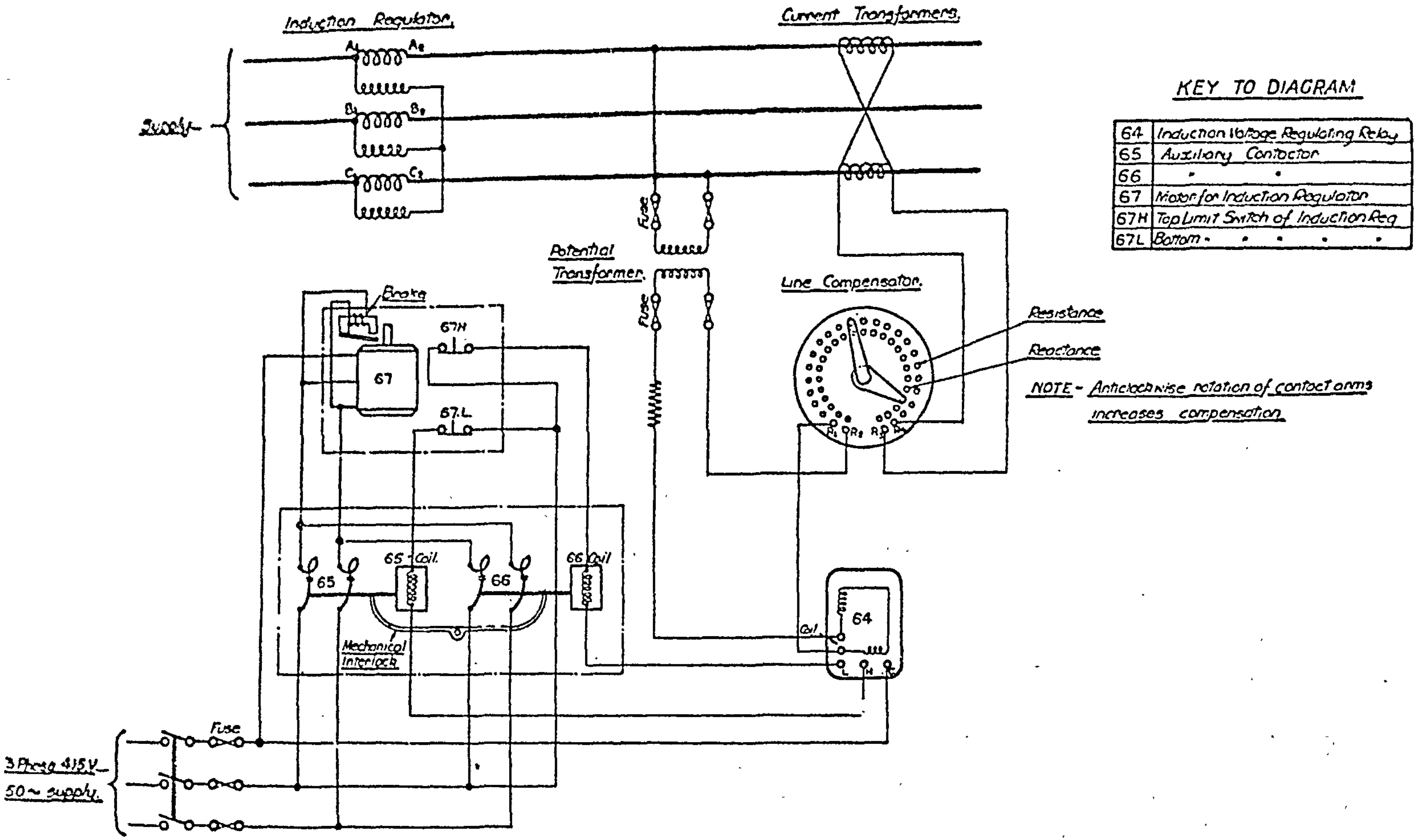


FIG. 13.13.—AUTOMATIC CONTROL OF INDUCTION REGULATOR BY MEANS OF LINE-DROP COMPENSATOR.
(*Metropolitan-Vickers Electrical Co., Ltd.*)

When the voltage at the far end of a long feeder is to be held constant, this can be accomplished very conveniently by means of an induction regulator. The regulator is placed at the supply end, and not at the delivery end, in order to avoid sending the regulator losses and line current along the line. The connection diagram for a fully automatic scheme of this nature is shown in Fig. 13.13. The position of the rotor is controlled by the voltage-regulating relay 64, which is energised by a potential transformer, and therefore sensitive to variations in line voltage. If the voltage falls by a predetermined amount the relay moves in such a direction that the operating coil 66 of the contactor 66 is energised, thereby closing this contactor. This causes the motor, 67, to move, so raising the voltage. At the same time the potential

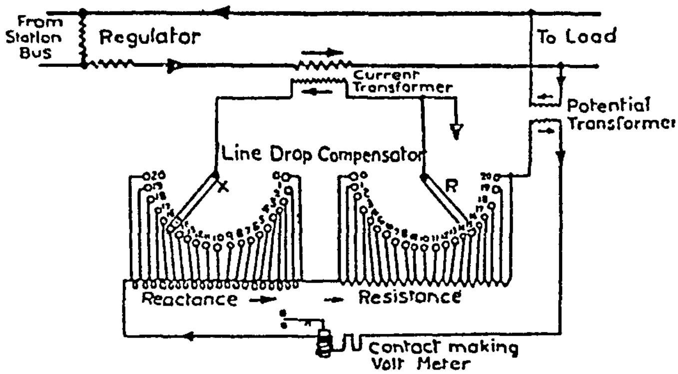


FIG. 13.14.—PRINCIPLE OF THE LINE-DROP COMPENSATOR.

of the potential transformer will rise until eventually the relay is brought back to the neutral position, the contactor, 66, is de-energised, and the motor stops in its new position. With a fall in voltage the relay moves in the opposite direction, thereby energising contactor 65, the connections being such that the motor now moves in the opposite direction, again coming to rest when the voltage is such as to bring the relay to its neutral position.

The appliance named the line compensator is included because the voltage at the delivery end of the line has to be kept constant and consequently the drop of volts in the line has to be allowed for. The compensator takes the form of a variable resistance R, Fig. 13.14, in series with a variable reactance X. The values of R and X are adjusted so as to be proportional to the actual resistance and reactance of the line, and the circuit is energised by current from the secondary of a current transformer. In other words, there is a circuit with constants proportional to the line constants carrying current proportional to the line current, the result being that the drop across R and X in series is proportional to the line drop. The ends of the circuit are also taken to the pressure transformer and the operating coil of the relay, with the result that the actual voltage across this operating coil is equal to the difference of the line voltage at the potential transformer and the line drop.

An extension of the above is the control of a feeder to compensate for the load drop between two distribution centres and

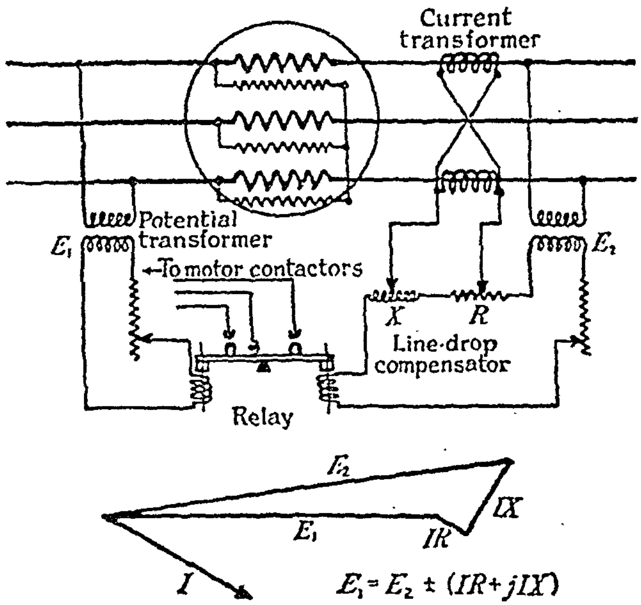


Fig. 13.15.—Automatic Control by Differential Relay.
(*English Electric Co., Ltd.*)

maintain the same voltage at either end irrespective of the supply voltage. This is accomplished by means of a differential relay supplied from two potential transformers and a line-drop compensator, as shown in Fig. 1315. The incoming volts plus the line-drop volts are balanced against the outgoing voltage and thus provide the necessary boost according to the load.

Another application of the induction regulator is to the operation in parallel of a number of regulated feeders. This does not involve the use of twin regulators if means are provided for maintaining the voltage phase angle the same in all the feeders. Where mechanical coupling of the regulators is possible the problem is automatically solved, but such mechanical coupling is often impossible, because of site limitations and necessity for future extensions. Generally, it is desired to balance the loads on feeders irrespective of the angular position of the regulator. Where line-drop compensation is used, this load-balancing may be roughly obtained by connecting the line-drop compensator of feeder 1 to the current transformer of feeder 2, and vice versa. It is pointed out by Ayres that this method is comparatively insensitive, as with 5 per cent. line-drop compensation and voltage relays set for ± 1¼ per cent., one feeder can fall to 75 per cent.

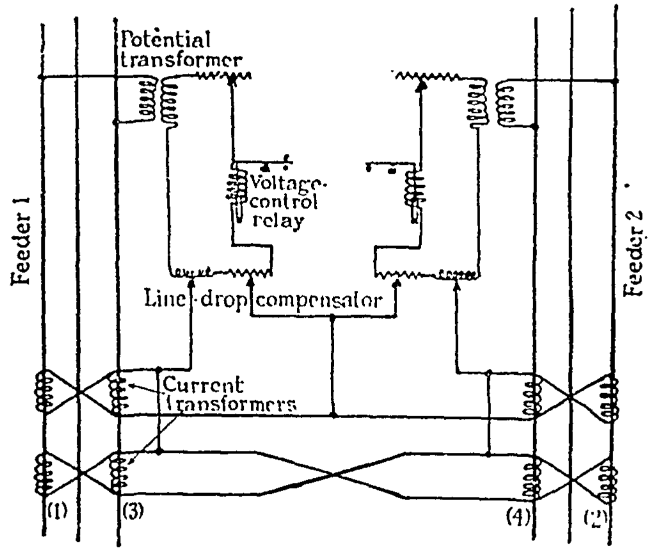


FIG. 13.16.—LOAD BALANCING ON PARALLEL FEEDERS.
(*English Electric Co., Ltd.*)

load and the other rise to 125 per cent. load before there is any tendency to restore balance. He therefore recommends the arrangement shown in Fig. 13.16, in which the induction regulators are omitted for simplicity. The current transformers (1) and (3) in one feeder are connected in series with (2) and (4) in the adjacent parallel feeder. If the currents in the two are balanced, the secondary currents circulate freely, but if one feeder has a higher current the excess secondary current cannot pass through the high impedance of the other transformer and is forced through the line-drop compensators, tending to raise the voltage of the lightly loaded feeder and depress the voltage of the heavily loaded feeder, until balance is restored.

Moving-coil Regulators

The moving-coil regulator, which has been introduced by Messrs. Ferranti Ltd., consists of a transformer core provided with a pair of fixed coils placed at either end of the limb, and a moving coil which can be given any desired position along the limb. One of the fixed coils is tapped and the moving coil is short-circuited, the scheme being as shown in Fig. 13.17. In the position shown, i.e. with the short-circuited coil in the middle of the limb, the fluxes due to the top and bottom coils take the dotted paths and the short-circuited coil has no effect. If this coil is moved

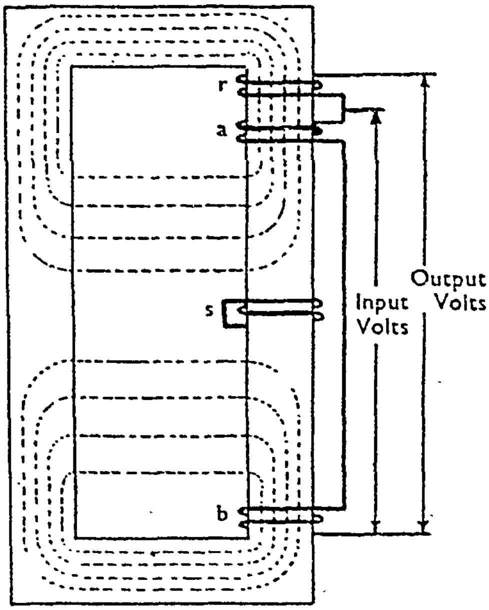


FIG. 13.17.—PRINCIPLE OF THE FERRANTI MOVING-COIL REGULATOR.

up the limb, then it comes within the action of the magnetic field due to the upper coil, and a current will be induced in it which will set up an opposing flux. The net result is a reduction of the flux through the upper coil which, in virtue of its tapping, can be regarded as an auto-transformer, and the consequence will be a reduction of the voltage induced in the portion *r*. As the voltage in this portion is added to the intake voltage to give the output voltage, it follows that such a movement of the short-circuited coil will result in a reduction of the output voltage. If this coil is now moved to the bottom, the flux linking with the coil *b* will be reduced, the effect of which is to transfer the bulk of the intake voltage to the portion *a* of the top coil. Hence, since portions *a* and *r* together constitute a transformer there will be an increase in the induced voltage in *r* and, consequently, an increase in the output voltage. Assuming a turns ratio between *a* and *r* of 10 : 1 and an intake voltage of 100, the range of regulation—boost only—will be from 100·5 to 109·5 volts, as shown in Fig. 13.18. Obviously this range is dependent on the turns ratio between *a* and *r*, and can therefore be given any desired value.

The appliance can be made to buck as well as boost by using a double coil for *b* so that transformer action can take place at this coil, and in order that the induced voltage shall be in apposition,

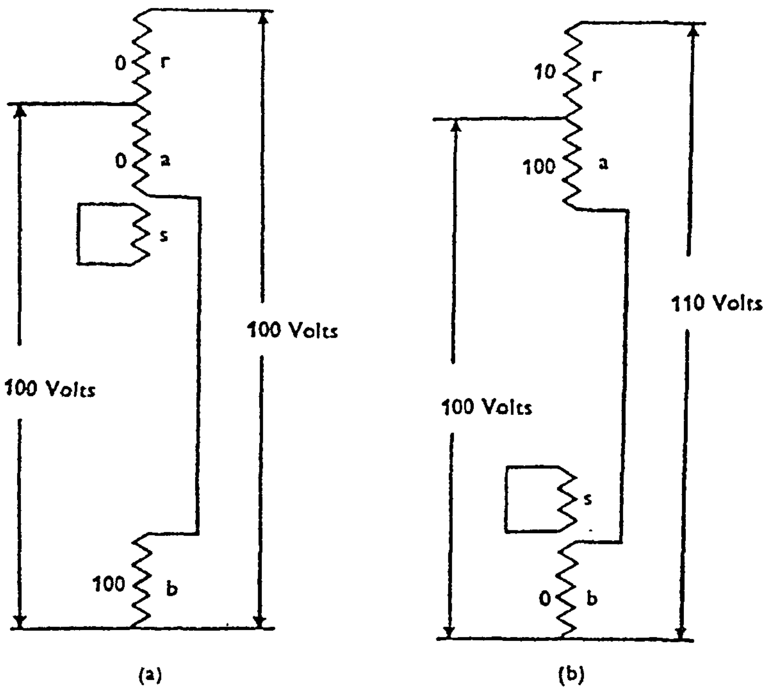


FIG. 13.18.—CONNECTIONS OF REGULATOR FOR BOOSTING.

the coil connections are reversed as shown in Fig. 13.19. With the same turns ratios and the same intake voltage, the figure shows that the output range is now from 91 to 109 volts.

The device, as described above, is a separate piece of apparatus which can be applied at any point in the system. It can also be incorporated with a double-wound transformer.

The regulator has the advantage of infinite variability of boost associated with the induction regulator, with the further advantage that there is no phase shift. This means that it can be utilised on interconnectors and on lines in parallel, the only proviso being that similar regulators shall be installed and that their operation shall be simultaneous. As the voltage variation is affected by the movement, up or down, of a single coil the regulator is very suited to automatic operation, as the power requirements for the moving of the coil are exceedingly small. It can be connected in either

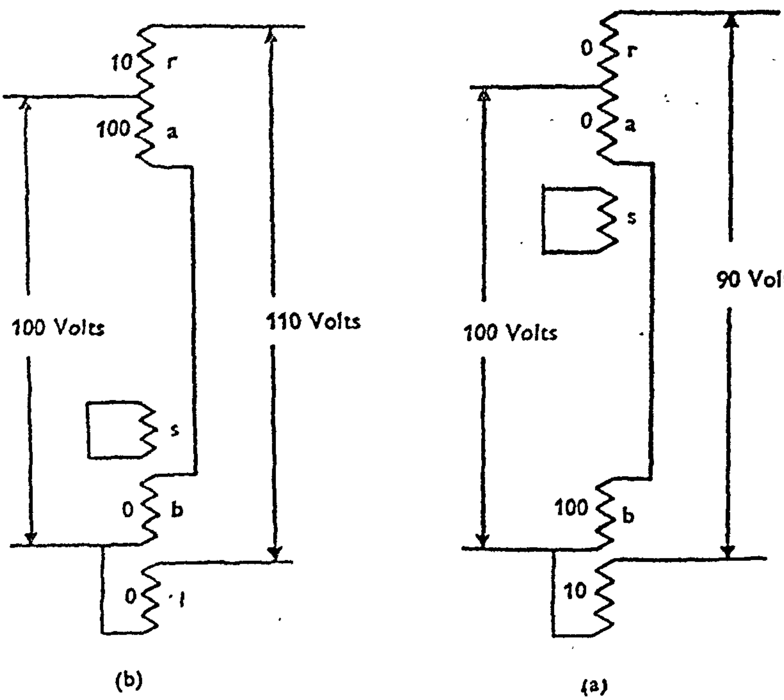


Fig. 13.19.—Connections of Regulator for Bucking and Boosting.

the H.T. or the L.T. side of a system, the usual ranges being from 8 to 20 per cent. on the H.T. and as much as 30 per cent. on the L.T. side.

Boosters

The double-wound transformer with on-load tap-changing gear performs two functions, viz. transforming the voltage, and bucking or boosting the voltage. The booster performs the second of these functions only, and can therefore be regarded as accessory to the double-wound transformer. It can be installed at a substation as an addition to the main transformer if voltage regulation is subsequently found to be necessary, or it can be installed as a separate piece of apparatus at any intermediate point in the line. The booster can take the form of an auto-transformer with tap-changing gear, or it can be a double-wound transformer with secondary in series with the line.

For small outputs and voltages not greater than 2,000, the simplest booster consists of an auto-transformer with the necessary tappings, and a face-plate type switch of the type developed for battery regulation, and with a resistance or a choker connected between the main and auxiliary brushes. Removable carbon tips are provided on the brushes, as with a sliding contact there is inevitable wear due to burning. This type of booster is only used

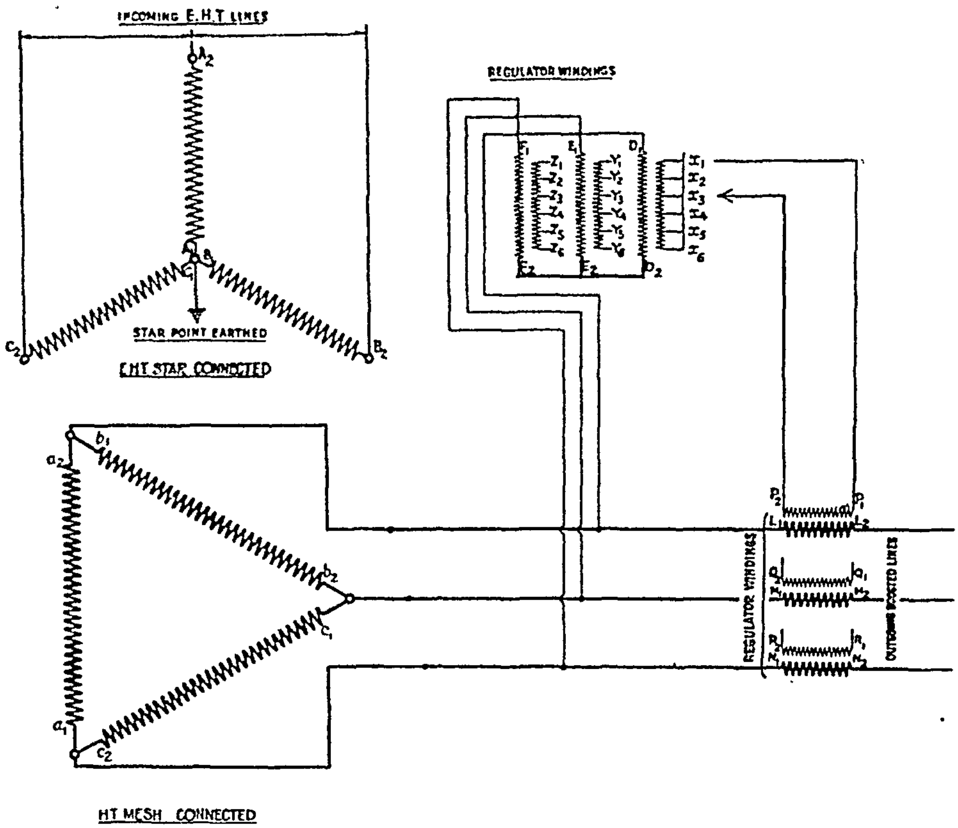


FIG. 13.20.—THREE-PHASE BOOSTER WITH TAP-CHANGING GEAR.

on single-phase circuits and it is often possible to operate the switch in the earthed line, the problem of insulation thus being simple.

For higher voltages and larger sizes it is necessary to utilise on-load tap-changing gear, and also to perform the switching in an isolated circuit, the voltage of which is only a fraction of the line voltage. One method is to energise the primaries of the boosting transformers by means of what is called a " teaser," or regulating transformer, the secondaries of which are provided with tappings, and the necessary tap-changing gear. The main connections of such a scheme are shown in Fig. 13.20, which should be self-explanatory. The voltage changes are made by means of a motor-operated controller (not shown) and arrangements are made to reverse the connections to the primaries of the regulating transformers so that both boost and buck can be obtained. The change from buck to boost is effected by a switch which connects

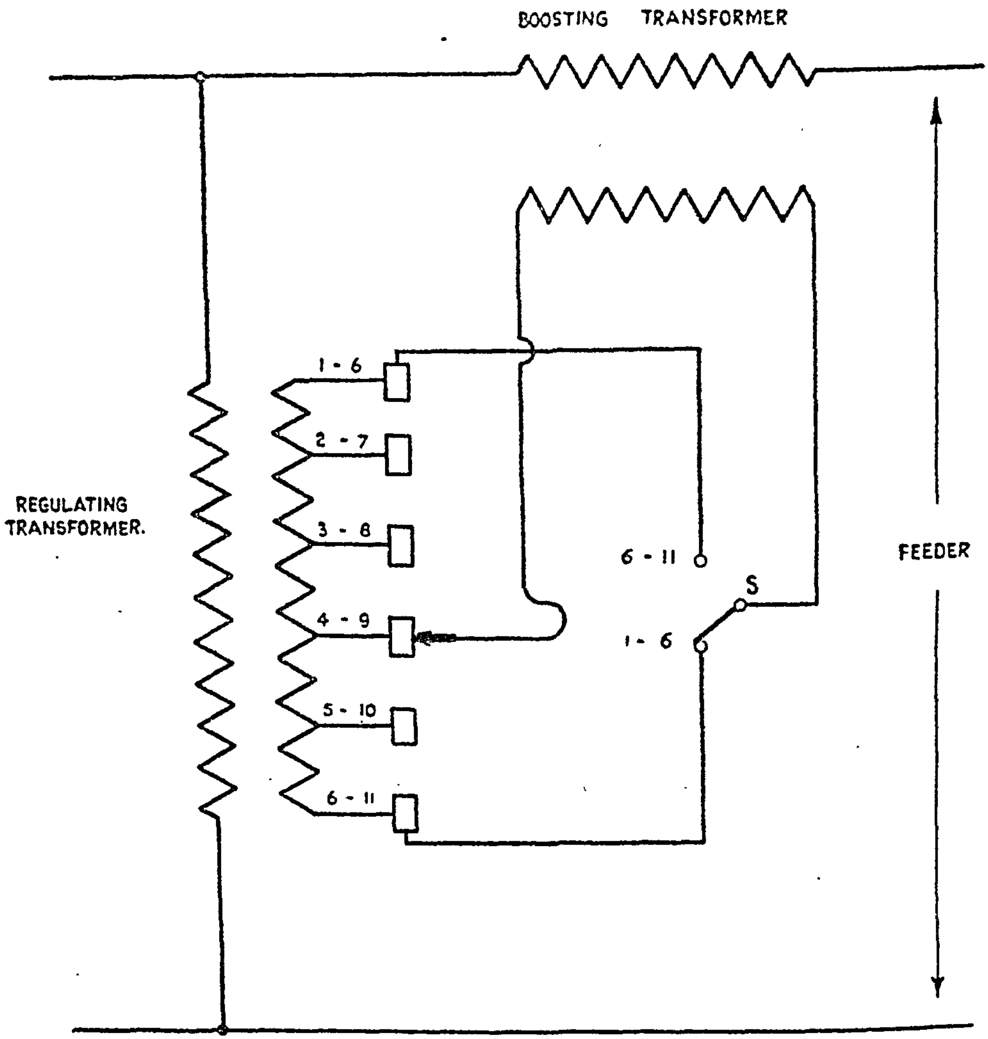


FIG. 13.21.—PRINCIPLE OF THE BOOSTING TRANSFORMER.

one end of the booster primary to either end of the regulating transformer secondary. This switch is shown at S in Fig. 13.21, which gives one phase only for simplicity.

The advantages of the booster can be summarised as follows:

1. Entirely separate from the transformer and therefore applicable to any point in the circuit.

2. When used as an addition to a fixed-ratio transformer the booster can be cut out for inspection or overhaul while the transformer remains in service.

3. The main transformer has no tappings and has no working parts, and is therefore as simple, and reliable, as possible.

The disadvantages of the booster, when used in conjunction with a transformer, are that transformer and booster together are more expensive than a transformer with on-load tap-changing gear, they are less efficient owing to the losses in the booster, and they take up more floor space.

Special Methods for Low-voltage Circuits

For low-capacity low-voltage circuits, gear of any of the types previously described is prohibitively expensive, and therefore

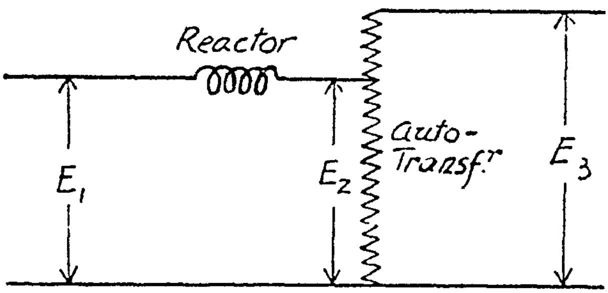


FIG. 13.22.—SELF-COMPOUNDING SINGLE-PHASE BOOSTER.

entirely different methods of voltage control, involving only simple and cheap apparatus, have to be adopted. The circuit diagrams explaining the methods are drawn as for single-phase circuits, but it is to be noted that, so far as this country is concerned, there is now very little single-phase, but a great deal of three-phase four-wire distribution. The principle is, of course, the same.

One method is to employ an auto-transformer with a single tapping only, to have the greater number of turns on the outgoing side, and to connect a reactor in series with the incoming line which is connected to the tapping. The scheme is shown in Fig. 13.22. The manner in which this apparatus effects the voltage regulation is similar to the reactance method of compounding a rotary converter and can be followed by the vector diagrams in Fig. 13.23. In Fig. A, which represents no-load conditions, the no-load current I_0 of the transformer is shown lagging nearly 90 degrees behind the incoming voltage E_1. The drop E_x across the reactor is small, and is in quadrature with I_0, and by deducting

E_x from E, we obtain the transformer primary voltage E_2. The secondary voltage E_3 is, of course, proportional to E_2. Fig. B represents the conditions under a load of power factor approaching unity. The primary current I will be nearly in phase with E_1 and the reactor drop E_x will now be increased. But, owing to the change in power factor, E_x will be nearly in quadrature with E_1 so that, when deducted from E_1 as before, the difference E_2, and therefore E_3, is much greater than before. By suitably choosing the turns ratio and the impedance of the reactor, the increase in

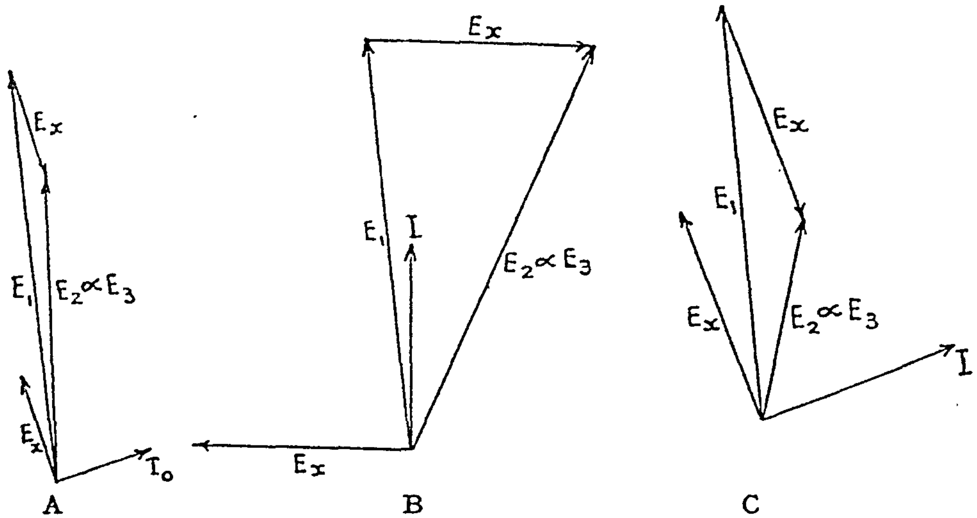


FIG. 13.23.—VECTOR DIAGRAM OF SELF-COMPOUNDING BOOSTER.

E_3 can be given any desired value. Fig. C shows that this method is only applicable to power factors in the neighbourhood of unity, for if the power factor is low, then E_x becomes more nearly in phase with E_1, and the vector difference of E_1 and E_x decreases. As it happens, the major part of the load on the lines for which such a method is suitable is domestic and therefore of high power factor, and, in practice, the method is limited to lines in which the power factor is not less than 0·95 lagging. The maximum amount of boost which can be obtained is of the order of 6 per cent. The regulator can be connected at any point in the distributor, either the supply end or at some intermediate point, depending on the length of the distributor and the nature of the loading.

Power-factor Control

As pointed out previously, the voltage at the receiving end of a line can be controlled by means of a synchronous condenser whose wattless leading kVA. intake is automatically varied by the variation of its excitation, according to the load on the line. The efficiencies of these machines are very high, representative values of the losses as a percentage of the kVA. rating being about 6 per cent. for 500 kVA. machines and less than 4 per cent. for 2,000 kVA. machines. It is therefore justifiable to assume that the current taken by the machine leads the applied voltage by 90 degrees. For simplicity, consider a line having ohmic resistance and inductive reactance only, and work in terms of the voltage to neutral, E_p, which we will assume is to be the same at both ends of the line.

Let $\quad I_l$ = load current at a P.F. of $\cos \varphi$

and $\quad I_m$ = synchronous condenser current.

It is convenient to split I_l into in-phase and quadrature components, viz.:

$I_a = I_l \cos \varphi$, the in-phase component

and $\quad I_b = I_l \sin \varphi$, the quadrature component.

The three currents I_a, I_b, and I_m produce resistance and reactance drops per phase of I_aR, I_aX; I_bR, I_bX; and I_mR, I_mX; the resistance drops being in phase with, and the reactive drops in

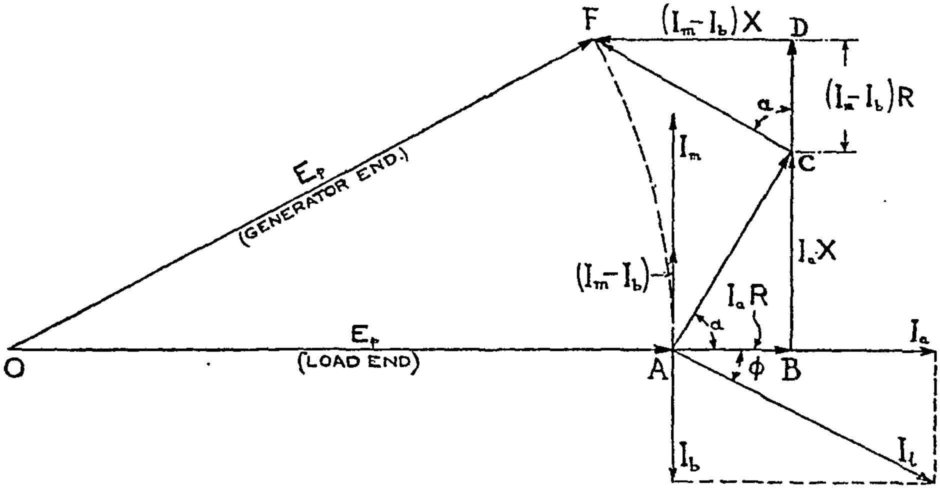


FIG. 13.24.—ILLUSTRATING POWER-FACTOR CONTROL.

quadrature leading with, the corresponding currents. The vector diagram is as shown in Fig. 13.24. Note that since I_m and I_b are in direct opposition and that I_m must be greater than I_b, the four drops due to these two currents simplify to

$(I_m - I_b)R$ in phase with I_m

and $\quad (I_m - I_b)X$ in quadrature leading with I_m.

Since the lengths OA and OF are equal, we have from

$$OF^2 = (OA + AB - DF)^2 + (BC + CD)^2$$

$$\therefore\ E_p^2 = \{E_p + I_a R - (I_m - I_b) X\}^2 + \{I_a X + (I_m - I_b)R\}^2$$

This is an equation with only one unknown, viz. I_m, and its solution therefore gives the necessary wattless leading current to be taken by the synchronous condenser. Finally,

$$\text{Wattless leading kVA. capacity of condenser} = \frac{3E_pI_m}{1000} \text{ or } \frac{\sqrt{3}EI_m}{1000}$$

where E is the line voltage.

Example

A load of 12,000 kW. at power factor 0·8 lagging is supplied by a three-phase line whose voltage has to be maintained at 40,000 at each end. If the line resistance and reactance per phase are 5·5 ohms and 10 ohms respectively, calculate the capacity of the necessary synchronous condenser.

$$I_l = \frac{12000 \times 1000}{\sqrt{3} \times 40000 \times \cdot 8} = 217 \text{ amps.}$$

$$\therefore\ I_a = 217 \times \cdot 8 = 174 \text{ amps.}$$

$$\text{and}\ I_b = 217 \times \cdot 6 = 130 \text{ amps.}$$

$$E_p = 40000/\sqrt{3} = 23100 \text{ volts}$$

$$R = 5\cdot 5 \text{ and } X = 10$$

$$\therefore\ 23100 = \sqrt{\{23100 + 174 \times 5\cdot 5 - (I_m - 130) \times 10\}^2 + \{(174 \times 10) + (I_m - 130) \times 5\cdot 5\}^2}$$

the solution of which is

$$I_m = 234 \text{ amps.}$$

Hence, wattless leading reactive kVA. of the synchronous condenser:

$$\frac{3E_pI_m}{1000} = \frac{3 \times 23100 \times 234}{1000} = 16200$$

The kVA. capacity of the condenser increases rapidly as the power factor of the load decreases, and the following table gives the values of I_m and kVA. for different power factors, the kVA. of the load, and therefore the load current I_l, being the same in all cases.

P.F. of Load.	I_m	kVA. of Synchronous Condenser.
1·0	108	7,500
·8	234	16,200
·6	339	23,500
·4	506	35,000

In the example we see that the kVA. capacity of the synchronous condenser is considerably greater than the kVA. of the load, viz. 16,200 against 12,000/·8, i.e. 15,000. Owing to the high cost of the condenser and its control gear this would not be an economical proposition, and in practice it would be preferable to allow a small line drop. For example, suppose we allow a line drop of 1,000 volts, thus requiring a voltage of 41,000 at the sending end, then in Fig. 13.24 the length OF will be $41000/\sqrt{3}$, i.e. 26,800 instead of 23,100. The equation for I_m with I_l at the full value of 217 amps, and power factor 0·8 is thus

$$23600 = \sqrt{\{23100 + 174 \times 5{\cdot}5 - (I_m - 130) \times 10\}^2 + \{174 \times 10 + (I_m - 130)5{\cdot}5\}^2}$$

the solution of which is

$$I_m = 95 \text{ amps. and kVA.} = 6580$$

We see that this gives a considerable reduction in the kVA., and this solution would, in consequence, be more acceptable than the former as the line drop is only 1,000 in 41,000 volts.

The variation in condenser kVA. with load current—assuming the load power factor to remain constant—can be calculated in the above manner, or it can be determined graphically as follows. The total vectorial drop in the line is the sum of the drops due to the load current I_l and the current I_m. Each of these has component resistance and reactance drops, and the drop triangles due to I_l and I_m are similar, their base angle being given by

$$\alpha = \text{arc tan}\,\frac{X}{R} = \text{arc tan}\,\frac{10}{5{\cdot}5} = 61^\circ\,12'$$

Hence, draw OA, Fig. 13.25, equal to $40000/\sqrt{3}$ volts to represent the phase voltage at the receiving end, and with O as centre strike an arc with radius $41000/\sqrt{3}$, the phase voltage at the sending end. Draw the line drop triangle $ABC_{4/4}$ for full-load line current of 217 amps., and from $C_{4/4}$ draw a line $C_{4/4}D_{4/4}$ inclined α (= 61° 12′) to the vertical to cut the circle at $D_{4/4}$. Then the length $C_{4/4}D_{4/4}$ represents the additional line drop due to the current I_m in the synchronous condenser and we have, for full load

$$I_m = 217 \times \frac{C_{4/4}D_{4/4}}{AC_{4/4}}$$

At three-quarters of full load the line drop will be $AC_{3/4}$, and for the corresponding value of the synchronous condenser current we have

$$I_m = 217 \times \frac{C_{3/4}\,D_{3/4}}{AC_{3/4}}$$

and similarly for other fractions of the full load. After the circle cuts the hypotenuse $AC_{4/4}$, the value of I_m will be negative, which

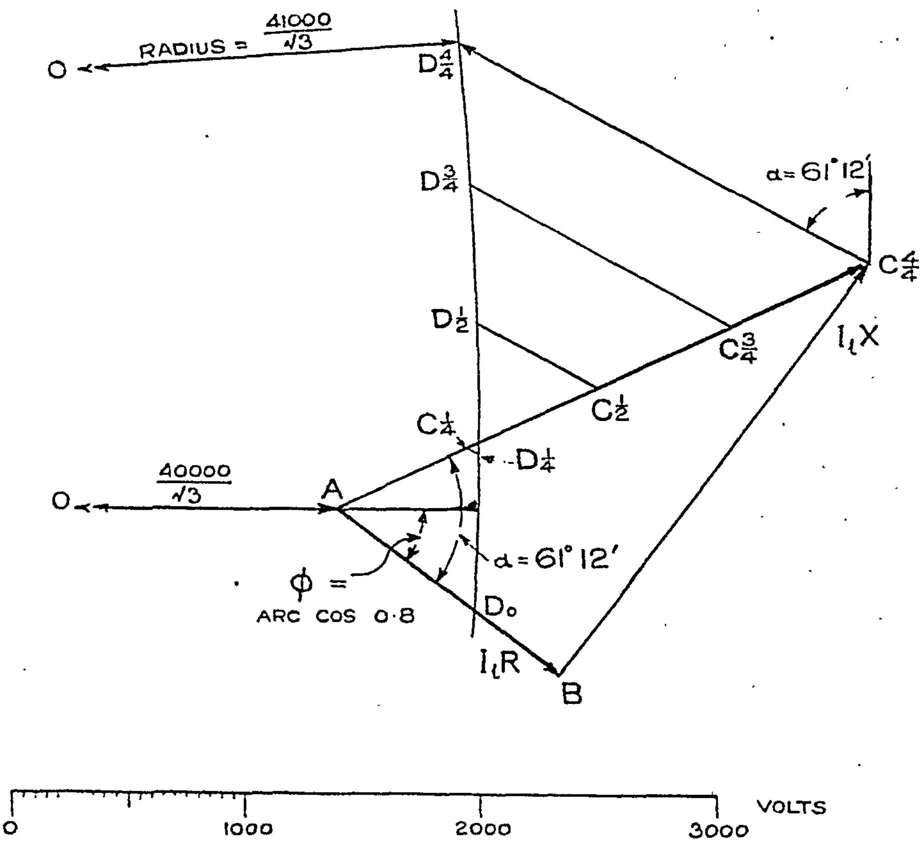


FIG. 13.25.—POWER-FACTOR CONTROL.

means that the synchronous condenser will have to supply a lagging, instead of a leading, current. Fig. 13.25 is drawn to scale and from it we have:

Load, as a Fraction of Full Load.	I_m.
$\frac{4}{4}$	169·5
$\frac{3}{4}$	147·3
$\frac{2}{4}$	102·2
$\frac{1}{4}$	— 21·7

Circle Diagram

For a comparatively short line the above elementary discussion is sufficiently accurate for practical purposes as the capacitance of the line is very small. With a long high-voltage line, the line capacitance can no longer be neglected, and its effect is to reduce the wattless leading kVA. output of the synchronous condenser at full load, but to increase its wattless lagging kVA. output at light loads, as shown in Chapter IV. Any line, including terminal transformers, if any, can be reduced to a simple π line as indicated in Fig. 13.26. Woodruff designates the element of this equivalent line as the architrave, the sending-end, and the receiving-end, or

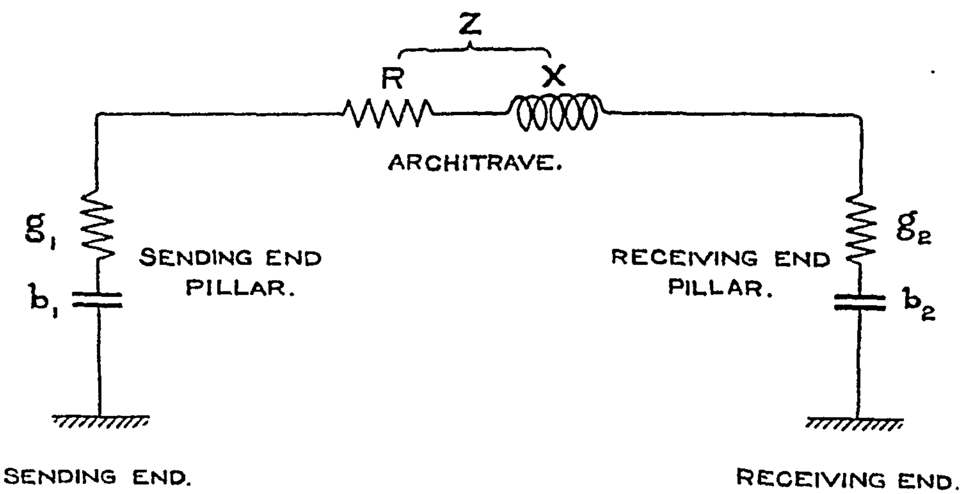


FIG. 13.26.—EQUIVALENT π OF A TRANSMISSION NETWORK.

the load-end, pillar. For a line without terminal transformers the constants of the two pillars will be the same, but if there are transformers either at one end only or at both, then the two pillars will, in general, be different. Referring everything to the phase voltage at the receiving end we can write

Receiving-end voltage	$E_{p.2} = x + j \times 0$
Sending-end voltage	$E_{p.1} = x' + j \times y'$
Load current, plus synchronous condenser current	$I_l = x'' + j \times y''$
Architrave current	$I_a = x''' + j \times y'''$

The sending-end voltage is constant in magnitude, but it will vary in phase with respect to the receiving-end voltage. The load current and the architrave current are, of course, both variable.

Now, obviously

$$E_{p.1} = E_{p.2} + ZI_a \text{ (vector sum)}$$

$$\begin{aligned} \therefore\ x' + jy' &= x + (R + jX)(x''' + j\cdot y''') \\ &= x + Rx''' + jRy''' + jXx''' - Xy''' \\ &= (x + Rx''' - Xy''') + j(Ry''' + Xx''') \end{aligned}$$

The square of the left-hand side is $E_{p.1}^2$, numerically

$$\begin{aligned} \therefore\ E_{p.1}^2 &= (x + Rx''' - Xy''')^2 + (Ry''' + Xx''')^2 \\ &= x^2 + R^2x'''^2 + X^2y'''^2 + 2Rxx''' - 2Xxy''' - 2RXx'''y''' \\ &\quad + R^2y'''^2 + X^2x'''^2 + 2RXx'''y''' \end{aligned}$$

Substituting $E_{p.2}$ for x, as we are now dealing with numerical values only, we have

$$E_{p.1}^2 - E_{p.2}^2 = x'''^2(R^2 + X^2) + y'''^2(R^2 + X^2) + 2E_{p.2}\,x'''R - 2E_{p.2}\,y'''X$$

Dividing by $Z^2(= R^2 + X^2)$ and rearranging, we have

$$\frac{E_{p.1}^2 - E_{p.2}^2}{Z^2} = \left(x'''^2 + \frac{2E_{p.2}x'''R}{Z^2}\right) + \left(y'''^2 - \frac{2E_{p.2}\,y'''X}{Z^2}\right)$$

We can complete the squares of the two expressions in the bracket by adding: $E_{p.2}^2\,R^2/Z^4$ and $E_{p.2}^2\,X^2/Z^4$ respectively, giving

$$\frac{E_{p.1}^2 - E_{p.2}^2}{Z^2} + \frac{E_{p.2}^2}{Z^4}(R^2 + X^2) = \left(x'''^2 + \frac{E_{p.2}R}{Z^2}\right)^2 + \left(y'''^2 - \frac{E_{p.2}X}{Z^2}\right)^2$$

$$\therefore\ \frac{E_{p.1}^2}{Z^2} = \left(x'''^2 + \frac{E_{p.2}R}{Z^2}\right)^2 + \left(y'''^2 - \frac{E_{p.2}X}{Z^2}\right)^2$$

This is the equation to a circle of radius $E_{p.1}/Z$, whose centre is defined by

$$x''' = -\frac{E_{p.2}R}{Z^2};\quad y''' = +\frac{E_{p.2}X}{Z^2}$$

Again, the receiving-end pillar carries a constant current, owing to the voltage $E_{p.2}$ being constant. But the load current is the difference of the architrave current and the receiving-end pillar current $E_{p.2}(g_2 + jb_2)$, with the result that the load current is represented by a circle of the same radius as the architrave current, but with its centre displaced $(-E_{p.2}\,g_2;\ -E_{p.2}\,b_2)$ with respect to the centre of the circle of architrave current, giving the co-ordinates

$$-E_{p.2}\left(\frac{R}{Z^2} + g_2\right);\quad +E_{p.2}\left(\frac{X}{Z^2} - b_2\right)$$

By taking a series of values of $E_{p.2}$ we can draw a family of circles, and we see that, although the co-ordinates of the centres of the various circles will be different, their radii will all be the same, viz. $E_{p.1}/Z$.

Example

$$E_1 = 132000 \quad \therefore \; E_{p.1} = \frac{132000}{\sqrt{3}} = 76300$$

$$\text{Architrave impedance } Z = 45 + j \times 250$$

Receiving-end pillar admittance $Y_2 = 1{\cdot}2 \times 10^{-5} + j \times (60 \times 10^{-5})$

Then the radius of each circle is given by

$$\frac{E_{p.1}}{Z} = \frac{76300}{\sqrt{45^2 + 250^2}} = \frac{76300}{254} = 300 \text{ amps.}$$

$$\frac{R}{Z^2} + g_2 = \frac{45}{64530} + 1{\cdot}2 \times 10^{-5} = 71{\cdot}7 \times 10^{-5}$$

$$\frac{X}{Z^2} + b_2 = \frac{250}{64530} - 60 \times 10^{-5} = 32{\cdot}8 \times 10^{-4}$$

Hence, taking a series of values of $E_{p.2}$ of 76,300, 76,000, 75,000, 74,000, 73,000, and 72,000, we have for the co-ordinates of the centre :

$E_{p.2}$	$-E_{p.2}\left(\frac{R}{z^2} + g_2\right)$	$E_{p.2}\left(\frac{X}{z^2} - b_2\right)$
76,300	− 54·5	250
76,000	− 54·2	249
75,000	− 53·8	246
74,000	− 53	243
73,000	− 52·1	239·9
72,000	− 51·5	236
70,000	− 50·2	229·6

The circles are drawn to scale in Fig. 13.27, and the diagram is used as follows :

Suppose that the load current is one of 100 amps at a power factor of cos φ = 0·8 lagging, then a current vector OA is drawn, of length 100 amps. and inclined φ = arc cos 0·8 to OX as shown. A vertical ABC is drawn from A, its intersections with the various circles giving the leading current I_m to be taken by the synchronous condenser. Thus for a receiving-end voltage of $E_{p.2} = 70{,}000$, the point of intersection is B and the current is $I_m = AB = 19{\cdot}5$ amps. For a voltage $E_{p.2}$ of 76,300, the point is C, and the value of I_m is increased to 42 amps. Similarly with other receiving-end voltages.

If synchronous condensers are not used the diagram gives the maximum load current which can be delivered for a given receiving-end voltage. Thus, at a load power factor of 0·8, this current is OF $= 94{\cdot}8$ amps. for $E_{p.2} = 70{,}000$, and OD $= 64{\cdot}8$ amps. for $E_{p.2} = 76{,}300$. Similarly for other power factors.

Frequently the constants of the diagram are multiplied by $\frac{3}{1000} \times E_{p.2}$, in which case the vector OA will be the load kVA., ordinates will be in reactive kVA., and abscissæ in kW.

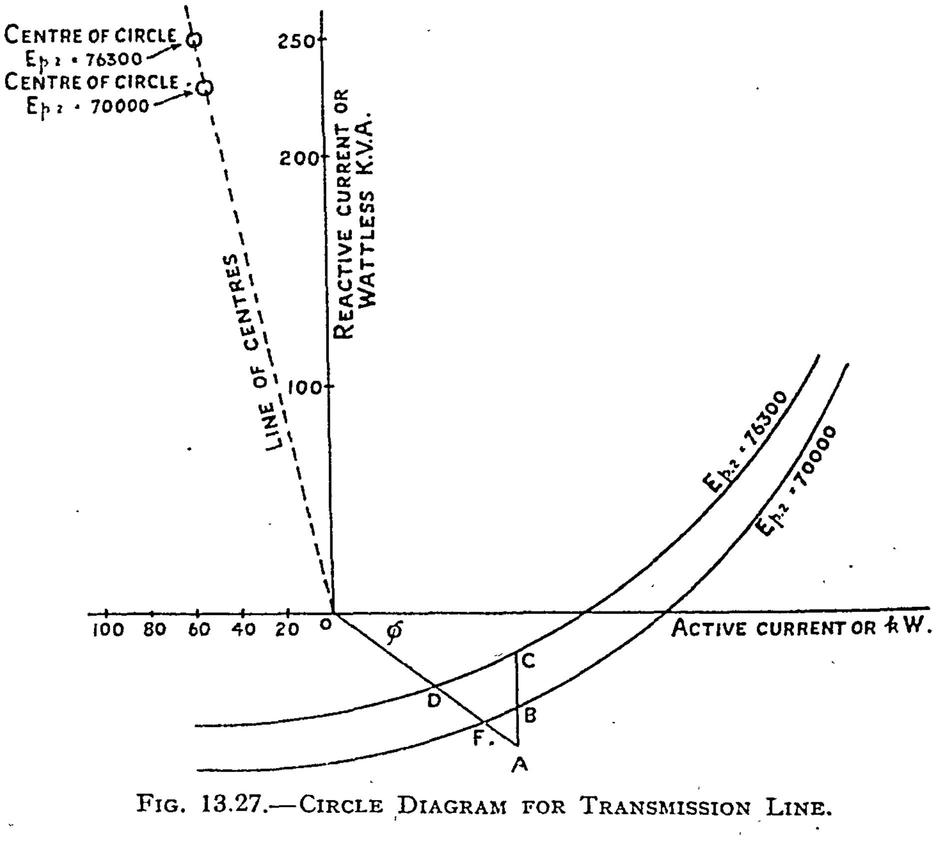


FIG. 13.27.—CIRCLE DIAGRAM FOR TRANSMISSION LINE.

It is also apparent from the diagram that the use of a synchronous condenser not only improves the line regulation but enables a given line to carry a much greater load than would otherwise be possible. Thus a line can be designed from a purely economical point of view, i.e. irrespective of line drop, and synchronous condenser plant installed to keep the resultant drop within practical limits. Where there is no synchronous condenser plant but there are several lines radiating from a station it is sometimes the practice to utilise the capacitance of an idle line to improve the regulation, but obviously the applications of such a method are decidedly limited.

The nature of the performance of this class of machine can be gathered from the following figures which refer to a 20,000-kVA. synchronous condenser :

Iron loss	66 kW.
Friction and windage	83 kW.
Field copper	55 kW.
Exciter	6 kW.
Stator copper	68 kW.
Total	278 kW.

The total loss is thus only 1·4 per cent. of the kVA. rating. The machine has six poles, the speed thus being 1,000 r.p.m., and in order to dispense with pole-face starting windings, the pole faces are solid. The method of starting is by applying a reduced voltage to the stator, the changes in voltage being effected by tap-changing gear of the type previously described. In order to maintain the line voltage constant during variations in load, it is necessary that the excitation shall be capable of adjustment over a wide range, and that this adjustment shall be effected by changes in voltage. The regulator operates on the principle of a set of contacts alternately inserting and short-circuiting a resistance in the exciter shunt field circuit, as in the well-known Tirrill automatic voltage regulator as used with alternators. The regulator, as shown in Fig. 13.28, has two sets of contacts, one set connected from the " all-in " end of the rheostat to the slider, and the other set connected from the slider to a point near the " all-out " end of the rheostat. The former set of contacts is normally closed, and the latter normally open. At any one time one set only is vibrating, the other set being in its normal position, the operating advantages of this arrangement are as follows: (1) Smoother control, since the full exciter field rheostat is not under the control of one set of contacts, and variations of the field current above and below its average value is less than with the older types of regulator. (2) A wide range of control is possible, enabling rapid response to be obtained to load changes, a feature which is essential for the stable operation of large interconnected systems. (3) On constant load the slider can be adjusted to the correct position, thus throwing both sets of contacts out of operation, with resulting increase in their life.

A special feature of the regulator is the use of a small shaded pole induction motor driving a cam to impart a vibrating motion to the small contacts controlling the relay which operates the main contacts. This ensures constant frequency and amplitude of vibration, resulting in consistent and reliable operation of the regulator. The cam is made of insulating material.

Interconnectors

An interconnector, or tie line, is a cable or overhead line connecting two generating stations so that one of the stations can

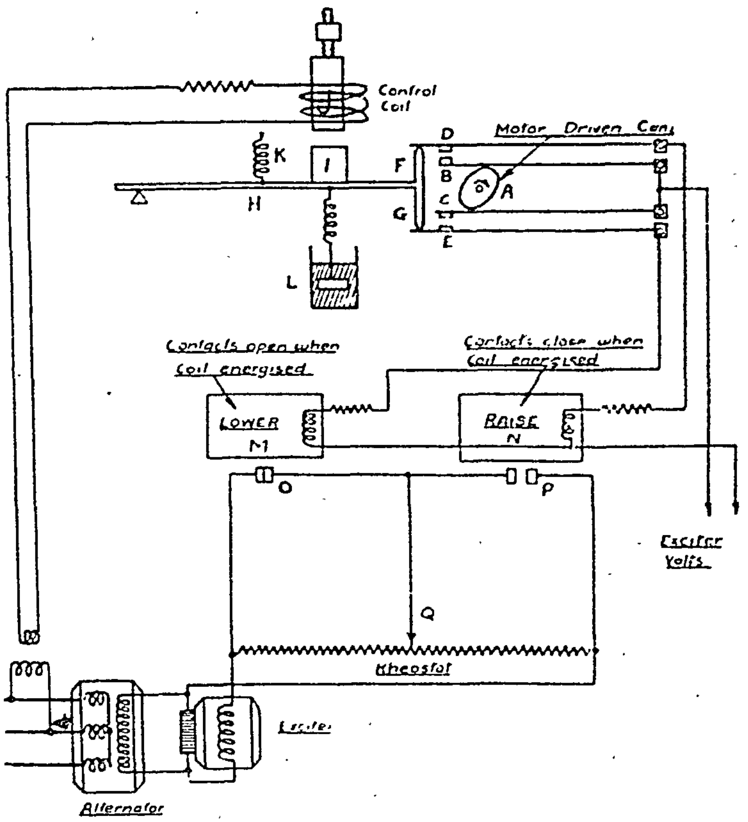


FIG. 13.28.—AUTOMATIC REGULATOR FOR SYNCHRONOUS CONDENSER
(*Metropolitan-Vickers Electrical Co., Ltd.*)

take over a portion of the load on the other. Power may flow in either direction according to the desired division of the total load between the two stations. Thus each section of a ring main can be regarded as an interconnector. The advantages of interconnection are both technical and financial, and may be summarised as follows: In the first place, peak loads can be reduced and therefore plant load factors improved, since heavy overloads can be distributed over the system and the loads at the various power stations kept fairly steady. Any small generating stations can be used as stand-by stations for operation during the hours of peak load, the advantage of this being that small stations have, as a rule, higher running costs than large stations. In countries where there are both steam and hydro-electric stations linked together, it is possible to draw most of the power from the most economical source of supply, at any time of the day or at any season of the year, and to distribute the load so that the whole of the available water supply is utilised. In this way it has been possible to utilise the water supply of some variable regime rivers which, without interconnection, could not have been developed as a paying proposition. It is also possible for one station to deal with the whole of the wattless kVA. of the load, so that the stations supplying the true kW. will be running with power factors in the neighbourhood of unity, and therefore under the most favourable conditions.

An interconnector is essentially a means of enabling two generating stations to operate in parallel, and as the principles are essentially the same as for two alternators in parallel we will briefly review this problem first. When two D.C. generators are paralleled any desired division of load can be obtained by the simple adjustment of excitation, since the current delivered by either machine is proportional to the difference between its induced and terminal voltages. Also, since the speeds of the two machines need not be identical, the governors can automatically take care of the change in load imposed by the new conditions of excitation. With two alternators in parallel the speeds are of necessity identical and the division of the kW. load is therefore unrelated to the excitations (except in a minor degree) but is dependent on the amounts of steam admitted to the prime movers. In other words, the required division of kW. loading is obtained by direct adjustment of the stop valves. If the excitation of one machine is raised, it will not cause that machine to take more of the total kW. load but will produce a wattless circulating current which will lower its own power factor, at the same time raising the power factor of the other machine. The power factors of the alternators, or in other words, the amounts of wattless kVA. which they deliver, are thus controlled by the excitation.

Consider first of all two identical alternators I and II, Fig. 13.29A, with identical steam admissions and identical excitations. Then they divide both the working component $2I_a$ and the wattless component $2I_b$ of the load current I equally between them, and they each work at the same power factor as the load (neglecting the effect of the internal impedance). If we draw a vertical vector,

Fig. 13.30A, to represent the kW. of the load and a horizontal to represent the wattless kVA., then the base angle gives the load phase angle. Each machine carries 50 per cent. of both kW. and wattless kVA. loading, the shaded triangles in the figure thus representing the conditions in the two machines.

Now suppose that the steam admissions are unaltered so that each machine takes one-half of the kW. loading as before, but let the excitation of No. I be greater than that of No. II. Then the

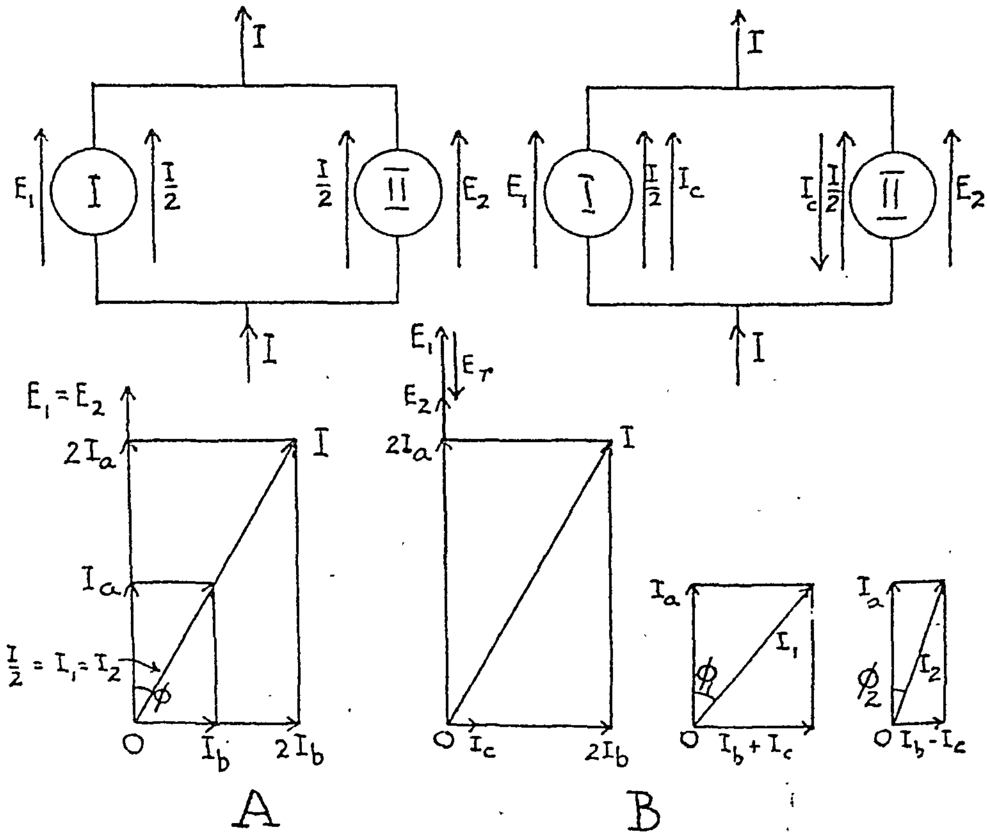


FIG. 13.29.—OPERATION OF ALTERNATORS IN PARALLEL.

induced E.M.F. E_1 will be greater than E_2 with the result that the resultant E_r $(= E_1 - E_2)$ will set up a circulating current I_c round the armatures and bus-bars. This current I_c will be nearly wattless, owing to the very small armature resistance, and it will be added to the load wattless current in machine I, and subtracted from that in machine II. The conditions are shown in Figs. 13.29B and 13.30B, and we see that the power factor of machine I will now be lower than that of machine II. This shows that the division of the load kVA. between the two machines can be given any desired value by an adjustment of the difference between the two excitations, and by giving this difference the appropriate value one machine can, if desired, be operated at unity power factor.

The third case is given by a difference in the steam supplies, the excitations being the same. In this case the machine with the greater steam supply, say No. I, must take the greater portion of the kW. loading, and this is accomplished by a forward rotation of E_1 with respect to E_2, as shown in Fig. 13.31. The resultant E_r is now practically in quadrature with both E_1 and E_2, so that the circulating current I_c is, with respect to the local circuit, almost in

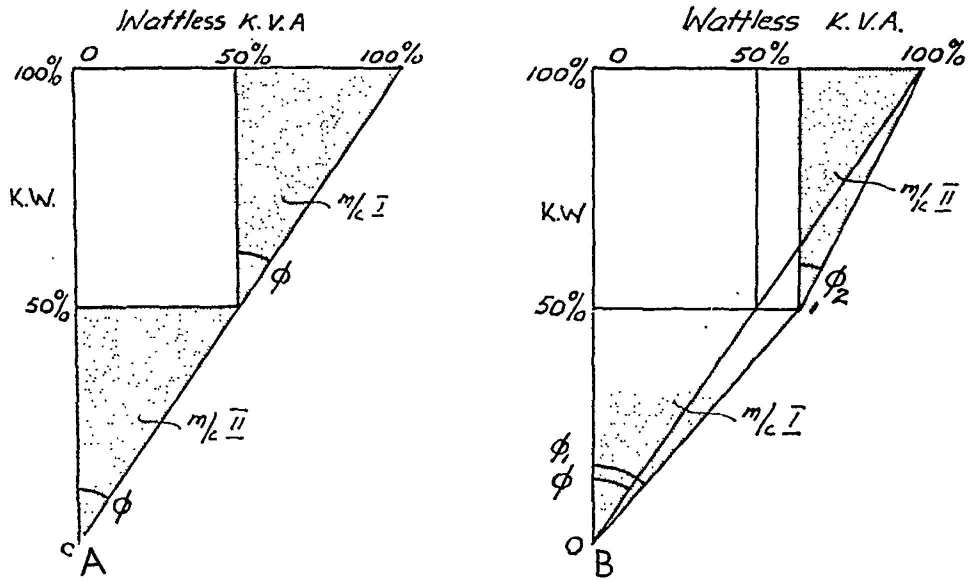


FIG. 13.30.—DIVISION OF LOAD.

phase with E_1 and in phase apposition to E_2. Hence the current I_c is added to the one-half of the useful component of the load current carried by No. I, and deducted from that carried by No. II. The conditions in the two machines are represented by the vector diagrams in Fig. 13.31 and by Fig. 13.32.

Summarising, we see that (*a*) the division of the power component, or kW., between the two machines is effected by an adjustment of the steam supplies, while (*b*) the division of the wattless kVA. is effected by adjustment of the excitations. The problem of the interconnector is essentially the same except that in the case of alternators in parallel the bus-bars can be regarded as of zero resistance and reactance, while with an interconnector there are both resistance and reactance present. Again, each station will supply its own local load, independent of the interconnector, so that a condition of operation is that the station voltages shall each be kept constant. Consequently, voltage regulators are required for the interconnector to give the necessary amount of

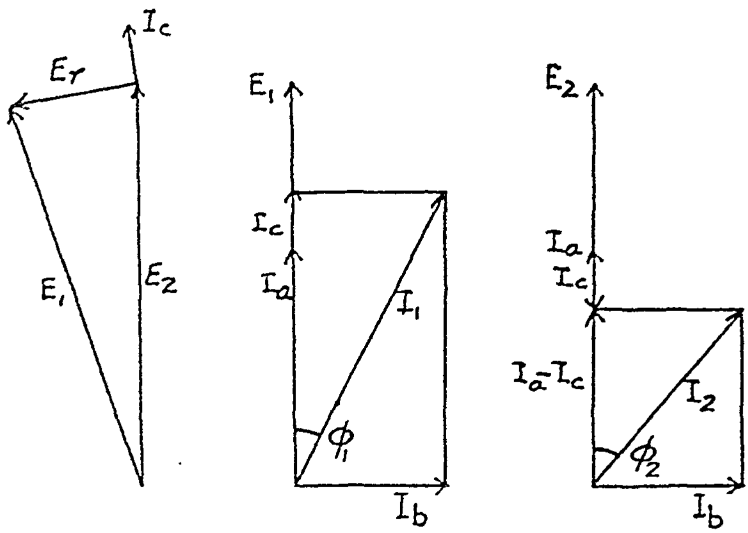


FIG. 13.31.—VECTOR DIAGRAM OF TWO ALTERNATORS IN PARALLEL.

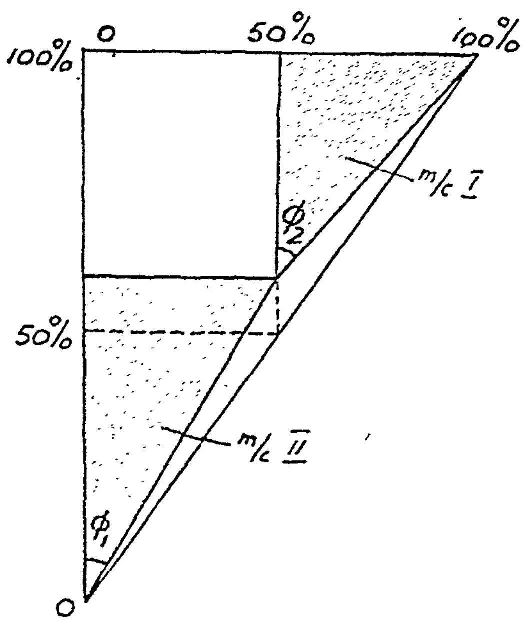


FIG. 13.32.—DIVISION OF LOAD.

boost or buck to supply the necessary line drop between I and II. Let station I supply a current of I amps. to station II along the interconnector, the power factor at the receiving end being cos φ.

Then the vector diagram is as shown in Fig. 13.33, and we have:

$$\text{In-phase voltage drop} \quad AN = RI\cos\varphi + XI\sin\varphi$$
$$\text{Quadrature voltage drop } CN = XI\cos\varphi - RI\sin\varphi$$

Let the percentage impedance be $(r + jx)$, then with full-load current flowing:

$$\text{In-phase voltage drop} = (r\cos\varphi + x\sin\varphi) \text{ per cent.}$$
$$\text{and quadrature voltage drop} = (x\cos\varphi - r\sin\varphi) \text{ per cent.}$$

Assume that the interconnector is carrying its full-load current but that the power factor, $\cos\varphi$, at the receiver end is varied over

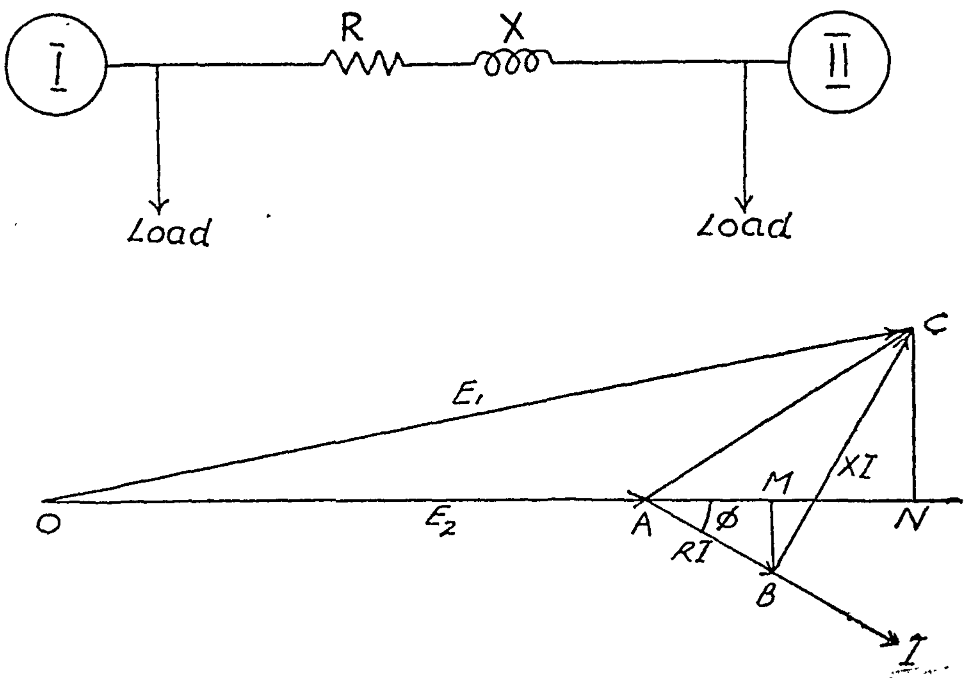


FIG. 13.33.—REGULATION OF AN INTERCONNECTOR.

wide limits, then the above expressions will give the voltage drop and the phase shift respectively since, very approximately, $(r\cos\varphi + x\sin\varphi)$ is the percentage voltage drop, while $(x\cos\varphi - r\sin\varphi)$ gives the length CN, and therefore the angle through which E_2 is rotated with respect to E_1. As a numerical example suppose that r and x are 1 per cent. and 10 per cent. respectively, then

$$(r\cos\varphi + x\sin\varphi) = \cos\varphi + 10\sin\varphi$$

and
$$(x\cos\varphi - r\sin\varphi) = 10\cos\varphi - \sin\varphi$$

The performance can therefore be calculated in tabular form as follows:

Cos ϕ.	ϕ	% drop. Cos ϕ + 10 sin ϕ	% phase shift. 10 cos ϕ − sin ϕ
·9 lead	− 25° 50′	− 3·457	9·44
·995 lead	− 5° 43′	0	10·05
1·0	0°	1·0	9·4
·95 lag	18° 12′	4·073	9·19
·925 ,,	22° 20′	4·725	8·87
·9 ,,	25° 50′	5·3	8·6
·8 ,,	36° 52′	6·8	7·4
·6 ,,	53° 8′	8·6	5·2
·2 ,,	78° 28′	10·0	1·02
·0995 ,,	84° 17′	10·05	0

Starting from the bottom of the table we deduce the following conditions of operation:

(1) With a P.F. of 0·0995, i.e. practically 0·1 or 10 per cent., we see that if the steam supplies at the two stations are so adjusted that the interconnector is fully loaded on a kVA. basis but only carries 10 per cent. of the maximum possible kW. load, and at the same time the voltage boost at station I is adjusted to 10·05, then there will be no phase shift and E_1 and E_2 will be in phase. Thus, if the voltage boost is effected by on-load tap-changing gear with the taps on the usual windings—in-phase tap-changing—then not more than 10 per cent. of the maximum possible kW. can be transmitted without phase shift.

(2) Referring to a P.F. of 0·995 leading, i.e. a power transmitted of 99·5 per cent. of the maximum possible we see that there is zero voltage drop, but that there is a phase shift of 10·05 per cent. In other words, if quadrature boost controlled by tap-changing gear were available then the interconnector could deliver practically the whole of its possible kW. loading with such a phase shift of 10·05 per cent.

(3) At first sight the conditions in case (2) appear desirable, but in practice it is preferred that the power factors of the two stations and of the interconnector shall all be of about the same value. The table shows that the power factor of the load transmitted is dependent on the amount of quadrature boost, and consequently any desired power factor can be obtained by an adjustment of this boost. Although such a boost can be provided, the cost of the necessary apparatus would be considerable, but, fortun-

ately, there is a simpler solution as follows: each main winding of the transformer supplying the interconnector is connected in series with another winding placed on another limb, and it is this auxiliary portion which is tapped. The arrangement is shown in Fig. 13.34. In this way the tap-changing has the double effect of altering the transformation ratio and of shifting the relative phase angle between the primary and secondary open-circuit voltages. Suppose we take the same total boost of 10·05 per cent. as in the

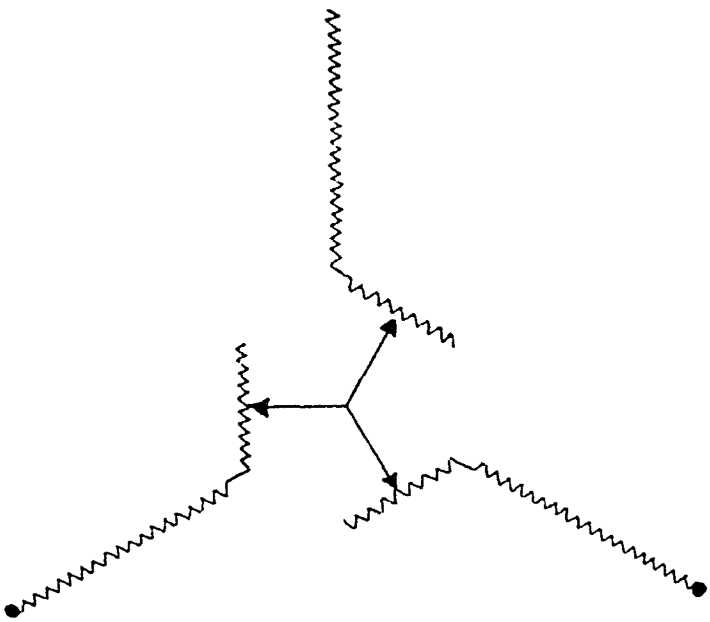


FIG. 13.34.—TAP-CHANGING TRANSFORMER FOR PHASE-SHIFTING.

other cases considered, then since this boost is at 60 degrees instead of in quadrature, the quadrature component is 8·7 per cent. and the in-phase component 5·03 per cent. Hence from

$$\cos \varphi + 10 \sin \varphi = 5{\cdot}03$$
$$\varphi = 24^\circ\ 17'$$
$$\text{and P.F.} = \cos \varphi = {\cdot}9115$$

With such a boost the interconnector will carry its full kVA. loading at a power factor of just over 0·91, and there will be zero total phase shift because the quadrature component of the 60-degrees boost will just compensate the lagging quadrature component due to the impedance. Also the in-phase boost will just balance the in-phase voltage drop. The method of operation for a system with the above constants would therefore be to adjust the steam supplies so as to give a power flow in kW. of 0·91 times the kVA. capacity, and also to adjust the tapping so as to give a power factor in the interconnector of 0·91 lagging.

In the above discussion we have considered the interconnector impedance only, but obviously the transformer impedances can be included without altering the principle.

Synchronising Power of an Interconnector

It is proved in the text-books on Electrical Technology that the synchronising power for two alternators in parallel is a maximum when the armature resistance and armature reactance are equal. Similarly with two stations joined by an interconnector there will be a flow of synchronising power if the phase angle of one station relative to the other is suddenly changed by a sudden large change in load. This synchronising power will, for a given increment of phase angle, be a maximum when the resistance and reactance of the interconnector are numerically equal.

Let W_S = power delivered to the sending end
W_R = power received at receiving end
and β = phase angle between $E_{p.1}$ and $E_{p.2}$
Then

$$W_R = 3E_{p.2}\, I \cos \varphi_2$$
$$= \frac{3E_{p.2}}{Z} \cdot IZ \cos \varphi_2$$

But, from the vector diagram of Fig. 13.35 :

$$IZ = \frac{E_{p.1} \sin \beta}{\sin \varepsilon_2}$$

$$\therefore W_R = \frac{3E_{p.2}}{Z} \cdot \frac{E_{p.1} \sin \beta}{\sin \varepsilon_2} \cdot \cos \varphi_2$$

Now, $\varphi_2 = \alpha - (180 - \varepsilon_2)$
and $\cos(\alpha - 180 + \varepsilon_2) = \sin \alpha \sin \varepsilon_2 - \cos \alpha \cos \varepsilon_2$

$$\therefore W_R = \frac{3E_{p.1} E_{p.2}}{Z} \cdot \frac{\sin \beta}{\sin \varepsilon_2} (\sin \alpha \sin \varepsilon_2 - \cos \alpha \cos \varepsilon_2)$$
$$= \frac{3E_{p.1} E_{p.2}}{Z} \cdot \sin \beta\, (\sin \alpha - \cos \alpha \cot \varepsilon_2)$$

Again, $\cot \varepsilon_2 = \dfrac{E_{p.2} - E_{p.1} \cos \beta}{E_{p.1} \sin \beta}$

$$\sin \alpha = \frac{X}{Z} \text{ and } \cos \alpha = \frac{R}{Z}$$

$$\therefore W_R = \frac{3E_{p.1}E_{p.2}}{Z}\left\{\frac{X}{Z}\cdot\sin\beta - \frac{R}{Z}\left(\frac{E_{p.2}-E_{p.1}\cos\beta}{E_{p.1}}\right)\right\}$$

$$= \frac{3E_{p.1}E_{p.2}}{Z^2}\left(X\sin\beta + R\cos\beta - R\cdot\frac{E_{p.2}}{E_{p.1}}\right)$$

Similarly, $W_S = \frac{3E_{p.1}E_{p.2}}{Z^2}\left(X\sin\beta - R\cos\beta + R\cdot\frac{E_{p.1}}{E_{p.2}}\right)$

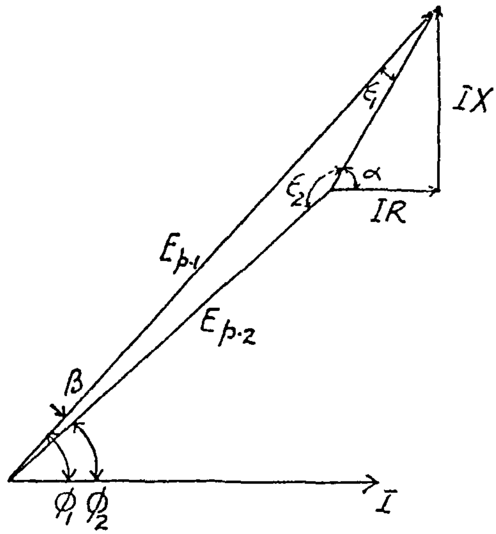


FIG. 13.35.—REGULATION OF AN INTERCONNECTOR.

If the phase angle β be increased to $(\beta + d\beta)$, then

$$W_R = \frac{3E_{p.1}E_{p.2}}{Z^2}\left\{X\sin(\beta+d\beta) + R\cos(\beta+d\beta) - R\cdot\frac{E_{p.2}}{E_{p.1}}\right\}$$

and $W_S = \frac{3E_{p.1}E_{p.2}}{Z^2}\left\{X\sin(\beta+d\beta) - R\cos(\beta+d\beta) + R\cdot\frac{E_{p.1}}{E_{p.2}}\right\}$

$\therefore$ Increase in W_R

$$dW_R = \frac{3E_{p.1}E_{p.2}}{Z^2}\left\{X\sin(\beta+a\beta) - X\sin\beta + R\cos(\beta+d\beta) - R\cos\beta\right\}$$

And increase in W_S

$$dW_S = \frac{3E_{p.1}E_{p.2}}{Z^2}\left\{X\sin(\beta+d\beta) - X\sin\beta - R\cos(\beta+d\beta) + R\cos\beta\right\}$$

$\therefore\ dW_S - dW_R =$ increase in losses

$$= \frac{6RE_{p.1}E_{p.2}}{Z^2}\left\{\cos\beta - \cos(\beta+d\beta)\right\}$$

Hence, increase in power flowing through the line after deducting this increase in losses

$$= \frac{3 X E_{p.1} E_{p.2}}{Z^2} \left\{ \sin (\beta + d\beta) - \sin \beta \right\}$$

Now $\sin (\beta + d\beta) = \sin \beta . \cos d\beta + \cos \beta . \sin d\beta$.
Since $d\beta$ is small we can write :

$$\cos d\beta \simeq 1 \text{ and } \sin d\beta \simeq d\beta$$

The increase in power for a given increment $d\beta$ in the angle β is thus

$$\frac{3 X E_{p.1} E_{p.2}}{Z^2} \cos \beta$$

$$= 3E_{p.1}E_{p.2} \cos \beta \times \frac{X}{R^2 + X^2}$$

$$= 3E_{p.1}E_{p.2} \cos \beta \times \frac{1}{\frac{R^2}{X} + X}.$$

Regarding X as a variable, the product of the two terms in the denominator is independent of the variable, showing that the expression is a maximum when R and X are numerically equal.

Stability of an Interconnector

An interconnector, with a generator or a power-station at either end, presents a problem very similar to that of two generators in parallel, the difference being the impedance of the tie. The magnitude and direction of power flow is decided by the angle

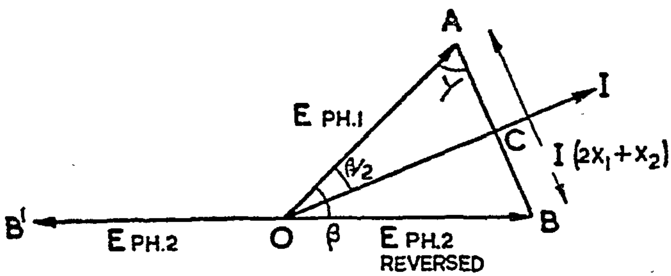


FIG. 13.36.

between the induced E.M.F.s at the two ends. In the following discussion the resistance of all parts of the circuit is neglected, and the circuit assumed to have inductive reactance only.

Let $E_{ph.1}$ and $E_{ph.2}$ be the induced E.M.F. per phase at the two ends, as before, and let the angle between them be β. If $E_{ph.1}$ leads $E_{ph.2}$ we can reverse the vector of $E_{ph.2}$ giving the voltage triangle OAB, Fig. 13.36. AB is the drop in the two generators, the two transformers if present, and the interconnector. Let X_1 be the reactance of each generator plus transformer and X_2 the reactance of the line. Then

$$AB = (2X_1 + X_2)\,I.$$

If we assume $E_{ph\cdot 1}$ and $E_{ph\cdot 2}$ to be numerically equal then, since the current I is in quadrature with the reactive drop, the current vector OI must bisect the angle AOB.

Power per phase delivered by generator 1—

$$P_1 = E_{ph\cdot 1}\,I\cos\beta/2$$

$$\cos\beta/2 = OC/OA = \sin\gamma$$

$$\text{and } \sin\gamma/\sin\beta = E_{ph\cdot 2}/(2X_1 + X_2)\,I$$

$$= E_{ph\cdot 1}/(2X_1 + X_2)\,I$$

$$\therefore P_1 = \frac{E_{ph\cdot 1}\,I \times E_{ph\cdot 1}\sin\beta}{(2X_1 + X_2)\,I}$$

$$= \frac{E_{ph\cdot 1}^2}{2X_1 + X_2}\sin\beta \text{ watts per phase}$$

$$\text{Torque } T = \text{watts/radians per sec.}$$

and therefore, if ω is the angular velocity of the generator,

$$T = \frac{1}{\omega}\cdot\frac{3E_{ph\cdot 1}^2}{2X_1 + X_2}\sin\beta \text{ newton metres}$$

or $T \propto \sin\beta$.

This shows that the torque is a sinusoidal function of the angle β, between the induced E.M.F.s at the two ends of the line. The maximum torque is given by $\beta = \pi/2$ radians. Such a value would obviously lead to instability because an increase in β would result in a decrease in torque. Thus in practice, the angle β is appreciably less than $\pi/2$ radians.

Now let v_x denote the total percentage reactance of the system with respect to the current I, and v_x' that to the working component I_a, then—

$$v_x = \frac{100(2X_1 + X_2)\,I}{E_{ph\cdot 1}}$$

$$\text{and } v_x' = v_x\cos\beta/2$$

$$\text{Now } (2x_1 + x_2) = E_{ph\cdot 1}^2\sin\beta/P_1$$

$$= \frac{E_{ph\cdot 1}^2\sin\beta}{E_{ph\cdot 1}\,I\cos\beta/2}$$

$$= \frac{E_{ph\cdot 1}\sin\beta}{I\cos\beta/2}$$

$$\therefore v_x' = \frac{100}{E_{ph\cdot 1}} \times I \cos \beta/2 \times \frac{E_{ph\cdot 1}}{I \cos \beta/2} \cdot \sin \beta$$
$$= 100 \sin \beta$$

Example.—The 40,000-V. line of p. 308 for which

$$E_{ph} = 23{,}100 \text{ V}$$
$$I_a = 174 \text{ A}$$
$$X_L = 10 \text{ ohms per line}$$

Hence with respect to I_a the percentage reactance of the line alone is—

$$v_L = \frac{100 \times 174 \times 10}{23100} = 7{\cdot}54$$

Take the generator and transformer reactances as 15 per cent. and 5 per cent. respectively, then—

$$v_x = 2(15 + 5) + 7{\cdot}54$$
$$= 47{\cdot}54$$
$$\therefore \sin \beta = 47{\cdot}54/100 = 0{\cdot}4754$$
$$\therefore \beta = 28^\circ\, 25'$$

Suppose that the maximum allowable value for β is 35°, then the maximum allowable value for v_2 is—

$$v_2 = \sin 35^\circ = 0{\cdot}5736$$

The maximum allowable value of I_a is thus—

$$I_a = 174 \times 0{\cdot}5736/0{\cdot}4754$$
$$= 210{\cdot}2 \text{ amps.}$$
$$\therefore P = 3 \times 23100 \times 210{\cdot}2/1000$$
$$= 14600 \text{ kW.}$$

From the expression—

$$v_x = 100 \sin \beta$$

we see that, the greater the value of v_x the greater will be the phase difference β, showing that it is desirable to keep the value of v_x down. For generators and transformers of given characteristics this can be achieved only by keeping the line reactance as small as possible. Extensions to the power plant will reduce the total machine reactance, but where this cannot be done the simplest solution is to run a second line in parallel with the first.

Frequency Ties

Although standardisation of frequency to 50 cycles has eliminated many of the frequency differences between neighbouring systems, there are still cases of two systems operating at different frequencies being linked together by interconnectors. In such cases there must be some form of frequency tie since, obviously, the interconnector can only operate at the frequency of one of the stations. The frequency tie takes the form of a frequency converter, of which there are several types, viz.:

1. A synchronous set, comprising a synchronous motor driving a synchronous alternator.
2. An induction motor/synchronous alternator set.
3. A speed-regulating induction motor/synchronous alternator set.
4. Induction synchronous set.
5. Synchronous converter set.

All of the above sets are reversible in that they enable the flow of power to take place in either direction. The main characteristics of the different methods are as follows:

Synchronous Set

This is the most common form of frequency converter, and its chief characteristic is that it forms a rigid frequency tie between the two systems, the ratio of the two frequencies being absolutely fixed. Momentary fluctuations of frequency may exist, but these must be insufficient to pull the machines out of step. This type of set does not provide a voltage tie, since an adjustment of the voltage may be made on one side without affecting the other side in any way, and the load over the set is controlled by an adjustment of the steam supplies at the two stations. Its advantages are simplicity of construction and operation with a high efficiency. If the interconnector is not in use the synchronous motors can be run as synchronous condensers.

Induction Motor Set

The operation is very similar to the above, with the exception that the frequency ratio varies slightly with changes in load owing to the slip of the induction motor. The power factor and efficiency are not so good as with the synchronous set, and the application has been limited to cases where fluctuations in frequency may be too great for the synchronous set.

Example

Two generating stations, A and B, have full-load capacities of 10,000 kW. and 5,000 kW. respectively. They are connected by an interconnector containing an induction motor/synchronous generator set C having a full-load capacity of 2,500 kW. Percentage changes of speed of A, B, and C are 5, 4, and 3 respectively. There are loads of 5,000 kW. and 2,500 kW. on

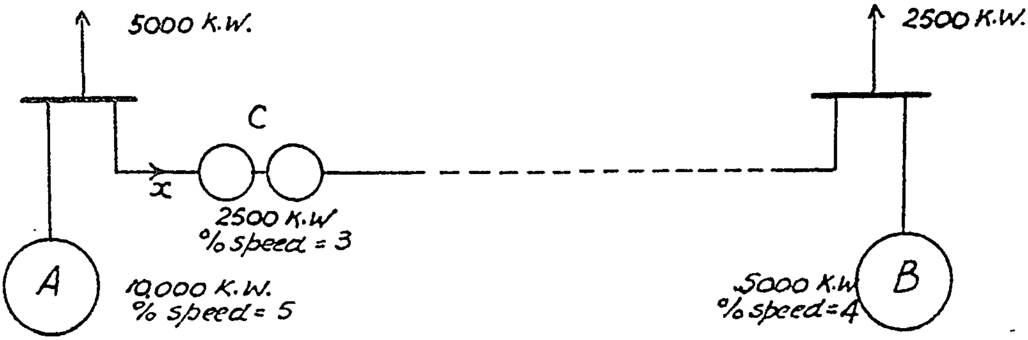


FIG. 13.37.

the bus-bars of A and B respectively. What load is taken by the set C, and in what direction is the energy passing ?

The system is represented in Fig. 13.37. Assume that x kW. flow in the direction shown, then

$$\text{Load on station B} = (2500 - x)\text{ kW.}$$

$$\therefore \text{ Per cent. drop in speed} = 4 \times \frac{2500 - x}{5000}$$

$$\text{Load on station A} = (5000 + x)\text{ kW.}$$

$$\therefore \text{ Per cent. drop in speed} = 5 \times \frac{5000 + x}{10000}$$

Now, this fixes the frequency of supply to the motor-generator set, and the frequency of the supply to B will be modified by the drop in speed through C. But since the frequencies of the right-hand machine of C and of B will be the same, the drop in speed of B will be equal to the combined drops in speed of A and C.

$$\text{Per cent. drop in speed of C} \simeq 3 \times \frac{x}{2500}$$

$$\therefore 5 \times \frac{5000 + x}{10000} + 3 \times \frac{x}{2500} = 4 \times \frac{2500 - x}{5000}$$

which gives $x = -200$

Hence, the load taken by the set C is 200 kW. in the direction B to A.

Speed-regulating Induction Motor Set

If there are variations in frequency in one or both of the systems which are too great to be handled by either of the previously described types an induction motor set with Scherbius speed regulation can be used in connection with the induction motor portion of the set. In this way the speed of the motor can be adjusted to be either below or above synchronism. Also, this machine will act as a generator or a motor according to the direction of power flow. For motor operation below synchronism or asynchronous generator operation above synchronism, the energy which is taken from the slip rings drives the speed-regulating set in such a way as to return the slip energy to the line. With the main motor operating as a motor above synchronism or as a generator below synchronism the power flow through the regulating set is reversed. The objections of the induction motor set are removed by the addition of the speed-regulating gear, since both of the connected systems can operate at exactly synchronous speed. Also the power factor of the induction motor is improved. A feature of the method is that it allows of control of the load on the frequency converter set instead of at the governors of the turbines in the two-power stations.

Induction Synchronous Set

This consists of a wound-rotor induction motor operated at a definite slip, determined by the speed of the synchronous machine to which it is direct-connected, the higher-frequency system being connected to its stator circuit, and the lower frequency circuit connected to the rotor of the induction machine, and also to the synchronous machine. Hence, although it makes use of an induction machine, this type of set is entirely synchronous in action.

Synchronous Converter Set

This consists of two rotary converters operated in series, the first converting A.C. to D.C. and the second D.C. to A.C. of the frequency of the second system. First cost and high maintenance have limited the application of the type, which has the operating advantages of wide range of frequency ratio, load control at the converter instead of at the station governors, and provision of a voltage tie.

With the advent of Grid control of mercury-arc rectifiers, it is possible to use two such rectifiers, one at each end, to supply a D.C. link between two power stations. Grid control gives complete control over the flow of power in either direction, but it is obvious that the D.C. interconnector cannot carry any wattless current. At the present time the highest D.C. voltage at which a single unit has been operated is in the neighbourhood of 30,000, and this is not sufficient for long-distance transmission, but if this voltage can be increased to the neighbourhood of 100,000, either in a single unit or by the operation of rectifiers in series, then this type of interconnector will be a possibility. At the receiving end the rectifier will act as an inverter, and synchronous machinery is therefore necessary to supply the wattless kVA. This might be an objection to transmission as distinct from interconnection by this system, but with the latter application the synchronous machinery is naturally present and the objection does not hold. It has been suggested that if the properties of the Grid-controlled rectifier could have been anticipated when the National Grid System was designed, the transmission would have been by D.C. instead of A.C. since, in effect, each section of the Grid is more in the nature of an interconnector than a straight transmission line.

Ferranti Effect

The effect of the line capacitance on the line regulation in the case of a long high-voltage line has already been discussed by exact methods, but the following approximate solution is of value in that it brings out the nature of the problem rather more clearly. If we consider a line on no-load with the normal voltage $E_{p.1}$ maintained at the sending end, then the only current flowing will be the distributed current due to line capacitance. As an approximation we can neglect the line resistance and assume that the current is uniformly distributed, the load centre then being at the middle of the line. We can thus imagine the charging current I_c concentrated at the middle of the line, and since it will then flow through only one half of the line inductance, we have :

$$\text{Inductive drop} = \tfrac{1}{2} I_c X$$

Also, since the current I_c is in quadrature leading, this drop will actually be a rise in voltage. Again, if C is the capacitance in farads per mile, and l the length in miles

$$I_c = E_{p.1} C l \omega$$

Let L be the inductance per mile, then $X = Ll\omega$

$$\begin{aligned} \therefore \tfrac{1}{2} I_c X &= \tfrac{1}{2} E_{p.1} C l \omega \times L l \omega \\ &= \tfrac{1}{2} E_{p.1} \omega^2 l^2 \times LC \end{aligned}$$

Now we have already seen that the product LC is a constant, being equal to $2{\cdot}9 \times 10^{-11}$

$$\therefore \text{ Rise in voltage} = 1{\cdot}45 \times 10^{-11}\omega^2 l^2 E_{p.1}$$

This expression gives, approximately, the excess of voltage at the receiving end over that at the sending end when the line is open-circuited. The effect is sometimes called the Ferranti effect. If the rise in voltage refers to the voltage between conductors instead of voltage to neutral, we have

$$\text{Rise in line voltage} = 1{\cdot}45 \times 10^{-11}\omega^2 l^2 E_1$$

As a numerical example consider a 100,000-volt line, 100 miles long, the frequency being 50. Then

$$\omega = 314$$

and rise in voltage

$$= 1{\cdot}45 \times 10^{-11} \times (3{\cdot}14)^2 \times 10^4 \times 10^4 \times 10^5$$
$$= 1430 \text{ volts}$$

The above simple discussion brings out the important fact that the excess of voltage at the receiving end of an open line is proportional to the square of the length of the line. A simple calculation will show that quite a small load will eliminate the effect, and where there are transformers in circuit at the receiving end, the magnetising current will act in such a way.

Suggestions for Further Reading

Dannatt, C., *Electrical Power Transmission and Interconnection* (Pitman).

Rissik, H., *Power System Interconnection* (Pitman).

Woodruff, L. F., *Principles of Electric Power Transmission and Distribution* (Wiley).

Young, H. P., *Electrical Power System Control* (Chapman & Hall).

CHAPTER XIV

SYMMETRICAL COMPONENTS

IT has been shown by Fortescue that any three vectors can be resolved into three sets of components, viz. two sets of balanced three-phase vectors, one of positive and the other of negative phase sequence, and a set of three vectors which are equal in magnitude and in phase. These various sets of vectors are called the symmetrical components of the original unbalanced system. Let the operator "*a*" rotate a vector in a counter-clockwise direction through 120 degrees, then it follows from Fig. 14.1 that, for a vector of length I:

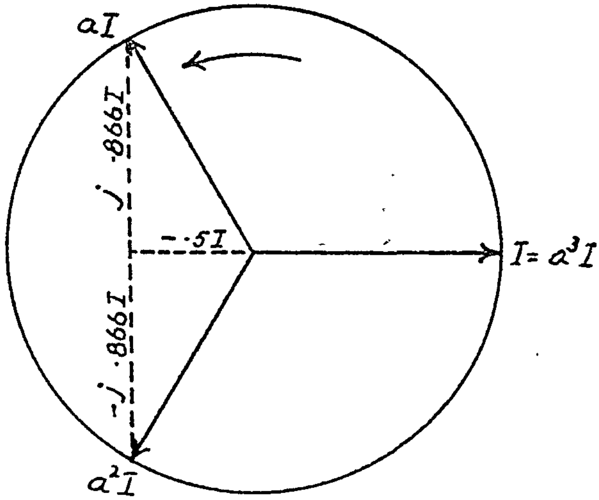


FIG. 14.1

for one operation $aI = -\cdot 5I + j \times \cdot 866I$

or $a = -\cdot 5 + j \times \cdot 866$ . . . . . . (1)

for two operations $a^2I = -\cdot 5I - j \times \cdot 866I$

or $a^2 = -\cdot 5 - j \times \cdot 866$ . . . . . . (2)

for three operations $a^3I = I$

or $a^3 = 1$ . . . . (3)

Also $1 + a + a^2 = 0$ . . . . (4)

Now consider a three-phase system with earthed neutral: let the actual phase currents be I_a, I_b, and I_c, and the neutral current I_0 (all maximum values). Let the positive sequence components be represented by I_{a1}, I_{b1}, I_{c1}. Then, since these are all equal in magnitude and have the three-phase displacement of 120 degrees, we have for a phase sequence a, b, c, I_{b1} lags I_{a1} by 120 degrees and I_{c1} lags I_{b1} by 120 degrees.

$$\therefore \left.\begin{aligned} I_{a1} &= aI_{b1} = a^2 I_{c1} \\ I_{b1} &= a^2 I_{a1} = aI_{c1} \\ I_{c1} &= aI_{a1} = a^2 I_{b1} \end{aligned}\right\} \quad \ldots\ldots \quad (5)$$

Also from equations (4) and (5)

$$I_{a1} + I_{b1} + I_{c1} = 0 \quad \ldots\ldots \quad (6)$$

Next consider the negative sequence components: let their peak values be I_{a2}, I_{b2}, and I_{c2}, then, since their phase sequence is $a - c - b$, instead of $a - b - c$, we have

$$\left.\begin{aligned} I_{a2} &= a^2 I_{b2} = aI_{c2} \\ I_{b2} &= aI_{b2} = a^2 I_{c2} \\ I_{c2} &= a^2 I_{a2} = aI_{b2} \end{aligned}\right\} \quad \ldots\ldots \quad (7)$$

and from equations (4) and (7) we have

$$I_{a2} + I_{b2} + I_{c2} = 0 \quad \ldots\ldots \quad (8)$$

Finally consider the zero sequence components: let the component currents in the phases be I_{a0}, I_{b0}, and I_{c0}. These components are all equal and in phase with one another so that:

$$I_{a0} = I_{b0} = I_{c0} \quad \ldots\ldots \quad (9)$$

From the relationships of equations (6) and (8) we see that there are no positive or negative sequence neutral currents, the whole of the neutral current I_0 thus being derived from the zero sequence components.

$$\therefore I_0 = I_{a0} + I_{b0} + I_{c0} \quad \ldots\ldots \quad (10)$$

also

$$I_0 = I_a + I_b + I_c \quad \ldots\ldots \quad (11)$$

and

$$I_0 = 3I_{a0} = 3I_{b0} = 3I_{c0} \quad \ldots\ldots \quad (12)$$

The total current in any phase is the vector sum of the three components in that phase:

$$\therefore \left.\begin{aligned} I_a &= I_{a0} + I_{a1} + I_{a2} \\ I_b &= I_{b0} + I_{b1} + I_{b2} \\ I_c &= I_{c0} + I_{c1} + I_{c2} \end{aligned}\right\} \quad \ldots\ldots \quad (13)$$

Expressing the three total currents in terms of the components in phase a, with the help of equations (5), (7), and (9), we have:

$$I_a = I_{a0} + I_{a1} + I_{a2} \quad \ldots\ldots \quad (14)$$

$$I_b = I_{a0} + a^2 I_{a1} + aI_{a2} \quad \ldots\ldots \quad (15)$$

$$I_c = I_{a0} + aI_{a1} + a^2 I_{a2} \quad \ldots\ldots \quad (16)$$

To solve these equations for I_{a0}, I_{a1}, and I_{a2}, multiply (14), (15), and (16) through by the appropriate operator, either 1, a, or a^2, and add, remembering that $1 + a + a^2 = 0$, $a^3 = 1$, and $a_4 = a$.

Hence

$$I_{a0} = \tfrac{1}{3}(I_a + I_b + I_c) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$
$$I_{a1} = \tfrac{1}{3}(I_a + aI_b + a^2I_c) \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$
$$I_{a2} = \tfrac{1}{3}(I_a + a^2I_b + aI_c) \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Knowing the three components for phase a, we know the components for phases b and c.

We are now in a position to calculate the negative sequence component for a line-to-line short-circuit. The first step is the calculation of I_{a1}, and this is given by

$$I_{a1} = \frac{E_a}{Z_1 + Z_2},$$

where Z_1 is the transient impedance of the network to positive phase-sequence currents and Z_2 the impedance to negative phase-

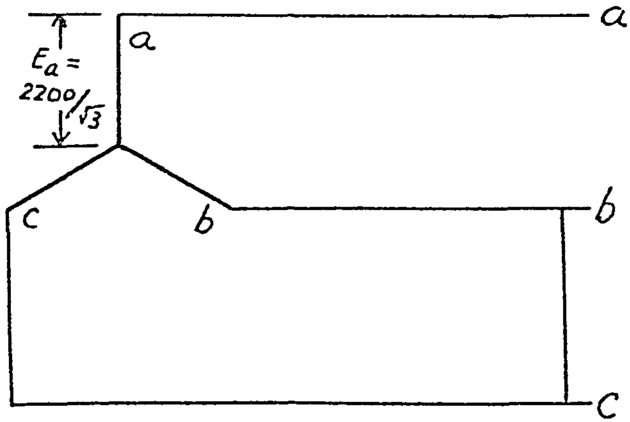


FIG. 14.2.

sequence currents. Suppose there is a short-circuit between phases b and c (Fig. 14.2) (thereby introducing the lack of symmetry which is responsible for the existence of the negative sequence components); then there will be no total current in phase a:

$$\therefore\ I_{a1} + I_{a2} + I_{a0} = I_a = 0$$
$$\text{and} \quad I_{a2} = -I_{a1} \text{ (since } I_{a0} = 0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

As a numerical example take the case of a three-phase 2,200-volt alternator of negligible resistance, and reactance per phase of 10 ohms: let the resistance and reactance per phase of the transmission line up to the point of short-circuit be 4 ohms and 6 ohms respectively. Then

$$Z = 0 + j \times 10 \text{ for alternator}$$
$$Z = 4 + j \times 6 \text{ for line}$$

The positive phase-sequence transient impedance up to the fault is

$$Z_1 = (0 + j \times 10) + (4 + j \times 6)$$
$$= 4 + j \times 16 \text{ ohms}$$

The negative phase-sequence transient impedance is also $(4 + j \times 6)$ for the line, but for the alternator it is 73 per cent. of the positive phase-sequence impedance, viz. $(0 + j \times 7{\cdot}3)$. Hence for the total negative phase-sequence transient impedance up to the fault, we have

$$Z_2 = (0 + j \times 7{\cdot}3) + (4 + j \times 6)$$
$$= 4 + j \times 13{\cdot}3 \text{ ohms}$$

also $$E_a = \frac{2200}{\sqrt{3}} = 1270 \text{ volts}$$

$$\therefore I_{a1} = \frac{E_a}{Z_1 + Z_2} = \frac{1270}{(4 + j \times 16) + (4 + j \times 13{\cdot}3)}$$
$$= \frac{1270}{8 + j \times 29{\cdot}3}$$
$$= \frac{1270(8 - j \times 29{\cdot}3)}{8^2 + 29{\cdot}3^2}$$
$$= 11 - j \times 40{\cdot}3 \text{ amps.}$$

Hence for the *numerical* values of the positive sequence components we have

$$I_{a1} = I_{b1} = I_{c1} = \sqrt{11^2 + 40{\cdot}3^2} = 41{\cdot}8 \text{ amps.}$$

For the negative sequence components we have, since the zero phase-sequence components are zero and also the total current in phase a is zero :

$$I_a = I_{a1} + I_{a2} + I_{a0}$$
$$\therefore 0 = I_{a1} + I_{a2} + 0$$
$$\text{or } I_{a2} = - I_{a1}$$
$$= - 11 + j \times 40{\cdot}3 \text{ amps.}$$

Hence for the numerical values of the negative sequence components we have

$$I_{a2} = I_{b2} = I_{c2} = \sqrt{11^2 + 40{\cdot}3^2} = 41{\cdot}8 \text{ amps.}$$

The complete vector relationship is shown in Fig. 14.3, from which we see that

$$I_{a1} = -I_{a2}\,;\; I_{b1} = -I_{c2}\,;\; I_{c1} = -I_{b2}$$

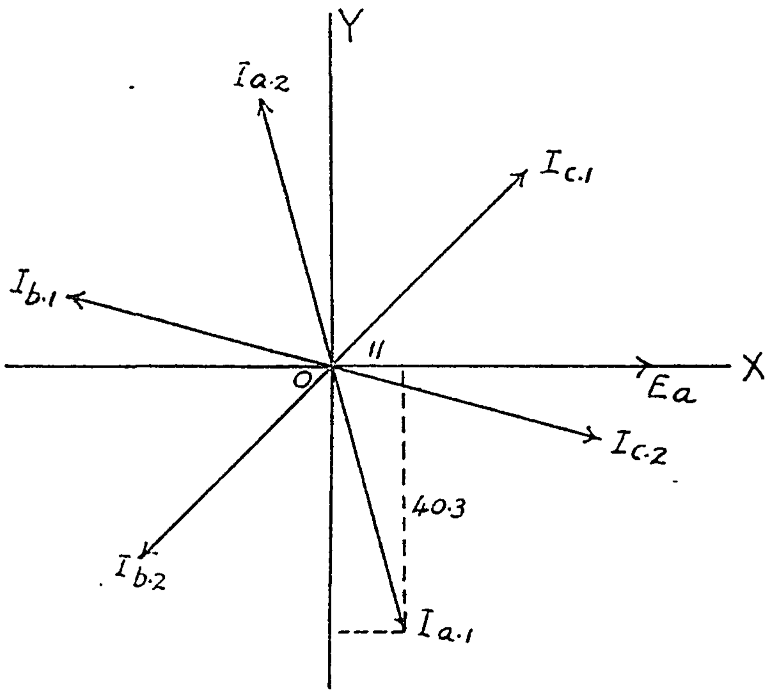


FIG. 14.3.

To find the total currents in phases b and c, we have from equations (13)

$$\begin{aligned} I_b &= I_{b0} + I_{b1} + I_{b2} \\ &= 0 + I_{b1} + I_{b2} \\ &= I_{b1} - I_{c1} \\ &= a^2 I_{a1} - aI_{a1} \text{ from equations (5)} \\ &= (11 - j \times 40{\cdot}3)(-{\cdot}5 - j \times {\cdot}866 + {\cdot}5 - j \times {\cdot}866) \\ &= -1{\cdot}732\, j(11 - j \times 40{\cdot}3) \\ &= -69{\cdot}7 - j \times 19{\cdot}1 \text{ amps.} \end{aligned}$$

and

$$\begin{aligned} I_c &= I_{c0} + I_{c1} + I_{c2} \\ &= 0 + I_{c1} + I_{c2} \\ &= I_{c1} - I_{b1} \\ &= -I_b \\ &= 69{\cdot}7 + j \times 19{\cdot}1 \end{aligned}$$

For the magnitudes of the total currents we thus have

$$\begin{aligned} I_a = 0\,;\; I_b = I_c &= \sqrt{69{\cdot}7^2 + 19{\cdot}1^2} \\ &= 72{\cdot}4 \text{ amps.} \end{aligned}$$

Case of an Alternator with Earthed Neutral

Denoting the induced E.M.F.s per phase by E_a, E_b, and E_c and the terminal P.D.s by V_a, V_b, and V_c, we have—

$$\begin{aligned}
V_a &= E_a - [Z_0 I_{a0} + Z_1 I_{a1} + Z_2 I_{a2}] \\
V_b &= E_b - [Z_0 I_{b0} + Z_1 I_{b1} + Z_2 I_{b2}] \\
&= E_b - [Z_0 I_{b0} + a^2 Z_1 I_{a1} + a Z_1 I_{a2}] \\
V_c &= E_c - [Z_0 I_{c0} + Z_1 I_{c1} + Z_2 I_{c2}] \\
&= E_c - [Z_0 I_{c0} + a Z_1 I_{a1} + a^2 Z_2 I_{a2}] \\
\therefore V_{a0} &= \tfrac{1}{3}[V_a + V_b + V_c] \\
&= - Z_0 I_{a0} \\
V_{a1} &= \tfrac{1}{3}[V_a + aV_b + a^2 V_c] \\
&= E_a - Z_1 I_{a1} \\
V_{a2} &= \tfrac{1}{3}[V_a + a^2 V_b + aV_c] \\
&= - Z_2 I_{a2}
\end{aligned}$$

Case (1).—One line earthed (Fig. 14.4). The conditions of the fault are—

$$\begin{aligned}
I_b = I_c &= 0 \\
V_a &= 0 \\
I_{a0} &= \tfrac{1}{3}(I_a + I_b + I_c) \\
&= \tfrac{1}{3} I_a \\
I_{a1} &= \tfrac{1}{3}(I_a + aI_b + a^2 I_c) = \tfrac{1}{3} I_a \\
I_{a2} &= \tfrac{1}{3}(I_a + a^2 I_b + aI_c) = \tfrac{1}{3} I_a
\end{aligned}$$

also $$V_{a0} + V_{a1} + V_{a2} = V_a = 0$$

Since the sequence currents I_{a0}, I_{a1}, and I_{a2} are all equal and the sum of the sequence P.D.s V_{a0}, V_{a1}, and V_{a2} is zero, it follows that the three sequence-impedances act as though connected in series, as shown in Fig. 14.4(b).

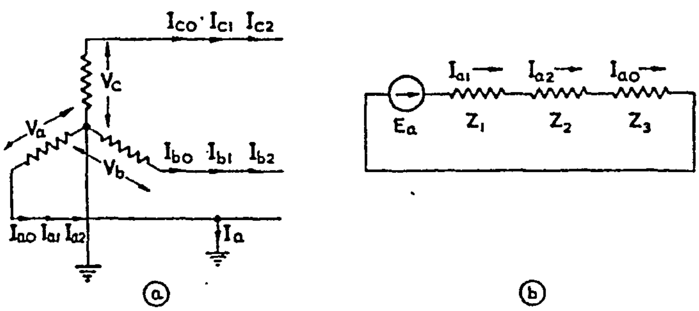


FIG. 14.4.—ALTERNATOR WITH MARKED NEUTRAL AND ONE LINE EARTHED.

Since $V_a = 0$

$$E_a - (Z_0 I_{a0} + Z_1 I_{a1} + Z_2 I_{a2}) = 0$$

$$\therefore E_a = \tfrac{1}{3}(Z_0 + Z_1 + Z_2)\, I_a$$

$$I_a = \frac{3E_a}{Z_0 + Z_1 + Z_2}$$

$$\begin{aligned} I_b &= I_{b0} + I_{b1} + I_{b2} \\ &= I_{a0} + a^2 I_{a1} + a I_{a2} \\ &= (1 + a^2 + a)\, I_{a0} \\ &= 0 \end{aligned}$$

Similarly $I_c = 0$

$$V_{a0} = 0 - Z_0 I_{a0} = -Z_0 \times \tfrac{1}{3} I_a = -\frac{Z_0}{Z_0 + Z_1 + Z_2} \cdot E_a$$

$$V_{a1} = E_a - Z_1 I_{a1} = \left(1 - \frac{Z_1}{Z_0 + Z_1 + Z_2}\right) E_a$$

$$= \frac{Z_0 + Z_2}{Z_0 + Z_1 + Z_2} \cdot E_a$$

$$V_{a2} = 0 - Z_2 I_{a2} = -Z_2 \times \tfrac{1}{3} I_a$$

$$= -\frac{Z_2}{Z_0 + Z_1 + Z_2} \cdot E_a$$

$$\therefore V_a = V_{a0} + V_{a1} + V_{a2} = 0$$

$$\begin{aligned} V_b &= V_{b0} + V_{b1} + V_{b2} \\ &= V_{a0} + a^2 V_{a1} + a V_{a2} \\ &= \frac{-Z_0 + a^2(Z_0 + Z_2) - aZ_2}{Z_0 + Z_1 + Z_2} \cdot E_a \\ &= \frac{(a^2 - 1)\, Z_0 + (a^2 - a)\, Z_2}{Z_0 + Z_1 + Z_2} \cdot E_a \end{aligned}$$

$$\begin{aligned} V_c &= V_{a0} + a V_{a1} + a^2 V_{a2} \\ &= \frac{-Z_0 + a(Z_0 + Z_2) - a^2 Z_2}{Z_0 + Z_1 + Z_2} \cdot E_a \end{aligned}$$

Case (2).—Two lines earthed (Fig. 14.5(a)). The conditions at the fault are—

$$I_c = 0$$

$$V_a = V_b = 0$$

$$V_{c0} = V_{a0} = \tfrac{1}{3}(V_a + V_b + V_c) = \tfrac{1}{3}V_c$$

$$V_{c1} = aV_{a1} = \frac{a}{3}(V_a + aV_b + a^2 V_c) = \tfrac{1}{3}V_c$$

also $I_c = I_{c0} + I_{c1} + I_{c2} = 0$

Thus, in this case the three sequence voltages are equal and it is the sum of the sequence currents which is zero. It follows that the three impedances act as though connected in parallel, as in Fig. 14.5(*b*).

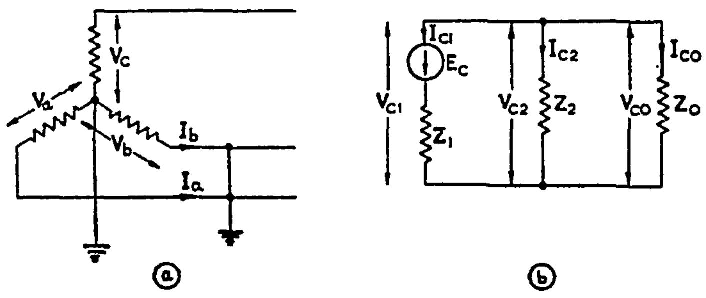


FIG. 14.5.—TWO LINES EARTHED.

$$I_{c1} = \frac{E_c}{Z_1 + \dfrac{Z_0 Z_2}{Z_0 + Z_2}}$$

$$= \frac{Z_0 + Z_2}{Z_0Z_1 + Z_1Z_2 + Z_2Z_0} \cdot E_c$$

$$I_{c2}Z_2 = I_{c0}Z_0$$

$$= -(I_{c1} + I_{c2})\,Z_0$$

$$\therefore I_{c2} = -\frac{Z_0}{Z_0 + Z_2} \cdot I_{c1}$$

$$= -\frac{Z_0}{Z_0Z_1 + Z_1Z_2 + Z_2Z_0} \cdot E_c$$

$$I_{c0} = -I_{c1} - I_{c2}$$

$$= -\frac{Z_2}{Z_0Z_1 + Z_1Z_2 + Z_2Z_0} \cdot E_c$$

$$I_b = I_{b0} + I_{b1} + I_{b2}$$

$$= I_{c0} + aI_{c1} + a^2I_{c2}$$

$$= \frac{-Z_2 + a(Z_0 + Z_2) - a^2Z_0}{Z_0Z_1 + Z_1Z_2 + Z_2Z_0} \cdot E_c$$

$$I_a = I_{c0} + a^2I_{c1} + aI_{c2}$$

$$= \frac{-Z_2 + a^2(Z_0 + Z_2) - aZ_0}{Z_0Z_1 + Z_1Z_2 + Z_1Z_0} \cdot E_c$$

$$V_{c0} = 0 - I_{c0}Z_0$$
$$= \frac{Z_0Z_2}{Z_0Z_1 + Z_1Z_2 + Z_2Z_0} \cdot E_c$$
$$\therefore V_c = V_{c0} + V_{c1} + V_{c2}$$
$$= 3V_{c0}$$
$$= \frac{3Z_0Z_2}{Z_0Z_1 + Z_1Z_2 + Z_2Z_0} \cdot E_c$$
$$V_b = V_{b0} + V_{b1} + V_{b2}$$
$$= V_{c0} + aV_{c2} + a^2V_{c1}$$
$$= V_{c0}(1 + a + a^2)$$
$$= 0$$

Measurement of Symmetrical Components

1. *Currents*

(*a*) Three-wire circuit with insulated neutral (Fig. 14.6).

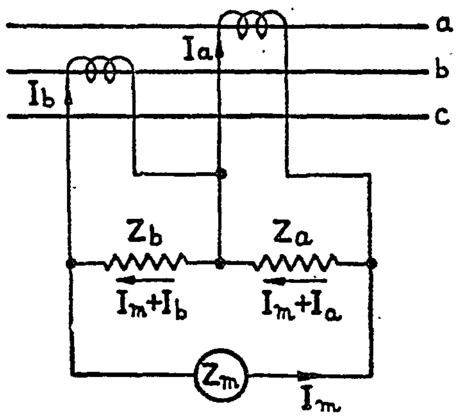


FIG. 14.6.—MEASUREMENT OF SYMMETRICAL COMPONENT CURRENTS.

Two current-transformers act as summation transformers for an ammeter. If I_a, I_b, and I_m are the two secondary currents and the meter current; then, using the notation of the figure—

$$(I_m + I_a) Z_a + (I_m + I_b) Z_b + I_mZ_m = 0$$
$$\therefore I_m = -\frac{I_aZ_a + I_bZ_b}{Z_a + Z_b + Z_m}$$
$$= -\frac{(I_{a1} + I_{a2}) Z_a + (I_{b1} + I_{b2}) Z_b}{Z_a + Z_b + Z_m}$$

$$= -\frac{(I_{a1} + I_{a2})Z_a + (a^2 I_{a1} + a I_{a2})Zb}{Z_a + Z_b + Z_m}$$

$$= -\frac{I_{a1}(Z_a + a^2 Z_b) + I_{a2}(Z_a + a Z_b)}{Z_a + Z_b + Z_m}$$

Suppose that Z_a and Z_b are such that $Z_a = -aZ_b$, then—

$$I_m = \frac{-I_{a1}(Z_a + a^2 Z_b)}{Z_a + Z_b + Z_m}$$

and is therefore proportional to I_{a1}. Let Z_a be a pure resistance R, then for the above condition to be fulfilled we have—

$$\begin{aligned} Z_b &= -\frac{1}{a}R \\ &= -\frac{a^2}{a^3} \cdot R \\ &= -a^2 R \\ &= -R(\cos 240° + j \sin 240°) \\ &= R(0{\cdot}5 + j\,0{\cdot}866) \end{aligned}$$

Hence Z_b must be an inductive impedance of characteristic angle 60°. The ammeter can then be calibrated to read positive sequence currents. Now let—

$$Z_a = -a^2 Z_b$$

then I_m becomes proportional to I_{a2}. Let Z_b now be a pure resistance, then—

$$\begin{aligned} Z_a &= -a^2 R \\ &= R\,(0{\cdot}5 + j\,0{\cdot}866) \end{aligned}$$

and, with this condition fulfilled, the ammeter can be calibrated to read negative-sequence currents. Thus, to read first positive, and then negative-sequence currents Z_a and Z_b are in the position shown, and are then interchanged. As there is no fourth wire, there is no zero-sequence current.

(*b*) Four-wire circuit, or three-wire with earthed neutral (Fig. 14.7).

The circuit for the positive and negative sequence currents is basically the same as Fig. 14.6, but provision has to be made to keep the zero-sequence component away from the ammeter. This is done by connecting a current transformer in the neutral as shown in Fig. (a), thereby providing a closed path for the circulation of the current I_0. The procedure is identical to that for case (*a*).

zero-sequence voltages cancel out. The zero-sequence voltage is equal to one-third of the sum of the line-to-earth voltages, and can therefore be measured as shown.

Influence of Transformer Saturation and Connection on Zero-sequence Currents

Zero-sequence currents are, by definition, in phase with each other in the three windings of any three-phase apparatus. They may be caused by:

(*a*) Magnetic saturation in transformers;

(*b*) Unbalanced loading, including unbalance due to faults to earth in the case of systems with earthed neutral.

As is well known, the effect of saturation in transformers is to cause the magnetising current to depart from the sinusoidal, and take on a peaked, shape. Typical relative values of fundamental and harmonic currents for transformers of 50–1000 kVA., working with medium-core densities are, as percentages of the fundamental:

Fundamental	3rd-harmonic	5th	7th	9th	11th
100	44·5	22·3	11	1·7	0·22

Usually it is only the third-harmonic which is considered. In a three-phase system, these harmonics are all in phase with one another, and consequently they result in a zero-sequence system of triple frequency.

If a current of sinusoidal wave shape is passed through the windings of a choke coil—a current-transformer with open-circuit secondary, for example—then the volt drop across the winding will be peaked and, according to the degree of saturation, may have a very high third-harmonic content. The flux, under the above conditions, will be flat-topped and this corresponds to the presence of a third-harmonic flux. Hence, in the case of three-phase apparatus the harmonics of triple frequency form a zero-sequence system, and the nature of the distortion—e.g. whether of current, E.M.F., or flux—depends on the method of interconnection of the phase windings.

Star

(*a*) *Three-wire.*—Third-harmonics of current cannot flow. Third-harmonics of voltage may exist between lines and neutral or ground, but not between the lines themselves.

(*b*) *Four-wire.*—Third-harmonics of current may flow through the phases and through the line wires and load, and back through the fourth wire to the neutral. Thus, if the flux wave can be restored to an approximate sinusoidal shape, the voltage harmonics will be largely suppressed and the line-to-neutral voltage approximate to a sine wave.

Delta

The third-harmonic E.M.F.s are in phase with respect to the closed circuit round the delta, and hence cause a circulating current of triple frequency. The resulting impedance drop round the circuit utilises the whole of the induced E.M.F., with the result that the third-harmonic voltage is suppressed both in the phase voltage and in the terminal voltage. Consequently, there are no third-harmonic currents in the lines.

The above remarks apply to three-phase apparatus of any description, and, in the case of a transformer, to primary and secondary windings; also, if the transformation is by a three-phase shell-type transformer or a bank of single-phase transformers. This is because, in each case, the magnetic circuit of each phase is complete in itself and no interchange of M.M.F. or flux occurs between the phases. With the three-phase, three-limb core-type, any third-harmonic fluxes would have the same instantaneous directions up or down each limb, and would have to find a return path of high reluctance outside the core. Hence, third-harmonics in the flux and induced E.M.F. waves are small. The effect of magnetic saturation is annulled by the effect of the interlinking of the magnetic circuits, and no third-harmonic currents can flow either in the primary or secondary phase windings, or in the lines.

Tertiary Delta Winding

This is an independent three-phase winding provided for the purpose of giving:

(*a*) A path for the flow of zero-sequence current, especially where such a path does not otherwise exist as in the case of star-star connection.

(*b*) A local supply at low tension for either power or metering purposes.

Objectionable Features of Third-harmonics

Due to third-harmonic currents:

(*a*) Overheating of transformer windings and load.

(*b*) Disturbance to telephone gear and discriminative protective gear.

(*c*) Increased iron loss in transformers.

To measure the zero-sequence current when the neutral wire is available, it is merely necessary to connect a current trans-

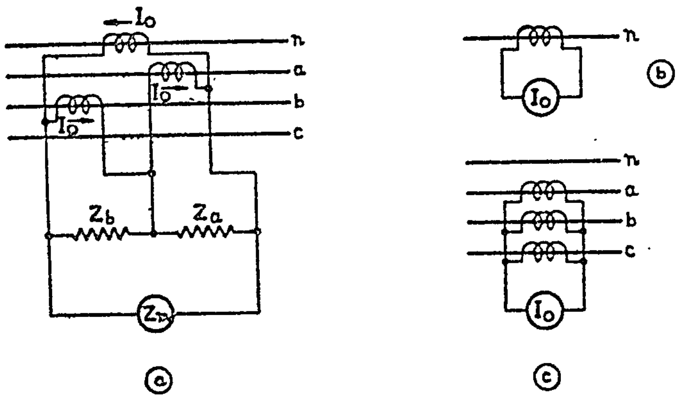


FIG. 14.7.—MEASUREMENT OF SYMMETRICAL COMPONENT CURRENTS.

former in this wire as shown in Fig. (b). If the neutral is not available, then three current transformers as in Fig. (c) are necessary. We then have—

$$I_0 = \tfrac{1}{3} \times \text{Current in neutral}$$
$$= \tfrac{1}{3}(I_a + I_b + I_c)$$

2. *Voltages*

(*a*) Three-wire circuit with insulated neutral (Fig. 14.8).

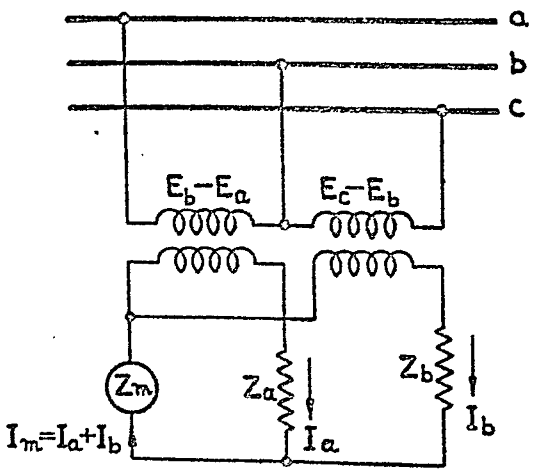


FIG. 14.8.—MEASUREMENT OF SYMMETRICAL COMPONENT VOLTAGES.

Assume for simplicity a 1/1 ratio for the potential transformers. The resultant primary voltage of transformer *ab* is $(E_b - E_a)$ and of transformer bc $(E_c - E_b)$. Hence—

$$E_b - E_a = Z_a I_a + Z_m I_m$$
$$E_c - E_b = Z_b I_b + Z_m I_m$$
$$\therefore Z_b(E_b - E_a) + Z_a(E_c - E_b) = Z_a Z_b(I_a + I_b) + I_m(Z_b Z_m + Z_a Z_m)$$
$$\therefore I_m = \frac{Z_b(E_b - E_a) + Z_a(E_c - E_b)}{Z_a Z_b + Z_a Z_m + Z_b Z_m}$$

Again,

$$E_b - E_a = (E_{b0} + E_{b1} + E_{b2}) - (E_{a0} + E_{a1} + E_{a2})$$
$$= E_{a0} + a^2 E_{a1} + a E_{a2} - E_{a0} - E_{a1} - E_{a2}$$
$$= (a^2 - 1)\, E_{a1} + (a - 1)\, E_{a2}$$
$$E_c - E_b = (E_{a0} + a E_{a1} + a^2 E_{a2}) - (E_{a0} + a^2 E_{a1} + a E_{a2})$$
$$= (a - a^2)\, E_{a1} + (a^2 - a)\, E_{a2}$$

Hence

$$I_m = \frac{a(1 - a)(aZ_b + Z_a)}{Z_a Z_b + Z_a Z_m + Z_b Z_m} \cdot E_{a1} + \frac{(a - 1)(Z_b + aZ_a)}{Z_a Z_b + Z_a Z_m + Z_b Z_m} \cdot E_{a2}$$

The coefficient of E_{a2} will be zero if

$$Z_b = -aZ_a$$

Let $Z_b = R$

$$\therefore Z_a = -\frac{1}{a} \cdot R$$
$$= -a^2 R$$
$$= R(0{\cdot}5 + j0{\cdot}866)$$

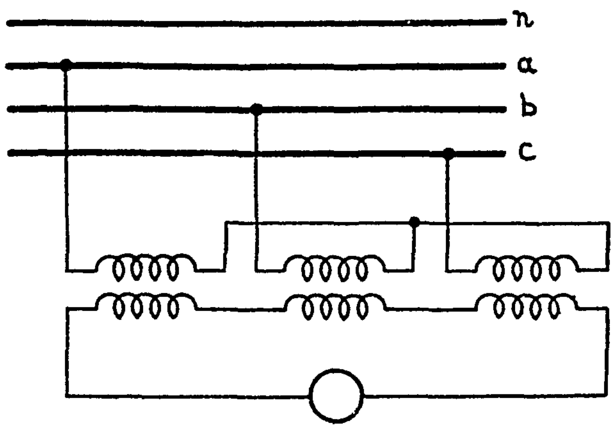


Fig. 14.9.—Measurement of Symmetrical Component Voltages.

With this condition fulfilled the meter indicates positive-sequence voltages. With Z_a and Z_b interchanged it indicates negative-sequence voltages.

(*b*) Four-wire circuit, or three-wire with earthed neutral (Fig. 14.9).

The method under (*a*) applies to this case also in the measurement of the positive- and negative-sequence voltages. The network is supplied by line-to-line voltages and therefore the

Due to third-harmonic voltages:

(*a*) Increased transformer insulation stress.

(*b*) Electrostatic charging of adjacent lines and telephone lines.

(*c*) Possible selective resonance at third-harmonic frequency of transformer reactance and line capacitance.

(*d*) Oscillating neutral. This occurs in a bank of three single-phase transformers connected star-star, or in a three-phase shell-type transformer with these connections. It may be present whether the neutrals are earthed or not. The third-harmonic E.M.F.s in each phase are in phase with one another, with the result that there is one third-harmonic vector only. This rotates at three times the angular velocity of the three vectors of E.M.F.

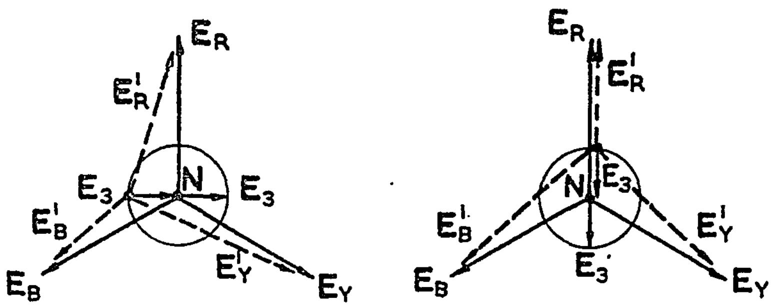


FIG. 14.10.—TO ILLUSTRATE OSCILLATING NEUTRAL.

of fundamental frequency. If we bring the fundamental vectors to rest by imposing an angular velocity of $-\omega$ on both fundamental and third-harmonic vectors, then the latter will rotate at angular velocity ω, its extremity therefore tracing out a circle having its centre at the neutral point N which would exist if the third-harmonic E.M.F. were not present. Two possible cases of the E.M.F. diagram are therefore represented in Fig. 14.10. In general, this causes a triple-frequency fluctuation of the line-to-ground voltage, and also of the neutral-to-ground when the neutral is not earthed.

The oscillating neutral effect may also occur in a D.C. three-wire system with static balancer (Fig. 14.11). The out-of-balance current I divides at the balancer, the two halves of I/2 flowing in opposite directions in the balancer winding. The alternating magnetising current $I\mu$ does not divide in this way, and therefore, at any instant, it will flow in the same direction as the I/2 in one-half of the winding, and in the opposite direction in the other half. Hence, in succeeding half-cycles each half of the winding carries alternately, the sum and the difference of the currents I/2 and $I\mu$. The P.D. across the half-winding carrying the sum is obviously greater than the P.D. across the half carrying the difference, because of the greater volt drop, and for this reason the potential of the neutral wire

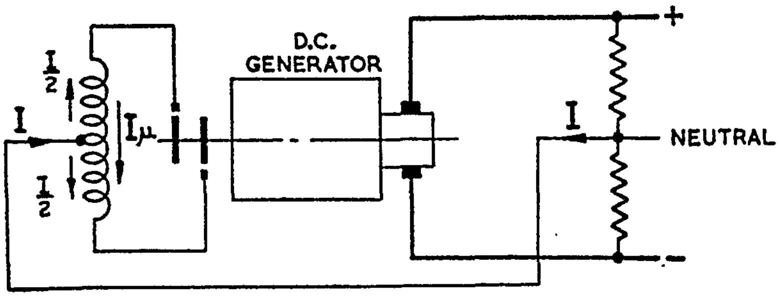


FIG. 14.11.—D.C. GENERATOR WITH STATIC BALANCER.

oscillates about the true mean value. To avoid this effect, static balancers—little used at the present time—are worked at a low flux density and low current density, so that both magnetising current and volt drop are small.

Effect of Unbalanced Currents

If the secondary circuit of a transformer is a three-wire circuit, any unbalanced load will resolve into two balanced systems—one positive sequence, the other negative.

Each of these will be transformed independently as a balanced load and give rise to balanced voltage drops in the transformer. The secondary terminal voltages may be slightly unbalanced by the combination of the two sets of balanced reactance drops of opposite sequence, but this effect will be very small. The most likely case of an unbalanced load is that of a distribution transformer and these are usually designed with a close voltage regulation, the full-load reactive drop being not more than 5 per cent. The primary and secondary ampere-turns are balanced in each phase so that there is no abnormal excitation.

Consider the case of the star-delta transformer of Fig. 14.12, with a load of I amps. across the B and Y lines. For convenience, unity transformation ratio is assumed, and the various circuits in the phase windings on both primary and secondary sides are as shown. We have for the primary side—

$$
\begin{aligned}
I_{R1} &= \tfrac{1}{3}(I_R + aI_Y + a^2 I_B) \\
&= \frac{1}{3}\left(-\frac{2}{3}I + a\frac{I}{3} + a^2\frac{I}{3}\right) \\
&= \frac{I}{9}(-2 + a + a^2) = -\frac{I}{3} \\
I_{R2} &= \tfrac{1}{3}(I_R + a^2 I_Y + aI_B) \\
&= -\frac{1}{3}\left(-\frac{2}{3}I + a^2\frac{I}{3} + a\frac{I}{3}\right) \\
&= \frac{I}{9}(-2 + a^2 + a) = -\frac{I}{3}
\end{aligned}
$$

On the secondary side the phase currents are in the same relative ratios as the primary phase currents, namely 1:1:2.

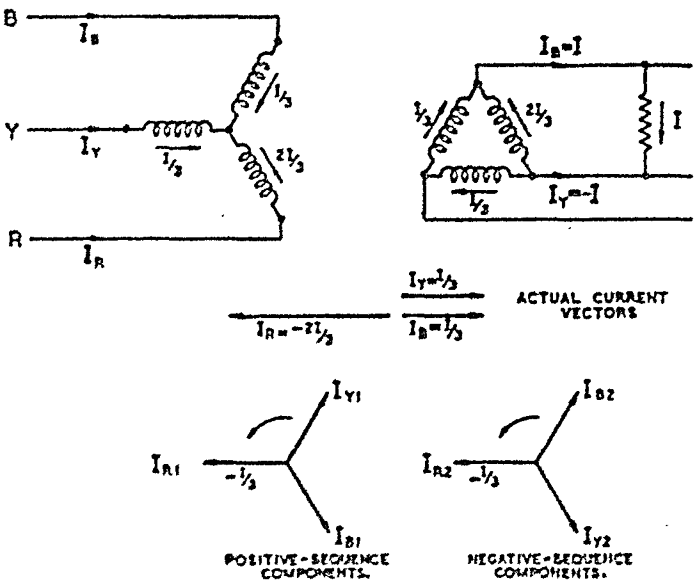


FIG. 14.12.—UNBALANCE IN A STAR-DELTA TRANSFORMER.

Hence, their symmetrical components are similar to those of the primary.

Secondary supplying a Four-wire Line

If the secondary circuit is four-wire, then the above remarks again apply as far as the positive- and negative-sequence systems are concerned. In addition, there is now a zero-sequence system, and the effect of this on the voltage regulation depends to some extent on the transformer connections.

(*a*) *Delta-star**

The delta winding permits the circulation of a zero-sequence current to balance the ampere-turns produced by the zero-sequence currents in the secondary phases. Hence, the only unbalancing of the secondary voltages is that due to the combination of the reactive drops due to the positive- and negative-sequence currents in the windings. This result also follows from the fact that each primary phase is connected directly across two supply lines and can therefore carry whatever current may be required to balance the ampere-turns of the corresponding secondary-phase winding, independently of what may be taking place in the other phases.

(*b*) *Star-star*

The zero-sequence system of currents in the secondary windings cannot be compensated by corresponding currents in the primaries since there is no closed path for the circulation of what, after all, amounts to a single-phase current. Hence, there can be no primary currents to give an M.M.F. balance in the case of the secondary zero-sequence currents, with the result that the secondary line voltages to neutral are unbalanced, the extent of the unbalance being governed by the magnetising effect of the zero-sequence currents. This will be large where there is an independent magnetic circuit to each phase, as in the case of a bank of single-phase transformers, or of a three-phase shell transformer. In the case of the three-phase core-type transformer the zero-sequence fluxes must complete their circuit from top to bottom of each limb of the core by way of some high reluctance path external to the core. Consequently, in such a case, the unbalancing of the secondary voltages to neutral is small. We have previously seen that the third-harmonic fluxes are also negligible, and therefore a star-star core-type transformer with four-wire secondary loading is able to give a stable neutral point.

With a bank of three single-phase transformers, or a single three-phase shell-type transformer, even a slight unbalance in a four-wire load will result in a large unbalance in the secondary voltages to neutral. In the extreme case of a single-phase load between one line and neutral, only a small current will flow even if the load impedance is reduced to zero. This is because of the large reduction in voltage that results from a small increase in current on the loaded phase. This is also apparent from the fact

*Note that in (*a*) and (*b*) the star side is 4-wire.

that the primary current of the loaded phase has to flow through the primary windings of the other two phases, thereby causing a large reactive drop in them. Thus in the above case the star-star connection should not be used unless special means are provided to avoid neutral instability.

(*c*) *Star-delta-star* (4-wire secondary)

In this case the presence of the tertiary delta windings provides a path for the circulation of zero-sequence current, the neutralisation of the ampere-turns of zero-sequence currents flowing in the secondary winding thus being accomplished. It also provides

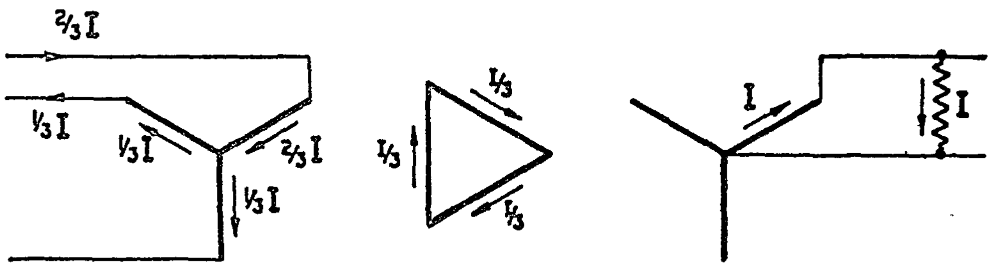


FIG. 14.13.—STAR-DELTA-STAR TRANSFORMATION.

a path for the flow of third-harmonic magnetising current, and hence gives an approximate sinusoidal voltage between line and neutral on the secondary side. The current circulation in a star-delta-star transformer with a single-phase load from line to neutral is shown in Fig. 14.13.

(*d*) *Star-interconnected star*

Each secondary phase consists of two halves which are connected in such a way that the total induced E.M.F. results from the action of two primary phases. This is made clear by the connection scheme of Fig. 14.14. It follows that the zero-sequence currents flowing in the secondary windings are self-neutralising as regards their magnetic effect. There is no path for the flow of third-harmonic magnetising current and therefore there will be a third-harmonic in the induced E.M.F. in each section of the secondary windings, and they will all be in phase with one another. Hence they will cancel out between line and neutral.

An interconnected star (zigzag) choke coil is sometimes used to earth a system, as explained on p. 22. While taking small three-phase magnetising currents, it offers a low impedance to the flow of single-phase fault current, which is limited by means of resistance connected either in each line or in the earth connection to the neutral.

The above discussion shows that, provided there is a path for the flow of zero-sequence currents there will be little trouble with either waveform or voltage unbalance. This means that the zero-sequence impedance of a transformer or bank of trans-

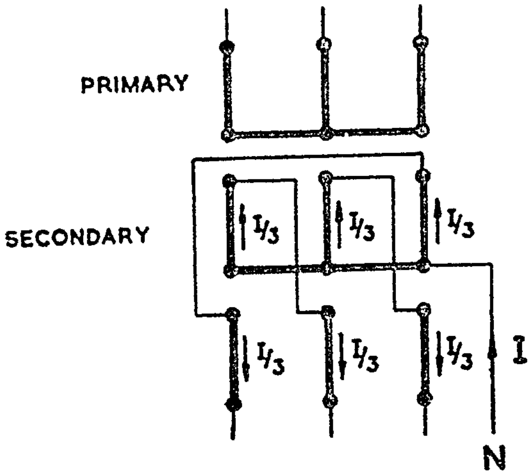


FIG. 14.14.—STAR-INTERCONNECTED STAR TRANSFORMATION.

formers must be small. This provision is unnecessary in the special cases of (*a*) a three-phase core-type transformer, which offers a very high reluctance to zero-sequence fluxes, (*b*) an inter-connected star winding, in which the zero-sequence ampere-turns on each limb are always zero.

SUGGESTIONS FOR FURTHER READING

BLUME, L. F., *Transformer Engineering* (N.Y., Wiley).
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NEUMANN, R., *Symmetrical Component Analysis of Unsymmetrical Polyphase Systems.*
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CHAPTER XV

CIRCUIT-BREAKERS

THE function of a circuit-breaker is to make and break a circuit under light-load, heavy-load, or short-circuit conditions; although the making and breaking of short-circuit currents has the major influence on the size and rating of the circuit-breaker, the interruption of light-load currents at low leading or lagging power factors may be difficult and has a significant influence on the design.

Calculation of Short-circuit Current

A short-circuit on a power system may be symmetrical, in which all three phases are short-circuited, or it may be unsymmetrical, e.g. between one or two phases and earth or between two phases. Although unsymmetrical faults are more prevalent, the symmetrical fault usually gives the more severe duty on the circuit-breaker and is used in the determination of circuit-breaker ratings. In this chapter attention is therefore confined to symmetrical three-phase faults in which the currents in each of the three phases are identical and only one phase need be considered in the calculations.

Short-circuit of Simple Inductive Circuit

Consider a symmetrical short-circuit at the terminals of a circuit having a constant E.M.F. E and a resistance and self-inductance per phase of R and L respectively. Working in terms of phase voltage and current:

$$Ri + L\frac{di}{dt} = E_{max} \sin \omega t$$

The solution of this for steady-state conditions is:

$$i = I_{max} \sin (\omega t - \phi)$$

$$\text{where } I_{max} = \frac{E_{max}}{Z}$$

$$\phi = \text{arc cos} \frac{R}{Z}$$

At the moment of closing the circuit this solution is not complete, since it does not take into account the initial conditions which decide the magnitude of the transient current which almost always occurs before the steady-state conditions are reached. The transient current is given by the solution of the complementary function, viz.:

$$Ri + L\frac{di}{dt} = 0$$

$$\therefore\ i = A\varepsilon^{-\frac{R}{L}.t}$$

Hence the complete solution is:

$$i = I_{max}\sin(\omega t - \phi) + A\varepsilon^{-\frac{R}{L}.t}$$

Let $t = 0$ when $e = 0$, and let the circuit be closed when t has any value t_0: then, at the moment of switching on, since obviously at this moment $i = 0$, the equation to i is:

$$0 = I_{max}\sin(\omega t_0 - \phi) + A\varepsilon^{-\frac{R}{L}.t_0}$$

$$\therefore\ A = -\ I_{max}\sin(\omega t_0 - \phi)\varepsilon^{+\frac{R}{L}.t_0}$$

The equation for i thus becomes:

$$i = I_{max}\sin(\omega t - \phi) - I_{max}\sin(\omega t_0 - \phi)\varepsilon^{-\frac{R}{L}(t - t_0)}$$

If the short-circuit occurs close to the alternator terminals, the resistance is small compared with the reactance, so that:

$$\phi \simeq \frac{\pi}{2}$$

$$\therefore\ i = I_{max}\sin\left(\omega t - \frac{\pi}{2}\right) - I_{max}\sin\left(\omega t_0 - \frac{\pi}{2}\right)\varepsilon^{-\frac{R}{L}(t - t_0)}$$

Consider three cases.

Case I.—Circuit closed when $e = 0$, $\therefore\ t_0 = 0$

$$\therefore\ i = I_{max}\sin\left(\omega t - \frac{\pi}{2}\right) + I_{max}\,\varepsilon^{-\frac{R}{L}t}$$

showing that the current is a sinusoidal current of amplitude I_{max} on which is superposed an exponentially decreasing direct current of initial value I_{max}. The changes of current with time are shown in Fig. 15.1, from which it is seen that very shortly after the closing of the circuit the current rises to very nearly $2I_{max}$, thus giving rise to the so-called doubling effect.

Case II.—Circuit closed when $e = E_{max}$. This is a quarter period later than Case I, so that for t_0 we have:

$$\omega t_0 = \frac{\pi}{2}, \text{ or } t_0 = \frac{\pi}{2\omega}$$

$$\therefore\ i = I_{max} \sin\left(\omega t - \frac{\pi}{2}\right) - I_{max} \sin\left(\frac{\omega\pi}{2\omega} - \frac{\pi}{2}\right)\varepsilon^{-\frac{R}{L}\left(t - \frac{\pi}{2\omega}\right)}$$

$$= I_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$$

Hence there is no transient in this case, and the current is purely sinusoidal right from the beginning.

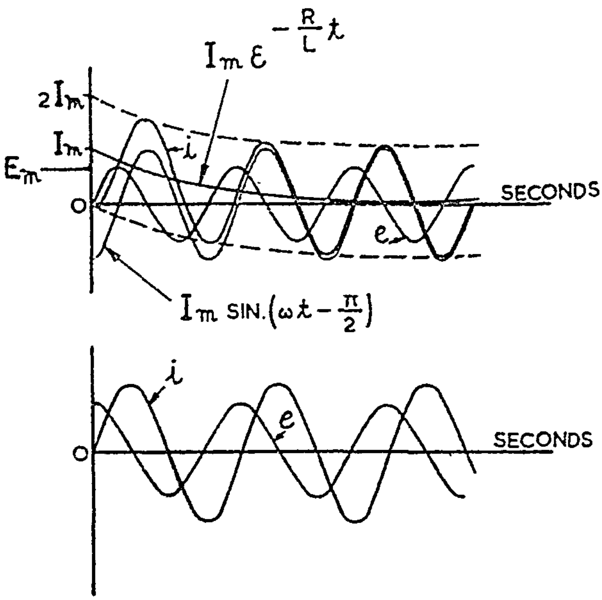

FIG. 15.1.—VOLTAGE AND CURRENT CHANGES FOLLOWING THE SHORT-CIRCUIT OF AN ALTERNATOR.

Case III.—Circuit closed when $e = 0$, but is changing from positive to negative. This corresponds to one half of a period later than Case I, so that for t_0 we have :

$$\omega t_0 = \pi, \text{ or } t_0 = \frac{\pi}{\omega}$$

$$\therefore\ i = I_{max} \sin\left(\omega t - \frac{\pi}{2}\right) - I_{max}\ \varepsilon^{-\frac{R}{L}\cdot t}$$

The transient is now negative, its initial value being $-\ I_{max}$, so that the conditions are similar to those of Case I except that the current rises to very nearly $-2I_{max}$ instead of $+2I_{max}$. Thus cases I and III give the extreme values for the total instantaneous current attained. Closing the circuit at all other instants gives a smaller transient, and in particular the transient is zero if the circuit is closed at the instant when the E.M.F. is a maximum, either positive or negative.

Short-circuit of an Alternator

The case of an alternator is not so simple as the above, because, in addition to possessing resistance and inductance, the armature, by virtue of its reaction on the main field, can control the resultant E.M.F. acting in it. The first point to notice is that, since the transient is not an alternating current but a direct current of exponentially decreasing magnitude, it will set up in the armature an M.M.F. which is stationary with respect to the armature itself. Hence with respect to this M.M.F. the field system will rotate at synchronous speed, and will therefore have induced in it an E.M.F. of fundamental frequency but of gradually diminishing amplitude. This E.M.F. will be superposed on the steady E.M.F. derived from the exciter. In the second place, the true alternating current will set up an armature M.M.F., which will rotate at synchronous speed and therefore keep pace with the field system. If we imagine the alternator delivering its normal load at normal power factor, the armature reaction M.M.F. will be proportional to the current and, owing to the load power factor, will be partially cross-magnetising and partially demagnetising. The excitation will have been adjusted to overcome the demagnetising component.

Now let the alternator be short-circuited suddenly. First of all the load current will increase in view of the fact that the only impedance to be overcome is that of the alternator windings. Hence, in view of the large ratio of reactance to resistance, this current will no longer have the angle of lag corresponding to the power factor of the external load, but will lag by nearly $\pi/2$. A direct consequence of this is that the whole of the armature M.M.F. due to the greatly increased sinusoidal component of the total current will be almost entirely demagnetising in action. But it is impossible suddenly to wipe out part of the main flux by the establishment of the demagnetising effect, for two reasons: (1) the field winding of a large alternator has a large self-induction and therefore a very long time constant, and consequently some appreciable time is taken to effect any change in the total flux; (2) any change in flux will alter the flux linkages with the solid pole faces and will, in consequence, induce heavy currents in the pole faces by transformer action.

If E is the E.M.F. generated by the main flux in the air gap before the short-circuit it might be thought that the initial short-circuit current would be $I = E/x_L$ where x_L is the leakage reactance. This, however, is not strictly true because, as the short-circuit current in the armature grows during the first 1/4 cycle, it tends, by its demagnetising action, to reduce the main flux. In conformance with the constant-linkage theorem, however, the flux linked with the field winding tends to remain constant and a current is induced in the field winding tending to maintain it; this induced current sets up additional field leakage flux so that there is a corresponding initial reduction in the main air-gap flux and a consequent reduction in the initial generated E.M.F. to, say, E′. The initial short-circuit current is thus $I = E'/x_L$;

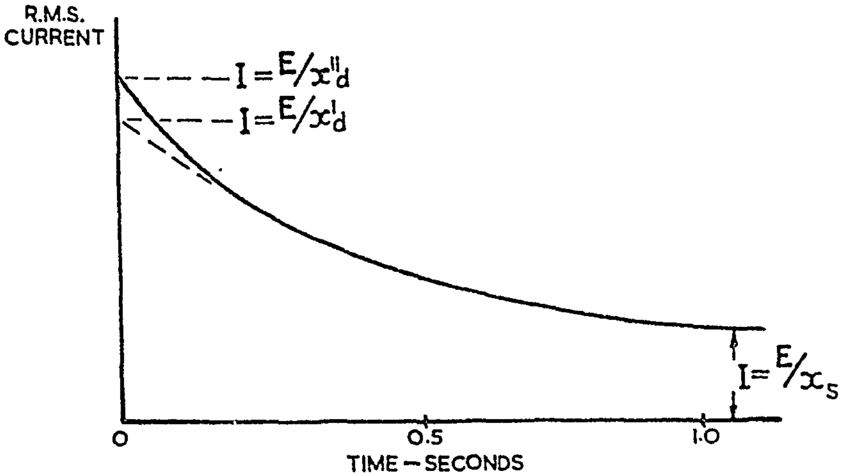

FIG. 15.2.—TYPICAL DECREMENT CURVE FOR A SYNCHRONOUS MACHINE.

for convenience it is usual to write this $I = E/x'_d$ where x'_d is a fictitious reactance known as the *transient reactance* and is slightly greater than x_L.

If damper windings are fitted or if the machine has solid pole faces as in turbo-alternators, currents are induced in these rather than in the field winding. As the leakage flux associated with such currents is very small, due to their physical position close to the air gap, there will be a negligible drop in the main flux and the initial short-circuit current will be practically $I = E/x_L$; it is customary, however, to write it as $I = E/x''_d$, where x''_d is known as the *sub-transient reactance*. For all practical purposes x''_d may be taken as equal to x_L.

A curve showing how the short-circuit current varies with time, and known as a *decrement curve*, is shown in Fig. 15.2. Modern practice is to interrupt short-circuit currents within a few cycles of their occurrence so that current decrement due to the action of the alternator has hardly time to occur. Normal procedure in calculating the R.M.S. value of short-circuit currents is therefore to use the transient reactance of the alternator together with the other circuit reactances; if the initial peak value is to be found, the sub-transient value may be used together with a factor of 1·8 to allow for the doubling effect (Case I, p. 355) and a factor of $\sqrt{2}$ to give the peak value. For a sustained short-circuit, during which the full demagnetising action of the armature has time to become fully effective, the current is governed by the synchronous reactance, x_s. Typical values of these reactances are given in Table I.

TABLE I

TYPICAL VALUES OF REACTANCE FOR SYNCHRONOUS MACHINES (PER-UNIT VALUES)

	Sub-transient Reactance. x''_d	Transient Reactance. x'_d	Synchronous Reactance. x_s
Turbo-generator (up to 13 kV.)	0·1 –0·15	0·12–0·26	0·95–1·75
Turbo-generator (22–33 kV.)	0·17–0·2	0·2 –0·3	0·95–1·75
Salient-pole generator (no dampers)	0·21–0·34	0·25–0·4	0·6 –1·5
Salient-pole generator (with dampers)	0·17–0·28	0·25–0·4	0·6 –1·5
Synchronous capacitor	0·16–0·37	0·25–0·6	1·5 –2·2

General Methods of Calculation

Two methods for the calculation of the short-circuit current or short-circuit kVA. at any point in a power system are available, the *ohmic-value* and the *per-unit-value** methods.

In the ohmic-value method the impedance in ohms (phase-to-neutral value) of each item is determined and the total impedance, Z, from the generator to the fault calculated. If, due to transformers, parts of the system are at different rated voltages, then all impedances must be referred to a common basic voltage, V_B (phase-to-neutral value), which may be any suitable value but is generally selected to be the rated voltage of some part of the system. Thus—

$$\text{Referred impedance } z' = z \,.\, (V_B/V_{actual})^2$$

The short-circuit current is then $I_{sc} = V_B/Z'$, referred to the basic voltage, and the short-circuit kVA. is $3 \,.\, V_B \,.\, I_{sc} 10^{-3}$ kVA.

* Impedance and other data are still frequently quoted as percentage values, but for calculation purposes per-unit (p.u.) values are often more convenient. $x\% = 100 \,.\, x_{pu}$ or $x_{pu} = x\%/100$

In the per-unit method all impedances are expressed in per-unit values from the defining equation—

$$\text{Per-unit impedance, } z_{pn} = I_f\, z/V_{ph}$$

where I_f is the full-load current, V_{ph} the rated phase voltage, and z the ohmic impedance per phase of the equipment in question.

The total per-unit impedance, Z_{pn}, from the generator to the fault can then be found by combining per-unit impedances by addition, parallel combinations, or star-delta transformations, exactly as with ohmic values. Where, however, the system comprises equipment having different full-load ratings, the per-unit values must all be referred to the full-load current corresponding to a common basic rating, $kVA._B$; this may be selected as the total generated kVA. or as any arbitrary value such as 100,000 kVA. Thus—

$$\text{Referred per-unit impedance } z'_{pn} = z_{pn}(kVA._B/kVA._{actual})$$

The short-circuit kVA. is then $kVA._{sc} = kVA._B/Z_{pn}$ and the short-circuit current is $kVA._{sc}/3V._{ph}$.

More complete details of the above procedures are given elsewhere,* but examples illustrating the per-unit methods are given below.

Example 1 (Fig 15.3)

Generator 10,000 kVA., 6,600 volts, 0·12 p.u. reactance.
Fault F close to bus-bars.

$$\text{Normal current} = \frac{10000 \times 1000}{\sqrt{3} \times 6600} = 878 \text{ amps.}$$

$$\text{Short-circuit kVA.} = \frac{10000}{0{\cdot}12} = 83300 \text{ kVA.}$$

$$\text{Short-circuit current} = \frac{83300 \times 1000}{6600 \times \sqrt{3}} = 7280 \text{ amps.}$$

This is the initial value of the short-circuit current; if the interruption were delayed by a few cycles, the alternator decrement would reduce, by an appreciable amount, the current that actually had to be interrupted by the circuit-breaker. If, however, the circuit-breaker were to be closed on to the short-circuit, the initial peak-making current would be 7280 1·8 $\sqrt{2}$ = 18,600 A and it would have to be designed to withstand the electromagnetic forces due to this current.

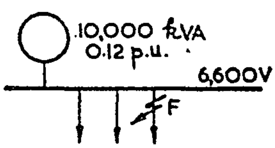

FIG. 15.3.

* Taylor, E. O., *Distribution and Utilisation of Electrical Energy* (Blackie). Lackey, C. H. W., *Fault Calculations* (Oliver & Boyd).

Example 2 (Fig. 15.4)

The same system as in Fig. 15.3, but with the fault F at the far end of one of the feeders. The full-load capacity of the feeder is 5,000 kVA. and its reactance 0·05 p.u.

$$\therefore \text{ Feeder reactance based on 10000 kVA.} = \frac{10000}{5000} \times 0{\cdot}05$$
$$= 0{\cdot}1 \text{ p.u.}$$

Hence total reactance based on 10000 kVA.

$$= 0{\cdot}12 \text{ (generator)} + 0{\cdot}1 \text{ (feeder)} = 0{\cdot}22 \text{ (total)}$$

$$\text{Short-circuit kVA.} = \frac{10000}{0{\cdot}22} = 45400 \text{ kVA.}$$

$$\text{Short-circuit current} = \frac{45400 \times 1000}{6600 \times \sqrt{3}} = 3970 \text{ amps.}$$

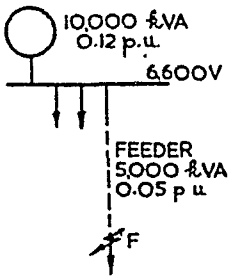

Fig. 15.4.

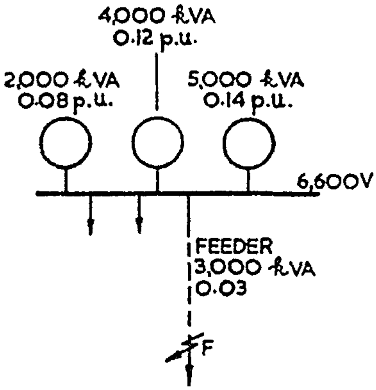

Fig. 15.5.

Example 3 (Fig. 15.5)

Three alternators in parallel:

A 2000 kVA. 0·08 p.u. reactance
B 4000 kVA. 0·12 p.u. reactance
C 5000 kVA. 0·14 p.u. reactance

Fault at far end of a feeder of 3,000 kVA. capacity and 0·03 p.u. reactance.

Total machine capacity = 2000 + 4000 + 5000 = 11000 kVA.

$$\therefore \text{ Normal current based on 11000 kVA.} = \frac{11000 \times 1000}{\sqrt{3} \times 6600}$$
$$= 965 \text{ amps.}$$

$$\text{Reactance of A based on 11000 kVA.} = \frac{11000}{2000} \times 0{\cdot}08 = 0{\cdot}44 \text{ p.u.}$$

Reactance of B based on 11000 kVA. $= \frac{11000}{4000} \times 0{\cdot}12 = 0{\cdot}33$ p.u.

Reactance of C based on 11000 kVA. $= \frac{11000}{5000} \times 0{\cdot}14 = 0{\cdot}31$ p.u.

These three reactances are in parallel, the total machine reactance to the same base thus being:

$$\frac{1}{\frac{1}{0{\cdot}44} + \frac{1}{0{\cdot}33} + \frac{1}{0{\cdot}31}} = 0{\cdot}117 \text{ p.u.}$$

Feeder reactance based on 11000 kVA.

$$= \frac{11000}{3000} \times 0{\cdot}03 = 0{\cdot}11 \text{ p.u.}$$

$\therefore$ Total reactance up to fault $= 0{\cdot}117 + 0{\cdot}11 = 0{\cdot}227$ p.u.

$$\text{Short-circuit kVA.} = \frac{11000}{0{\cdot}227} = 48300 \text{ kVA.}$$

$$\text{Short-circuit current} = \frac{48300 \times 1000}{6600 \times \sqrt{3}} = 4230 \text{ amps.}$$

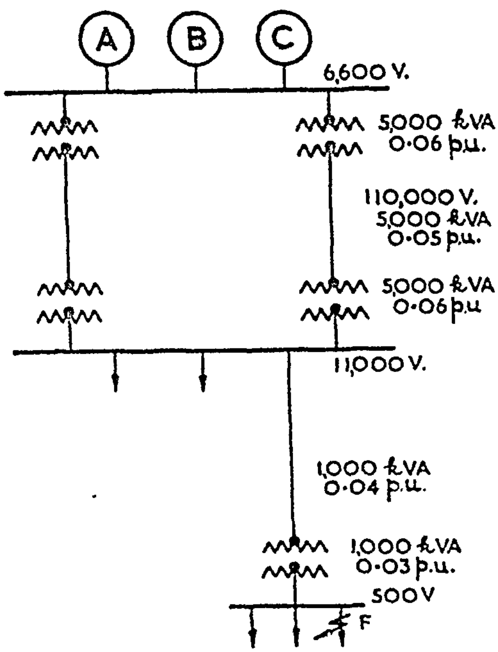

FIG. 15.6.

Example 4

The system is shown in Fig. 15.6, comprising three alternators, two step-up transformers, two transmission lines in parallel, two step-down transformers, secondary feeders each with step-down transformers supplying distributors and a fault on one of the distributors near the transformer.

Let the three alternators have the same capacities and reactances as in Example 3. Then machine capacity = 11,000 kVA., and the normal current based on this capacity is 965 amps.

Reactance of alternators based on 11000 kVA. $= 0{\cdot}117$ p.u.

Reactance of step-up transformer to same base $= \frac{11000}{5000} \times 0{\cdot}06 = 0{\cdot}132$ p.u.

Reactance of line to same base $= \frac{11000}{5000} \times 0{\cdot}05 = 0{\cdot}11$ p.u.

Reactance of step-down transformer to same base $= \frac{11000}{5000} \times 0{\cdot}06 = 0{\cdot}132$ p.u.

∴ Total reactance of one line with transformers $= 0{\cdot}132 + 0{\cdot}11 + 0{\cdot}132$

$= 0{\cdot}37$ p.u.

∴ Total reactance of two lines in parallel $= 0{\cdot}185$ p.u.

Reactance of secondary line to 11000 kVA. base $= \frac{11000}{1000} \times 0{\cdot}01 = 0{\cdot}44$ p.u.

Reactance of 1000 kVA. transformer to same base $= \frac{11000}{1000} \times 0{\cdot}03 = 0{\cdot}33$ p.u.

Hence total reactance reckoned up to the fault:

$$= 0{\cdot}117 + 0{\cdot}187 + 0{\cdot}44 + 0{\cdot}33$$
$$= 1{\cdot}07$$

Hence the short-circuit kVA.

$$= 11{\cdot}000/1{\cdot}07 = 10{\cdot}220 \text{ kVA.}$$

and the short-circuit current

$$= 10{\cdot}220 \;.\; 1000/\sqrt{3} \;.\; 500 = 11800\text{A.}$$

With a fault so far from the generators, the generator reactance is only a small proportion of the total and the generator decrement will have only a negligible effect. It will also be clear from Example 4 that lines or transformers of small capacity present a special feature, since their reactance when expressed on the basis of total plant capacity is so high. Thus, suppose one of the distributors had a transformer of capacity 100 kVA. and reactance 0·05 per unit, then this transformer could not pass more than $\frac{100}{0{\cdot}05} = 2{,}000$ kVA., no matter how great the total capacity of the generating plant might be.

The Use of Reactors

A reactor is a coil designed to have a large inductive reactance ($L\omega$) in comparison with its ohmic resistance. The purpose of reactors in transmission and distribution systems should thus be obvious from the previous section dealing with the short-circuit currents to be interrupted by the circuit-breakers. In a system of small extent and small capacity the inherent reactance of generators, transformers, and lines may be amply sufficient to limit the short-circuit kVA. at any point to a value which can be handled by the circuit-breakers. If such a system is extended by the installation of additional feeders and distributors in parallel with those already existing, and with corresponding additions to the generating plant, then a fault at the same location may result in the flow of a much greater current, and the current to be interrupted by the same circuit-breaker will be much greater than before. On the other hand, the current to be carried by this circuit-breaker during normal operation will be the same as before, and this illustrates the difference between the "carrying" capacity and the "breaking" capacity of a circuit-breaker.

With systems having large concentrations of generating plant and short interconnecting feeders it may therefore be necessary to increase the total reactance of the system by the inclusion of reactors at "strategic" points. The possible locations for such reactors are illustrated in Figs. 15.7 and 15.8.

Generator Reactors

Modern generators have sufficient reactance to safeguard the machines even in the event of a dead three-phase short-circuit at the terminals, and consequently in any new scheme external generator reactors should be unnecessary. If an old station is being remodelled and it is intended to retain the older machines along with their switchgear, then it will be justifiable to include a reactor in series with each old machine. It is impossible to give a figure for the magnitude of such a reactor, as it depends on the impedance of the alternator with which the reactor is associated, but 0·05 or 0·06 per unit* might be taken as a rough representative value. Except in such cases generator reactances are undesirable because of their large I^2R loss, and their effect on voltage regulation.

Feeder Reactors

Although the per-unit reactance of a feeder reactor may be small with respect to the kVA. of the feeder (or group of feeders) with which it is associated, this value will be large when referred to the total plant kVA., and consequently a small reactor will be effective in limiting the fault current in the event of a fault

* Reactors are normally rated by quoting the kVA. or MVA. that they are designed to carry and the per-unit or percentage impedance drop when carrying this rated load.

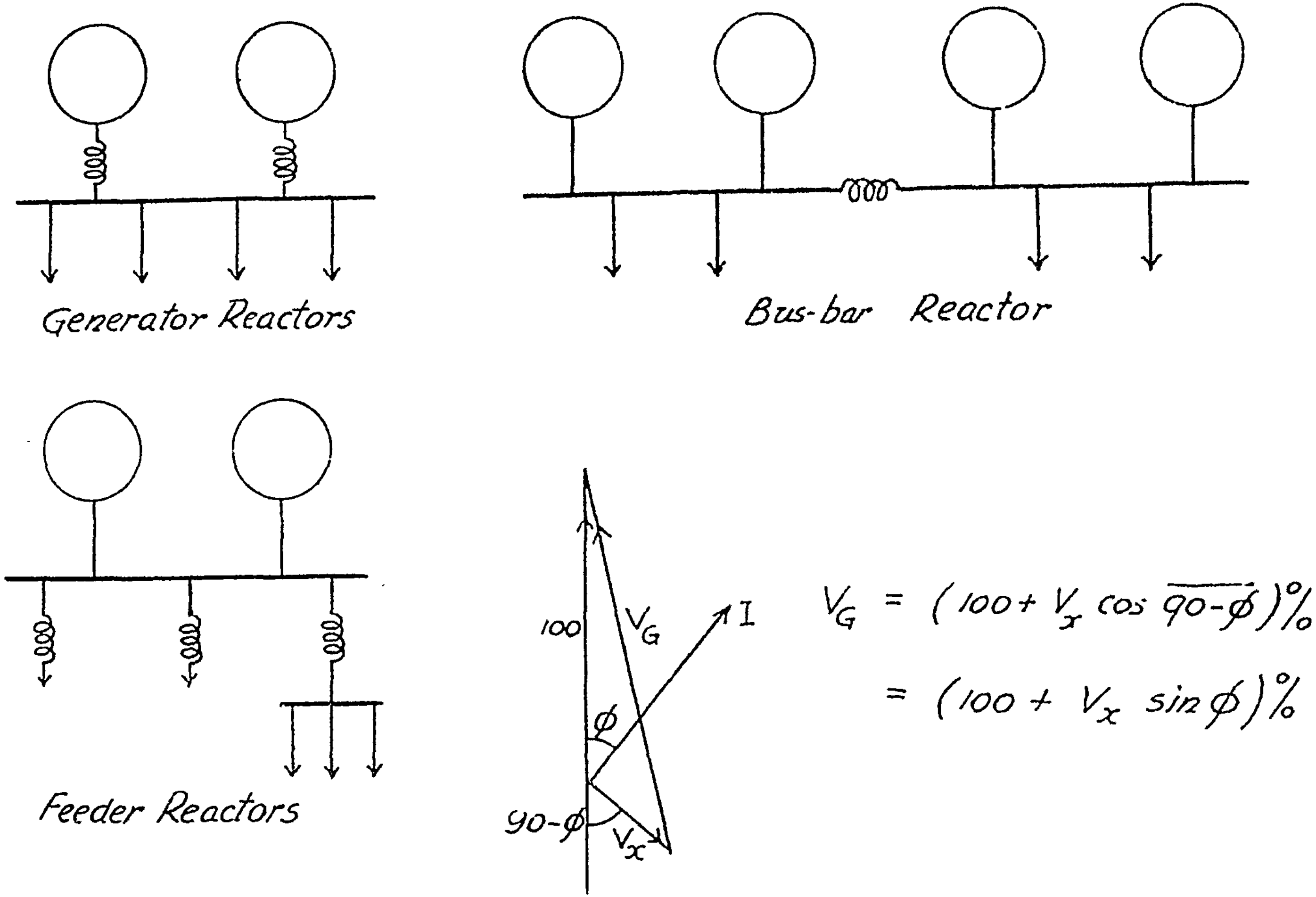

FIG. 15.7.—LOCATION OF REACTORS.

close to the generating station. Without this feeder reactance a fault in such a location would bring the bus-bar voltage almost down to zero, and as a result the various generators would probably fall out of parallel. The use of the reactor will permit the bus-bar voltage to be maintained and the generating plant to remain in parallel so long as the fault is on the load side of the

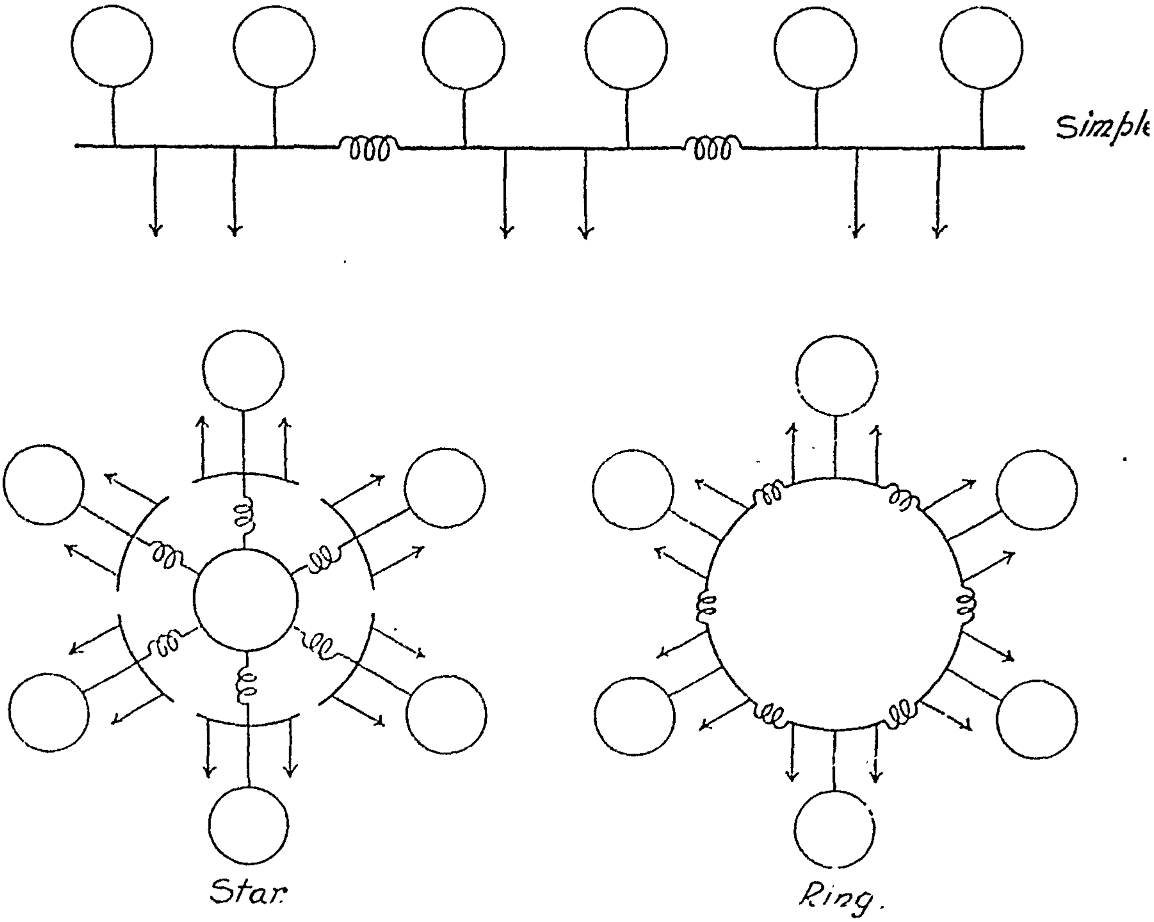

FIG. 15.8.—LOCATION OF REACTORS.

reactor. In a very valuable paper on the use of reactors, Major David* points out that an alternator takes 0·5 sec. to achieve full reactance value with dead short-circuit, but takes 3 to 5 sec. to recover after the short-circuit. With partial short-circuit, as with feeder faults, it takes up to 1·5 sec. to reach full reactance. If the feeder circuit-breaker opens rapidly, the bus-bars recover

* "Protection of A.C. Systems against Short-circuits," *The Mining Electrical Engineer*, November and December 1926.

voltage very quickly and synchronous machinery does not fall out of step.

It has already been seen that the quicker a circuit-breaker opens, the greater will be the current to be interrupted, and consequently to secure this quick opening and at the same time reduce the current interrupted, the feeder must be associated with additional reactance.

If the voltage at the load is 1·0 per unit, the per-unit reactance of the line x, and that of the reactor x', then, neglecting resistance, as is usual, we have for the generator voltage when the feeder is delivering its full kVA. at a power factor of $\cos \phi$:

$$1{\cdot}0 + (x + x') \sin \phi \qquad \text{per unit}$$

If the power factors of the various feeder groups are appreciably different, then for the same total reactance $(x + x')$ the regulations of the groups will be different. It is therefore advisable to group the high-power-factor feeders together and the low-power-factor feeders together. Also, it is not possible to compensate for the voltage regulation of the low-power-factor feeders by raising the bus-bar voltage, because this will make the voltage of the high-power-factor feeders too high. Hence, if the voltage regulation is required to be the same for the two groups, reactances of different value must be used. Let $\cos \phi_1$ and $\cos \phi_2$ be the power factors, and x'_1 and x'_2 the feeder per-unit reactances; then for equal voltage regulation, assuming equal feeder reactances:

$$x'_1 \sin \phi_1 = x'_2 \sin \phi_2$$

$$\therefore \frac{x'_1}{x'_2} = \frac{\sin \phi_2}{\sin \phi_1}$$

The magnitude of the feeder reactor should be such that in the event of a short-circuit close to the reactor, the bus-bar voltage will not be pulled down so low as to cause the generators to fall out of synchronism. Average values are from 0·05 to 0·12 per unit.

Bus-bar Reactors

Fig. 15.8 shows that generating-station bus-bars can be sectionalised and the various sections joined by bus-bar reactors. This application does not strictly come within the province of transmission, but it is given here for completeness. The presence of such reactors will also have some effect on the current flowing in the event of a fault external to the station, as well as an internal fault. It is clear that if the magnitude and phase of the bus-bar voltages are the same for the various sections, then the reactors will carry no current and the sections can be regarded as independent. If only the generators in one group are running, as in times of light load, then the reactors can be short-circuited and the station will then be operating without bus-bar reactors. The function of the bus-bar reactors is to confine any trouble as far as possible to one section, so that in the event of a very severe fault, say, a short-circuit on a bus-bar section, there is no danger of shutting down the whole station. A bus-bar reactor must therefore be large enough to protect adjacent sections, but not so large as to interfere with the successful parallel running of the various generator groups. A reactor which gives a drop of 30 to 50 per cent. of normal voltage when carrying the full current of one bus-bar section is suitable. Hence, referred to the kVA. rating of one section, the value will be 0·3 to 0·5 per unit.

There are three methods of connecting bus-bar reactors, as shown in Fig. 15.8. The simple system is suitable for stations of moderate output, the output of each group of generators being of the order of 30,000 to 50,000 kVA. Where the size of the station justifies a large number of groups, the reactors can be connected on the star or on the ring system. The point to be noted in connection with bus-bar reactors is that any transfer of power through a reactor necessitates a voltage drop and therefore a difference in the voltages of adjacent bus sections. If the current is at unity power factor, this drop will be in quadrature and the voltage difference between the bus sections will be very small. If, however, the current is wattless, then the drop across the reactor will be in phase with the bus-bar voltages and the voltage difference between the bus sections will be considerable. Since the allowable voltage difference between bus sections is very limited, it follows that large currents at unity power factor can be transferred with little disturbance to the bus-bar voltages, but that very little magnetising current can be transferred. Consequently, if it is desired to transfer power from Section I to Section II, the machines connected to Section II should, by excitation adjustment, be made to provide all the magnetising current required by the load on Section II.

With the star arrangement the whole of the current transferred from one section to another flows through two reactors in series whereas in the ring arrangement it flows through two paths in parallel. For the same protection there is little difference in size and cost except that, in the limit, the advantage is slightly with the star. The star design is also more flexible and for this reason is generally preferred unless there is some difficulty in accommodating the necessary tie-bar. There is also the possibility that with the ring system it may be necessary to open the ring at some time: in such a case there will be one path only for the current transferred, the magnitude of the maximum possible transfer thus being reduced.

Construction of Reactors

There are two types of reactor in use, viz. air-insulated and oil-immersed types. The type of air-insulated construction usually employed in this country is to embed circular coils of stranded copper conductor in a series of concrete pillars arranged in circular

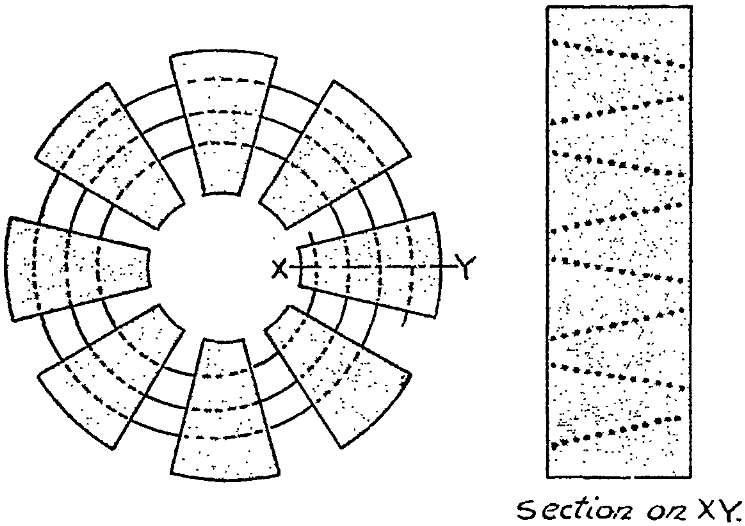

FIG. 15.9.—CONSTRUCTION OF AIR-CORED CONCRETE REACTOR.

formation, as shown in Fig. 15.9. The whole is placed on a concrete slab, and mounted on porcelain footstep insulators. The conductor is usually bare, so that the reactor must be placed in a stone or brickwork cell, as the conductor is at full phase potential to earth.

The oil-immersed type employs insulation and cooling arrangements similar to those of the ordinary transformer, as shown in Fig. 15.10. If an air-cored construction is used, there must be laminated-iron shields or copper shields around the outside of the conductors in order to prevent the magnetic flux entering the tank walls and producing excessive losses and heating. With the iron-cored type air gaps are introduced in the core to prevent saturation and to give a magnetising current of the desired value.

So far as the comparison of the two types is concerned, it will be obvious that there is a voltage limit of about 33 kV. to the air-insulated type so that for very high voltages the oil-immersed type is necessary. The air-insulated type is very simple from the constructional point of view and is robust, but the room taken up is rather large because of the large clearances and also stray flux may cause heating in adjacent metalwork. The air-cored type, having no iron, has a constant reactance at all currents, but the reactance of the iron-core or iron-shielded types may drop by about 10 per cent. due to saturation at the highest currents.

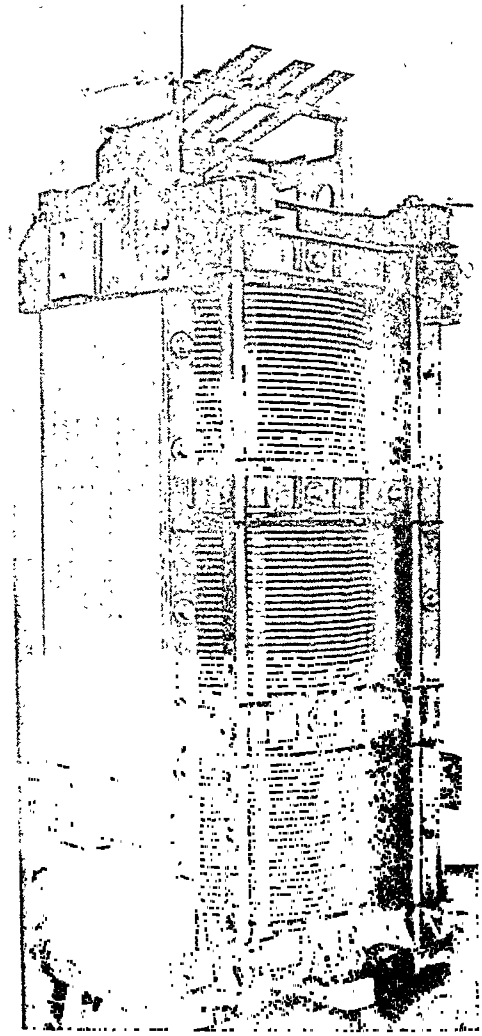

FIG. 15.10.—IRON-CORED, OIL-IMMERSED REACTOR.
(*Bruce Peebles & Co. Ltd.*)

Circuit-breaker Principles

When current-carrying contacts are parted, an arc is formed and this arc plays an essential part in the circuit-interrupting process. The arc provides a gradual transition from the current-carrying to the voltage-isolating states of the contacts, but it is dangerous on account of the energy generated in it in the form of heat which may result in explosive forces. The science of circuit-breaker design is therefore governed by the necessity of so controlling the arc that it can cause no damage to equipment or danger to personnel.

Two methods are available for extinguishing the arc—the *high-resistance method*, in which the arc resistance is made to increase rapidly by lengthening and cooling so that the current is quickly reduced to a value which is insufficient to maintain the arc; and the *low-resistance* or *zero-point method*, applicable only to A.C. circuits, in which the resistance is kept low and the arc prevented from restriking after it has gone out at a current zero. The high-resistance method is employed in D.C. circuit-breakers and medium-power industrial-type air-break circuit-breakers; low-resistance zero-point interruption is used for oil and air-blast circuit-breakers as used on transmission and distribution systems. Only the latter method is discussed in what follows.

Low-resistance Zero-point Interruption

After the arc has gone out at the end of a half-cycle of the current, the space between the contacts contains ions and electrons so that it has a finite resistance (*post-zero* resistance) and can fairly easily be broken down again by the rising contact voltage; if such a breakdown does occur, the arc will persist for another half-cycle, when the process will be repeated. The problem is therefore, immediately after the current zero, to remove the ions and electrons either by causing them to recombine into neutral molecules or by sweeping them away so that the rising contact voltage cannot break down the contact space.

The recombination occurs naturally and very rapidly but can be accelerated by cooling the arc space and also by putting it under mechanical pressure; with high-power circuit-breakers it is also necessary to sweep away the ionised gases either by a blast of cool oil or by a blast of air. These two processes lead to the two main types of circuit-breaker for transmission and distribution purposes, i.e. the oil circuit-breaker and the air-blast circuit-breaker.

The Oil Circuit-breaker

It was found very early in the history of circuit-breaker development that immersing the contacts in oil had an advantageous effect. The arc decomposes the oil into its constituent gases so that it actually takes place in a gas bubble; this process absorbs some of the arc energy and so keeps the arc space cool, and at the current zero the pressure due to head of oil above the contacts causes cool oil to be forced into the contact space, thus interposing an insulating barrier between the contacts and preventing the restriking of the arc. When interrupting heavy short-circuit currents, the pressure due to the head of oil is insufficient and a suitable pressure must be produced artificially.

Three types of oil circuit-breaker thus emerge—

(i) Plain-break Type, in which the oil pressure is due solely to the head of oil above the contacts.

(ii) Self-generated-pressure Type, in which the arc itself is used to set up the necessary pressure (Explosion-pot Type).

(iii) Externally-generated-pressure Type, in which the pressure is produced by an external device (Impulse Type).

The Plain-break Circuit-breaker

The usual arrangement is shown in Figs. 15.11 and 15.12, two contacts being enclosed in an earthed metal tank containing oil. It can be seen that by using two contacts in this way it is not necessary to lead current into any moving part resulting in a

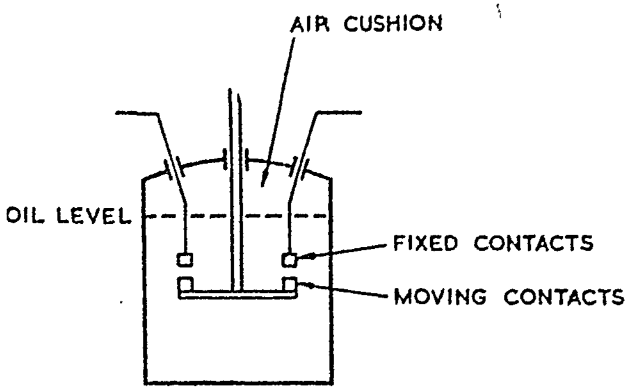

FIG. 15.11.—DIAGRAM OF A PLAIN-BREAK BREAKER

simple and very widely-used construction. The oil acts both as an arc-extinguishing medium and as an insulator between the live parts and earth.

The relationship between the volume of oil and the breaking capacity is linear, the volume required being practically proportional to the breaking capacity. For a breaking capacity of 400 MVA. the volume is about 150 gallons.

The air cushion acts as a buffer and affects the tank pressures, oil throw, arc stabilising, secondary explosions, and gas ignition. It is thus bound up with the head of oil, since, with a small head, there may be a continuous stream of liberated gas from the contacts to the oil surface, giving rise to the so-called "chimney" effect. The diffusion of such gases into the air cushion may form an explosive mixture which may be fired by compression or by a hot gas bubble on a second fault. The volume of the air cushion is usually about one-eighth of the tank volume. A vent pipe should always be employed with circuit-breakers above, say, 100-MVA. breaking capacity, but is not necessary for sizes smaller than this. It is obvious that the position of the outlet should be chosen carefully, otherwise a fire may be easily started.

Past experience with circuit-breaker explosions has demonstrated that a strong tank is a necessity, and in this country it is

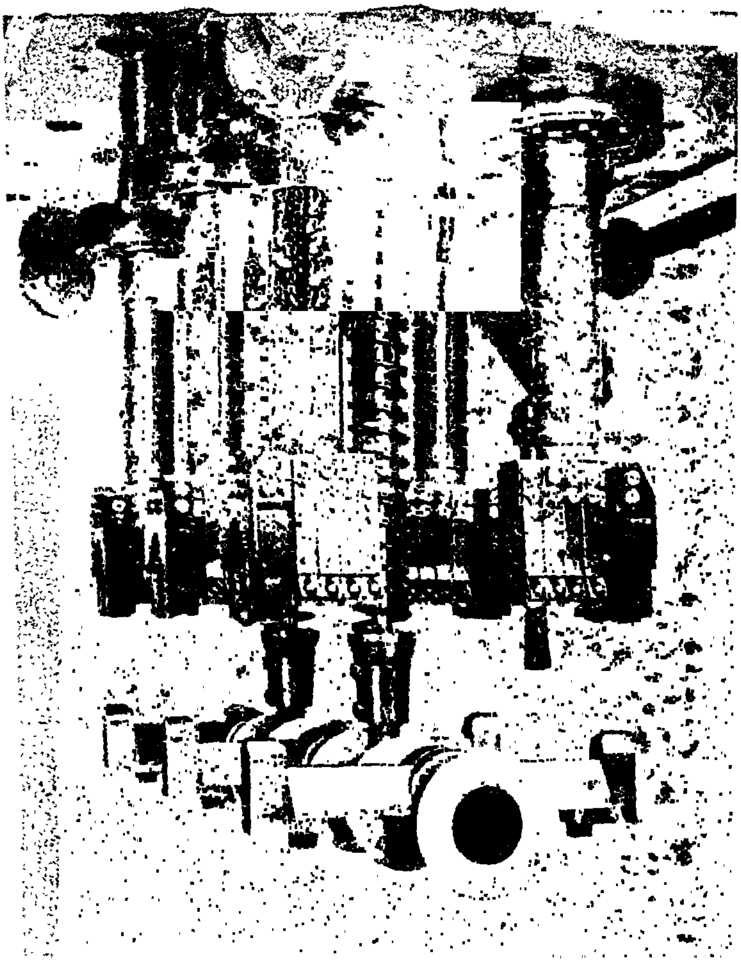

FIG. 15.12.—VIEW OF FIXED AND MOVING CONTACTS.
(*Reyrolle & Co. Ltd.*)

usual to make the tanks and the top-plates of welded sheet steel or boiler plate.

The distance between phases, and the clearances between live metal and earthed metal, are of importance from the point of view of normal operation, and are of still greater importance when abnormal conditions occur, since with inadequate clearances a short-circuit between phases, or between one phase and earth, may be set up and the circuit-breaker destroyed. Inside the tank the clearances can be artificially increased by interposing insulated barriers.

Circuit-breakers of the plain-break type are not now considered suitable for short-circuit ratings above about 150 MVA. at 11 kV.

Self-generated-pressure Circuit-breakers (Explosion-pot Type)

In order to accentuate the movement of cool oil into and across the arc space, the contacts may be surrounded by a pressure

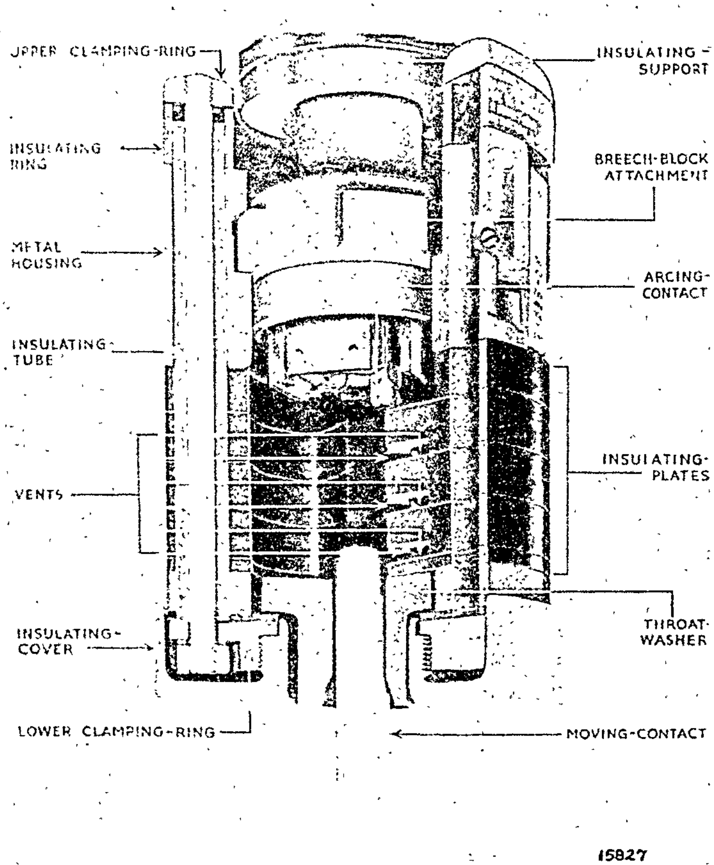

FIG. 15.13.—CONSTRUCTION OF ONE FORM OF EXPLOSION POT.
(*Reyrolle & Co. Ltd.*)

chamber. The first device of this nature was the explosion pot, which was merely a cylinder enclosing the contacts, closed at the top but with a restricted opening at the bottom through which the moving contact passed. The arc was established in a confined space and the products of the arc were violently expelled through the lower opening. The device has the effect of controlling the nature of the turbulence to a certain extent, thereby giving a certain degree of consistency. A modern form of explosion pot is illustrated in Fig. 15.13. The pot consists of a stack of plates of insulating material shaped so as to form a series of channels lying at right angles to the normal arc path, i.e. horizontally when the moving contact moves vertically downwards. The moving contact just clears these plates where it passes through the central holes, with the result that there will be little tendency for any arc products to be expelled downwards. There is a side pocket called the pressure chamber, and when an arc is drawn by the opening of the contacts the high pressure generated in this restricted chamber causes the arc products to be violently expelled through the horizontal channels. The multiplication of these channels divides the arc into sections, each of which is literally blown out by the lateral motion of the arc products, their place being taken by cool dissociated oil.

Since the pressure is set up by the action of the arc itself, it will be greater with heavy currents. The problem in designing an explosion-pot device is therefore to ensure sufficient pressure to extinguish the arc at low currents without having excessive pressures, which might burst the chamber, at heavy currents. The solution of this problem has led manufacturers to design chambers differing in detail and given various trade names such as Cross-jet, Turbulator, Deion-grid, Self-compensated, etc.

These arc-control chambers give excellent service in circuit-breakers up to the highest ratings that have been called for; they may be applied equally well to the earthed-tank type of circuit-breaker as shown in Fig. 15.12 or to the live-tank types described later.

Externally-generated-pressure Circuit-breakers (Impulse Type)

Since the pressure produced in the self-generated-pressure circuit-breaker is low at low currents, arc extinction at such currents may take longer than at heavy currents. To avoid this possible inconsistency at low currents, designs have been developed which incorporate a piston which gives a positive oil movement across the contact space quite independently of the current magnitude. In some designs this movement is relied upon at all currents, while in others it is relied on at low currents and the self-generated-pressure principle used at high currents. Owing to the excellence of the performance of the self-generated-pressure type, together with the added complication of the piston mechanism, the externally-generated-pressure type is not frequently used.

The Air-blast Circuit-breaker

The disadvantages of the oil circuit-breaker are the fire risk due to the inflammable oil, the deterioration of the oil, necessitating periodic replacement, and the difficulty of reaching the contacts for maintenance purposes. This led to the development, initially in Europe, but subsequently in this country, of circuit-breakers using air as the interrupting medium. To secure effective results with medium and high-power circuit-breakers, a fairly high air pressure (up to about 300 lb./in.^{2}) must be used, and this can be practicably obtained from only an external source. Although the self-generated-pressure principle has been used for circuit-breakers having low ratings—e.g. for interrupting load currents—the externally-generated-pressure principle, using a separate air-compressor system, is generally employed.

The flow of the blast of compressed air around the contacts may be axial, radial, or cross, as shown diagrammatically in Fig. 15.14. Axial or radial blast construction is most common,

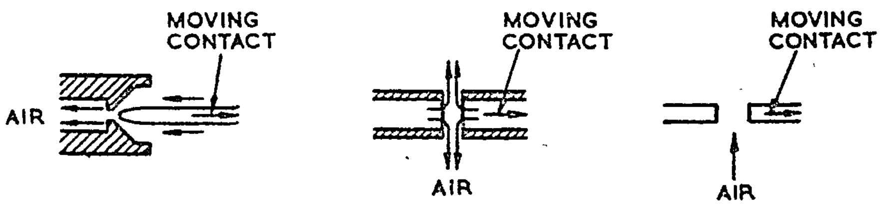

FIG. 15.14.—CONTACT ARRANGEMENT FOR AIR-BLAST BREAKERS.

although the cross-blast arrangement has been commonly used by American manufacturers for medium-voltage (up to 15 kV.) high-power units.

When interrupting low currents, the very forcible blast of air may cause the arc to be blown out before the current zero is reached, a process known as *current chopping*, and this sudden cessation of current may set up high transient voltages in the system. Provided, however, that the distance between the contacts is small, the contact space will break down again before the transient voltage has reached a dangerous value. It is therefore necessary that the distance between the contacts should not exceed a few cm. Such a short break would not be sufficient to withstand the normal system voltage unless at a high air pressure. It is therefore necessary to maintain the contact space continuously under pressure when the circuit-breaker is open or, alternatively, to provide an additional break, in the form of a simple isolating switch, in series with the main break, this being automatically opened as soon as the arc has been extinguished at the main break; the air pressure in the main contact chamber need then be maintained for only the one or two seconds

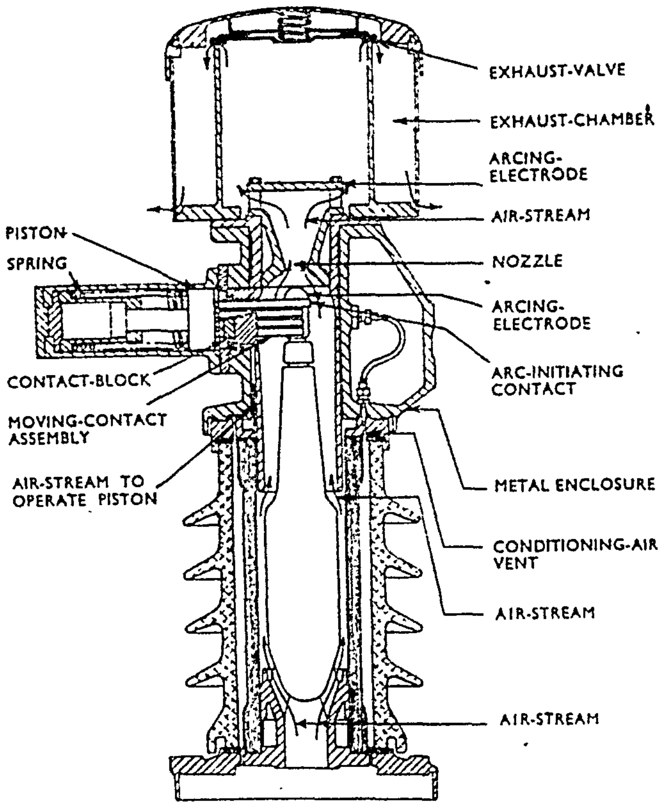

FIG. 15.15.—INTERRUPTOR HEAD FOR AIR-BLAST BREAKER. (*Reyrolle & Co. Ltd.*)

necessary to open the isolating switch. Most air-blast circuit-breakers employ the isolating switch, although successful designs have been made which operate with the contacts continuously under pressure.

In the usual construction the moving contact is normally held closed by a spring. When opening is required, compressed air is admitted which forces the moving contact to the open position, provides the air blast for extinguishing the arc, and opens the isolating switch; as soon as the isolator is fully open, the air supply is cut off and the main contacts reclose under the action of the spring. Making of the circuit, when the circuit-breaker has to be closed, is done on the isolator.

A cross-section of a typical interrupting head is shown in Fig. **15.15**, two or more such heads being arranged in series on a

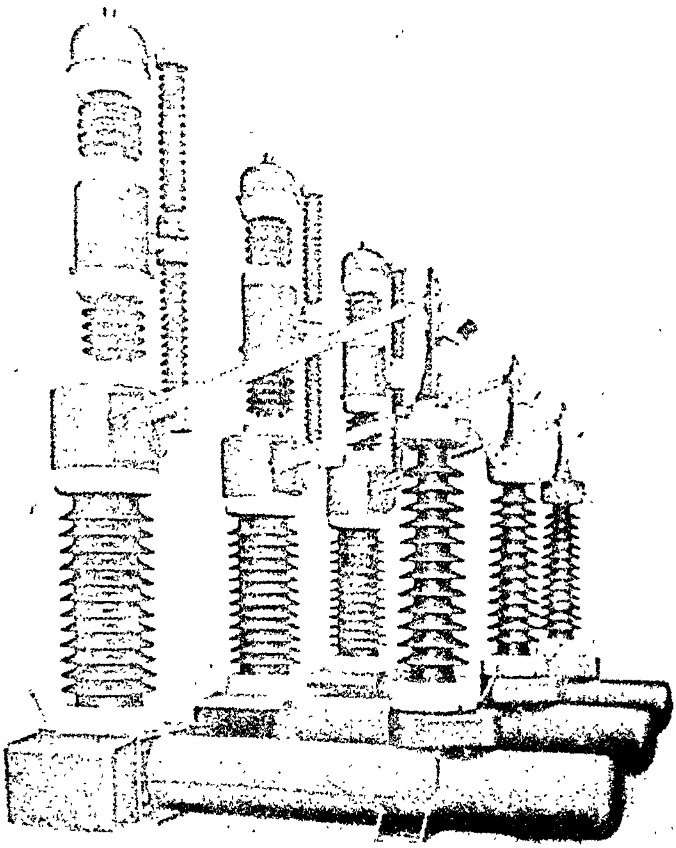

FIG. 15.16.—VIEW OF INTERRUPTOR HEAD SHOWING ISOLATING SWITCH.
(*English Electric Co. Ltd.*)

high-power circuit-breaker. The general arrangement, including the isolating switch, is shown in Fig. 15.16.

Effect of Circuit Conditions on Behaviour

It became clear many years ago that the circuit in which a circuit-breaker was connected had an important influence on its behaviour. The development of the cathode-ray oscillograph, which permitted the recording of voltages immediately after the current zero, showed that the rate of rise of the restriking voltage was of extreme importance.

A typical oscillogram showing currents and voltages during the interruption of a short-circuit current is shown in Fig. 15.17. In this case there is no current until the short-circuit occurs, when it rises to a high value. As soon as the contacts separate, an arc is formed and the *arc voltage* appears across the contacts. When the arc goes out at the current zero, the circuit voltage, which is near its peak value since a short-circuit current is, generally almost 90° lagging, begins to appear across the contacts. This rising voltage is the *restriking voltage* and will probably restrike the arc so that it persists for another half-cycle. At some subsequent current zero, however, the contacts will have separated sufficiently far, and the ionised gas between them will have been removed, so that the dielectric strength of the contact space will exceed the restriking voltage and the gap will not break down. Interruption is then complete and the normal circuit voltage, possibly modified by armature reaction and other effects, will appear across the contacts; this normal frequency voltage is known as the *recovery voltage*.

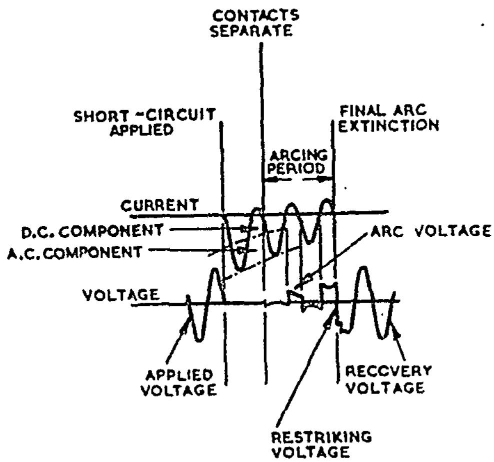

FIG. 15.17.—VARIATIONS OF CURRENT AND VOLTAGE DURING OPERATIONS.

The mechanism of the interruption has been described as a race between the building up of the dielectric strength in the contact space and the building up of the restriking voltage across it; if the former increases sufficiently rapidly, then interruption is effective.

An alternative explanation is given by the energy-balance theory. Immediately after the current zero, the contact space contains ionised gas and therefore has a finite post-zero resistance. The rising restriking voltage causes a current to flow between the contacts so that power is dissipated (as heat) in the contact space. Initially, when the restriking voltage is zero, the power is zero and it is again zero when the space has become fully de-ionised and the resistance is infinitely high; between these two limits the power dissipated rises to a maximum. If the heat so gener-

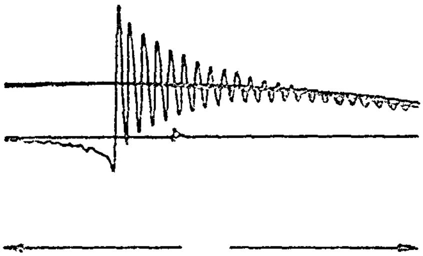

FIG. 15.20.—OSCILLOGRAM OF RESTRIKING VOLTAGE.

Assuming zero time to be at the current zero, when $t = 0$, $di/dt = w\, i_{max} \sin wt = E_{max}/L$

$$\therefore \frac{E_{max}}{L} = \frac{e}{L} + C\frac{d^2e}{dt^2}.$$

The solution to this standard differential equation is

$$e = E_{max}\,[1 - \cos\,(t/\sqrt{LC})]$$

Inserting the numerical values

$$E = \sqrt{2}\cdot 191\,[1 - \cos\,(t/\sqrt{10\cdot 10^{-3}\, 0\cdot 0210^{-6}})]\ \text{kV}.$$
$$= 27\,[1 - \cos\,(t.10^5/1\cdot 414)]\ \text{kV} \quad \text{if } t \text{ is in seconds}$$
$$= 27\,[1 - \cos\,(0\cdot 0707t)]\ \text{kV} \quad \text{if } t \text{ is in } \mu\text{sec.}$$

Plotting this gives the curve of Fig. 15.21, from which it is seen that the restriking voltage rises to $2E_{max}$. In the absence of any circuit resistance this oscillatory voltage would continue indefinitely, the frequency being $f = (1/2\pi)(1/\sqrt{LC}) = (1/2\pi)(1/\sqrt{10\cdot 10^{-3}\, 0\cdot 0210^{-6}}) = 11200$ c/s. Resistance is, however, always present, e.g. the post-zero resistance of the arc, and may be represented by a resistor R in parallel with the L and C already considered. Thus $i = i_R + i_L + i_C$ leading to a damped oscillatory voltage given by

$$e = E_{max}\,\{1 - \varepsilon^{-at}\,[\cos\sqrt{b}\,t + (a/\sqrt{b})\sin\sqrt{b}\,t]\}$$

where $a = 1/2CR$ and $b = 1/LC - 1/4C^2R^2$. Such a voltage is shown in Fig. 15.19.

If $R^2 < 0.5\sqrt{LC}$ (critical damping) the restriking voltage is non-oscillatory and rises relatively slowly to the value of the recovery voltage.

It is normally only the first half-cycle of the restriking voltage that is of interest so that a calculation neglecting the resistance is usually adequate. The rate of rise of restriking voltage may be specified as being the average rate from zero to the first peak or by the maximum rate represented by the maximum slope of the voltage curve. No formal standard has yet been laid down.

The average rate is the rise from O to A in 44·4 μsec, i.e. a rate of 54000/44·4 = 1220 V/μsec.

The maximum rate is the slope of the line BC which is 3820 V/μsec.

Determination of Rate of Rise of Restriking Voltage

Calculation of rates of rise of restriking voltage may be difficult, partly on account of the complicated nature of the network and partly because of the difficulty of determining the values of the L and C parameters.* They can, however, be measured experimentally on actual systems, and a considerable quantity of data has been amassed by the Electrical Research Association in this country and by similar organisations abroad.† These measurements can be carried out by means of a high-voltage cathode-ray oscillograph connected across the circuit-breaker contacts when it is interrupting a short-circuit on an actual system or by means of a device known as a *restriking-voltage indicator*. It has already been explained that conditions immediately following the current zero result from the sudden application of a voltage E_{max} across the circuit-breaker contacts. Assuming all circuit constants to

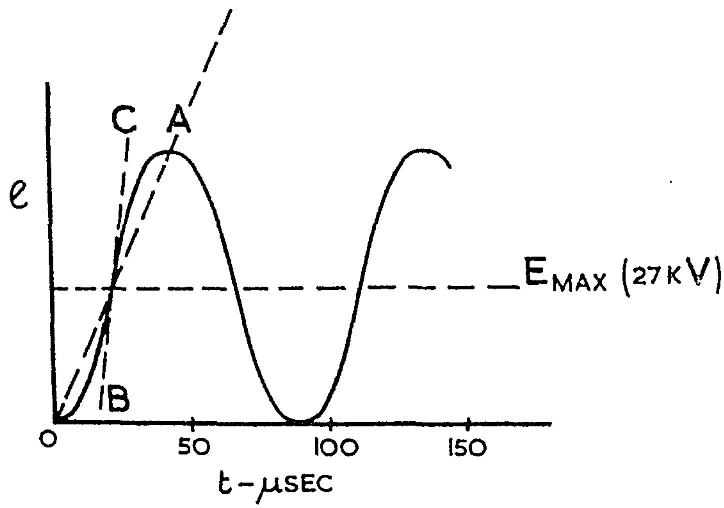

FIG. 15.21.—RESTRIKING VOLTAGE CALCULATION.

be linear, the magnitude of this voltage will not influence the shape of the resulting oscillatory current, so that if the system can be made dead a low voltage (100–200 V.) can be applied across the circuit-breaker contacts and the resulting transient voltage recorded. Furthermore, the voltage can be applied many times per second, using a recurrent surge-voltage generator, so that a visual picture of the restriking voltage can be seen on the screen of an ordinary low-voltage cathode-ray oscillograph. The complete assembly of the recurrent surge generator and the oscillograph forms the restriking voltage indicator which is widely used for determining rates of rise of restriking voltage.

* Taylor, E. O., and others, *Power System Transients*, Newnes (1954). Mortlock, J. R., "Evaluation of Restriking Voltage," *Journ. I.E.E.*, 92 (II), p. 562 (1945).

† Trencham, H., *Circuit-Breaking*, Butterworth Scientific Publications.

ated exceeds the rate at which heat can be dissipated from the contact space, ionisation will persist and breakdown will occur, giving an arc for another half-cycle.

Both explanations are to some extent true, and research work on the precise phenomena occurring at and immediately after current zero is still being actively pursued. It appears, however, that the energy-balance theory is more effective in explaining the behaviour of the oil circuit-breaker and the voltage-race theory is preferable for the air-blast circuit-breaker.

With both theories it is evident, however, that the rate at which the restriking voltage rises is of fundamental importance. This rate of rise of restriking voltage is a property of the circuit to which the circuit-breaker is connected, and not of the circuit-breaker itself.

Rate of Rise of Restriking Voltage

Consider a simple circuit containing a circuit-breaker as shown in Fig. 15.18 where L is the circuit inductance and C the capacitance to earth, e.g. of cables, switch bushings, etc. If the circuit-breaker is closed, the capacitance will have no voltage across it and the short-circuit current will lag the voltage by 90°; at current zero the E.M.F. will therefore be at its peak value E_{max}. If the current ceases at this moment, the conditions are thus as if the voltage E_{max} were applied to a circuit comprising L and C in series; as is well known this results in an oscillatory current of frequency $1/2\pi\sqrt{LC}$, giving a peak voltage across the capacitor of 2 E_{max} as shown in Fig. 15.19. This oscillatory voltage across the capacitor also appears across the circuit-breaker contacts and is, in fact, the restriking voltage. Due to circuit resistance the oscillatory current is quickly damped out, leaving the normal-frequency recovery voltage across the contacts.

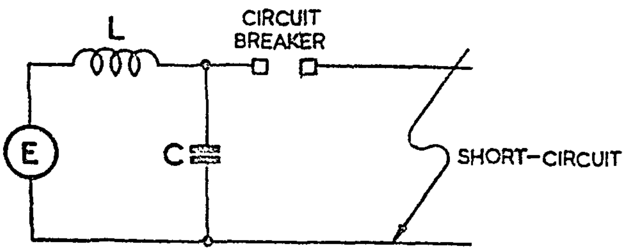

FIG. 15.18.—INTERRUPTION OF SIMPLE SHORT-CIRCUIT.

The important features of the restriking voltage are, firstly, that it rises to a value of about 2 E_{max} and, secondly, that it rises extremely rapidly. The rate of rise depends partly on the recovery voltage which governs the value that it ultimately reaches and partly on the above-mentioned natural frequency of the system. Typical values of this rate of rise of restriking voltage vary between about 500 V/μs for an extensive system, where C is relatively large, to about 5000 V/μs when the short-circuit and the circuit-breaker are near the terminals of a generator and C is small. A typical oscillogram of a restriking transient is shown in Fig. 15.20. An actual system is less simple than that shown in Fig. 15.18, and the restriking voltage may be made up of two or more voltages of different frequencies.

A simple illustration of a restriking voltage calculation is given in example 5.

Example 5. In a system that can be reduced to the circuit of Fig. 15.18 the voltage is 19·1 kV., L is 10 mH and C is 0·02 μF. Determine the rate of rise of restriking voltage when the circuit-breaker opens.

If the generator voltage is $E = E_{max} \sin wt$ the short-circuit current before the circuit-breaker opens is $i = (E_{max}/wL) \cos w t$. Assuming the current to be interrupted at the zero of the current wave the opening condition may be obtained by assuming a voltage, e, to be applied between the switch contacts such that a current equal and opposite to the short-circuit current would result. The voltage necessary to set up this current is then identical with that which would occur as the circuit is interrupted.

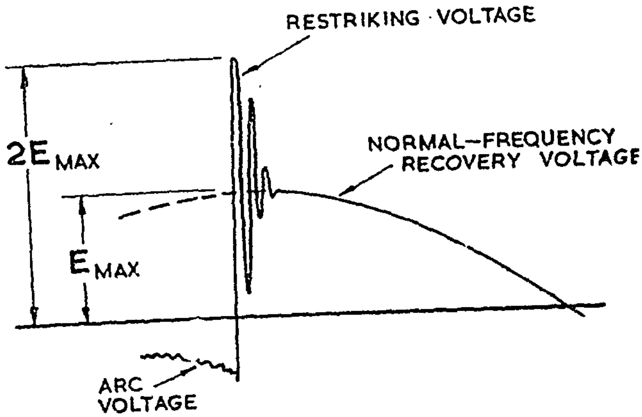

FIG. 15.19.—ILLUSTRATING RESTRIKING VOLTAGE AND RECOVERY VOLTAGE.

The current that has to be injected at the switch terminals is thus $i \cos wt$ and this current is made up of two components i_L through the inductor L and i_c through the capacitor C, these being in parallel when viewed from the switch terminals. Thus

$$i = i_L + i_C$$

and if e is the voltage setting up these currents

$$i = \frac{1}{L}\int e\,dt + C\frac{de}{dt}$$

$$\therefore \frac{di}{dt} = \frac{e}{L} + C\frac{d^2e}{dt^2}$$

It is important to realise, however, that the restriking voltage indicator gives the inherent rate of rise for the system; when the circuit-breaker is interrupting a short-circuit, the rate of rise may be modified by the post-zero resistance between the contacts since this is in parallel with the circuit. This post-zero resistance has a damping effect on the transient voltage and tends to reduce its rate of rise; if the resistance is less than $0{\cdot}5\sqrt{(LC)}$ the transient may be completely damped out.

In the plain-break oil circuit-breaker the post-zero resistance is generally fairly low (hundreds of ohms), so that the rate of rise of restriking voltage is usually damped down to a fairly low value;

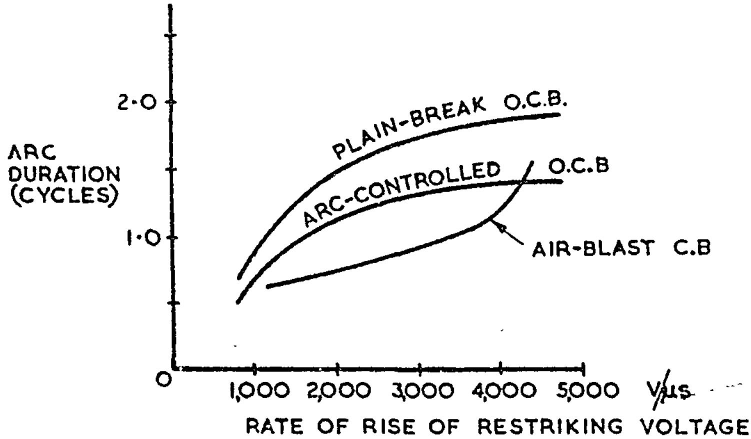

FIG. 15.22.—EFFECT OF RATE OF RISE OF RESTRIKING VOLTAGE ON ARC DURATION.

the inherent rate of rise thus has little effect on the behaviour of such a circuit-breaker, i.e. the arc duration when interrupting a given current does not increase appreciably with increasing rates of rise as shown in Fig. 15.22. A similar shape of curve obtains, although for a different reason, with oil circuit-breakers having explosion pots; in this case the post-zero resistance is relatively high so that there is little damping effect. The very high pressure set up when interrupting large currents can, however, satisfactorily interrupt the current whatever the rate of rise of the restriking voltage within the normal limits. The rate of rise may, however, have some influence when interrupting smaller currents. With the air-blast circuit-breaker the arc-extinguishing effect is constant and independent of the current; to avoid difficulties at low currents, e.g. current chopping, the blast effect must be limited so that it can be no more than adequate at the maximum current. A high rate of rise of restriking voltage may thus cause failure as shown by the upward bend of the curve in Fig.

15.22. This sensitivity to rate of rise of restriking voltage is more pronounced with medium-voltage air-blast circuit-breakers (e.g. 33 kV.) where the currents are relatively high.

Resistance Switching

Since the post-zero resistance between the contacts has been shown to reduce the rate of rise of restriking voltage it might be thought desirable deliberately to connect across the contacts a resistor having a value approximately equal to the above critical value of 0·5 $\sqrt{(LC)}$ ohms as shown in Fig. 15.23. The current passing through such a resistor after the main contacts have been opened must, of course, be subsequently broken at another contact; as the critical resistance is generally of the order of hundreds of ohms, this current may be of the same order as the normal load current.

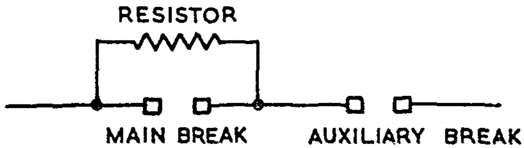

FIG. 15.23.—TO ILLUSTRATE RESISTANCE SWITCHING.

Since the air-blast circuit-breaker is sensitive to rate of rise of recovery voltage, this resistance-switching procedure is commonly used with such circuit-breakers, particularly at the lower voltages. The isolating switch, which has already been seen to be necessary with most air-blast circuit-breakers, can be used to break the resistor current. Alternatively, the arc within the arcing chamber can be " blown " from the main contact on to an auxiliary contact connected in series with the resistor.

At high currents the oil circuit-breaker is not sensitive to rate of rise of restriking voltage, so that it would not appear to be necessary to employ resistance switching. The performance at low currents can, however, be improved by the use of resistors, and they are sometimes used for this reason; when interrupting a low current, the value of reactance in the circuit will tend to be high so that the inductance L in the expression for the critical resistance will be larger, resulting in resistors of the order of thousands of ohms.

Wire-wound resistors are normally used. Resistors employed for the above purposes must not be confused with those used for voltage equalisation.

Circuit-breaker Construction

Although the circuit-breaker contacts are the essential part of the circuit-breaker, a considerable amount of ancillary apparatus must be incorporated in its construction; this includes current and voltage transformers, closing and tripping mechanisms, and suitable enclosure to ensure safety to operators and to prevent danger of fire. The appearance of a complete circuit-breaker unit therefore assumes many different forms.*

Dead- and Live-tank Units

In the simple construction shown in Figs. 15.11 and 15.12 the contacts are immersed in oil contained in an earthed tank; this tank serves to contain the oil and also provides a safe enclosure for the live parts. The oil in the tank also serves two purposes: firstly, that of an arc-extinguishing medium as already explained and, secondly, as an insulator for the live parts. As the quantity of oil actually needed for arc extinction is only about one-tenth of the total, the rest being used for insulation, the possibility of using an alternative form of insulation was studied. This led to the live-tank or low-oil-content circuit-breaker in which the arc chamber contains a minimum quantity of oil and is mounted on a porcelain insulator to insulate it from earth; the arc chamber itself is, in fact, of bakelised paper enclosed in porcelain, so that such a circuit-breaker appears as in Figs. 15.24 and 15.25, the lower porcelain being the support and the upper porcelain enclosing the contacts. This mode of construction is also used for the majority of air-blast circuit-breakers.

For oil circuit-breakers, both the earthed-tank (bulk-oil types) and the low-oil-content type are used up to the highest ratings, although for medium and low ratings the bulk-oil type is more common.

Single- and Multi-break Units

A single pair of contacts would appear to be the simplest arrangement, since if two or more pairs are used there may be difficulty in ensuring that each break performs its proper share of the interrupting duty.

The two-break earthed-tank construction illustrated in Fig. 15.11 has the merit of simplicity in that current is not led into any moving part. Due to stray capacitances to earth and between contacts the restriking voltage does not divide equally across the breaks, so that one break may take more than half—e.g. up to 70 or 80 per cent.—of the interrupting duty. In spite of this,

* It is not possible to give detailed descriptions of switchgear without overloading this treatise. Very complete descriptions, with admirable illustrations, are given in manufacturers' publications, and in the technical Press.

however, and on account of the simple construction, this type of circuit-breaker is more common than any other, except for the highest voltages and interrupting capacities.

The live-tank arrangement lends itself particularly to the multi-break construction, since the interrupting chambers can be mounted one above the other as shown in Fig. 15.24, or in Vee formation as in the 380 kV. breaker of Fig. 15.26 for an air-blast unit. Multi-break construction can, however, also be incorporated in the bulk-oil type. Since the largest rating that can

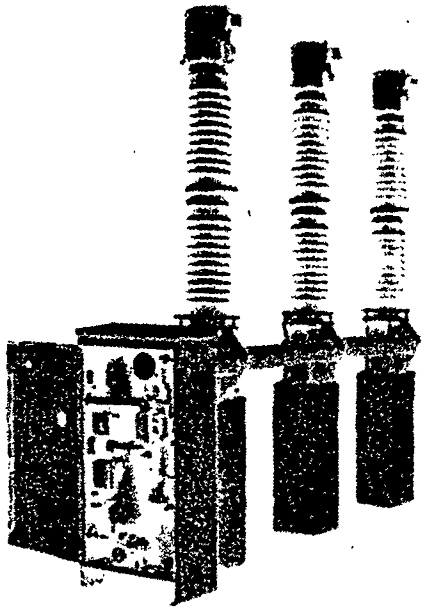

FIG. 15.24.—LOW OIL-CONTACT CIRCUIT-BREAKER.
(*Reyrolle & Co. Ltd.*)

conveniently be handled by a single break is about 2,500 MVA. at 132 kV., the multi-break construction must be used for the largest ratings. A further advantage of the multi-break construction is that it enables the individual breaks to be tested separately, thus demanding a much smaller output from the circuit-breaker test plant.

With the multi-break construction the stray capacitances cause considerable differences in the distribution of the restriking voltages across the various units, resulting in a very uneven sharing of the total interrupting duty. It may be noted that this uneven voltage distribution is similar to, and occurs for a similar reason to, that across a long suspension insulator string as described on p. 173.

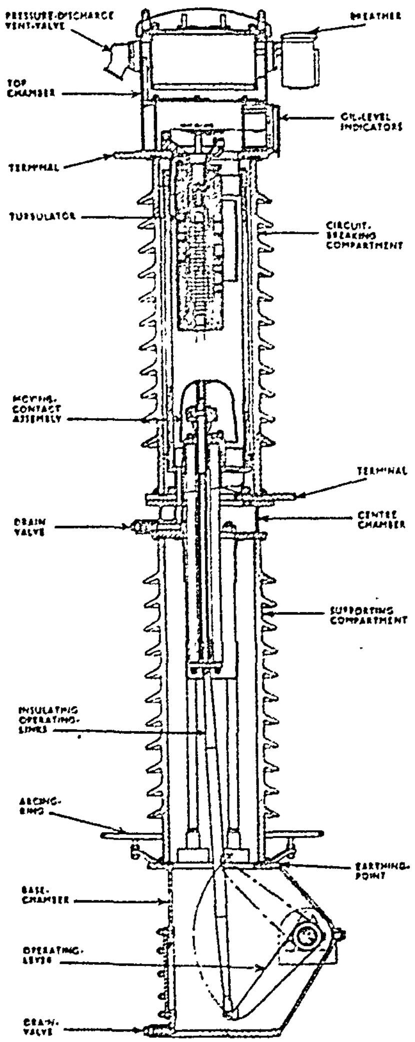

Fig. 15.25.—Low Oil-content Circuit-breaker.
(*Reyrolle & Co. Ltd.*)

To secure an economical design it is necessary to take steps to equalise this voltage distribution, and this is generally done by connecting impedances across each of the breaks, these being of sufficient magnitude to swamp out the effects of the various stray capacitances. Resistors of the order of hundreds of thousands of ohms are commonly used for the purpose. The function of these voltage-equalising resistors must not be confused with those described on p. 371 for resistance switching. The current carried in this case is very small and can quite easily be interrupted by an isolating switch; in view of the small currents, non-linear resistors can be used which have a higher value at the normal voltage than at the possibly higher restriking voltage.

Capacitors have been used in place of resistors, but the largest capacitance that can be conveniently accommodated is 500–600 pF. and this may not be enough to give the required swamping effect.

Mounting and Enclosure

Circuit-breakers are normally connected electrically to a common bus-bar as shown in Fig. 15.27. Isolating switches or other means of isolation must be provided in order to enable the circuit-breaker to be made dead for maintenance purposes without shutting down the rest of the substation. Current transformers always and voltage transformers frequently have to be provided as shown. Electrical or mechanical interlocks must be provided to ensure that the circuit-breaker is open before its isolators are operated and that the

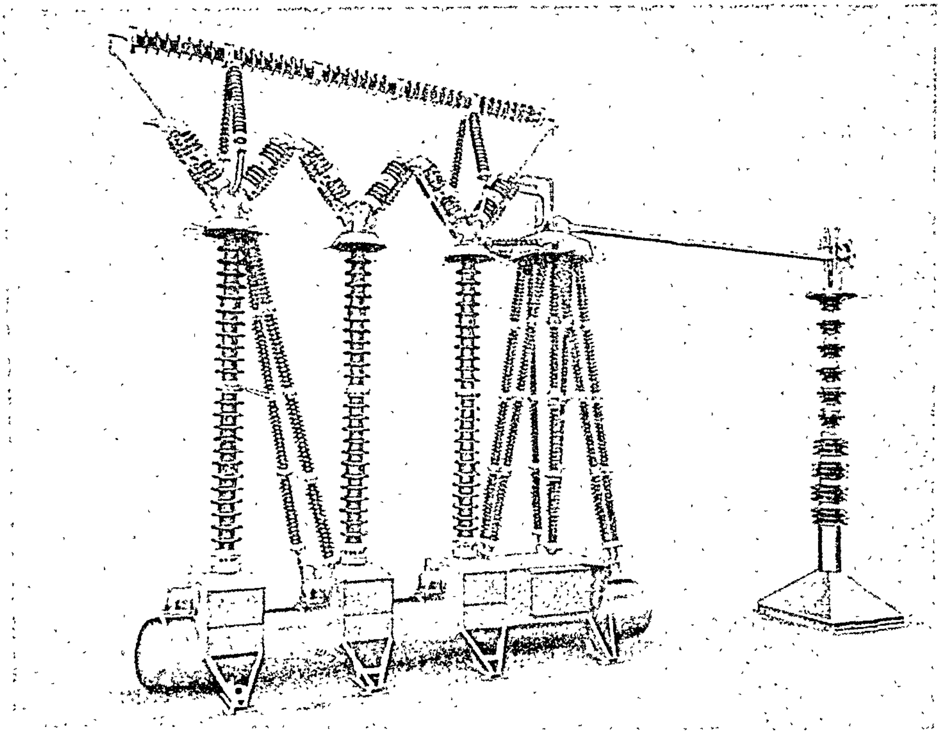

FIG. 15.26.—GENERAL ARRANGEMENT OF 380 KV. AIR-BLAST BREAKER.
(*ASEA Electric Ltd.*)

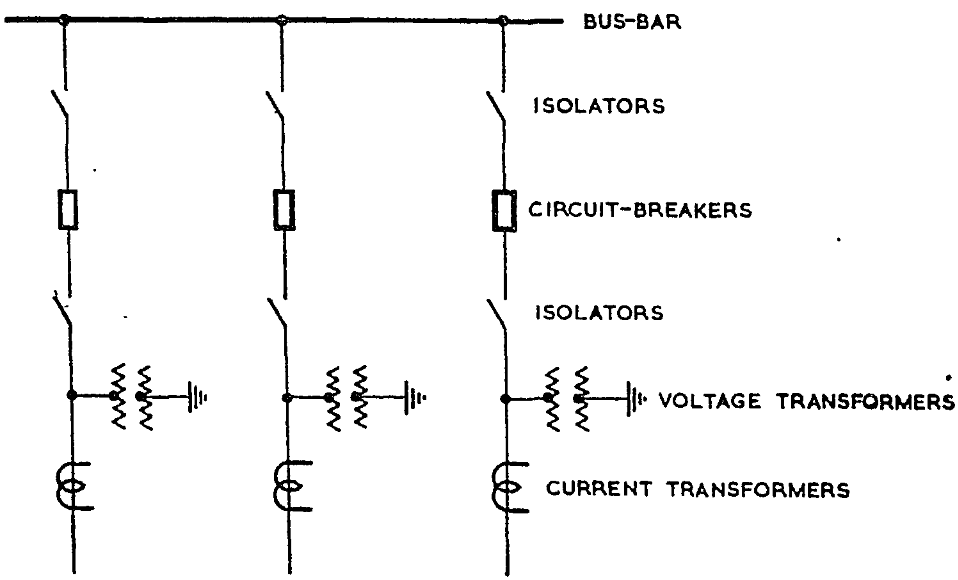

FIG. 15.27.—ONE-LINE DIAGRAM OF SUBSTATION.

isolators are open before any door or barrier around live parts can be opened or removed.

Types of circuit-breaker mounting are as follows:

Stonework cubicle	Metal-clad
Truck	Open (outdoor)

Cubicle Type

In the cubicle arrangement the circuit-breaker and its ancillary equipment is mounted in a cell of brickwork or concrete slabs with sheet-steel doors (cellular type) or in a completely sheet-steel cubicle. The very desirable feature of phase isolation is secured for bus-bars and all apparatus with the exception of the circuit-breaker and voltage transformer. Every part is accessible, and any visual damage or deterioration is easily ascertained.

Truck Type

In the truck type of equipment the circuit-breaker and other apparatus is enclosed in a sheet-steel cubicle but is mounted on a

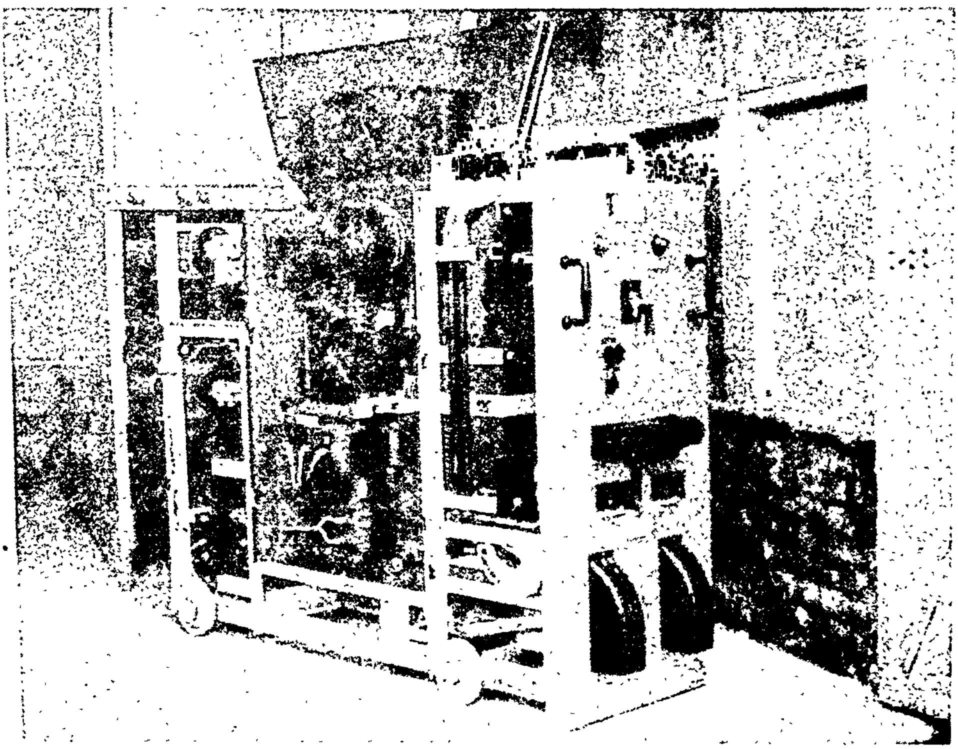

FIG. 15.28.—TRUCK-TYPE SWITCHGEAR.
(*English Electric Co. Ltd.*)

movable truck which can be withdrawn from the cubicle as shown in Fig. 15.28 for maintenance purposes. Plugs on the truck connected to the circuit-breaker enter sockets on the cubicle connected to the bus-bars and feeder circuits so that no isolating switches are required; shutters must, however, be arranged automatically to cover the live sockets whenever the truck is withdrawn, and the truck must be locked in position whenever the circuit-breaker is closed. The largest practicable size of truck limits this type to a voltage of 11 kV. and a short-circuit rating of 250 MVA. The arrangement described above is referred to as a horizontal-isolation unit; in an alternative arrangement the circuit-breaker is lowered vertically on to a truck for withdrawal, the instrument transformers being contained in the cubicle. Such a unit has the advantage over the horizontally-isolated arrangement of requiring less floor-space.

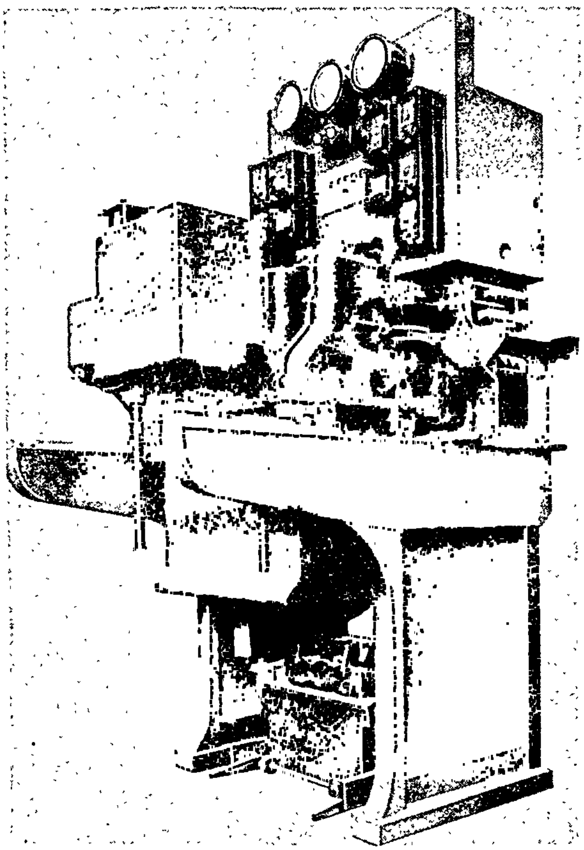

FIG. 15.29.—METAL-CLAD SWITCHGEAR FOR HORIZONTAL ISOLATION. (*Reyrolle & Co. Ltd.*)

Metal-Clad Type

With the metal-clad type of equipment all live parts are completely enclosed in earthed metal, the enclosures being air-, oil-, or compound-filled. Maximum safety is thus ensured, trouble due to dirt or vermin is eliminated, and the space occupied is reduced to a minimum. The circuit-breaker itself is arranged so that it can be moved away from the bus-bars either horizontally as in Fig. 15.29 or vertically as in Fig. 15.30. Metal-clad circuit-breakers can be built in sizes up to 1,000 MVA. at 66 kV., although the widest application is for medium-sized units.

Open Type

For the largest ratings, outdoor installation is usual in order to avoid building costs. Figs. 15.31 and 15.32 show substations containing outdoor bulk-oil, low oil-content, and air-blast circuit-breakers respectively. In certain instances it has been found

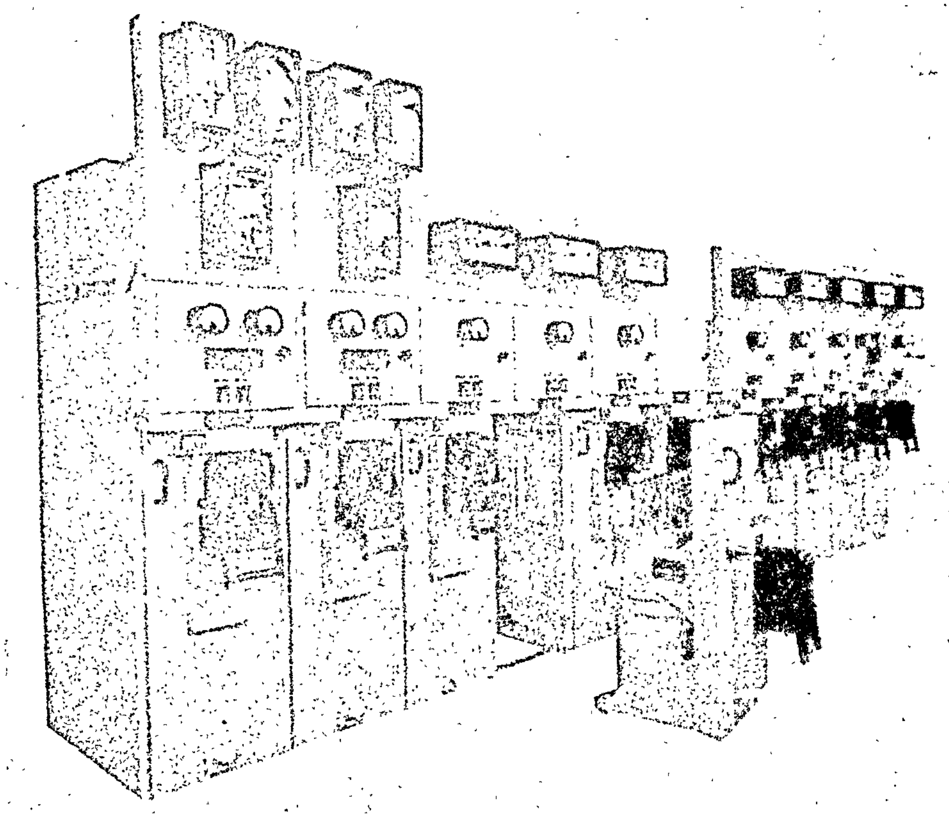

FIG. 15.30.—METAL-CLAD SWITCHGEAR FOR VERTICAL ISOLATION. (*English Electric Co. Ltd.*)

economical to use open-type switchgear in a building, the switchgear in this case being of the low oil-content type.

Operating Mechanisms

The force for opening a circuit-breaker must be instantaneously available on receipt of the tripping impulse from the protective gear, and the universal practice is to use a previously-charged spring for oil circuit-breakers and compressed air for air-blast circuit-breakers.

With spring-opening, the circuit-breaker closing mechanism compresses the spring and latches the moving contacts in the closed position. The tripping impulse then energises a small solenoid that releases the latch and allows the spring to exert its

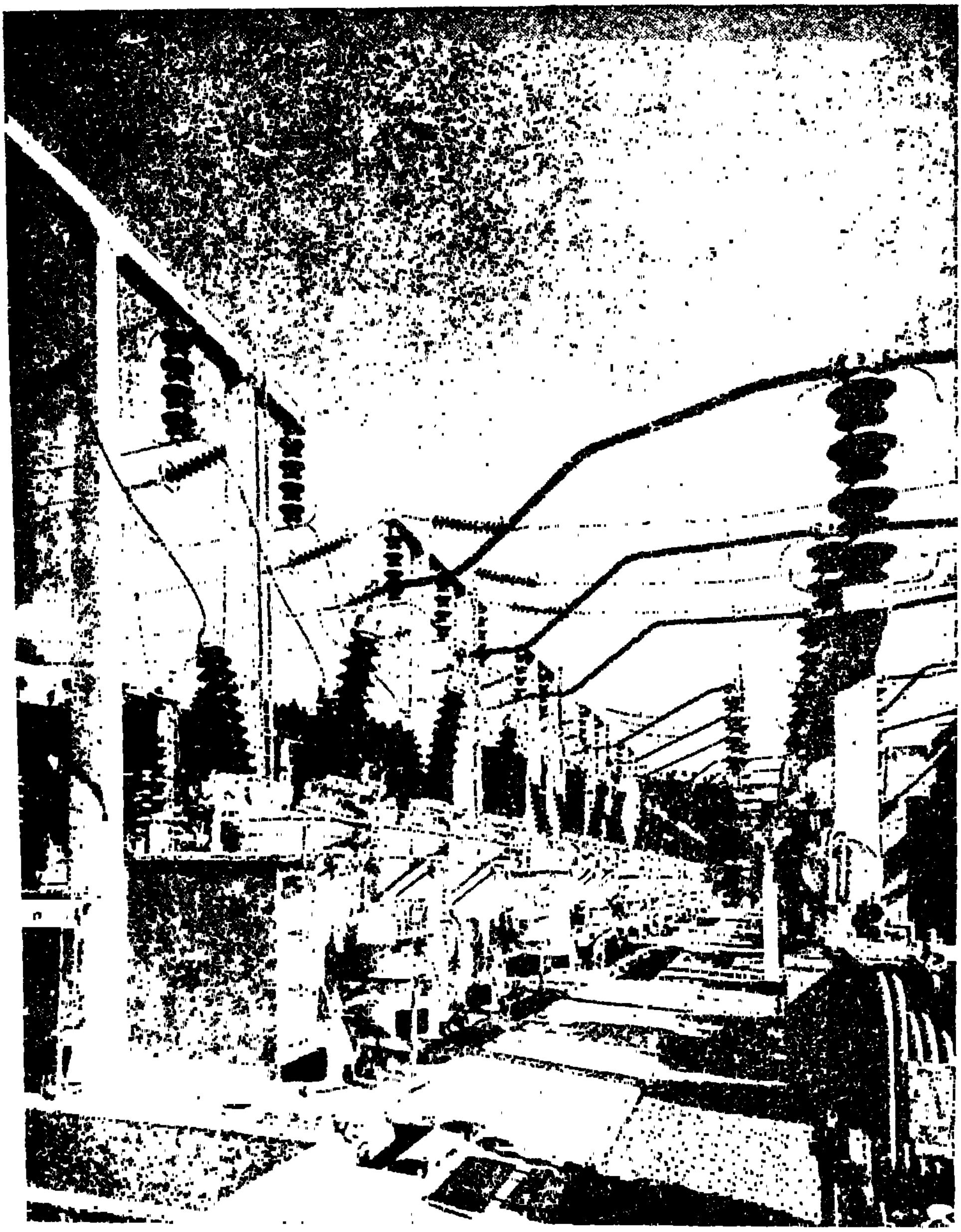

FIG. 15.31.—OUTDOOR SUBSTATION WITH LOW OIL-CONTENT BREAKERS. (*English Electric Co. Ltd.*)

full force to open the contacts; suitable buffers must be provided to absorb the kinetic energy of the moving parts at the end of the stroke.

With an air-blast circuit-breaker the contacts are normally held closed by a spring and forced open by the admission of compressed air, the compressed air thus performing the double function of opening the contacts and extinguishing the arc. The air supply must therefore be maintained until after the auxiliary isolating switch (also operated by compressed air) has opened, after which it is cut off and the main contacts return to the closed position under the action of their spring.

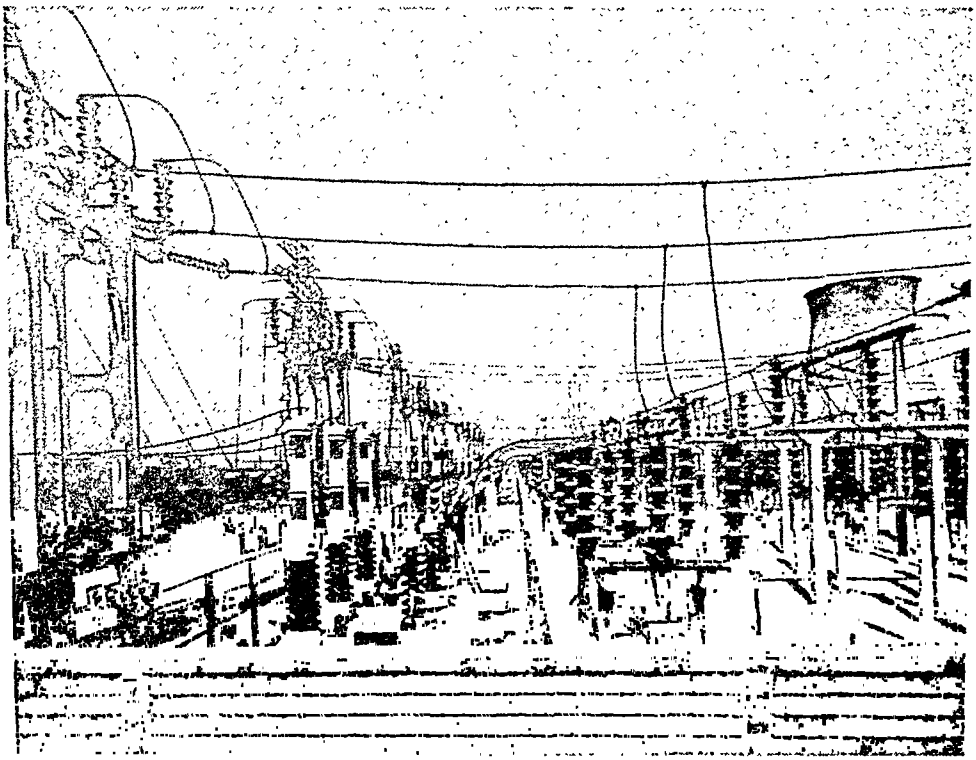

FIG. 15.32.—OUTDOOR SUBSTATIONS WITH AIR-BLAST BREAKERS. (*English Electric Co. Ltd.*)

The force for closing a circuit-breaker may be applied by hand, by a solenoid, by a spring or by compressed air. With spring-opened circuit-breakers, the closing force has to compress the spring and must therefore be considerable. For a large circuit-breaker, the power required for closing may be 25 kW., but the time occupied is only a fraction of a second so that the energy is relatively small—e.g. 6·25 kW.-sec. (= 4,600 ft.-lb.). Manual operation is thus impracticable for circuit-breakers having short-circuit ratings above about 150 MVA.

Solenoid closing is the most widely used method, the solenoid being fed by a battery, usually of 110 or 220 V. For large circuit-breakers, the current may be 200 to 300 A.

For medium-sized circuit-breakers, where the provision of a battery may be uneconomic, spring-closing may be employed, the spring being precharged by hand or by a small motor.

The use of compressed air in connection with the air-blast circuit-breaker has been referred to above, closing of such circuit-breakers being effected by the isolating switch. With oil circuit-breakers in extensive substations, where the heavy cabling and voltage drop resulting from the solenoid currents might cause difficulties, compressed-air closing is sometimes used.

In all cases the closing mechanism must be trip-free—i.e. if a tripping impulse comes during the closing operation the circuit-breaker must immediately trip even though the closing force be still maintained; with spring-opened circuit-breakers this is effected by a mechanical toggle mechanism, while with air-blast circuit-breakers it is inherent in their operation.

Testing and Rating

Testing circuit-breakers at their full short-circuit rating is more difficult than testing other equipment such as machinery or transformers on account of the very large MVA. required; furthermore, no fully satisfactory method of reduced-power testing, analogous to the no-load and short-circuit tests of a transformer, has yet been devised.

Short-circuit Test Plant

The major item of a short-circuit test plant is a specially-designed generator of a physical size comparable to that of a normal 60–100 MVA. generator. When short-circuited, such a machine can give about 2,000 MVA. The generator is driven by a motor of about 1,000 h.p., this being usually disconnected from its supply immediately prior to the application of a short-circuit so that the energy in the short-circuit is supplied from the kinetic energy of the rotor. A large separately-driven exciter with a high ceiling voltage is required in order to prevent excessive current decrement during the short-circuit. Transformers, resistors, and reactors are employed to control the voltage and short-circuit current, and capacitors are used to give any desired rate of rise of restriking voltage.

During the actual test, electromagnetic oscillograms are taken of current, voltage in each phase, and also such other items as contact travel, tank pressure, and trip-coil current; cathode-ray oscillograms are taken of the restriking transient in each phase. A typical series of electromagnetic oscillograms is shown in Fig.

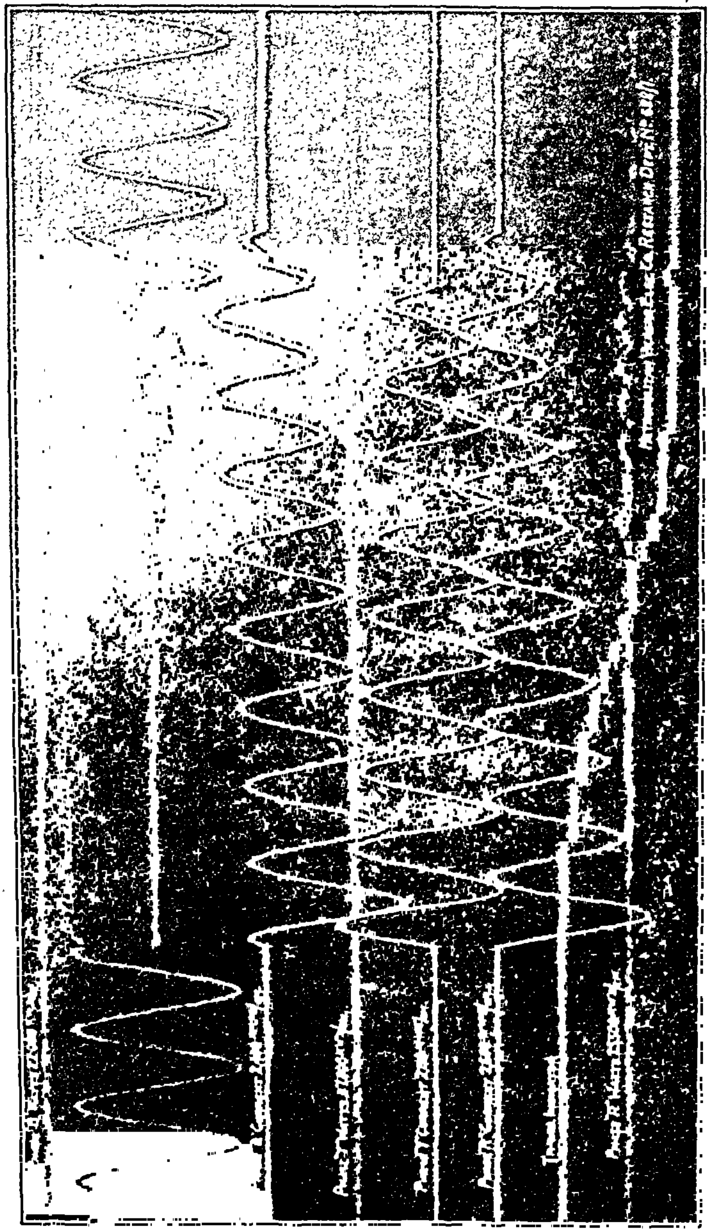

FIG. 15.33.—OPERATION OF CIRCUIT-BREAKER WITHOUT ARC-CONTROL DEVICE. (*Metropolitan-Vickers Electrical Co., Ltd.*)

15.33, while a cathode-ray oscillogram is shown in Fig. 15.20 (p. 382).

Type tests—i.e. tests on one item of a particular type—are taken on oil circuit-breakers in accordance with the British

Standards Specification 116 (1952). This lays down the following test cycle*—

Tests 1, 2, and 3.—B–3–B–3–B at 10 per cent., 30 per cent., and 60 per cent. of rated symmetrical breaking capacity.

Test 4.—B–3—MB—3–MB at 100 per cent. rated breaking and making capacity.

Test 5.—B–3–B–3–B at 100 per cent. rated asymmetrical breaking capacity with at least 50 per cent. D.C. component in one phase.

The power factor in the above tests must not exceed 0·15 lag, and the recovery voltage must be maintained for 5 cycles.

Other type tests include mechanical reliability tests, power-frequency, and impulse overvoltage tests and short-time current tests, full details being given in the specification.

A separate specification is under consideration for air-blast circuit-breakers; owing to the dependence of such circuit-breakers on rate of rise of restriking voltage, a clause governing this will be included.

Rating

A circuit-breaker is normally rated in terms of its number of poles, service voltage, normal current, frequency, making capacity, breaking capacity, and short-time current and operating duty, the first four of these items being self-explanatory. Making and breaking capacities are expressed as the making and breaking currents, these being determined from test oscillograms by the following procedure as laid down in the British Standards Specification.

Referring to Fig. 15.34 the R.M.S. values of breaking current at the instant of contact separation are—

$$\text{Symmetrical breaking current} = x/\sqrt{2}$$

$$\text{Asymmetrical breaking current} = \sqrt{(x/\sqrt{2})^2 + y^2}$$

The breaking capacity may also, for convenience, be expressed in MVA., the value being, for a three-phase circuit-breaker—

Breaking capacity (symmetrical or asymmetrical)

$$= \sqrt{3}\ .\ \mathrm{I}\ .\ \mathrm{V}\ .\ 10^{-6}\ \text{MVA.}$$

where I is the breaking current (symmetrical or asymmetrical), and V is the rated service voltage (line value).

* B refers to a breaking operation, MB to making a short-circuit with immediate breaking, and 3 to a 3-minute interval between the operations.

These breaking capacities are quoted under specified conditions of severity—e.g. power factor, recovery voltage and, possibly, rate of rise of restriking voltage.

The making current is, again referring to Fig. 15.36, given by z.

The short-time rated current is the current that can be safely applied, with the circuit-breaker in its normal condition, for 3

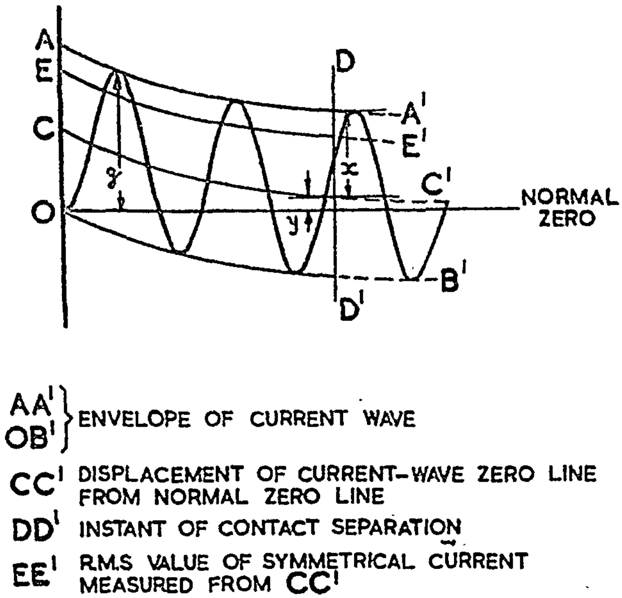

FIG. 15.34.—DETERMINATION OF BREAKING CURRENT AT INSTANT OF CONSTANT SEPARATION.

seconds, if the ratio of symmetrical breaking current to normal current is less than 40, or for 1 second otherwise.

Any circuit-breaker must be capable of the following short-circuit operating duty—

B–3–MB–3–MB

Substation Connections

The aim of any particular arrangement of substation connections is to achieve adequate operating flexibility, sufficient reliability and minimum cost, the latter being obtained by keeping the number of circuit-breakers to a minimum; it must be remembered, however, that reducing the number of circuit-breakers usually adds to the cost and complication of the protective gear. Typical connections are shown in Fig. 15.35.

In Fig. 15.35*a* is shown a duplicate bus-bar arrangement with section circuit-breakers and two circuit-breakers per each circuit;

in practice there would, of course, normally be more than four incoming and outgoing circuits. The chief features of this arrangement are that each circuit can be changed from one set of bus-bars to the other without making it dead, that any circuit can be fed from any other independently of the rest and that a fault on one section of bus-bars can be isolated without shutting down any other part of the substation. Since each circuit requires two circuit-breakers, the cost of this arrangement is very high and it can be justified only in exceptionally important cases such as a large power-station or a major switching station.

A more common arrangement is shown in Fig. 15.35*b* where each circuit has only one circuit-breaker, this being connected to either bus-bars by closing the appropriate isolators; changing a circuit from one set of bus-bars to the other can normally only be made with the circuit dead. Some of the truck and metal-clad types of circuit-breaker, particularly the vertical-isolation type, are so arranged that the change-over can be effected by partially withdrawing the circuit-breaker and moving it to a new position where it can be engaged with the second set of bus-bars. Considerable saving in costs results from this arrangement, although at some sacrifice of flexibility.

The ring bus-bar of Fig. 15.35*c* and the $1\frac{1}{2}$-breaker arrangement of Fig. 15.35*d* (developed in U.S.A.) give reasonable cost with adequate flexibility for many circumstances.

Simple substations may use a single bus-bar as shown in Fig. 15.35*e*; flexibility can be increased by the use of the mesh arrangement of Fig. 15.35*f* if the number of circuits does not exceed five or six.

Where the substation is to supply transformers from a ring main, the arrangements of Fig. 15.35*g*, *h*, *i*, or *j* may be used. The 3-circuit-breaker arrangement of Fig. 15.35*g* is widely used on the 132-kV. system in Great Britain and can be seen to be actually the mesh arrangement of Fig. 15.35*f* with three circuits. The arrangements of Fig. 15.35*i* and *j* are commonly used for primary distribution circuits at 33 and 11 kV.

Substation Layout

The choice between outdoor and indoor substations depends on their relative costs and the atmospheric conditions. An outdoor substation saves building costs but requires a larger site and more maintenance; a major factor in the design is the provision of adequate clearance so that maintenance work can be carried out without making more of the substation dead than is absolutely

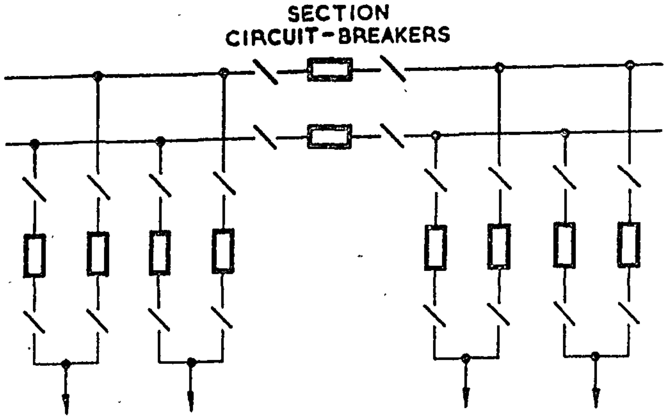

FIG. 15.35*a*.

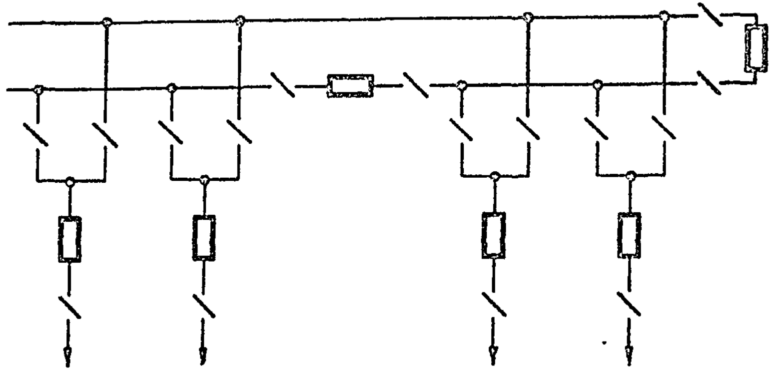
FIG. 15.35*b*.

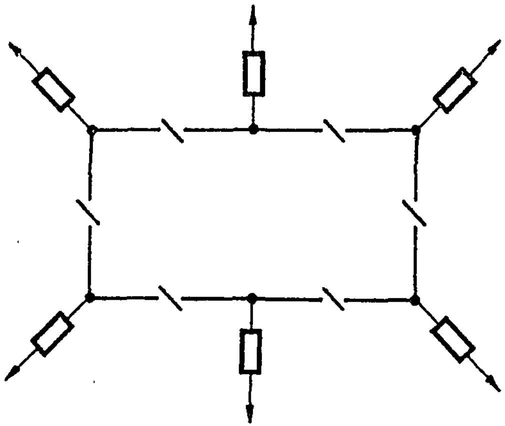
FIG. 15.35*c*.

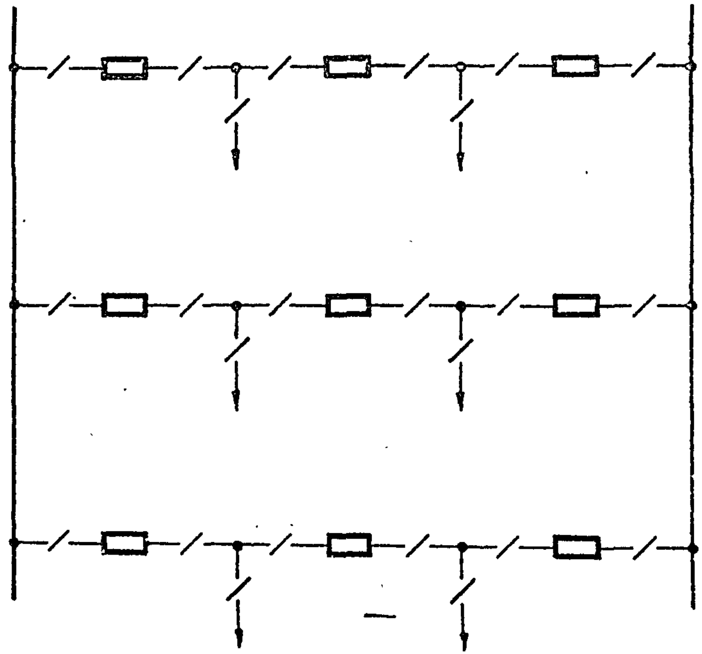
FIG. 15.35*d*.

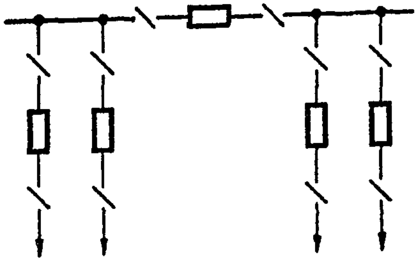

Fig. 15.35e.

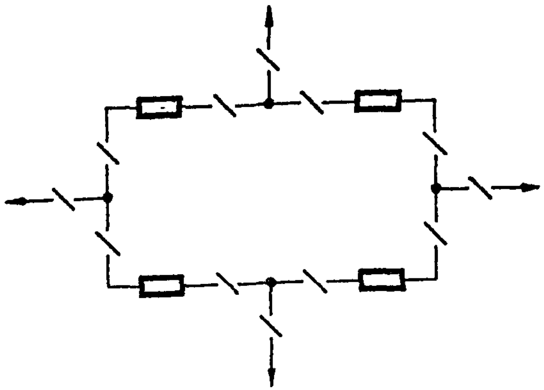

Fig. 15.35f.

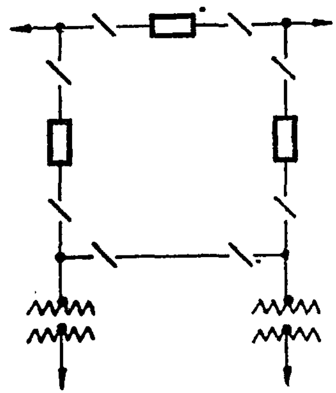

Fig. 15.35g.

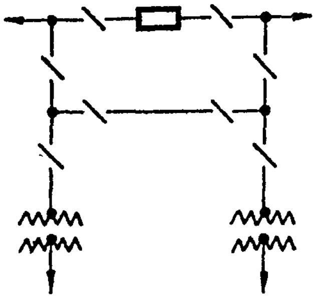

Fig. 15.35h.

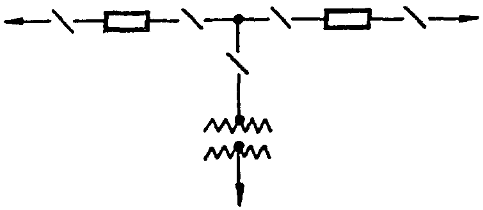

Fig. 15.35i.

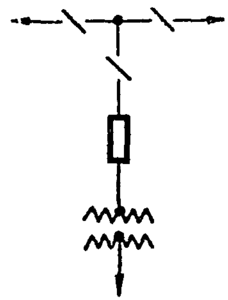

Fig. 15.35j.

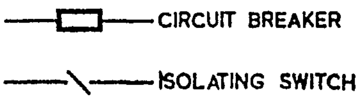

Fig. 15.35.—Systems of Substation Connections.

necessary. At and above 132 kV., outdoor construction is almost universal, typical stations being shown in Figs. 15.31 and 15.32. Care must also be taken to ensure that all metal work (structures, fencing, circuit-breaker tanks, etc.) is effectively bonded to the earthing system, the latter comprising a network of wires earthed by buried rods or plates at a number of points. Indoor stations are usual in urban areas where sites are expensive and difficult to obtain and where voltages are not usually above 66 kV. or occasionally 132 kV. A major factor in their design is the minimisation of the fire risk; sectionalising by fire-resisting walls is usually necessary and fire-extinguishing apparatus must be installed. Carbon-dioxide is a very effective fire-extinguishing medium and causes no damage to sound equipment; the gas is stored in cylinders and released automatically by fusible plugs. Where extremely rapid extinction is essential—e.g. to prevent damage to adjacent premises—chemical foam equipment is effective although the plant may suffer further damage from its use.

The following Books and Papers are recommended for Further Reading.

Books

GERSONOWICZ, S., *High-Voltage A.C. Circuit-breakers.* (Contains 600 references.) (Constable.)

MARTLOCK, J. R., *A.C. Switchgear.* Vols. I and II. (Chapman and Hall.)

I.E.E. Papers

These papers with their subsequent discussions give a full review of modern British circuit-breaker practice.

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CHRISTIE, J., LEYBURN, H., and FENN, R. W., " A New Testing Station for High-Power Circuit Breakers." *Proc. I.E.E.*, Vol. 102, Pt. A, 1955.

Numerous papers relating to modern American practice are to be found in the *Transactions of the American I.E.E.—Power Apparatus and Systems.*

CHAPTER XVI

PRINCIPLES OF FEEDER PROTECTION

In the previous chapter we have seen that the protection of the system against damage is the function of the circuit-breaker. Before the circuit-breaker can open, its operating coil must be energised, and this, in turn, requires that certain relays shall also be energised. There are many systems of feeder protection, and they can be classified broadly according to the type of relay used. With all systems it can be laid down that the fundamental requirement is that a faulty feeder shall be isolated from the system without any interruption of current in the rest of the system. This property is called " discrimination."

Under fault conditions there will be a difference between the currents at the two ends of the feeder. The minimum value of this difference current which will cause the system to operate is called the " sensitivity " of the system.

The maximum through current which will flow through the feeder without operating the system when there is no fault is called the " stability." It is obvious that stability is an essential property in order that the system may not operate under ordinary short-duration overload conditions. The ratio of the stability to the sensitivity is called the " stability ratio," and this ratio is a measure of the discriminating power of the system. The stability ratio can also be defined in terms of the setting encroachment, which is the value of the current which tends to inadvertent operation. The stability ratio in terms of the setting encroachment is the ratio of the setting for operation to the setting encroachment.

It goes without saying that a *sine qua non* is reliability of operation, robustness of the component parts and maximum simplicity consistent with the required operating features.

The various systems of feeder protection can be classified as follows :

1. *Graded Time-lag Systems* employing time-lag relays with a directional feature.
2. *Differential Systems* in which the relays are responsive only

to a difference in the magnitudes of currents which are normally equal.

3. *Systems employing Special Cables.*

4. *Distance Systems* in which the time delay of the relays is automatically made to depend upon their distance from the fault.

This list can be extended by the addition of the following:

5. *Negative-sequence Systems* which utilise the negative-sequence components set up by the unbalancing of the currents produced by a single-phase short-circuit.

6. *Zero-sequence Systems* which utilise the zero-sequence components set up by leakage to earth. These systems are more commonly termed leakage-protective systems.

7. *Signal-current and Carrier-current Systems.*

Graded Time-lag Systems

When feeders are not connected in parallel the necessary protection can be given by a series of over-current relays which are so arranged that the farther the distance of the relay from the generating station the greater is its time to operate. Consider the scheme shown in Fig. 16.1, in which a generating station GS

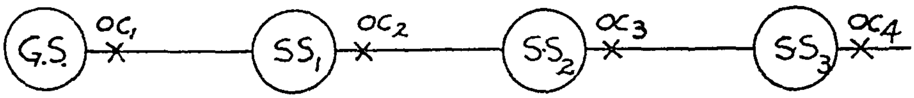


FIG. 16.1.—SERIES SYSTEM WITH GRADED TIME-LAG PROTECTION.

supplies three transforming substations SS_1, SS_2, SS_3 in series. If a fault occurs on the far side of SS_3 it is obvious that only the over-current relay OC_4 should operate, and not any of the others, and consequently the time taken to operate OC_4 should be less than that of any of the others. Similarly the time taken should increase progressively as the generating station is approached, and this is the principle of the graded time-lag system. For example, under severe short-circuit conditions the operating times might vary from, say, 0·4 sec. for OC_4 to 1.6 secs. for OC_1.

In addition to this time discrimination it is essential that the time of operation of any one of the relays should be dependent on the severity of the fault in such a way that the more severe the fault the less the time to operate, this being called an inverse-time characteristic. This is essential in order that a relay shall not operate under normal overload conditions of short duration. A typical set of time-current characteristics for the relay described on p. 407 is shown in Fig. 16.2, and the relay can be made to work

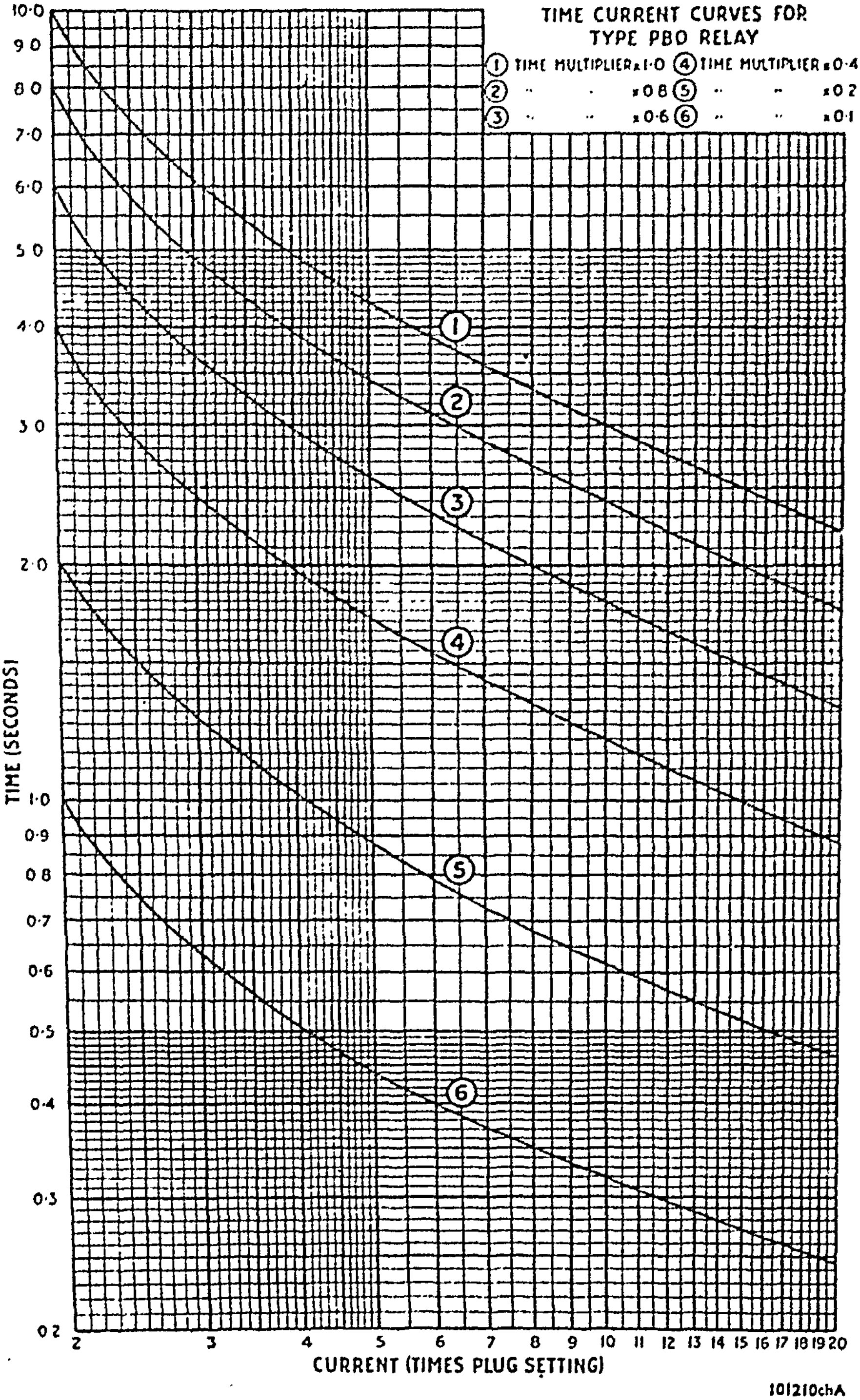


FIG. 16.2.—CHARACTERISTICS OF OVER-CURRENT RELAY.
(*Metropolitan-Vickers Electrical Co., Ltd.*)

on any one of the curves by an appropriate setting of the over-current element of the relay. For the circuit shown in Fig. 15.1 four similar relays could be used with the over-current elements set to give characteristic curve 4 for OC_1, curve 3 for OC_2, and so on. The equation to the characteristic curve is represented approximately by

$$t = \frac{a}{I^2} + b$$

where t = time to operate, in seconds
I = current
a and b = constants for a given relay and given setting.

It is essential also that there shall be a definite minimum time of operation, which can be adjusted to suit the requirements of the particular installation.

An over-current relay of this type, i.e. giving inverse-time operation with a definite minimum time characteristic, is illustrated

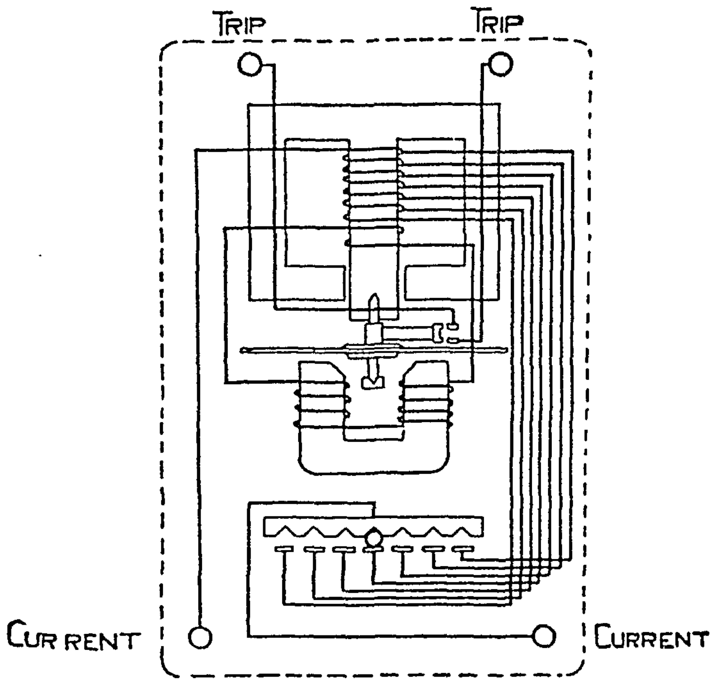


FIG. 16.3.—CONNECTIONS OF INDUCTION-TYPE OVER-CURRENT RELAY.
(*Metropolitan-Vickers Electrical Co., Ltd.*)

diagrammatically in Fig. 16.3. It will be seen that it consists essentially of an A.C. meter mechanism modified to give the required characteristics. The upper electromagnet has two windings, of which one, the primary, is connected to a current transformer in the line to be protected, and is tapped at intervals. The tappings are connected to a plug setting bridge by which the number of turns in use can be adjusted, thereby giving the desired current setting. The second winding is energised by induction from the primary, and is connected to the winding of the lower electromagnet. By this means the leakage flux from the upper and the flux produced by the lower are displaced sufficiently in phase to set up a rotational torque on the metal disc suspended between the two magnets. The disc spindle carries a moving contact which bridges two fixed contacts when the disc has rotated through an angle which can be adjusted to any value between 0 degrees and 360 degrees. By adjusting this angle the relay can be given any desired time setting.

The definite minimum-time characteristic is obtained by saturating the iron in the upper electromagnet so that there is practically no increase in flux after the current has reached a certain value. This results in the flattening-out of the current-time characteristic.

For very small amounts of power an inverse time element can be given by shunting the breaker overload coils with fuses. The disadvantage of the time-limit fuse is that fusing current and time lag cannot be varied independently, since until the fuse has actually blown it carries practically the whole of the current. Since it is undesirable to use fuses whose normal fusing current is more than about 200 amps., it is necessary to operate the trip coil through a current transformer when the time element is obtained by means of fuses.

Note on the Current-time Characteristics

When the protective system consists of a series of over-current relays with graded operating times, it is essential that the curves of all the relays shall flatten out in the same manner and that there shall not be any crossing. In this way the operation of the required relay is ensured, no matter what the severity of the fault may be. For imagine a system consisting of generating station, GS, and two substations, SS_1 and SS_2, in series as shown in Fig. 16.4, and suppose that the relay OC_1 has the dotted characteristic of curve 4 instead of curve 1. Then up to point P all the relays will act in the correct order, but beyond P the relay OC_1 will have a shorter time lag than OC_2, thereby shutting down the whole system in the event of a fault between SS_1 and SS_2. With similar characteristics such a fault will operate OC_2, thus leaving the substation SS_1 in operation.

Protection of Parallel Feeders

With an important installation continuity of supply is a matter of vital importance, and consequently if a feeder is automatically disconnected by the protective gear on the occurrence of a fault, it is essential that supply should be available via some other route.

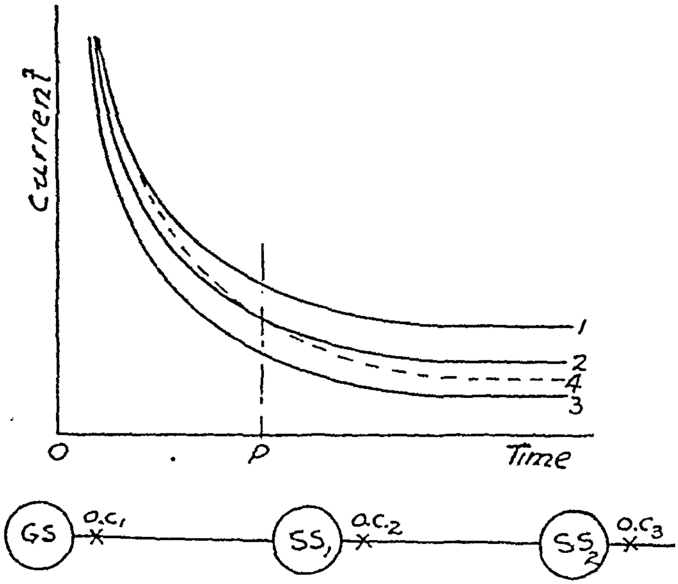


FIG. 16.4.—CHARACTERISTICS OF A SERIES OF GRADED OVER-CURRENT RELAYS.

This can be provided either by running important feeders in parallel or by means of a ring main. Consider first of all the case of duplicated feeders: with these reverse-power protection, in

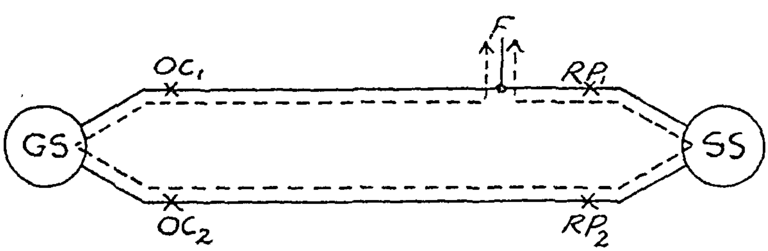


FIG. 16.5.—PROTECTION OF DUPLICATE FEEDERS.

addition to over-current protection, is necessary. For consider the duplicate feeders joining station GS and substation SS in Fig. 16.5, in which OC are the over-current relays and RP the reverse-power relays. If a fault occurs at F then current will flow into the fault via the two paths indicated by the dotted lines. The first path is the section of faulty feeder between GS and F; the second the whole of the sound feeder and the remainder of the faulty feeder. Obviously the over-current relay OC_1 will operate, thereby isolating the faulty feeder at the generating end, and this will have the effect of cutting off the supply to the fault except by the second path, which is through the reverse-power relay RP_1. This will operate and so isolate the faulty feeder at the substation end as well. The fault power flowing to F via RP_2 will do so in a forward direction relative to this relay, so that there will be no operation and the sound feeder will be left in service. This illustrates very well the meaning of discrimination. Obviously, inverse time lags are required on OC_1 and OC_2, but the reverse-power relays are set to operate either instantaneously or with a short fixed time lag. When one line is out of action due to a fault, then, of course, the system reverts to single-line working. For this reason, back-up protective gear is necessary in addition.

The construction of one form of directional over-current relay is shown in Fig. 16.6, from which it will be seen that it is a combination of an over-current element of the type previously described, and a wattmeter element actuated by both current and voltage; this latter element gives the relay its directional feature. The wattmeter element is prevented from rotating when the power flow is normal, but with a reversal of power rotation can take place. This completes the circuit of the over-current element, which then operates in the manner previously described.

An over-current relay of somewhat different type, although working on the same fundamental principles, is shown in Fig. 16.7. The fixed system embodies a U-shaped electromagnet, O, carrying the main winding, and an internal core, I, of cruciform shape, on one of the polar projections of which is a winding energised by induction from the main winding. The moving system is a light aluminium cylinder, D, free to rotate in the air gaps between O and I. The spindle to D carries a pinion which engages with a toothed sector carrying the bridging piece of the relay contacts, and the time setting can be varied by varying the initial position of the pinion on the sector. The action is as follows: the flux F_p, due to the primary winding P, circulates along the path indicated by the chain line, thus acting on the rotor D in a horizontal direction. The flux F_c, due to the current induced in the secondary winding S, takes the double path indicated by the dotted line, thus acting on the rotor in a vertical direction. The resistance R ensures a time phase in addition to the space displacement of these two fluxes, with the result that a turning moment is exerted on the rotor.

As with the previous type, the current setting is varied by plugging in the appropriate number of primary turns.

Fig. 16.8 shows a combination of over-current and directional elements suitable for the protection of parallel feeders. The main winding of the electromagnet is voltage operated, the terminals being V_+ and V, and one polar extension of the cruciform core

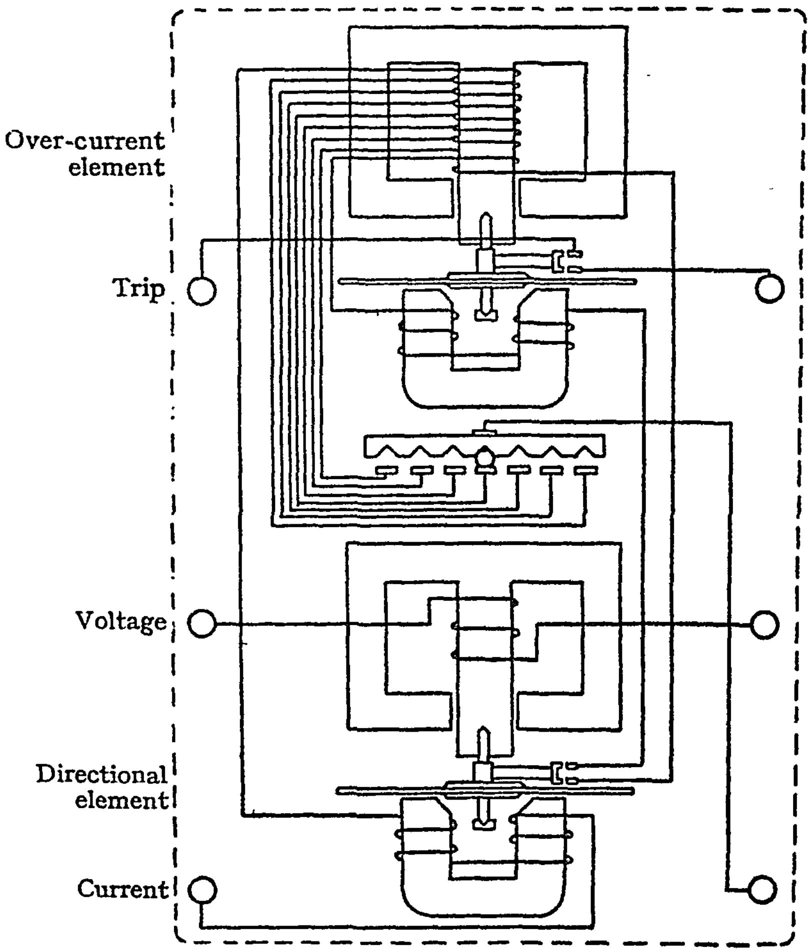


FIG. 16.6.—CONNECTIONS OF DIRECTIONAL OVER-CURRENT RELAY.
(Metropolitan-Vickers Electrical Co., Ltd.)

carries a winding V_1, which is energised by the secondary V_s. The opposite extension carries a current winding. There is an additional short-circuited winding, F, on the external magnet, whose function is that of power-factor compensation. With normal power flow the wattmeter element does not move, but with reversed flow movement takes place, thereby closing the contacts E and energising the over-current element. The figure shows the toothed sector by means of which the time delay can be controlled.

Since faults are often accompanied by a considerable reduction in line voltage, it is essential that the directional element shall be

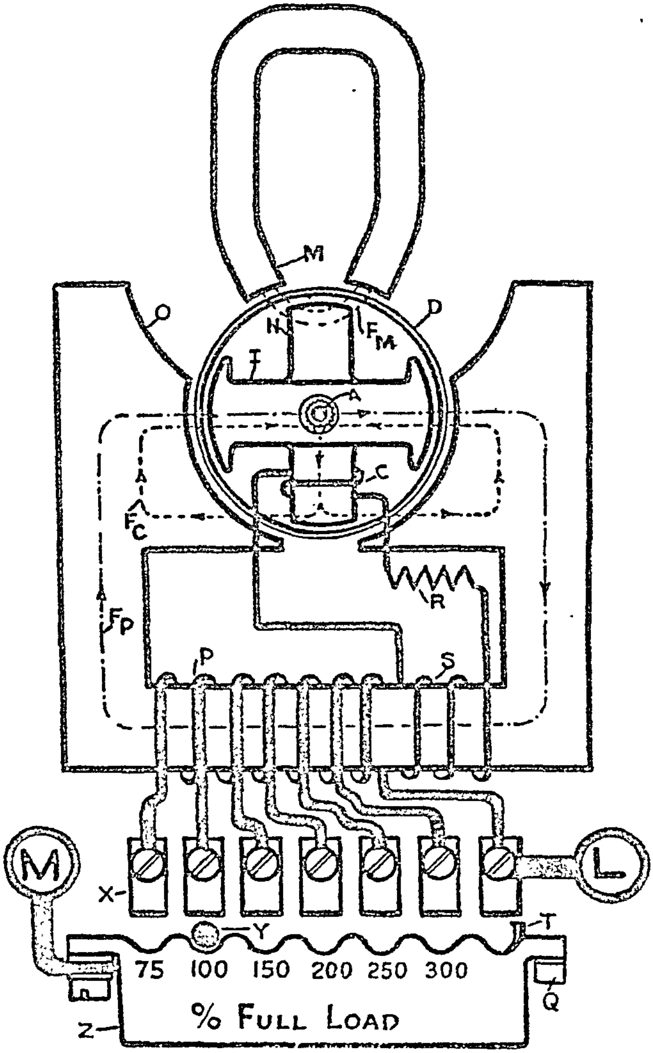


Fig. 16.7.—Inverse Time-limit Over-current Relay. (*N.C.S.*)

independent of voltage as far as possible ; thus with the relay just described the reverse-current setting is practically the same for all voltages from 100 per cent. down to 20 per cent. of normal, and for voltages as low as 2 per cent. the current to operate is only about twice normal.

Another method of protecting duplicate feeders is to employ differential over-current relays. Each relay is a centre-zero induction type and consists of a pivoted spindle carrying two aluminium discs. The torque on one disc is produced by the current in one feeder, and the torque on the other disc by the current in the parallel feeder, and these torques act in opposition. With equal currents in the two feeders, the total torque on the relay will be zero and the moving system will not be actuated. Should a fault develop on one feeder then this feeder will carry a heavier current than the sound feeder, and if the out-of-balance current reaches

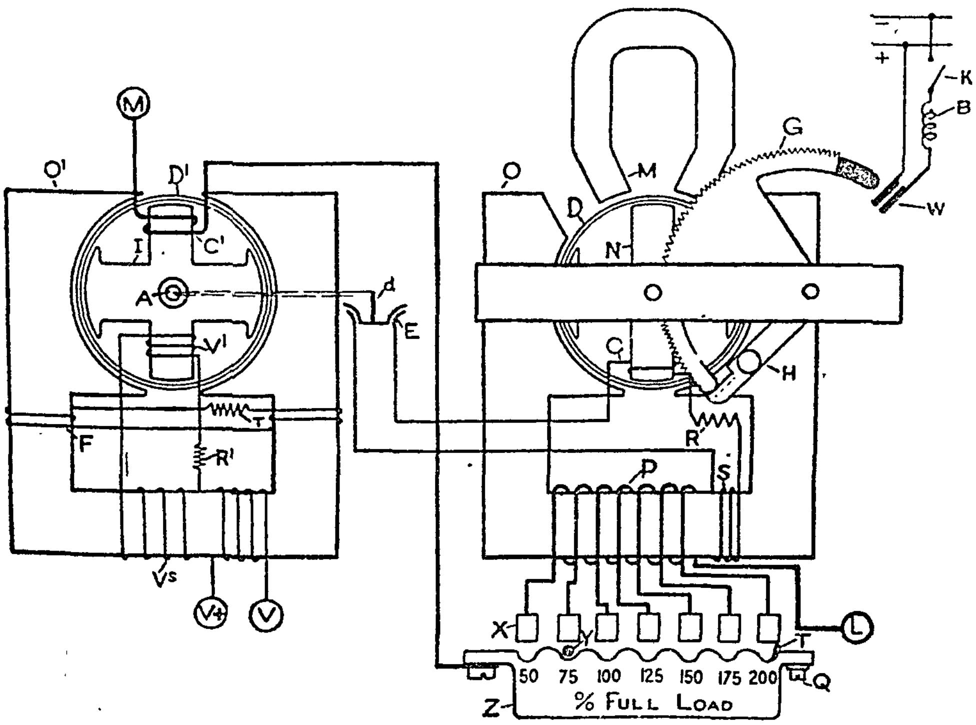


FIG. 16.8.—CONNECTIONS OF NALDER-LIPMAN DIRECTIONAL OVER-CURRENT RELAY.

the value for which the relay is set then the relay will operate. A feature of this type of relay is that it will take very heavy straight-through overload currents without operating.

The current through the reverse-power relay RP_1, Fig. 16.5, is the difference between the true-load current and the fault current, and therefore in the event of very high, but legitimate, loads it may be difficult to obtain discrimination between sound and faulty feeders. More reliable operation is obtained by making the relays operate with difference currents, instead of merely with reversals of power, and this is accomplished by operating the relays through cross-connected current transformers, as shown in Fig. 16.9. CT_1 and CT_2 are current transformers whose secondaries are cross-connected, and the two relays RP_1 and RP_2 are connected in series to what are points of equal potential in the secondary circuit when there is no fault, their operating coils then carrying no current. On the occurrence of a fault at F (Fig. 16.5)

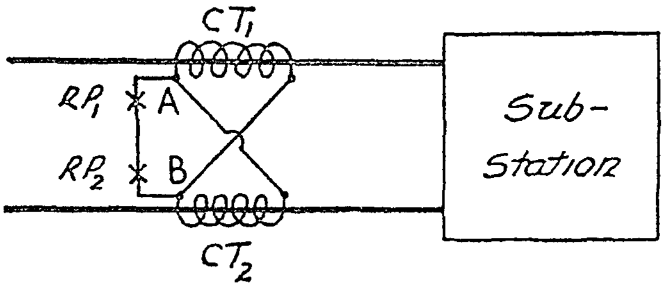


FIG. 16.9.—CROSS-CONNECTED CURRENT TRANSFORMERS.

the current in CT_1 will diminish, or may even reverse, while that in CT_2 will increase. The difference between the induced secondary currents will have to flow through the relays, and as these are directional, RP_1 will operate, but not RP_2. On the other hand, if the fault occurs in the other feeder, this difference current will be reversed and only RP_2 will operate.

Fig. 16.10 shows the essential circuit diagram for such a system, and it will be seen that back-up over-current relays are provided for single-circuit working, as explained previously. There is discrimination between internal and external faults, provided that both feeders are switched in at both ends. When only one feeder is in commission, a secondary current corresponding to the total current carried by the feeder flows through both sets of relays, tending to operate those of the feeder in commission. It is then necessary to make these relays inoperative, and this is done automatically by auxiliary switches in the voltage-supply leads to the directional relays. The auxiliary switches of one circuit-breaker are associated with the relays of the other feeder, and open with the circuit-breaker.

If both feeders are carrying the normal full load, and one of them is switched out at one end, the total load is carried by the other feeder, and in order to preserve stability the phase-fault setting of the relays must not be less than 200 per cent. of the normal full load carried by one feeder when both are in commission. This setting may be too high to deal adequately with earth faults, and then an earth-leakage relay must be used in addition to, or in place of, one of the phase-fault relays at each end of the feeder. The earth-fault settings so obtainable are from 20 to 70 per cent. of the current-transformer rating.

Parallel-feeder protection necessitates circuit-breakers at both ends of each feeder, for if a pair is controlled by a single breaker

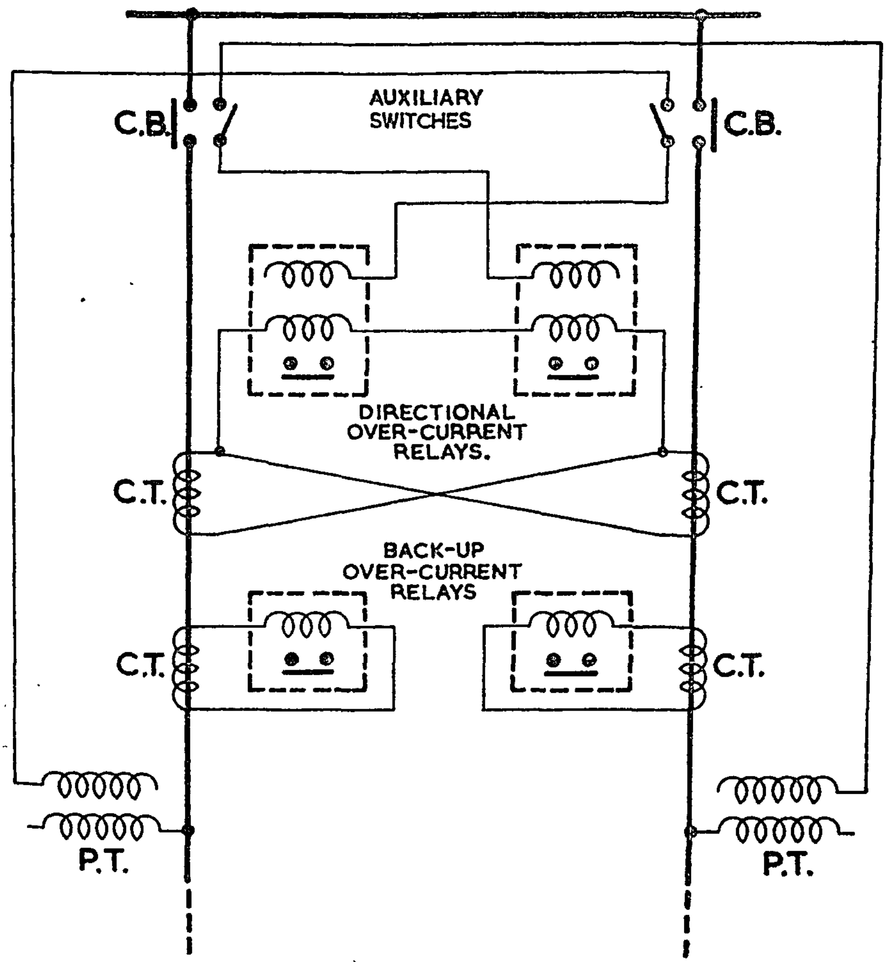


FIG. 16.10.—CROSS-CONNECTED CURRENT TRANSFORMERS WITH BACK-UP OVER-CURRENT RELAYS.

at one end, a fault near that end may not cause any difference current at the other end. With duplicate bus-bars the feeders must both be connected to the same set. Although a high phase-fault setting is used to maintain stability under switching conditions, there may be instability if a through fault occurs with one of the circuit-breakers at the receiving end open.

The possibility with unbiassed relays is that a slight want of balance in the current of the two transformer secondaries may set up sufficient current through the relays to operate one of them in the event of heavy through currents, even if there is no fault. For the protection of the outgoing end of two or more feeders in

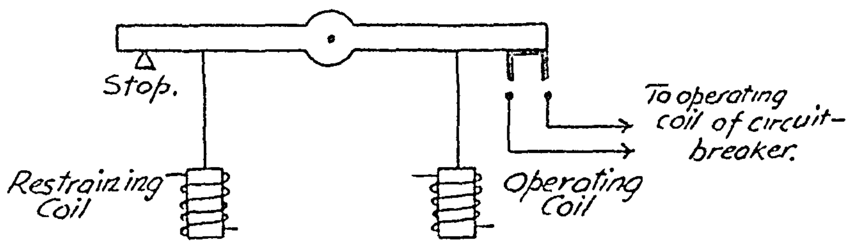


FIG. 16.11.—BEAM RELAY.

parallel, this difficulty can be overcome by giving the relays a 5 per cent. or 10 per cent. bias, so that unless the difference exceeds this percentage of the load currents, the relays will not act. One method of obtaining this bias is to use beam relays, in which the

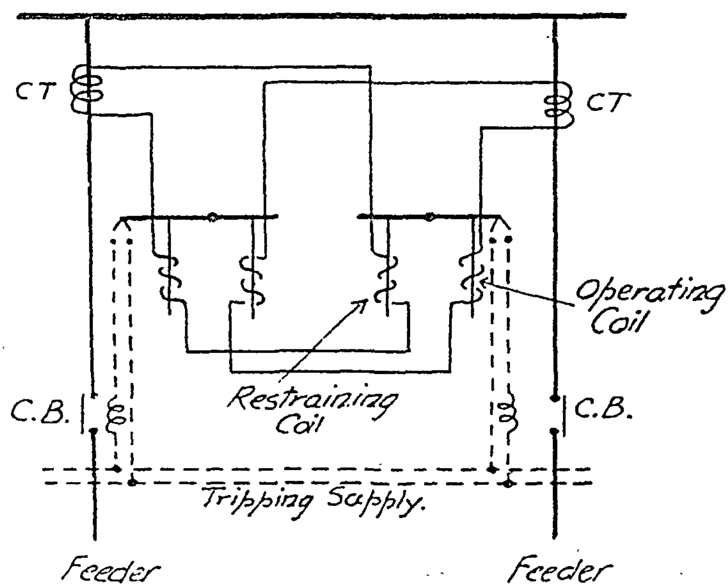


FIG. 16.12.—PROTECTION OF DUPLICATE FEEDERS BY MEANS OF BIASED BEAM RELAYS.

operating coil has to work against the pull of a restraining coil, as shown in Fig. 16.11. The current through the operating coil is derived from the feeder to be protected, whereas the current through the restraining coil is derived from the feeder in parallel. The restraining coil has 5 or 10 per cent. more turns than the operating coil, so that the relay will act only when the current through the operating coil is more than this percentage greater than the current through the restraining coil. Alternatively, the coils may have the same number of turns, the bias being obtained mechanically by shifting the fulcrum slightly to one side. In both cases the relationship will hold good, no matter what the magnitude of the total current may be, and this is of great practical importance, as it does away with the necessity for accurate matching of the current transformers. The application of such beam relays to the protection of two parallel feeders is illustrated in Fig. 16.12. Bias can also be provided on an induction-relay, and Fig. 16.13 shows a Reyrolle relay of this type. The figure

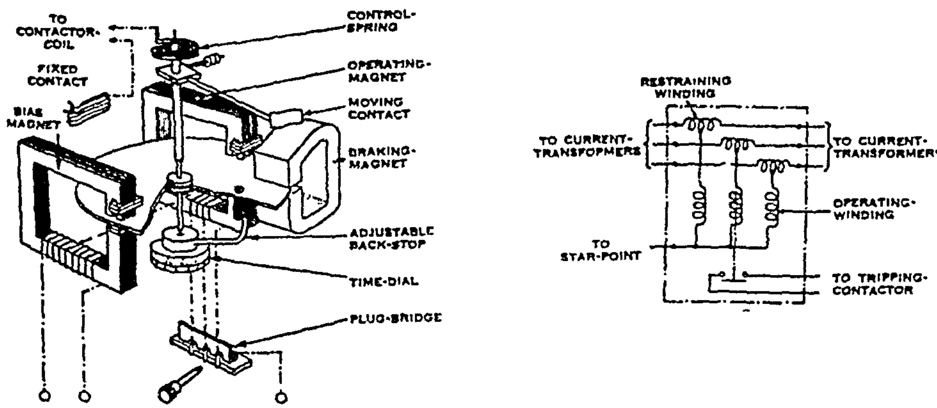


FIG. 16.13.—BIASED INDUCTION RELAY.
(*A. Reyrolle & Co. Ltd.*)

shows that there are two shaded-pole electromagnets acting on one disc as well as the permanent braking magnet. The operating torque comes from one of the electromagnets and the disc turned against the control torque of a hair spring. The second electromagnet is a biasing device which produces restraint by acting on the disc in opposition to the operating magnet, one end of the coil of which is connected to the mid-point of the bias winding.

Protection of Ring Mains

The ring main is a system of interconnection between a series of stations by means of which provision is made for alternative routes of power supply without the necessity for running feeders in parallel. The peculiarity of the ring main is that it may be a normal operating condition for the flow of power in some of the sections to be reversed at will, in which case reverse-power relays will not be applicable. If, however, the daily load curves of the various substations fed by the ring main are such that, under healthy conditions, the flow of power in any section is never reversed, then the use of reverse-power relays is possible. First consider the effect of using graded over-current relays only, the grading being carried out in both clockwise and counter-clockwise directions round the ring, as in Fig.16.14. If a fault occurs between GS and SS_3 then current will flow to the fault from GS through the faulty feeder, and also from GS via the substations SS_1, SS_2, and SS_3. Now, since the relays at C and 3 have the

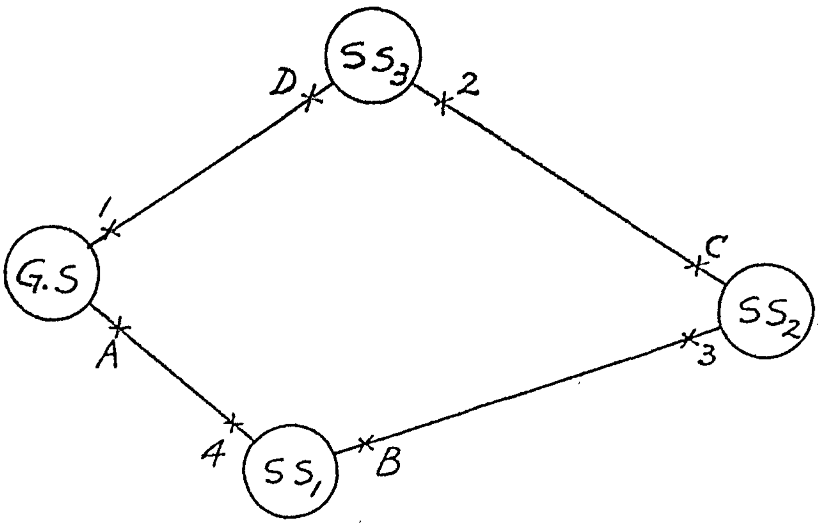


FIG. 16.14.—PROTECTION OF A RING MAIN.

shortest time lag, they will open first, and that at 1 will open next, thereby shutting down everything but the substation SS_1. Thus the above system is not practicable.

If there is no reversal of power in any section under normal operating conditions, then a series of reverse-power relays with graded time lags can be adopted, the grading being in both clockwise and counter-clockwise directions. At the generating station there can be no reversal of direction, the two relays at this station thus being of the non-directional, over-current type. Fig. 16.15 shows such a system, suitable minimum operating times being indicated. Since non-directional relays are less expensive and simpler than the directional type, an alternative scheme in which directional features are omitted at points C, E, F, and H can be adopted as in the following table. The necessary stability of operation is now achieved at the expense of slightly longer time intervals between adjacent relays. Note that the times given in the figure and in the table are for illustration only. In any particular installation they would be chosen to suit that installation.

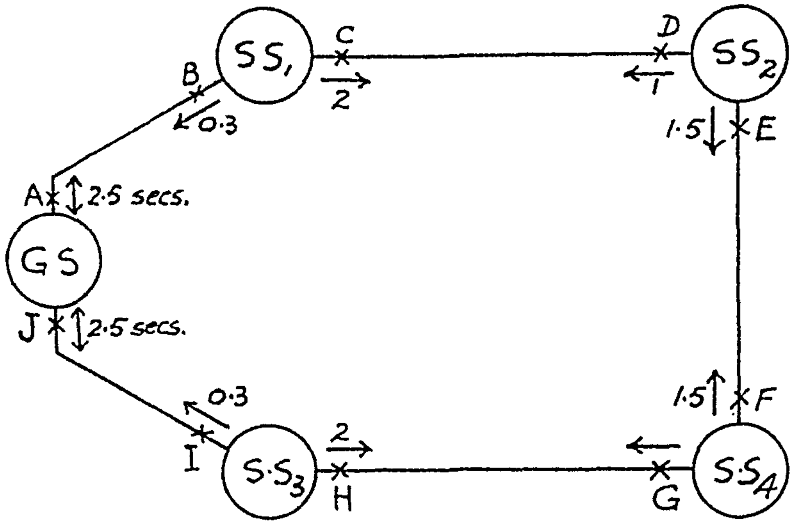


FIG. 16.15.—PROTECTION OF A RING MAIN.*

Station:		GS	SS_1		SS_2		SS_3		SS_4		GS
Oil switch:		A	B	C	D	E	F	G	H	I	J
Scheme of Fig. 16.15	Directional over-current relay: Clockwise	—	—	2	—	1·5	—	1	—	0·3	—
	Counter-clockwise	—	0·3	—	1	—	1·5	—	2	—	—
	Non-directional over-current relay	2·5	—	—	—	—	—	—	—	—	2·5
Alternative scheme	Directional over-current relay: Clockwise	—	—	—	—	—	—	1	—	0·3	—
	Counter-clockwise	—	0·3	—	1	—	—	—	—	—	—
	Non-directional over-current relay	2·8	—	2·2	—	1·6	1·6	—	2·2	—	2·8

Where the flow of power may be reversed at will, as in the case of a ring interconnecting a series of generating stations instead of a series of substations, it is necessary to consider each separate feeder without any reference to the others, and to use one of the forms of protection described later.

* The setting time at G is 1·0 sec.

Differential Systems

In these systems the relays respond only to the difference between currents which are normally equal. The biased-relay method of protection of parallel feeders, as previously described, is, strictly speaking, within this category. With differential systems the operation can depend on either voltage balance or on current balance, the difference between these two methods being illustrated in Fig. 16.16. With voltage balance the secondaries of two current transformers are connected in opposition, so that current only flows when there is any difference in the secondary-induced voltages e_1 and e_2. With current balance the secondaries are connected in series, but not in opposition, so that a circulating current is a normal feature. Two points normally

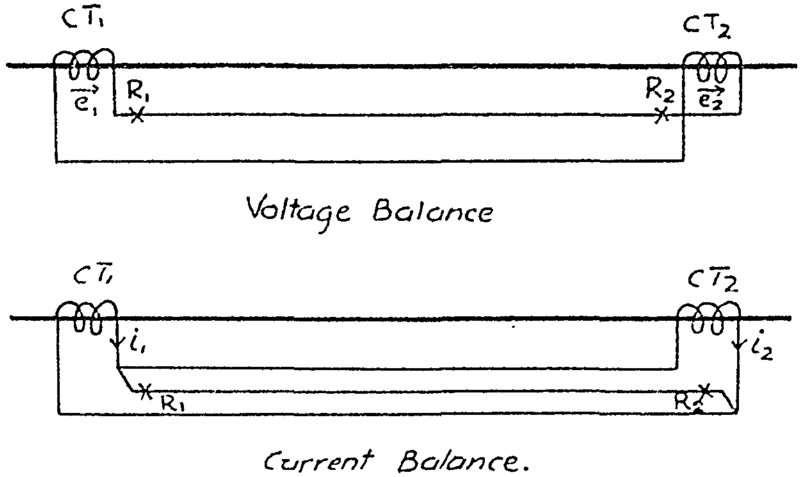


FIG. 16.16.—ILLUSTRATING THE PRINCIPLES OF VOLTAGE AND CURRENT BALANCE.

at the same potential are joined together, but this cross circuit will only carry current when the secondary-induced currents i_1 and i_2, and therefore the primary currents, are different.

Probably the best known of the differential systems is the Merz-Price system, which, as applied to feeder protection, utilises the principle of voltage balance. The general principle will be apparent from Fig. 16.17, in which CT_1 and CT_2 are current transformers located at the ends of the feeder to be protected, R relays, and T.C. the circuit-breaker trip coils. In the event of a fault at F and flow of power in the direction AB, CT_1 will carry a greater current than CT_2 with the result that the secondary voltage of CT_1 will be greater than that of CT_2, and a current will flow through the pilot wires and the breaker trip coils, thus bringing out both breakers at the same instant. For a three-phase feeder it is usual to connect the current transformers in star at each end, a three-conductor pilot cable thus being necessary, as shown in Fig. 16.18.

It will be clear that the current transformers are a critical feature of this system, since they have to be balanced exactly, not

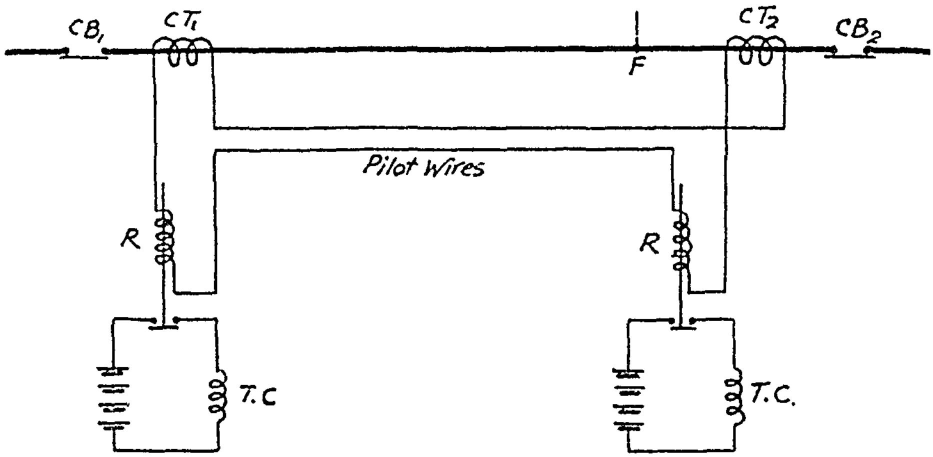


FIG. 16.17.—PRINCIPLE OF THE MERZ-PRICE SYSTEM.

only initially but permanently. In order that the induced voltage shall be proportional to the line current, it is essential that the magnetic circuit shall not reach saturation, and this is accomplished by providing a number of narrow air gaps in the

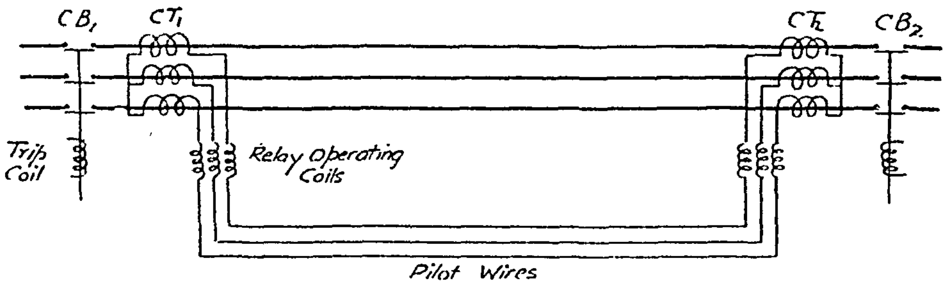


FIG. 16.18.—ESSENTIAL CONNECTIONS FOR THE MERZ-PRICE SYSTEM.

iron core. To secure initial matching, the transformers are balanced against a standard, and to ensure that there shall be no change of characteristics in service they are enclosed within a magnetic shield which prevents neighbouring iron affecting the distribution of flux.

The pilot wires are usually in the form of a three-core cable, which can be buried in the ground in the case of an insulated cable system, or slung from the towers in the case of an overhead system. Apart from the high cost of these pilot cables, which must be used on all systems depending on a balance between the two ends of a feeder and not employing special cables, the most serious disadvantage of the Merz-Price system, as described above, is the possibility of operation by heavy through currents due to the capacitances of the pilot wires. Such currents may induce voltages of the order of 1,000 volts in the pilot-wire circuit, and to prevent the resulting capacitance current operating the relays, the setting of the relays must be higher than is desirable. This difficulty is overcome in the Beard-Hunter modification, in which each pilot wire is compensated for capacity current by means of a metallic sheath which diverts this current from the relays. To prevent the sheath from forming a closed path for the true tripping current it is necessary to provide a break, as indicated in Fig. 16.19.

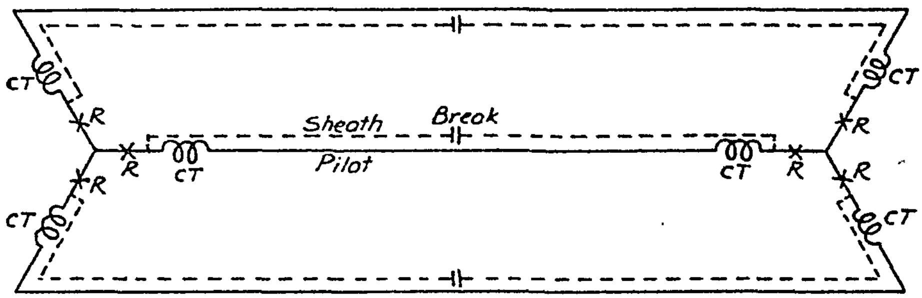
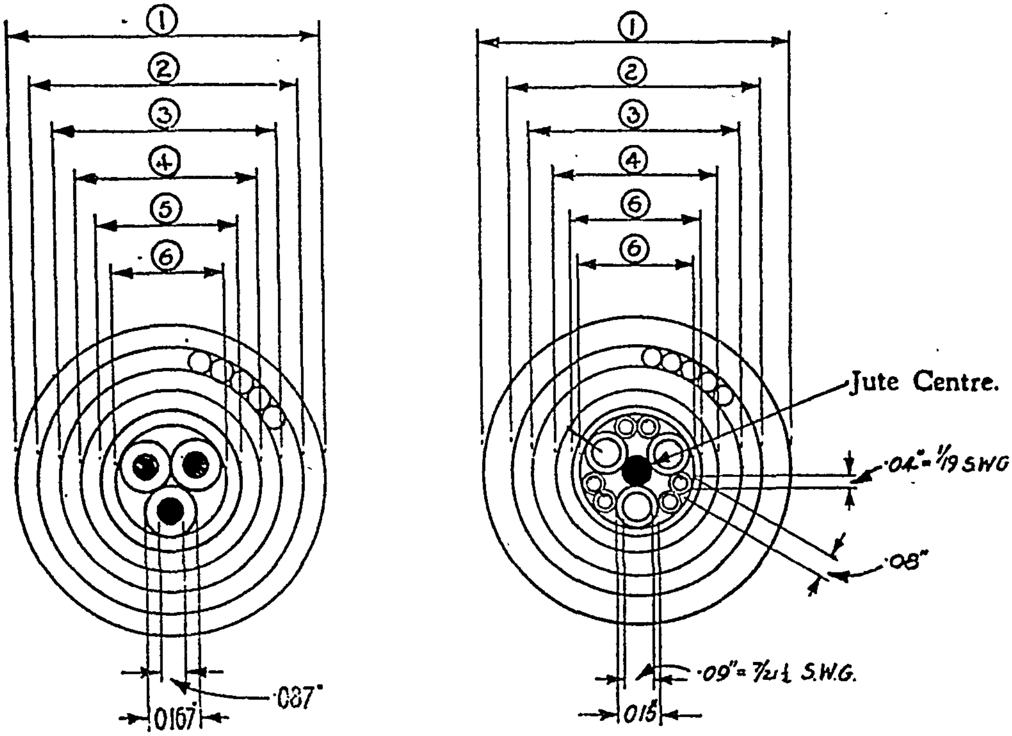


FIG. 16.19.—PRINCIPLE OF THE SHEATHED-PILOT SYSTEM.

The advantage of this method over the original Merz-Price system is that the relays can be of a more sensitive type and can have lower fault settings. The disadvantage is the extra cost of the pilot cable, the three-core sheathed type being roughly twice the cost of the plain three-core type. When telephone wires are incorporated in the pilot cable, then the difference in cost is not so great. Fig. 16.20 shows typical constructions for plain three-core pilot cables and pilot cables with telephone pairs.

An entirely different method of differential protection, due to McColl, is to employ biased relays at the two ends of the feeder to be protected, the bias being either mechanical, as with the beam relay, or electrical, as with a biasing transformer. Fig. 16.21 shows the essentials of the system when beam relays are employed. Each operating coil is energised from the corresponding current transformer, and has in series with it a duplicate resistance of the same value as one of the pilot wires. The restraining coils and the pilot wires form a series circuit in which the principle of current



(1) 1·124″ dia. over two hessian tapes.
(2) 0·924″ ,, ,, galvanised-steel wire.
(3) 0·78″ ,, ,, two hessian tapes.
(4) 0·58″ ,, ,, lead ·07″ thick.
(5) 0·44″ ,, ,, insulation ·04″ thick.
(6) 0·36″ ,, ,, cable cores.

(1) 1·194″ dia. over two hessian tapes.
(2) 0·954″ ,, ,, galvanised-steel wires.
(3) 0·81″ ,, ,, two hessian tapes.
(4) 0·61″ ,, ,, lead ·06″ thick.
(5) 0·49″ ,, ,, insulation ·03″ thick
(6) 0·43″ ,, ,, cable cores.

FIG. 16.20.—THREE-CORE PILOT CABLE, PLAIN AND WITH TELEPHONE PAIRS. (*A. Reyrolle & Co., Ltd.*)

balance is used, and since the relay is mechanically balanced in favour of the restraining coil, as explained previously on p. 416, through currents will give the restraining coils the greater leverage

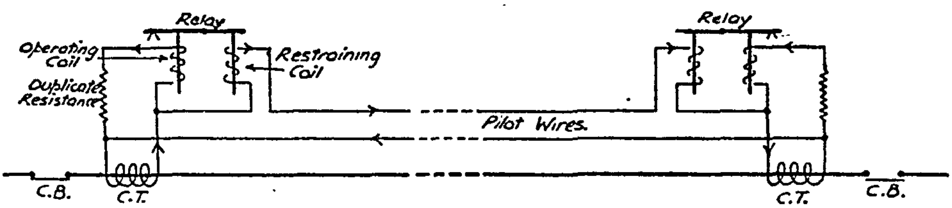
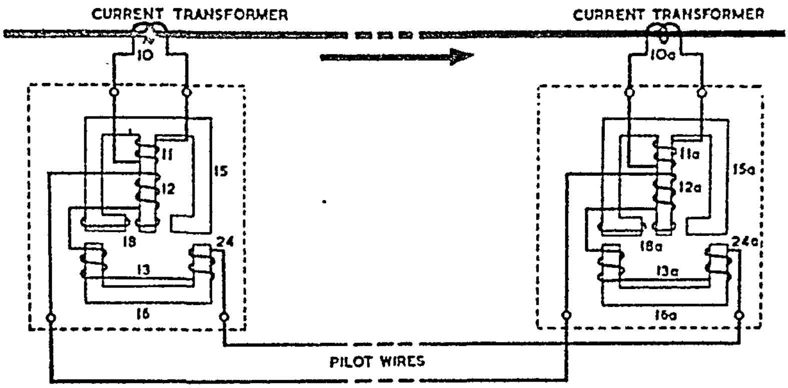


FIG. 16.21.—PRINCIPLE OF THE MCCOLL PROTECTIVE SYSTEM.

and thus prevent the relays from operating. If a fault occurs at any point between the two current transformers, the current at the sending end will be greater than that at the receiving end, and consequently there will be an excess of voltage in the transformer secondary at the sending end. This excess voltage will set up two currents, one through the operating coil and duplicate resistance at the sending end, and the other through both pilot wires, operating coil, and duplicate resistance at the receiving end. The ratio of the resistances is 1 : 3, and consequently the ratio of the currents in the two paths will be 3 : 1. If the fault current is of such a value that the increased current in the operating coil can overcome the restraining coil, the relay will operate and trip the breaker at the sending end. The circulating current through the pilot wires is of the order of 0·25 amp. at full load. Ordinary pilot cables without sheathing are employed; in fact, the capacitance currents flow through the restraining coils and thus help to give



(Discs and contacts omitted for clearness).

FIG. 16.22.—METROPOLITAN-VICKERS TRANSLAY SYSTEM.

the system stability. The above description applies to the three phase system equally well, except that the duplicate resistance has now a resistance equal to one-half of one of the pilot wires.

In the Metropolitan-Vickers system known as the Translay system, voltage balance is used, the fault setting is not fixed but is variable, and the relays are biased electrically. The essentials of the system are illustrated in Fig. 16.22, in which the tripping circuits have been omitted for clearness. So long as the feeder is healthy the line-current transformers 10 and 10*a* at the two ends of the feeder carry equal currents, and the coils 11 and 11*a* induce equal E.M.F.s in the windings 12 and 12*a*. These latter coils are connected in opposition by means of the pilot wires, with the operating windings 13 and 13*a* in series with them. Compensating devices neutralise the effect of capacity currents in the pilot wires and any lack of balance in the current transformers, which are of the ordinary instrument type and need not be balanced.

In the event of a fault on the protected feeder the line current at one end must carry a greater current than that at the other end, with the result that the E.M.F.s induced in coils 12 and 12*a* will be different and a current will flow through the pilot circuit. This current flows through the operating windings 13 and 13*a*, thereby setting in motion the moving elements of the relays. The spindle of the contact arm is spring controlled, and the setting can be varied by adjusting this spring. The relay automatically resets itself when the fault is cleared.

The compensation is effected by the short-circuited copper loops 18 and 18*a*. The pilot capacitance currents set up a flux in the lower magnet 16, which is approximately in phase with the leakage flux from the upper magnet 15, these currents thus setting up no torque on the moving element. To effect compensation the loops 18 and 18*a* are adjusted in position until the torque due to capacitance currents is a backward torque, a forward torque being produced only when the pilot current is in phase with or lagging behind the pilot voltage. This backward torque is also sufficient to compensate for the lack of balance of the current transformers 10 and 10*a*, and in addition it automatically adjusts itself to the needs of the occasion: it is negligible when the fault is on the protected feeder, but reaches a maximum under through-fault conditions.

It will be seen that operation is produced by two co-operating currents, the operating torque being proportional to their product. One of these is supplied by the local current transformers, and the other, which flows in the pilot circuit, can, in consequence, be relatively weak. Hence the ohmic resistance of the pilots has little effect on the sensitivity of the gear: the pilots can therefore be of small cross-section.

Since the pilot capacitance currents are compensated, inexpensive pilots without sheaths can be used, and the pilot voltage is limited by saturation of the iron circuits of the relays to about 130 volts.

The Reyrolle Solkor System

One of the main difficulties associated with the differential systems of feeder protection has been the current transformer. With the earlier systems it was necessary that these should be balanced and that they should maintain their balance in service. It is this which led to the systems employing ordinary unbalanced transformers described above. A further development is the Solkor system, which works on the balanced-voltage principle.

It is a pilot-wire system, these wires being supplied from summation transformers having their primary windings connected to current transformers of normal design, this being possible because the magnetising current of the summation transformer serves as a permanent burden. The summation transformers are designed to saturate at a low value of the secondary voltage and consequently unscreened pilot cables are possible. In fact, the pilot voltage is limited to about 150, thus limiting the pilot capacitance current and counteracting the effect of differences in the current transformer characteristics at high fault currents.

The effect of the saturable summation-transformers is also to distort the pilot-voltage waveform under heavy through-fault conditions, so that the difference between the voltages acting at the two ends of the pilot circuit, due, for example, to dissimilarity of the current-transformers at the two ends, is largely composed of higher harmonics. The resulting out-of-balance current in the pilot circuit is consequently due to higher harmonics and therefore cannot cause inadvertent operation since the relays are tuned to the supply frequency. This tuning, which is effected by shunting the primary of the relay-transformer with a condenser, also renders the relays immune from operation by high-frequency transient currents which may, under certain conditions, flow in the main cable. Stability is further improved by the use of restraining coils in the relays.

The connections for single-feeder protection are shown in Fig. 16.23. The current transformers at each end of the feeder are connected to tappings on the primary winding of the summation transformer, the secondary winding of which is in series with the pilots. The relay is a single-pole electromagnetic type, the operating coil being energised by the secondary winding of a relay transformer,the primary of which (shunted by the tuning condenser) is in series with the pilot circuit. The restraining coil is energised from a tertiary winding on the summation transformer, and carries a current which increases with the primary current.

Imagine an external earth fault on the red phase; then current flows out of the red-phase current-transformer secondary at the end X, into the primary winding of the summation transformer at D, through the whole winding, out at A, and back to the current transformer, inducing a voltage in the secondary winding as shown by the arrow V. At the end Y an identical current flows into the summation transformer at A′ and out at D′, inducing the secondary voltage V′. The voltages V and V′ are substantially equal in magnitude, since both are produced by the same primary current, but they are in opposition. The effect on the relays of any small pilot current (due, for example, to any small difference in the current-transformer characteristics and to pilot capacitance current) is inoperative because of the characteristics described above. With an internal fault fed from one end of the feeder, one summation transformer is energised, and current circulates in the pilots since there is no opposing voltage at the other end. The relays at both ends are thus operated. If an internal fault is fed from both ends of the feeder, the two primary

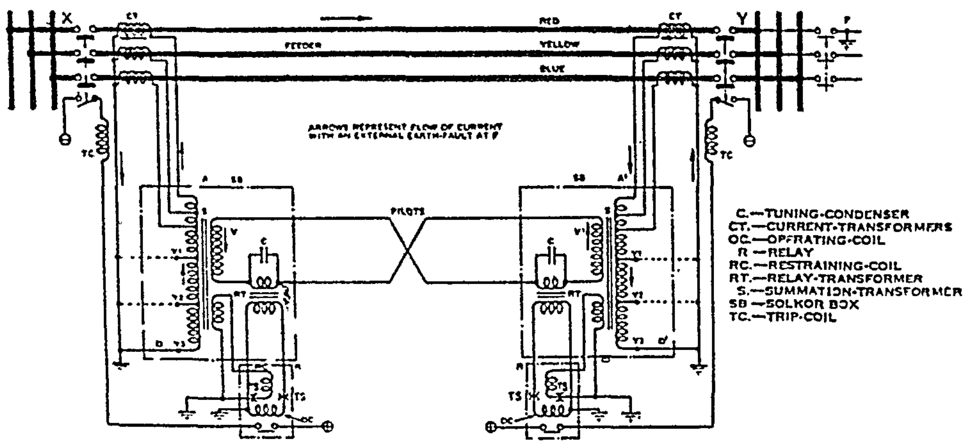


FIG. 16.23.—SOLKOR PROTECTIVE AS APPLIED TO A SINGLE FEEDER.
(*A. Reyrolle & Co. Ltd.*)

currents are in opposite directions and the induced secondary voltages thus in phase with one another, pilot current thus being circulated.

Fig. 16.24 shows typical stability characteristics, from which it will be seen that the curve for the out-of-balance current rises quickly to a maximum, and then flattens off. A similar characteristic obtains for the current required to operate the relays, so that a nearly constant factor of safety for stability is achieved.

It is now common practice to connect a high-voltage feeder directly to the terminals of a power transformer, the unit being controlled by one circuit-breaker at each end. One method of protection was a pilot system for the feeder, Merz-Price or combined over-current and earth-leakage protection for the transformer, together with intertripping to insure that both circuit-breakers would open in the event of a transformer fault. The Solkor system can be used for the protection of the zone including both feeder and transformer. A typical example is given in Fig. 16.25. It is, for all practical purposes, a combination of the system described above with instantaneous earth-fault protection.

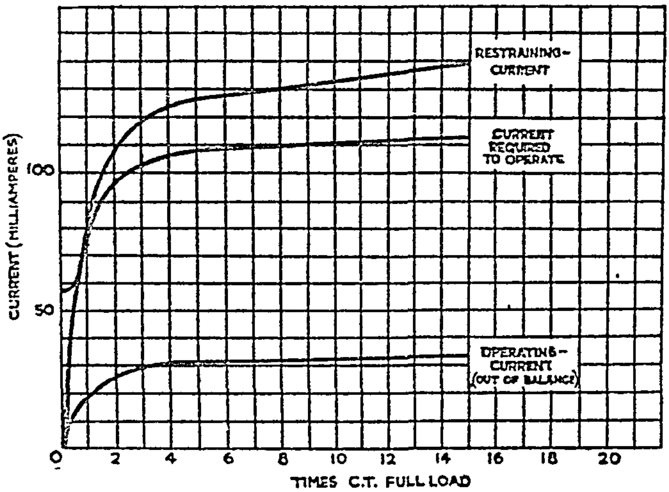


FIG. 16.24.—STABILITY CHARACTERISTICS OF THE SOLKOR SYSTEMS. (*A. Reyrolle & Co. Ltd.*)

As stated, one of the important requirements of an overall system of protection is that an earth-fault on either side of the transformer shall cause tripping of both circuit-breakers. This is

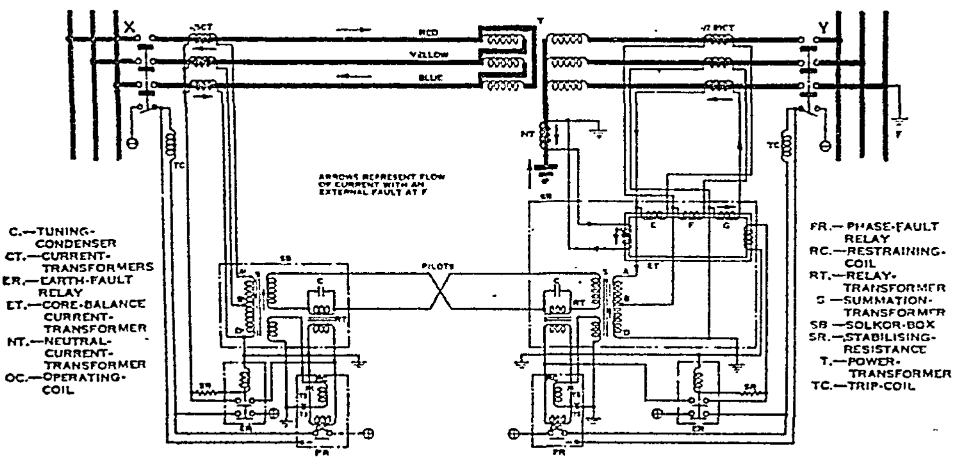


FIG. 16.25.—SOLKOR PROTECTIVE AS APPLIED TO A FEEDER AND TRANSFORMER. (*A. Reyrolle & Co. Ltd.*)

accomplished by arranging for the earth-fault relay at either end to inject its associated residual current into the pilot circuit through the relay transformers and so bring about the operation of the equipment at the other end. Suppose that this is not done and an earth-fault occurs on the star-connected winding of the transformer, then this fault will be cleared as a phase-fault at the other end of the feeder. However, any earth-current is limited by the neutral earthing resistor, and consequently the phase-fault setting might be too high to ensure protection of an adequate percentage of the star-connected winding. In such a case intertripping would be necessary. The Solkor system figured avoids the necessity for intertripping, since the earth-fault current at either end is made to operate directly the equipment at the other end.

Distance Systems

With the balanced systems it is possible to obtain discrimination with almost instantaneous operation of the relays, but with other systems time grading is essential. The time grading previously described is, to some extent, arbitrary, but in the distance systems the time delay is automatically made proportional to the distance between the relay and the fault. The principle of operation is illustrated by the diagram of Fig. 16.26, which shows the distribution of voltage along a feeder from the switch, where the voltage is a maximum, to the fault, where the voltage may be

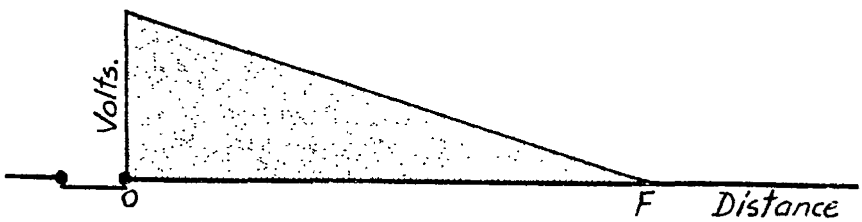


FIG. 16.26.—PRINCIPLE OF THE DISTANCE SYSTEM OF PROTECTION.

zero. The whole of the available voltage at the relay end is thus utilised in overcoming the impedance of the section of feeder OF. Denoting the impedance of this section by Z, we thus have the relationship

$$\frac{E}{I} = Z \propto d$$

Hence if the time delay is to be proportional to d, the relay must be so constructed that this delay is proportional to E and inversely proportional to I.

Fig. 16.27 shows a system consisting of two generating stations, two substations, and an interconnector between the substations. In the event of a fault at F on the interconnector, possible distributions of voltage and current will be as indicated, and by dividing the voltage by the current at any point we obtain the impedance between that point and the fault, and therefore a measure of the distance between that point and the fault. The

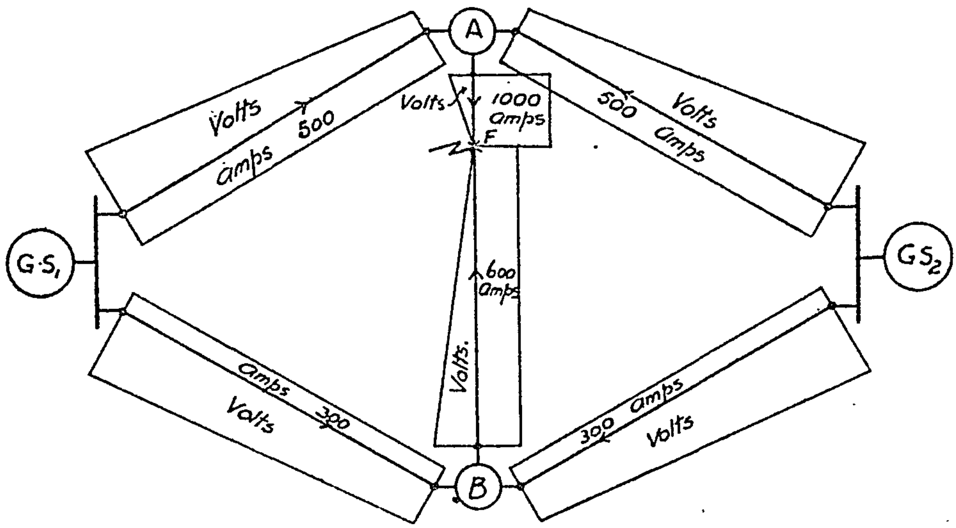


FIG. 16.27.—APPLICATION OF THE DISTANCE SYSTEM.

impedance at substation A is lower than at any other part of the system, and consequently with relays operating in the above manner the interconnector will be disconnected at A, and immediately afterwards at B.

Littlejohn also points out that under some circumstances the relay must also be provided with directional properties, for consider the section of a ring main in Fig. 16.28 comprising two generat-

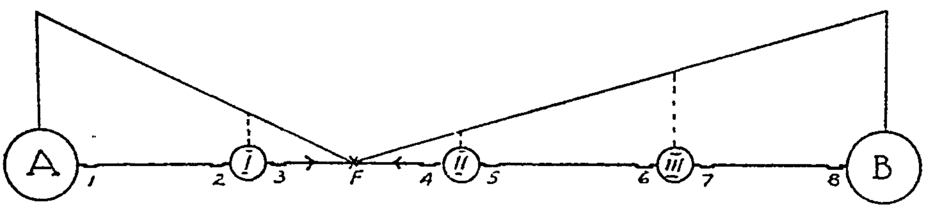


FIG. 16.28.—APPLICATION OF THE DISTANCE SYSTEM.

ing stations A and B, and three substations I, II, and III. With a short-circuit at F the voltage distribution will be as shown, and to clear the faulty section switches 3 and 4 must be tripped. But switch 2 has the same current and voltage as switch 3 and thus a directional element is necessary to ensure that switch 2 will not be opened.

The operation of this system on a network similar to that of

Fig. 16.27 will be made more clear by a numerical example. Suppose the system voltage is 30,000, and all cables have an impedance of 1·05 ohms per mile. The distances and actual impedances Z_1 are indicated in Fig. 16.29. Suppose that the potential trans-

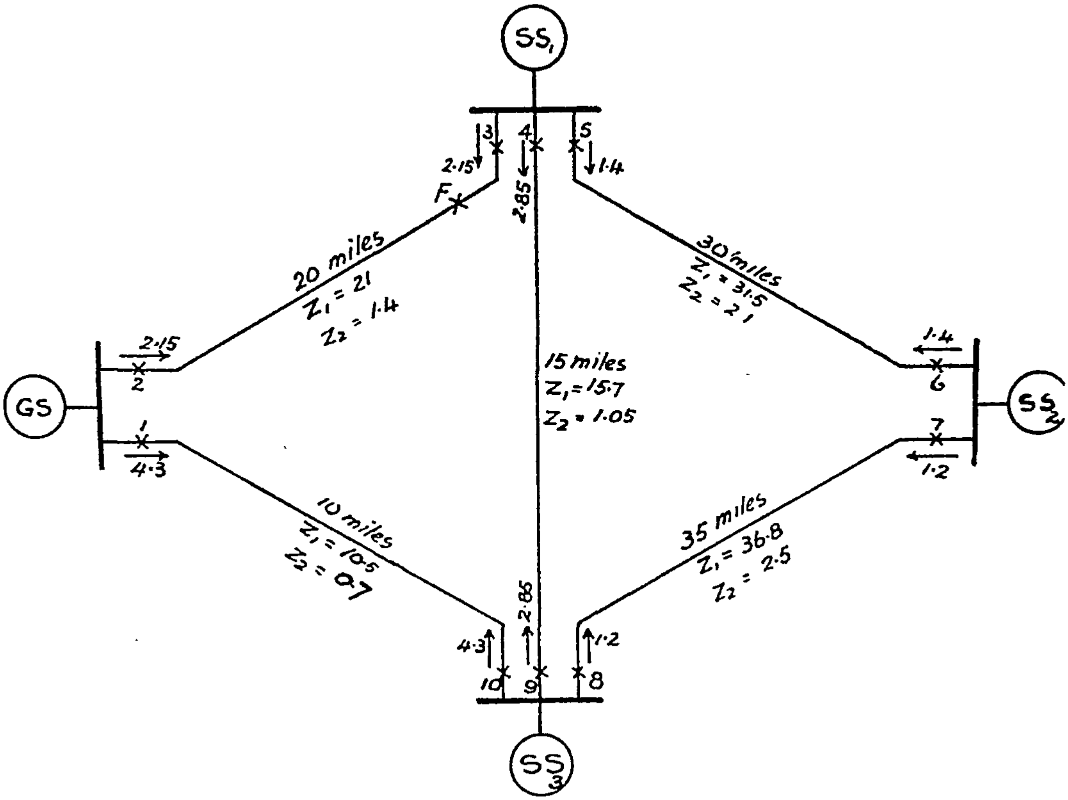


FIG. 16.29.—NUMERICAL EXAMPLE.

formers have a ratio of 30,000/100, and the current transformers a ratio of 100/5. The tripping time

$$t = k \cdot \frac{E}{I} = kZ$$

obviously refers to the volts and amps. in the secondaries of the instrument transformers, i.e. the secondary impedance Z_2, which is given by

$$Z_2 = Z_1 \times \frac{100}{30000} \times \frac{100}{5} = \frac{Z_1}{15}$$

These secondary impedances are indicated by Z_2 in the figure. The characteristic of the relay is given by

$$k = \frac{t}{Z_2}$$

Suppose the maximum permissible time is chosen as 3 secs. in all cases, then

$$k = \frac{3}{Z_2}$$

and the characteristics are as indicated by the figures adjacent to the various relays. Thus for relays Nos. 2 and 3

$$k = \frac{3}{1 \cdot 4} = 2 \cdot 15$$

for relays 4 and 9

$$k = \frac{3}{1 \cdot 05} = 2 \cdot 85, \text{ and so on.}$$

If a short-circuit occurs at F, the relay No. 3, which has a basic time of 2·15 secs., will trip its breaker in time T_3 (Fig. 16.30), and

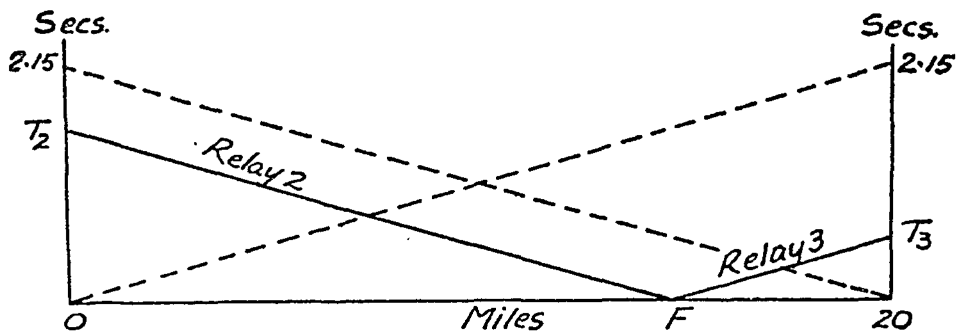


FIG. 16.30.—RELAY SETTINGS.

relay No. 2 will follow in time T_2. No other relay will follow ; for consider No. 9. The secondary impedance between it and the fault is just over 1·05 ohms, say 1·1 ohms, so that the time is

$$t = kZ_2 = 2 \cdot 85 \times 1 \cdot 1 = 3 \cdot 15 \text{ secs.},$$

by which time the faulty section has been cleared. Relays 4 and 5 are prevented from operating along with 3, by making them directional as indicated by the arrows.

Now consider a series circuit consisting of a single generating station supplying a number of substations, then the directional feature can be omitted and the locations of the relays will be as shown in Fig. 16.31. The setting of the relays will be as follows : the minimum time in the case of a fault adjacent to the relay cannot be zero, but will be a small fraction of a second, say 0·2. The maximum time for a fault at the far end of the section being pro-

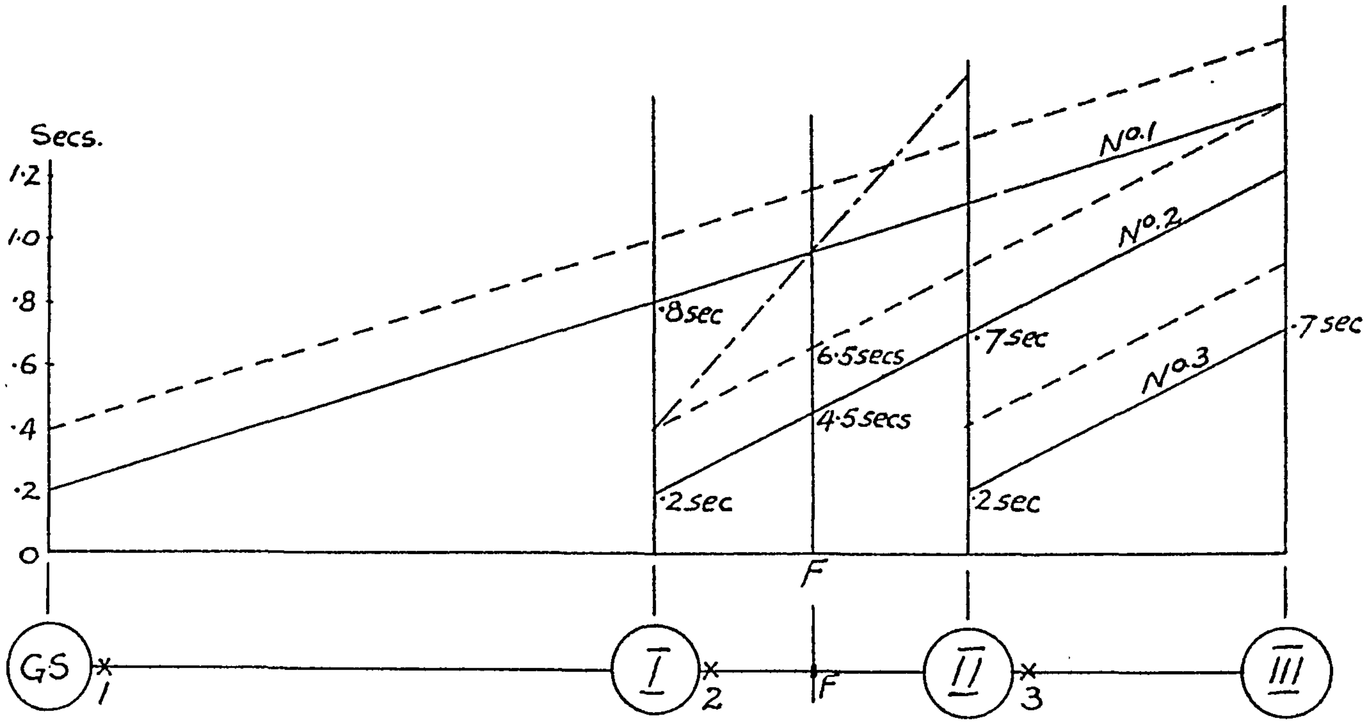


FIG. 16.31.—RELAY SETTINGS.

tected will be not more than, say, 0·8 sec. Using these two values the characteristics of the three relays will be as shown in the figure, and if a short-circuit occurs at F then relay No. 2 will operate in 0·5 sec., that is, before No. 1 can operate. As there can be no reversal of power in this system, No. 3 will not be affected.

To obtain the complete performance of the system it is necessary to take into account the time of opening of the breaker contacts: suppose this is 0·2 sec., then the operating characteristics of relays plus breakers are indicated by the dotted lines, from which we see that as none of the characteristics intersect one another the operation will be correct. Thus for a fault at F breaker No. 2 would open in 0·7 sec. Suppose the characteristic of relay 2 (plus breaker) were as shown by the chain-dotted curve, then for faults beyond F breaker No. 1 would be opened, thus showing that such intersection of the characteristics is not permissible.

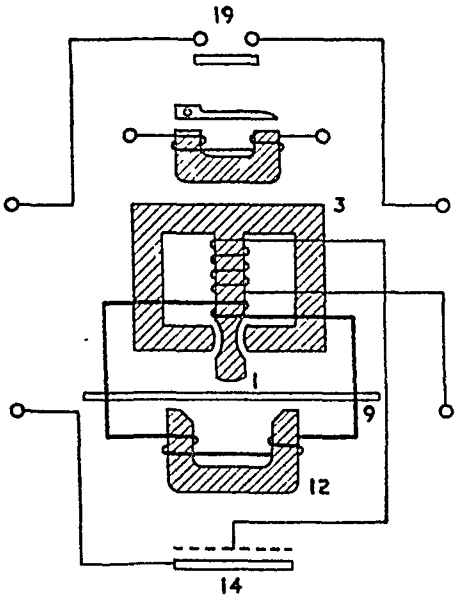


FIG. 16.32*a*.—CIRCUIT OF IMPEDANCE-TIME RELAY.

The impedance-time relay manufactured by the Metropolitan-Vickers Co., Ltd., consists of a combination of an over-current element of the type illustrated in Fig. 16.3 with a voltage-restraint element, the circuit being illustrated in Fig. 16.32*a*, in which 3 and 12 are the electromagnets excited exactly as in the over-current relay, 9 the rotating disc, and 14 the plug bridge for adjustment of the current setting. The action of the impedance element is illustrated by Fig. 16.32*b*. The spindle of the rotating disc is geared down to a second spring-controlled shaft, the top spring of which is attached to a bent lever whose movement controls the position of an insulated arm mounted on a separate shaft coaxial with the spring-controlled shaft, as shown in the figure. This second shaft also carries a flat armature of soft iron which is normally held against the face of the voltage element consisting of an electromagnet energised by a voltage coil. The relay contacts are so arranged that their position is controlled by the insulated arm, one being mounted on a flexible strip, and moved into, or out of, contact with the second by means of the arm. The fixed contacts are indicated by 19 in the figure, and the voltage element with its armature is immediately below these.

The operation is as follows: on the occurrence of a fault the over-current element rotates the disc at a speed depending on the magnitude of the current, thereby winding up the spring and so setting up a pull on the armature of the voltage element. When this pull is greater than the pull of the restraining magnet the armature leaves the face of this magnet and the contacts are closed

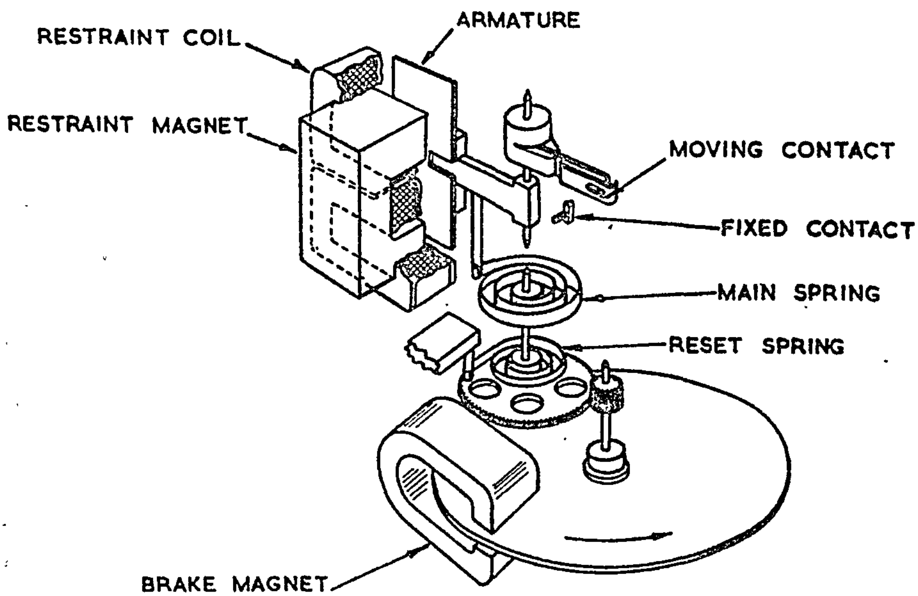


FIG. 16.32*b*.—ARRANGEMENT OF BIASED INDUCTION RELAY.
(*Metropolitan-Vickers Electrical Co. Ltd.*)

by the sudden movement of the insulated arm. For a given current the time taken by the above operation is proportional to the pull of the voltage coil, and therefore to the line voltage at the point where the relay is connected. Also the speed of the disc is proportional to the current, the time of operation thus being inversely proportional to the current. The necessary condition of time proportional to the quotient E/I is thus fulfilled.

The relay as described above is non-directional and when directional operation is required a wattmeter type of element is added.

The impedance-time relay gives correct discrimination if the impedance of the fault is zero; if the fault impedance is appreciable then the combined effect of line and fault impedance will be to make the relay operate as though there were a fault of zero impedance at some more distant point, with a consequent increase in operating time. In such a case each relay will act as a "back-up" relay to the next in the sequence so that, even if the correct circuit-breaker fails, the fault will still be isolated in a slightly longer time. Ross and Bell show how this effect can be compensated by means of resistances connected in series with the current coils of the relay. A second method is to make the relay a reactance-time, instead of impedance-time, relay, since the impedance of an arc is mainly due to resistance, there being very little inductance.

There are two types of distance or impedance relay, viz. the impedance-time or reactance-time relay as described above, and the definite impedance relay. This latter type is designed to operate when the impedance of the circuit falls below a specified value. The impedance of a circuit (reckoned from any point such as a generating station or substation) is proportional to the length of the circuit, and if there is a fault at some intermediate point the impedance between the station and the fault will be less than that of the complete line. Consequently if the relay is set so that it will operate when the line impedance becomes less than its total impedance under conditions of no fault, it is clear that it will operate under fault conditions. As the setting of such a relay can be based on the greatest length of line which, when short-circuited, will cause operation, the name distance relay is also used. The distance relay can either be of the induction type or the beam type, the latter being illustrated in Fig. 16.33. The beam is actuated by two magnets energised respectively by the current and voltage of the circuit, and the tripping contacts are placed at the current magnet end so that current tends to trip the breaker, and voltage tends to keep the breaker closed. The beam has also a slight mechanical bias to ensure that the trip circuit will remain open when neither current coil nor voltage coil carry current. Assuming that the pulls of the current magnet and voltage magnet are proportional to I^2 and E^2 respectively, these two pulls, or rather the turning moments produced by them, can be expressed by k_1I^2 and k_2E^2 respectively, where k_1 and k_2 are constants. Then the relay will be on the point of operating when

$$k_1I^2 = k_2E^2$$

$$\text{when } \frac{E}{I} = \sqrt{\frac{k_1}{k_2}} = \text{a constant}$$

$$\text{when } Z = \text{a constant}$$

Thus by a suitable choice of the numbers of turns on the two magnets, and of the ratios of the instrument transformers, the relay can be set to operate at any desired impedance. If there is a fault within the length of line corresponding to this setting, the pull of the current magnet will predominate and the tripping contacts will be closed.

One form of induction-type impedance relay is of the attracted disc type. The disc is acted on by two electro-magnets, one

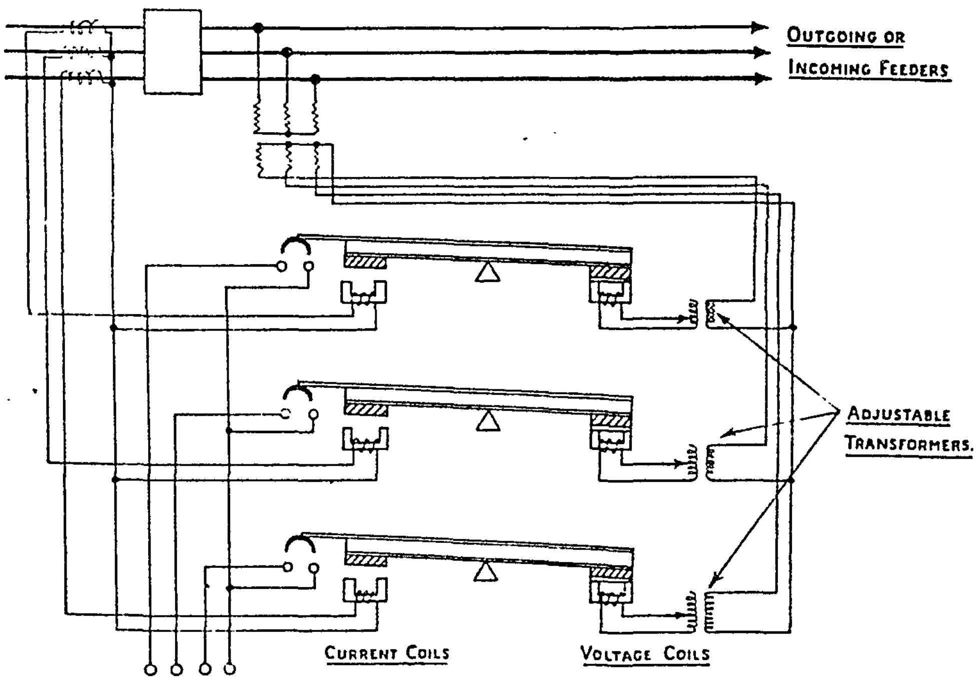


FIG. 16.33.—DEFINITE-IMPEDANCE SYSTEM USING INSTANTANEOUS BEAM RELAYS. (*Metropolitan-Vickers Electrical Co., Ltd.*)

current-operated, the other voltage-operated. Assuming a square law, the closing torque due to the current element and the restraining torque due to the voltage element, the torque is given by—

$$T = k_1 I^2 - k_2 V^2$$

The relay will thus be on the point of operating when—

$$k_1 I^2 = k_2 V^2$$

exactly as with the beam type. The angular travel for operation is made small, thereby ensuring rapid operation for near faults. For a distant fault the reduction in the operating torque increases the closing time, with the result that near the far end of the protected zone the closing time increases rapidly as shown in Fig. 16.34, which should be compared with Fig. 16.31. Because of this peculiarity the relay can be set for an impedance greater than that of the zone it protects without any risk of inadvertent operation because of a fault in the following section.

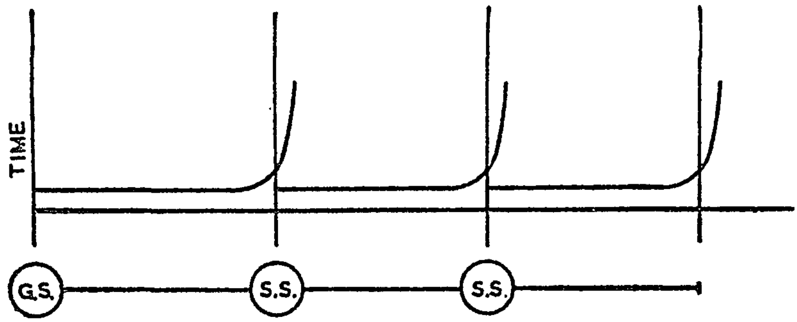


FIG. 16.34.—TIME-DISTANCE CURVE FOR DEFINITE IMPEDANCE RELAY.

This type of relay is applicable to the case shown in Fig. 16.35, in which a station GS feeds a substation SS through a single transmission line. The substation can be protected locally by an inverse-time overload relay, but we cannot use an overcurrent relay to protect the feeder because the current which flows on the occurrence of a fault is not a constant, but varies with the amount of plant feeding into the system and with the number of sections

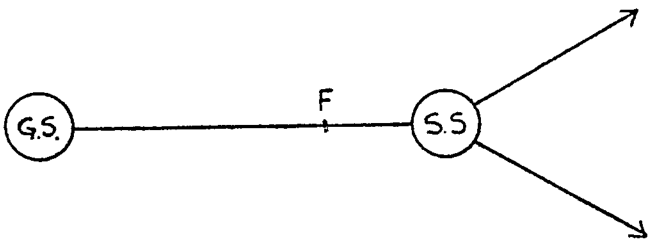


FIG. 16.35.

of the network in commission. Thus the current in the line on the occurrence of a fault at F can vary within wide limits, but the ratio of the voltage at GS to the current in the line will be roughly constant and equal to the impedance. Thus a definite-impedance relay is applicable, and for such a system its action will be instantaneous.

It is possible to combine definite-impedance and impedance-time relays in the same system. The former protect the greater length of the section—say, 75 per cent.—the advantage being that for the whole of this length the operating time is small and constant—i.e. not progressively increasing as with the impedance-time relay. Therefore 25 per cent. is protected by the latter, which can act in a dual role by providing back-up protection as well.

A development of this is the stepped characteristic of Fig. 16.36. This is preferable to the others because (*a*) it gives minimum operating times over the feeder as a whole, and (*b*) it is the simplest to grade. The curves above the base line apply to the relays at A and B when fault power flows from A to C, and those below the base line apply to the relays at C and B when fault power flows from C to A. Thus, on the occurrence of a fault at F in the feeder BC, fault power flows from A to C, and the relays at A and B

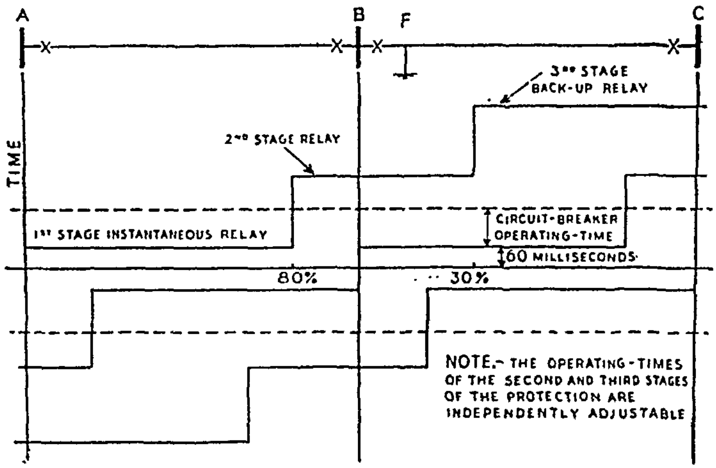


FIG. 16.36.—STEPPED CHARACTERISTICS.

begin to operate in accordance with their characteristics. Hence, the relay at B trips its circuit-breaker instantaneously, but that at A does not trip because the fault is cleared before the relay at A has completed its operation. The margin of discrimination is given, as before, by the vertical intercept at F between the characteristics for the relays at A and B. It is sufficient for the circuit-breaker at B to clear the fault before the relay at A has closed its tripping circuit. The curves show that about 80 per cent. of the length of each feeder is in the instantaneous zone. For faults outside this, the time of operation is increased in order to provide discrimination between adjacent feeders, as explained previously.

A more recent type of impedance relay which can fulfil the above requirements is the magnetically polarised relay of Fig. 16.37, which shows a single element. It has a permanent magnet core and a cvlindrical yoke, this giving a uniform radial magnetic field. The current and voltage coils are wound on a horizontal cylindrical metal former E which is supported in the radial field between the concentric poles of a permanent magnet. The coil former is carried on a horizontal spindle D extended at both ends and suspended on vertical leaf springs, which permit it to move axially but keep it concentric with the magnet poles. The assembly carries a moving contact C which closes on to a fixed contact B when the relay operates. The moving-coil type of construction has the inherent advantage that the operating forces produced by the two coils are independent of the position of the moving parts, whereas in an attracted armature or beam type of relay any small displacement of the armature from the voltage electromagnet greatly offsets the restraining force and may lead to false tripping.

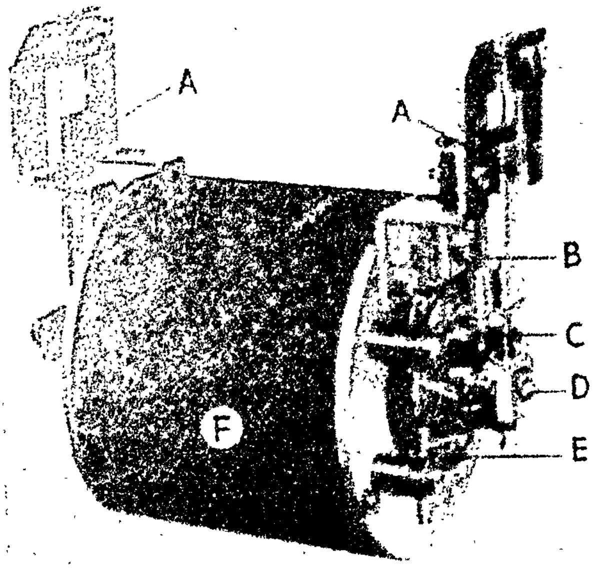


FIG. 16.37.—"STABILAY" MOVING-COIL ELEMENT. (*Metropolitan-Vickers Electrical Co. Ltd.*)

A. Flexible leaf suspension.
B. Fixed contact with backing strips.
C. Moving contact.
D. Spindle carrying moving coil.
E. Moving coil.
F. Magnetic element.

Reactance Protection

Impedance relays measure the impedance of the fault and the earth return. This impedance is due to (*a*) resistance in the fault arc, (*b*) tower footing resistance. With small fault currents, this being more probable with earth-faults than with phase-faults, the arc may expand to considerable length. With the substantial faults usually experienced in practice, the arc resistance may be of little importance. Tower footing resistance is of considerable importance in the case of earth-faults—for example, with high soil resistivity—particularly for feeders not provided with an earth wire. Thus, these additional resistances, which modify the impedance but not the reactance, are of importance only with earth-faults, so that there is some advantage in using a reactance-measuring relay for protection against earth-faults, while retaining impedance relays for phase-faults.

In the ratio-balance system introduced by Messrs. A. Reyrolle &

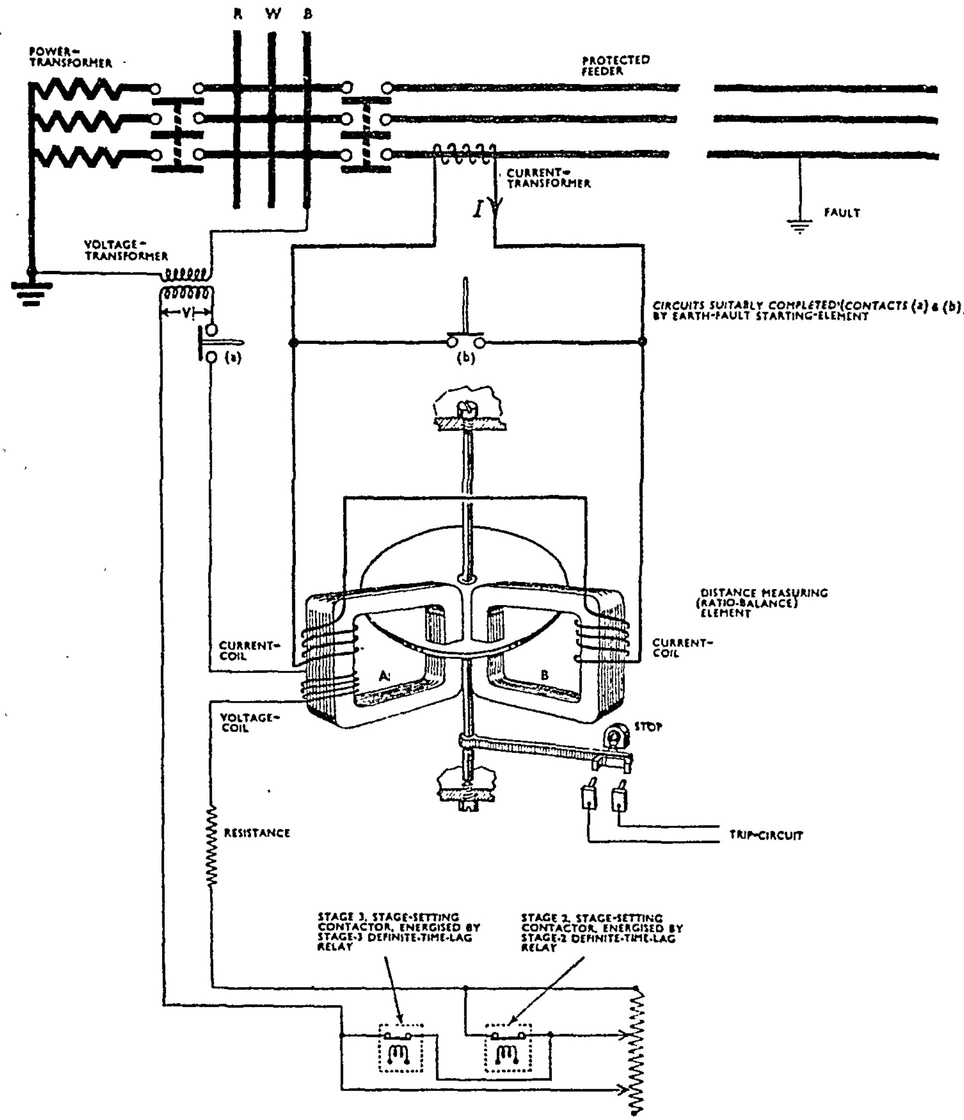


FIG. 16.38.—REACTANCE-TIME RELAY.
(*A. Reyrolle & Co. Ltd.*)

Co., Ltd., the relays are of the reactance-time type. The general arrangement is shown in Fig. 16.38, from which it will be seen that the moving system is actuated by two elements, the time-measuring element, which is both current and voltage operated, and the distance-measuring element. Normally the moving system of the distance-measuring element holds the contact-making member against a stop and leaves the tripping circuit open. When a fault occurs the voltage and current-coil circuits are energised by the closing of contacts *a* and the opening of contacts *b*, and the operation then depends upon the position of the fault. If the fault is in the instantaneous zone, the distance-measuring element (which is set to operate at, or below, a definite value of reactance) closes the tripping circuit instantaneously. If the fault is outside the instantaneous zone, a definite time-lag relay energises the stage-2 stage-setting contactor which then inserts resistance in the voltage-circuit of the reactance-measuring element and thus increases its setting to the stage-2 value. If the reactance relay now operates, the time taken to complete the tripping circuit has been increased by the operating time of the definite time-lag relay.

If the reactance element still does not operate, however, another definite time-lag relay energises the stage-3 stage-setting contactor after a further time-delay. This causes a further increase in the setting of the reactance element to its stage-3 value, and the operating time will have been further increased if the relay now operates.

The following theory of the instrument has been supplied by the makers. The torque on the induction disc is produced by the interaction of the fluxes due to A and B. The flux in magnet system A is proportional to the vector sum of the currents in the two windings accommodated on A. Since these are energised by the current I and voltage V, and since it is possible to adjust the phase relation between these two quantities by a suitable phase-shifting device, the flux in A may be made proportional to

$$I - \frac{V}{k}\sin\phi$$

The flux in B is proportional to I. Since, however, in any watt-metrical movement, the torque is proportional to the product of the two fluxes and a function of the phase angle between them (the latter being assumed constant in the case under consideration since phase angle ϕ between V and I is already contained in the expression $I - \frac{V}{k}\sin\phi$), the torque T upon the induction disc is given by

$$T = c(I - \frac{V}{k}\sin\phi)I$$

where c and k are constants.

$$\therefore T = cI^2 - \frac{c}{k} VI \sin \phi$$

The expression thus consists of two terms of opposite sign, and the relay operates and closes the tripping contacts when the first term exceeds the second term: it is restrained against the stop when the second exceeds the first. It is just on the point of operating when

$$cI^2 - \frac{c}{k} VI \sin \phi = 0$$

when

$$\frac{V \sin \phi}{I} = k$$

In other words, the relay operates at, or below, a constant value of $V \sin \phi/I$, i.e. a constant reactance.

Considering now the complete operation of the reactance-measuring unit, we have seen that the stage 2 and stage 3 setting contactors insert resistance in the voltage circuit, and therefore reduce the flux produced by V. If we assume that this reduced flux is proportional to V/K, where K is a constant depending on the magnitude of resistance inserted, we have the following expression:

$$\frac{V \sin \phi}{IK} = k$$

Thus the relay still operates at or below a constant value of reactance equal to $K \times k$ ohms, the value of K being different for time intervals corresponding to stage 2 and stage 3 time-settings.

Interlocking Systems

Provided that the cost of the pilot wires is not prohibitive, the systems of protection previously described have proved themselves to be thoroughly satisfactory. In the case of the very long lines which are becoming increasingly common, the cost of the pilot wires becomes prohibitive. One solution is to install pilot wires in the same channel as telephone lines, running, more or less, along the same route as the feeder to be protected, and this solution may be facilitated by the fact that, for the purpose of supervision, power-stations and substations are linked by telephone. A difficulty is that with A.C. operation it is necessary to separate the pilot wires from the telephone wires by means of metallic screens and special insulation. Thus, the Post Office requires 15-kV. insulation for this purpose. This difficulty is overcome by the use of D.C. for the pilot current.

In the D.C. "lock-in" system there is a locking relay which, when required, prevents the inadvertent opening of one of the circuit-breakers, thereby, as it were, locking the feeder to the circuit-breaker. The following description of the operation of the system, which is due to Kaufmann (*The Protective Gear Handbook*), can hardly be bettered. "Direction-comparison lock-in systems, as the name implies, operate on the simple principle of comparing the relative directions of power flow at the two ends of a section. The comparison takes place only when the magnitude of the power flowing exceeds the rated full-load power of the circuit, to which end the relays which do the comparing are given suitable settings in terms of the nominal full-load current. At values of power less than the setting, the equipment as a whole remains static, notwithstanding that the components responsible for detecting fault-power direction (the directional elements) may respond.

"At higher values, the equipment at the sending end of the feeder, where the power flow is *away* from the bus-bars, becomes operative and tends to trip the circuit-breaker at that end. At the same time the equipment at the receiving end, where the power flow is normally *towards* the bus-bars, operates to connect a local battery across the pilot wires, and a 'locking' relay at the sending end is thereby operated. The operation of the locking relay prevents the sending-end equipment from tripping the circuit-breaker, and thus the feeder is 'locked in.'

"The foregoing is a description of the sequence of events under the conditions when either excessive load power or fault power is fed into the feeder at one end and out at the other; in other words, under healthy conditions on the feeder concerned.

"When that feeder is faulty, the sending-end equipment tends to operate as before. At the receiving end (assuming the feeder is in a ring main or is otherwise supplied with power at both ends) the power flow is now *away* from the bus-bars, i.e. towards the fault. The equipment thus behaves as a sending-end equipment and operates to trip the local circuit-breaker, and no locking impulse is initiated. The result is that the sending-end equipment is allowed to complete its operation and to trip the circuit-breaker at that end, thereby isolating the fault.

"In the absence of a power source at one end, as, for example, in the case of an open ring main (a fault close to the sending-end bus-bars would have the same effect with the ring closed), the relays at the receiving end would not operate. The circuit-breaker at that end would therefore not be tripped; but neither would a locking impulse be transmitted, so that the sending end would operate to isolate the fault. From this it is easy to realise how important it is that the directional relays should faithfully respond to the direction of the *fault* power, as opposed to the load power."

The application of the system to radial feeders is illustrated in Fig. 16.39, in which the directional relays are omitted for simplicity. This is justifiable since the contacts of the directional element close in order that the circuits of the overcurrent element may be completed, as in Fig. 16.6. Suppose that a fault takes place at F, beyond the substation no. 2. Then, initially, both sets of protective gear will begin to operate and, when the overcurrent element at no. 2 closes its contacts *p* and *q*; the former

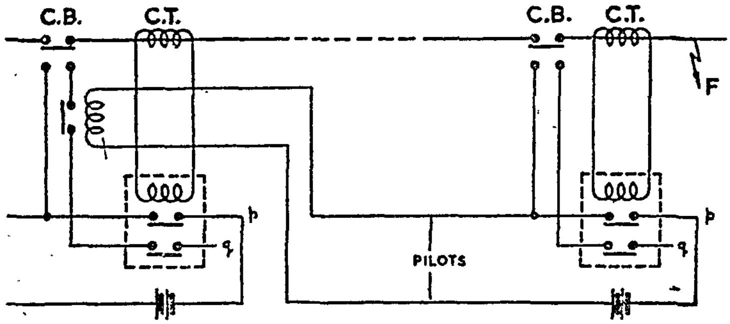


FIG. 16.39.—PRINCIPLE OF THE INTERLOCKING SYSTEMS OF FEEDER PROTECTION.

are closed first. This energises the locking relay at station no. 1, opens the contacts of this relay, which are normally in the closed condition, and so locks in the circuit-breaker at this station. Contacts *q* close later on, and thereby energise the tripping circuit of circuit-breaker no. 2. In a similar fashion all circuit-breakers between substation no. 2 and the generating station are locked in, and only the faulty section of the line is isolated. The only essential delay with this system is the time interval between the closing of contacts *p* and *q*, and thus short clearance times are possible. It is possible to use as pilots an ordinary telephone pair rented from the G.P.O. In such a case, in the event of a fault a relay is installed to disconnect the pair from the telephone equipment during the sending of the locking signal. The operating time is about 0·25 second.

The lock-in system using pilot wires can be modified for high-speed operation—that is, capable of operating in two or three cycles, and thereby taking advantage of the high-speed of modern arc-controlled circuit-breakers which can give arc extinction in less than five cycles.

Carrier-current Systems

These are essentially lock-in systems in which a high-frequency current is used for the sending of the locking signal. This signal can be injected into the line wires, pilot wires being thereby eliminated. The coupling of the signal circuits to the power lines is effected by means of large capacitors in the form of porcelain insulators, and these also serve in the dual role of capacitor-transformers by means of which a fraction of the voltage can be tapped off for application to the voltage coils of, for example, relays. They then take the place of the ordinary potential transformer. Line couplers of this type are illustrated in Fig. 16.40. These capacitors are tuned to the signal frequency by means of series-connected inductors. The other major items of equipment are the high-frequency generator and receiver units (which need not be described here as, apart from features of precaution, they follow conventional lines) and the fault-detecting and direction-comparing relays. To confine the carrier current to the required path, tuned line traps, consisting of chokes shunted by capacitors, are inserted in the two lines acting as the pair. Their impedance to the carrier frequency is very high, but, being tuned to the supply frequency, they offer no obstacle to the normal flow of line-frequency current.

Between the coupling capacitors and the high-frequency cable to the transmitting and receiving equipment a protective filter unit is included. This contains tuned circuits which bypass to earth line-frequency currents from the low-voltage taps of the coupling capacitors. It is also necessary to protect the equipment from high-voltage surges on the line, this requiring apparatus similar to that of Fig. 9.6. This includes drainage resistors which offer a low, non-inductive path to earth for high-voltage surges, and a centre-tapped high-frequency transformer with the two halves of the secondary winding acting in opposition. In this way disturbances which may affect the line voltages tend to cancel out. The connection scheme for this system is shown in Fig. 16.41. The operation is simliar to that of the D.C. lock-in system previously described, apart from its greatly increased speed. Normally the transmitter is inoperative because of the short-circuiting by relay, of the tuned circuit of the master oscillating valve. The locking signal is initiated by the operation of this relay and the immediate generation of the high-frequency current.

There were several problems to overcome with the above method of protection. (*a*) Bouncing relay-contacts caused delay in the locking signal and so jeopardised stability. (*b*) The re-

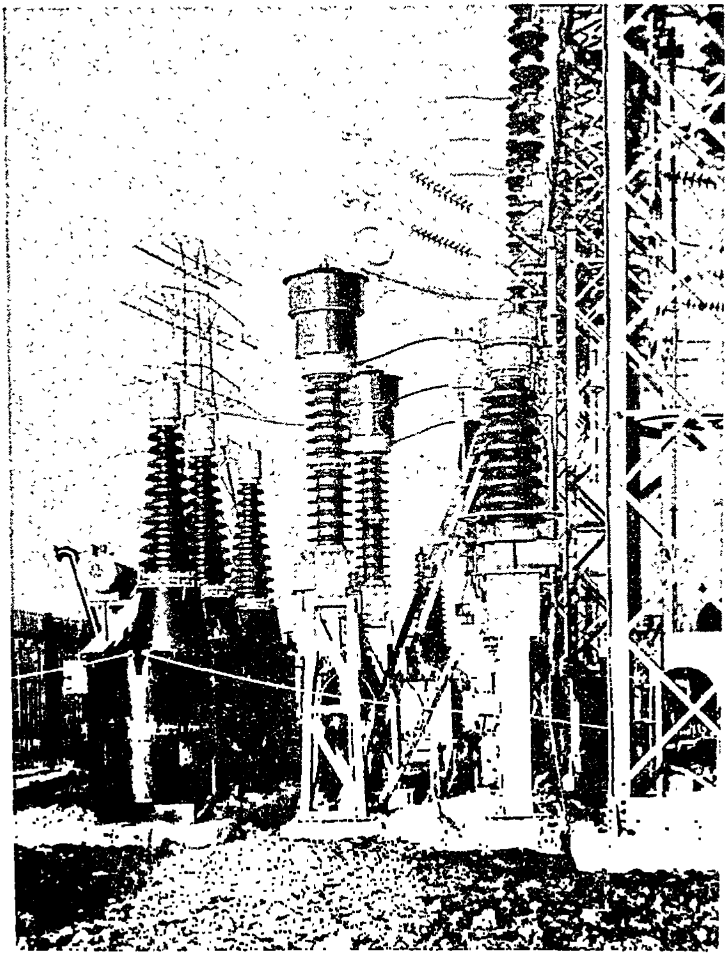


FIG. 16.40.—LINE COUPLERS FOR USE WITH CARRIER-CURRENT PROTECTION.

striking phenomena in the circuit-breaker produced an interfering high-frequency voltage during the clearing of the fault and could, on occasion, suppress the locking signal. This was remedied by giving a slight delay to the time of operating of the lock-out relay. (*c*) If insulated cable is connected in series with the overhead line, the high capacitance may bypass the high-frequency currents and so reduce the strength of the locking signal. This can be remedied by an increase in the power of the transmitter, but G.P.O. regulations limit the extent to which this can be done. An alternative remedy is to bypass the cable by means of a high-frequency cable, but this, in turn, increases the cost, not only because of the high-frequency cable but because of the associated

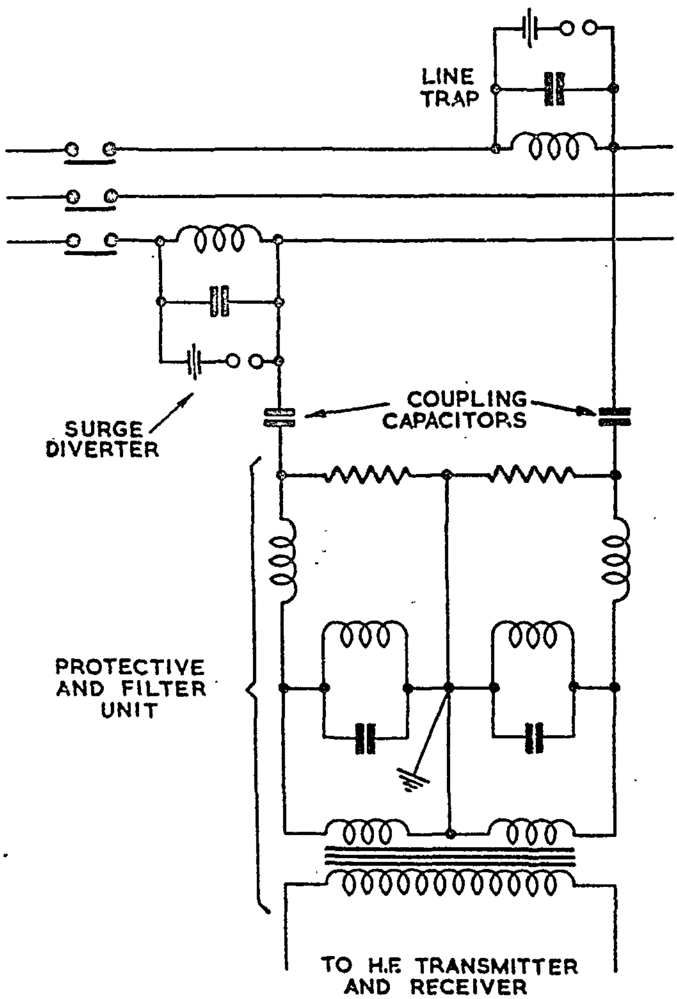


FIG. 16.41.—CIRCUIT A LINE-COUPLING EQUIPMENT FOR CARRIER-CURRENT PROTECTION.

terminal apparatus also. A more sensitive receiver might be used but, with certain types of fault, undesirable signals may be set up. The possibility of cable insertions in any but short lengths is thus limited by G.P.O. restrictions. (*d*) The relatively indeterminate operation of phase-fault directional relays in the case of a three-phase fault near the terminals of the potential transformer or divider.

Telephase Protection

The basic principle of this system is that of determining the phase angle between the currents at the two ends of a protected feeder. The line-current transformers are so connected that their secondary currents are 180° out of phase when normal current is flowing in the feeder. With an external fault the directions of current flow are as for normal operation, the phase difference of 180° is retained and there is no operation. With an internal fault, fed from both ends, the phase angle between the secondary currents is zero, or very small, and a tripping signal is transmitted. This is illustrated diagrammatically in Fig. 16.42. It is to be

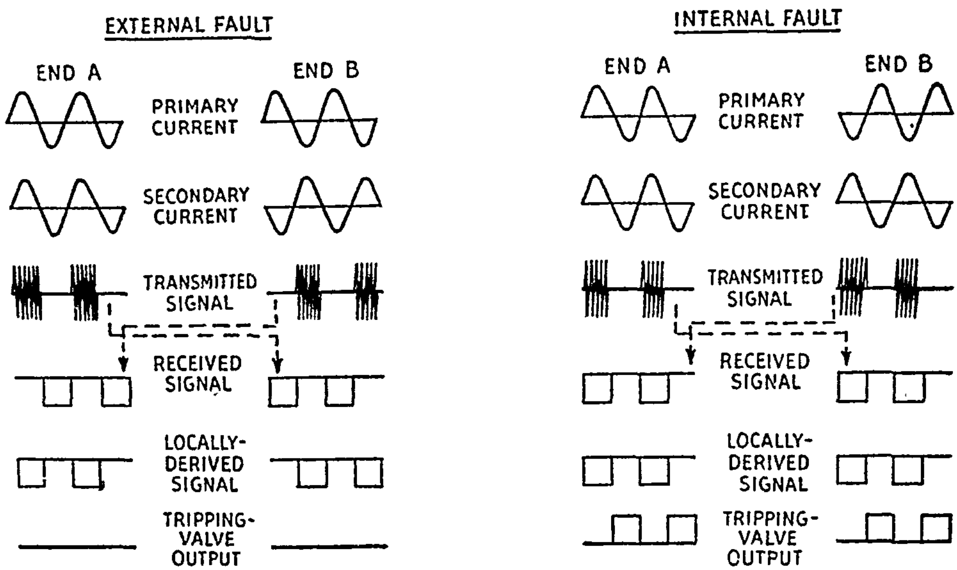


FIG. 16.42.—PRINCIPLE OF TELEPHASE PROTECTION.
(*A. Reyrolle & Co. Ltd.*)

noted that it is only phase angles which are compared, not the magnitudes of the currents.

The operation of the scheme, which can only be given in outline here, is as follows: Fig. 16.43 gives a block diagram of the equipment at one end, A, of the protected zone. The line current-transformers feed a sequence network which is insensitive to the zero phase-sequence component of the fault current (see p. 452), partly sensitive to the positive phase-sequence component, and mainly sensitive to the negative phase-sequence component. The output from the sequence network is fed into the starting circuit where, under fault conditions, it operates two starting relays. The contacts of stage 1 starting relay allow the 50-cycle output from the sequence network to be fed into the transmitter through a low-pass filter. In the transmitter this 50-cycle input modulates the high-frequency input from the oscillator. The output of the modulator is then divided, part going directly into the mixer-circuit in the local receiver, and the remainder to the line through an amplifier, a band-pass filter, and a coupling equipment.

The transmitted signal passes down through the line-coupling equipment at the other end, B, into the receiver, being accepted

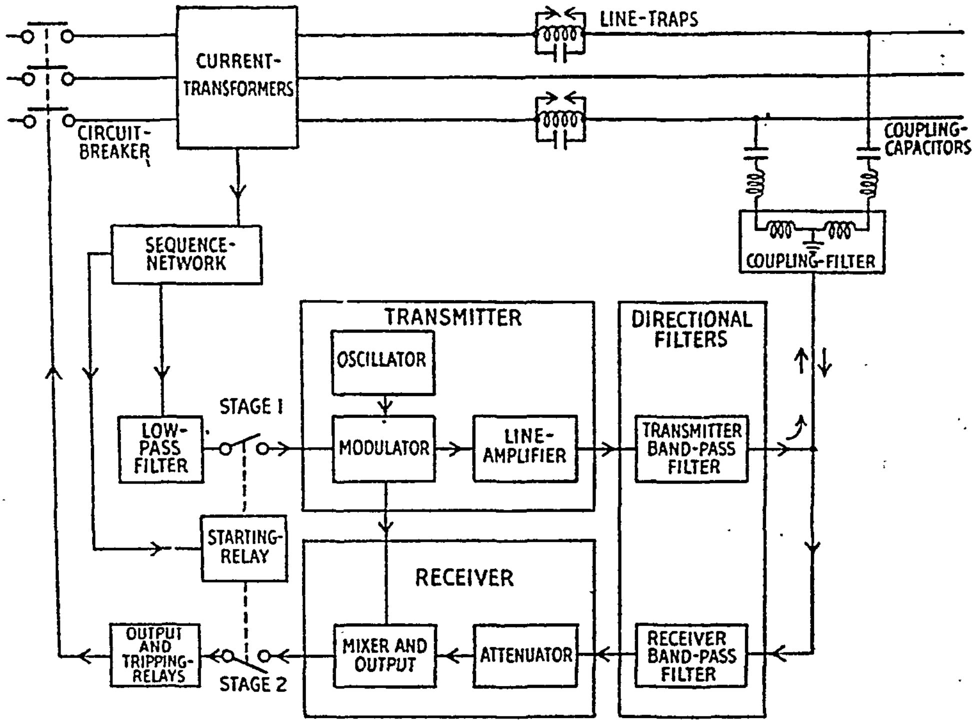


FIG. 16.43.—BLOCK DIAGRAM OF EQUIPMENT USED IN TELEPHASE PROTECTION.

by the band-pass filter. It is then passed on to the attenuator, where it is corrected for amplitude, and finally passed into the mixer-circuit.

End B transmits a signal simultaneously, and this reaches the end A mixer-circuit in exactly the same way. The mixer-circuits at each end thus receive two signals. They are so arranged that the maximum output is obtained from the receiver when the two received signals are in phase, or when only one signal is received. The receiver output decreases as the phase-angle between the two signals is increased, until there is no output when the two signals are 180° out of phase. If the fault current is large enough to operate the stage 2 relay in the starting panel, the output from the receiver is applied to the output relay, which operates the tripping relays.

We have already seen that the arrangements are such that there is tripping at both ends of the line with an internal fault, but no tripping with an external fault. If, under internal fault conditions, fault-current is fed from one end only—say, end A—signals are fed from the transmitter into the receiver at end A, and

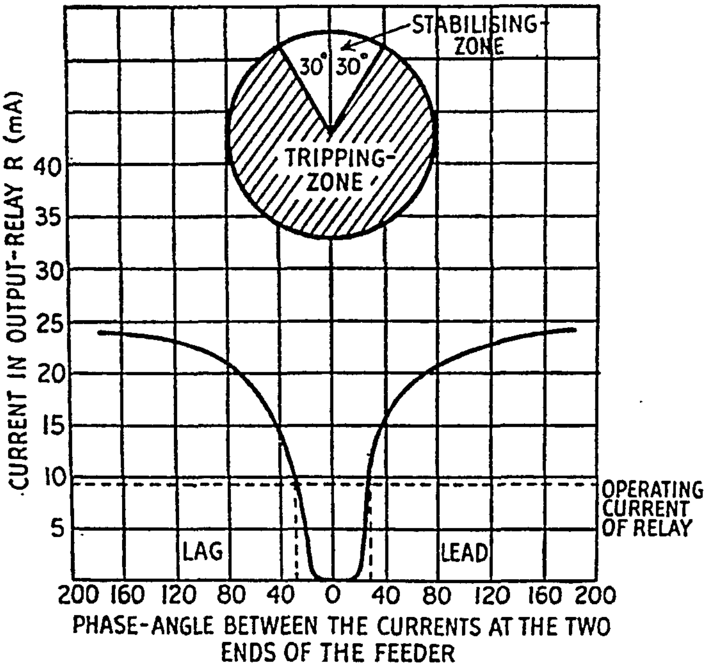


FIG. 16.44.—STABILITY DIAGRAM FOR TELEPHASE PROTECTION.

transmitted over the line to end B. There is no transmission from end B, since the starting relays there do not operate, and so the mixer circuits, both at A and B, receive only one signal. Consequently there is an output from the output-transformers at both ends of the line, but tripping occurs only at end A, since at end B the stage 2 relay has not operated to complete the tripping circuit.

The variation of the output current, as a function of the phase angle between the *line* currents at the two ends, is shown in Fig. 16.44. It shows that there is no tripping unless the phase-angle difference exceeds 30°.

The sequence segregating network which, as explained previously, is insensitive to zero-sequence, partially sensitive to positive-sequence, and mainly sensitive to negative-sequence currents, is shown in Fig. 16.45. As will be seen, it consists of two resistors R_1, R_2, and a three-winding air-gap reactor X. There is also an autotransformer connected across the output of the network. Fault-settings corresponding to a range of load currents are obtained by selecting the required tapping of the autotransformer. The operation of the network is as follows: the voltage developed across resistor R_1 depends solely on the red-phase current I_R, while that across resistor R_2 is proportional

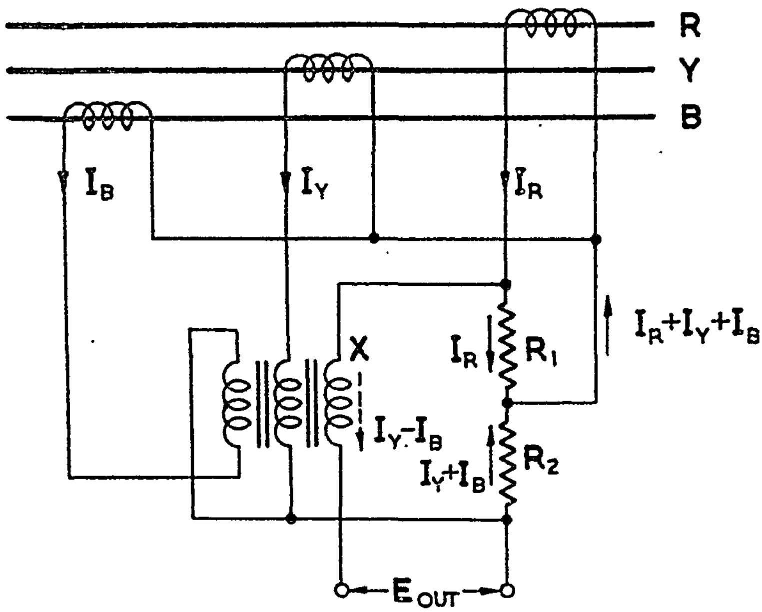


FIG. 16.45.—CONNECTIONS OF THE SEQUENCE NETWORK.

to the vector sum of the currents I_Y and I_B. The voltage across the reactor X is an induced voltage proportional to the vector difference of $(I_Y - I_B)$. Adding these voltages round the output circuit gives the equation—

$$E_{out} = -I_R R_1 + (I_Y + I_B) R_2 + jx(I_Y - I_B) \quad . \ (1)$$

Now express this in terms of the symmetrical components of the various currents. We have:

1. Zero-phase sequence currents

$$I_0 = I_{R0} = I_{Y0} = I_{B0}$$

Substituting these in equation (1) gives the output available for zero-phase sequence components of the fault current

$$E^0{}_{out} = -I_0 R_1 + 2I_0 R_2 \quad . \quad . \quad . \ (2)$$

From which it follows that no output of this particular sequence is obtained when R_1 is made equal to $2R_2$.

2. Positive-phase sequence currents

$$I_{R1} = I_1 \ ; \ I_{Y1} = a^2 I_1 \ ; \ I_{B1} = aI_1$$

Substituting these values in equation (1) gives the output available for positive-phase sequence components of the fault current—

$$E^+_{out} = - I_1R_1 + (a^2 + a) I_1R_2 + jx(a^2 - a) I_1$$

or, $$E^+_{out} = - I_1R_1 - I_1R_2 + \sqrt{3}\, I_1X \quad . \quad . \quad . \quad (3)$$

3. Negative-phase sequence currents

$$I_{R2} = I_2 \ ; \ I_{Y2} = aI_2 \ ; \ I_{B2} = a^2 I_2$$

$$\therefore \ E^-_{out} = - I_2R_1 + (a + a^2) I_2R_2 + jx(a - a^2) I_2$$

or, $$E^-_{out} = - I_2R_1 - I_2R_2 - \sqrt{3}\, I_2X \quad . \quad . \quad . \quad (4)$$

The total output voltage of the network is therefore—

$$\begin{aligned} E_{out} &= E^0_{out} + E^+_{out} + E^-_{out} \\ &= - I_0(R_1 - 2R_2) - I_1(R_1 + R_2 - \sqrt{3}X) \\ &\qquad - I_2(R_1 + R_2 + \sqrt{3}X) \\ &= R_2[I_0 (2 - k_1) + I_1(\sqrt{3}k_2 - k_1 - 1) \\ &\qquad - I_2(\sqrt{3}k_2 + k_1 + 1) \quad . \quad . \quad . \quad (5) \end{aligned}$$

where $k_1 = R_1/R_2$, and $k_2 = X/R_2$

We have already seen that k_1 is made equal to 2. The value of k_2 is made such that the output voltage for the positive-sequence component is small compared with that due to the negative-sequence component. This enables unbalanced faults of less than normal full-load current to be detected, and also insures that, although the protection operates with a three-phase fault, it does not operate with normal load. Hence the network governs the fault-settings of the protection.

The practical utilisation of negative sequence components is limited to those cases where the fault current may be small, possibly even less than the normal full-load current. It will be appreciated that in such circumstances it will be very difficult to apply protective gear of any of the types previously described. A typical example is the case of a step-down transformer connected to a transmission line with no intervening oil-switch. If the line forms part of a ring main the full-load of the transformer will probably be considerably less than that of the line, but it is necessary to include in the zone protected by the relays at adjacent stations the low-voltage winding of the transformer. If the lines are protected on the impedance principle then satisfactory protection of the lower voltage windings will not be obtained, for :

(*a*) Due to the reactance of the transformer the apparent impedance between fault and relay will be very high, and even assuming that operation is possible, the time delay will be unduly long.

(*b*) If the line current due to the fault is less than the full-load line current, neither impedance relays nor overload relays will operate at all.

The problem might be solved by one of the balanced systems, but in the nature of the case the cost of pilot wires will not be

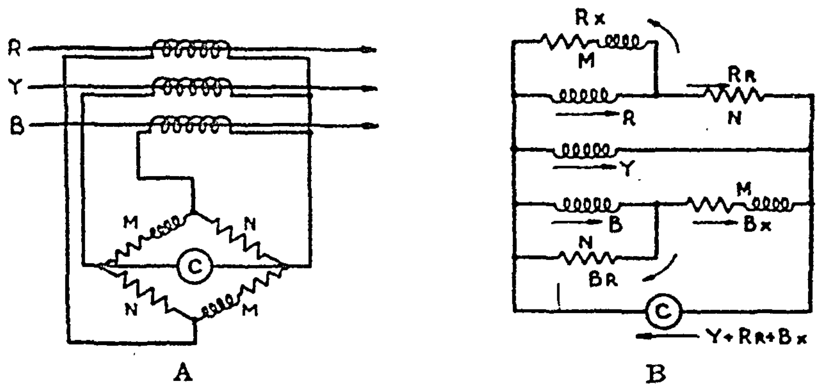


FIG. 16.46.—NEGATIVE PHASE-SEQUENCE CONNECTIONS.
(*Metropolitan-Vickers Electrical Co., Ltd.*)

justifiable. A satisfactory solution is provided by negative phase-sequence protection. The protective relay devised by the Metropolitan-Vickers Electrical Co., Ltd., for use with this system is an over-current relay connected in a bridge arrangement of resistances and reactances, as shown in Fig. 16.46(A), and represented diagrammatically in Fig. 16.46(B). The relay is indicated by C, while M and N are adjusted so that the currents in the R and B phases, and designated by R and B respectively, divide as follows :

R_r leads R by 30°, R_x lags R by 30°, $R_r + R_x = R$
B_r leads B by 30°, B_x lags B by 30°, $B_r + B_x = B$

From the circuit diagram it will be seen that the current through the relay coil C is, at any time, the sum of Y, R_r and B_x. Under normal balanced conditions, the relationships are as shown in Fig. 16.47, from which it will be seen that $(Y + R_r + B_x)$ is zero, and the relay does not tend to operate. Should a reversal of phase rotation take place, as, for example, on the establishment of negative sequence components under unsymmetrical fault conditions, the relationships become as in Fig. 16.48, the current through C is no longer zero, and the relay operates.

The relay used is a single-pole instantaneous relay of the attracted armature pattern. In order to secure discrimination

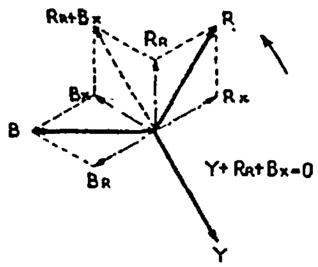


Fig. 16.47.—Vector Diagram for Negative Phase-sequence Protective System.

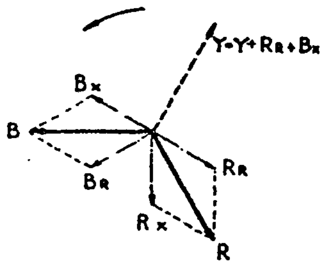


Fig. 16.48.—Vector Diagram for Negative Phase-sequence Protective System.

with the protective gear on other parts of the system the relay does not trip the circuit-breaker directly, but energises a separate timing relay as shown in Fig. 16.49.

In the G.E.C. negative phase-sequence system also, the principle of operation is the error in balance existing between the currents in two phases, when negative phase-sequence currents are present. But the relay consists of three biased units which directly compare the currents in the red and yellow, yellow and blue, and blue and red phases respectively, as shown in Fig. 16.50; when one exceeds another by 10 per cent., the corresponding relay element trips and effects isolation. The relays are of the disc and shaded-pole principle, and a mechanical bias is achieved by moving the pivot support of the disc. This system has been installed in many parts of the South-West England Grid, the peculiarity of this area being the restriction of the fault currents by the high values of the earthing resisters employed.

Zero Phase-sequence or Earth-leakage Systems

With faults between phases but no earth connection of any phase conductor, there can be no zero phase-sequence components, but only positive and negative phase-sequence components. If, however, a line becomes earthed, then with an earthed neutral system a path is provided and the zero phase-sequence components are established. Consider again the system of Fig. 14.2, and let there be a line-to-ground short-circuit on phase *a*, instead of the short-circuit between phases *a* and *b*. Then the conditions are as shown in Fig. 16.51, and we have for the total currents:

$$I_a = I_g,\ I_b = 0,\ I_c = 0$$

Hence
$$I_{a1} = \tfrac{1}{3}(I_a + aI_b + a^2I_c) = \frac{I_a}{3} = \frac{I_g}{3}$$

$$I_{a2} = \tfrac{1}{3}(I_a + a^2I_b + aI_c) = \frac{I_a}{3} = \frac{I_g}{3}$$

and
$$I_{a0} = \tfrac{1}{3}(I_a + I_b + I_c) = \frac{I_a}{3} = \frac{I_g}{3}$$

$$\therefore\ I_{a1} = I_{a2} = I_{a0} = I_{b0} = I_{c0} = \frac{I_a}{3} = \frac{I_g}{3}$$

The various components are therefore represented vectorially in Fig. 16.52, from which it will be seen that the components in phase

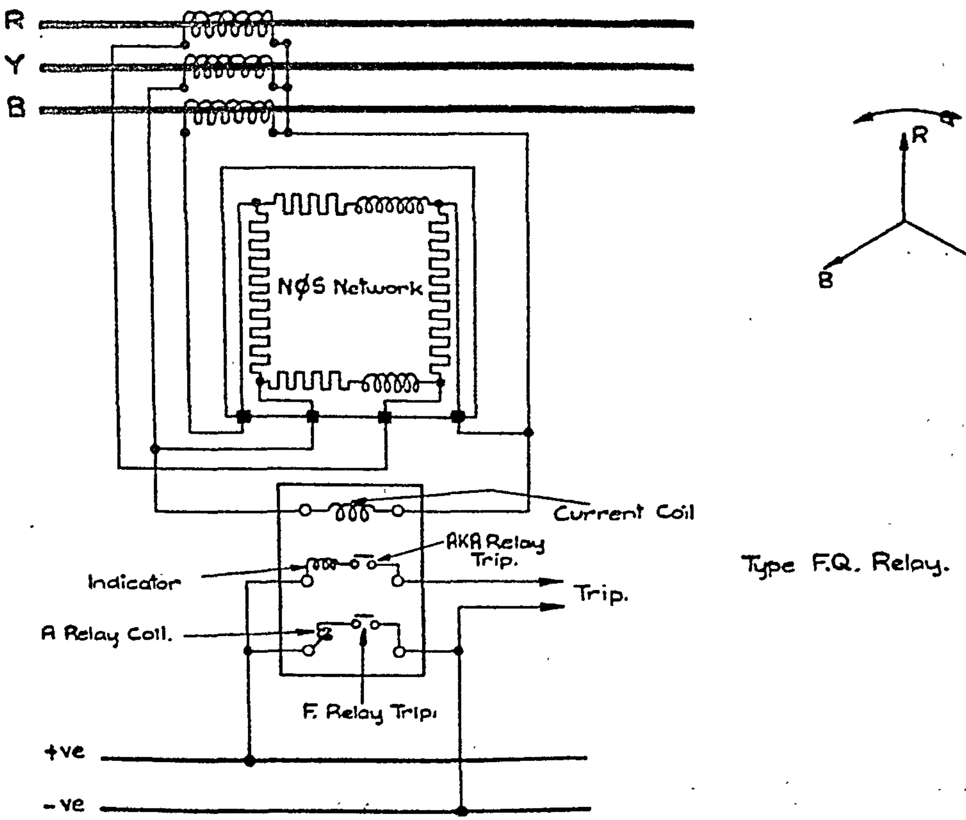


Fig. 16.49.—Negative Phase-sequence Connections.
(*Metropolitan-Vickers Electrical Co., Ltd.*)

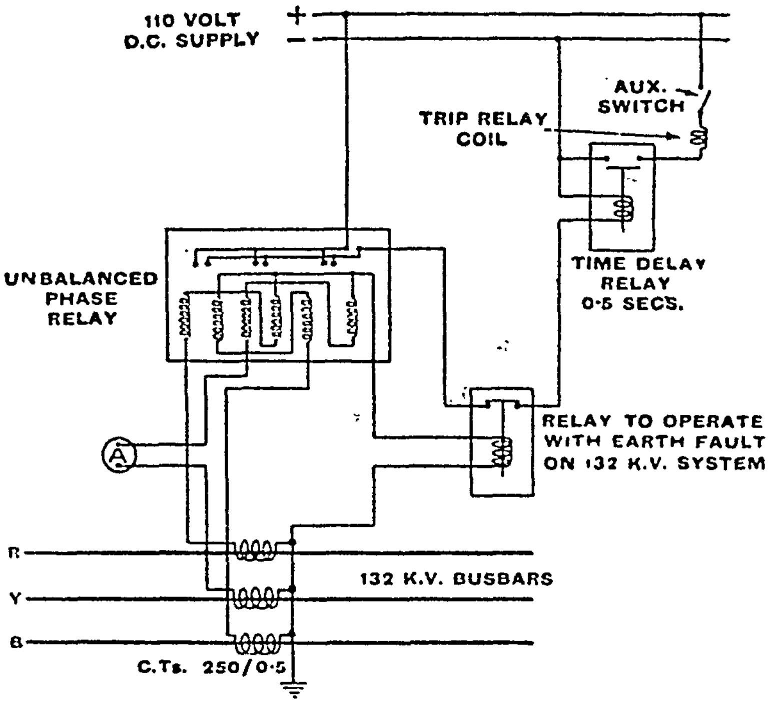


FIG. 16.50.—NEGATIVE PHASE-SEQUENCE CONNECTIONS.
(*G.E.C. Co., Ltd.*)

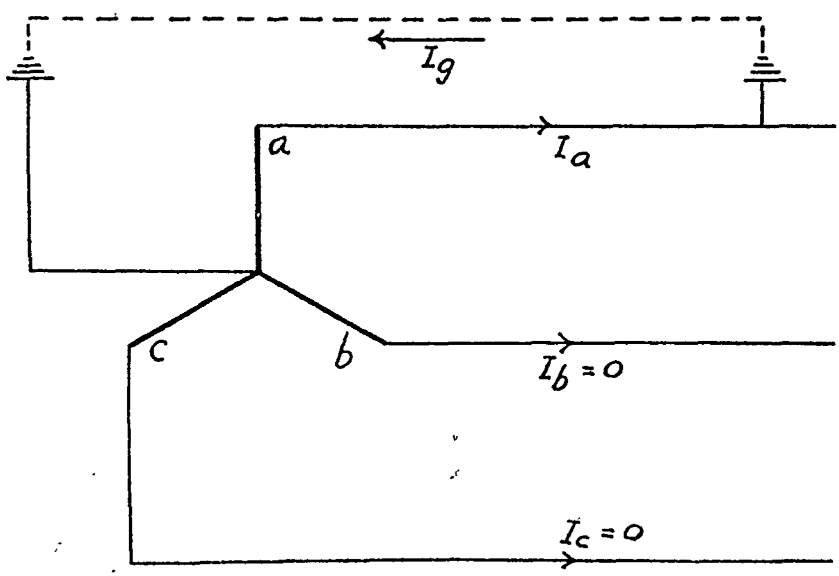


FIG. 16.51.

a add up to I_a or I_g, while those in phases *b* and *c* add up to zero.

The value of I_{a1} is again the starting-point in a numerical calculation, and it is determined as follows : Let Z_1, Z_2, and Z_0 be the positive, negative, and zero phase-sequence impedances of the circuit from the alternator neutral to the fault, then the voltage

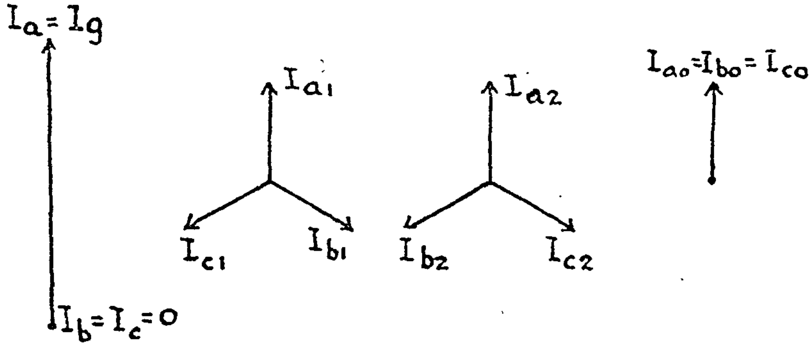


FIG. 16.52.

drop due to the component I_{a1} flowing through impedance Z_1 is $I_{a1}Z_1$. Similarly, the drop due to I_{a2} is $I_{a2}Z_2$, and that due to I_{a0} is $I_{a0}Z_0$. Thus the total drop in phase *a* is

$$I_{a1}Z_1 + I_{a2}Z_2 + I_{a0}Z_0$$

Since phase *a* is grounded, the whole of the above drop must be equal to the phase voltage E_a, giving

$$E_a = I_{a1}Z_1 + I_{a2}Z_2 + I_{a0}Z_0$$

But

$$I_{a1} = I_{a2} = I_{a0}$$

$$\therefore E_a = I_{a1}(Z_1 + Z_2 + Z_0)$$

$$\therefore I_{a1} = \frac{E_a}{Z_1 + Z_2 + Z_0}$$

and

$$I_g = 3I_{a1}$$

For the same system as before we have

$$Z_1 = 4 + j \times 16 : Z_2 = 4 + j \times 13{\cdot}3$$

For the zero phase-sequence impedance we can take the line reactance as four times the ordinary line reactance, and alternator reactance at 27 per cent. of the ordinary reactance.

$$\therefore Z_0 = (0 + j \times 10 \times {\cdot}27) + (4 + j \times 6 \times 4)$$
$$= 4 + j \times 26{\cdot}7$$
$$\therefore Z_1 + Z_2 + Z_3 = (4 + j \times 16) + (4 + j \times 13{\cdot}3) + (4 + j \times 26{\cdot}7)$$
$$= 16 + j \times 56$$

$$\therefore\ I_{a1} = \frac{1000}{16 + j \times 56}$$
$$= \frac{1000(16 - j \times 56)}{16^2 + 56^2}$$
$$= 4{\cdot}72 - j \times 16{\cdot}5$$
$$\therefore\ I_g = 14{\cdot}16 - j \times 49{\cdot}5$$

Its numerical value is thus

$$I_g = \sqrt{14{\cdot}16^2 + 49{\cdot}5^2} = 51{\cdot}5 \text{ amps.}$$

Since the zero phase-sequence components are in the nature of single-phase currents they can be detected by means of single-phase relays. In the core-balance system all three conductors are passed through a single transformer core, the secondary winding of which is a single winding connected to a single-pole relay. Under healthy conditions the M.M.F.s due to the three-line currents balance, and under line-to-line fault conditions the M.M.F.s due to the positive phase-sequence components balance, as also do those due to the negative phase-sequence components. Hence in both cases there will be no flux in the transformer core and therefore no E.M.F. induced in the secondary. But with an earth fault on any conductor the three components I_{a0}, I_{b0}, and I_{c0} are all in phase, as far as the transformer is concerned, being together equivalent to a single-phase through current of I_g. Thus the secondary is energised and the relay operated. This system is very suitable for circuits carrying small amounts of power, and has been widely used for colliery work, where leakage to earth is of special importance. For this application the whole of the gear is incorporated in an ironclad switch unit, the connections for such a unit being shown in Fig. 16.53, which refers to a Reyrolle mining pillar with this form of protection. There are two points of interest in the diagram shown. Firstly, the breaker trip-coil is energised from a pressure transformer worked from one phase, since very little energy can be obtained from the secondary of a transformer operating in such a manner as the core-balance transformer. Secondly, the switch being of the draw-out type it is obvious that the transformer secondary circuit must be open when the main switch is open. This is ensured by the small trip-coil switch, which closes only during the closing of the main switch, and opens when the main switch opens. Thirdly, protection against overloads or faults between phases is ensured by the provision of overload trips as shown.

There are some collieries at which the neutral is insulated, the result being that there is no path to the alternator neutral for the flow of zero phase-sequence components. This difficulty is overcome by installing a one-to-one transformer with star-connected secondary and connecting the secondary neutral point to earth: the system as a whole is still completely insulated, but a path is provided for the zero phase-sequence components, and therefore leakage protection is possible.

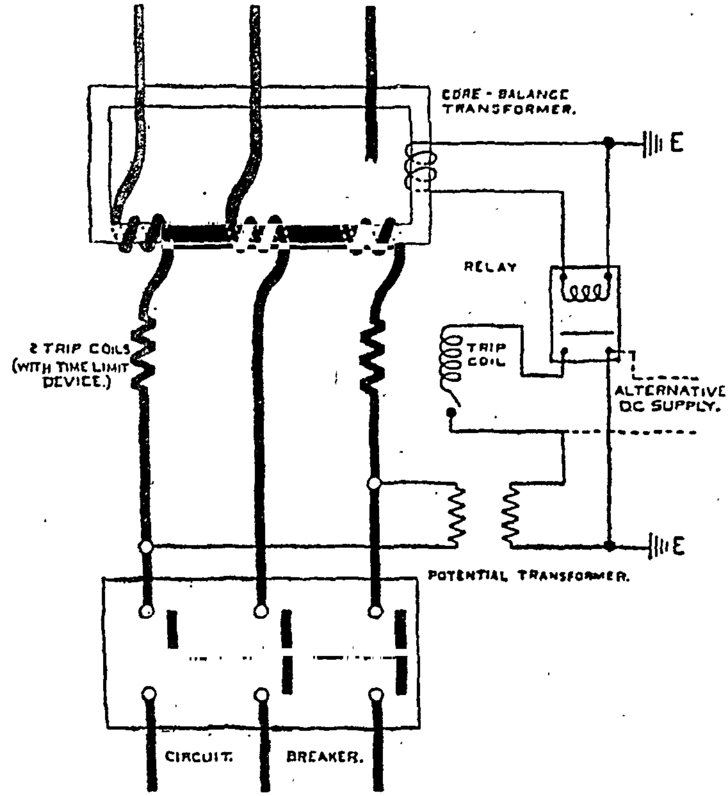


FIG. 16.53.—CONNECTIONS OF SWITCH UNIT INCORPORATING CORE-BALANCE PROTECTION.

(*Messrs. A. Reyrolle & Co., Ltd.*)

It is also necessary to realise that leakage protection is not suitable for parallel feeders because, with an earth on one feeder, zero phase-sequence currents will flow, not only in the faulty feeder, but also in the sound feeder, via the bus-bar, as shown in Fig. 16.54. This also applies to a ring main. As it happens, the underground installation at any colliery consists of a series of entirely separate districts, each supplied by its own feeder: there are thus no parallel paths for zero phase-sequence components, and leakage protection is entirely satisfactory.

It is not necessary in all cases to keep the earth leakage and over-current systems of protection entirely separate, as in the above example. Since the total earth current I_g is the sum of the

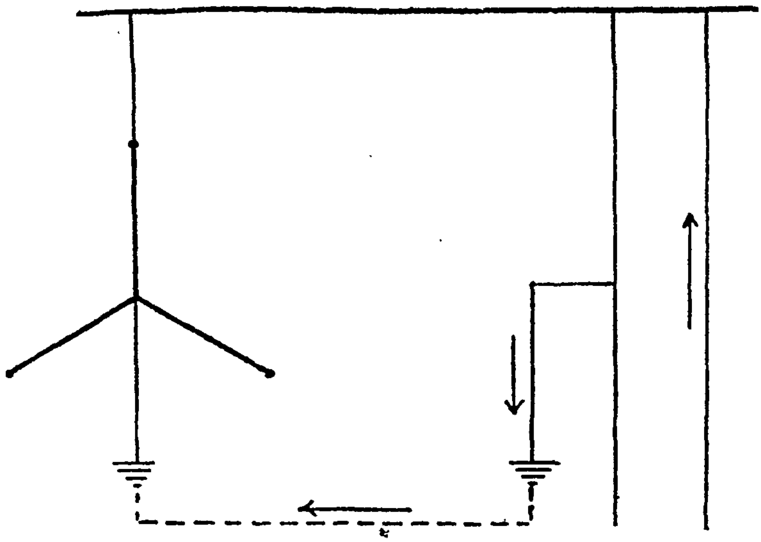


FIG. 16.54.

components I_{a0}, I_{b0}, and I_{c0}, a relay actuated by the current I_g will give protection against earth leakage, and it can be connected so as to operate the same trip circuit as the over-current relays. One method is shown in Fig. 16.55, from which it will be seen that

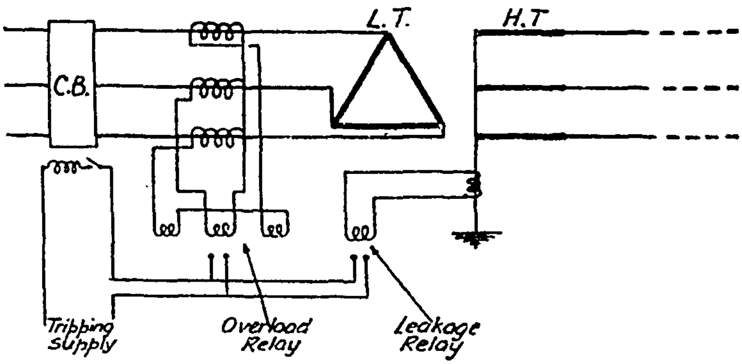


FIG. 16.55.—COMBINED OVER-CURRENT AND EARTH-LEAKAGE PROTECTION

there are three over-current relays and one leakage relay, the current transformer for the latter being on the H.T. (star-connected) side of the power transformer. With this system the relays at the far end will be combined over-current, leakage, and reverse power relays, and the operation is as follows: Faults in the low-tension side of the power transformer are cleared by the leakage relay or by the over-current relays, according to the nature of the fault. With an earth fault on the transmission line or in the high-tension winding the leakage relay will operate. This has a short time setting and will thus prevent any of the over-current relays on the low-tension side from operating: the line will then be disconnected at the far end by means of the reverse power relays.

Since over-current protection can be obtained by means of two

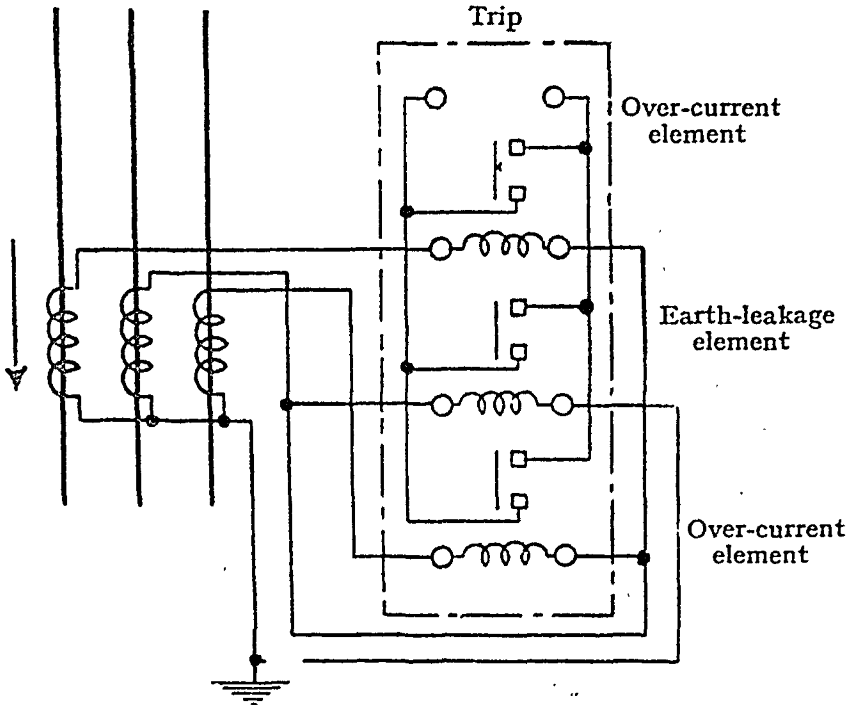


FIG. 16.56.—THREE-POLE RELAY CONNECTED FOR COMBINED OVER-CURRENT AND EARTH-LEAKAGE PROTECTION.
(*Metropolitan-Vickers Electrical Co., Ltd.*)

over-current relays, a combination of two such relays with one leakage relay can be adopted, as shown in Fig. 16.56.

In mining work, underground, the motor, if at the working face, is connected to a control panel called a "gate-end" panel, through a flexible "trailing" cable. This cable is connected to both gate-end panel and motor by means of special plugs. All operations, whether for the purpose of normal running or protection of the equipment under abnormal conditions, are performed at the gate-end panel, and to this end the trailing cable is provided with earth- and pilot-cores, in addition to the three power cores. The switch on the panel is of the contactor type, and all arcing takes place at the panel and not at, or near, the motor. The following conditions have to be fulfilled:

1. The trailing cable must be dead when the motor is not running.

2. Operators must not be able to leave the trailing cables alive at the end of a shift.

3. The machine can be re-started after a trip out on over-current without the operator having to travel to the panel, which may be some distance away.

4. Electrical interlock ensures that if the plug carrying the

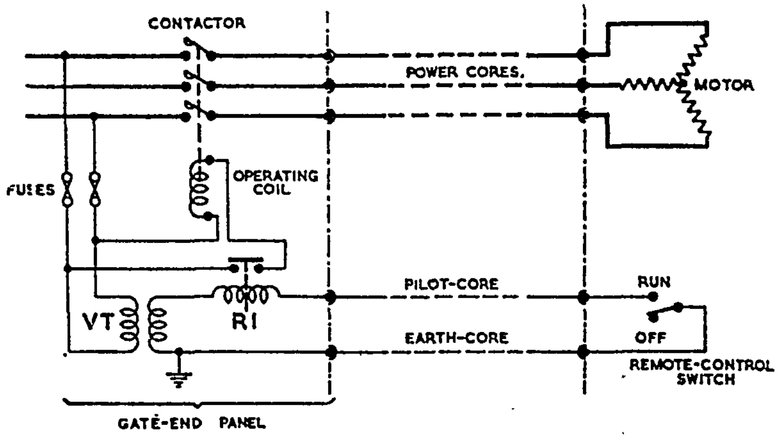


FIG. 16.57.—CONNECTION SCHEME FOR PROTECTION OF MINING EQUIPMENT UNDERGROUND.

trailing cable is withdrawn, the panel will trip and the exposed contacts will be dead.

5. Earth continuity is provided.

Basically, the scheme of protection is illustrated by Fig. 16.57. It will be seen that the pilot-core and earth-core comprise a separate circuit energised from an "intrinsically safe" transformer VT giving a secondary voltage of not more than 30. In this circuit is the operating coil of a pilot relay RI, and the circuit can be completed only by the closing of the remote-control switch at the motor end. If either pilot or earth-core is broken, the control circuit is opened and the main supply to the motor automatically cut off; similarly, if the plug is withdrawn. The lengths of the pins and the positioning of the sockets are such that the pilot- and earth-core contacts are properly made before the pins for the power cores begin to engage.

The above simple scheme does not give protection if the control switch is left in the "start" position after a failure of supply.

Restoration of the supply would reclose the contactor and start the motor, a condition which could be dangerous with the circuit shown. To prevent this a resistance or an inductance is inserted in the pilot circuit, and a modified remote-control switch, on the lines of a simple drum controller, adopted. A practical scheme, with this device incorporated, is shown in Fig. 16.58. The feature of the scheme is a differentially wound transformer with two windings, X and Y, supplied from the voltage-transformer in the panel. The winding X is in series with the pilot-control circuit, this circuit now including an impedance at the remote control switch. This circuit is earth, via the earth-core, at the panel.

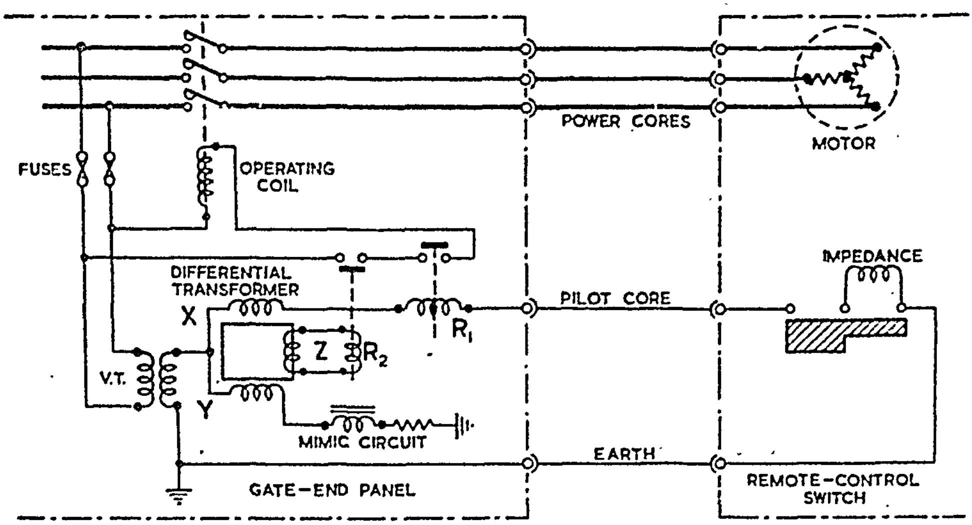


Fig. 16.58.—Remote-control Scheme with Pilot-core Protection. (*Reyrolle & Co. Ltd.*)

The winding Y supplies a mimic circuit, corresponding to the pilot-operating circuit, and also earthed at the panel. When the values in the two circuits are equal, the M.M.F.s of the windings X and Y neutralise one another, no voltage is induced in the winding Z, and therefore the relay R2 remains inoperative with its contacts closed.

When the remote-control switch is moved to the "start" position, the pilot circuit is completed and the relay R1 energised. As the impedance is short-circuited by the remote-control switch, the two circuits of the differential transformer are not balanced, and consequently the relay R2 is energised. Its contacts open and thus break the circuit of the contactor operating-coil. When the control switch is in the "run" position, the impedance is put in the pilot circuit, the two circuits balance, the contacts of relay

R2 close, and the panel operates, thereby connecting the motor to the supply.

If the pilot- and earth-cores become joined at any intermediate point due to mechanical damage, or if the pilot-core becomes earthed, the impedance at the motor end becomes short-circuited by the fault, unbalance is produced, and the contactor is tripped out. The impedance also ensures that protection is given if the control switch is left in the " start " position after a failure of supply. To this end the magnitude of the impedance is such that, when it is in circuit, the pilot relay will not pick up and close its contacts, but sufficient current flows in the pilot circuit to keep this relay closed once it has attained that condition. If the contactor trips out for any reason, it is necessary to bring the control switch to the " off " position before it can be reclosed.

Phase-faults or faults to earth involving the power-cores, as well as ordinary overloads, are cleared by switchgear in the permanent installations, as, for example, a switch unit incorporating core-balance protection.

The " Ohm " Reactance Relay*

This is a high-speed relay whose characteristic when plotted on an impedance diagram is a straight line, parallel to the R axis; Fig. 16.59. The " ohm " unit is of the induction type, a 4-pole electromagnet carrying operating, polarising, and restraining coils; Fig. 16.60. From this figure it is clear that the torque is proportional to the product of the flux due to the polarising current and the resultant flux due to the opposing current and voltage coils. Hence the equation for the operating point which determines the pick-up characteristic is:

$$KI^2 = VI \cos (c\phi - \theta)$$

where K = a relay constant

ϕ = phase angle of protected circuit

θ = phase angle of the relay characteristic

$$\therefore K = (V/I) \cos (\phi - \theta) = Z \cos (\phi - \theta)$$

* Warrington, A. R. van C., "Application of Ohm and Mho Principles to Protective Relays". *Trans. Amer. I.E.E.*, 65, 1946, p. 378.

Warrington, A. R. van C., " Graphical Method for Estimating the Performance of Distance Relays during Power Swings." *Amer. I.E.E., Technical Paper, No.* 49–154.

Cordray, R. E., and Warrington, A. R. van C., " The Mho-carrier Relaying Scheme." *Trans. Amer. I.E.E.*, 63, 1944, p. 228.

Adamson, C., and Wadepope, L. M., " Power System Protection." *Journ. I.E.E.*, 103 Pt. A, 1956, p. 379.

Adamson, C., and Wadepope, L. M., " A Dual-Comparator Mho-Type Distance Relay." *Journ. I.E.E.*, 103, Pt. A, 1956, p. 509.

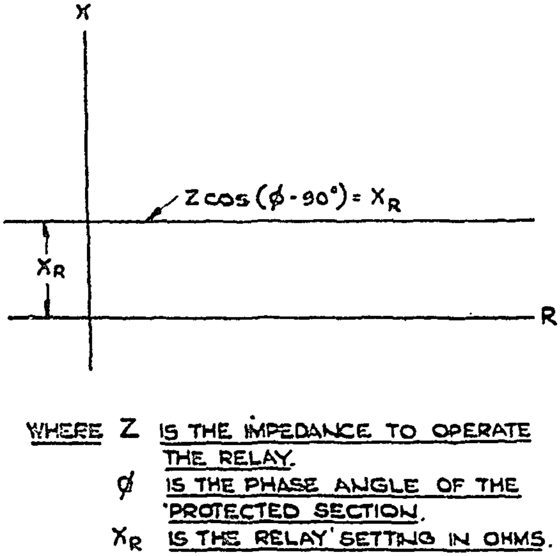


FIG. 16.59.—CHARACTERISTIC OF "OHM" UNIT.

A capacitor in the potential circuit provides a pick-up characteristic of

$$K = Z \cos(\phi - 90°) = Z \sin \phi = X_R,$$

the reactive component of the faulted circuit, and it is this that the relay measures; because of this, the "ohm" unit above would operate as high power-factor load currents since they would appear as low, or negative, reactance. It is therefore used with a directional fault detector which ensures that fault conditions actually exist. This is a "mho" type relay described below.

In a three-step distance relay scheme the "ohm" unit measures the reactance from the relay to the fault and determines in which of the three zones of protection the fault lies. The relay operates when the fault reactance is less than the relay setting. If the fault lies beyond the first zone of protection, the setting of the "ohm" unit is increased to zone 2 by a timing relay. The three-step characteristic is shown in Fig. 16.61. With this three-step distance protection the relay is especially applicable to short overhead lines where fault resistance may affect the reach of impedance relays. It can also be used as a one-step directional impedance relay on very short overhead lines, and so used, the time settings can be reduced

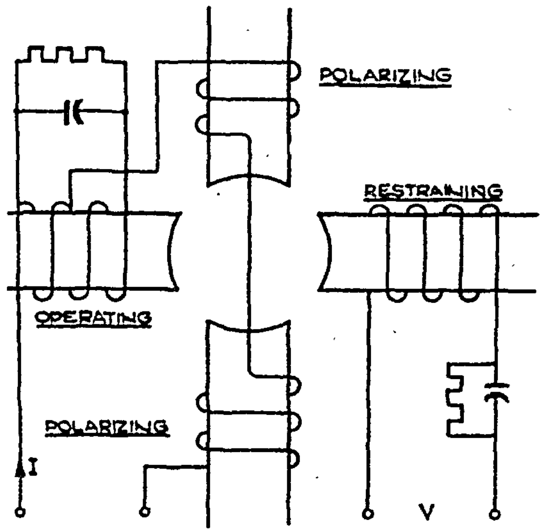


FIG. 16.60.—SCHEMATIC CONNECTIONS OF "OHM" UNIT.
(*English Electric Co. Ltd.*)

so as to ensure quick clearance anywhere on the system. Due to a high torque/friction ratio, there is (*a*) extremely fast operation

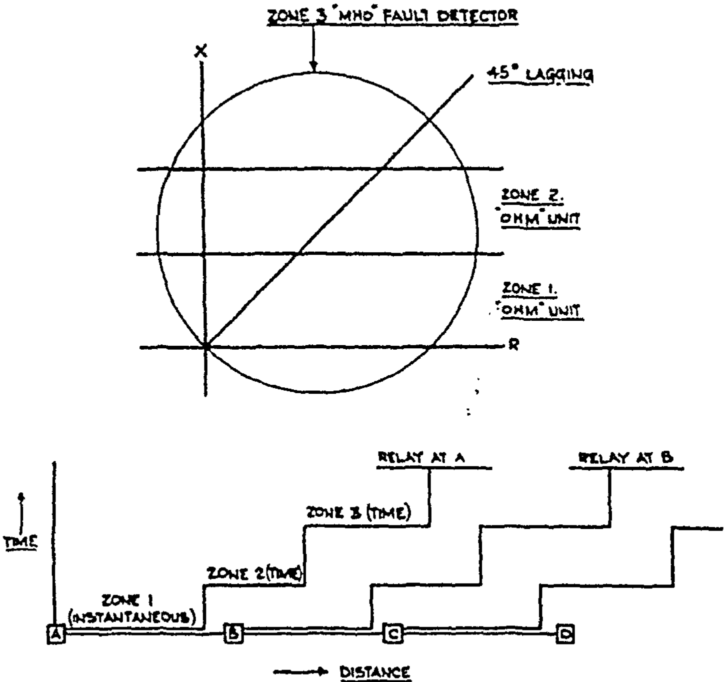


FIG. 16.61.—(Upper figure.) THREE-STEP IMPEDANCE CHARACTERISTICS. (Lower figure.) THREE-STEP TIME/DISTANCE CHARACTERISTICS.

(0·010 sec. minimum time), (*b*) great accuracy, within ± 2 per cent. over a wide range of currents.

The " Mho " Directional Impedance Relay

This is a high-speed relay whose characteristic when plotted on an admittance diagram is a straight line. Its characteristic on an impedance diagram is therefore a circle through the origin; Fig. 16.62. It operates when the impedance presented to it falls within this circle. It is inherently directional, so that a single " mho " relay can replace both impedance and directional units employed by other methods of directional distance protection. This leads to fast tripping and fault clearance, and reduced VA burdens.

If Z_R = ohmic setting of the relay
ϕ = angle by which fault current lags relay voltage V
and θ = angle by which I lags V for maximum operating torque and maximum impedance reach

then, from the elementary connection scheme of Fig. 16.63 we have

$V[I \cos(\phi - \theta) - V/Z_R] = 0$ as the equation for pick-up which reduces to

$$Y \cos(\phi - \theta') = 1/Z_R$$

which is a straight line on an admittance diagram. Inverting both sides, we have

$$Z = Z_R \cos(\phi - \theta)$$

which is the equation to a circle of diameter Z_R and which passes through the origin. It is the fact that the circle passes through the origin which makes the relay inherently directional. The

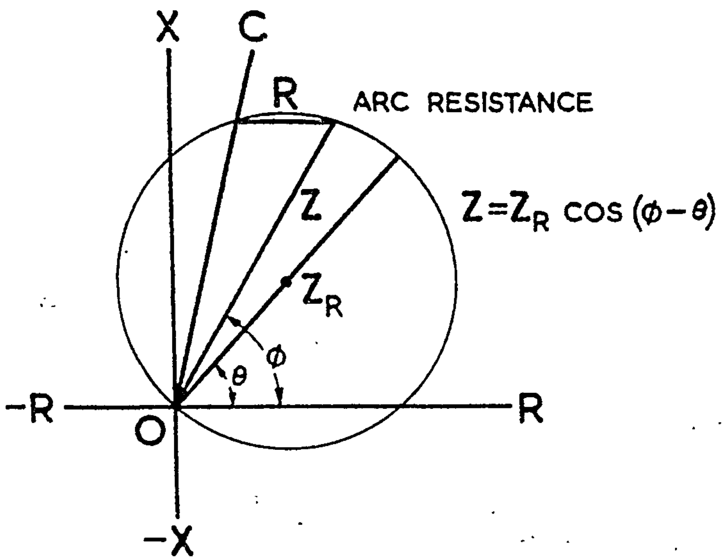


FIG. 16.62.—CHARACTERISTIC OF "MHO" UNIT.

impedance of the protected line is represented by the slope of its characteristic OC, and consequently the arc resistance R, is represented by the length of the horizontal to OC from the extremity of the chord Z. By making θ equal to, or a little less lagging than, ϕ, the circle is made to fit very closely around the fault area so that the relay is insensitive to power swings and therefore particularly applicable to the protection of long or heavily loaded lines. For one-step application the "mho" relay is usually set to cover 90 per cent. of the protected section.*

It follows from the torque equation that the torque will be very small when V is small. The circuit of the polarising coils is

* For three-step application see Hutchinson, R. M., "The Mho Distance Relay," *Trans. Amer. I.E.E.*, 65, 1946, p. 353.

therefore tuned so that the voltage is maintained for several cycles after the line voltage has dropped because of a fault.

Comparison of the Different Systems

Having reviewed the various methods of protection, with descriptions of typical systems, it is now possible to make some comparison between them.

1. The graded time-lag systems possess the advantage that no pilots are required. They also give good discrimination, and pro-

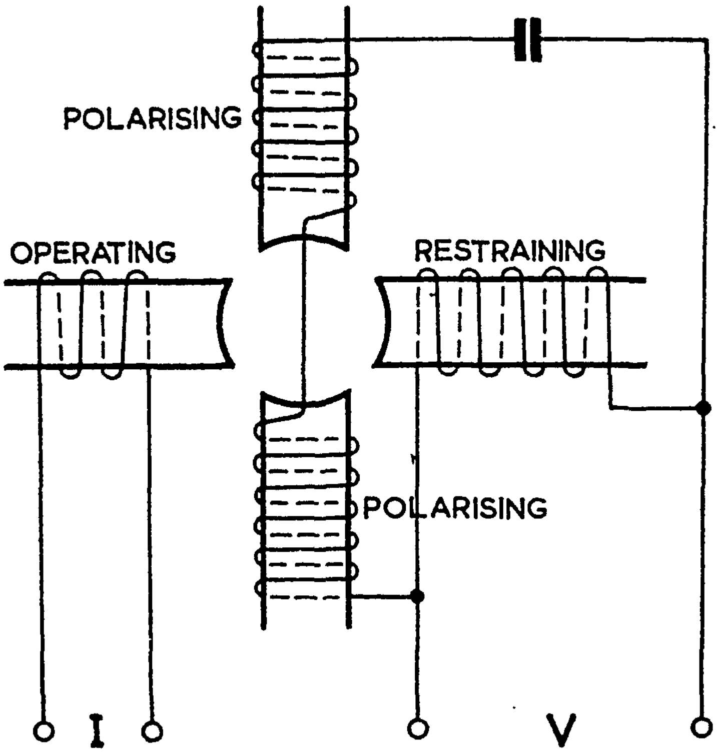


FIG. 16.63.—SCHEMATIC CONNECTIONS OF "MHO" UNIT.

vide back-up protection; that is, the settings of adjacent relays in the sequence overlap, so that if one fails to operate, the next in the sequence will operate. They suffer from the great disadvantage that, for more than, say, four sections, the time delay at the generating stations becomes very long, say, 1·6 secs. or more. For radial feeders no voltage transformers are required, and the current transformers are of the ordinary (not D.A.G.) type, since matching is unnecessary. For ring-main protection voltage transformers are required, since some of the relays must be directional. For the above reasons the graded time-lag systems are suited only to networks of comparatively small extent and therefore of comparatively low voltage. If possible, the time delay at the generating station should not exceed 1 sec., and this puts a limit to the number of sections possible.

The meanings of setting encroachment and stability are well illustrated by the stability diagram of Fig. 16.64, which refers to the graded time-lag system of protection applied to a radial feeder

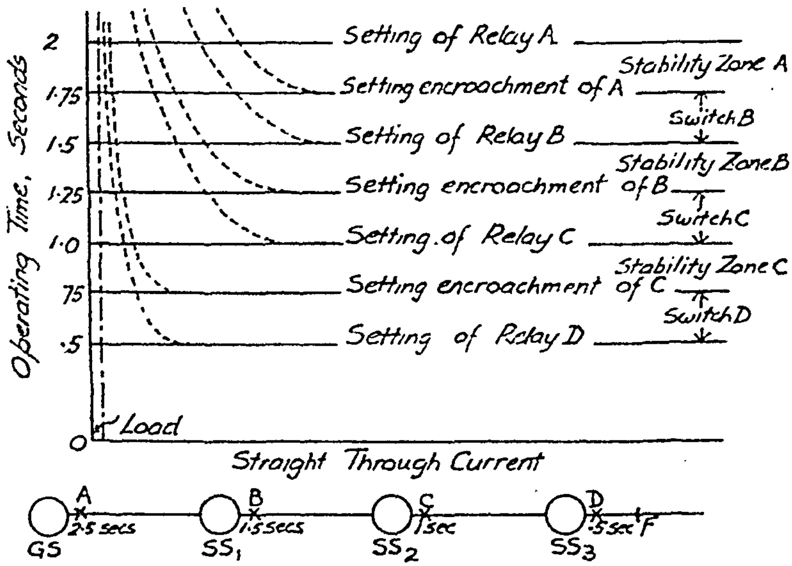


FIG. 16.64.—STABILITY DIAGRAM FOR GRADED TIME-LAG PROTECTIVE SYSTEM.

with three sections, the time lags ranging from 0·4 sec. at the most remote station, by equal intervals of 0·4 sec. to 1·6 secs. at the power station. The setting encroachments are given by the characteristic curves, the operating times of all the switches being 0·2 sec. For the stability ratios we have:

$$\text{Stability ratio at A} = \frac{\text{Time setting of relay at A}}{\text{Time setting of relay at B} + \text{operating time of switch B}}$$

$$= \frac{1{\cdot}6}{1{\cdot}2 + 0{\cdot}2} = 1{\cdot}14$$

$$\text{Stability ratio at B} = \frac{\text{Time setting of relay at B}}{\text{Time setting of relay at C} + \text{operating time of switch C}}$$

$$= \frac{1{\cdot}2}{0{\cdot}8 + 0{\cdot}2} = 1{\cdot}2$$

$$\text{Stability ratio at C} = \frac{\text{Time setting of relay at C}}{\text{Time setting of relay at D} + \text{operating time of switch D}}$$

$$= \frac{0{\cdot}8}{0{\cdot}4 + 0{\cdot}2} = 1{\cdot}33$$

These low stability ratios are a definite disadvantage, as other systems will give values as high as 2.

2. The apparatus associated with the balanced systems employing pilot wires is, on the whole, simpler than that used in the graded time-lag systems and, in addition, the balanced systems give perfect discrimination with practically instantaneous operation. Great stability in the case of through faults has been secured by biased opposition in the tripping relay in the McColl system, by elimination of the effect of capacity currents in the translay system, and by neutralising the effects of capacity, vagabond, and other harmful currents in the split-pilot and Solkor system. A stability ratio as high as 2 can be obtained. These systems are applicable to all types of feeder, whether radial or ring main, and whether underground cable or overhead line. A further advantage is that each section of the feeder system is protected as an individual unit, there being no dependence on the apparatus associated with other sections.

The chief disadvantage is the cost and disposal of the pilot cables, this cost being very heavy in the case of long feeders. The original Merz-Price system suffered from the interference of pilot capacity currents with relay operation, but with the modern derivatives this no longer applies. Another disadvantage of the original system was that associated with the balancing of the current transformers. At one time current transformers were balanced against a standard, and not against one another, and the maintenance of balance is assured by the D.A.G. construction and by magnetic screening. This is no longer necessary. Thus in the Solkor and other systems, standard current transformers may be used, balance being obtained by other means, as, for example, between the saturable summation transformers in the case of the Solkor system. A much more important point is the robustness of the relays and the absence of voltage transformers, this latter being especially important in the case of very high voltage systems. Because of the cost of the pilots these systems are best suited to relatively short sections of feeder, say, up to 15 miles in close proximity to heavy power distributing centres.

3. Of the self-balancing systems employing special cables, those

6. Systems giving protection on the zero phase-sequence system only are also limited in application, since they require that there shall be no interconnection either in the form of parallel feeders or ring mains. They are peculiarly suited to the protection of underground colliery feeders. The relays used are instantaneous, but over-current protection is provided by separate over-current elements, and these usually have an inverse time lag. With the other protective systems suited to more important feeders leakage protection can be added by means of a separate relay.

7. The interlock systems were developed to provide unit protection and to enable the signals provided for discrimination to

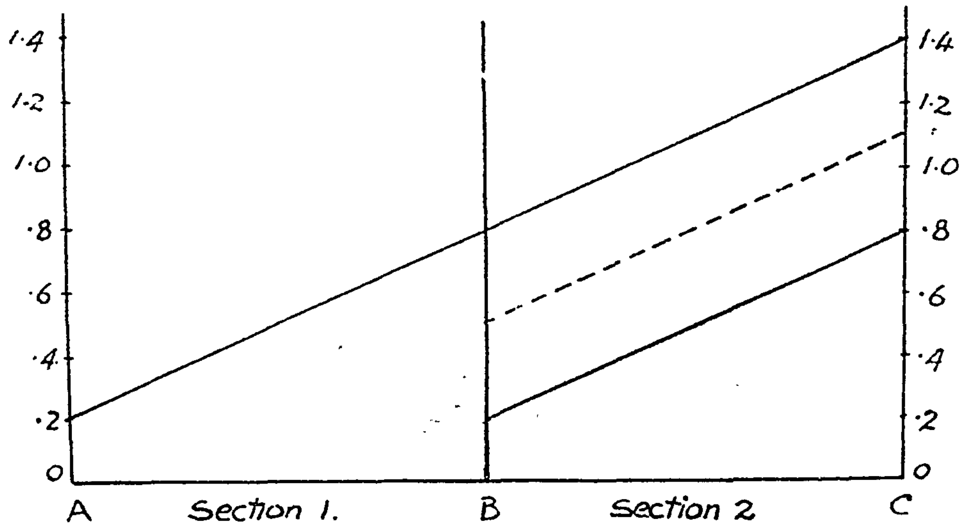


FIG. 16.67.—STABILITY DIAGRAM FOR DISTANCE SYSTEM.

be transmitted over any form of pilot, including telephone pairs which are used normally for communication purposes. In this way the pilot is not used for protection alone, thereby effecting a very considerable economy. Where telephone circuits are not available, the cost of hiring these from the Post Office is considerably less than the cost of pilot cables. Even the momentary borrowing of telephone pairs is eliminated in the superimposed interlock and phase-comparison systems employing high-frequency signal currents. These systems are thus applicable to very long lines.

8. The "mho" unit is a directional impedance unit whose accuracy is comparable with that of the reactance "ohm" unit used in a distance relay. It is ideally suited to the protection of long or heavily loaded lines because of its unlikelihood of tripping

on power swings. The fact that the "mho" unit combines sensitive directional action with accurate ohmic measurement means that one unit will do the work of two conventional units. This simplifies the circuits and eliminates the possibility of "contact race" when two units are used. There is a lower limit to the impedance for which the relay is suitable. Lines too short for the "mho" unit can be protected by the reactance "ohm" unit.

Supply to Relay Voltage Elements

On system voltages up to about 60,000 no difficulty is experienced with direct-connected voltage transformers of ordinary design, but for higher voltages the cost of such transformers having the necessary performance characteristics is very high. There is also the fear that such transformers, with their primary windings consisting of fine wire connected directly to a high-voltage network, constitute a weak link in the insulation security of the network as a whole. There are two alternatives:

1. The use of voltage compensators, in which the voltage transformer has its primary connected to the low-voltage side of the network and the voltage drop is compensated by additional apparatus. The chief objection to this appliance is that, although it can be made accurate under conditions of normal load, it is inaccurate under short-circuit conditions.

2. Capacitative couplers consisting of condensers whose capacity current is utilised in an auxiliary transformer. Since the windings of this transformer are only energised at a fraction of the voltage of the high-voltage network, and since the only portion connected directly to the network is a condenser bushing, the risk of insulation breakdown is negligible, and the need for high-voltage fuses, resistors, and isolating switches is eliminated.

Protection of Direct-current Systems

The left-hand diagram of Fig. 16.68 shows a two-wire D.C. system with leaks f_1 and f_2 on the two sides. The algebraic sum of the earth currents must be zero, and therefore if E is the system voltage, V_p, the voltage of the positive side with respect to earth, and $(E - V_p)$, the voltage of the negative side with respect to earth:

$$\frac{V_p}{f_1} + \frac{V_p - E}{f_2} = 0$$

$$\therefore V_p = E \times \frac{f_1}{f_1 + f_2}$$

still in use now, to a very limited extent, are the split-conductor and the Merz-Hunter systems.

Very briefly these are as follows: In the split-conductor system each line conductor consists of two conductors in parallel, the two half-conductors per phase passing through the same current transformer, but in opposite directions (Fig. 16.65). Under

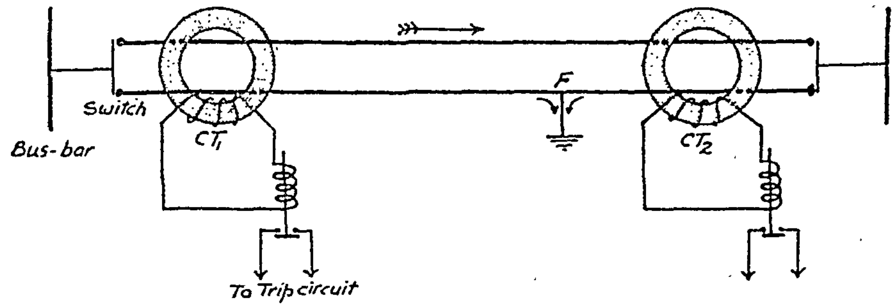


FIG. 16.65.—SPLIT-CONDUCTOR SYSTEM OF FEEDER PROTECTION.

normal conditions there is no primary M.M.F., but under fault conditions this balance is destroyed and the transformer secondaries energised. The splits have to be taken right back to the switches themselves, which therefore have to be of special construction. In the Merz-Hunter system a four-core cable is used to give a combination of the Merz-Price and split-conductor systems (Fig. 16.66). All three phases are brought through a

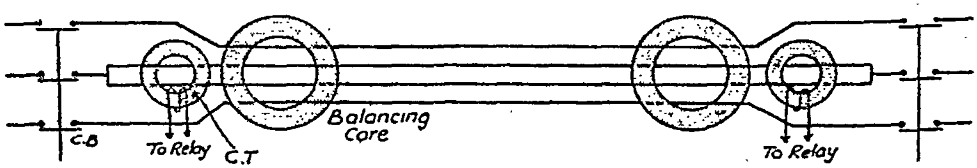


FIG. 16.66.—MERZ-HUNTER PROTECTIVE SYSTEM.

common " balancing " transformer, and one core is split, and has the current-transformers of the simple split-conductor system. The balancing transformers have no secondaries. The two plain cores can be regarded as the primaries and the split cores as the secondary of a balancing transformer. The primaries induce in the split voltages which are normally equal and opposite. In the event of a fault on one of the plain cores, the induced voltages in the split will no longer be equal, a circulating current will be set up, and the relays energised. A fault on a split will cause operation after the manner of the split-conductor system. With this system, ordinary circuit-breakers can be used.

These systems save the expense of pilot cables, the additional cost of the special designs of cable not being very serious. They provide unit protection, give excellent discrimination with almost instantaneous operation, and excellent stability with low fault settings. They also dispense with voltage transformers. Protection is afforded against earth leakage, as well as against faults between phases, without the necessity for separate earth-leakage relays. In the past the split switch has been regarded with disfavour and this objection still applies, but for the important reason of the limited breaking capacity available.

4. The development of the distance class has been mainly due to the long lines associated with very high voltage transmission, the cost of pilots used solely for protective purposes being absolutely prohibitive for long distances. These systems have thus an advantage in this respect over the various Merz-class systems. They have good discrimination and also provide back-up protection. Time delays are an essential feature with the consequent stresses on plant and network, but these time delays are much shorter than in the graded time systems. In addition, the time delays are automatically adjusted by the position of the fault, so that faults adjacent to the generating station are cleared as expeditiously as remote faults. A serious disadvantage was that voltage transformers were necessary, the cost of these being very high, since the distance systems are best suited to long high-voltage lines.

For two equal sections in series, a time lag varying from 0·2 to 0·8 sec., and a switch operating time of 0·3 sec., the stability ratio of section 2 will vary from

$$\frac{\cdot 8}{\cdot 2 + \cdot 3} = 1{\cdot}6 \text{ to } \frac{1{\cdot}4}{\cdot 8 + \cdot 3} = 1{\cdot}28$$

according to whether the fault is near B or C (Fig. 16.67). This should be regarded only as an illustrated example. The ratio is determined in exactly the same way for other shapes of time-distance characteristics. This is overcome in the more recent systems such as the Telephone system previously described, which gives rapid clearance of faults, with remote control and communication between switching stations by means of carrier-current transmitted by the overhead lines themselves. Capacitor couplers eliminate the necessity for potential transformers.

5. The negative phase-sequence systems are necessarily limited in application, and a comparison with the other systems is hardly possible. The fields of application have already been given on p. 453.

Thus $V_p = E$ if $f_2 = 0$; $V_p = 0$ if $f_1 = 0$, and V_p can have any intermediate value according to the relative values of f_1 and f_2.

The right-hand figure shows the conditions when P and N are joined by a high resistance, the middle point of which is connected to earth through a relay of resistance r. The possible potentials

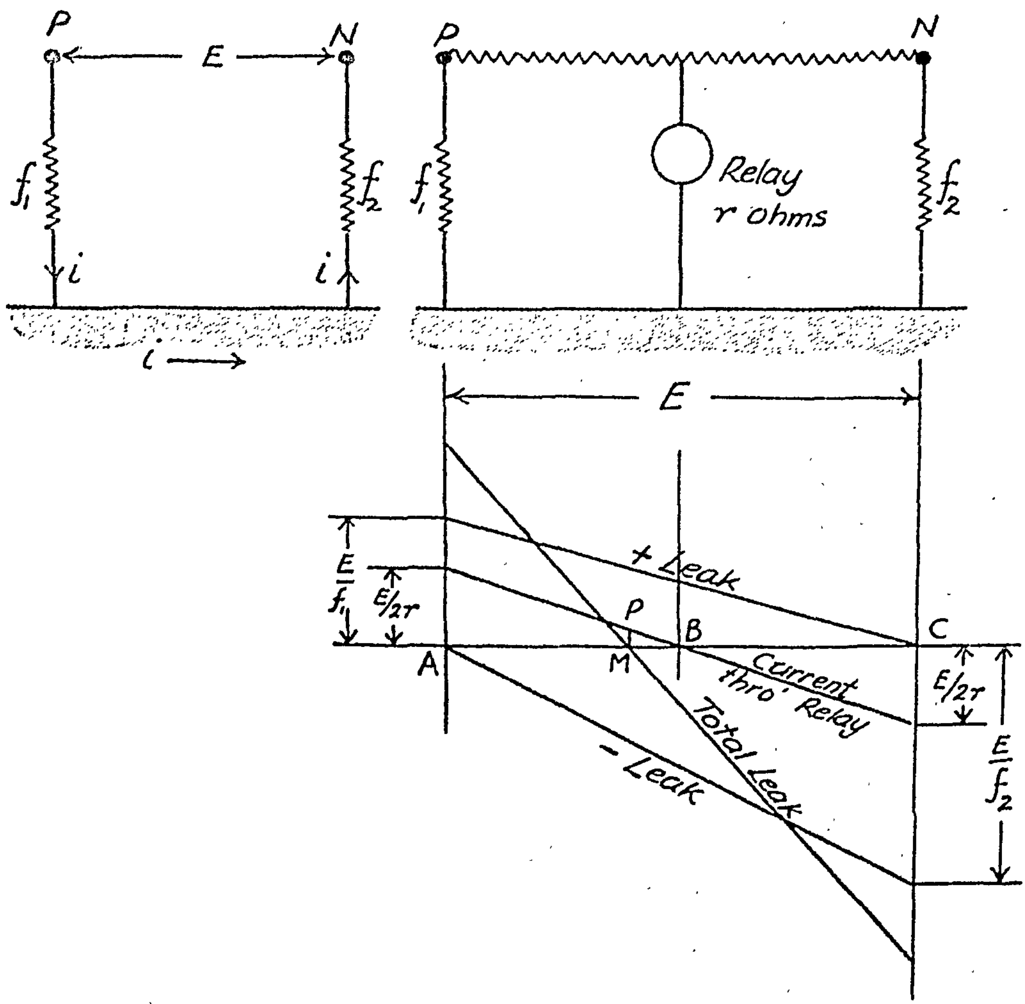


FIG. 16.68.—TO ILLUSTRATE LEAKAGE IN A D.C. SYSTEM.

of the system are best illustrated by a diagram similar to the Taylor diagram. The base line AC represents the system voltage E, and vertical distances represent the leakage currents. Thus, suppose P were at earth potential, then the negative leak would be E/f_2, and the current through the relay $E/2r$, these two currents being set off vertically downwards from C. If N were at earth potential, the positive leak would be E/f_1 and the current through the relay again $E/2r$, these two currents being set off vertically upwards from A. For any other potentials of these points with respect to earth, the leakage currents will be represented by the straight lines joining these extreme points, while the total leakage current will be the sum of all three. From this diagram it will be seen that for the conditions shown (i.e. $f_2 > f_1$) the relay current will be given by the ordinate PM, which makes the total leak, namely, the algebraic sum of the three earth currents, zero. It

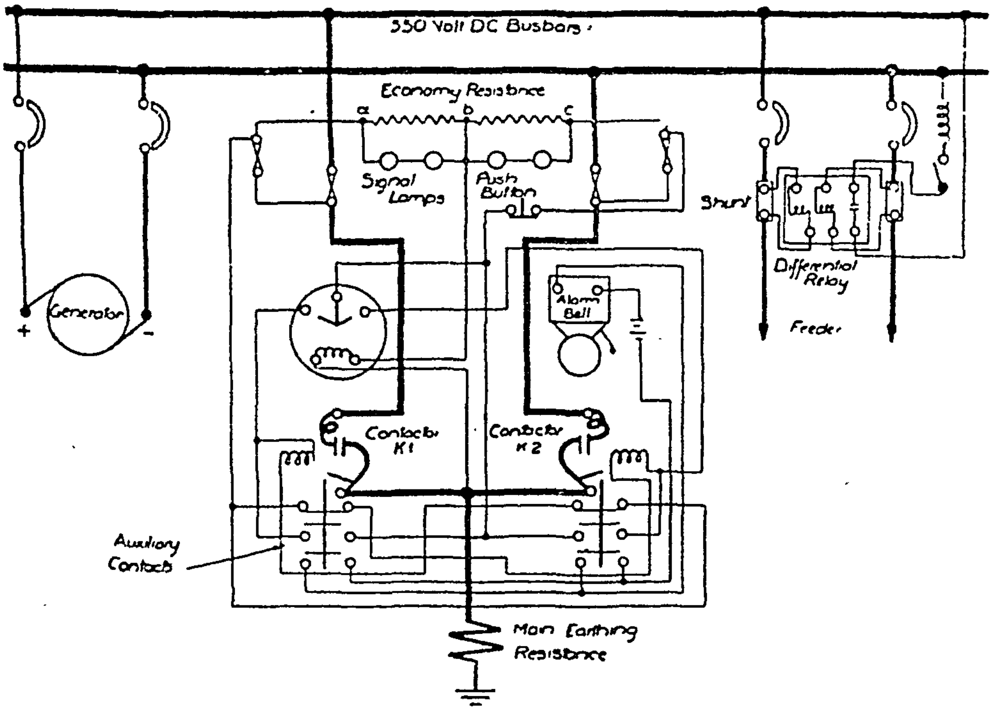


FIG. 16.69.—SCHEME OF PROTECTION FOR A D.C. SYSTEM. (*B.T.-H. Co., Ltd.*)

will be obvious that the current through the relay can never be zero except when f_1 and f_2 are equal, and this is the principle of the system of protection.

The essentials of the system are shown in Fig. 16.69, from which it will be seen that each side of the feeder to be protected has a shunt, the leads from which are taken to the two windings of the feeder relay, which is differential in action. Hence, when the currents in the positive and negative sides of the feeder are equal the relay will not operate, but if through leakage on one side these currents are not equal, one relay coil will overcome the other and, in consequence, the relay will operate. Actually, the out-of-balance between the two sides of the feeder might not be sufficient to operate the directional relay, and therefore to ensure that the feeder switch will trip, it is necessary to increase this out-of-balance until the switch does trip. This is accomplished by the master relay connected in the circuit of the earthing resistance. The fault on the feeder will set up a current in this earthing resistance, as explained above, with the result that current will flow in the operating coil of the master relay. This relay is polarised, i.e. the movement is to one side or the other according as the current through the resistance is from earth or to earth, the relay consisting essentially of a zero-centre millivoltmeter movement with contacts mounted on the pointer. The effect of the fault will thus be to move this relay to one side or the other according to which side of the system is the faulty one. The operation of the relay energises the operating coil of a single-pole contactor which, on closing, puts the bus-bar on the healthy side to earth through the main resistance. This raises the potential of the faulty side with respect to earth, and so increases the leakage current to such an extent that the differential relay will operate with certainty. The earth connection of the healthy bus-bar can then be removed, since the gear includes an alarm bell, which warns the attendant that one side of the supply is earthed through the resistance, and that a faulty feeder has been disconnected.

The above system isolates a faulty feeder almost instantaneously, while the fault current is only a small fraction of the normal full-load current, and gives discrimination between healthy and faulty feeders. With plain overload protection by means of over-current relays leakage currents may continue for many months and serious damage may result through electrolysis and corrosion. Again, faulty feeders are cleared by ordinary overload protection only when the earth current has reached a high value, and by that time the fault has usually assumed the proportions of a short-circuit on the system. By disconnecting the feeder at the inception of a fault, the above core-balance gear reduces material damage at the fault and makes it unnecessary for the circuit-breakers to interrupt excessive fault currents. Ordinary leakage indicators can detect very small earth-leakage currents, but the faulty feeder can be identified only by disconnecting the feeders in turn until the fault is removed. This tedious operation is generally deferred until a period of light load, with the danger that the fault may develop.

The system is of peculiar interest to D.C. colliery installations, where the avoidance of earth leakage is of great importance.

Further Reading

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Mathews, P., *Protective Current Transformers and Circuits* (Chapman & Hall).
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CHAPTER XVII

TRAVELLING WAVES

THE establishment of a potential difference between the conductors of a cable or a transmission line is accompanied by the creation of an electrostatic flux, while the flow of current is accompanied by a magnetic field. The capacitances which are the seat of the electrostatic fields are in the nature of shunt capacitances, whilst the inductances due to the magnetic fields are in series with the line. From this it follows that if there is any sudden change in either voltage or current at any point of the line, then some definite time must elapse before other points in the line are affected.

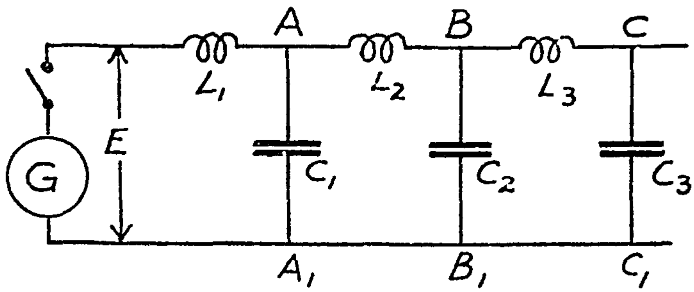

FIG. 17.1.—DISTRIBUTED CHARACTERISTICS OF A TRANSMISSION LINE.

For consider the section of a line adjacent to the generator G, Fig. 17.1, and let the voltage be raised suddenly to E by the closing of the switch. This voltage will be utilised initially in sending a current i through the circuit consisting of L_1 and C_1 in series. This current will build up the voltage across C_1, and therefore across the line at AA_1, according to the law:

$$i_1 = C_1 \frac{de_1}{dt}$$

Only after this voltage e_1 has become appreciably high can there be any similar charging of C_2, with consequent establishment of voltage e_2 across BB_1. The establishment of this voltage e_2 is followed in the same way by a voltage e_3 across CC_1, and so on, showing that the greater the distance from the generator G, the greater will be the time elapsed from the closing of the switch to the establishment of the line voltage. It is also clear that voltage and current are intimately associated, and that any voltage phenomenon is associated with an attendant current phenomenon.

The gradual establishment of the line voltage can be regarded as due to a voltage wave travelling from the end G towards the far end, and the progressive charging of the line capacitances will account for the associated current wave. Suppose that, at any instant, both voltage and current waves have reached some point P, Fig. 17.2; then there will be a dielectric flux as represented by

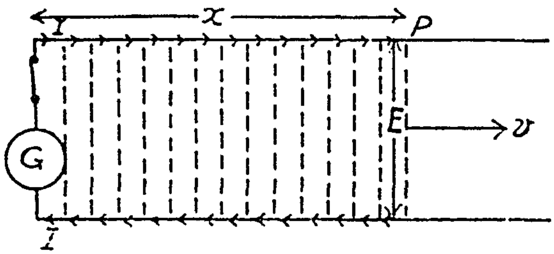

FIG. 17.2.—PRODUCTION OF TRAVELLING WAVES.

the dotted electrostatic lines of force, and, from the equation $i = C\frac{de}{dt}$, the whole of the current at the point P will be utilised in setting up the electrostatic flux at P. Similarly, the voltage at P will be utilised in overcoming the back E.M.F. due to the increase in the magnetic flux at P. Let the point P be distant x miles from G, and let the line inductance be L henries per mile.

$$\therefore \text{ Magnetic linkage up to point P} = ILx$$
$$\therefore \text{ In time } dt, \text{ change in linkage} = IL\,dx$$
$$\text{and rate of change} = IL\frac{dx}{dt}$$

But the rate of change of linkage is equal to the voltage E.

$$\therefore E = IL\frac{dx}{dt}$$
$$\text{or } E = IL\,v$$
$$\text{where } v = \text{velocity of propagation of wave.}$$

Again, if the line capacitance is C farads per mile, electrostatic flux up to point P

$$= EC\,x$$

$$\therefore \text{ Rate of change of electrostatic flux} = EC\frac{dx}{dt} = EC\,v$$

Hence $$I = EC\,v$$

Multiplying the two expressions for v we obtain the relationship:

$$v = \sqrt{\frac{1}{LC}} \text{ miles per sec.}$$

Again $$\frac{E}{I} = L\,v = \sqrt{\frac{L}{C}} = Z_n \text{ say.}$$

Now, obviously, the quantity E/I is in the nature of an impedance, and consequently $\sqrt{\frac{L}{C}}$ is called the natural impedance of the line. The adjective natural is used because this impedance has nothing to do with any load, but depends only on the line constants.

In a single-phase line we have

$$L = 14{\cdot}8 \times 10^{-4} \log_{10} \frac{d}{r} \text{ henries per mile of loop}$$

$$C = 19{\cdot}4 \times 10^{-9} \div \log_{10} \frac{d}{r} \text{ farads per mile of loop}$$

$$\therefore\ v = \frac{1}{(14{\cdot}8 \times 10^{-4} \times 19{\cdot}4 \times 10^{-9})^{\frac{1}{2}}} \text{ miles per sec.}$$
$$= 187000 \text{ miles per sec.}$$

For a three-phase line we have

$$L = 7{\cdot}4 \times 10^{-4} \log_{10} \frac{d}{r} \text{ henries per mile of conductor}$$

$$C = 38{\cdot}8 \times 10^{-9} \div \log_{10} \frac{d}{r} \text{ farads per mile of conductor}$$

giving $v = 187000$ miles per sec., as before.

Since the product LC is the same for all overhead lines, it follows that the velocity of propagation is also the same. This velocity is the same as the velocity of light, but as we have assumed a resistanceless line in the above investigation, the velocity in practice will be from 5 to 10 per cent. less than this.

The ratio L/C is not a constant, and therefore different lines will have different natural impedances.

Alternating Voltage applied

In the above treatment the voltage E has been assumed constant, and in practice such an assumption is generally sufficient, owing to the enormously high velocity of propagation. Assuming that $v = 180000$ in an actual case, at 50 cycles the first impulse will have travelled a distance of

$$\frac{180000}{50} = 3600 \text{ miles}$$

by the end of the first cycle, which means that a line would have to be 3,600 miles long to carry the whole of the voltage distribution corresponding to one cycle. Such a line would be a line of full-wave length. A line of such length is obviously impossible. We shall see that a type of resonance called quarter-wave resonance is possible with a line of one-quarter of the full-wave length, but as this length is 900 miles, such resonance is not likely at commercial frequencies. But suppose that there are harmonics present in the voltage wave. As these are of higher frequency than the fundamental, the line length for quarter-wave resonance is reduced in proportion to the order of the harmonic. Thus the lengths are:

For the 3rd harmonic, 300 miles
For the 5th harmonic, 180 miles
For the 7th harmonic, 129 miles, and so on.

These values show that with modern long-distance transmission quarter-wave resonance with one of the harmonics comes within the bounds of possibility.

Case of an Open-circuited Line

Let an open-circuited line be switched suddenly on to a source of constant voltage E. Then, neglecting for a moment the effect of line resistance and leakance, a rectangular voltage wave of amplitude E and its associated current wave of amplitude $I = E/Z_n$ will travel with velocity v_n towards the open end. Fig. 17.3A shows the conditions at the instant that the waves have reached the open end, the whole line being at voltage E and carrying current I. Now, at the open end the current must of necessity fall to zero, and consequently as each element dx of the current wave reaches this point the energy stored in the magnetic field corresponding to the length dx must be transformed in some way. In the case under consideration there is only one possibility, and that is the production of an equal amount of energy in the electrostatic field. This will raise the voltage by some amount e, so that we have

$$\tfrac{1}{2}Ce^2dx = \tfrac{1}{2}LI^2dx$$

$$\therefore e = \sqrt{\frac{L}{C}}\,I = Z_nI = E$$

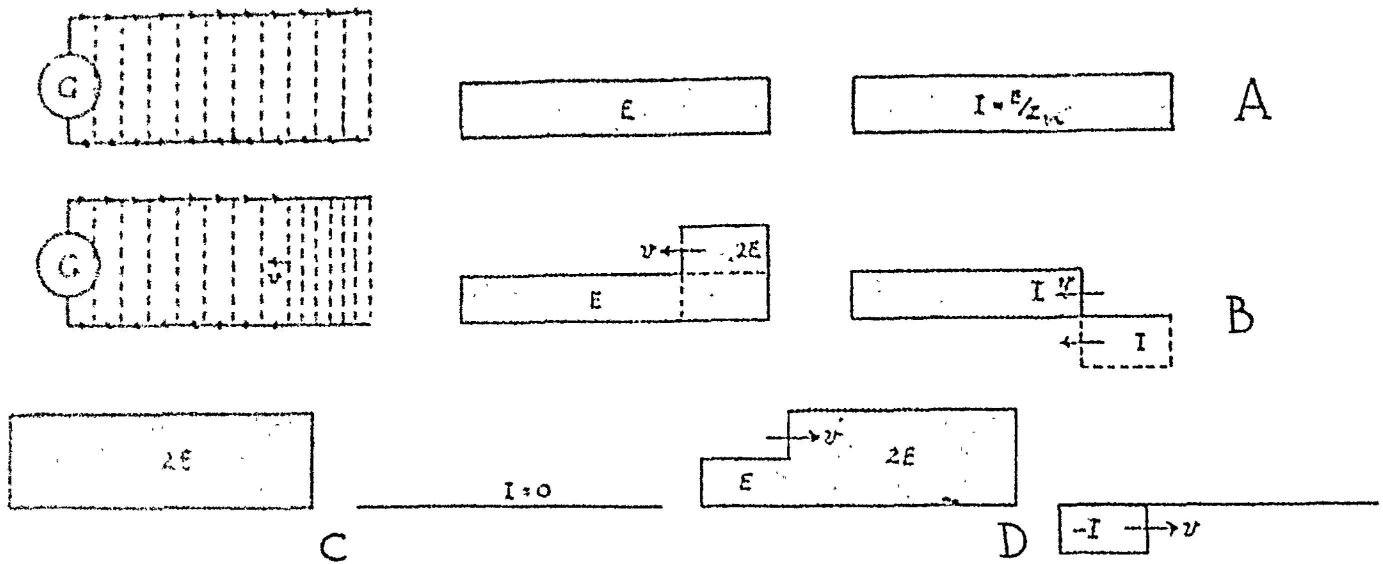

Fig. 17.3.—Travelling Waves in an Open-circuited Line.

Hence the voltage E is increased by an equal amount, or, in other words, it is doubled, thus becoming 2E. The open end of the line can thus be regarded as the origin of a second voltage wave of amplitude E, this second wave travelling back with the same velocity v. Hence at some time subsequent to the instant of Fig. 17.3A, the incoming and reflected voltage waves will superpose and give a total voltage of 2E, as in Fig. 17.3B. The step in the voltage wave will travel with velocity v towards G. Now, the raising of the voltage E by an additional E must be associated with the disappearance of the current, and this is equivalent to the establishment of a reflected current wave of negative sign, as shown in the diagram of current distribution in Fig. B. At the instant of this figure the distributions of voltage and current are therefore both of them stepped, that of the voltage up to 2E, and that of the current down to zero. An open-ended line thus reflects a voltage without change of sign, and a current with reversal of sign. At the instant the reflected waves reach the end G, the distribution along the whole line will be voltage 2E and current zero, as in Fig. C. But as at G the voltage is held to the value E, it follows that there must be a reflected voltage of — E, and associated with it there will be a current wave of — I. After these have travelled a little way along the line the conditions will therefore be as in Fig. D. When these reach the open end the conditions along the line will be voltage E and current — I. The reflected waves due to these will be — E and + I, and when these have travelled to the end G they will have wiped out both voltage and current distributions, leaving the line for an instant in its original state. The above cycle is then repeated. We see that this cycle occupies the time taken for a wave to travel four times the length of the line, the frequency of the cycle thus being

$$\frac{v}{4l} = \frac{1}{4l\sqrt{LC}}$$

If we now consider the case of an open-ended line subjected to an alternating voltage we see that if, at the moment the reflected wave of voltage reaches G, a second impulse is sent into the circuit, the incident and reflected waves will be added together, and this addition will be continued at successive reflections. Whatever the nature of the impressed voltage, the " switch-in wave " must travel four times the length of the line during one cycle of changes, and consequently for resonance to occur the cycle of impressed voltage must correspond to four times the line length. In other words, the line length must correspond to one-quarter of the impressed voltage cycle. Hence the name quarter-wave resonance.

Case of a Short-circuited Line

In this case the voltage at the far end of the line must of necessity be zero, so that as each element of the voltage wave arrives at the end there is a conversion of electrostatic energy into electromagnetic energy, instead of the reverse case, as with an open-circuited line. Hence it is the voltage which is reflected with reversal of sign, while the current is reflected without any change of sign : thus on the first reflection the current builds up to 2I. Successive stages of the phenomenon are represented in Fig. 17.4.

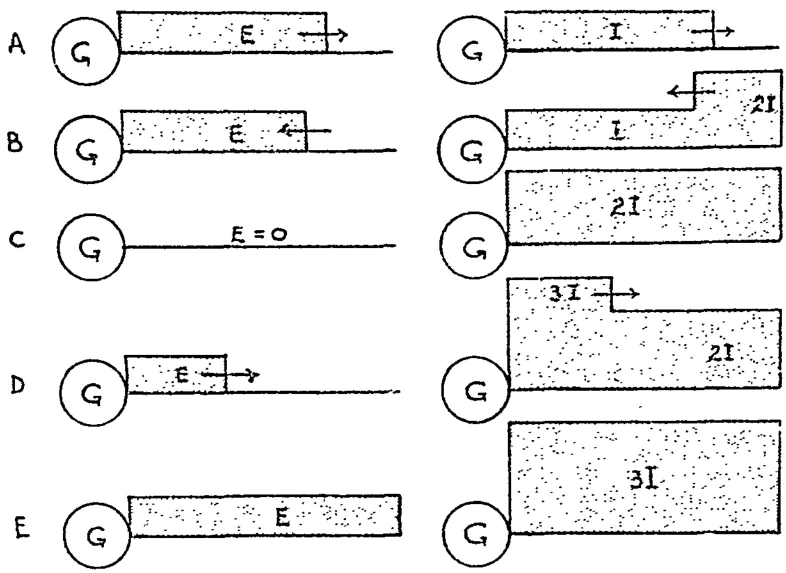

FIG. 17.4.—TRAVELLING WAVES IN A SHORT-CIRCUITEL INE.

A. Original voltage and current waves just prior to the first reflection.

B. Distributions just after the first reflection.

C. Distributions at the instant the first reflected wa s have reached the generator : note that the whole of the line is at zero voltage.

D. Distributions after the first reflection at the generator end.

E. Distributions at the instant the first reflected waves from the generator reach the far end.

It will be seen that the line voltage is periodically reduced to zero, but that at each reflection at either end the current is built up by the additional amount $I = E/Z_n$. Thus theoretically the current will eventually become infinite, as is to be expected in the case of a resistanceless line. In practice the resistance of the line produces attenuation, so that the amplitude of each wave-front gradually diminishes as it travels along the line, and the ultimate effect of an infinite number of reflections is to give the steady Ohm's law current of E/R. We therefore see that the establishment of the current in a long short-circuited line is not a continuous process, as is the establishment of current in a circuit with lumped constants, but that it takes place as a series of steps, each step corresponding to a new reflection. As these steps diminish progressively owing to attenuation they give a convergent series, the sum of an infinite number of terms of which is E/R.

Line closed through a Resistance

If a non-inductive resistance R is connected across the line at the far end, the voltage of the reflected wave at this point depends upon the magnitude of R. With $R = \infty$ we have the case of the open-circuited line; with $R = 0$ the case of the short-circuited line. With $R = Z_n$, the resistance can be regarded as a natural continuation of the circuit, and the current and voltage waves pass into R without any change. For any value of R different from the above special cases, there will be partial reflection of the current and voltage waves. Thus with $R > Z_n$ there will be partial reflection with reversal of current, and with $R < Z_n$ partial reflection with reversal of voltage.

Let E and I refer to the incident waves,
E′ and I′ to the reflected waves,
and E″ and I″ to the waves which penetrate the resistance.

Then
$$\frac{E}{I} = Z_n$$

Also, since the reflection is accompanied by a change in sign of *either* voltage or current, but not both:

$$\frac{E'}{I'} = -Z_n$$

The voltage in the receiving circuit at any instant will be the algebraic sum of the incident and reflected voltages.

$$\therefore E'' = E + E'$$

The difference between the incident current I and the current I″

penetrating the resistance R is the reflected current I′: but this current is reflected with change of sign,* so that we have

$$-I' = I - I''$$

or
$$I'' = I + I'$$

Again,
$$E'' = I''R$$

$$\therefore I''R = E + E' = Z_nI - Z_nI'$$

$$\therefore I''R = Z_nI - Z_n(I'' - I)$$

giving
$$I'' = \frac{2IZ_n}{R + Z_n} = \frac{2E}{R + Z_n}$$

Hence
$$I' = I'' - I$$

$$= \frac{2IZ_n}{R + Z_n} - I = -I\left(\frac{R - Z_n}{R + Z_n}\right)$$

Also
$$E' = -Z_nI' = +Z_nI\left(\frac{R - Z_n}{R + Z_n}\right)$$

$$= +E\left(\frac{R - Z_n}{R + Z_n}\right)$$

Summarising, we have for the reflected wave:

$$E' = E\left(\frac{R - Z_n}{R + Z_n}\right); \quad I' = -I\left(\frac{R - Z_n}{R + Z_n}\right)$$

And for the wave penetrating the resistance R:

$$E'' = \frac{2ER}{R + Z_n}; \quad I'' = \frac{2E}{R + Z_n}$$

For the particular case in which $R = Z_n$ we see that both E′ and I′ become zero, and there is thus no reflection. If the resistance has series capacitance the effect of this capacitance is at first zero because it is initially uncharged.

The Bewley Lattice Diagram

This is a very convenient diagram devised by Bewley, which shows at a glance the position and direction of motion of every incident, reflected, and transmitted wave on the system at every instant of time. By its means the difficulty of keeping track of the multiplicity of successive reflections is considerably simplified. As a first example we will take the case of an open-circuited line having series resistance as well as inductance, and shunt leakance as well as capacitance. With such a line there is attenuation of

* This only applies when $R > Z_n$ which is assumed in the argument. In the general case $I'' = I + I'$ where I′ carries its own sign.

both voltage and current waves as they travel along the line. In the Heaviside distortionless line the voltage and current waves remain similar in spite of attenuation, and the condition for such a distortionless line is

$$RC = GL$$

In such a line it can be shown that if a wave is of amplitude A_1 at any point of the line, the amplitude A_2 at some point distant x is

$$A_2 = A_1 e^{-\beta x}$$

where β is a constant called the attenuation constant. For the distortionless line the value of this constant is $\sqrt{RG}$. With such a line any wave travelling the whole length of the line will suffer a definite percentage attenuation, and, knowing this attenuation, it is possible to determine the amplitude of any wave at any instant. Consider for example the case of an open-circuited line in which

$$R = \cdot 5 \text{ ohm per mile; } G = 10 \times 10^{-7} \text{ mho per mile; } l = 400 \text{ miles}$$

$$\therefore \ \beta = \sqrt{RG} = \sqrt{\cdot 5 \times 10 \times 10^{-7}} = 0{\cdot}000707$$

$$\therefore \ e^{-\beta x}, \text{ when } x = l, = 400 \text{ miles}$$

$$= e^{-\cdot 2828} = \cdot 754$$

If we denote this quantity by α and the initial value of the voltage at the generating end by unity, then we have the following sequence of events, as far as the reflected wave is concerned. Let t' be the time taken to make one tour of the line, i.e. $400/186000 = 0{\cdot}00215$ sec. in the present case. At zero time a wave of amplitude 1 starts from G. At time t' a wave of amplitude $\alpha = 0{\cdot}754$ strikes the open end and a reflected wave of $+\alpha = 0{\cdot}754$ commences the return journey. At time $2t'$ this reflected wave is attenuated to $\alpha^2 = 0{\cdot}5685$ and has reached G. Here it is reflected to $-\alpha^2 = -0{\cdot}5685$, and after a time $3t'$ it reaches the open end attenuated to $-\alpha^3 = -0{\cdot}4287$. It is then reflected without change of sign and reaches G after a time $4t'$, with an amplitude of $-\alpha^4 = -0{\cdot}3232$. It is then reflected with change of sign, thus starting off with an amplitude of $+\alpha^4 = +0{\cdot}3232$, and so on. The Bewley lattice diagram is a space-time diagram with space measured horizontally and time vertically, and the lattice for the above example is shown in Fig. 17.5.

Now, the increment of voltage at the receiving end due to any reflection is twice the amplitude of the incident wave, because of the reflection without change of sign. Also, the final voltage at this end is the sum to infinity of all such increments. Thus, in the above example, it is

$$2(\cdot 754 - \cdot 4287 + . \quad . \quad .)$$

It is simpler to express the series generally in terms of α, thus

$$2(\alpha - \alpha^3 + \alpha^5 - \alpha^7 + . \quad . \quad .)$$

$$= 2\{(\alpha + \alpha^5 + \alpha^9 + . \quad . \quad .) - (\alpha^3 + \alpha^7 + \alpha^{11} + . \quad . \quad .)\}$$

$$= 2\left[\frac{\alpha}{1 - \alpha^4} - \frac{\alpha^3}{1 - \alpha^4}\right]$$

$$= \frac{2\alpha}{1 + \alpha^2}$$

$$= \frac{2 \times \cdot 734}{1 + \cdot 5685} = \cdot 93$$

Thus, even when open-circuited, such a line gives a far-end

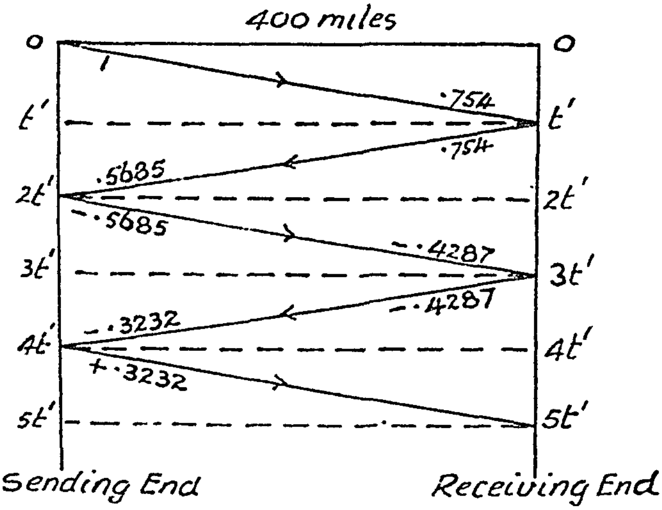

Fig. 17.5.—Lattice Diagram for Line with Attenuation.

voltage less than the sending-end voltage, the reason being that the shunt leakance causes a drop along the series resistance.

As far as the sending end is concerned, the voltage at the moment of incidence of a wave sent from the far end is unity plus the amplitude of this wave. The lattice shows the amplitudes of these incident waves to be $+ \alpha^2$, $- \alpha^4$, and so on, successive sending-end voltages thus being 1, $1 + \alpha^2$, 1, $1 - \alpha^4$, 1, and so on. Thus the voltage oscillates about unity, the amplitude of the oscillation rapidly diminishing so that eventually the voltage becomes the generator voltage of unity, as would be expected.

Now consider the case of the ideal resistanceless and leakance-less line joined at the far end by a resistance R. Then there is no attenuation as a wave travels along the line, but at the receiving end there is a reflection operator of $\frac{R - Z_n}{R + Z_n}$, which is less than unity. We can thus use this in place of α in the lattice diagram at the receiving end, and put $\alpha = -1$ at the sending end. Suppose that $\frac{R - Z_n}{R + Z_n} = 0 \cdot 5$, then the lattice diagram will be as shown in Fig. 17.6. At the receiving end the increment of voltage is the

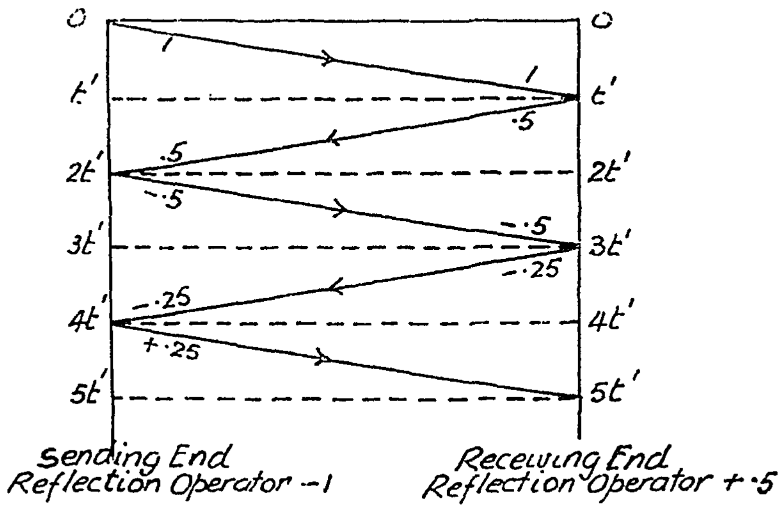

FIG. 17.6.—LATTICE DIAGRAM FOR LINE JOINED THROUGH A RESISTANCE.

sum of the incident and reflected waves at each reflection, so that the ultimate voltage at this point is the sum to infinity of the series :

$$(1 + \alpha) - (\alpha + \alpha^2) + (\alpha^2 + \alpha^3) - (\alpha^3 + \alpha^4) + \ldots$$
$$= (1 + \alpha)(1 - \alpha + \alpha^2 - \alpha^3 + \ldots)$$
$$= (1 + \alpha)\,\{(1 + \alpha^2 + \alpha^4 + \alpha^6 + \ldots) - (\alpha + \alpha^3 + \alpha^4 + \ldots)\}$$
$$= (1 + \alpha)\left[\frac{1}{1 - \alpha^2} - \frac{\alpha}{1 - \alpha^2}\right]$$
$$= 1$$

Thus the voltage at the receiving end finally settles down to that at the sending end, and consequently the current settles down to the simple Ohm's law value of E/R. The increments of current are obviously proportional to the increments of voltage at the receiver end, and therefore the voltage-time and current-time curves for this end for $\alpha = 0 \cdot 5$ are as shown in Fig. 17.7. The tabulated values are as shown, and it will be seen that both

Time	Increment of Voltage or current	Sum of Increments
0	0	0
t'	$1+\alpha \quad = 1\cdot5$	$1\cdot5$
$3t'$	$-(\alpha+\alpha^2) \quad = -\cdot75$	$0\cdot75$
$5t'$	$\alpha^2+\alpha^3 \quad = \cdot375$	$1\cdot125$
$7t'$	$-(\alpha^3+\alpha^4) \quad = -\cdot188$	937
$9t'$	$\alpha^4+\alpha^5 \quad = \cdot094$	$1\cdot031$
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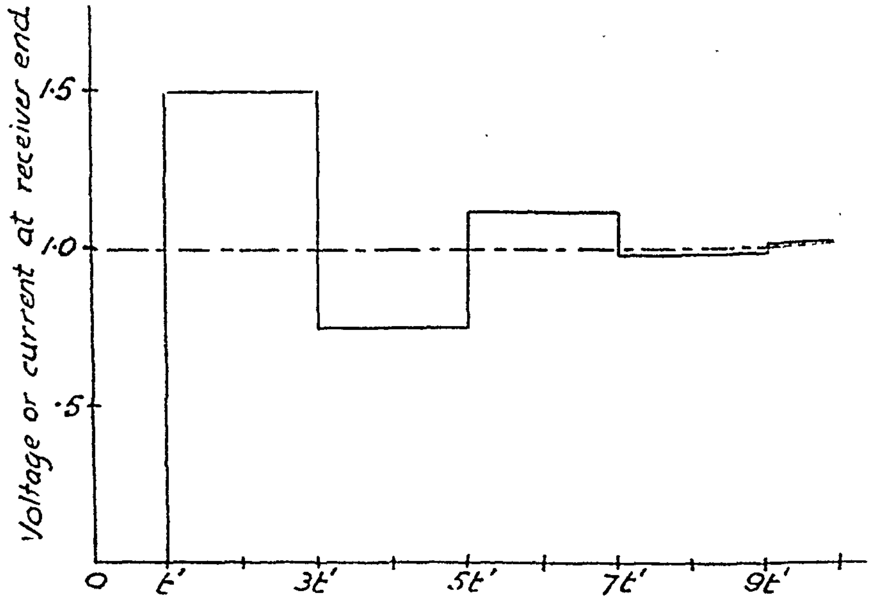

FIG. 17.7.—BUILDING UP OF CURRENT AND VOLTAGE IN A LINE JOINED THROUGH A RESISTANCE.

voltage and current oscillate about the value unity and finally settle down to this value.

Time.	Increment of Voltage or Current.	Sum of Increments.
0	0	0
t	$1 + a = 1\cdot5$	$1\cdot5$
$3t'$	$-(a + a^2) = -\cdot75$	$\cdot75$
$5t'$	$a^2 + a^3 = \cdot375$	$1\cdot125$
$7t'$	$-(a^3 + a^4) = -\cdot188$	$\cdot937$
$9t'$	$a^4 + a^5 = \cdot094$	$1\cdot031$
—	—	—

Junction of a Cable and Overhead Line

Let a wave originate in the overhead line and travel towards the junction. Then, since the values of Z_n may be considerably different in the overhead line and cable, there will be partial reflection and partial transmission of both voltage and current waves. Let the constants be:

$$\text{Overhead line } L_1, C_1, \therefore Z_1 = \sqrt{\frac{L_1}{C_1}}$$

$$\text{Cable } \quad L_2, C_2, \therefore Z_2 = \sqrt{\frac{L_2}{C_2}}$$

Let the incident waves be E, I; the waves reflected back into

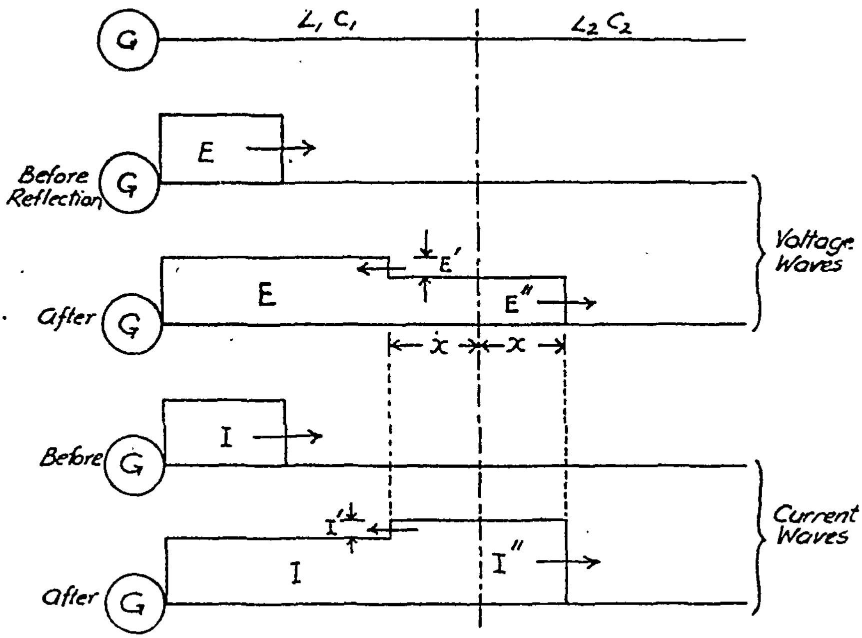

FIG. 17.8.—EFFECT OF A CHANGE IN LINE CONSTANTS.

the overhead line E′, I′; and the waves transmitted into the cable E″, I″.

Then $$I = \frac{E}{Z_1}, \; I_1 = \frac{E'}{Z_1}, \text{ and } I_2 = \frac{E''}{Z_2}$$

The conditions just before and just after reflection are shown in Fig. 17.8, and we have by an argument identical with that on p. 487, but substituting Z_2 for R:

Reflected waves: $E' = E\dfrac{Z_2 - Z_1}{Z_1 + Z_2}$; $I' = -I\left(\dfrac{Z_2 - Z_1}{Z_1 + Z_2}\right)$

Transmitted waves · $E'' = \dfrac{2EZ_2}{Z_1 + Z_2}$; $I'' = \dfrac{2E}{Z_1 + Z_2}$

As $Z_1 > Z_2$ the term $(Z_2 - Z_1)$ is negative, with the result that the voltage wave is reflected with change of sign and the current wave without change of sign. The voltage reflection operator α_1 is thus negative, while the current reflection operator α_2 is positive. The voltage transmission (or refraction) operator and the current operator, β_1 and β_2, are both positive. With the wave originating in the overhead line we see that the total voltage at the junction is reduced at the first reflection in the ratio $(1 - \alpha_1)$, and for this reason it is sometimes the practice to terminate an overhead line near to a station and to connect the line to the station plant by a short length of insulated cable. In this way the danger of building up a dangerous voltage is minimised. Actually, if the cable length is short compared with the length of the incident wave, there will be a building up of voltage at the junction because of successive reflections, but this building up will be at a slower rate than the rate of arrival of the wave front. As the slowing down of the wave front is also a safety measure, in that the rate of increase of voltage is reduced, we see that the cable is still effective.

If the wave originates in the cable, then if we now denote the natural impedance of the cable by Z_1 and that of the overhead line by Z_2 the above equations still hold good. But $(Z_2 - Z_1)$ is now positive, so that on reflection the voltage suffers no change of sign, but the current is reversed. In this case the total voltage at the junction is increased by reflection and there is thus the possibility of building up a high voltage. The voltage reflection operator α_1 is now positive, while the current reflection operator α_2 is negative. The refraction operators β_1 and β_2 are both positive, as before.

As a numerical example, take the case of a long overhead line joined to a short length of cable which is open-circuited at its far end. Let the wave originate in the line, and let the ratio Z_1/Z_2 be equal to 10, a fairly representative value. Then for the numerical values of the operators on the voltage wave we have:

1. Line to cable:

$$\text{Reflection operator } \alpha_1 = \frac{1 - 10}{1 + 10} = -\cdot 818$$

$$\text{Refraction operator } \beta_1 = \frac{2 \times 1}{1 + 10} = \cdot 182$$

2. Cable to line:

$$\text{Reflection operator } \alpha_2 = \frac{10 - 1}{1 + 10} = + \cdot 818$$

$$\text{Refraction operator } \beta_2 = \frac{2 \times 10}{1 + 10} = 1 \cdot 818$$

On first striking the junction there is a reflection of − 0·818 and a refraction of 0·182. This voltage of 0·182 travels to the open end of the cable, where it is reflected with a reflection operator of plus 1,

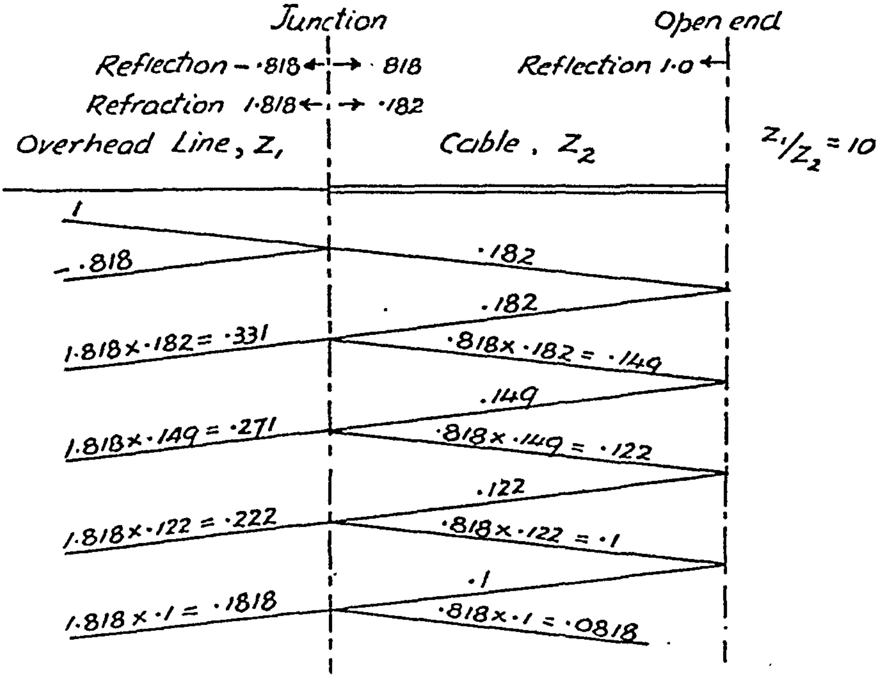

FIG. 17.9.—LATTICE DIAGRAM FOR A LINE CONSISTING OF TWO SECTIONS WITH DIFFERENT CONSTANTS.

so that the same voltage of 0·182 returns towards the junction. At this point the reflection operator is now plus 0·818, while the refraction operator is plus 1·818. Hence, at this stage:

$$\text{Reflected wave} = \cdot 818 \times \cdot 182 = \cdot 149$$
$$\text{Refracted wave} = 1 \cdot 818 \times \cdot 182 = \cdot 331$$

And similarly with successive reflections. The amplitudes of the successive reflected and refracted voltage waves at both junction and open end are shown in the Bewley lattice of Fig. 17.9.

As a second numerical example consider the case of an overhead line for which $L_1 = 1 \cdot 5$ millihenry per mile and $C_1 = 0 \cdot 015$ mfd. per mile joined to a cable for which $L_2 = 0 \cdot 25$ millihenry per mile and $C_2 = 0{\cdot}45$ mfd. per mile. If a surge of 10 kV. originates in the line and enters the cable, calculate the voltage and current in the cable.

$$Z_1 = \sqrt{\frac{1{\cdot}5 \times 10^{-3}}{{\cdot}015 \times 10^{-6}}} = 316 \text{ ohms}$$

$$Z_2 = \sqrt{\frac{{\cdot}25 \times 10^{-3}}{{\cdot}45 \times 10^{-6}}} = 23{\cdot}6 \text{ ohms}$$

Original current in overhead line $I = \frac{E}{Z_1} = \frac{10000}{316} = 31{\cdot}7$ amps

$$\text{Voltage in cable } E'' = \frac{2EZ_2}{Z_1 + Z_2} = \frac{2 \times 10000 \times 23{\cdot}6}{316 + 23{\cdot}6} = 1395 \text{ volts}$$

$$\text{Current in cable } I'' = \frac{2E}{Z_1 + Z_2} = \frac{2 \times 10000}{316 + 23{\cdot}6} = 58{\cdot}9 \text{ amps.}$$

Now consider the case of a cable insertion between two overhead lines (Fig. 17.10), and let the impedances, in order, be Z_1, Z_2, and Z_3.

Let a wave of vertical front A and tail attenuating with *time* according to the law $A^{-}e^{\beta t}$ be created in the first section and let it

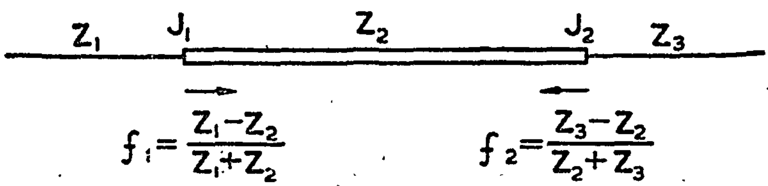

FIG. 17.10.

travel towards the first junction J_1. The amplitude of the wave which enters the cable at junction J_1 is, as before—

$$\frac{2AZ_2}{Z_1 + Z_2}$$

First of all, neglect attenuation: on reaching the junction J_2 where the reflection factor is—

$$f_2 = \frac{Z_3 - Z_2}{Z_2 + Z_3}$$

the total instantaneous voltage at that point is—

$$\frac{2AZ_2}{Z_1 + Z_2}(1 + f_2)$$

the amplitude of the wave reflected back being—

$$\frac{2AZ_2}{Z_1 + Z_2} f_2$$

This reaches the junction J_1 and is again reflected back with reflection factor—

$$f_1 = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

so that, after the double reflection, and at the commencement of the second tour of the cable the amplitude is—

$$\frac{2AZ_2}{Z_1 + Z_2} f_1 f_2 = \frac{2AZ_2}{Z_1 + Z_2} \cdot f \text{, say,}$$

and this will travel to the end J_2.

At the end of the second tour the amplitude of the step reaching J_2 is—

$$\frac{2AZ_2}{Z_1 + Z_2} f^2$$

In general, at the end of the *n*th tour the amplitude of the step reaching J_2 is—

$$\frac{2AZ_2}{Z_1 + Z_2} f^n$$

Now consider the effect of attenuation at the moment of creation of this *n*th step at J_2. The previous step—i.e. the $(n - 1)$th step—has, at the same moment, made the double journey, its actual amplitude thus being—

$$\frac{2AZ_2}{Z_1 + Z_2} f^{(n-2)} e^{-\beta T}$$

where T is the time taken to make the double journey. Replace $e^{-\beta T}$ by α, then the amplitude becomes—

$$\frac{2AZ_2}{Z_1 + Z_2} \alpha f^{(n-2)}$$

The $(n - 2)$ step has, at the same moment, made the double journey, so that its amplitude is—

$$\frac{2AZ_2}{Z_1 + Z_2} \alpha^2 f^{(n-3)}$$

and so on. Hence, for the total voltage at junction J_2 at the nth step we have—

$$E_{J2} = \frac{2AZ_2}{Z_1 + Z_2} [f^{(n-1)} + \alpha f^{(n-2)} + \alpha^2 f^{n-3} + \ldots + \alpha^{(n-1)}]$$

$$= \frac{2AZ_2}{Z_1 + Z_2} \times \frac{f^n - \alpha^n}{f - \alpha}.$$

$$= \frac{2AZ_2}{Z_1 + Z_2} \times \frac{f^n - e^{-n\beta T}}{f - e^{-\beta T}}$$

The denominator is a constant, and therefore we find the value of n to give the maximum voltage at J_2 by differentiating the numerator with respect to n and equating to zero. This gives—

$$n = \frac{\log\left(\frac{-\beta T}{\log f}\right)}{\beta T + \log f}$$

It is obvious that there must be such a maximum since, immediately after the commencement, there will have been no build up of voltage at J_2 due to the arrival of successive steps, and, after sufficient time, the whole phenomenon will have died down because of attenuation.

The phenomena at junction J_1 can be investigated in the same way.

The effect of a cable insertion between an overhead line and an alternator or transformer is summarised by Thompson* as follows:

" A length of cable between a generator and transformer and the overhead line has three effects. One is to retard the front of an impulse wave, and a second is to absorb an amount of energy from the wave (in accordance with the attenuation equation $E_1 = E_0 \varepsilon^{-\beta}$). But the third effect, which is not beneficial, is the building up (by successive reflections) of the voltage wave within the cable until the transmitted portions reach values high enough to cause breakdowns of insulation. This is at its worst when long-tailed waves traverse a short length of cable, pass through a transformer or generator winding that is long electrically, and meet a neutral point either insulated or earthed. All these features contribute to aggravate a condition that can result in electrical breakdown. The long-tailed wave maintains the surge energy supply. The short length of cable has a " badminton "

* Thompson, W. H., " Lightning Protection with Surge Divertors. *Metro-Vic. Engineering Data Leaflet*, 285/0–4.

effect on the wave: having once got into play it works backwards and forwards within the length of the cable, piling up the voltage at each change in direction. The greater the electrical length of the transformer or generator winding, the longer the delay in the return of any negative wave that might come from an earthed neutral point and counteract the cable reflections; indeed, if the neutral point is insulated (or earthed through a high surge impedance), there will be no negative return wave but a positive recoil that will make matters worse."

Reflection and Refraction at a T

Let a line of natural impedance Z_1 bifurcate into two branches of natural impedances Z_2 and Z_3, then, as far as the voltage wave is concerned, the refracted portion will be the same for both branches, since they are in parallel. On the other hand, the refracted currents will be different in the general case of $Z_2 \neq Z_3$. A short time after reflection the conditions will be as shown in Fig. 17.11, in which it is assumed that the voltage is reflected with reversal of sign, which is very probable in such a case.

Let E_1, I_1 be the incident voltage and current
E', I' be the reflected voltage and current
E'', I_2 be the refracted voltage and current along Z_2
E'', I_3 be the refracted voltage and current along Z_3

Then
$$I_2 = \frac{E''}{Z_2} \text{ and } I_3 = \frac{E''}{Z_3}$$

also
$$\frac{E_1}{Z_1} - \left(\frac{E'' - E_1}{Z_1}\right) = \frac{E''}{Z_2} + \frac{E''}{Z_3}$$

the solution of which is
$$E'' = \frac{\dfrac{2E_1}{Z_1}}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \dfrac{1}{Z_3}}$$

Knowing E_1, all the other quantities can be calculated.
If we put $Z_3 = \infty$ in the above expression we have

$$E'' = \frac{2EZ_2}{Z_1 + Z_2}$$

the case becoming that of a simple junction of two lines having different characteristics.

Example.—An overhead transmission line having a surge impedance of 450 ohms runs between two substations A and B; at B it branches into two lines C and D, of surge impedances 400 and 50 ohms respectively. If a travelling wave of vertical front

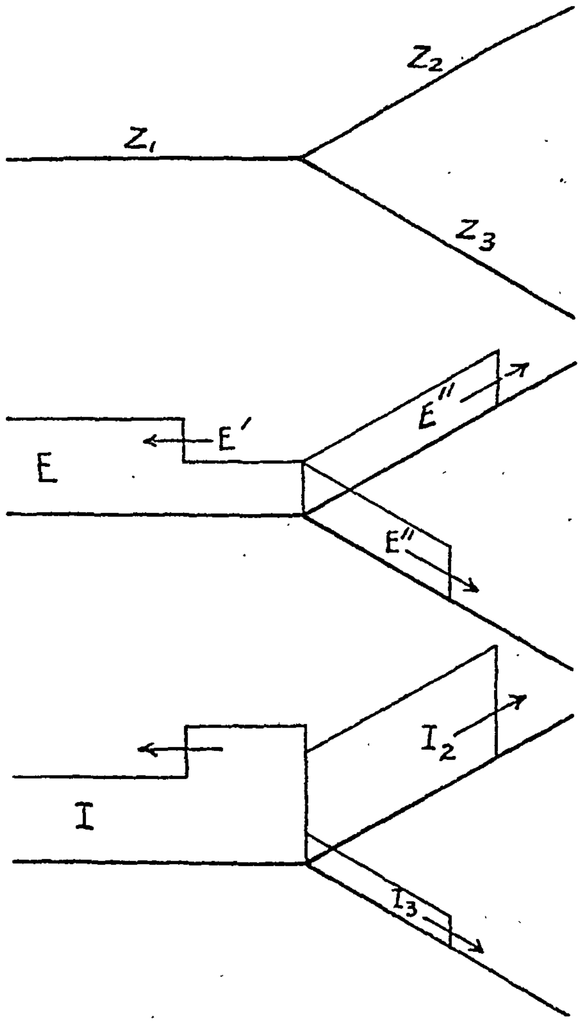

FIG. 17.11.—EFFECT OF A BIFURCATION ON THE TRAVELLING WAVES

and magnitude 25 kV. travels along the line AB, calculate the magnitude of the voltage and current waves which enter the branches C and D.

Using the previous notation (Fig. 17.12), we have—

$$Z_1 = 450 \text{ ohms}$$
$$Z_2 = 400 \text{ ohms}$$
$$Z_3 = 50 \text{ ohms}$$

Incident voltage $E_1 = 25{,}000$ V.
Incident current $I_1 = E_1/Z_1$
$= 25{,}000/450$
$= 55{\cdot}6$ A.

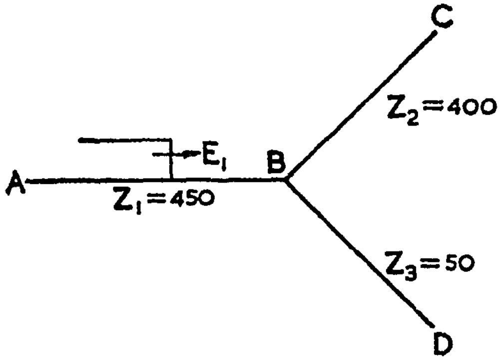

FIG. 17.12.

Transmitted voltage along BC and BD—

$$E'' = \frac{\dfrac{2E_1}{Z_1}}{\dfrac{1}{Z_1}+\dfrac{1}{Z_2}+\dfrac{1}{Z_3}}$$

$$= \frac{\dfrac{2 \times 25}{450}}{\frac{1}{450}+\frac{1}{400}+\frac{1}{50}}$$

$$= 4{\cdot}5 \text{ kV.}$$

Transmitted current along BC—

$$I_2 = \frac{E''}{Z_2}$$

$$= \frac{4500}{400} = 11{\cdot}25 \text{ A.}$$

Transmitted current along BD—

$$I_3 = \frac{E''}{Z_3}$$

$$= \frac{4500}{50} = 90 \text{ A.}$$

Thus current reflected back into the line AB—

$$= 90 + 11{\cdot}3 - 55{\cdot}6$$

$$= 45{\cdot}7 \text{ A.}$$

Power and Energy associated with a Travelling Wave

Because of the distributed inductance and capacitance of a line, a travelling wave is in the nature of a double-energy transient, the current wave being associated with the electromagnetic energy, and the voltage wave with the electrostatic energy. If, for unit length of the line the resistance, inductance, and capacitance are R, L, and C respectively, and the leakance G mhos, we have for a point at which E.M.F. and current are e volts and i amps. respectively,

Energy per unit length $w = \frac{1}{2}Li^2 + \frac{1}{2}Ce^2$ joules.

We have already seen that—

$$\frac{e}{i} = \sqrt{\frac{L}{C}}$$

$$\therefore Ce^2 = Li^2$$

$$\therefore w = Li^2 = Ce^2 \text{ joules.}$$

Now consider the power. The power loss per unit length is the sum of the I^2R and E^2G losses.

$$\therefore dp = i^2R\,dx + e^2G\,dx$$

again
$$p = ei = i^2Z_n$$

$$\therefore dp = 2iZ_n di$$

This change in dp must be reckoned negative because it is a diminution in the power in the element of length

$$\therefore -2iZ_n di = (i^2R + e^2G)\,dx$$

$$= i^2(R + Z_n^2G)\,dx$$

$$\therefore \frac{di}{i} = -\frac{1}{2}\left(\frac{R + Z_n^2G}{Z_n}\right)dx$$

$$= -\frac{1}{2}\left(\frac{R}{Z_n} + GZ_n\right)dx$$

The solution is, in terms of the amplitude i_0 at the wave front—

$$i = i_0\,\varepsilon^{-\frac{1}{2}\left(\frac{R}{Z_n} + GZ_n\right)x}$$

and similarly—

$$e = e_0\,\varepsilon^{-\frac{1}{2}\left(\frac{R}{Z_n} + GZ_n\right)x}$$

Put
$$a = \frac{1}{2}\left(\frac{R}{Z_n} + GZ_n\right)$$

then
$$i = i_0\,\varepsilon^{-ax}$$

$$e = e_0\,\varepsilon^{-ax}$$

Hence, power per unit length at a point distance x from the wave front—

$$p = ei = e_0 i_0 (\varepsilon^{-ax})^2$$
$$= e_0 i_0 \, \varepsilon^{-2ax}$$

Example.—A voltage wave of 10^6 volts at the wave front is established on a line having the following parameters—

$$R = 0{\cdot}45 \ \Omega/\text{mile}$$
$$L = 2{\cdot}21 \times 10^{-3} \ \text{H/mile}$$
$$C = 1{\cdot}385 \times 10^{-8} \ \text{F/mile}$$
$$G = 5 \times 10^{-9} \ \text{mho/mile}$$
$$\therefore Z_n = \sqrt{\frac{L}{C}} = \left(\frac{2{\cdot}21 \times 10^{-3}}{1{\cdot}385 \times 10^{-8}}\right)^{\frac{1}{2}} \simeq 400 \text{ ohms}$$

The resistance to a travelling wave will be very much greater than that to a current of normal line frequency, and we will assume that it is fifty times as great, giving R = 22·5.

$$\therefore a = \tfrac{1}{2}\left(\frac{22{\cdot}5}{400} + 5 \times 10^{-9} \times 400\right)$$
$$= \tfrac{1}{2}(5{\cdot}625 \times 10^{-2} + 0{\cdot}002 \times 10^{-3})$$

We see that for the particular values given the term due to leakance is negligible, and—

$$a = 2{\cdot}82 \times 10^{-2}$$

Suppose that $e_0 = 10^6$ volts

then $i_0 = 10^6/400 = 2{\cdot}5 \times 10^3$ amps.

$$\therefore e = 10^6 \, \varepsilon^{-2{\cdot}82 \times 10^{-2}x}$$
$$i = 2{\cdot}5 \times 10^3 \, \varepsilon^{-2{\cdot}82 \times 10^{-2}x}$$

Energy per mile length for the first mile—

$$= Li_0^2$$
$$= 2{\cdot}21 \times 10^{-3} \times 6{\cdot}25 \times 10^6$$
$$= 13{\cdot}8 \times 10^3 \text{ joules}$$

On the other hand, the power at the wave front is—

$$(10^6 \times 2{\cdot}5 \times 10^3) \times 10^{-3} = 2{\cdot}5 \times 10^6 \text{ kW.}$$

For the amplitude to be reduced to one-half we have—

$$0{\cdot}5 = \varepsilon^{-2{\cdot}82 \times 10^{-2}x}$$
$$\therefore -2{\cdot}82 \times 10^{-2} \times 0{\cdot}4343x = -0{\cdot}3010$$
$$x = 24{\cdot}5 \text{ miles}$$

It will have been obvious that the attenuation, calculated on the above basis, is decided by the value chosen for the resistance of the line to a travelling wave. This value depends, not only on the conductors themselves, but also on the shape and length of the travelling waves. A useful formula is given by Foust and Menger in the form—

$$e_0 = \frac{e}{1 - kxe}$$

$$\text{or} \quad e = \frac{e_0}{1 + kxe_0}$$

where x is in miles, and e and e_0 in kV.

The constant k is the attenuation constant of value

$$k = 0{\cdot}0006 \text{ for chopped waves}$$
$$k = 0{\cdot}0003 \text{ for short waves}$$
$$k = 0{\cdot}00016 \text{ for long waves}$$

In the numerical example we were considering a long wave, so that $k = 0{\cdot}00016$. Hence, according to this formula the distance over which the amplitude is reduced to a half is given by—

$$0{\cdot}5 = \frac{1}{1 + 0{\cdot}00016e_0x}$$

Since e_0 appears in the denominator, it follows that the attenuation depends on e_0 also. The greater the value of e_0, the smaller the distance x to produce a given attenuation. Thus, with $e_0 = 10^6$ volts $= 10^3$ kV., we have—

$$0{\cdot}5 = \frac{1}{1 + 0{\cdot}16x}$$

$$x = \frac{0{\cdot}5}{0{\cdot}08} = 6{\cdot}26 \text{ miles}$$

If we can take $e_0 = 10^5$ volts $= 10^2$ kV., then—

$$0{\cdot}5 = \frac{1}{1 + 0{\cdot}016x}$$

$$x = \frac{0{\cdot}5}{0{\cdot}008} = 62{\cdot}6 \text{ miles}$$

Thus the resistance per unit length to a travelling wave varies along the whole length of the wave, the resistance at any particular point at the instant the voltage is some specified value, being a function of that value. In other words, the resistance per unit length varies in space and in time so that it is practically impossible to assign a value to it.

Suggestions for Further Reading

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CHAPTER XVIII

PROTECTION AGAINST OVERVOLTAGES

OVERVOLTAGES in electrical systems may be caused by the following:

(*a*) *External Causes*.—These are mainly due to atmospheric conditions.

1. Direct lightning stroke.
2. Statically induced charges on the conductors due to the proximity of charged clouds.
3. Dynamically induced currents due to a lightning discharge which strikes the ground near a line. Such a discharge is called a "side," as distinct from a "direct," stroke.
4. Electrostatic charges imparted to a transmission line whose altitude above sea-level varies considerably along the route.
5. Electrostatic charges imparted to the line due to the friction of dust or fine snow blown past the conductors.

(*b*) *Internal Causes*.—Under this heading may be classed the various pressure rises due to switching operations, e.g. switching-in of a non-loaded line; sudden changes in the load; the very abrupt circuit interruption due to circuit-breaker opening under conditions of fault or severe overload; bad paralleling; arcing grounds, etc. It is also convenient to class under this heading the resonance effects which may be set up by the inherent capacitance and inductance of lines and apparatus.

The disturbances due to external causes may be in the nature of impulses, e.g. an almost instantaneous building up of voltage or current, followed by a comparatively long tail. Bewley points out that the great majority of lightning waves are of comparatively simple shape, and can be represented as the difference of two exponentials, thus:

$$e = E(\varepsilon^{-at} - \varepsilon^{-bt})$$

where a and b are constants which determine the shape. The shape used for most calculations is similar to that given in Fig. 18.1, and the designation of the wave is made dependent on the time p taken to attain maximum value, and the time q taken for the tail to fall to 50 per cent. of maximum value. Thus, suppose that p is 1 microsecond and q is 10 microseconds, then the wave is a 1/10 wave. In some cases external causes may result in oscillatory waves of very high frequency, say, several million cycles per second. On the other hand, those produced by internal causes have a moderately high frequency of the order of several thousand cycles per second.

Because of this multiplicity of possible types of overvoltage, it follows that any protective device which functions differently at different frequencies may give protection against one set of causes, but be quite useless against others. Again, the internal causes,

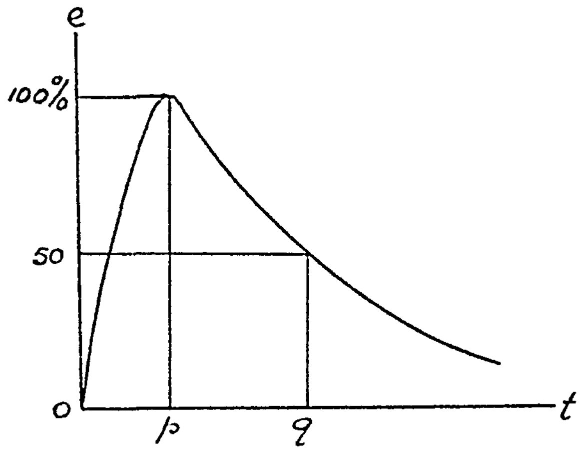


FIG. 18.1.

with the exception of arcing grounds, produce overvoltages between line and line, whereas external causes produce overvoltages between line and earth. Thus, different dispositions of the apparatus may be required for protection against internal and external causes.

Lightning

Lightning discharges have been classified by Sir Oliver Lodge into two kinds, namely, A and B strokes. The A stroke is produced by a charged cloud, which induces a charge on upstanding objects such as church spires, etc. This induced charge is distributed in such a way as to cause a concentration of potential at the upper end of the object, with the result that the electrostatic stress there is very great. This causes the air in the immediate neighbourhood to be ionised very rapidly, streams of charged particles being expelled from the pointed end. This produces a gradual lowering of the resistance of the discharge path between the cloud and the conductor until eventually the lightning discharge takes place, as in Fig. 18.2*a*. It will therefore be seen that this type of flash takes some time to produce, and when it does take place it will most probably be directed to the most sharply pointed and upstanding object in the neighbourhood.

The B stroke is a discharge which is induced by a previous A stroke. Thus, in Fig. 18.2*b*, if an A stroke takes place between the two charged clouds 1 and 2, a B stroke may be immediately produced between loud 3 and the conductor, there being no time lag associated with the B strokes. The B stroke, being an induced stroke, takes place, in fact, with absolute suddenness, and

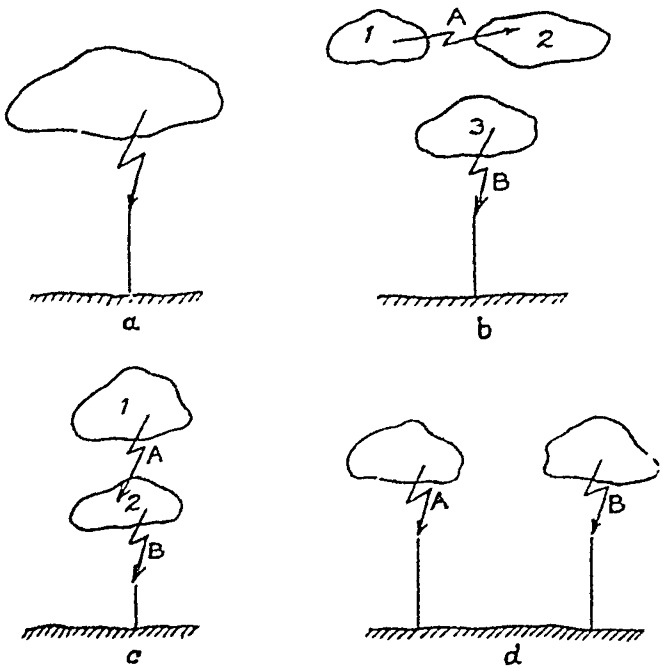


Fig. 18.2.—Production of Lightning Discharges.

it does not seem to obey any definite laws as does the A stroke. It may completely ignore a well-designed lightning conductor and shatter a neighbouring building, or it may strike the ground even if there are tall objects such as buildings or trees near by. The B stroke is as a result very much more dangerous than an A stroke as far as the protection of buildings or structures is concerned. The figures 18.2*c* and 18.2*d* show two other ways in which B strokes can be produced. In Fig. 18.2*c* an A stroke takes place between the clouds 1 and 2, thereby inducing a B stroke between the lower cloud 2 and the tall object. In Fig. 18.2*d* both the A stroke and the induced B stroke take place to earth.

If a transmission line is struck direct by either an A stroke or a B stroke, it is practically certain that considerable damage will be done. If the line is struck a long distance from a station or substation the overvoltage and associated current will flow along the line in both directions, shattering insulators and even wrecking poles until its energy is spent up in the paths to earth so formed. If it strikes the line immediately adjacent to a station then damage to plant is almost certain, since it is doubtful whether any ordinary arrester could divert to earth such a powerful discharge. Creighton states that an arrester could be designed to do this, but considers that it would not be good engineering practice, because most direct strokes take place sufficiently far from a station to be relieved locally, the extra interest on the bigger equipment therefore not being justified. Fortunately, direct strokes are of infrequent occurrence compared with side strokes, the effects of which are not so severe. Russell has computed that the pressure behind a lightning stroke may be anything up to one thousand million volts.*

Nature of the Lightning Discharge

A lightning discharge takes place when a cloud is raised to such a high potential with respect to earth, or to a neighbouring cloud, that the insulating property of the neighbouring air is destroyed. The generation of an atmospheric charge of electricity is considered to be due to the friction of wind on drops of rain, and this, in all probability, is the cause, or one of the causes, of the thunderstorms associated with a V-shaped barometric depression, in which there is an abrupt change in the direction of the isobars. In the neighbourhood of Great Britain V-depressions are often the southernmost extension of cyclonic barometric distributions. With strong winds, the wind direction is more or less along the isobars, and consequently a V-depression is a region in which a north-westerly air stream is approaching one from the south-west. The former will be cold and relatively dry, the latter warm and moist. In addition to gradient wind effects,

* An objection to the theory of a very high voltage between clouds and earth is the effect they would have on the velocity of fall of raindrops, which, as Simpson has shown (*Mem. Indian Met. Dept.*, Simla, 1910, pt. 8, and *Phil. Mag.*, 1915), often have a charge of as high as 6 e.s. units. With a potential gradient of 30,000 volts per cm. a force of six-tenths that due to gravity would act on each drop, thereby causing a great difference in the velocity. If directed vertically downwards, this force would therefore cause the drop to break up into smaller drops. Actually, the drops during a thunderstorm are larger than the average, but it must also be remembered that under an intense potential gradient the surface tension is diminished, thus increasing the tendency to coalesce when drops collide.

there will also be the raising of the south-westerly air, accompanied by adiabatic cooling and rain production. The vertical currents can be very strong, and the frictional effects of this motion—a component of the total velocity not directly due to the atmospheric gradient—is also an important factor in the generation of atmospheric electricity. In summer, with high atmospheric temperatures, an atmospheric gradient may be almost entirely absent, and electrification is then entirely due to upward currents. The violence of upward currents produced in this way is demonstrated by the laminated structure of hailstones, a structure due

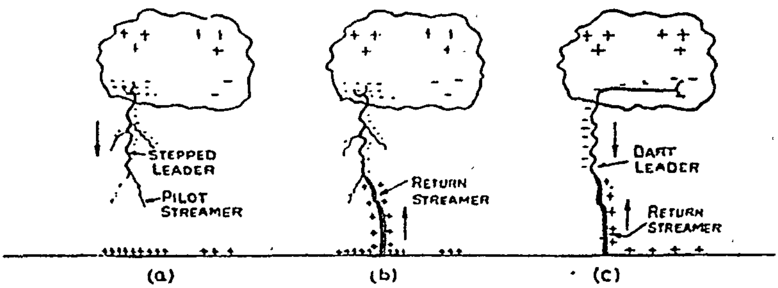


FIG. 18.3.—FORMATION OF A LIGHTNING STROKE.
(*English Electric Co. Ltd.*)

to alternate falling under gravity and lifting by the vertical current, until eventually the weight becomes too great for the process to be continued.

The first process in the actual lightning discharge is the production of a small corona discharge called a " pilot streamer," which detaches a small charge from the cloud and moves it towards the earth, or other body, to which the main discharge will ultimately take place. This pilot streamer ionises the air in its neighbourhood, and eventually there is sufficient destruction of the insulation to produce a path along which an arc, the " stepped leader," can take place. It is this which gives rise to the first visible phenomenon of the discharge. The stepped leader derives its name from its zigzag shape; it consists of a series of steps of about 150 feet in length. It is also associated with side streamers and retains the pilot streamer as a kind of spearhead. One stage in the process is illustrated in Fig. 18.3*a*, there being so far no discharge to ground. The path of the pilot streamer is a path of ionisation and therefore of complete breakdown of insulation, and consequently, when it reaches the earth, there is already a path from the earth for an arc called the " return streamer " (Fig. 18.3*b*). In the figure the under side of the cloud is shown as negatively charged, and therefore the induced charge on the earth is positive. It therefore follows that, in the conventional sense, the current directions in stepped leader and return streamer are the same. It is the return streamer which gives rise to what is called the lightning flash. With the resulting neutralisation of much of the negative charge on the cloud, any further discharge from the cloud may have to originate from some other portion of it. Such a discharge will, however, make use of the already ionised path, and consequently it will be non-branching and will be associated with a high current. It is called a " dart leader " (Fig. 18.3*c*). It is not invariable for a lightning discharge to have this multiple nature, but generally it is so, and as many as forty discharges have been recorded in a single stroke. The time interval between successive components may be anything from one-half to one-thousandth of a second.*

From the above description it will be seen that, in spite of the fact that there are discharges both from the cloud and from the earth, the complete discharge is, from the point of view of current flow, unidirectional in nature. Of strokes to earth, the only kind which concern the transmission engineer, 90 per cent. are due to negative electrification of the cloud.

Steinmetz has computed that the current in a lightning discharge may amount to 10,000 amperes, but from the melting effect of a lightning rod Humphreys estimates that the current may be of the order of 90,000 amperes. On the other hand, photographs taken of lightning flashes, and, later, cathode-ray oscillograms, have proved that the duration of the flash is exceedingly short, of the order of one-millionth to about one ten-thousandth of a second, from which it follows that although the power may be very great, the quantity of electricity and the energy dissipated will be small. Thus, suppose we take as representative values :

$$\begin{aligned}
\text{Voltage} &= 2 \times 10^8 \\
\text{Current} &= 4 \times 10^4 \dagger \\
\text{Duration} &= 10^{-5} \text{ sec.} \\
\therefore \text{ kW.} &= 2 \times 10^8 \times 4 \times 10^4 \times 10^{-3} = 8 \times 10^9 \\
\text{But kWH.} &= 8 \times 10^9 \times 10^{-5} \div 3600 \\
&= 22
\end{aligned}$$

* For the complete accounts of the most recent theories of the nature of the lightning discharge see

Allibone, T. E., and Schonland, B. F. J., " Branching of Lightning." *Nature*, 128, 1931, p. 794.

Meek, J. M., " The Electric Spark." *Journ. I.E.E.*, 89, Pt. I, 1942.

† The instantaneous current in a discharge may attain the order of 100,000A.

This amount of energy could be dissipated by any transmission line with ease if given sufficient time, and the danger of lightning thus consists in the exceedingly short duration of the discharge. Expressed in another way, the wave fronts of the travelling waves set up by lightning are very steep, this meaning simply that from the moment the wave reaches any given point in the line the voltage or current rises to its maximum value in an exceedingly short time. In this respect a direct stroke is more serious than a side stroke, the wave front due to the latter being the less steep.

Fig. 18.4 shows how a conductor may be statically charged by a neighbouring charged cloud. If the cloud is charged positively,

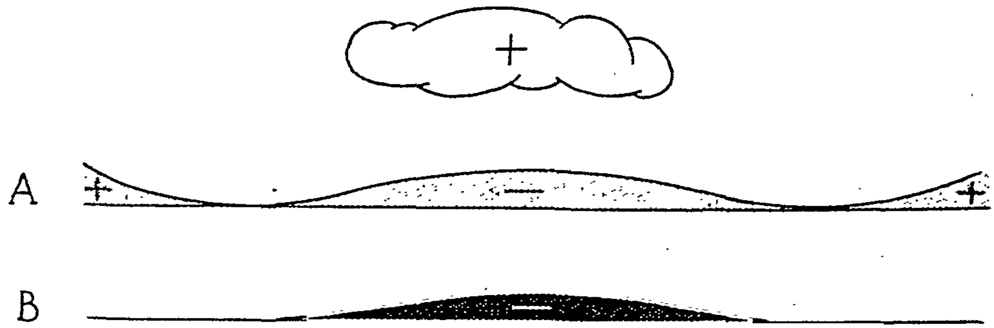


FIG. 18.4.—STATIC CHARGING OF THE CONDUCTORS OF AN OVERHEAD LINE.

as shown, the neighbouring section of the line will acquire an induced negative charge, while with perfect insulation a positive charge will be produced at the far ends of the conductor, as in Fig. 18.4A. Actually, owing to inevitable leakage, the positive charges are sure to leak to earth, leaving the negative charge in the middle bound, as in Fig. 18.4B. While this state of affairs lasts no travelling waves will be produced, but the line insulation in the neighbourhood of the cloud may be so severely stressed as to cause a spill-over, with resulting short-circuit on one or more phases if the neutral is earthed, and an arcing ground if the neutral is insulated. If the cloud quickly passes away, or it suddenly loses its charge, the induced charge on the line will be released and travelling waves will be produced. The wave front due to these may also, in some cases, be very steep fronted.*

The electrostatic charging of a line whose altitude varies considerably is due to the fact that the equipotential surfaces of the natural electrostatic field above the earth's surface are roughly

* The instantaneous voltage on the line may reach 2,000 kV., and this is sufficient to cause flashover on lines insulated for 220 kV.

parallel over plains, but have a tendency to crowd together in mountainous regions. As a result, a gradually ascending transmission line passes progressively from regions of low to regions of high atmospheric potential. It is unlikely that travelling waves can be set up in this way, but there may be important static increases in potential.

The static charges due to the friction of fine sand or snow are only of importance on low-voltage or telegraph lines. Thus, on the latter rises of 13,000 volts due to frictional effects have been observed.

Switching

We have seen that when an open-ended line is connected to a source of voltage, a voltage wave travels along the line, and that, on reaching the far end, it is reflected without change of sign, thereby producing voltage-doubling at that end. This reflected wave travels back to the supply end, giving rise to further reflections. Actually, owing to losses, the wave is attenuated and the line settles down to its normal voltage. If E is the R.M.S. supply voltage to neutral, then, because of voltage-doubling, the instantaneous voltage which can be attained under conditions of no attenuation is $2\sqrt{2}E$.

Now consider the case of opening a line which is carrying a current of I amps., R.M.S. Current waves of amplitude $-i$, according to the magnitude of the current at the instant of opening, will be generated in the line for the purpose of bringing the total line current to zero. These will travel along the line in both directions from the point at which the line is opened, and associated with them will be voltage waves of magnitude $-Z_n i$ in the forward direction and $+Z_n i$ in the opposite direction, the total voltage across the break thus being $2Z_n i$. Thus, if I = 200A, the break occurs at a moment of current maximum, and the natural impedance of the line is 500 ohms, the voltage across the break will be—

$$2 \times \sqrt{2} \times 200 \times 500/1000 = 141{\cdot}4 \text{ kV}.$$

If the rupture takes place in a switch, the sudden elimination of such a current is impossible because of arcing at the switch contacts. The possibility of trouble is more likely to be due to re-striking phenomena in the switch. Thus, consider the case of an open-ended line connected to an alternator; the current in the line will be the charging current. If the switch is opened for the purpose of disconnecting the line, an arc will take place between the contacts, and this arc will be extinguished at an instant of current zero. Since the current is a charging current, this instant will be one of maximum generator voltage E_{max}. After one half-cycle the voltage at the generator will be $-E_{max}$, but the line will have retained its potential distribution because of the line capacitance, with the result that the actual voltage at the switch will be $2E_{max}$. If, as the result of this, the switch contacts arc across, the open-ended line will be suddenly re-connected to a system which is now at $2E_{max}$ volts. Hence a travelling wave of amplitude $2E_{max}$ will be set up and, on reflection at the open end, the voltage attained will (neglecting attenuation) be of amplitude $4E_{max}$.

This is the worst case, and, in practice, the voltage $4E_{max}$ cannot be attained. It does, however, illustrate the possibility of serious overvoltage due to this cause. Air-blast breakers are more prone to restriking phenomena than oil-breakers owing to the slower rate of build-up of dielectric strength of the gap, in the absence of the gases generated in an oil-breaker. Furthermore, the air pressure (300 to 400 lb. per sq. in.) in an air-blast breaker is considerably less than the gas pressure usually set up in the arc-control device of a modern oil-breaker. The significance of this lies in the fact that the dielectric strength of air or of a gas increases as the pressure is increased.

Voltage oscillations can also be set up by what is called "current chopping" in a circuit-breaker during the process of opening of a comparatively small current. The pressure and the volume of air used for the purpose of arc extinction in an air-blast breaker are a constant for a given breaker, being related to its designed mVA rupturing capacity, and therefore the same whatever the value of the current to be interrupted. Hence, if the current is small, the blast, being designed for maximum short-circuit conditions, is able to extinguish the arc before the moment of natural current zero. This will cause a voltage rise of $2Z_n i$, as explained previously, but, in order that such rises may be prevented from attaining their prospective value, the gap between the contacts is kept relatively small. Thus the gap breaks down each time such a voltage rise occurs, a current is produced, and this, in turn, is chopped until, towards the natural current zero, the circuit is finally broken. These phenomena are illustrated in Fig. 18.5. The breaker under these conditions functions as a surge diverter, and the system is protected if the dielectric strength of the gap is less than the impulse breakdown voltage (see p. 525) of the system. In the case of the oil circuit-breaker, the volume and

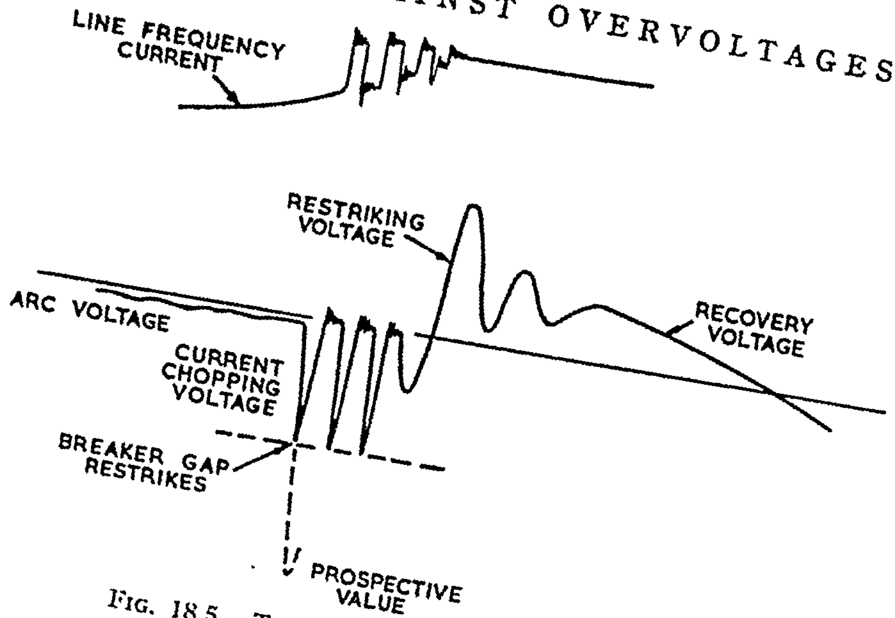


FIG. 18.5.—TO ILLUSTRATE CURRENT CHOPPING.

pressure of the gases generated by the arc, and utilised for arc extinction, depend on the value of the current, so that with small currents it is unlikely that the arc will be extinguished except at, or near, a natural current zero.

Insulation Failure

The failure of the system insulation may take place between the conductors of an overhead line or the cores of an insulated cable; or between one conductor, or core, and earth. The failure to earth is the more frequent. Suppose that a line is at an instantaneous voltage $2e$ with respect to earth, when there is a sudden breakdown of insulation to earth. The potential at the fault suddenly falls to zero and therefore a negative voltage, of amplitude e, and of very steep front, travels from the fault in both directions. The reduction of voltage to one-half is due to the equal dimension of the energy into electromagnetic and electrostatic energy. Each voltage wave is accompanied by a current wave of amplitude e/Z_n, as shown in Fig. 18.6, and consequently the current to earth, if there is an extremely low tower footing resistance—in the case of an overhead line—is $2e/Z_n$. The phenomena of reflection and attenuation are now governed by the principles previously described. Provided the voltage causing breakdown is the normal system voltage, the voltage waves set up may be serious only because of their steep wave fronts. The really dangerous condition is a breakdown of insulation due to an induced overvoltage set up by a lightning discharge or, of course, a direct stroke to the line. Protection against these is considered later.

Arcing Grounds

The previous discussion of insulation failure was based on the assumption of an earthed neutral. If the neutral is insulated—that is, the whole system normally unearthed—then the effects

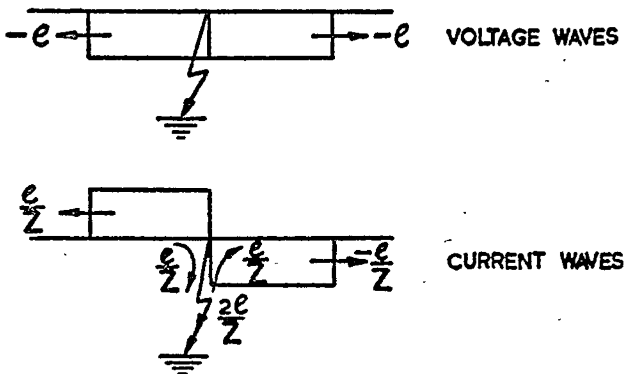


FIG. 18.6.—TRAVELLING WAVES SET UP BY A FAULT TO EARTH.

of a breakdown of insulation to earth are modified. Consider the circuit of Fig. 18.7, which represents an alternator connected to a line with distributed inductance, and distributed capacitance to earth. The only connection of the alternator windings to

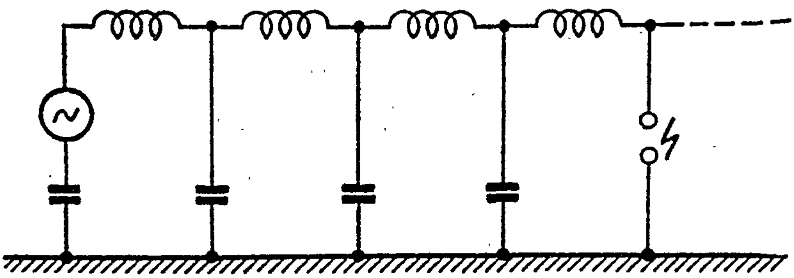


FIG. 18.7.—CASE OF THE INSULATED NEUTRAL.

earth is through its capacitance to earth, as shown. Let a spark gap be connected between line and earth. The alternating E.M.F. impressed on the circuit by the alternator will charge the line at each half-wave, and if the line P.D. becomes high enough the gap will break down and the circuit then resonate at its own natural frequency.

Similar conditions exist if an earth-fault develops on a transmission line connected to an all-insulated system. When a flashover occurs—as, for example, by the failure of an insulator, accidental contact with trees, and so on, or by the breakdown of a weak spot in the case of an insulated cable—the conductor is discharged through the agency of a transient high-frequency oscillation. The faulty conductor is thus brought to earth potential, with the result that the two other conductors, normally at phase voltage to earth in the case of balanced capacitances, are suddenly raised to the line voltage to earth.

When the line is discharged to earth, the gap opens, the line charges up again, and the cycle is repeated. In themselves, oscillations such as the above are not particularly dangerous, but, if continued, the resulting pressure rise may result in a breakdown of the insulation at some other point, and possibly on another conductor. In such a case a double fault to earth will be formed, and it will be equivalent to a short-circuit.

Surges and other transient phenomena due to switching operations, or taking place in such a way that the initiating voltage is the system voltage, are rarely dangerous in themselves, partly because attenuation may limit serious voltage rise, and partly because the associated energy is small. The danger is when a flashover may open a path for a power arc, much as the stepped leader of a lightning flash paves the way for the main discharge. An example is the double fault which may result from an arcing ground, as explained above.

Protection against Lightning Discharges and Other Atmospheric Disturbances

When the atmospheric conditions are those associated with lightning, cloud surfaces may be raised to potentials of several million volts with respect to earth, and consequently high static potential gradients may be set up. These may reach the value of 100 kV. per foot. A transmission line underneath such a charged cloud has induced on it an electric charge of opposite sign, as shown in Fig. 18.4, the distribution depending on the size of the cloud and the distribution of its electrification. The resulting increase in potential of the line may, in itself, be sufficient to cause arcing at an insulator, independently of a lightning discharge, or even of the production of travelling waves due, say, to a change in the distribution of charge.

The cloud may lose its charge (*a*) suddenly, due to a lightning stroke between positively and negatively charged portions of itself, or by a stroke to a neighbouring cloud or to earth; (*b*) slowly, by leakage to earth when there is rain. In the former cases the bound charge on the line is suddenly released and a travelling wave set up. As we have already seen, its energy becomes divided equally between magnetic and electrostatic energy, with the result that the amplitude of the travelling wave is one-half of the potential of the bound charge. If the discharge of the cloud were instantaneous, the shape of the travelling wave would be the same as that of the distribution of bound charge before release. Actually the time is finite although small, sometimes as little as one or two microseconds, and this time affects the shape of the wave front, the steepness decreasing as the time increases.

Ground Wire

The ground wire is a conductor run parallel to the main conductors of a transmission line supported on the same towers, and

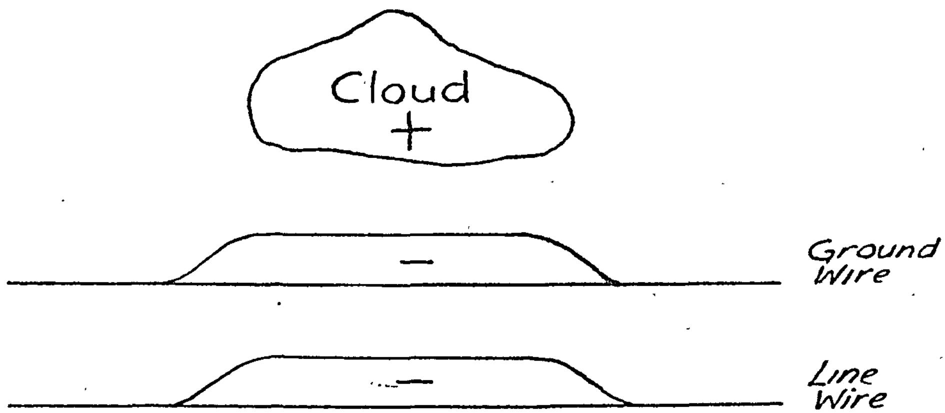


FIG. 18.8.—EFFECT OF A GROUND WIRE.

earthed at regular intervals. Sometimes the ground wire is run underneath the main conductors, but it is better above. Since a charged cloud will induce charges of the same sign on both line conductors and ground wire, as shown in Fig. 18.8, it is clear that the potential gradient in the neighbourhood of the lines is reduced, the magnitude of the resulting impulse thus being considerably reduced. In addition, if the ground wires are disposed correctly they will save the line from direct strokes.

The mechanism by which the line wire is protected is somewhat as follows: Assume a positively charged cloud, as in Fig. 18.8, and first of all no ground wire. A negative charge will be induced on the line, and in view of the small portion of line directly underneath the cloud, the actual charge will be largely obtained from the remote parts of the line. The potential of the line with respect to earth will be positive, its magnitude being equal to the potential due to the cloud above, less the potential due to the bound charge, if this could exist alone. With perfect insulation this condition would persist as long as the position of the cloud and its state of electrification remained the same. Because of leakage across insulators there will be a continual leakage of positive charge to earth so long as the potential difference between line and earth exists, and, as a result, initial negative electrification of the line is augmented and the line brought more nearly to earth potential. Owing to the very low value of the leakage conductance of a healthy line, this process is very slow, but if it could continue long enough it would ultimately bring the line almost to earth potential. If the cloud discharges, its field disappears, and the line then takes up the potential corresponding to its own charge, because this charge, previously bound, is not able to escape to earth instantaneously. There are two possibilities: (*a*) The immediate rise in line potential may, in the case of a low- or medium-voltage system, cause flashover at one or more places, and this will act as a safety-valve. (*b*) There may be no flashover, and therefore there will be the production of travelling waves, as explained previously. These also will not produce flashover, but if they reach terminal apparatus such as transformers before sufficient attenuation has taken place, they may cause damage to end-windings. This kind of danger is therefore increased by the proximity of the charged cloud to the line termination.

Now consider the effect of the ground wire, carried above the line wire and efficiently earthed at the towers or poles which carry it. A neighbouring positively charged cloud produces bound negative charges on both wires, as in Fig. 18.8. The ground wire is continuously at earth potential, and the line wire will, because of leakage across the insulators, tend to earth potential. Because of the introduction between the cloud and line wire of the ground wire's negative charge, the resultant electric field intensity at the latter due to the combined action of cloud and ground wire is reduced. Consequently, the negative charge on the line wire necessary to reduce its potential to zero is less than in the case of no ground wire. Thus, prior to any discharge of the cloud, the ground wire reduces the possibility of flashover, particularly in the case of low- and medium-voltage systems where it would not be economical to insulate for potentials much higher than the system voltage to earth.

Now consider the effect of a discharge of the cloud. The bound charge on the earth wire flows to earth so that the conditions are now similar to those without an earth wire except that the negative charge on the line wire is less. Thus, any travelling waves set up are of smaller amplitude. There is, however, an additional effect. When the earth wire has lost its previously bound negative charge, it will have induced on it by the charge on the line wire, a charge of opposite sign, the result of which will reduce the field intensity between line wire and earth. This will reduce still further (*a*) the likelihood of flashover, and (*b*) the amplitude of the travelling waves. In brief, the function of the ground wire is to reduce the potential appearing on a line wire just after a lightning discharge takes place but before the charges begin to travel along the line in the form of travelling waves.

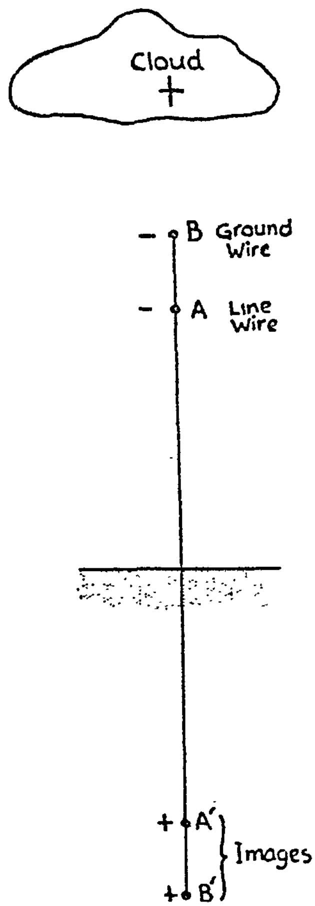


FIG. 18.9.

It is possible to calculate with reasonable accuracy the degree of protection afforded by a ground wire because (*a*) leakage reduces the static potential of a conductor almost to zero before the cloud discharge, (*b*) the height of the cloud relative to the height of the conductor above earth justifies the assumption of an electrostatic field which is vertical in the neighbourhood of the wire and of earth. Consider the case of a single line conductor and single ground wire, as shown in Fig. 18.9.

Let G = average potential gradient due to cloud between the conductor and earth.

q_1 = charge per unit length in the ground wire before cloud discharge.

q_2 = charge per unit length on conductor, also before cloud discharge.

h_1 = height of ground wire.

h_2 = height of conductor.

In terms of Maxwell's potential coefficients,* we have before the

* The introduction of the image changes at A′ and B′ in Fig. 28.9 is to facilitate the determination of the potential coefficients.

cloud discharges—

$$0 = Gh_1 + p_{11}q_1 + p_{12}q_2 \quad . \quad . \quad . \quad (1)$$

$$0 = Gh_2 + p_{21}q_1 + p_{22}q_2 \quad . \quad . \quad . \quad (2)$$

Immediately after the cloud has discharged, the charge q_2 on the conductor remains unaltered, but the charge q_1 on the ground wire becomes changed to some new value q_1'; also q_1' is now of opposite sign to q_2. The conductor will also have its electrostatic potential raised from the assumed value zero to V, and we now have—

$$0 = p_{11}q_1' + p_{12}q_2 \quad . \quad . \quad . \quad . \quad (3)$$

$$V = p_{21}q_1' + p_{22}q_2 \quad . \quad . \quad . \quad . \quad (4)$$

The solution of equations (1) and (2) is

$$q_2 = \frac{\begin{vmatrix} p_{11} & -Gh_1 \\ p_{21} & -Gh_2 \end{vmatrix}}{\begin{vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{vmatrix}}$$

$$= -G\left(\frac{p_{11}h_2 - p_{21}h_1}{p_{11}p_{22} - p^2_{12}}\right)$$

Equation (3) gives—

$$q_1' = -\frac{p_{12}}{p_{11}}q_2$$

Hence, from equation (4)—

$$V = p_{21} \times \left(-\frac{p_{12}}{p_{11}}q_2\right) + p_{22}q_2$$

$$= (p_{11}p_{22} - p_{12}^2) \times q_2/p_{11}$$

$$= G\left(\frac{p_{12}}{p_{11}} \,.\, h_1 - h_2\right) \quad . \quad . \quad . \quad (5)$$

If the ground wire is absent, $p_{12} = 0$ and the equation for V reduces to—

$$V = -Gh_2$$

The ratio of the conductor electrostatic potentials immediately after the cloud has discharged—(*a*) with ground wire, (*b*) without ground wire—is called the protective ratio (P.R.) of the ground wire. In the case considered its value is—

$$P.R = \frac{\frac{p_{12}}{p_{11}}h_1 - h_2}{-h_2}$$

$$= 1 - \frac{p_{12}h_1}{p_{11}h_2}$$

As a numerical example, let $h_1 = 50$ ft., $h_2 = 40$ ft., $r = 0{\cdot}25$ in.

$$p_{11} = 2 \log h \frac{2h_1}{r}$$

$$= 2 \log h \frac{2 \times 100 \times 12}{0{\cdot}25} = 2 \log h\, 4800$$

$$p_{12} = 2 \log h \frac{h_1 + h_2}{h_1 - h_2}$$

$$= 2 \log h \frac{90}{10}$$

$$\therefore \frac{p_{12}}{p_{11}} = \frac{\log h\, 9}{\log h\, 4800} = \frac{\log_{10} 9}{\log_{10} 4800} = 0{\cdot}259$$

$$\therefore \text{P.R} = 1 - 0{\cdot}259 \times \frac{50}{40}$$

$$= 0{\cdot}676$$

This ratio can be increased by increasing the number of ground wires, but it is not proportional to the number, the contribution of each additional wire decreasing very materially.

Early Forms of Protection.

It is of interest to review, very briefly, a few of the devices previously used.

1. *The Water-jet Arrester*

This was a continuous leak to earth and was naturally favoured at hydro-electric stations. An empirical rule was a leakage current varying from 0·25 amp. for a 4,400-volt system to 0·08 amp. for a 77,000-volt system. These give a power loss of 1·9 kW. for the 4,400-volt and 10·7 kW. for the 77,000-volt, and jet resistances of 10,200 and 555,000 ohms respectively. A representative natural impedance for a 77,000-volt overhead line is 800 ohms, and on comparing this with the jet resistance we see that the latter is so high as to be quite useless as a means of protection against travelling waves. To give any degree of protection the leak resistance would have to be reduced to such a valuc that the value of the annual loss of energy would run into at least four figures. Thus the water-jet arrester can only deal with slowly accumulating, not bound, static charges.

2. *Earthing Choking Coil*

This has the advantage that it takes little wattful power and there is no waste of water which, in any but a hydro-electric station, might be serious. To provide a high inductance with minimum copper in the windings the coil has a laminated core. The very low winding resistance can drain even rapidly accumulating charges, and this is the only possible function, as the high inductance offers a very high impedance to travelling waves.

3. *Condensers*

A condenser of low capacitance, connected between line and earth, will take a low current at line frequency and a high current at high frequency. Thus, if the line voltage to earth is 10,000 and $C = 0{\cdot}003\ \mu f$, the current to earth is 0·0096 amp. at 50 cycles and 96 amps. at 500,000 cycles. Thus a condenser is very efficient in relieving a line from the effects of high-frequency disturbances. Its behaviour with travelling waves is discussed on p. 552.

4. *Spark Gaps*

Spark or arc gaps were introduced because the three types of apparatus above, either singly or in combination, were unable to give protection against unidirectional travelling waves. These were of two kinds: (*a*) multiple gaps, (*b*) horn gaps. The former consisted of a series of metallic cylinders insulated from one another and separated by a small interval. For protection against overvoltages due to external causes the series was connected between line and earth, and against internal overvoltages in star formation between line and line. The metal, an alloy of zinc, was such that an overvoltage of sufficient magnitude would cause sparkover, but a power arc had difficulty in following because the metal was such that the supply of electrons by field emission was difficult, and thermionic emission was prevented by cooling due to a knurled surface. In spite of this, a power arc did sometimes follow unless a series resistance was included, and this limited the degree of protection against travelling waves. This was remedied to a certain extent by shunting some of the gaps, as shown in Fig. 18.10. When conditions are

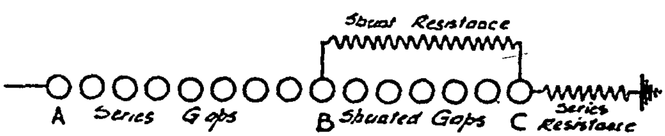


FIG. 18.10.—LOW EQUIVALENT ARRESTER.

normal, point B is at earth potential so that discharge will take place when an overvoltage is sufficient to break down the series gaps A to B. The impulsive rush of current following the breakdown will choose the straight-through path to earth via the shunted gaps B and C, instead of the alternative path through the shunt resistance, finally passing to earth via the small series resistance. When this current rush is over, the arcs B to C go out and any power current following the impulse is limited by the two resistances which are now in series. This current is too small to maintain the arcs in the gaps A to B, and normal conditions are thereby restored. This arrester has the property of extinguishing a current at a natural current zero and cannot, therefore, act as a generator of high-frequency oscillations.

The horn gap, illustrated in Fig. 18.11, consists of two horn-shaped rods separated by a small air gap and connected, one to the line and the other to earth, either with or without series

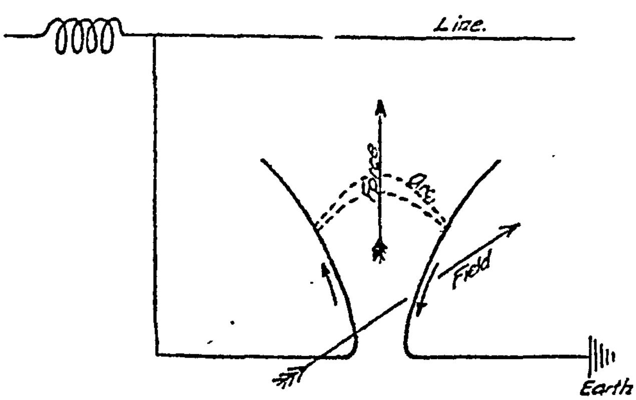


FIG. 18.11.—ACTION OF THE HORN GAP.

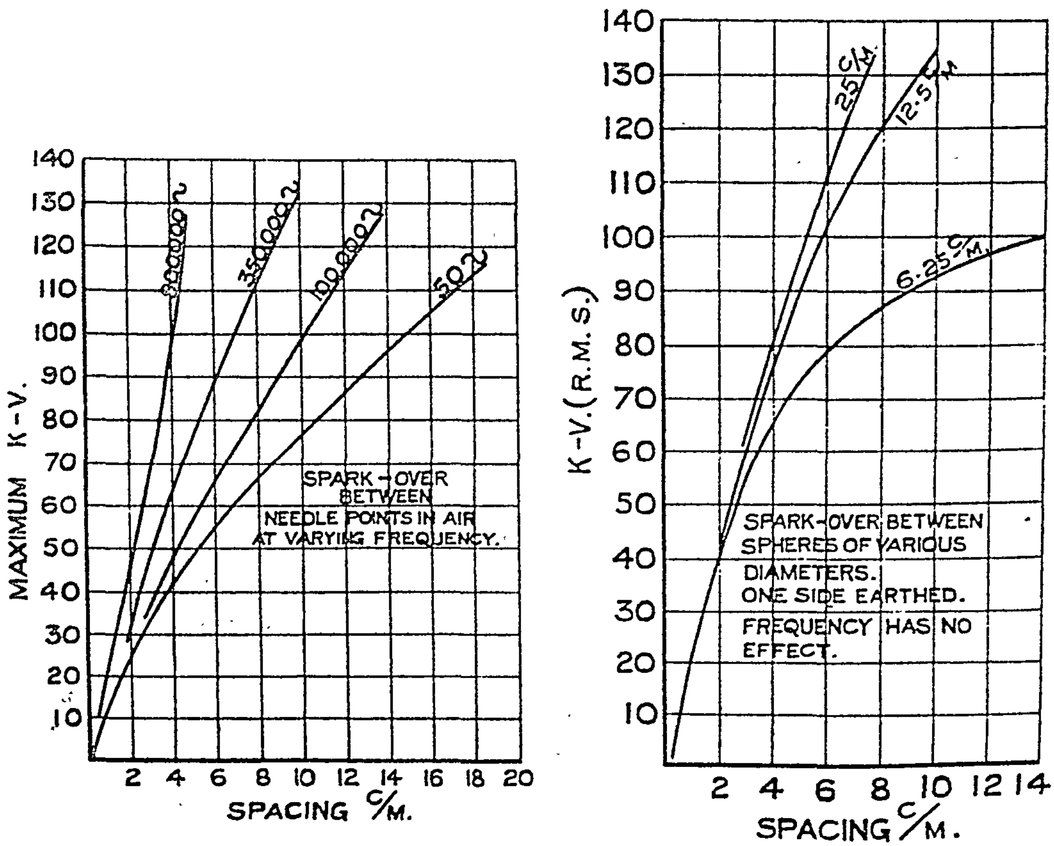
resistance. A choke is connected between the gap and the terminal apparatus to be protected in order to reflect travelling waves back on to the horns. When the voltage rises sufficiently to break down the gap, a discharge takes place and the disturbance is bypassed to earth. Dynamic current follows and the resulting arc, being a flexible conductor, rises under the influence of the electromagnetic forces and is eventually blown out. The main trouble with the horn gap is the time of operation: experiment made by the author on a gap 36 in. high operated at 10,000 volts gave an average time of 3 seconds. With very large currents and resulting large forces, the time would be reduced but, in view of the very short operating times of modern feeder protective gear, the time is far too long. It has therefore completely disappeared from important power lines. With an insulated neutral, the operation of a horn gap can constitute a particularly vicious kind of arcing ground.

Impulse Ratio

We have seen that an earth wire can reduce the potential to which an overhead line can be raised as the result of neighbouring atmospheric disturbances. This is the only function of the earth wire. The release of bound charges is always followed by travelling waves, and consequently other devices have to be adopted for protection against these. The breakdown voltage of any device may be a function of the time of application of the voltage, of its frequency if alternating, and of the shape of the wave if an impulse. The impulse ratio of any gap, or of any device, is defined as the ratio of breakdown voltage to an impulse of specified shape to the breakdown voltage at power frequency. The first portion of an impulse of steep front—namely, up to the point of maximum voltage—is analogous to the first quarter of a high-frequency wave, showing that, in a sense, the value of the impulse ratio is a measure of the effect of frequency on the breakdown voltage. The value of the impulse breakdown voltage for any gap or appliance is the peak value of an impulse of specified shape for which the number of breakdowns is 50 per cent. of the number of impulses applied. It is called the 50-per-cent. impulse flashover voltage.

Before any spark gap will operate, the air between the electrodes has to be ionised until the resistance is sufficiently reduced to allow the P.D. between them to cause breakdown. When it is realised that a travelling wave front has a velocity of about 186,000 miles a second, it will be seen that a lag of a very small fraction of a second may enable the wave front to pass before the gap can operate. Thus for a given shape and spacing of the electrodes an extremely high-frequency disturbance, or a steep-fronted impulse, may require a considerably greater voltage to produce breakdown than a power-frequency voltage. When the time is not limited, a fixed minimum voltage is required for the breakdown of a specified gap under specified atmospheric conditions. For a needle gap the impulse ratio is of the order 2 because of the comparatively long time taken to operate it, this being due to the building-up of a spherical corona on each electrode as a preliminary to discharge. A sphere gap, on the other hand, has an impulse ratio of unity, the breakdown voltage being independent of frequency. The difference between needle gaps and sphere gaps in this respect is illustrated very clearly by Fig. 18.12.

The conception of the impulse ratio is not confined to gaps which are normally expected to break down under certain abnormal conditions, but also to insulation and even windings of machines, which are expected not to break down. This is because an expenditure of energy is necessary to rupture any dielectric, whether gaseous, liquid, or solid, and energy involves time. During this time, the dielectric time lag, disruptive energy is supplied to the dielectric. Thus, if the P.D. across the gap increases at a very rapid rate, as in the case of an impulse, breakdown is not produced when the P.D. for breakdown corresponding



SPARKOVER VOLTAGES FOR NEEDLE GAPS.

SPARKOVER VOLTAGES FOR SPHERE GAPS.

FIG. 18.12.

to a slow increase in P.D. is reached. The P.D. has to rise above this value, and this excess voltage is greater (*a*) the greater the time lag of the discharge device; (*b*) the greater the rate of increase of the applied P.D. Electrodes giving a sensibly uniform field in the discharge path, such as spheres, have a minimum lag; needle gaps have a maximum lag.

Typical values of impulse ratio are as follows: sphere gap, unity; needle gap, 1·5 to 2·3, according to the frequency and gap length; suspension-type insulators, 1·2 to 1·6; pin-type insulators, 1·3 to 2·3. The impulse ratio of a gap of given geometry and dimensions is greater with a solid dielectric than with air. Insulators should be able to withstand a high impulse voltage without requiring a high factor of safety in relation to the working voltage of the line, and consequently a more economical design of insulator becomes possible if the impulse ratio is high. On the other hand, a surge diverter, formerly called lightning arrester, should have an impulse ratio as low as possible so that an incoming impulse may be bypassed to earth instead of passing on to the terminal apparatus and damaging transformer or machine windings.

An interesting example of the way in which the impulse ratio of a gap of given shape can be modified is afforded by the impulse horn gap used, at one time with electrolytic surge diverters. A sphere gap has an impulse ratio of unity but an arc, once formed, is not self-extinguishing. The horn gap is self-extinguishing but it has a high impulse ratio of 2 to 3 unless the setting is small, as with low voltages. When the gap is wide, its properties approximate to those of a needle gap. The impulse gap is a horn gap provided with spheres at the narrow portion (Fig. 18.13). A pointed electrode A is mounted between the main electrodes and is connected to one through condenser K_2 and the other through condenser K_1 in series with an unbalancing resistance UR. The pointed electrode is so adjusted that its position between the two main electrodes corresponds to the potential differences across K_2 and (K_1 + UR) at the normal line frequency. At very high frequency the P.D. across K_2 will be practically zero, thus concentrating the whole across the gap between A and S_1, which therefore breaks down, the rest of the gap between A and S_2 immediately following. Thus, initially an impulse ratio of unity is imparted by the spherical surfaces S_1 and S_2, then the arc is self-clearing because of the action of the horns. In the actual arrangement, ordinary line insulators are used for K_1 and K_2.

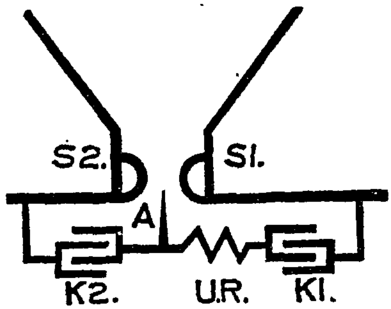


FIG. 18.13.—IMPULSE GAP.
(*Metropolitan-Vickers Electrical Co., Ltd.*)

The volt-time characteristics of gaps having one electrode earthed depend, to a certain extent, on the polarity of the applied impulse. This is illustrated by the characteristics of a rod gap

(Fig. 18.14), from which it will be seen that the breakdown voltage for a negative impulse is somewhat greater than for a positive. This disparity is not constant, but is influenced by the nearness of earthed metal, or of current-carrying conductors. For post

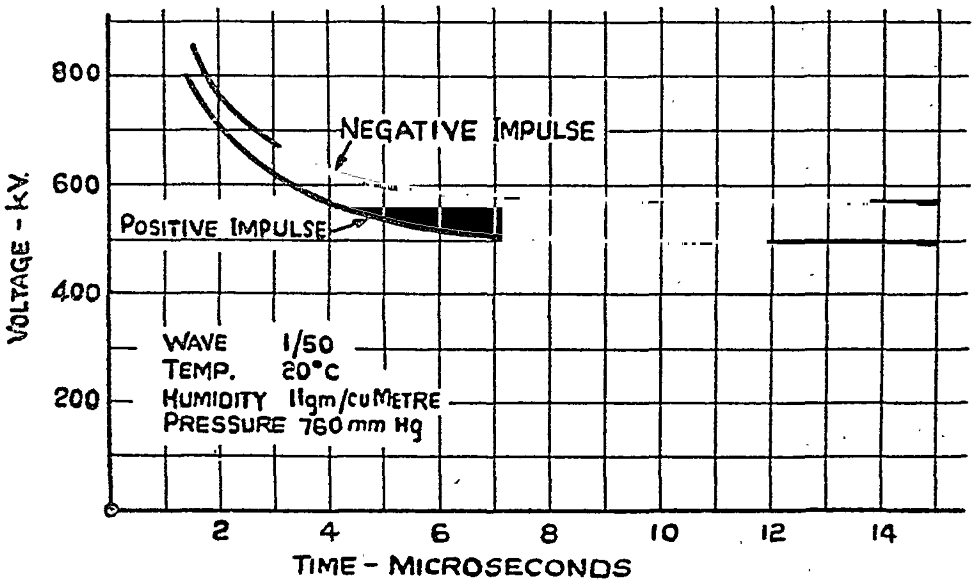


FIG. 18.14.—IMPULSE BREAKDOWN CHARACTERISTICS OF 30-INCH ROD GAP. (*English Electric Co. Ltd.*)

insulators the higher breakdown voltage is with a negative impulse, but in the case of suspension insulators it is the reverse, although the disparity between the two is smaller.

Methods of Protection

1. *The Peterson Coil*

This is a device for the protection of a system on which arcing grounds may be set up. It is essentially a linear reactor connected between the system neutral and the earth. It may be air-cored, in which case it is perfectly linear, or iron-cored: in the latter case the cross-section of the iron must be sufficient to prevent saturation. The inductance should be such that the resulting earth current at a fault is practically nil, thus suppressing the arc. This means that, in the event of an arcing ground, the coil will pass a reactive current equal in magnitude to the capacitance current which will flow from the two sound conductors to earth. The potential of each of these with respect to earth will, of course, be $\sqrt{3}V$ under fault conditions, where V is the phase voltage of the system. Let $L\omega$ be the inductive reactance of the coil and C the capacitance of each sound conductor to earth under the conditions imposed by the grounding of the other conductor. This capacitance can be calculated by means of the potential coefficients after the manner of the example on p. 520. The circuit diagram and vector diagram are shown in Fig. 18.15. It is required that—

$$0 = I_c + I_{23} + I_{13} = j\omega C(\overline{E_2 - E_3} + \overline{E_1 - E_3}) - \frac{j}{L\omega}(-E_3)$$

$$\therefore \omega CE(a^2 - a + 1 - a) + \frac{aE}{L\omega} = 0$$

$$-3a\omega C + \frac{a}{L\omega} = 0$$

$$L\omega = \frac{1}{3C\omega}$$

Note that in the vector diagram of Fig. 18.15 E_1 is taken vertically upwards for convenience. It is usual to provide tappings on the coil for tuning purposes, according to the amount of capacitance in the system. Again, if there are branch lines, and one or

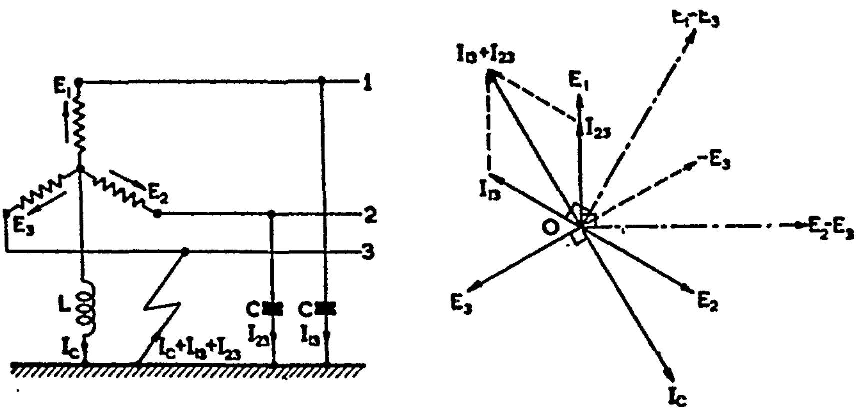


FIG. 18.15.—CIRCUIT AND VECTOR DIAGRAM OF THE PETERSON COIL.

more of these are disconnected, then the tapping must be changed to suit the new conditions. If, under healthy conditions, there is an unbalance in the voltages, this will send a circulating current through a local circuit in which the inductive and capacitance resistances are in series, their sum therefore being zero. Consequently, such unbalance may result in a heavy flow of circulating current. It is to be noted that the Peterson coil does not provide any protection in the case of a line-to-line flashover or a double line-to-earth flashover.

As an example of the order of inductance necessary, take the case of a 50-cycle system in which C is equal to 1·5 μF.

$$C = 1{\cdot}5 \times 10^{-6}\ \mathrm{F}$$

$$\omega = 314$$

$$\therefore\ L\omega = \frac{10^6}{3 \times 1{\cdot}5 \times 314}$$

$$= 704\ \Omega$$

$$\therefore\ L = \frac{704}{314}$$

$$= 2{\cdot}25\ \mathrm{H}$$

2. *The Surge Absorber*

This is a device for absorbing the energy content of a travelling wave. The energy dissipation, due to corona formation may act as a kind of natural safety valve in the case of travelling waves raising the line voltage above the corona level, and there have been cases in which a length of line adjacent to the terminal station has been worked at a lower critical corona voltage than the main length of the line. Another method is the installation of a length of steel conductor at the terminal end—say, one or two miles—the skin effect of the wire to steep fronted waves being equivalent to a great increase in the high frequency resistance. These are cheap compromises which give no protection against disturbances originating near the station. A better method is to terminate the line in a length of underground cable. This has three advantages: (*a*) the cable is shielded from all electrostatic phenomena, and from accidental earths of various causes; (*b*) travelling voltage waves entering the cable from the overhead line are reduced in magnitude to about one-fifth of their incident value because of the ratio of the natural impedances; (*c*) a travelling wave transmitted into the cable is attenuated rapidly because of the dielectric losses at high frequencies.

A more convenient application of the same principle is the insertion of a series resistance near the entrance to the terminal apparatus. A shunting inductance coil is then required to carry the load current, while acting to a steep wave front almost as an open circuit. An extension of this is the Ferranti surge absorber (Fig. 18.16). It consists of an air-cored inductor connected in series with the line and surrounded by, but insulated from, an earthed metallic sheet called a dissipator. For line voltages under 11 kV. it consists simply of a coil surrounded by a sheet-iron case; for voltages up to 33 kV. there are two or more disc-shaped coils with dissipators in the form of metal sheets placed between the coils; for voltages above 66 kV. the coil is disposed horizontally and is oil-immersed, the dissipator being a metal cylinder inside the oil.

The principle of all three forms is the same, viz. that the coil can be regarded as the primary of a transformer, and the dissipator as a short-circuited secondary of one turn. The energy of a surge is used up in the form of heat generated in the dissipator; firstly, due to the current set up in it by ordinary transformer action, and secondly, by eddy currents. The apparatus is used largely for the protection of transformers, and since there is distributed capacitance between coil and dissipator, just as there is distributed capacitance between transformer windings and earthed core, it is possible to regard the surge absorber as an extension of the transformer winding specially arranged to relieve the main winding from the initial very steep potential gradient. The makers state that a surge absorber will reduce the pressure rise across the end turns to 15 per cent. of the rise without the absorber.

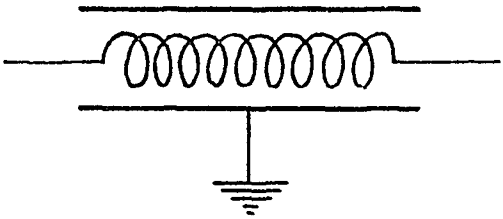


FIG. 18.16.—FERRANTI SURGE ABSORBER.

Surge absorbers of all types are analogous to a resistance insertion in the line, and their effect can be examined in the light of this analogy. The conditions after the arrival of a rectangular wave of voltage E and current I are shown in Fig. 18.17. The main portion of the line is of natural impedance Z_1 and the terminal portion Z_2. There is a reflected voltage wave E' with change of sign and current wave I' without change of sign. The transmitted current wave is I'' and this results in a uniform volt drop RI'' in the resistor. The relationships are thus—

$$E + E' = RI'' + E''$$

$$I + I' = I''$$

$$\therefore \frac{E}{Z_1} - \frac{E'}{Z_1} = \frac{E''}{Z_2}$$

whence

$$E'' = E \times \frac{2Z_2}{R + Z_1 + Z_2}$$

The maximum dissipation of energy occurs when RI''^2 is a maximum—

$$RI''^2 = R\left(\frac{E''}{Z_2}\right)^2 = \frac{4RE^2}{(R + Z_1 + Z_2)^2}$$

This approaches zero as R approaches zero or infinity, and is a maximum when—

$$R = Z_1 + Z_2$$

In the case of $Z_1 = Z_2 = Z$, say, the power of the oncoming surge is—

$$EI = \frac{E^2}{Z}$$

The power absorbed is $\frac{4RE^2}{(R+2Z)^2}$ and this is equal to $\frac{E^2}{2Z}$ if $R = 2Z$.

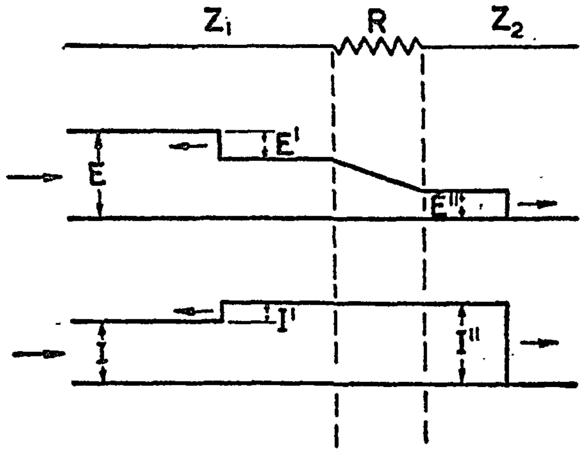


FIG. 18.17.—EFFECT OF AN INSERTED RESISTOR.

Under the conditions $R = 2Z$ the power absorbed is thus one-half of the power of the surge.

The transmitted voltage is—

$$E'' = E \times \frac{2Z}{R + 2Z} = \frac{E}{2}$$

The power of the transmitted wave is—

$\frac{E''^2}{Z} = \frac{E^2}{4Z}$ = one-quarter of the power of the incident wave. As one-half is absorbed in the resistor, this leaves one-quarter associated with the reflected wave.

3. *The Surge Diverter*

(*a*) Expulsion gaps

(*b*) Non-linear diverters

The surge diverter, often called a lightning arrester, consists essentially of a resistance with series spark gap connected between line and ground. This gap is usually set to break down at about a 50 per cent. overvoltage, and therefore the diverter does not commence to absorb the energy of the surge until the gap has actually flashed over. Owing to dielectric spark lag the surge will usually have passed some distance along the line before breakdown. Thus, if the lag is 3 microseconds, the front will have travelled—

$$186{,}000 \times 3 \times 10^{-6} = 0{\cdot}56 \text{ miles}$$

before breakdown.

At the moment of breakdown, travelling waves are set up in both directions, as shown in Fig. 18.18. Let the natural impe-

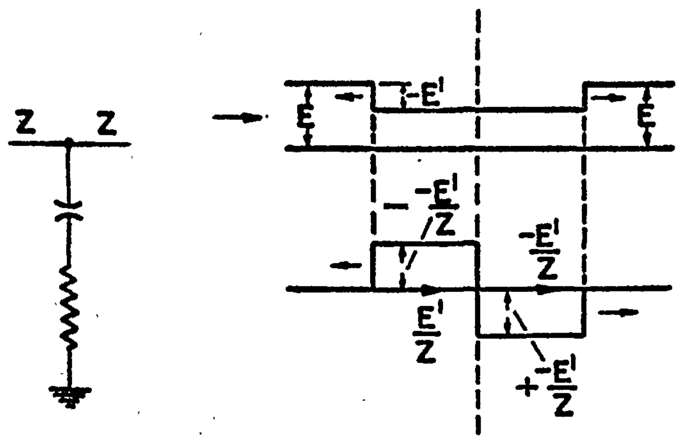


FIG. 18.18.—ACTION OF THE SURGE DIVESTOR.

dance of the line be Z and the resistance of the diverter R, and let there be an overpotential E which causes the gap to break down. If I is the current to earth, the line voltage to earth immediately falls to RI, and a negative voltage wave E′ now travels in both directions along the line. The conditions are—

$$E' = E - RI$$

$$I = \frac{E'}{Z} - \left(-\frac{E'}{Z}\right) = \frac{2E'}{Z}$$

$$\therefore E' = E - 2R \cdot \frac{E'}{Z}$$

$$= \frac{E}{1 + \frac{2R}{Z}}$$

and $$I = \frac{2E}{Z + 2R}$$

We see that if $R = 0$ the line voltage is immediately reduced to zero, but travelling waves of amplitude E, the value of the disturbance voltage, are set up. If R is large, the travelling waves are reduced in amplitude, but there is no marked reduction in the line potential with respect to earth. Thus an intermediate value of R is desirable, so that the line will be relieved of part of the excess voltage without setting up unduly high amplitudes in the travelling waves. This resistance R includes, of course, the tower footing resistance.

From results of tests made on lines equipped with expulsion gaps as surge diverters (see below), Bewley draws the following conclusions regarding the value of the tower footing resistance:

1. On lines equipped with expulsion gaps at every tower, grounding resistance is of little consequence—except that excessively high resistance may precipitate flashovers at so many gaps that the available follow current per gap may not be sufficient to interrupt the fault current.

2. On lines having expulsion gaps on some poles and not on others, in any alternating sequence, the grounding resistance, conductor spacing, insulation, and length of span become of paramount importance in reducing flashovers at unprotected poles. Only in the case of low ground resistance, high insulation, and short spans can an unprotected pole be justified.*

(*a*) *The expulsion gap.*—This consists essentially of a rod gap, in air, in series with a second gap which is enclosed within a fibre tube (Fig. 18.19). In the event of a sufficiently high overvoltage, both gaps break down simultaneously and, because of the low arc impedance, the current is limited only by the tower footing resistance. The internal arc vaporises a small portion

* There is a minimum current at power frequency below which an expulsion gap is unable to interrupt the follow-through current. For this reason the tower footing resistance should not be of such value as to prevent the current from attaining the necessary value for operation. The *desirability* of an expulsion gap at each tower is twofold: (*a*) Great differences in voltage can occur even on adjacent towers; (*b*) there is the possibility of a direct stroke at any tower.

of the material, the gas thus produced being an un-ionised mixture of water vapour and decomposition products of the fibre. This gas drives out the air ionised by the arc, with the result that when the follow-up current passes through its zero, the arc path is de-ionised. By the time the line voltage returns to normal, the space between the electrodes has recovered its insulating properties and no follow-up current can be re-established. This follow-up current may become very high, but as its duration does not exceed one or two half-cycles, and is generally less than a single half-cycle, no network disturbance is produced. Thus, for use

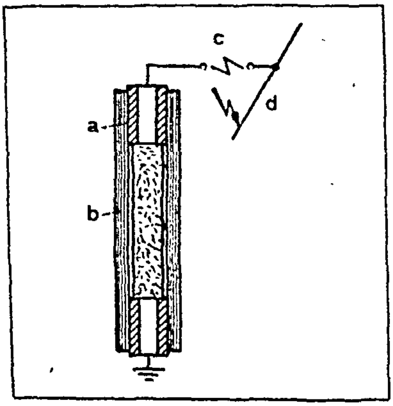


FIG. 18.19.—DIAGRAM OF EXPULSIVE GAP.

a. upper electrode.
b. fibre tube.
c. external gap.
d. line.

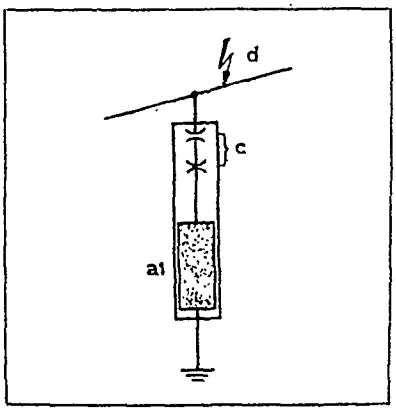


FIG. 18.20.—DIAGRAM OF NON-LINEAR DIVERTER.

a1. non-linear resistor.
c. divided spark gap.
d. line.

(*A.C.E.C.*)

on 230 kV. lines the current rating may be as high as 7,500 amps. Since the gases generated have to be expelled, one of the electrodes is hollow and the diverter is open at its lower end.

(*b*) *The non-linear diverter.*—This type of diverter consists essentially of a divided spark gap in series with a resistance element having non-linear characteristics. The gap breaks down on the arrival of an overvoltage, and it must then reopen in order to prevent the follow-up of dynamic current when the overvoltage is passed. The arrangement is shown diagrammatically in Fig. 18.20.

The ideal characteristic for the resistance elements would be RI = a constant, since such a material would maintain constant voltage by changing its resistance in inverse proportion to the current. The material used, which goes by such names as

" thyrite " and " metrosil," is a hard ceramic substance in the form of cylindrical blocks composed of small crystals of silicon carbide bound together by means of an inorganic binder, and the whole subjected to heat-treatment. The non-linear characteristic is attributed to the properties of the electrical contacts between the grains of silicon carbide. For standard thyrite discs 6 in. diameter and $\frac{3}{4}$ in. thick, the characteristics are of the type—

$$R = 580\, I^{-0{\cdot}72}$$
$$E = 580\, I^{-0{\cdot}28}$$
$$I = 1{\cdot}365 \times 10^{-10}\, E^{3{\cdot}57}$$

The factor 580 is decided by the grain characteristics, and also by the dimensions of the block, the mixing of the ingredients, and the heat treatment. The exponent 0·28 in the equation for E is also

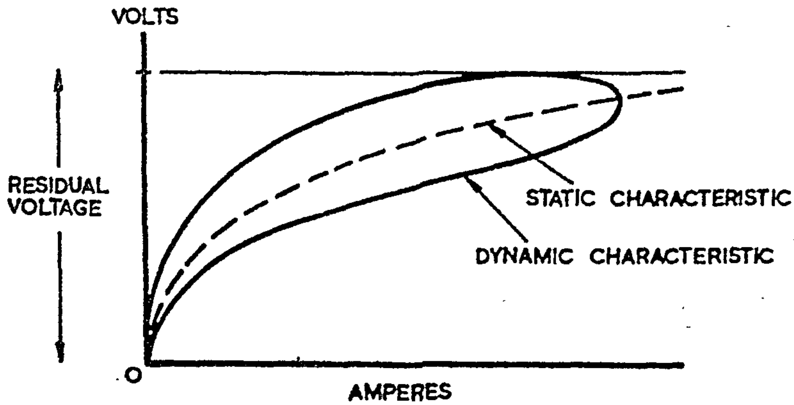


FIG. 18.21.—VOLT-AMPERE CHARACTERISTICS OF NON-LINEAR SURGE DIVERTER.

a function of the type and dimensions of the silicon carbide grains. It can be given different values, but it should never be less than 0·2 because it is necessary that the application of a surge should cause neither flashover nor puncture of the blocks. The low value of this exponent gives the smallest possible voltage for high current values, with resulting low value of the residual voltage.* Fig. 18.21 gives the volt-ampere characteristics of a non-linear resistance of the required type, the dotted curve being the static characteristic, and the closed curve the dynamic characteristic corresponding to the application of a voltage surge. If a horizontal tangent is drawn as shown to the dynamic charac-

* Complete particulars of these materials are given in Ashworth, F., Needham, W., and Sellars, R. W., " Silicon Carbide Non-Ohmic Resistors." *Journ. I.E.E* 93, Pt. I, 1946.

teristic, its intercept with the voltage axis gives the residual voltage. This voltage is defined as the crest value of the voltage appearing between the terminals of the surge diverter at the time of the discharge of a surge current wave. It varies from about 3 kV. to 6 kV. according to the type—i.e. whether station or line, the discharge current, and the rate of change of this current.

The spark gaps are designed to give the surge absorber arc impulse ratio of unity: as a consequence they are unable to interrupt high values of current, for which reason the follow-up current must be limited to 20 to 30 amps. The resistance units must therefore be arranged so as to give this characteristic. The ideal characteristic curve is one which rises very rapidly and then becomes horizontal but, so far, no material having such a property has been discovered.

The impulse breakdown voltage of a non-linear surge diverter is of very short duration and is generally less than the residual voltage. As a result, from the point of view of coordination o the insulation, it is the residual voltage which decides the protec tion level.

Although the follow-up current has to be limited to such a low value, the maximum value of the surge current which the diverter can pass without damage to itself is very high; up to 100,000 amps. for a station type and 65,000 amps. for a line type. The passage of smaller currents of, say, 100 to 1,000 amps. for longer periods than the normal duration of a heavy surge current submit the elements to more severe stress, and for this reason it is essential that breakdown should not occur with overvoltages due to internal causes and of amplitude insufficient to endanger the system insulations. The function of the diverter is the protection of the insulation against overvoltages high enough to be dangerous, and for this reason the breakdown voltage at system frequency is made greater than 1·8 times the nominal voltage.

The construction of a typical non-linear surge diverter is illustrated in Fig. 18.22*a*. The description is as follows: The discs are $3\frac{1}{2}$ in. diameter and 1 in. thick. The spark gap is a multiple assembly consisting of a number of electrodes in series, with fixed gap spacing between each pair. The electrodes are electro-tinned copper pressings, of disc formation with a semi-toroidal sparking surface. A mica disc provides insulation between the electrodes, and the resulting capacitance between each pair of electrodes grades the voltage evenly across the gap assembly as a whole. The discs and gaps are housed in a porcelain container, and, to rectify any lack of uniformity of voltage distribution among the large number of units required for a high-voltage diverter, a grading ring is used.

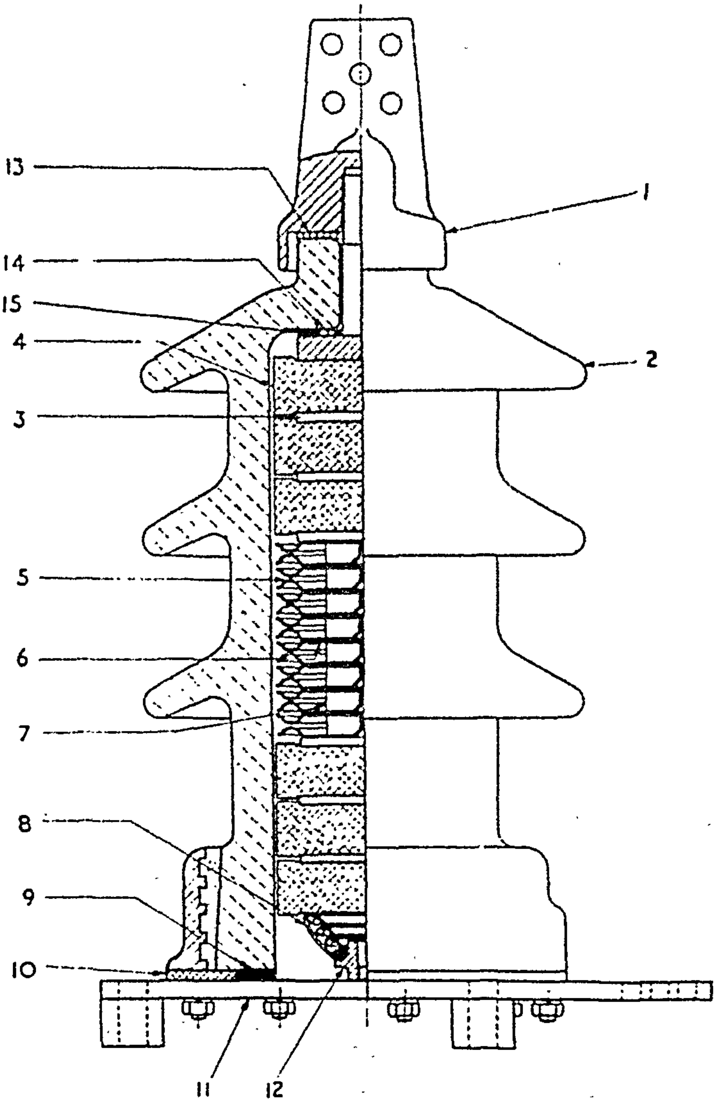


FIG. 18.22*a*.—NON-LINEAR SURGE DIVERTER.
(*B.T.H. Co. Ltd.*) or (*Metropolitan-Vickers Electrical Co. Ltd.*)

It is essential that no moisture shall enter. A leaking diverter may remain in service for some time provided it is not called upon to operate. On the occurrence of the first surge it will probably be destroyed, although, in the process, it will give protection against that one surge.

Diverters for very high voltages are of considerable physical is illustrated in Fig. 18.22*b*. This is the unit from which a high voltage diverter consisting of several in series is built up, and it will be seen that the non-linear element is in the form of a cylinder surrounding the spark gaps.

Comparison of the Two Types

The series gap of the expulsion-type diverter is an isolator for normal line conditions. If the diverter is called upon to operate, the impedance of the series gap is very low: that of the arc in the tube is also very low, with the result that the follow-up current is almost the same as that due to a connection of the line to earth,

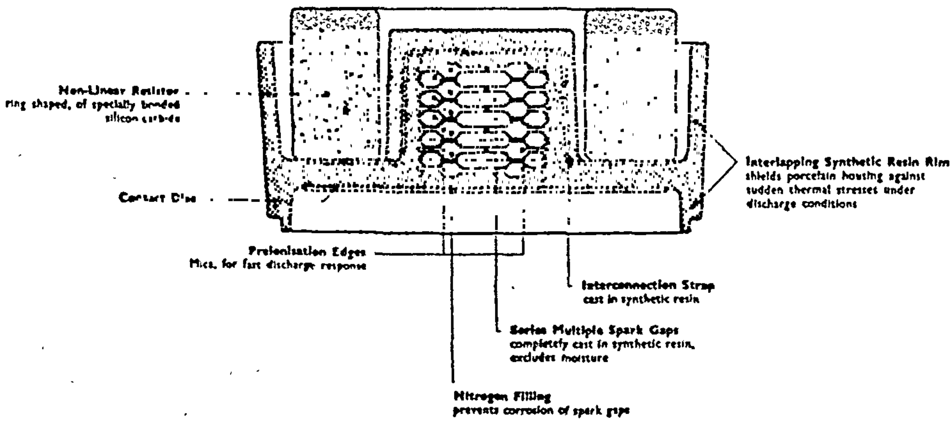


FIG. 18.22*b*.—ONE UNIT OF NON-LINEAR SURGE DIVERTER. (*Oerlikon.*)

its value therefore being decided by the factors used in the calculation of fault currents. Thus this current may become as high as several thousand amperes. Again, since the follow-up current is decided by the network under fault conditions, it is a reactive current, and passes through a zero value when the system voltage is a maximum. There is no phenomenon analogous to residual voltage, as the arc voltage, at the moment of breakdown, falls rapidly to a very low value. As a result, the level of protection is fixed solely by the value of the impulse breakdown voltage. Because of the small amount of de-ionising gas produced by it, the expulsion gap has difficulty in extinguishing currents of low value.

As we have seen, the subdivided series gap of the non-linear diverter, although an isolator during normal line conditions, actually plays a part in the diverter performance during operation.

The non-linear resistors limit the current to a comparatively low value, and this current is practically in phase with the system voltage. Thus a moment of current zero is also a moment of low system voltage, the subdivided gap thus interrupting this current at the first zero. Since the impulse breakdown voltage is usually less than the residual voltage, it is the latter which decides the level of protection given by the non-linear diverter.

J. S. Cliff* states that only the surge diverter gives consistent protection, for which reason its characteristics are taken as a basis for determining insulation requirements for apparatus exposed to lightning overvoltages. For systems up to 52 kV., the smallness of the substations and apparatus permit the installation of the surge diverter close to the protected apparatus. The insulation level can be based on the residual voltage of the diverter at a current of 20 kA. Above 52 kV. the substations are larger, the diverter may be a considerable distance from the protected apparatus, and there may be some voltage drop in the earth connection. Large currents will seldom exceed 5 kA., since the lines are usually shielded. To allow for these factors, the protection and insulation levels are based on the formula—

$$\text{Protection level} = 1{\cdot}15 \times (\text{residual voltage at 5 kA.}) + 30 \text{ kV.}$$

Location of Protective Apparatus

It will be obvious from the description of their action that surge absorbers of all kinds are installed at a line termination adjacent to a station. The case of a station with rotating machinery is different from that of the transformer station because machine windings often have several turns per slot, the inter-turn insulation strength therefore being relatively low. In such a case it is necessary, not only to reduce the amplitude of a voltage wave but also to reduce the shape of the front. The figure accepted is a 10 microsecond delay to the wave crest and this can be secured by an insulated cable termination, or, if this is not possible, by provision of additional capacitance to earth. In such a case the function of the surge absorber is that of reducing the slope of the wave front, and surge diversion is essential in addition.

The expulsion gap is not to be regarded as a competitor to the non-linear diverter. Its function is the protection of the line as a whole, and from the opinions of Bewley, reproduced above, it would appear that, unless the tower footing is low, there should

* "The co-ordination of insulation of high-voltage electrical installations." *Journ. I.E.E.*, 101, Part I, 1953, p. 2197.

be gaps at each pole or tower in localities where the probable number of thunderstorm occurrences per annum warrant this type of protection.

The non-linear surge diverter is an appliance used mainly for the protection of apparatus, not lines. The stations should, in

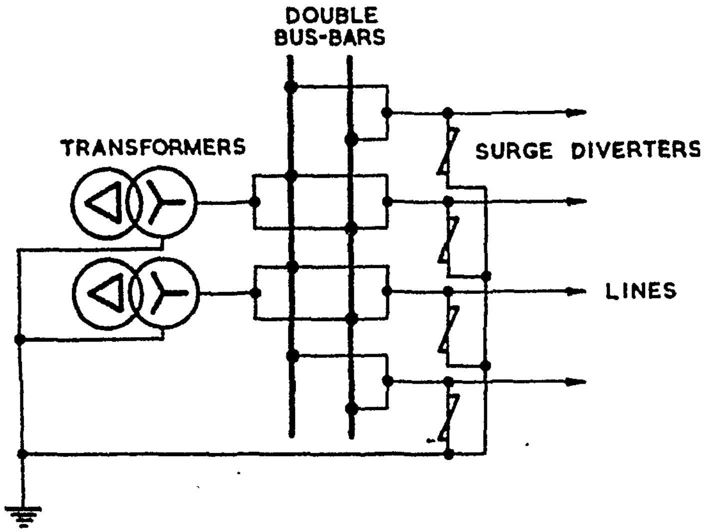


FIG. 18.23.—LOCATION OF SURGE DIVERTERS.

addition, be protected from direct lightning strokes by suitable ground wires. Obviously there are many possibilities and Fig. 18.23 shows a substation with diverters fitted to each feeder. In the case of a station with rotating machinery, the installation

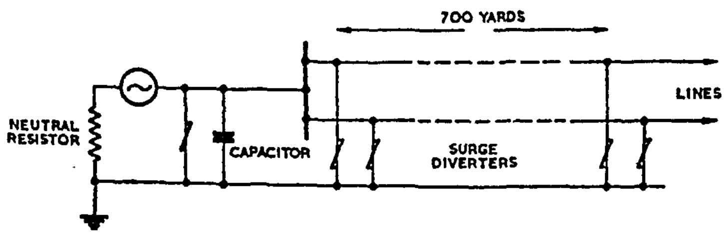


FIG. 18.24.—PROTECTION OF A STATION HOUSING ROTATING MACHINE.

will consist of (*a*) surge absorbers to reduce the slope of the wave front, (*b*) surge diverters to by-pass the surge to earth. A possible example is given in Fig. 18.24, in which the surge absorber consists of a capacitor to earth. In such a case it is advisable to have additional diverters 700 yards away from the station and also efficient ground wires. This is to control the rate of charging of the capacitors and to prevent waves reflected from them reaching the surge diverters and interfering with their discharge.

The Earth Connection

The earth connection of a surge diverter is not called upon to carry current of normal line frequency, its function being the dissipation to earth of current impulses of hundreds, of even thousands of amperes. It therefore by no means follows that low resistance to small steady-state currents affords a criterion for judgment of the performance under impulse conditions. Tests carried out in America have shown that earth connections have a dynamic characteristic in the form of a loop, but that it is associated with a mean value more nearly linear than the static characteristic of a non-linear surge diverter—i.e. it is not of the type shown by the dotted characteristic of Fig. 18.21. Consequently, there is no phenomenon like that of the residual voltage. One set of tests showed the initial slope of the characteristic to correspond with the measured resistance to power frequency current. The resistance at maximum voltage and current was found to be, on an average, about 60 per cent. of the power-frequency value. The greatest difference was found with high-resistance earth connections: with low resistance the difference between the two was found to be small. The lower value of resistance to impulse current was found to apply to all classes of soil, with or without special treatment—e.g. watering with brine—and for all shapes of electrodes.

Co-ordination of the System Insulation

The co-ordination of the insulation of the various parts of a system is somewhat analogous to the grading of the fuses or other current-limiting devices in the overall picture of protection against overcurrents. If there is an overcurrent in a subsidiary circuit, then it is essential that this circuit only shall be disconnected, and not the complete system. Similarly with protection against overvoltages. It is obvious that the over-insulation of one part of the system may, in the event of serious overvoltage, result in the breakdown of the insulation of some vital, and perhaps inaccessible part. Thus, it is preferable that line insulators should flash over rather than the bushings of transformers should break down. Again, it is preferable that a bushing should break down before the insulation of the transformer of which it forms a part. To take two simple examples: In the case of insulator strings, destruction of the string can be prevented by the provision of arcing horns and rings, while in the case of transformers, rod gaps, called co-ordinating gaps, can be installed to protect the apparatus (Fig. 18.25).

Co-ordination is carried out in the insulation design of the transformer itself. The bushings and leads have considerable capacitance to earth, and the windings themselves possess a distributed capacitance to earth. Consequently, an incident

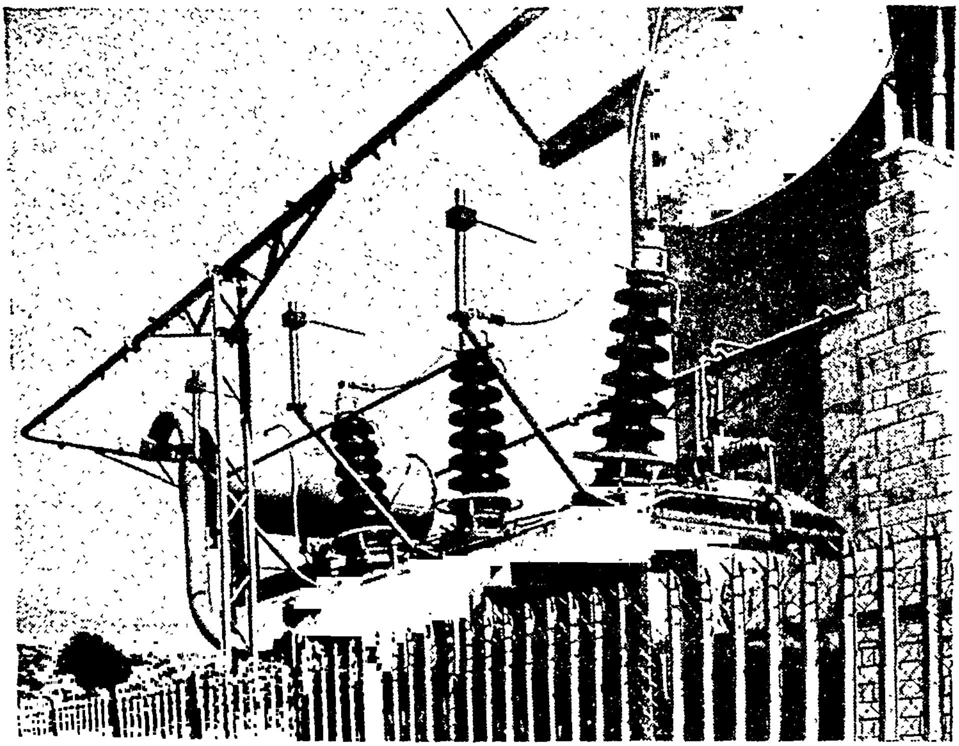


FIG. 18.25.—CO-ORDINATING GAPS MOUNTED ON A TRANSFORMER.
(*English Electric Co. Ltd.*)

travelling wave of voltage must charge up the capacitance it meets first of all before passing on to more remote portions. This modifies the shape of the wave front, the energy required to charge the successive capacitances being progressively abstracted from that of the wave, whose energy and potential are therefore reduced. The initial voltage distribution throughout the winding is a curve of the form shown in Fig. 18.26, from which it will be seen that the greater the value of a where—

$$a = \sqrt{C_g/C_s}$$

where C_g = capacitance per unit winding length to ground,

C_s = series capacitance per unit winding length, the greater will be the initial voltage concentration at the end-turns of the transformer. The straight line $\alpha = 0$ represents the case for $C_g = 0$, and also the steady state distribution. During the passage of the wave there is a transition from the initial curve to the straight line case, and this transition may be accompanied by severe oscillations in voltage.

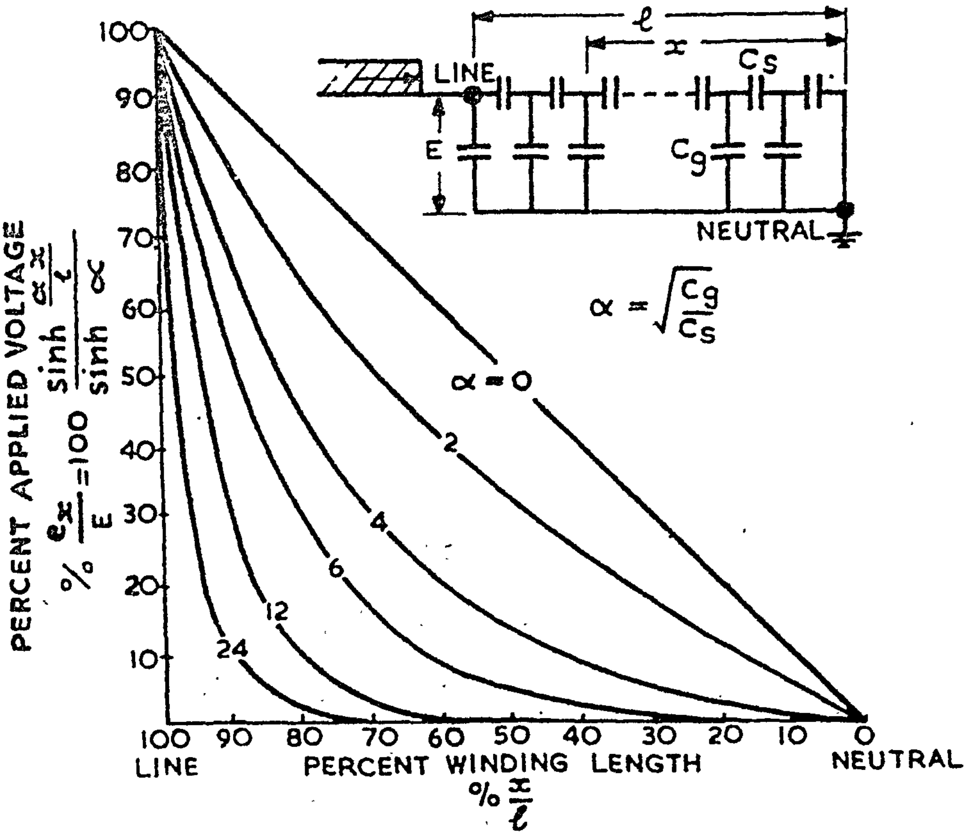


FIG. 18.26.—INITIAL VOLTAGE DISTRIBUTION AS A FUNCTION OF CAPACITANCE. Note the voltage concentration at the line-end of the windings for the higher values of α.

(*B.T.H. Co. Ltd.*)

The cost of high-voltage equipment is very high, and the insulation is responsible for a large portion of this. Transformers are particularly costly, and for these it is financially advantageous to reduce the insulation level on the equipment to be used in cases of a system voltage of 88 kV. and above. For outdoor type oil circuit-breakers and for transformers, the British Standards " impulse withstand " voltages are as follows:

Rated service Voltage. kV.	Impulse Withstand Test Voltage.	
	Non-effectively Earthed System. kV.	Effectively Earthed system. kV.
22	150	150
33	190	190
44	260	260
66	330	330
88	450	450
110	550	450
132	630	550
165	730	630
220	1050	890
275	—	1050

Where a reduced insulation level is used, the usual practice is to adopt the next lower insulation level. Thus a 132-kV. transformer with reduced insulation level would have a 550-kV. impulse level, corresponding to 100 kV. full insulation level. Gaps used for co-ordination are normally set to have an impulse flashover voltage of about 80 per cent. of the transformer impulse withstand level.

In some cases the number of units on a few insulator strings near the station have been reduced so as to provide a discharge path in a readily accessible position. It is clearly desirable to limit the length of surge which can enter terminal apparatus, and this length is decided by the distance between the surge diverter, of whatever form, and the apparatus. This is because the doubling effect of the incident voltage on reflection is of duration equal to the time taken for the reflected wave to travel back to the surge diverter and thence return to the apparatus.

The problem of insulation co-ordination has been defined in two parts thus:

(*a*) The choice of a minimum protective level compatible with the particular conditions of the network and with the characteristics of the protective devices available.

(*b*) The choice of a minimum protected level, which determines the basic insulation level of the equipment.

The second of these we have considered very briefly: now consider the first with particular reference to the probable frequency of lightning discharges in the particular area. For any given area it is found that the amplitude of a travelling wave due to a lightning discharge is related to the number of occurrences of lightning per annum, the magnitude decreasing as the number increases. In other words, high amplitudes occur relatively seldom, whereas low amplitudes occur frequently. The curve

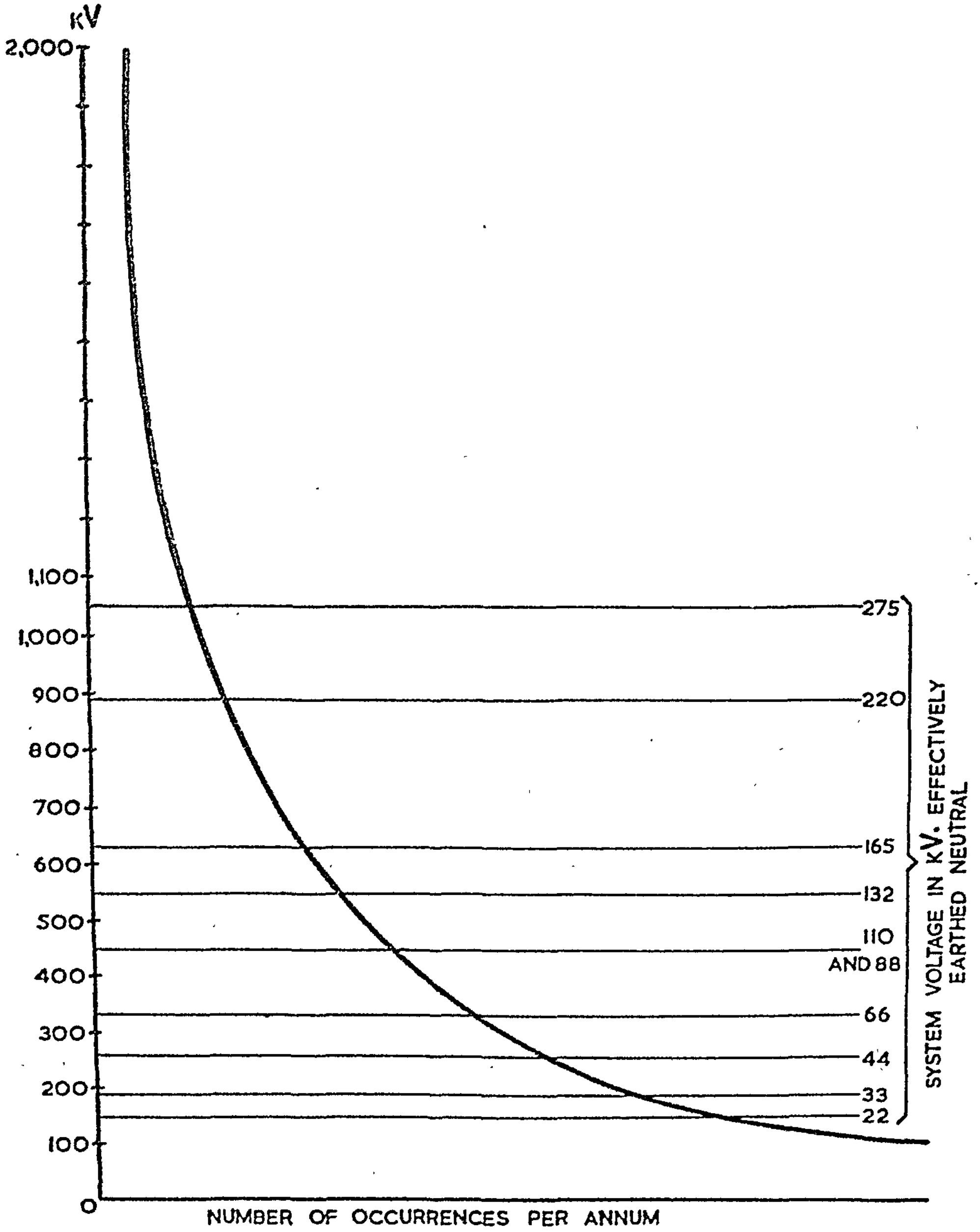


FIG. 18.27.—THE RELATIONSHIP OF AN OVERVOLTAGE DUE TO LIGHTNING TO THE NUMBER OF OCCURRENCES PER ANNUM.

is of the form shown in Fig. 18.27. If we plot a series of horizontal lines representing the impulse withstand voltages for the different system voltages, we obtain a series of intersections which indicate the degree of probable immunity from trouble. With very high voltages the probable number of shut-downs is small, but as the system voltage decreases the number increases rapidly. Hence, if a low-voltage line is situated in a thunderstorm area, reasonable immunity can only be secured (*a*) by increasing the insulation of the line, and/or (*b*) by the installation of surge diverters. If the line is relatively unimportant, the financial expense may not be justified, in which case frequent shut-downs will be inevitable.

A very useful summary of the aims of co-ordination is given by W. H. Thompson (M.V. leaflet 285/0–4): "... the various insulators on a system should be co-ordinated by means of rod gaps. The basis of this co-ordination should be the impulse strength of the transformers or machines. As a protection against a direct stroke to a transformer bushing, a rod gap should be fitted to this bushing and set to flash over in about 2 or 3 microseconds at a peak value that is about 80 per cent. of that of the standard test chopped wave* applied to the particular transformer. Rod gaps on the circuit-breakers and post and strain insulators in use within the switching station should have a setting that will flash over at a peak equal to that of the standard test chopped wave on the transformer. All gaps on apparatus within the station, other than transformer bushings, need not have this setting; if desired, rod gaps can be fitted solely to protect the porcelain from damage, and the setting can then be higher than stated above, the limit being that the gaps should always flash over at a value at least 10 per cent. higher than the clean wet impulse flashover of the porcelain to which it is fitted. Gaps (not necessarily with non-linear surge diverters) are sometimes fitted on the transmission lines to control the incoming waves to the station within a maximum limit, which is governed by the type of surge diverter used; these gaps (not to be confused with those used to protect rotating machines) should extend for at least half a mile out from the station to be effective. The other line insulators do not require gaps for co-ordinating purposes, although they may be fitted to protect the porcelain against the effect of flashover. If the remainder of the system is co-ordinated, this line insulation can be as high as economy permits."

* A chopped wave can be regarded as an impulse wave minus its tail, the application of such a wave therefore being analogous to the occurrence of a travelling wave of finite length. Owing to the sudden cut-off of the tail (the duration of the chopped wave being of the order of 3 or 4 microseconds), the stress between turns of the transformer under test is greater than with a normal impulse wave, because the slope of the chopped tail is steeper than the slope of the wave front.

The Series Inductance and Shunted Capacitance

A series inductance or a capacitance from line to earth installed at a substation line termination will reduce the amplitude of the

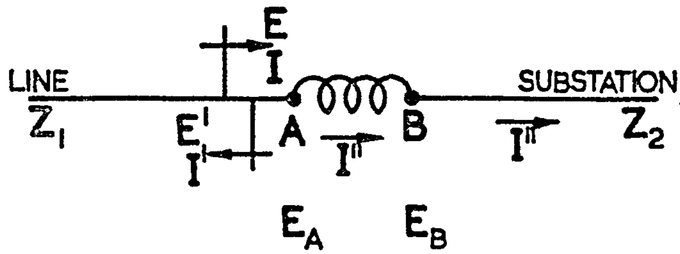


FIG. 18.28.—EFFECT OF SERIES INDUCTANCE.

wave reaching the apparatus, and will also reduce the slope of its front. Consider first of all the series inductance, Fig. 18.28. The equations are—

$$E = IZ_1$$
$$E'' = I''Z_2$$
$$E' = -\ I'Z_1$$
$$E_B = I''Z_2$$
$$E_A - E_B = L\frac{dI''}{dt}$$
$$E + E' = E_A$$
$$I + I' = I''$$

From the fifth of these equations we have—

$$E - I'Z_1 = E_A$$
$$\therefore E = E_A + I'Z_1$$
$$= E_A + (I'' - I)\,Z_1$$
$$= E_B + L\frac{dI''}{dt} + (I'' - I)\,Z_1$$
$$= I''Z_2 + L\frac{dI''}{dt} + I''Z_1 - E$$
$$\therefore 2E = I''(Z_1 + Z_2) + L\,\frac{dI''}{dt}$$

$$\frac{dI''}{2E - I''(Z_1 + Z_2)} = \frac{dt}{L}$$

$$\therefore -\frac{1}{Z_1 + Z_2}\log_e[2E - I''(Z_1 + Z_2)] = \frac{t}{L} + C$$

Put $$C = \frac{1}{Z_1 + Z_2}\log_e D$$

$$\therefore -\frac{1}{Z_1 + Z_2} \log_e [2E - I''(Z_1 + Z_2)] = \frac{t}{L} + \frac{1}{Z_1 + Z_2} \log_e D$$

$$\therefore [2E - I''(Z_1 + Z_2)]\, D = e^{-\frac{Z_1 + Z_2}{L}}$$

Now $I'' = 0$ when $t = 0$, so that $D = \frac{1}{2E}$

$$\therefore I'' = \frac{2E}{Z_1 + Z_2}\left[1 - e^{-\frac{Z_1 + Z_2}{L}.t}\right]$$

The voltage which arrives at the terminals of the substation apparatus is, at any instant, proportional to the value of I'' at that instant, and therefore the shape of the voltage wave which reaches the windings is of the same shape as the graph of I'' against time.

Example.

1.	2.
$L = 0{\cdot}02H$	$L = 0{\cdot}002H$
$Z_1 = 200$	$Z_1 = 200$
$Z_2 = 300$	$Z_2 = 300$
$E = 10^5$	$E = 10^5$

The currents are calculated for one-second intervals and plotted in Fig. 18.29. The ultimate current is the same in the two cases,

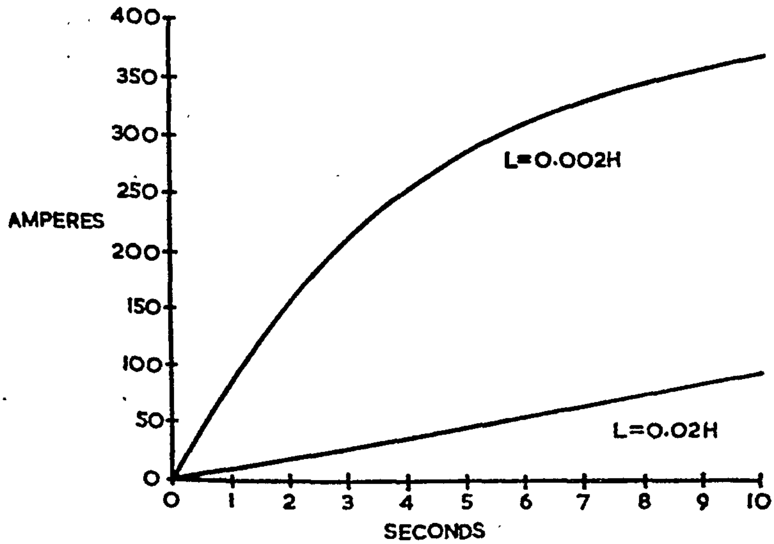


FIG. 18.29.—PLOT OF CURRENT V. TIME FOR THE TWO CASES IN THE NUMERICAL EXAMPLE.

and we see that the protective ability of the smaller inductance is negligibly small because of the rapid rise in the current. Thus, to effect any real protection, the inductance must be large and this is the disadvantage of the method.

Actually, the travelling wave produced by a lightning discharge is not infinitely long. Denote its length by l, then we can resolve the actual wave into two waves of infinite length, the first positive and the second negative. The relative positions of these two components will be as shown in Fig. 18.30. The time interval

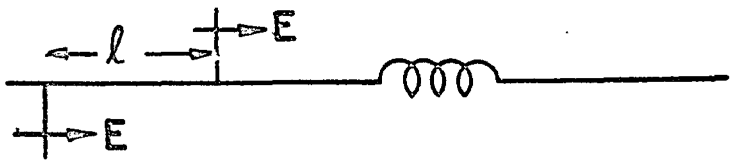


FIG. 18.30.—RESOLUTION OF A FINITE WAVE INTO TWO INFINITELY LONG COMPONENTS.

between these two fronts is v/l: denote it by T secs. Then from $t = 0$ to $t = T$, the positive wave acts alone and the above analysis applies. The transmitted current is—

$$I'' = \frac{2E}{Z_1 + Z_2}\left[1 - e^{-\frac{Z_1 + Z_2}{L}.t}\right]$$

and the voltage at the terminal B therefore—

$$E_B = I''Z_2 = \frac{2EZ_2}{Z_1 + Z_2}\left[1 - e^{-\frac{Z_1 + Z_2}{L}.t}\right]$$

From time $t = T$, and subsequently, the negative wave will give rise to a voltage at B of—

$$-\frac{2EZ_2}{Z_1 + Z_2}\left[1 - e^{-\frac{Z_1 + Z_2}{L}(t-T)}\right]$$

Thus, up to time $t = T$ the positive wave acts alone and the voltage rise will be a maximum at $t = T$. After this the voltage will decay, as shown in Fig. 18.31. We therefore see that

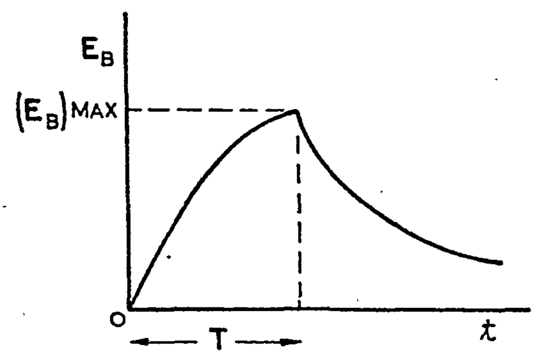


FIG. 18.31.—CURRENT V. TIME CHARACTERISTICS WITH SERIES INDUCTANCE.

conditions are ameliorated by the shortness of the wave. Actually it is more than the above analysis indicates because attenuation is not taken into account. According to the Foust and Menger formula (p. 504) the attenuation factor for a short wave is 0·0003 as against 0·00016 for a long wave, the attenuation thus being more rapid for a short wave.

Neglecting attenuation, the maximum value attained by E_B is—

$$(E_B)_{max} = \frac{2EZ_2}{Z_1 + Z_2}\left[1 - e^{-\frac{Z_1 + Z_2}{L}.T}\right]$$

$$\therefore e^{-\frac{Z_1 + Z_2}{L}.T} = 1 - \frac{(E_B)_{max}(Z_1 + Z_2)}{2EZ_2}$$

$$\therefore -\frac{Z_1 + Z_2}{L}.T = \log_e\left[1 - \frac{(E_B)_{max}(Z_1 + Z_2)}{2EZ_2}\right]$$

$$\therefore L = \frac{-(Z_1 + Z_2)T}{\log_e\left[1 - \frac{(E_B)_{max}(Z_1 + Z_2)}{2EZ_2}\right]}$$

Now put $$\frac{(E_B)_{max}}{E} = \frac{1}{m}$$

where $\frac{E}{m}$ is the allowable voltage rise at the terminal B.

$$\therefore L = \frac{-(Z_1 + Z_2)T}{\log_e\left[1 - \frac{(Z_1 + Z_2)}{2mZ_2}\right]}$$

This gives the value of L necessary to prevent the voltage rise at B from becoming greater than $\frac{E}{m}$.

Example

$$Z_1 = 500$$
$$Z_2 = 300$$
$$l = 3 \text{ miles}$$
$$m = 3$$

$$\therefore T = \frac{3}{186000} = 1{\cdot}61 \times 10^{-5} \text{ sec.}$$

$$\therefore L = -\frac{800 \times 1{\cdot}61 \times 10^{-5}}{2{\cdot}3 \log_{10}\left(1 - \frac{800}{2 \times 3 \times 300}\right)}$$
$$= 0{\cdot}022 \text{ H}$$

This is a very high value for an inductor which is connected in series with a power line and, except in the case of a current as low as 10 or 20 amps., would necessitate an inductor of large physical dimensions. This is the limitation on the use of the series inductor. There is a second consideration. The incident and reflected waves combine algebraically to give the transmitted wave, for both voltage and current. If, then, the slope of the front of the transmitted wave is reduced, that of the reflected wave will be increased. This may have a bearing on the performance on all the equipment in the vicinity of the inductor. We have—

$$E' = E\left[\frac{Z_2 - Z_1}{Z_1 + Z_2} + \frac{2Z_1}{Z_1 + Z_2}\varepsilon^{-At}\right] \text{ where } A = \frac{Z_1 + Z_2}{L}$$

$$\left(\frac{dE'}{dt}\right)_{t=0} = \frac{2EZ_1}{Z_1 + Z_2}(-A) = -\frac{2EZ_1}{L}$$

$$E'' = \frac{2EZ_2}{Z_1 + Z_2}[1 - \varepsilon^{-At}]$$

$$\left(\frac{dE''}{dt}\right)_{t=0} = -\frac{2EZ_2}{Z_1 + Z_2}(-A) = \frac{2EZ_2}{L}$$

This shows that the initial voltage gradient of both transmitted and reflected waves is reduced when L is increased.

Capacitance between Line and Earth

From Fig. 18.32 we see that the equations are—

$$I = \frac{E}{Z_1}$$

$$I'' = \frac{E''}{Z_2}$$

$$I' = -\frac{E'}{Z_1}$$

$$E'' = E + E'$$

$$I + I' = I'' + I_c$$

$$\frac{E}{Z_1} - \left(\frac{E'' - E}{Z_1}\right) = \frac{E''}{Z_2} + I_c$$

or

$$\frac{2E}{Z_1} = \frac{E''}{Z_1} + \frac{E''}{Z_2} + I_c$$

$$2E = \left(\frac{I''Z_2}{Z_1} + \frac{I''Z_2}{Z_2}\right)Z_1 + I_cZ_1$$

$$2E = I''(Z_1 + Z_2) + I_cZ_1$$

also

$$E'' = I''Z_2$$

Now $$I_c\,dt = dQ$$

But $$Q = CE''$$

$$\therefore I_c = C\frac{dE''}{dt} = CZ_2\frac{dI''}{dt}$$

$$\therefore I''(Z_1 + Z_2) + I_cZ_1 = 2E$$

$$\therefore I''(Z_1 + Z_2) + CZ_1Z_2\frac{dI''}{dt} = 2E$$

$$\therefore CZ_1Z_2\frac{dI''}{dt} = 2E - (Z_1 + Z_2)\,I''$$

$$\frac{Z_1Z_2\,dI''}{2E - (Z_1 + Z_2)\,I''} = \frac{dt}{C}$$

$$\frac{dI''}{\frac{2E}{Z_1 + Z_2} - I''} = \frac{dt}{C} \times \frac{Z_1 + Z_2}{Z_1Z_2}$$

$$= \frac{dt}{\alpha C}, \text{ say, where } \frac{1}{\alpha} = \frac{Z_1 + Z_2}{Z_1Z_2}$$

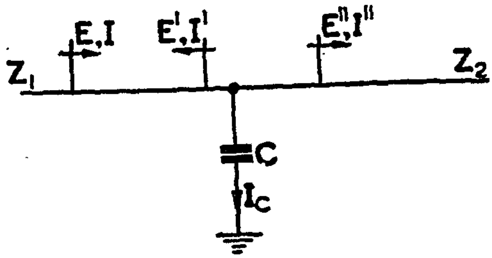


FIG. 18.32.—CAPACITANCE BETWEEN LINE AND EARTH.

Now $I'' = 0$ when $t = 0$, the solution therefore being—

$$I'' = \frac{2E}{Z_1 + Z_2}\left(1 - e^{-\frac{t}{\alpha c}}\right)$$

$$\text{and } E'' = I''Z_2 = \frac{2EZ_2}{Z_1 + Z_2}\left(1 - e^{-\frac{t}{\alpha c}}\right)$$

For the capacitance current, we have—

$$I_c = CZ_2\frac{dI''}{dt}$$

$$= \frac{2E}{Z_1}e^{-\frac{t}{\alpha c}}$$

Now suppose that the wave is of finite length, then, proceeding as in the case of the series inductance we have—

$$(E'')_{max} = 2E \cdot \frac{Z_2}{Z_1 + Z_2}\left(1 - e^{-\frac{T}{ac}}\right)$$

$$\therefore C = \frac{T}{a \log_e \left(\dfrac{\dfrac{2EZ_2}{Z_1 + Z_2}}{\dfrac{2EZ_2}{Z_1 + Z_2} - (E'')_{max}} \right)}$$

$$= \frac{T}{a \log_e \left(\dfrac{\dfrac{2Z_2}{Z_1 + Z_2}}{\dfrac{2Z_2}{Z_1 + Z_2} - \dfrac{(E'')_{max}}{E}} \right)}$$

Example.—$Z_1 = 500$; $Z_2 = 300$; $l = 3$ miles $\therefore T = 1{\cdot}61 \times 10^{-5}$ sec.; $m = 3$

$$a = \frac{500 \times 300}{800} = 187{\cdot}5$$

$$\therefore C = \frac{1{\cdot}61 \times 10^{-5}}{187{\cdot}5 \times 2{\cdot}3 \log_{10} \left(\dfrac{\dfrac{600}{800}}{\dfrac{600}{800} - 0{\cdot}333} \right)}$$

$$= 0{\cdot}145 \times 10^{-6}\ \text{F}$$

$$= 0{\cdot}145\ \mu\text{F}.$$

The early forms of condenser, such as those due to Moscicki and to Meirowski, had much smaller capacitances than this, of the order of 0·003 μF, with the result that they were practically useless as a protection against travelling waves. Their only function was protection against very high frequency oscillating disturbances.

FURTHER READING, IN ADDITION TO THAT SUGGESTED IN CONNECTION WITH CHAP. 16.

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