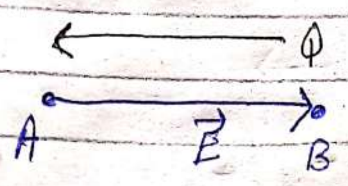
koracademy.com koracademy.com koracademy.com

BASIC ELECTRICAL ENGINEERING

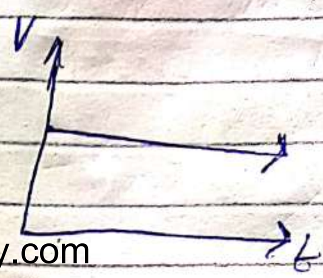
Voltage or Potential Difference



$V = \frac{W}{q}$

Energy required to move a charge of 1C from the initial position B to the final position A against the electric field.

Constant voltage
DC voltage



koracademy.com koracademy.com koracademy.com

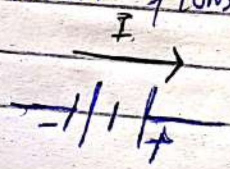
Time varying voltage
(or) Instantanous voltage.

Time varying electrical quantities are represented by small letters eg v, i etc and constant with capital eg V, I etc.

eg AC voltage
↳ mathematical equation $v = V_m \sin\omega t$

The mathematical equation in terms of time is known as instantanous equation.

Voltage Rise
Current flows from negative to positive.

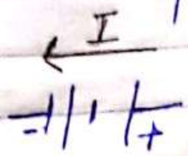


koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Voltage drop

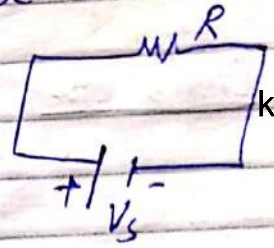
Current flows from positive to negative.



The circuit element that <u>delivers</u> energy is known as <u>active</u> circuit element. eg the voltage source V_s in this example.

The circuit element which <u>takes</u> energy from the source is known as <u>passive</u> circuit element.
eg resistor, capacitor, inductor etc.

koracademy.com koracademy.com koracademy.com



Ohm's Law

When we apply a voltage through a passive circuit element, there is a current flow through the passive circuit element.

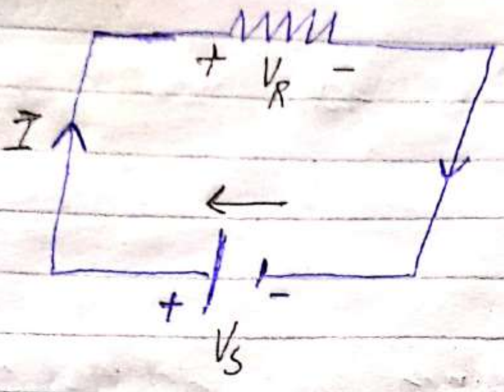
Whenever there is a current in any passive circuit element; it <u>usually</u> results in a voltage drop across that passive circuit element.

In this figure;
$V_s \rightarrow$ voltage rise, $V_R \rightarrow$ voltage drop

koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

$R \rightarrow$ linear circuit element.



Kirchoff's voltage Law.

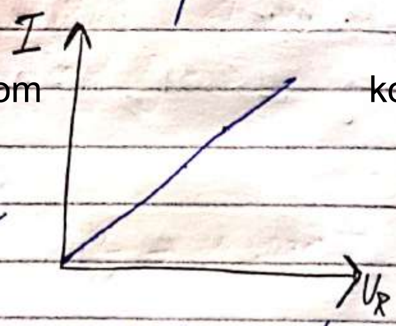
The sum of voltage rises in a loop will be equal to the sum of all the voltage drops in that loop.

As here $V_S = V_R$

koracademy.com koracademy.com koracademy.com

Ohm's law $I \propto V_R$

or $V_R \propto I$



Any circuit element which has a linear relationship b/w I and V is known as <u>linear</u> circuit element. (LCE)

eg R, C, L.

Capacitor $\rightarrow$ L.C.E $\rightarrow$ for AC voltage

Resistor $\rightarrow$ L.C.E $\rightarrow$ for any (AC or DC) voltage.

Inductor $\rightarrow$ L.C.E $\rightarrow$ for AC voltage.

A circuit element which allows current to flow through it many direction are called <u>bilateral</u> circuit elements

eg resistor, Inductor (AC V), capacitor (AC V).

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Diode is an example of unilateral circuit element, as it allows current in one direction and blocks in the other.

$$V_S = V_R = IR$$

The property due to which any passive ckt element opposes the flow of current through it is called resistance.

$$I = \frac{V_R}{R} = \frac{V_S}{R}$$

Electrical Energy:

The amount of energy required to move a charge of q coulombs against the field.

koracademy.com koracademy.com koracademy.com

$$W = QV.$$

Electrical Power.

The electrical energy per unit time.

$$P = \frac{W}{t} = \frac{QV}{t} = IV$$

According to law of conservation of energy

$$Power_{applied} = Power_{dissipated}$$

Resistor converts the electrical energy into heat energy and dissipates it into the surroundings.

$$P_S = P_R$$

$$V_S I = V_R I = P_R$$

$$\boxed{P_R = I^2 R} \quad \text{or} \quad P_R = \frac{V_R^2}{R}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

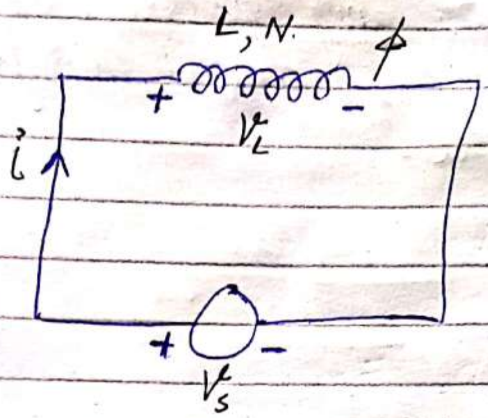
koracademy.com koracademy.com koracademy.com

Inductor

A straight conductor even acts as inductor.

conductor $N = 1$.

To enhance the property of inductivity, we make a coil of N turns.



By increasing i, ϕ increases and vice versa.

If current is constant, ϕ is also constant and if it is time varying, ϕ will also be varying.

koracademy.com koracademy.com koracademy.com

$$\phi \propto i$$

$$\phi = \frac{L}{N} i \qquad \frac{L}{N} \rightarrow \text{constant}$$

For a straight conductor or for a coil of 1 turn i.e $N = 1$

$$L = \frac{\phi N}{i} \Rightarrow L = \frac{\phi}{i}$$

From $\phi = \frac{L}{N} i \Rightarrow N\phi = Li$

$$V_L = N\frac{d\phi}{dt} = L\frac{di}{dt}$$

$$\boxed{V_L = L\frac{di}{dt} = V_s}$$

$V_L = N\frac{d\phi}{dt} \rightarrow$ voltage across inductor from Faraday's law.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

If DC is applied ; no voltage will be induced ie $V_L = 0$

This inductor behaves as short circuit for DC voltage.

To find current apply KVL;

$$V_s = V_L$$

As $V_s = V_L = L\frac{di}{dt} \Rightarrow di = \frac{1}{L} V_L dt$

or $$\boxed{i = \frac{1}{L}\int V_L dt}$$

koracademy.com koracademy.com koracademy.com

Also $P_s = P_L$

$$P_L = V_L i \Rightarrow \boxed{P_L = L i \frac{di}{dt}}$$

$$P_L dt = L i \, di \Rightarrow dW = L i \, di$$

$$W = L\int i \, di$$

$$\boxed{W = \frac{L i^2}{2}}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

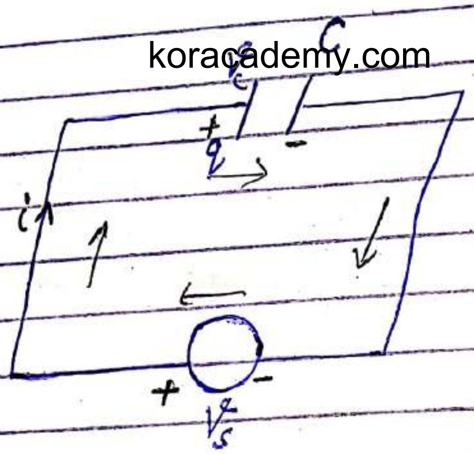
Capacitor:

Passive circuit element.
Also known as energy storing element.

Capacitance is the property of a capacitor to store electrical energy.

$$C = \frac{A\varepsilon}{d}$$

koracademy.com koracademy.com koracademy.com



When we apply a voltage through a passive circuit element, it results in a current through the passive circuit element. And the nature of the current depends on the nature of the applied voltage. as in this case time varying.

Applying KVL

$$v_s = v_c$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$v_s \rightarrow$ Active circuit element.
$C \rightarrow$ Passive circuit element.

$$q \propto v_s \quad \text{or} \quad q \propto v_c$$

$$q = C v_s$$

Differenciating

$$\frac{dq}{dt} = C \frac{dv_s}{dt}$$

$$\boxed{i = C \frac{dv_s}{dt}}$$

koracademy.com koracademy.com koracademy.com

If we apply a DC voltage across this capacitor, the voltage will not change;
ie $\frac{dv_s}{dt} = 0 \Rightarrow i = 0$

It means the DC current cannot flow through a capacitor.
In other words this capacitor offers infinite resistance to the flow of DC.
Or This capacitor behaves like an open circuit when connected across DC.

Finding voltage across the capacitor.

$$dv_c = \frac{1}{C} i \, dt$$

Integrate

$$\boxed{v_c = \frac{1}{C} \int i \, dt}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

According to law of conservation of energy

$$P_s = P_c$$

$$v_s i = v_c i \qquad P_c = v_c i$$

$$P_c = v_c \times C\frac{dv_c}{dt}$$

$$P_c\,dt = v_c \cdot C\,dv_c$$

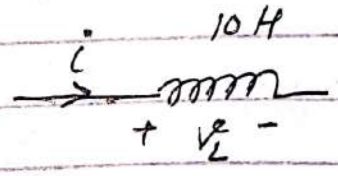
$$dW = C\,v_c\,dv_c \qquad \text{Integrate}$$

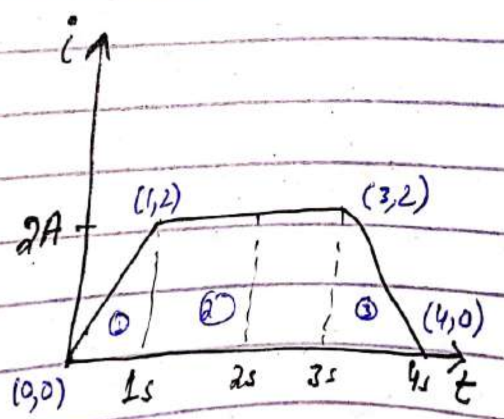
$$\int W = C\int v_c\,dv_c$$

koracademy.com koracademy.com koracademy.com

$$\boxed{W = \frac{1}{2} C v_c^2}$$

Example 1





v_L , P, w.

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Region 1. $0 \leq t \leq 1$.

from $y = mx \Rightarrow i = mt$

for m from $\frac{y_2 - y_1}{x_2 - x_1} \Rightarrow m = \frac{i_2 - i_1}{t_2 - t_1}$

$= \frac{2-0}{1-0} \Rightarrow m = 2 \Rightarrow i = 2t$

$V_L = L\frac{di}{dt} = 10 \times 2 = \boxed{20V}$

$P_L = V_L i = \boxed{40t}$

koracademy.com koracademy.com koracademy.com

$w = \frac{1}{2} L i^2 = 20t^2$

At $t = 0 \Rightarrow w = 0$ Ws
At $t = 0.5 \Rightarrow w = 5$ Ws
At $t = 1 \Rightarrow w = 20$ Ws.

Region 2. $1 \leq t \leq 3$

$i = 2A$

$V_L = L\frac{di}{dt} = 0$

$P_L = V_L i = 0$

$w = \frac{1}{2} L i^2 = 5 \times 4 = 20Ws$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Region 3 $3 \le t \le 4$

From $y = mx + C$
ie A line not passing through origin.
C B y intercept.

Here $i = mt + C$

$m = \frac{i_2 - i_1}{t_2 - t_1} = \frac{0-2}{4-3} \Rightarrow m = -2$

From graph and using eq

$0 = -2 \times 4 + C \Rightarrow C = 8$

koracademy.com koracademy.com koracademy.com

$\Rightarrow i = -2t + 8$

$V_L = L\frac{di}{dt} = 10 \times -2 = -20V$

$P_L = V_L i = 40t - 160$

At $t = 3 \Rightarrow P_L = -40$
At $t = 3.5 \Rightarrow P_L = -20$
At $t = 4 \Rightarrow P_L = 0$

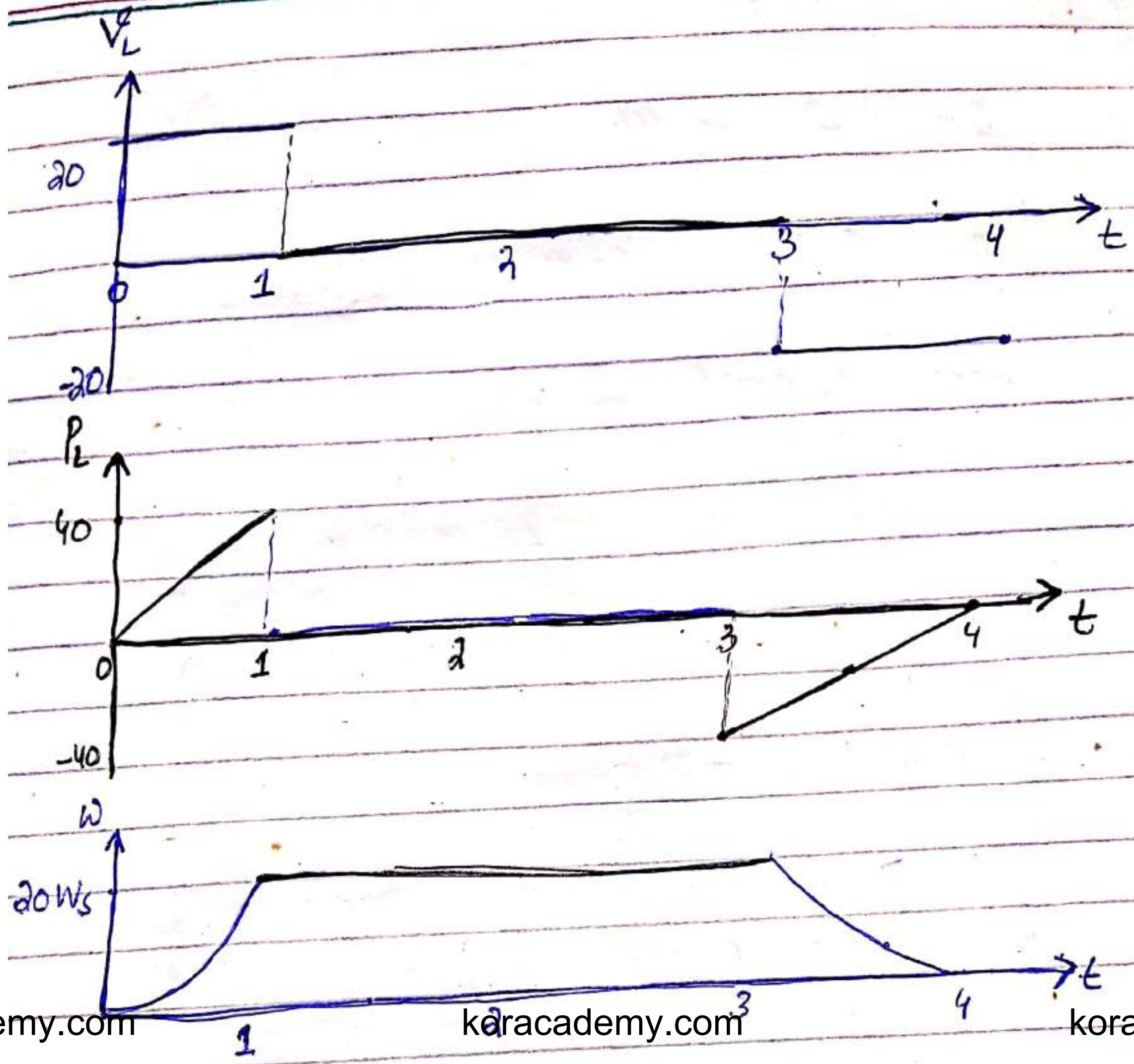
$w = \frac{1}{2} L i^2 = \frac{1}{2} \times 10 \times (-2t + 8)^2$

$w = 5(-2t + 8)^2$

At $t = 3 \Rightarrow w = 20$
At $t = 3.5 \Rightarrow w = 5\,WJ$
At $t = 4 \Rightarrow w = 0\,WJ$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

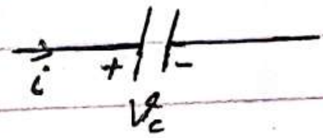
koracademy.com koracademy.com koracademy.com

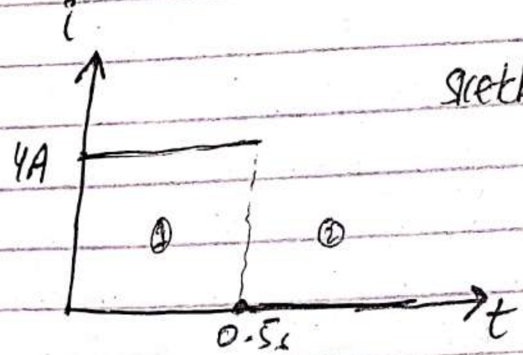


koracademy.com koracademy.com koracademy.com

Example 2:

$C = 0.1\ F$





sketch v_c, q, P_c, w_c as a function of time.

Region 1 $\quad 0 \leq t \leq 0.5$

$i = 4A$

$$v_c = \frac{1}{C}\int i\,dt = \frac{1}{0.1}\int 4\,dt = 40\int dt$$

koracademy.com koracademy.com koracademy.com

koracademy.com koracademy.com koracademy.com

$$v_c = 40t$$

$$q = C v_c = 4t$$

$$P_c = v_c i = 160t$$

$$w = \frac{1}{2} C v_c^2 = \frac{1}{2} \times 0.1 \times 1600 t^2$$

$$w = 80t^2$$

At $t=0 \Rightarrow w=0$
At $t=0.2 \Rightarrow w=5$
At $t=0.5 \Rightarrow w=20$

Region 2 $\quad t \geq 0.5$

koracademy.com koracademy.com koracademy.com

Under ideal condition, when a capacitor is charged ie it has energy; it will store this energy and does not lose it.

ie $v_c = 20\ V$

And the charges will also remain on the plates of capacitor forever, ie $q = 2C$

Also we know that $i = 0$

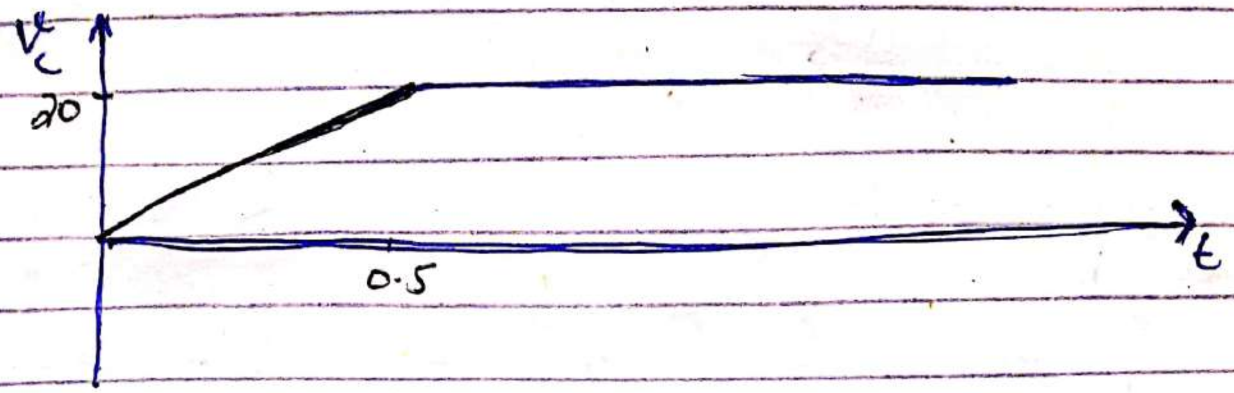
$$P_c = v_c i = 0$$

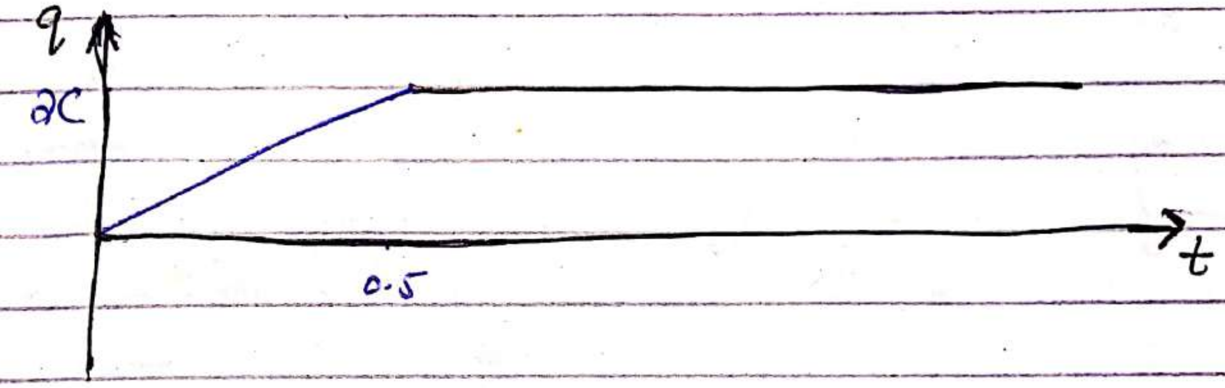
Also the energy remains the same.

$$w = 20\ Ws.$$

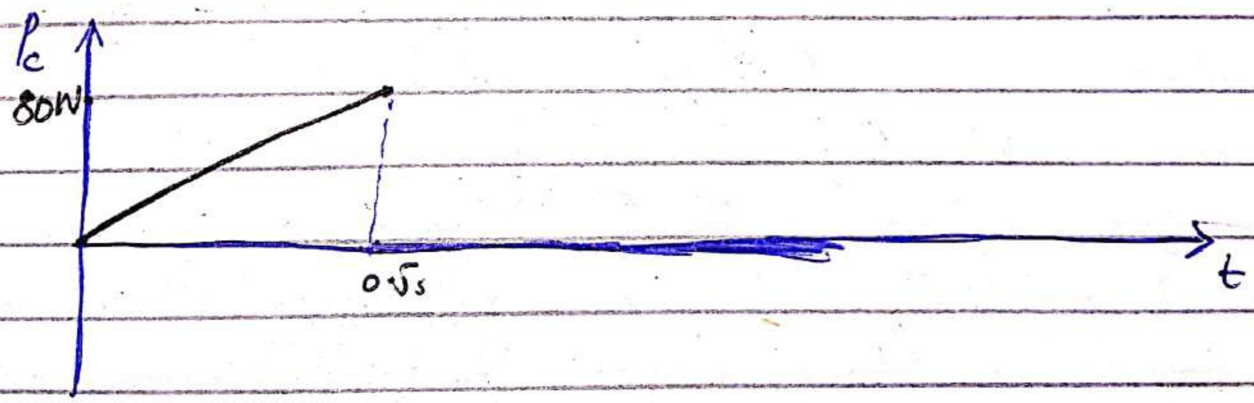
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

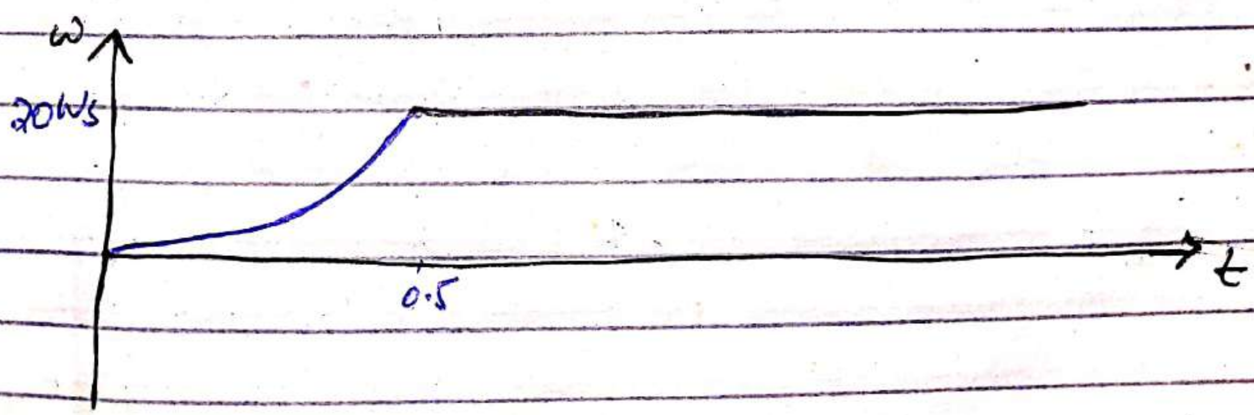
koracademy.com koracademy.com koracademy.com





koracademy.com koracademy.com koracademy.com





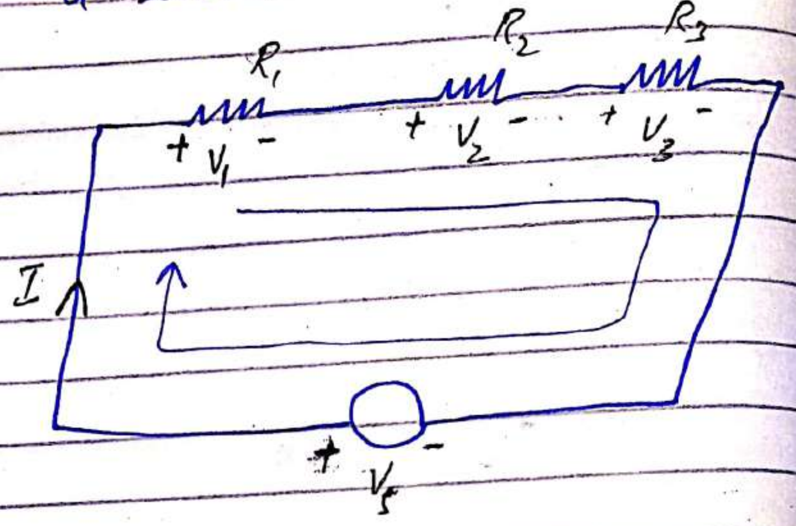
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Kirchoff's Voltage Law

Resistors In Series

Consider a series circuit.



According to KVL

Σ voltage rises = Σ voltage drops

$$V_s = V_1 + V_2 + V_3$$

koracademy.com koracademy.com koracademy.com

or $$0 = V_1 + V_2 + V_3 + (-V_s)$$

"The algebric sum of all the voltage drops in a loop in a specific direction is always equal to zero."

As $V_1 = IR_1 \quad V_2 = IR_2 \quad V_3 = IR_3$

$$\Rightarrow V_s = IR_1 + IR_2 + IR_3$$

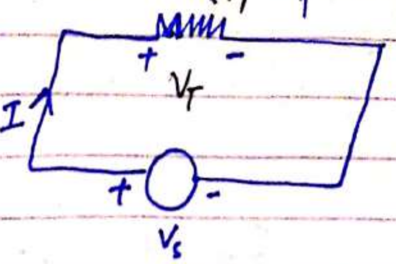
$$V_s = I(R_1 + R_2 + R_3) \rightarrow (I)$$

$$I = \frac{V_s}{R_1 + R_2 + R_3}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

R_T of Circuit.



Apply KVL

$$V_S = V_T$$

From ohm's law $V_T = I R_T$

$$\Rightarrow V_S = I R_T \rightarrow (II).$$

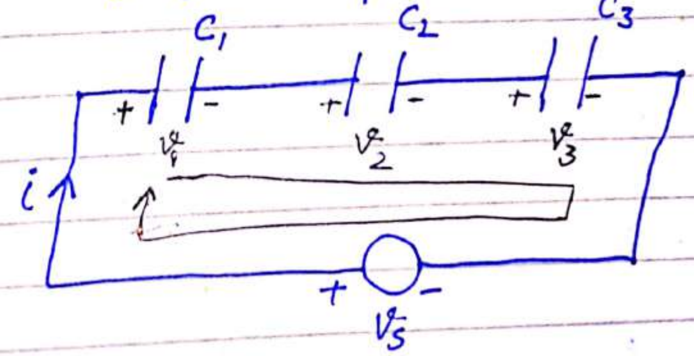
For Eq (I) = Eq (II)

$$\boxed{R_T = R_1 + R_2 + R_3.}$$

Capacitors In Series.

koracademy.com koracademy.com koracademy.com

As Capacitor blocks DC so we apply a time varying voltage.



Applying KVL

$$V_S^c = V_1^c + V_2^c + V_3^c$$

As Σ Voltage rises = Σ Voltage drops

$$\text{or}\quad 0 = V_1^c + V_2^c + V_3^c + (-V_S^c)$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

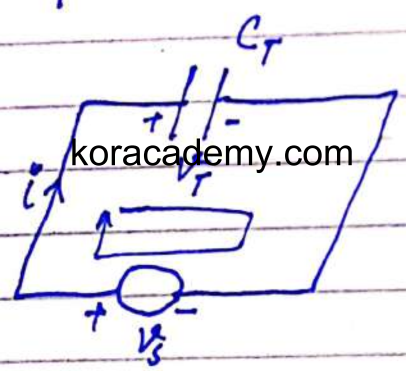
We know that

$$V_1 = \frac{1}{C_1}\int i\,dt$$

$$V_2 = \frac{1}{C_2}\int i\,dt \qquad V_3 = \frac{1}{C_3}\int i\,dt$$

$$\Rightarrow V_S = \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt + \frac{1}{C_3}\int i\,dt$$

Let capacitors are replaced by a single equivalent capacitor.



koracademy.com koracademy.com koracademy.com

The current in a capacitive circuit can be changed either by changing the source voltage or the capacitance of the circuit.

Apply KVL to second circuit

$$V_S = V_T$$

As $V_T = \frac{1}{C_T}\int i\,dt$

or $V_S = \frac{1}{C_T}\int i\,dt \rightarrow (II)$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$V_s = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\left(\int i\,dt\right) \rightarrow (II)$$

Eq I = Eq II if;

$$\boxed{\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Inductors In Series:

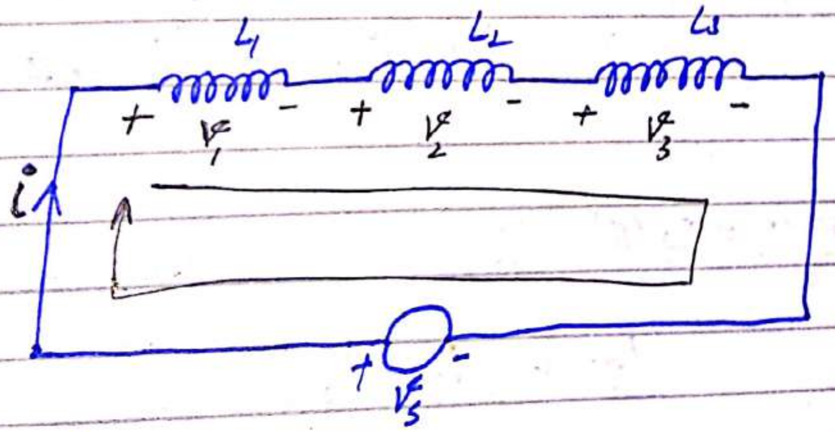
Assume that the mutual inductance of any two inductors is zero.

Apply a time varying voltage.

Under ideal condition, there will be infinite current in the circuit if DC voltage is applied.

Inductor do not provide opposition (resistance) to the flow of DC.

koracademy.com koracademy.com koracademy.com



Apply KVL

Σ voltage rise = Σ voltage drops

$$V_s = V_1 + V_2 + V_3$$

$$\Rightarrow 0 = V_1 + V_2 + V_3 + (-V_s)$$

"The algebric sum of all the voltage drops is zero."

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

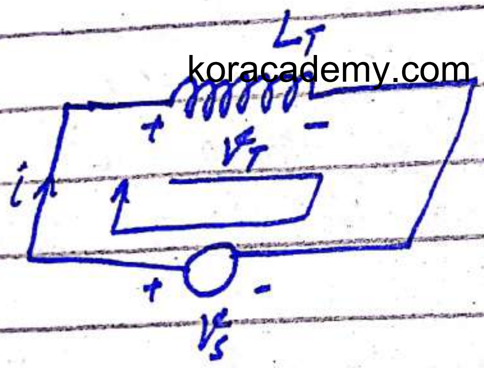
$$v_1 = L_1 \frac{di}{dt} \qquad v_2 = L_2 \frac{di}{dt} \qquad v_3 = L_3 \frac{di}{dt}$$

$$\Rightarrow v_s = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

$$v_s = L_1 + L_2 + L_3 \left(\frac{di}{dt} \right) \rightarrow (I)$$

Replace the combination of inductors with a single equivalent inductor.

koracademy.com koracademy.com koracademy.com



Apply KVL $\qquad v_s = v_T$

$$\Rightarrow \quad v_s = v_T = L_T \frac{di}{dt} \rightarrow (II)$$

Eq (I) = Eq (II) If

$$\boxed{L_T = L_1 + L_2 + L_3}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

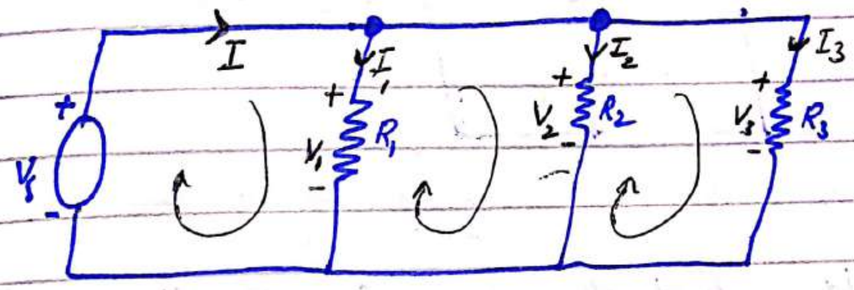
koracademy.com koracademy.com koracademy.com

Kirchoff's Current Law.

In parallel circuit, the voltage across all circuit elements is same.
In series circuit, the current across all the circuit elements is same.

Resistors In Parallel:

Apply constant voltage.



koracademy.com koracademy.com koracademy.com

There is no loop current. basically in any loop but for simple derivation we assume loop currents in the loops in clockwise direction.

Apply KVL
In loop 1: $V_s = V_1$
In loop 2: $V_1 = V_2$
In loop 3: $V_2 = V_3$

This implies that $V_s = V_1 = V_2 = V_3$

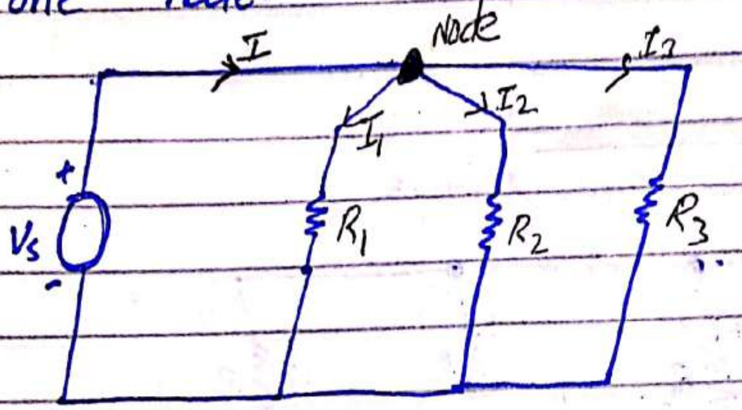
Junction or Node → point → joins two or more circuit elements → current divides at node.
. If these points (where current divides) have the same voltage, they are treated as a single node. If there voltages

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

are different, they are different nodes.

In the given circuit we have two points where current is dividing but they have the same potential so they are only one node.



koracademy.com koracademy.com koracademy.com

KCL states that the sum of all the currents moving towards a node will be equal to sum of all the currents moving away from the node.

$$I = I_1 + I_2 + I_3$$

Total current $(I) \rightarrow$ line current

$I_1, I_2, I_3 \rightarrow$ branch current

$$0 = I_1 + I_2 + I_3 + (-I)$$

$\Rightarrow$ KCl can also be stated as;

"The algebric sum of all the currents moving away from a node is always equal to zero."

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

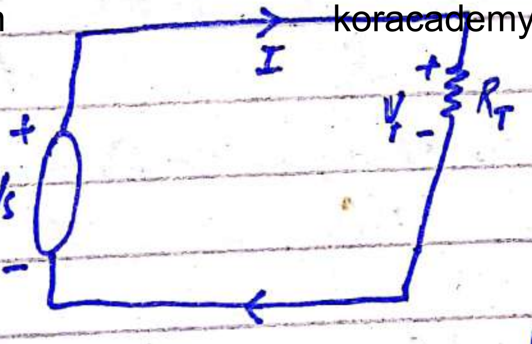
$$I_1 = \frac{V_1}{R_1} = \frac{V_s}{R_1} \qquad I_2 = \frac{V_2}{R_2} = \frac{V_s}{R_2} \qquad I_3 = \frac{V_3}{R_3} = \frac{V_s}{R_3}$$

$$\Rightarrow I = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3}$$

$$I = V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \rightarrow (I)$$

Replace the parallel combination by a single equivalent resistor.
Assume that the single resistor is equal to the total resistance of circuit 1.

koracademy.com koracademy.com koracademy.com



According to KVL, in circuit 2.

$$V_s = V_T$$

From ohm's law $V_T = IR_T$

or $V_s = IR_T \quad \Rightarrow I = V_s\left(\frac{1}{R_T}\right) \rightarrow (II)$

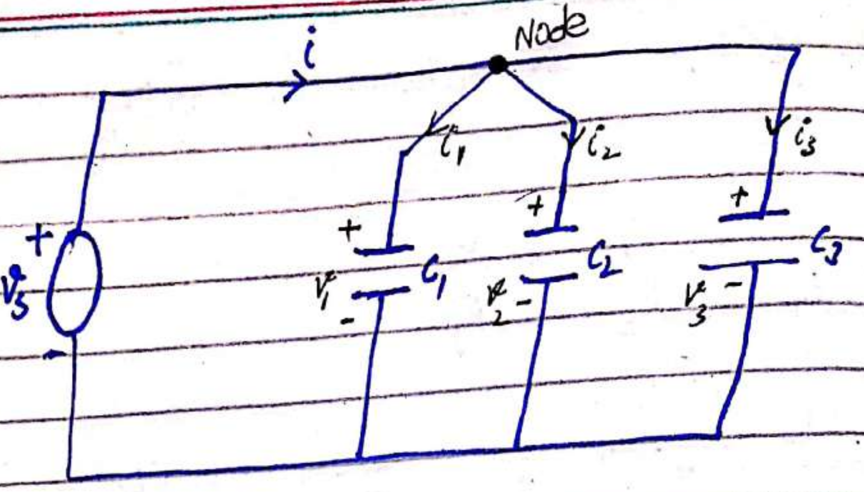
For Eq (I) = Eq (II) if $\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

Capacitors In Parallel

Apply time varying voltage.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com



Applying KCL

$$i = i_1 + i_2 + i_3$$

We know that in parallel, $V_s = V_1 = V_2 = V_3$

Also $i_1 = c_1 \dfrac{dV_1}{dt}$ $\quad i_2 = c_2 \dfrac{dV_2}{dt}$ $\quad i_3 = c_3 \dfrac{dV_3}{dt}$

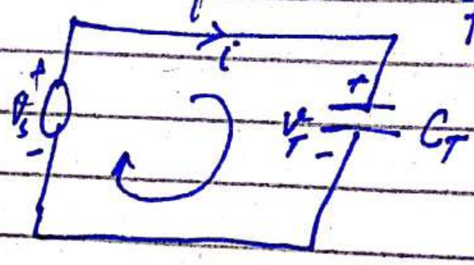
koracademy.com koracademy.com koracademy.com

or $i_1 = c_1 \dfrac{dV_s}{dt}$ $\quad i_2 = c_2 \dfrac{dV_s}{dt}$ $\quad i_3 = c_3 \dfrac{dV_s}{dt}$

$$\Rightarrow i = c_1 \frac{dV_s}{dt} + c_2 \frac{dV_s}{dt} + c_3 \frac{dV_s}{dt}$$

or $i = (c_1 + c_2 + c_3) \dfrac{dV_s}{dt} \rightarrow$ Eq (I)

Replace the parallel combination by a single capacitor; assuming that the capacitance of this single capacitor is equal to the total capacitance of circuit 1.



koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Apply KVL $\quad V_S = V_T$

We can say that current in circuit 2 is the current in the circuit.

$$i = C_T \frac{dV_T}{dt} = C_T \frac{dV_S}{dt} \rightarrow Eq\ (II).$$

For Eq (I) to be equal to Eq (II);

$$\boxed{C_T = C_1 + C_2 + C_3}$$

Example 1.I:

$V_s = 12\,V \quad I = 20\,A \quad t = 10\ hours$

koracademy.com koracademy.com koracademy.com

70% chemical energy is stored in the battery. Remaining 30% of energy is lost.

(a) Cost of charging of the electricity costs 5 cents / Kw hr.

(b) The amount of energy which is

$W = Pt$

$P = V_s I \quad = 12 \times 20 = 240$

$W = 240 \times 10 = 2400\ Whr = 2.4\ KWHr.$

The cost of 1 KW = 5 cents

Cost of 2.4 KW = 12 cents.

(b) $70\% = \frac{7}{100} \times 2400\ Whr$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

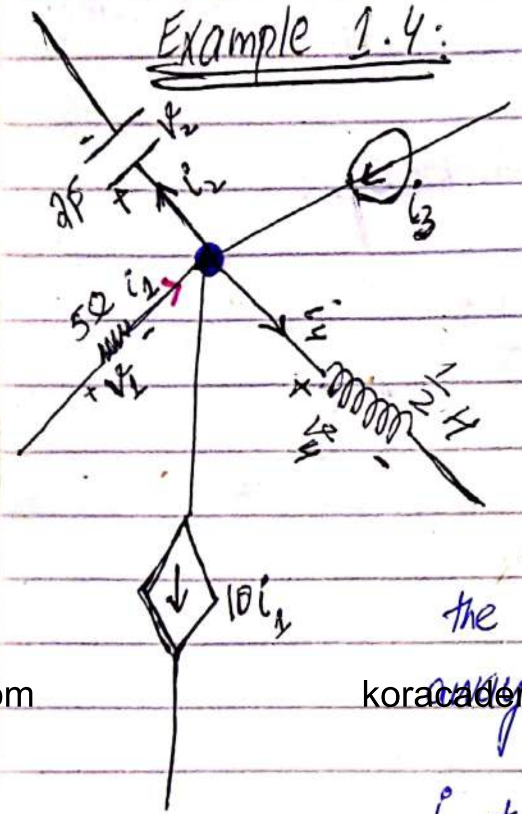
koracademy.com koracademy.com koracademy.com

$= 1680\,Wh$

$= 1680 \times 3600$ Wsec

$= 6048000\,J.$

Example 1.4:



Data

$v_1 = 10e^{-2t}$ Volts

$v_2 = 2e^{-2t}$ volts

$i_3 = 6e^{-2t}$ Amp

$v_4 = ?$

According to KCL, the current flowing towards the node is equal to the sum of currents flowing away from the node.

koracademy.com koracademy.com koracademy.com

$$i_1 + i_3 = i_2 + i_4 + 10i_1$$

$$i_4 = i_1 + i_3 - i_2 - 10i_1 \rightarrow (I)$$

$$v_1 = i_1 R_1 \Rightarrow i_1 = \frac{v_1}{R} = \frac{10e^{-2t}}{5}$$

$$i_1 = 2e^{-2t}$$

$$i_2 = C\frac{dv_2}{dt} = 2 \times -4e^{-2t} \qquad i_2 = -8e^{-2t} \text{ Amp}$$

This negative sign shows the direction of the current in the circuit is wrong. (But we dont change it).

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

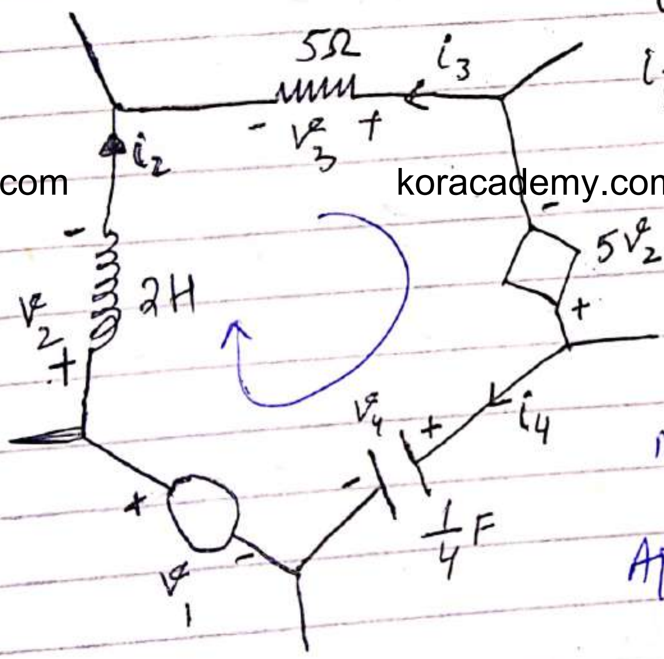
Eq (I) $\Rightarrow$ $i_4 = 2e^{-2t} + 6e^{-2t} + 8e^{-2t} - 20e^{-2t}$

$i_4 = -4e^{-2t}$

Again negative sign → assumed direction is wrong. → actually the current flows in the opposite direction.

$$v_4 = L\frac{di_4}{dt} = \frac{1}{2} \times 8e^{-2t} = \boxed{4e^{-2t}}$$ Ans

Example 1.5.



$v_1 = 12\sin 2t$ volts

$i_2 = 4\sin 2t$ A

$i_3 = 2\sin 2t - 4\cos 2t$ A

$i_4 = ?$

We assume that there is a loop current flowing in clockwise direction.

Apply KVL;

Σ voltage rises $= \Sigma$ voltage drop.

$5v_2 + v_1 + v_3 = v_2 + v_4$.

or $v_4 = 5v_2 + v_1 + v_3 - v_2 \rightarrow$ (I)

$v_2 = L\frac{di_2}{dt} = 2 \times 8\cos 2t \Rightarrow \underline{v_2 = 16\cos 2t}$.

$v_3 = i_3 R_3 = \cancel{5 \times 2}$ $10\sin 2t - 20\cos 2t$ V

koracademy.com koracademy.com koracademy.com
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

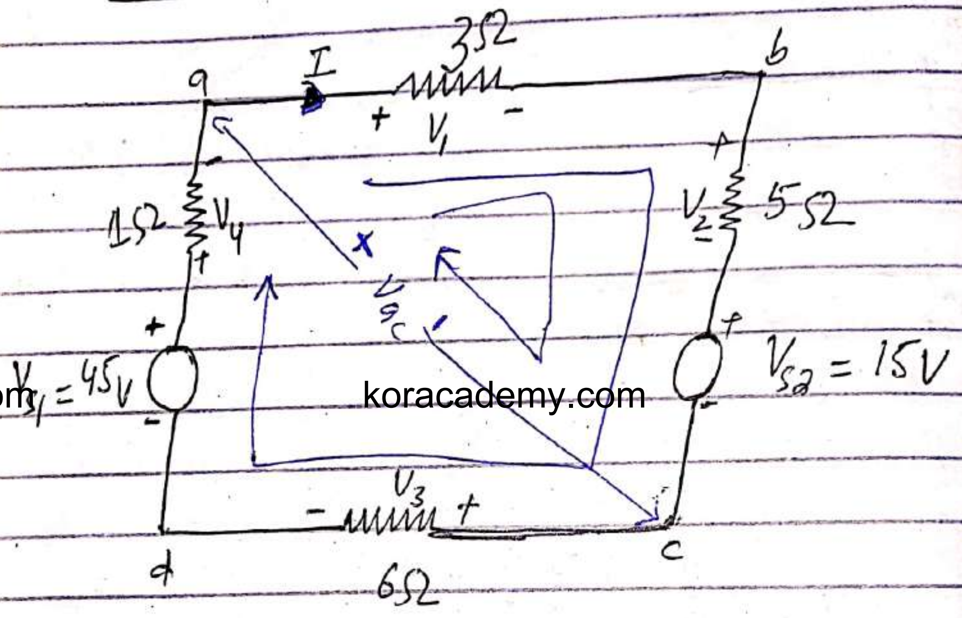
koracademy.com koracademy.com koracademy.com

$$Eq(II) \Rightarrow v_4 = 80\cos 2t + 12\sin 2t + 10\sin 2t - 20\cos 2t - 16\cos 2t$$

$$v_4 = 22\sin 2t + 44\cos 2t.$$

$$i_4 = C\frac{dv_4}{dt} \Rightarrow \boxed{i_4 = 11\cos 2t - 22\sin 2t}$$ Ans.

Exercise 1.2.21



koracademy.com koracademy.com koracademy.com

Calculate

(a) The current from a to b.

(b) The voltage V_{ac}.

(c) The power supplied by V_{s2}.

Apply KVL Σ voltage rise $= \Sigma$ voltage drop.

$$V_{s_1} = V_4 + V_1 + V_2 + V_{s_2} + V_3$$

$$V_1 + V_2 + V_3 + V_4 = V_{s_1} - V_{s_2}$$

$$V_1 + V_2 + V_3 + V_4 = 30V$$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

In series circuit, the current through all circuit elements is the same.

$$3I + 5I + 6I + I = 30$$

$$15I = 30$$

$$\Rightarrow I = 2A.$$

If voltage across source is voltage rise it will deliver power and if voltage across source is voltage drop it will absorb power.

(c) $P_{s_1} = V_{s_1} I = 45 \times 2 = 90\,W.$

koracademy.com koracademy.com koracademy.com

(b) Suppose there are two triangular loops (abc and acd). We consider the upper loop ie abc. and apply KVL to it.

$$V_{ac} = V_1 + V_2 + V_{s2}$$

$$V_{ac} = 6 + 10 + 15 = \underline{31 \text{ volts}}$$

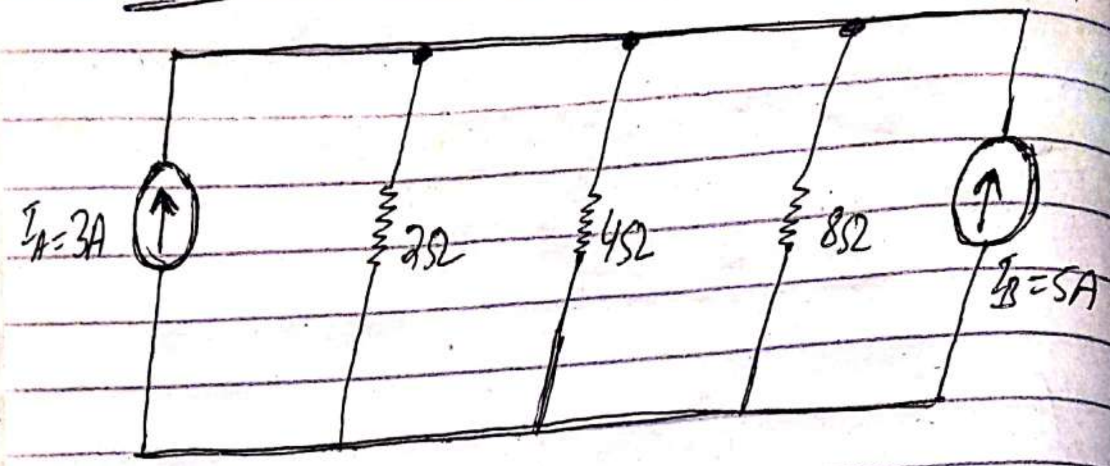
(d) Power absorbed by segment BC = 50 W

ie by resistor I^2R and the source also.

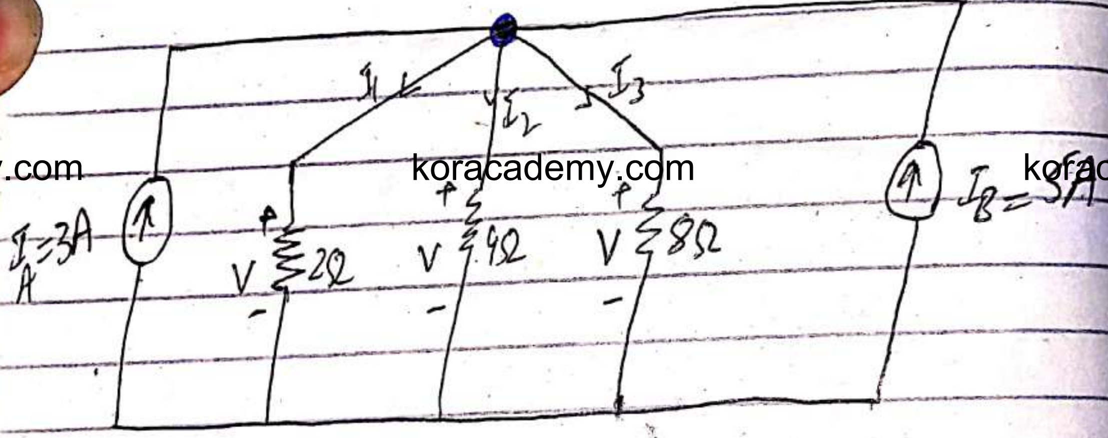
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Q23



There are three points in this circuit where the current divides; but they are at the same potential, so it is only a single node.



koracademy.com koracademy.com koracademy.com

In parallel circuit, voltage is same.

(a) $V = ?$ (b) $I_1, I_2, I_3 = ?$

(c) Power supplied by I_B.

Apply KCL

$$I_A + I_B = I_1 + I_2 + I_3$$

$$8 = \frac{V}{2} + \frac{V}{4} + \frac{V}{8}$$

$\Rightarrow$ 9.14 volts.

koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

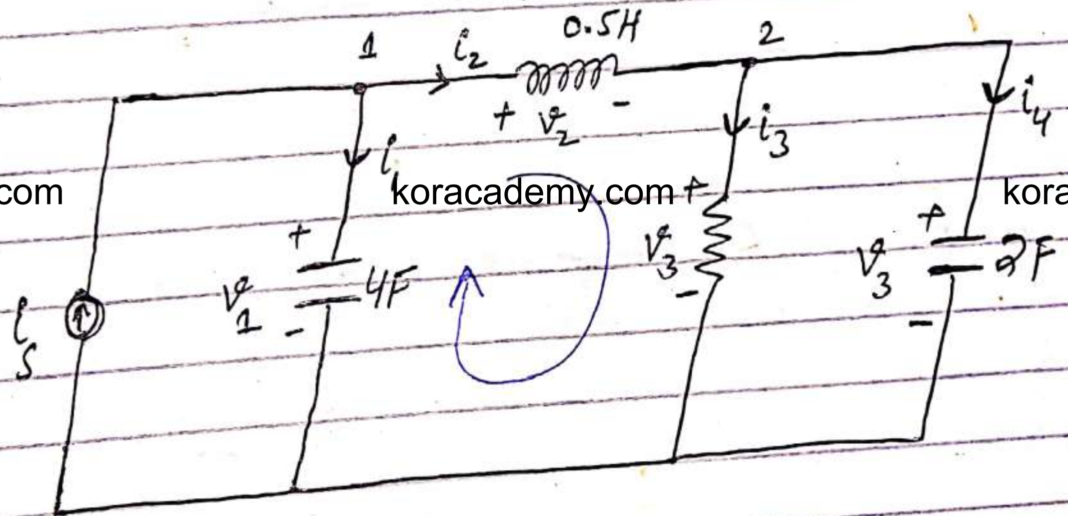
(b) $I_1 = \frac{V}{2} = \frac{9.14}{2} = 4.57\ A.$

$I_2 = \frac{V}{4} = \frac{9.14}{4} = 2.285\ A$

$I_3 = \frac{V}{8} = \frac{9.14}{8} = 1.1425$

(c) $P_B = V\,I_B = 9.14 \times 5 = 45.75$ watts.

Q29.



koracademy.com koracademy.com koracademy.com

$v_3 = 5e^{-t}$ volts

$i_4 = C\frac{dv_3}{dt} = -10e^{-t}\ A$

$i_3 = \frac{v_3}{R} = 15e^{-t}\ A$

Apply KCL to Node 2.

$i_2 = i_3 + i_4 = 5e^{-t}\ A$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$v_2 = L\frac{di_2}{dt} = -2.5e^{-t} \text{ volts}$$

let we suppose a loop current (which is actually not) and apply KVL.

$$v_1 = v_2 + v_3 = 2.5e^{-t} \text{ volts}$$

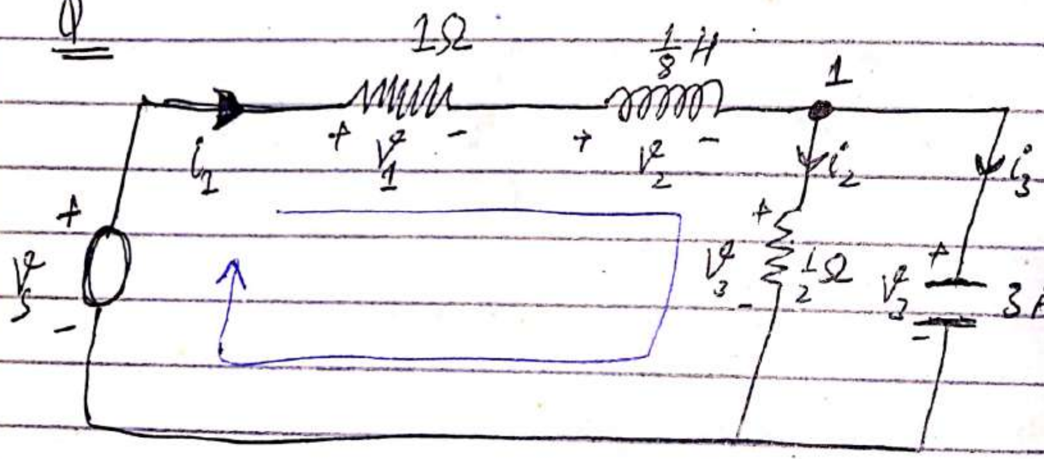
$$i_1 = C\frac{dv_1}{dt} = -10e^{-t} \text{ Amp.}$$

Apply KCL to node 1.

$$i_3 = i_2 + i_1 = -5e^{-t} \text{ Amp Ans}$$

koracademy.com koracademy.com koracademy.com

Q



$$i_2 = -20e^{-2t} \text{ Amp}$$

$$v_3 = i_2 R = -10e^{-2t} \text{ volts}$$

$$i_3 = C\frac{dv_3}{dt} = 60e^{-2t} \text{ Amp}$$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Apply KCL to node

$i_1 = i_2 + i_3 = 40e^{-2t}$ Amp

$v_1 = i_1 R = 40e^{-2t}$ volts

$v_2 = L\dfrac{di_1}{dt} = -10e^{-2t}$ volts.

Apply KVL to the loop.

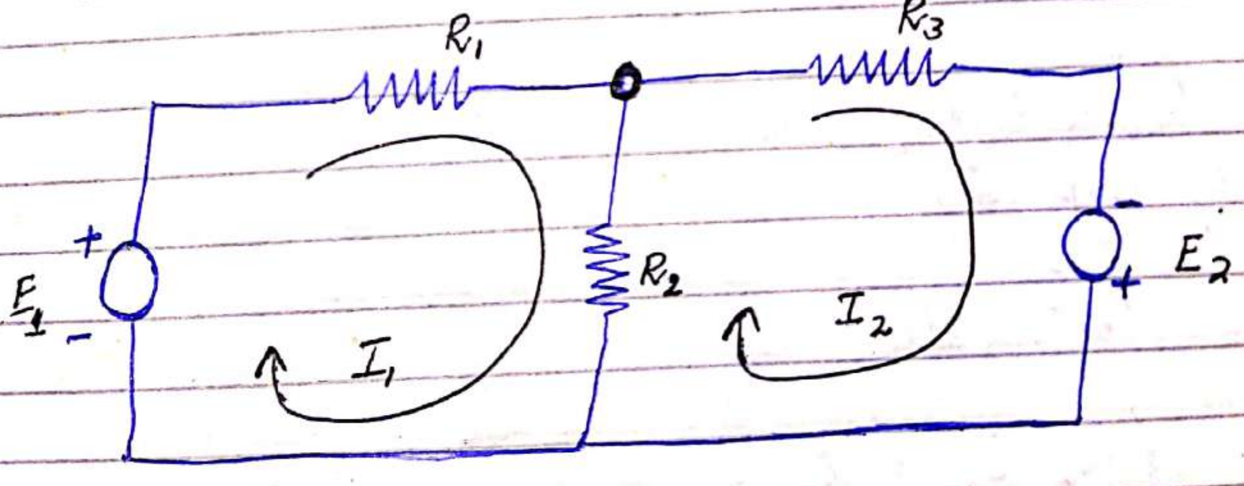
$v_s = v_1 + v_2 + v_3$

$= 40e^{-2t} - 10e^{-2t} - 10e^{-2t}$

$v_s = \underline{20e^{-2t}}$ volts.

koracademy.com koracademy.com koracademy.com

Standard loop Equations:



The current through R_2 is $I_1 - I_2$ whenever we consider it a member of loop 1. and is $I_2 - I_1$ when it is considered a member of loop 2.

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Apply KVL to loop 1.

Σ voltage rise = Σ voltage drop.

$$E_1 = I_1 R_1 + (I_1 - I_2) R_2$$
$$E_1 = I_1 R_1 + I_1 R_2 - I_2 R_2$$
$$E_1 = (R_1 + R_2) I_1 + (-R_2) I_2$$

$R_{11} = R_1 + R_2 \rightarrow$ The total self resistance of loop 1.

$$R_{12} = (-R_2)$$

$$\boxed{R_{11} I_1 + R_{12} I_2 = E_1} \rightarrow eq\,(A)$$

Apply KVL to loop 2.

koracademy.com koracademy.com koracademy.com

$$E_2 = I_2 R_3 + (I_2 - I_1) R_2$$
$$E_2 = I_2 R_3 + I_2 R_2 - I_1 R_2$$
$$E_2 = (R_3 + R_2) I_2 + (-R_2) I_1$$

$R_{22} = R_3 + R_2 \rightarrow$ the total self resistance of loop 2.

$$R_{21} = (-R_2)$$

$$\boxed{R_{21} I_1 + R_{22} I_2 = E_2} \rightarrow eq\,(B)$$

Eq (A) and Eq (B) are the standard loop equations.

The no. of standard loop equations depends upon the number of loops in the circuit.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

For 3 loops:

$$R_{11}I_1 + R_{12}I_2 + R_{13}I_3 = E_1$$
$$R_{21}I_1 + R_{22}I_2 + R_{23}I_3 = E_2$$
$$R_{31}I_1 + R_{32}I_2 + R_{33}I_3 = E_3$$

For n loops:

$$R_{11}I_1 + R_{12}I_2 + R_{13}I_3 + \cdots + R_{1n}I_n = E_1$$
$$R_{21}I_1 + R_{22}I_2 + R_{23}I_3 + \cdots + R_{2n}I_n = E_2$$
$$R_{31}I_1 + R_{32}I_2 + R_{33}I_3 + \cdots + R_{3n}I_n = E_3$$
$$\downarrow$$
$$\boxed{R_{n1}I_1 + R_{n2}I_2 + R_{n3}I_3 + \cdots + R_{nn}I_n = E_n}$$

koracademy.com koracademy.com koracademy.com

Eq(A) and Eq (B) in matrix form;

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

$$[R][I] = [V].$$

Size of R matrix $= 2\times2 \rightarrow 2$ loops.
$= 3\times3 \rightarrow 3$ loops
⋮

All the diagonal elements of R matrix will be positive.
The off diagonal elements may either be positive or negative depending on the direction of current.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

If the loop currents are in opposite direction → then negative.
If in the same direction → positive.

−ve to +ve → voltage rise →
→ + sign → with E.
+ve to −ve → voltage drop → −ve sign with E.

Cramer's Rule for finding current.

koracademy.com koracademy.com koracademy.com

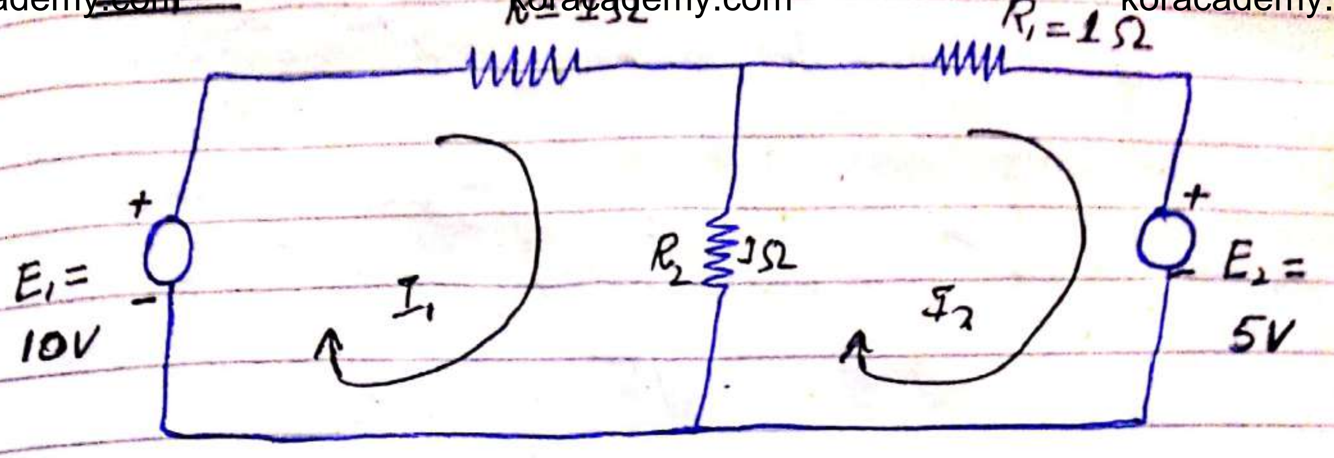
$$I_1 = \frac{\begin{vmatrix} E_1 & R_{12} \\ E_2 & R_{22} \end{vmatrix}}{\begin{vmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{vmatrix}}$$

$$I_2 = \frac{\begin{vmatrix} R_{11} & E_1 \\ R_{21} & E_2 \end{vmatrix}}{\begin{vmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{vmatrix}}$$

koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

Example :



$$R_{11} = 1 + 1 = 2\Omega$$
$$R_{12} = R_{21} = -R_2 = -1\Omega$$
$$R_{22} = 1 + 1 = 2\Omega$$

$E_1 \rightarrow$ voltage rise $\rightarrow 10V$

$E_2 \rightarrow$ voltage drop $\rightarrow -5V$

koracademy.com koracademy.com koracademy.com

$$I_1 = \frac{\begin{vmatrix} E_1 & R_{12} \\ E_2 & R_{22} \end{vmatrix}}{\begin{vmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{vmatrix}} = \frac{\begin{vmatrix} 10 & -1 \\ -5 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}} = \frac{15}{3} = \underline{\underline{5A}}$$

In matrix form

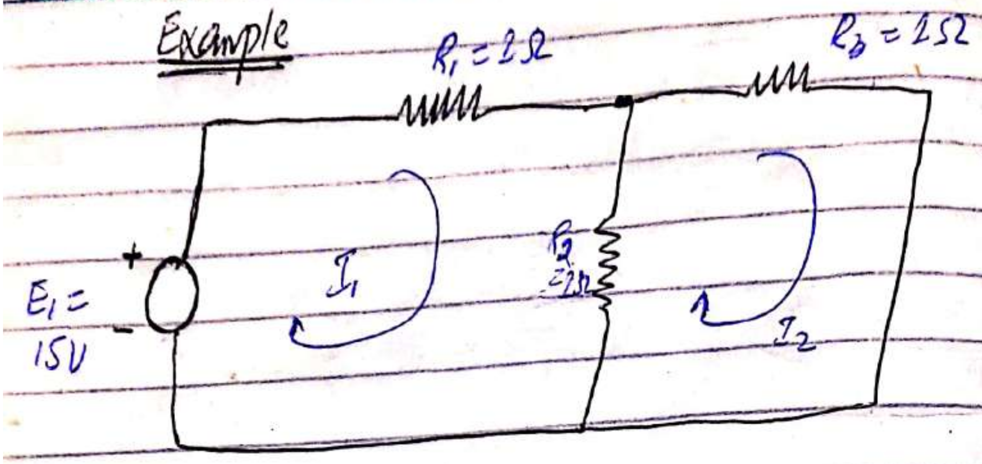
$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} E_1 = 10 \\ -5 \end{bmatrix}$$

$$I_2 = \frac{\begin{vmatrix} R_{11} & E_1 \\ R_{21} & E_2 \end{vmatrix}}{\begin{vmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{vmatrix}} = \frac{\begin{vmatrix} 2 & 10 \\ -1 & -5 \end{vmatrix}}{3} = \underline{\underline{0\ A}}$$

koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

Example



In matrix form;

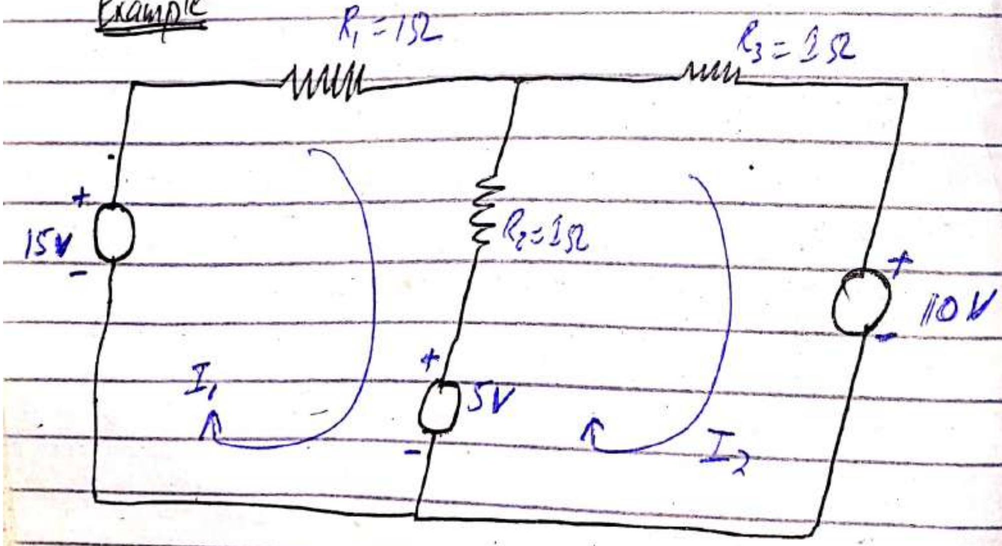
$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} 15 & -1 \\ 0 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}} = \frac{30}{3} = \underline{\underline{10A}}$$

koracademy.com koracademy.com koracademy.com

$$I_2 = \frac{\begin{vmatrix} 2 & 15 \\ -1 & 0 \end{vmatrix}}{3} = \frac{15}{3} = \underline{\underline{5A}}$$

Example



$E_1 \rightarrow$ total V in ~~exxx~~ loop 1
$E_2 \rightarrow$ total V in loop 2

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$E_1 = \overset{\text{rise }(+)}{15} - \overset{\text{drop }(-)}{5} = 10V$$

$$E_2 = -10 + 5 = -5V$$

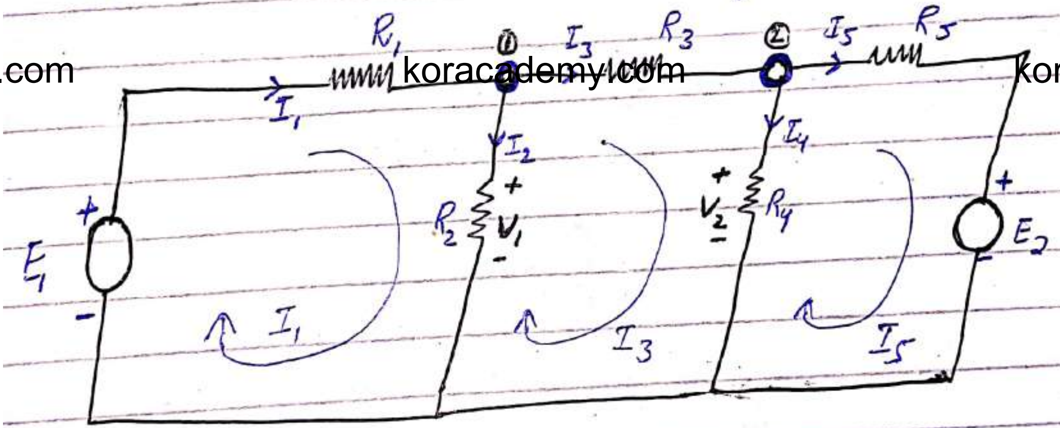
In matrix form

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ -5 \end{bmatrix}.$$

$\therefore$ Exple 2 $I_1 = 5A$ $I_2 = 0A$

Standard Node Equations:

Consider a circuit having 2 nodes.



koracademy.com koracademy.com koracademy.com

Apply KCL to node 1.

$$I_1 = I_2 + I_3 \rightarrow eq\ (i)$$

Apply KCL to node 2.

$$I_3 = I_4 + I_5 \rightarrow eq\ (ii)$$

Apply KVL to loop 1.

$$E_1 = I_1R_1 + V_1$$

$$I_1 = \frac{E_1 - V_1}{R_1}$$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$I_2 = \frac{V_2}{R_2}$$

Apply KVL to loop 2.

$$V_1 = I_3 R_3 + V_2$$

$$I_3 = \frac{V_1 - V_2}{R_3}$$

$$I_4 = \frac{V_2}{R_4}$$

Apply KVL to V_3

koracademy.com

$$V_2 = I_5 R_5 + E_2$$

koracademy.com koracademy.com

$$I_5 = \frac{V_2 - E_2}{R_5}$$

$$eq(i) \Rightarrow \quad \frac{E_1 - V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3}$$

$$\frac{E_1}{R_1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_1 + \left(\frac{-1}{R_3}\right)V_2$$

$\frac{E_1}{R_1} = \frac{\text{voltage}}{\text{resistance}}$ → gives unit of current → replace by I_{g1} → it is assumed → it does not exist in any part of the circuit.

conductance $G = \frac{1}{R}$ ℧ (mho)

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$I_{g_1} = (G_1 + G_2 + G_3)V_1 + (-G_3)V_2$$

The conductance of all resistors that are connected to node 1.

$$G_{11} = G_1 + G_2 + G_3$$

$G_3 \rightarrow$ common resistor $\rightarrow G_{12}$

$$\boxed{G_{11}V_1 + G_{12}V_2 = I_{g_1}} \rightarrow eq\,(A)$$

eq (ii) $\Rightarrow$ $$\frac{V_1 - V_2}{R_3} = \frac{V_2}{R_4} + \frac{V_2 - E_2}{R_5}$$

koracademy.com koracademy.com koracademy.com

$$\frac{E_2}{R_5} = \left(\frac{-1}{R_3}\right)V_1 + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)V_2$$

$$I_{g_2} = (-G_3)V_1 + (G_3 + G_4 + G_5)V_2$$

$G_{21} \rightarrow$ common resistor $\rightarrow G_3$

Total conductance in loop 2.

$$G_{22} = G_3 + G_4 + G_5$$

$$\boxed{I_{g_2} = G_{21}V_1 + G_{22}V_2} \rightarrow eq\,(B).$$

The number of node equations is equal to the number of nodes.

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

For 3 ~~loops~~ nodes

$$G_{11}V_1 + G_{12}V_2 + G_{13}V_3 = I_{g_1}$$
$$G_{21}V_1 + G_{22}V_2 + G_{23}V_3 = I_{g_2}$$
$$G_{31}V_1 + G_{32}V_2 + G_{33}V_3 = I_{g_3}$$

For n nodes

$$G_{11}V_1 + G_{12}V_2 + G_{13}V_3 + \ldots\ldots + G_{1n}V_n = I_{g_1}$$
$$G_{21}V_1 + G_{22}V_2 + G_{23}V_3 + \ldots\ldots + G_{2n}V_n = I_{g_2}$$
$$G_{31}V_1 + G_{32}V_2 + G_{33}V_3 + \ldots\ldots + G_{3n}V_n = I_{g_3}$$
$$\downarrow$$
$$\boxed{G_{n1}V_1 + G_{n2}V_2 + G_{n3}V_3 + \ldots\ldots + G_{nn}V_n = I_{g_n}}$$

koracademy.com koracademy.com koracademy.com

In matrix form;

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{g_1} \\ I_{g_2} \end{bmatrix}$$

$$[G][V] = [I]$$

Size of G matrix depends on number of nodes. The diagonal elements of G is always positive. The off diagonal elements will always be negative.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$\boxed{I_{g1} = \frac{E_1}{R_1}}$$ = voltage of voltage source connected to node 1 ÷ the resistance of series resistor

For standard node equation; there must be a series resistor b/w source and node.

$$\boxed{I_{g2} = \frac{E_2}{R_5}}$$

E_2 and second node are connected with the help of series resistor R_5.

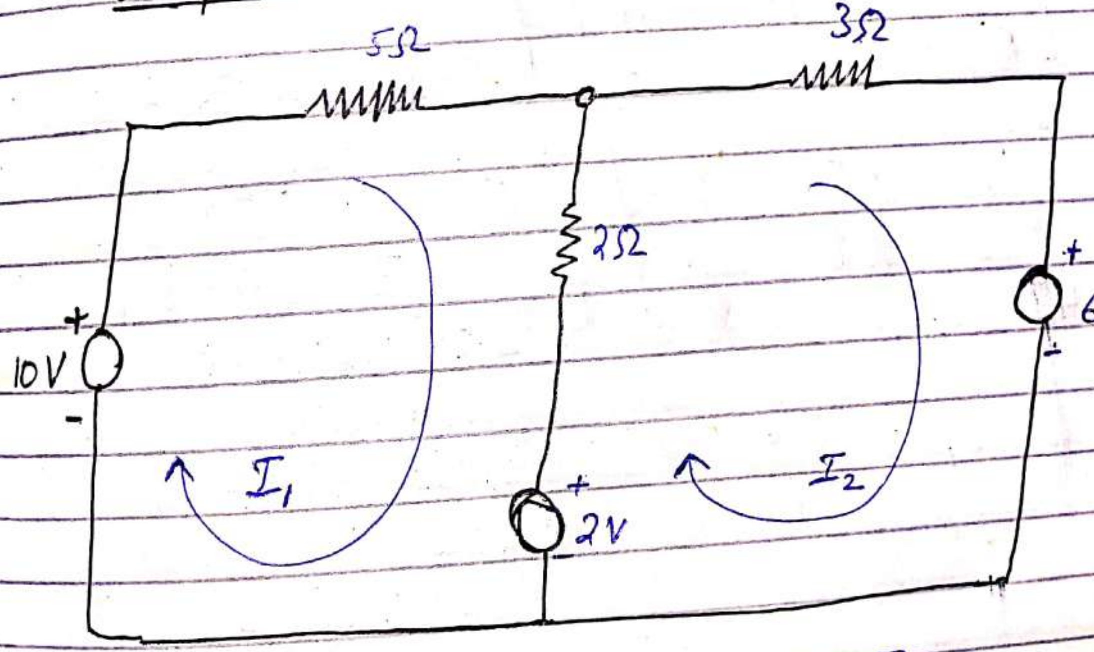
The + and − sign with I_{g1} and I_{g2} depends on the polarities.

If + polarity towards node → + sign

If − polarity towards node → − sign.

koracademy.com koracademy.com koracademy.com

Example



$$\begin{bmatrix} 7 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

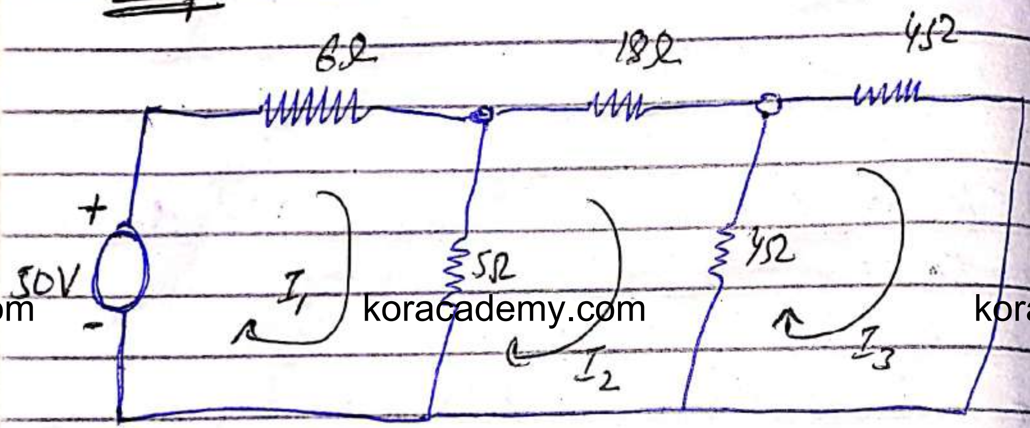
koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

$$I_1 = \frac{\begin{vmatrix} 8 & -2 \\ -4 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & -2 \\ -2 & 5 \end{vmatrix}} = \frac{32}{31} = 1.032\,A$$

$$I_2 = \frac{\begin{vmatrix} 7 & 8 \\ -2 & -4 \end{vmatrix}}{31} = \frac{-12}{31} = -0.387\,A$$

__Example__



koracademy.com koracademy.com koracademy.com

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

$$\begin{bmatrix} 11 & -5 & 0 \\ -5 & 27 & -4 \\ 0 & -4 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ 0 \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} 50 & -5 & 0 \\ 0 & 27 & -4 \\ 0 & -4 & 8 \end{vmatrix}}{\begin{vmatrix} 11 & -5 & 0 \\ -5 & 27 & -4 \\ 0 & -4 & 8 \end{vmatrix}} = \frac{10000}{2000} = 5A$$

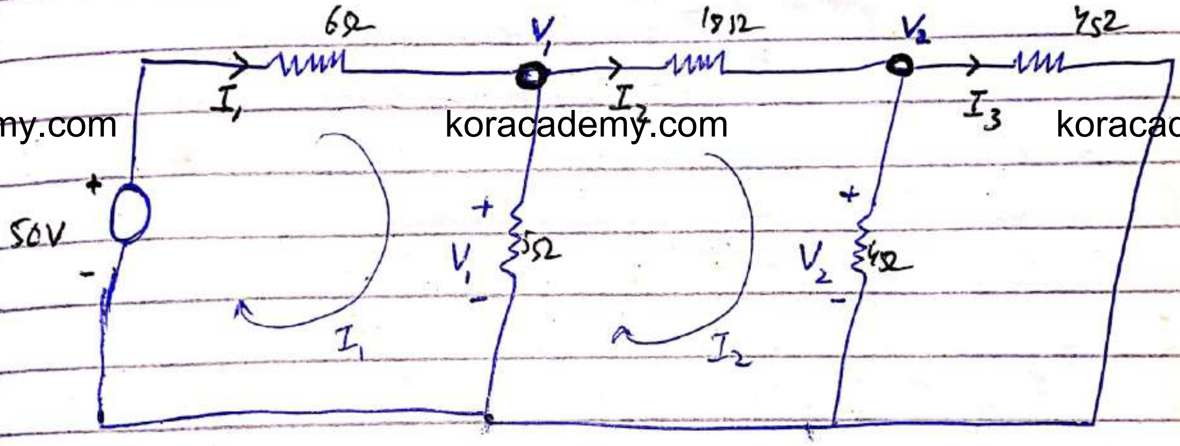
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$I_2 = \frac{\begin{vmatrix} 11 & 50 & 0 \\ -5 & 0 & -4 \\ 0 & 0 & 8 \end{vmatrix}}{2,000} = \frac{2000}{2000} = 1\,A$$

$$I_3 = \frac{\begin{vmatrix} 11 & -5 & 50 \\ -5 & 27 & 0 \\ 0 & -4 & 0 \end{vmatrix}}{2000} = \frac{1000}{2000} = 0.5\,A$$

The same example with standard node equations.



koracademy.com koracademy.com koracademy.com

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{g1} \\ I_{g2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{6} + \frac{1}{5} + \frac{1}{18} & -\frac{1}{18} \\ -\frac{1}{18} & \frac{1}{18} + \frac{1}{4} + \frac{1}{4} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 50/6 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.422 & -0.0555 \\ -0.555 & 0.555 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 8.33 \\ 0 \end{bmatrix}$$

Apply cramers rule to calculate V_1 and V_2.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$V_1 = \frac{\begin{vmatrix} 8.33 & -0.0555 \\ 0 & 0.555 \end{vmatrix}}{\begin{vmatrix} 0.422 & -0.0555 \\ -0.0555 & 0.555 \end{vmatrix}} = 20 \text{ Volts}$$

$$V_2 = \frac{\begin{vmatrix} 0.422 & 8.33 \\ -0.0555 & 0 \end{vmatrix}}{\begin{vmatrix} 0.422 & -0.0555 \\ -0.555 & 0.555 \end{vmatrix}} = 2 \text{ Volts.}$$

~~Apply KVL to loop 1~~

koracademy.com koracademy.com koracademy.com

$$50 = 6 I_1 + V_1$$

$$I_1 = \frac{50 - V_1}{6} = \frac{50 - 20}{6}$$

$$\boxed{I_1 = 5A}.$$

KVL to loop 2 $\Rightarrow$ $V_1 = 18 I_2 + V_2$

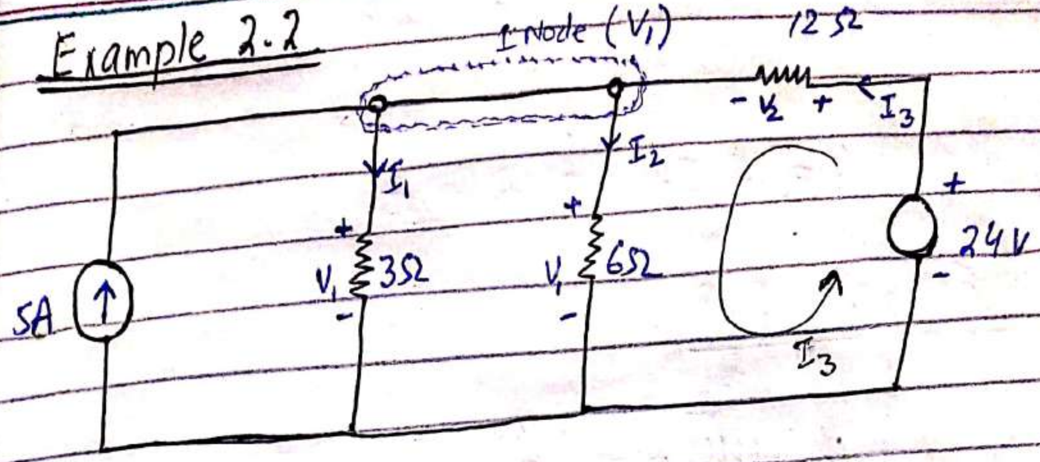
$$2 - 20 = 18 I_2 \Rightarrow \boxed{I_2 = 1A}$$

$$I_3 = \frac{V_2}{4} = \frac{2}{4} \Rightarrow \boxed{I_3 = 0.5A}$$

koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

Example 2.2



$$[G_{11}][V_1] = [I_{g1}]$$

$G_{11} \rightarrow$ the conductance of resistors connected to node 1.

if current is moving towards junction will be positive and vice versa. (for current source).

koracademy.com koracademy.com koracademy.com

$$I_{g1} = 5 + \frac{24}{12} = 7A$$

$$\left[\frac{1}{3} + \frac{1}{6} + \frac{1}{12}\right][V_1] = 7$$

$$V_1 = 12\ V$$

$$I_1 = \frac{V_1}{3} = \frac{12}{3} \Rightarrow I_1 = 4A$$

$$I_2 = \frac{V_1}{6} = \frac{12}{6} \Rightarrow I_2 = 2A$$

For calculating I_3, apply KVL to loop 3

$$24 = 12I_3 + V_1$$

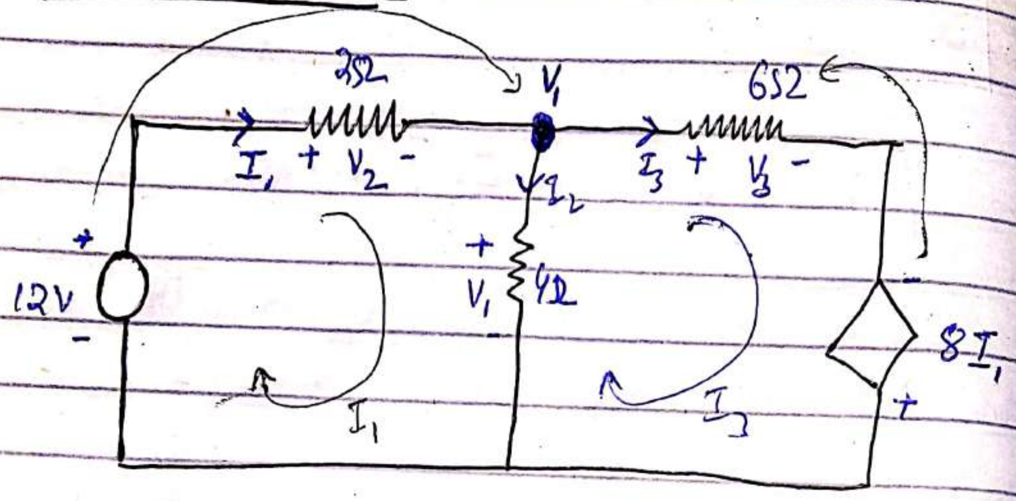
koracademy.com koracademy.com
Scanned with CamScanner
koracademy.com

koracademy.com koracademy.com koracademy.com

$$I_3 = \frac{24 - V_1}{12} = 1\,A$$

$$V_2 = 12\, I_3 = 12 \text{ volts.}$$

Example 2.3 :



koracademy.com koracademy.com koracademy.com

$$[G_{11}][V_1] = [I_{S1}]$$

$$\left[\frac{1}{2} + \frac{1}{4} + \frac{1}{6}\right][V_1] = \left[\frac{12}{2} - \frac{8I_1}{6}\right]$$

$$\left[\frac{1}{2} + \frac{1}{4} + \frac{1}{6}\right][V_1] = \left[6 - \frac{8}{6} I_1\right] \rightarrow eq(ii)$$

Apply KVL to loop 1 to find I_1

$$12 = 2 I_1 + V_1$$

$$\Rightarrow I_1 = \frac{12 - V_1}{2} \rightarrow eq(iii)$$

Put the value of I_1 in Eq(ii)

$$\left[\frac{1}{2} + \frac{1}{4} + \frac{1}{6}\right][V_1] = 6 - \frac{\cancel{8}^{4}}{6}\left(\frac{12 - V_1}{\cancel{2}}\right)$$

$$\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} - \frac{4}{6}\right)[V_1] = -2$$

$$V_1 = -8 \text{ volts.}$$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

eq (iii) ⇒ $I_1 = \frac{12+8}{2} = \underline{\underline{10\,A}}$

$I_2 = \frac{V_1}{4} = -2A$

Apply KVL to loop 2 & find I_3

$V_1 + 8I_1 = 6I_3$

$\frac{-8+80}{6} = I_3 = 12\,A$

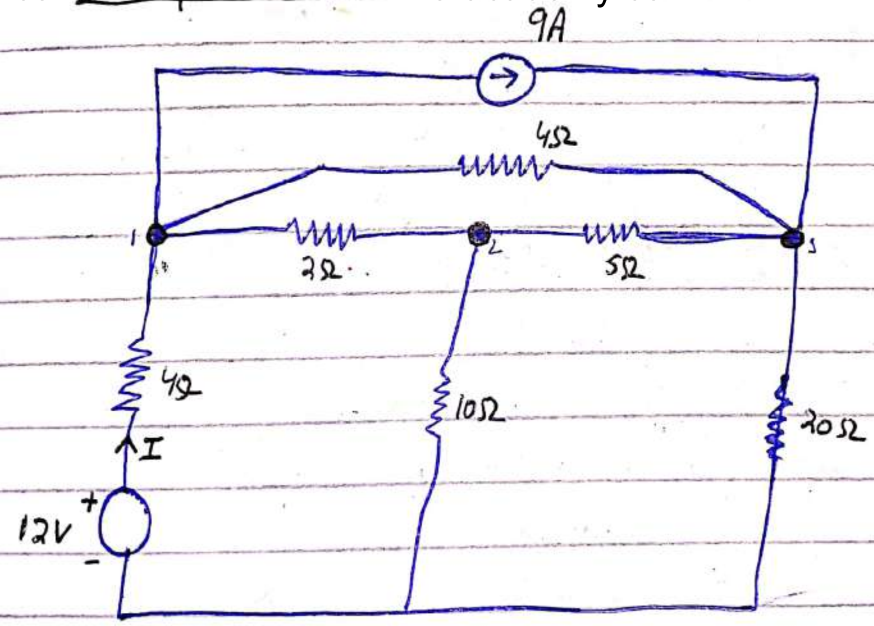
$V_2 = 2I_1 = 2 \times 10 = 20$ volts.

$V_3 = 6I_3 = 72$ volts.

koracademy.com

Example 2.5:

koracademy.com koracademy.com



$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_{s_1} \\ I_{s_2} \\ I_{s_3} \end{bmatrix}$$

$G_{11} = \frac{1}{4} + \frac{1}{4} + \frac{1}{2} = 1\ \mho$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$G_{12} = G_{21} = \frac{-1}{2} = -0.5\ \mho$$

$$G_{13} = G_{31} = \frac{-1}{4} = -0.25\ \mho$$

$$G_{22} = \frac{1}{2} + \frac{1}{10} + \frac{1}{5} = 0.8\ \mho$$

$$G_{23} = G_{32} = -\frac{1}{5} = -0.2\ \mho$$

$$G_{33} = \frac{1}{4} + \frac{1}{5} + \frac{1}{20} = 0.5\ \mho$$

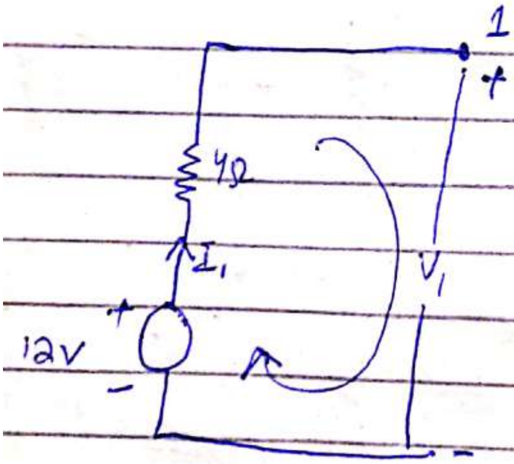
$$I_{g1} = \frac{12}{4} - 9 = -6A$$

$I_{g2} = 0A \qquad I_{g3} = 9A$

koracademy.com koracademy.com koracademy.com

$$\begin{bmatrix} 1 & -0.5 & -0.25 \\ -0.5 & 0.8 & -0.2 \\ -0.25 & -0.2 & 0.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 0 \\ 9 \end{bmatrix}$$

By cramers rule $V_1 = 4$ volts



Apply KVL to this loop

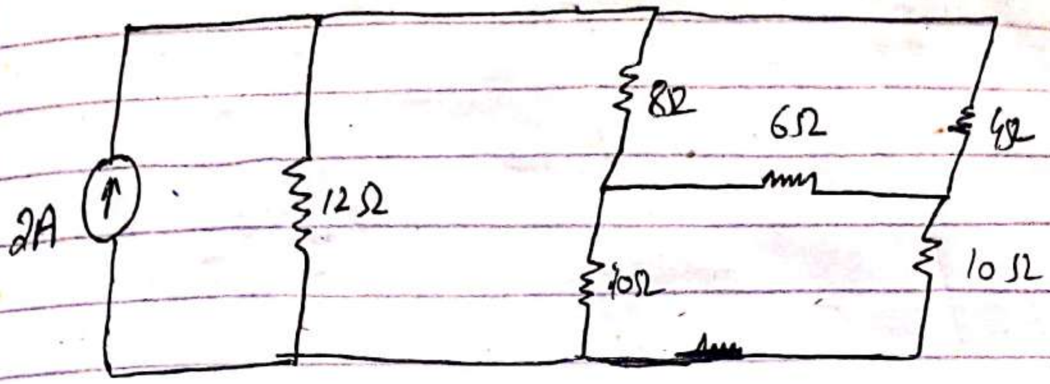
$$12 = 4I + V_1$$

$$I = \frac{12 - V_1}{4} = \frac{8}{4} = 2A$$

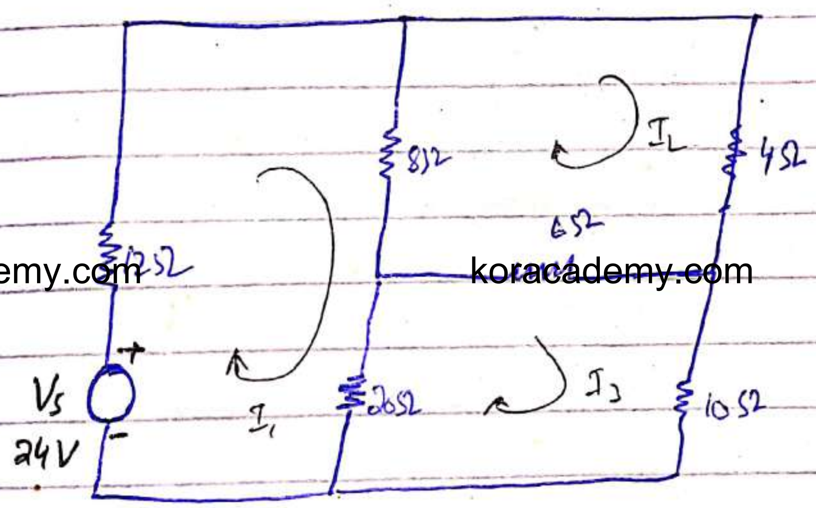
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 2.6:



We convert the current source into voltage source.



koracademy.com koracademy.com koracademy.com

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

$$\begin{bmatrix} 40 & -8 & -20 \\ -8 & 18 & -6 \\ -20 & -6 & 36 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 0 \\ 0 \end{bmatrix}$$

By Cramer's rule.

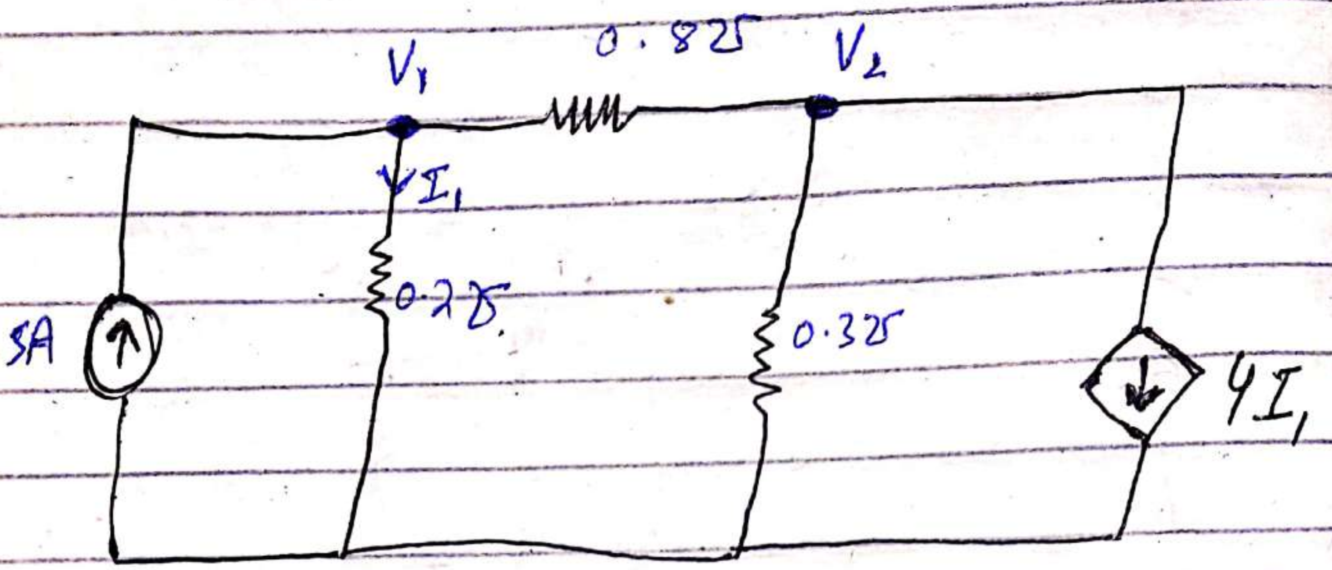
$I_1 = 1.125\ A \quad I_2 = 0.75\ A \quad I_3 = 0.75\ A.$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 2.7.

$V_{0.8\mho} = ?$



$$\begin{bmatrix} G_{11} = 1 & G_{12} = -0.8 \\ G_{21} = -0.8 & G_{22} = 1.1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{S1} = 5 \\ I_{S2} = -4I_1 \end{bmatrix}$$

$= -0.8V_1$

$$I_1 = \frac{V_1}{R} = GV_1 = (0.2)V_1$$

$$V_1 - 0.8V_2 = 5 \rightarrow (A)$$

$$-0.8V_1 + 1.1V_2 = -0.8V_1 \rightarrow (B)$$

$$V_2 = 0$$

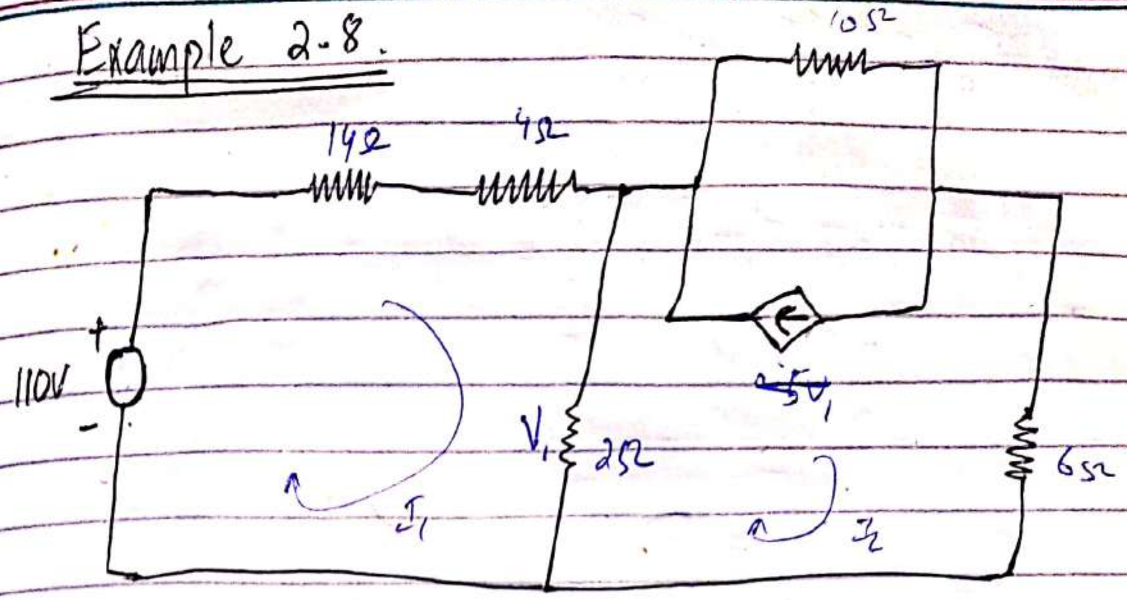
$$V_1 = 5V$$

$$V_{0.8\mho} = V_1 - V_2 = \underline{\underline{5V}}.$$

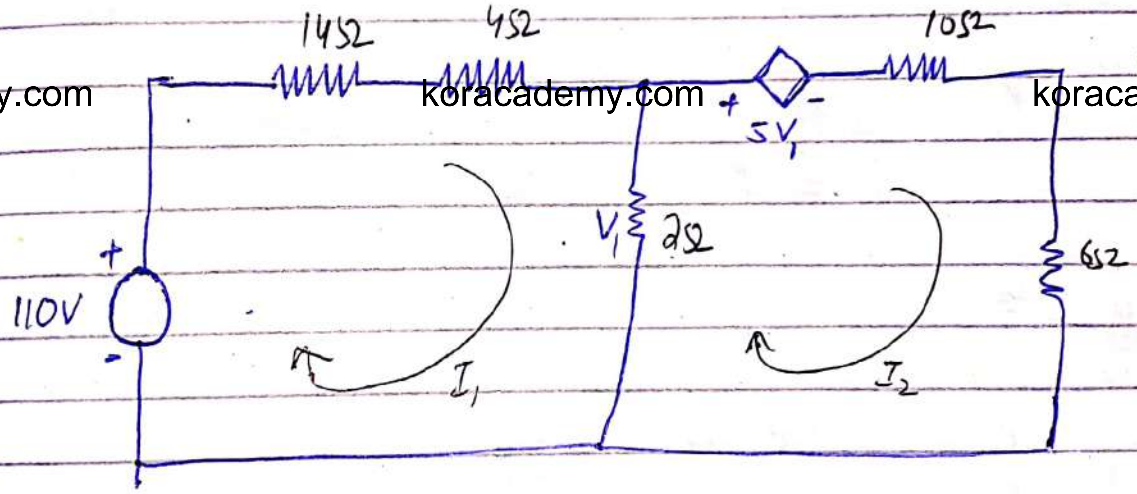
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 2-8.

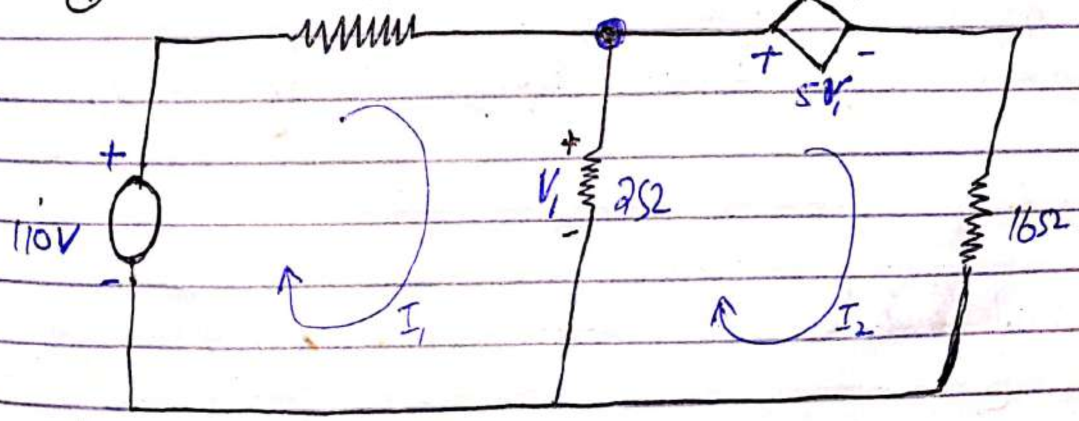


When there is a dependent current source and a resistor is connected to it in parallel, it can be converted to a dependent voltage source.



koracademy.com koracademy.com koracademy.com

or



$$[G_{11}][V_1] = [I_{S1}]$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$I_{g1} = \frac{110}{18} + \frac{5V_1}{16}$$

$$\left[\frac{1}{18} + \frac{1}{2} + \frac{1}{16}\right][V_1] = \left[\frac{110}{18} + \frac{5V_1}{16}\right]$$

$$0.306\,V_1 = \frac{110}{18}$$

$$\boxed{V_1 = 20\,V}.$$

Apply KVL to loop 1.

$$110 = 18 I_1 + V_1$$

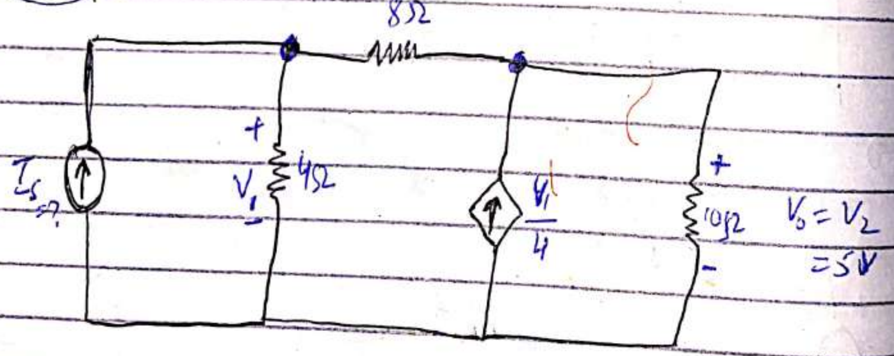
$$\Rightarrow I_1 = \frac{110 - 20}{18} \Rightarrow \boxed{I_1 = 5A}$$

koracademy.com koracademy.com koracademy.com

Apply KVL to loop 2

$$V_1 = 5V_1 + 16 I_2 \Rightarrow \boxed{I_2 = -5A}$$

Q1.33



$$\begin{bmatrix} 1/4 + 1/8 & -1/8 \\ -1/8 & \frac{1}{8} + 1/10 \end{bmatrix}\begin{bmatrix} V_1 \\ 5 \end{bmatrix} = \begin{bmatrix} I_s \\ V_1/4 \end{bmatrix}$$

$$\begin{bmatrix} 0.375 & -0.125 \\ -0.125 & 0.225 \end{bmatrix}\begin{bmatrix} V_1 \\ 5 \end{bmatrix} = \begin{bmatrix} I_s \\ 0.25V_1 \end{bmatrix}$$

koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com

$0.375 V_1 - 0.625 = I_s \rightarrow (1)$

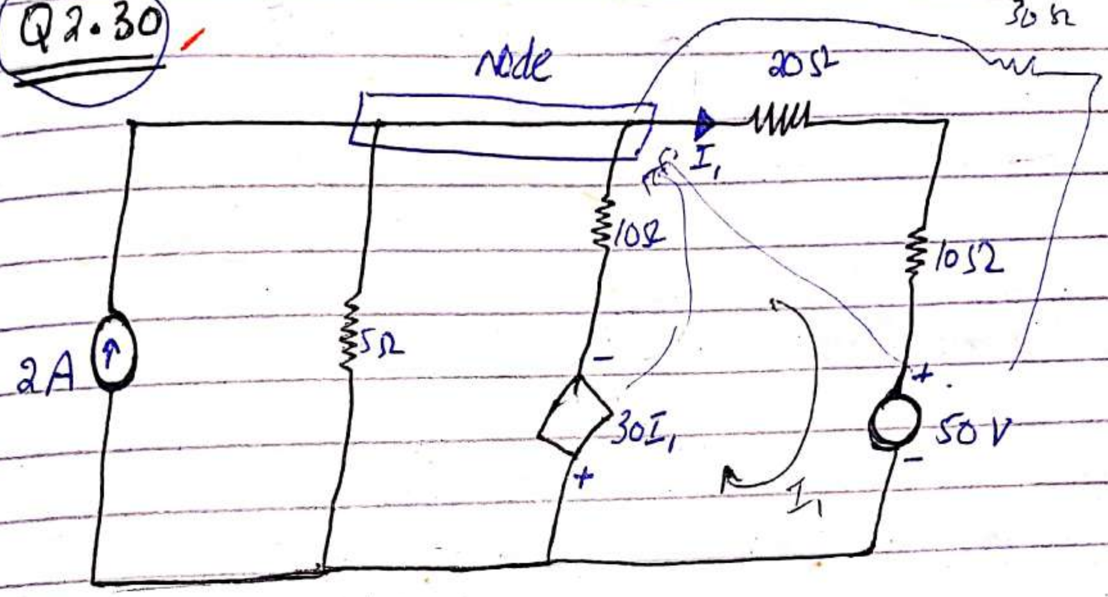
koracademy.com koracademy.com

$-0.125 V_1 + 1.125 = 0.25 V_1 \rightarrow (2)$

$(2) \Rightarrow V_1 = 3\,V$

$(1) \Rightarrow I_s = 0.5\,A$

Q2.30



koracademy.com koracademy.com koracademy.com

Replace the series combination of 20Ω and 10Ω by a single equivalent resistor

$$[G_{11}][V_1] = [I_g]$$

$$\left[\tfrac{1}{5} + \tfrac{1}{10} + \tfrac{1}{30}\right][V_1] = \left[2 + \tfrac{5}{3} - 3I_1\right]$$

Apply KVL to loop 3

$$V_1 = 30 I_1 + 50 \Rightarrow I_1 = \frac{V_1 - 50}{30}$$

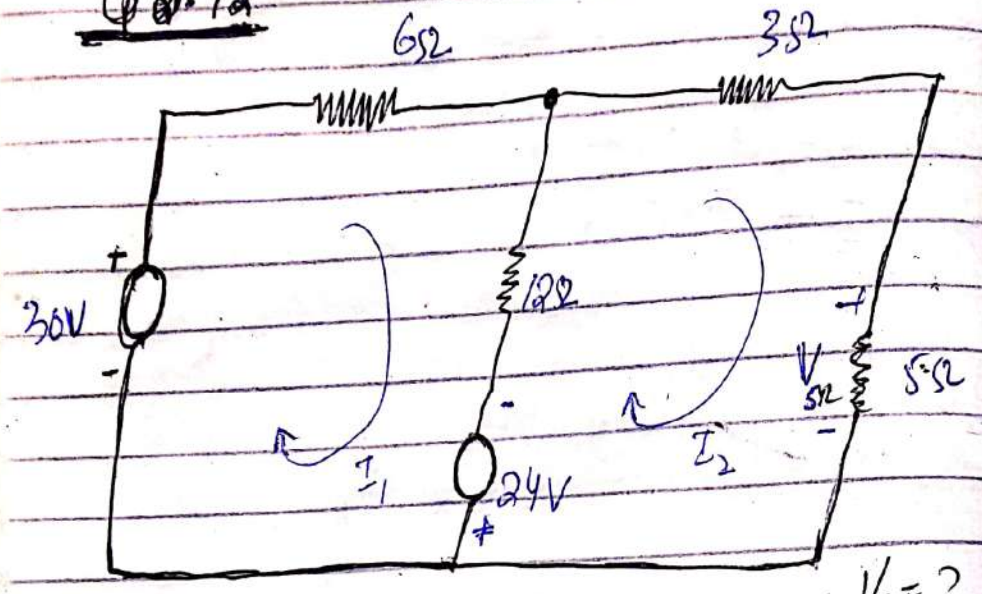
$$\left[\tfrac{1}{5} + \frac{1}{10} + \frac{1}{30}\right][V_1] = \left[2 + \frac{5}{3} - \left(\frac{V_1 - 50}{30}\right)\right]$$

$$\Rightarrow V_1 = 20\text{ volt}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Q 2.12



$V_{5\Omega} = ?$

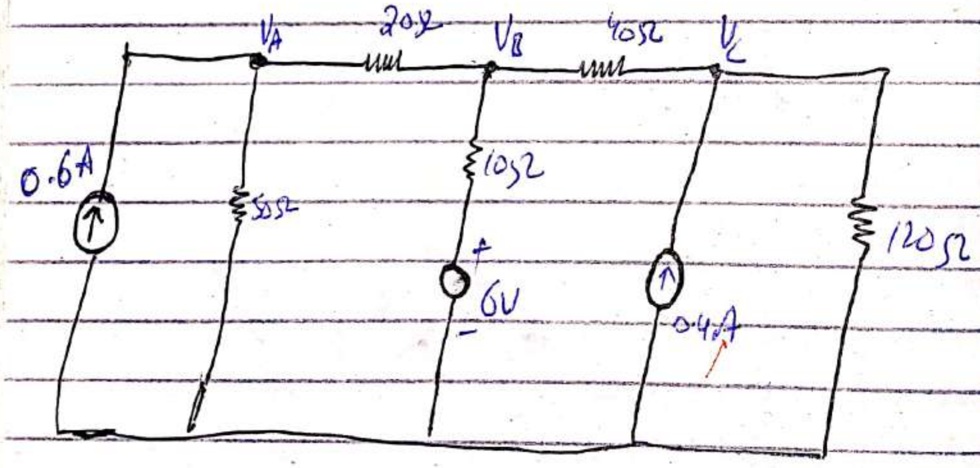
$$\begin{bmatrix} 18 & -12 \\ -12 & 20 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 54 \\ -24 \end{bmatrix}$$

$I_2 = 1A \qquad V_{5\Omega} = I_2 R$

$= \underline{5\ \text{volt}}$

koracademy.com koracademy.com koracademy.com

Q:2-20



$$\begin{bmatrix} 1/50 + 1/20 & -1/20 & 0 \\ -1/20 & 1/20 + 1/10 + 1/40 & -1/40 \\ 0 & -1/40 & 1/40 + 1/120 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.6 \\ 0.4 \end{bmatrix}$$

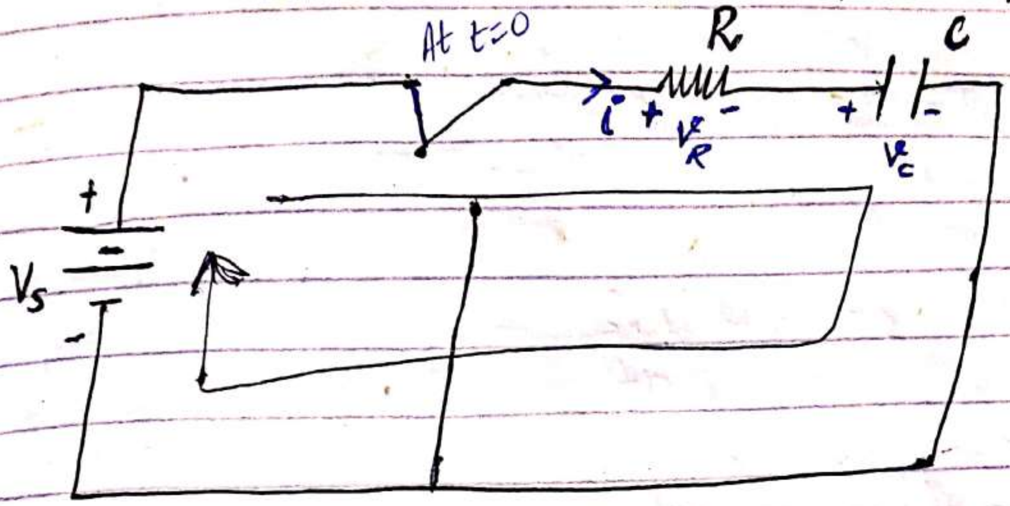
$V_B = ?$

By cramers rule.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Charging And Discharging of a Capacitor



When the capacitor is fully charged then it blocks DC.

let initially there is no voltage across this capacitor.

koracademy.com koracademy.com koracademy.com

At $t=0$, $V_c = 0$

when the switch is closed, current flows through the circuit (call it charging current). When the capacitor fully charges; there is no current in the circuit; so call it a time varying current.

Apply KVL to the only loop.

$$V_s = V_R + V_c \rightarrow ①$$

$$V_R = iR$$

Same current in all element (as series circuit)

$$i = C\frac{dV_c}{dt}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$V_R = RC\frac{dV_C}{dt}$$

$RC \to$ time constant $= \tau$

fully charging $5\tau - 6\tau$

$$① \Rightarrow V_s = RC\frac{dV_C}{dt} + V_C$$

$$\int -\frac{dV_C}{V_s - V_C} = \int -\frac{dt}{RC}$$

multiply by -1 and integrate.

$$\boxed{\ln(V_s - V_C) = \frac{-t}{RC} + K} \to ②$$

koracademy.com koracademy.com koracademy.com

Use the initial condition ie, At $t=0$, $V_C=0$

$$\ln(V_s) = 0 + K$$

$$② \Rightarrow \ln(V_s - V_C) = \frac{-t}{RC} + \ln(V_s)$$

$$\ln\left(\frac{V_s - V_C}{V_s}\right) = \frac{-t}{RC}$$

Antilog

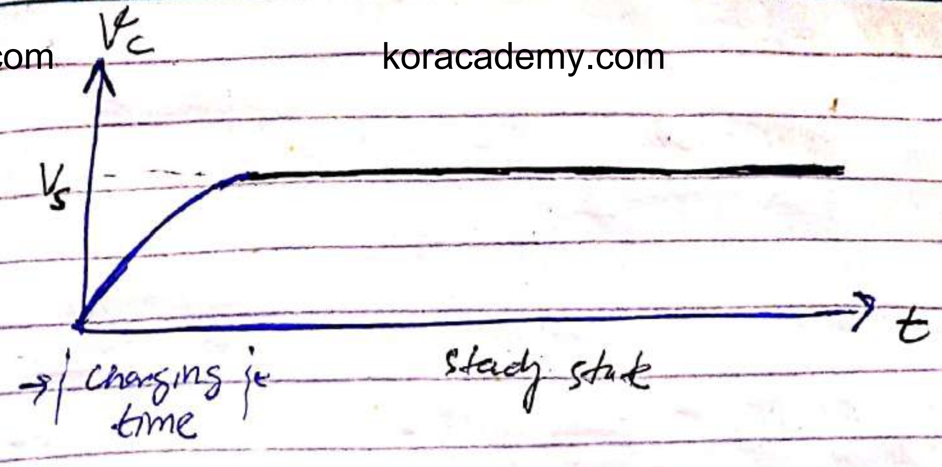
$$\frac{V_s - V_C}{V_s} = e^{-t/RC}$$

$$V_C = V_s - V_s e^{-t/RC}$$

$$\boxed{V_C = V_s\left(1 - e^{-t/RC}\right)}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

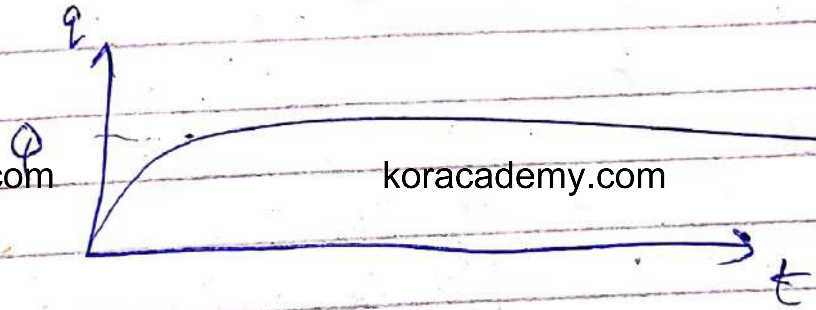


multiply by c

$$CV_c = CV_s\left(1 - e^{-t/RC}\right)$$

$$\boxed{q = Q\left(1 - e^{-t/RC}\right)}$$

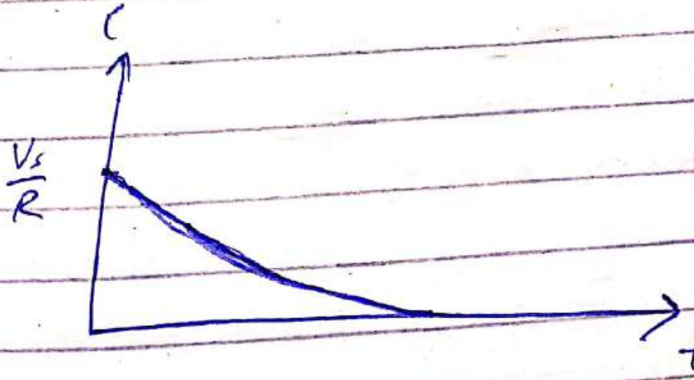
q → instantaneous charge   Q → max const charge



koracademy.com koracademy.com koracademy.com

$$i = \frac{dq}{dt} = \frac{CV_s}{RC}\, e^{-t/RC}$$

$$\boxed{i = \frac{V_s}{R}\, e^{-t/RC}}$$



koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 1.

$R = 1\times10^{6}\ \Omega \quad C = 2\times10^{-6} F \quad V_s = 100V$

(i) The initial value of i.
(ii) The time constant. (iii) The charging current after 2s. (iv) The V_c after 2s.
(v) The time taken by C so that $V_c = 80V$

(i) $i = \frac{V_s}{R} e^{-t/RC}$ At $t=0$, $e^{0} = 1$

$\Rightarrow i = \frac{V_s}{R} = \frac{100}{1}\times10^{-6} = \boxed{100\,\mu A}$

(ii) $\tau = RC = \boxed{2s}$

koracademy.com koracademy.com koracademy.com

(iii) $t = 2s \Rightarrow i = \frac{100}{1}\times10^{-6}\times e^{-2/2}$

$\Rightarrow \boxed{i = 36.78A}$

(iv) $V_c = V_s(1 - e^{-t/RC}) = 100(1 - e^{-1})$

$\boxed{V_c = 63.21 \text{ volts}}$

(v) $V_c = V_s(1 - e^{-t/RC})$

$80 = 100(1 - e^{-t/2})$

$1 - e^{-t/2} = 0.8 \Rightarrow e^{-t/2} = 0.2$

Taking log $\frac{-t}{2} = \ln(0.2)$
$\downarrow$
-1.6

$\Rightarrow \boxed{t = 3.2s}$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

(vi) The voltage V_c after $5\tau = 10\,sec$

$$V_c = 100\left(1 - e^{-5}\right) = \boxed{99.3 \text{ volts}}$$

Example 2

$R = 1 \times 10^{6}\,\Omega \quad C = 2 \times 10^{-6}\,F \quad V_s = 100V$

At $t = 0$, $V_c = V_0 = 10$ volts.

$$\ln\left(V_s - V_c\right) = \frac{-t}{RC} + K \rightarrow (i)$$

Initial condition, $V_c = V_0$ at $t = 0$

$$\ln\left(V_s - V_0\right) = 0 + K$$

$$(i) \Rightarrow \ln\left(V_s - V_c\right) - \ln\left(V_s - V_0\right) = \frac{-t}{RC}$$

koracademy.com koracademy.com koracademy.com

$$\ln\left(\frac{V_s - V_c}{V_s - V_0}\right) = -\frac{t}{RC}$$

Take Antilog

$$\frac{V_s - V_c}{V_s - V_0} = e^{-t/RC}$$

$$V_s - V_c = \left(V_s - V_0\right)e^{-t/RC}$$

$$\boxed{V_c = V_s - \left(V_s - V_0\right)e^{-t/RC}}$$

$$\boxed{CV_c = CV_s - C\left(V_s - V_0\right)e^{-t/RC}}$$

Differentiate it w.r.t t

$$\boxed{i = \frac{V_s - V_0}{R}\, e^{-t/RC}}$$

koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

Example 2 solution

(i) $\tau = RC = 2\text{sec}$

(ii) $i = \frac{V_s - V_0}{R} e^{-t/RC}$

At $t=0 \Rightarrow e^0 = 1$

$i = \frac{V_s - V_0}{R} = \boxed{90\,\mu A}$

(iii) Current after 2s

koracademy.com koracademy.com koracademy.com

$i = \frac{90}{1} \times 10^{-6} \times e^{-2/2} = \boxed{33\,\mu A}$

(iv) V_c after 2s.

$V_c = V_s - (V_s - V_0)\, e^{-t/RC}$

$V_c = 100 - 90\, e^{-1} = \boxed{66.89V}$

koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

Phasor

Phasor is a complex quantity.
They behave like vectors.
They may be vector or scalar depending on the original quantity they are representing. If the original quantity is scalar, the phasor representing it will also be a scalar and hence so for vectors.

Phasor → complex quantity → Real component $= a$
→ Imaginary component $= b$

$$z = a + jb$$

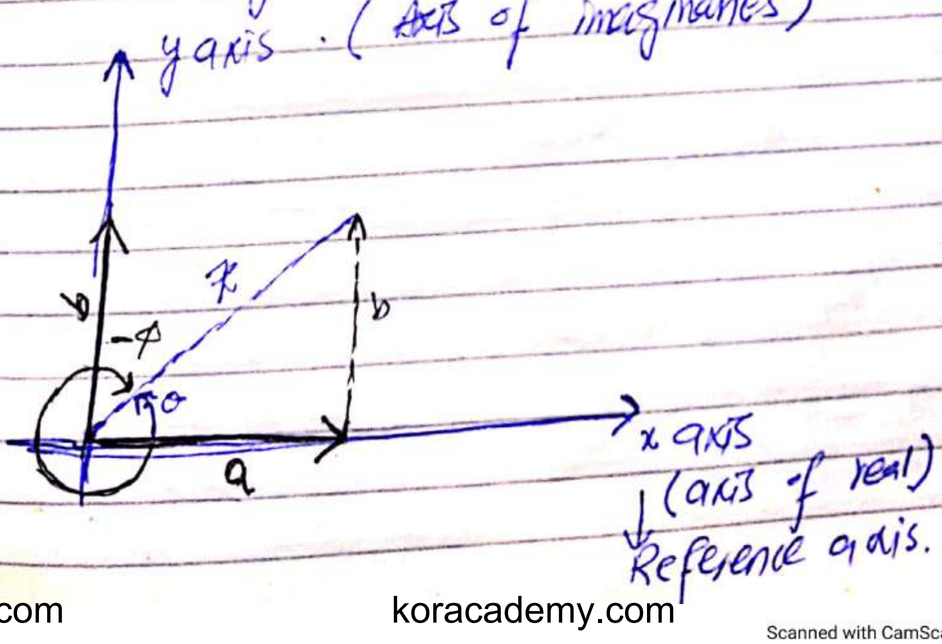
$$j = i = \sqrt{-1} \rightarrow \text{operator}$$

$$j^2 = i^2 = -1$$

koracademy.com koracademy.com koracademy.com

If the entire phasor behaves as vector quantity, both its components will behave like vectors.

Phasor Diagram



koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Any phasor (vector) has two important parameters ; the magnitude and direction (angle with the x axis).
clockwise angle → -ve anticlockwise → +ve

Representation of A Phasor.

(i) Rectangular form (ii) Trignometric or circular form (iii) Exponential form.
(iv) Polar form.

(i) Rectangular form
Real component and imaginary component.

$$\boxed{z = a + bj}$$

koracademy.com koracademy.com koracademy.com

(ii) Trignometric form

$|z|$ = magnitude of the phasor.
$\theta \rightarrow$ Angle of the phasor.

$a = |z| \cos\theta \qquad b = |z| \sin\theta$

$$z = a + jb \Rightarrow z = |z|\cos\theta + j|z|\sin\theta$$

$$\boxed{z = |z|\left[\cos\theta + j\sin\theta\right]}$$

$$|z| = \sqrt{a^2+b^2}$$

$$\tan\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right).$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

(iii) Exponential Form

Consider the trignometric form.

$$z = |z| \left[\cos\theta + j\sin\theta\right].$$

From Eueler's equations;

$$\cos\theta + j\sin\theta = e^{j\theta}.$$

$$\cos\theta - j\sin\theta = e^{-j\theta}.$$

$$\boxed{z = |z|\, e^{j\theta}}$$

(iv) Polar Form

The magnitude and angle ~~and~~ are given independently.

$$\boxed{|z| \angle \theta}$$

(this is not a product)

koracademy.com koracademy.com koracademy.com

Example

$z = 3 + j4$ → rectangular

$|z| = 5 \qquad \theta = 53.1^\circ$

$z = 5\left[\cos 53.1^\circ + j\sin 53.1^\circ\right]$ → trignometric

$z = 5e^{j53.1}$ → exponential

$z = 5 \angle 53.1^\circ$ → polar.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Addition of Phasors.

$$Z_1 = |Z_1|e^{j\theta_1} = |Z_1|\angle\theta_1 = a_1 + jb_1$$
$$Z_2 = |Z_2|e^{j\theta_2} = |Z_2|\angle\theta_2 = a_2 + jb_2$$

$$Z_a = Z_1 + Z_2 = |Z_1|e^{j\theta_1} + |Z_2|e^{j\theta_2}$$
$$= |Z_1|\angle\theta_1 + |Z_2|\angle\theta_2$$

$$Z_a = Z_1 + Z_2 = (a_1 + jb_1) + (a_2 + jb_2)$$
$$= (a_1 + a_2) + j(b_1 + b_2)$$ Rectangular form.

$$|Z_a| = \sqrt{(a_1+a_2)^2 + (b_1+b_2)^2}$$

koracademy.com koracademy.com koracademy.com

$$\theta_a = \tan^{-1}\frac{(b_1+b_2)}{(a_1+a_2)}$$

$$Z_a = |Z_a|e^{j\theta_a} = |Z_a|\angle\theta_a$$

Substraction of Phasors

$$Z_s = (a_1 - a_2) + (b_1 - b_2)j$$

$$|Z|_s = \sqrt{(a_1-a_2)^2 + (b_1-b_2)^2}$$

$$\theta_s = \tan^{-1}\frac{(b_1-b_2)}{(a_1-a_2)}$$

$$Z_s = |Z_s|e^{j\theta_s} = |Z_s|\angle\theta_s$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Multiplication of Phasor

$$Z_1 = |Z_1| e^{j\theta_1} = |Z_1| \angle \theta_1 = a_1 + jb_1$$
$$Z_2 = |Z_2| e^{j\theta_2} = |Z_2| \angle \theta_2 = a_2 + jb_2$$

$$Z_m = Z_1 \times Z_2 = (a_1 + jb_1)(a_2 + jb_2)$$
$$= (a_1 a_2 - b_1 b_2) + j(a_1 b_2 + a_2 b_1)$$

$$|Z_m| = \sqrt{(a_1 a_2 - b_1 b_2)^2 + (a_1 b_2 + a_2 b_1)^2}$$

$$\theta_m = \tan^{-1} \frac{(a_1 b_2 + a_2 b_1)}{(a_1 a_2 - b_1 b_2)}$$

koracademy.com koracademy.com koracademy.com

$$Z_m = |Z_m| \angle \theta_m = |Z|_m e^{j\theta_m}$$

In exponential form

$$Z_m = Z_1 \times Z_2 = |Z_1| e^{j\theta_1} \times |Z_2| e^{j\theta_2}$$
$$= |Z_1||Z_2| e^{j\theta_1 + j\theta_2} = \underbrace{|Z_1||Z_2|}_{|Z_m|} e^{j\overbrace{(\theta_1+\theta_2)}^{\theta_m}}$$

In polar form

$$Z_m = Z_1 \times Z_2 = |Z_1| \angle \theta_1 + |Z_2| \angle \theta_2$$
$$= |Z_1||Z_2| \angle \theta_1 + \theta_2$$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Division of Phasors

$$Z_d = \frac{Z_1}{Z_2} = \frac{(a_1 + jb_1)}{(a_2 + jb_2)} \times \frac{(a_2 - jb_2)}{(a_2 - jb_2)}$$

$$= \frac{(a_1 a_2 + b_1 b_2)}{(a_2^2 + b_2^2)} + j\left(\frac{a_2 b_1 - a_1 b_2}{(a_2^2 + b_2^2)}\right)$$

In exponential form.

$$Z_d = \frac{Z_1}{Z_2} = \frac{|Z_1| e^{j\theta_1}}{|Z_2| e^{j\theta_2}} = \frac{|Z_1|}{|Z_2|} e^{j\theta_1 - j\theta_2}$$

$$= \underbrace{\frac{|Z_1|}{|Z_2|}}_{|Z_d|} e^{j\overbrace{(\theta_1 - \theta_2)}^{\theta_d}}$$

koracademy.com koracademy.com koracademy.com

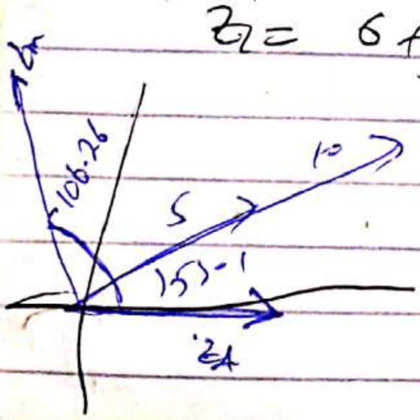
In polar form

$$Z_d = \frac{Z_1}{Z_2} = \frac{|Z_1| \angle \theta_1}{|Z_2| \angle \theta_2} = \underbrace{\frac{|Z_1|}{|Z_2|}}_{|Z_d|} \angle \underbrace{\theta_1 - \theta_2}_{\theta_d}$$

Q/

$Z_1 = 3 + j4 = 5 \angle 53.1^\circ$

$Z_2 = 6 + j8 = 10 \angle 53.1^\circ$



$Z_a = Z_1 + Z_2 = 9 + j12$

$= 11.18 \angle 53.1^\circ$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$Z_S = Z_2 - Z_1 = (6+j8) - (3+j4) = 3+j4$

$Z_S = 5 \angle 53.1^\circ$

Q3. $Z_m = Z_1 \times Z_2 = (3+j4)(6+j8)$

$= (18-32) + j(24+24) = -14 + j48$

$|Z_m| = \sqrt{196 + 2304} = 50$

$\theta_m = \tan^{-1} \frac{48}{14} = 73.73$

koracademy.com koracademy.com koracademy.com

$\Rightarrow 180 - 73.78 \Rightarrow \theta_m = 106.26^\circ$

$Z_m = 50 \angle 106.26^\circ$

In polar form; $Z_m = 5 \angle 53.1 \times 10 \angle 53.1^\circ$

$Z_m = 50 \angle 106.26^\circ$

Q4. $Z_d = \frac{Z_2}{Z_1} = \frac{10 \angle 53.1^\circ}{5 \angle 53.1^\circ} = 2 \angle 0^\circ = 2$

koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

AC FUNDAMENTALS

Generation of AC voltage

Faraday's law

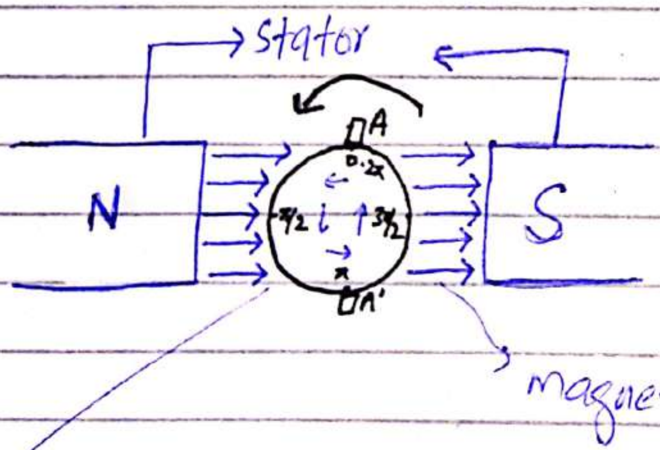
① When a conductor moves in a constant magnetic field

④ Whenever a conductor cuts the magnetic flux, voltage is induced across that conductor.

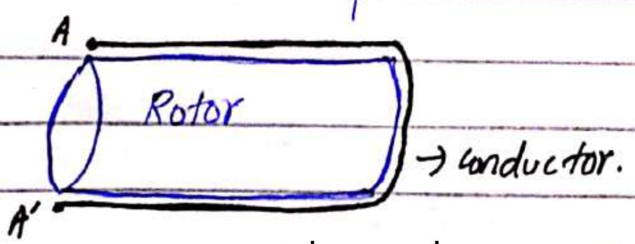
② If we place a conductor in a time varying magnetic field; voltage is induced across that conductor.

koracademy.com koracademy.com koracademy.com

→ If we want to generate voltage, we need:
a) magnetic field b) conductor (wiring)

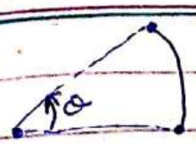


Rotor → the part that rotates.
Stator → The part in static condition.



koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

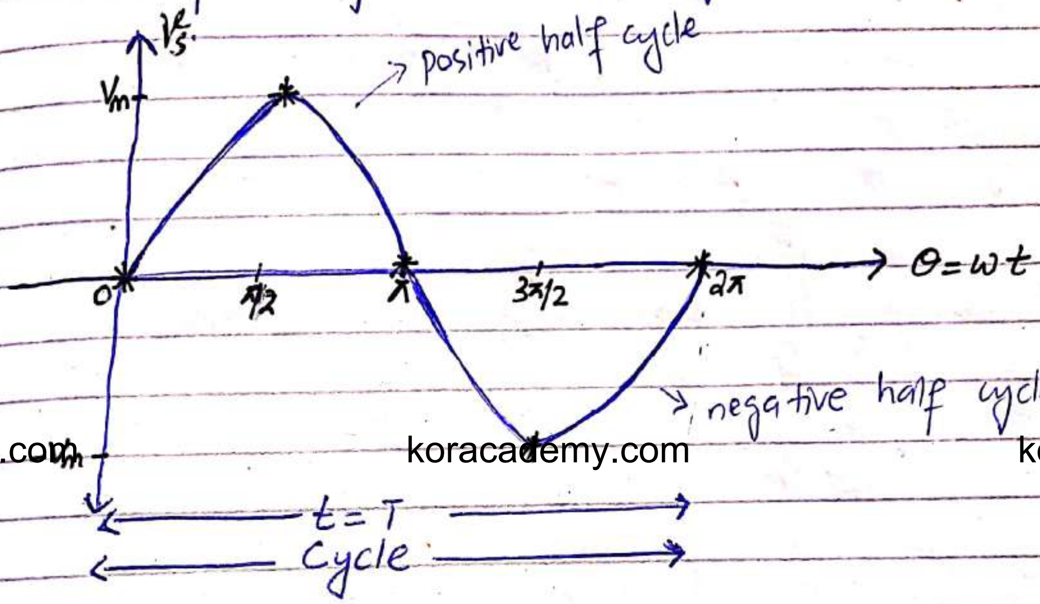


ω → angular velocity in rad/sec.
θ → angular displacement in "t" seconds.

$$\omega = \frac{\theta}{t} \rightarrow \text{rad/sec}$$

$$\theta = \omega t.$$

Waveform of the AC voltage



At $\theta = \omega t = 0 \Rightarrow V_s = 0$.
The direction of motion of the conductor is parallel to the flux, hence the conductor does not cut any line → no voltage.

At $\theta = \pi/2$ rad $V_s \Rightarrow V_m$ = Maximum voltage or peak value.
Direction of motion perpendicular to flux, so cut maximum lines and hence maximum voltage.

At $\theta = \omega t = \pi$ rad $\Rightarrow V_s = 0$
Direction of motion of conductor is ∥ to flux → does not cut any line → no voltage

koracademy.com koracademy.com koracademy.com
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

At $\theta = 3\pi/2$ $\quad -V_m \rightarrow$ maximum or peak value

Direction of motion $\perp \rightarrow$ maximum lines so maximum voltage.

The waveform satisfies the equation;

$$\boxed{V_s = V_m \sin \omega t}$$

which is the instantanous equation for the AC voltage.

$V_m \rightarrow$ maximum or peak value $\rightarrow$ which is constant

The time taken by one complete cycle of the AC voltage is known as time period. T.

koracademy.com Number of cycles per second is called koracademy.com koracademy.com frequency. f (cycles/sec or Hz).

In Pakistan it is 50 Hz.

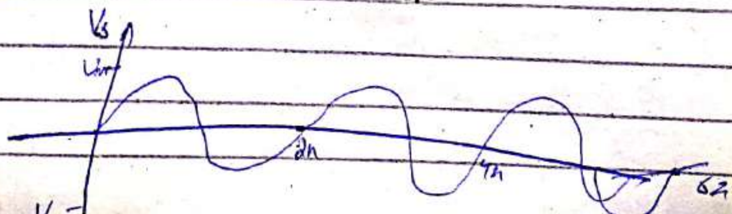
or The time taken by waveform to repeat itself.

In T seconds (time period) we get 1 cycle of AC voltage.

In 1 second we get $= \frac{1 \times 1}{T} = \frac{1}{T}$ cycles.

linear frequency $\leftarrow f = \frac{1}{T}$ cycles/sec

The time period of AC voltage in Pakistan is 0.02 seconds.



koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

The cycle repeats itself at $\omega t = 2\pi, 4\pi, 6\pi, 8\pi \ldots\ldots 2m\pi$ where $m = 1, 2, 3 \ldots$

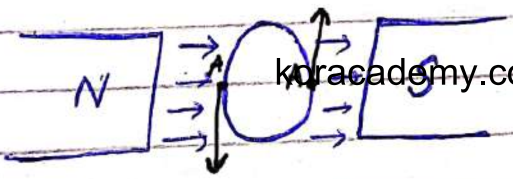
when $m = 1$, $\omega = 2\pi \Rightarrow t = T$, $\boxed{\omega T = 2\pi \downarrow \omega = \frac{2\pi}{T}}$

Angular frequency $\boxed{\omega = 2\pi f} \leftarrow$

The ω in the instantaneous expression is the angular frequency. $V_s = V_m \sin \omega t$

~~Root Mean Squared value or Effective value of AC current.~~

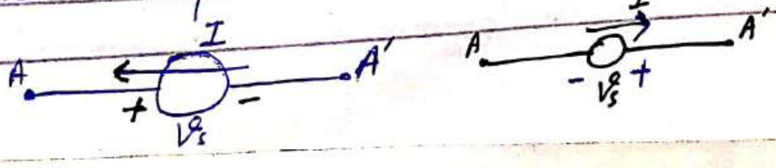
Fleming's Right Hand Rule



thumb → direction of motion of conductor.
index → direction of magnetic field / flux.
middle → direction of induced current in the conductor.

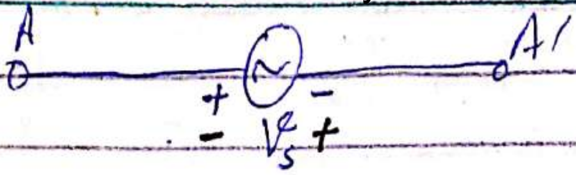
The conductor behaves as source.

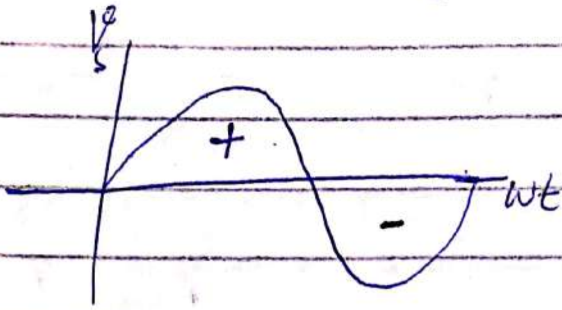
The conventional direction of current inside the source is from -ve to +ve.



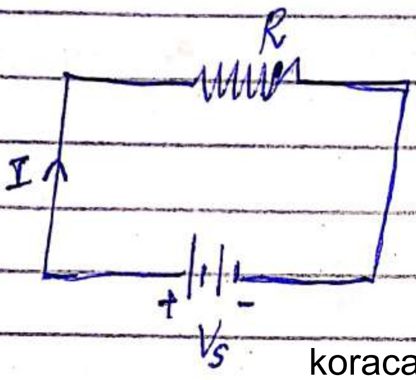
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com



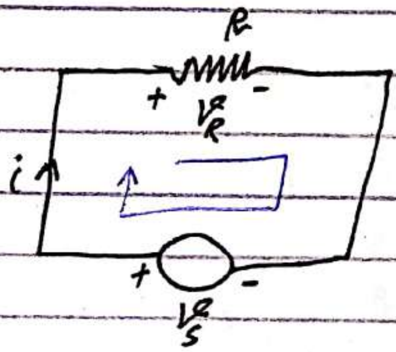


RMS value or Effective value of AC



$$P_R = I^2 R$$

koracademy.com koracademy.com koracademy.com



$v_s = V_m \sin\omega t$

Apply KVL to this loop.

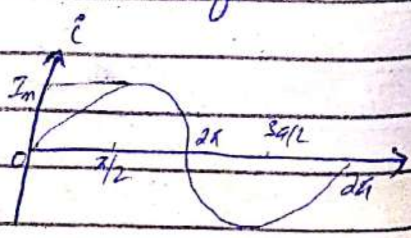
$v_R = v_s = V_m \sin\omega t$

$v_s = v_R = iR$

$$i = \frac{v_s}{R} \Rightarrow i = \frac{V_m}{R}\sin\omega t$$

$I_m = \frac{V_m}{R}$ → max or peak value of AC crt

$$\boxed{i = I_m \sin\omega t}$$



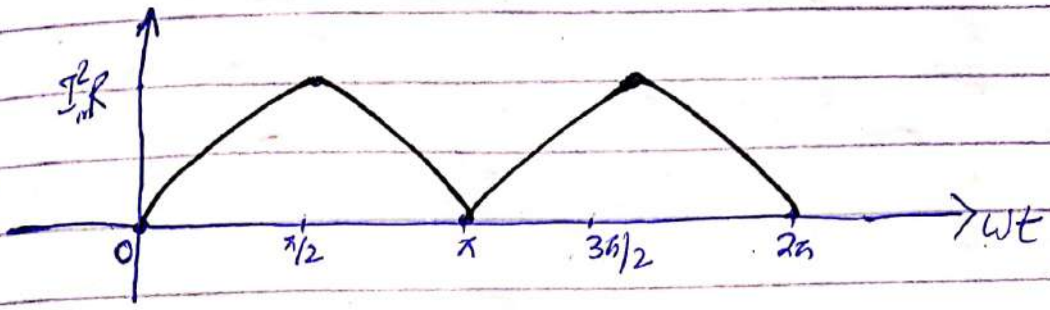
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

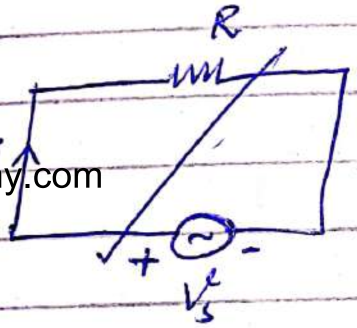
The instantaneous power, $P_R = i^2 R$

$\Rightarrow P_R = I_m^2 R \sin^2 \omega t$

$I_m^2 R \rightarrow$ max or peak value of AC power.



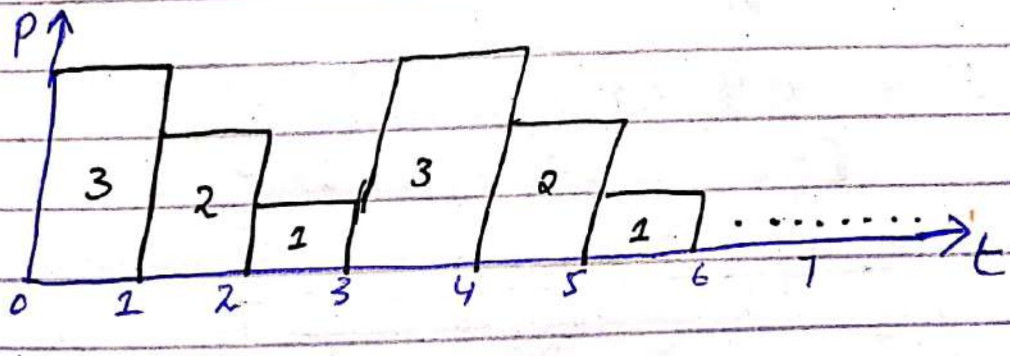
RMS Value Of ~~AC~~ current



~~$v_s = v_R = V_m \sin \omega t$~~

~~$i =$~~

koracademy.com koracademy.com koracademy.com



$$P_{average} = \frac{3+2+1+3+2+1}{6} = 2 \text{ watts.}$$

for one cycle ie, $T = 3$ s.

$$P_{av} = \frac{3+2+1}{3} = 2 \text{ Watts.}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$\boxed{P_{avg} = \frac{1}{\pi}\int_0^{\pi} P_R \, d\omega t}$$

$$\boxed{P_{avg} = \frac{1}{T}\int_0^{T} P_R \, dt}$$

As $P_R = i^2R \Rightarrow P_{avg} = \frac{1}{\pi}\int_0^{\pi} i^2R \, d\omega t$

The effective value of AC current is equal to DC current and effective value of AC voltage is equal to DC voltage if $P_R = P_{avg}$

$$I^2R = \frac{1}{\pi}\int_0^{\pi} i^2R \, d\omega t$$

koracademy.com koracademy.com koracademy.com

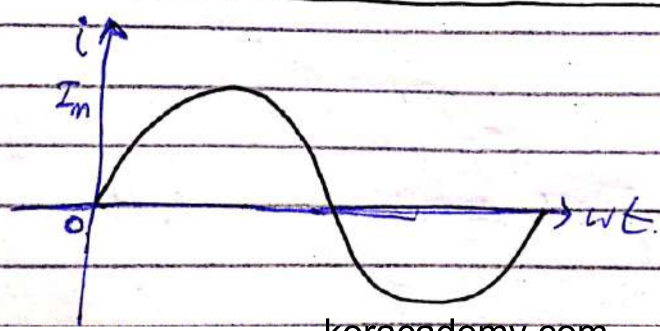
let us consider two cycles

$$I^2R = \frac{1}{2\pi}\int_0^{2\pi} i^2R \, d\omega t$$

As R is constant.

$$I^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 \, d\omega t$$

$$\boxed{I_{effective} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 \, d\omega t}} \rightarrow \text{RMS value}$$



koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$\boxed{I_{mean} = \frac{1}{2\pi}\int_0^{2\pi} i\, d\omega t}$$

If we have time along the horizontal axis,

$$\boxed{I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt}}$$

Put $i = I_m \sin\omega t$ in I_{eff} or I_{rms}.

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2\omega t\, d\omega t}$$

As I_m is a constant value

koracademy.com koracademy.com koracademy.com

$$I_{rms} = \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi} \sin^2\omega t\, d\omega t}$$

As $\sin^2\omega t = \dfrac{1-\cos 2\omega t}{2}$

$$I_{rms} = \sqrt{\frac{I_m^2}{4\pi}\left\{\int_0^{2\pi} d\omega t - \int_0^{2\pi} \cos 2\omega t\right\}}$$

$\downarrow \frac{\sin \omega t}{2} \rightarrow$ limits $\rightarrow 0$

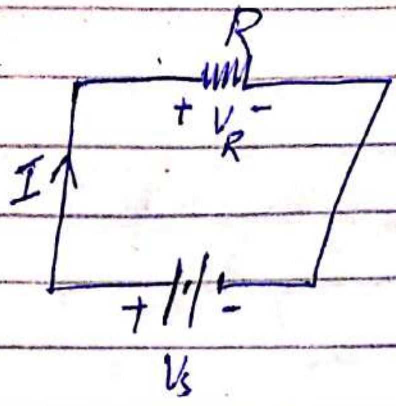
$$I_{rms} = \sqrt{\frac{I_m^2}{4\pi}(\omega t)\Big|_0^{2\pi}}$$

$$\boxed{I_{rms} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}}$$ or $\boxed{I_{rms} = 0.707\, I_m}$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

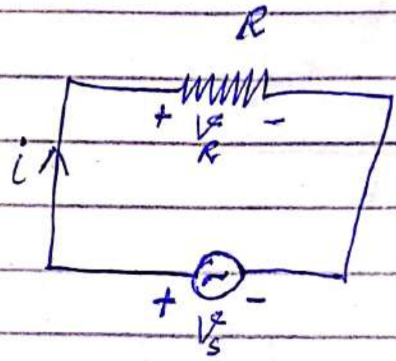
RMS Value of AC voltage.



According to KVL

$$V_s = V_R$$

$$P_R = \frac{V_s^2}{R} = \frac{V_R^2}{R}$$



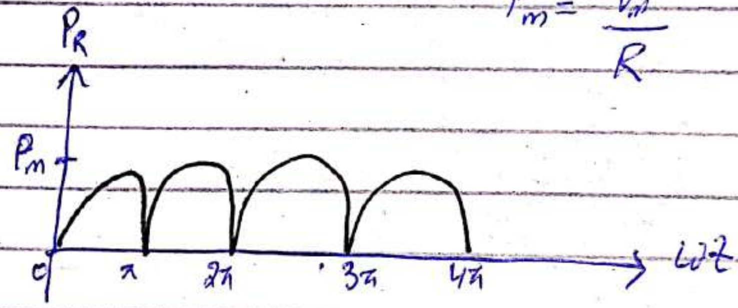
$$v_s = v_R = V_m \sin\omega t$$

$$P_R = \frac{v_R^2}{R} = \frac{v_s^2}{R}$$

$$P_R = \frac{V_m^2}{R} \sin^2\omega t$$

koracademy.com koracademy.com koracademy.com

$$P_m = \frac{V_m^2}{R}$$



$$P_{avg} = \frac{1}{\pi}\int_0^{\pi} P_R \, d\omega t$$

$$P_{avg} = \frac{1}{\pi}\int_0^{\pi} \frac{v_s^2}{R} \, d\omega t$$

$\underline{P_R = P_{avg}}$ → for AC to be equal to DC

$$\frac{V_s^2}{R} = \frac{1}{\pi}\int_0^{\pi} \frac{v_s^2}{R} \, d\omega t$$

(R is constant)

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

We can consider it for two cycles;

$$V_s^2 = \frac{1}{2\pi}\int_0^{2\pi} v_s^2 \, d\omega t$$

$$\boxed{V_{s\,rms} = \frac{1}{2\pi}\int_0^{2\pi} v_s^2 \, d\omega t}$$

If we have time along horizontal axis;

$$\boxed{V_{s\,rms} = \frac{1}{T}\int_0^{T} v_s^2 \, dt}$$

Put value of v_s in equation of $V_{s\,rms}$.

$$V_{s\,rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\omega t \, d\omega t}$$

koracademy.com koracademy.com koracademy.com

$$V_{s\,rms} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \sin^2\omega t \, d\omega t}$$

As $\sin^2\omega t = \frac{1 - \cos 2\omega t}{2}$

$$V_{s\,rms} = \sqrt{\frac{V_m^2}{4\pi}\left\{\int_0^{2\pi} d\omega t - \int_0^{2\pi} \cos 2\omega t \, d\omega t\right\}}$$

$$V_{s\,rms} = \sqrt{\frac{V_m^2}{4\pi}\left(\omega t \Big|_0^{2\pi}\right)}$$

$$\boxed{V_{s\,rms} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 1

$V_{s_{rms}} = 220\ V \quad f = 50\,Hz$

(Main AC source) $\quad v_s = ?$

$v_s = V_m \sin \omega t \rightarrow (i)$

$V_{s_{rms}} = \frac{V_m}{\sqrt{2}} \Rightarrow V_m = V_{s_{rms}} \sqrt{2}$

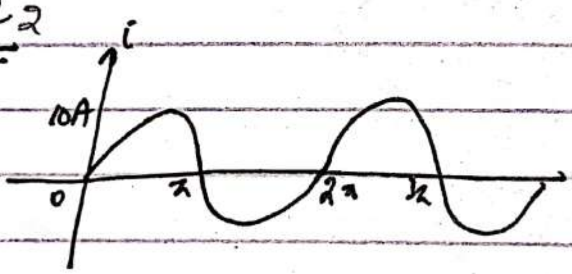
$V_m = 220\sqrt{2}$ volts.

$\omega = 2\pi f = 100\pi$ rad/sec

$(i) \Rightarrow \boxed{v_s = 220\sqrt{2} \sin 100\pi t}$

koracademy.com koracademy.com koracademy.com

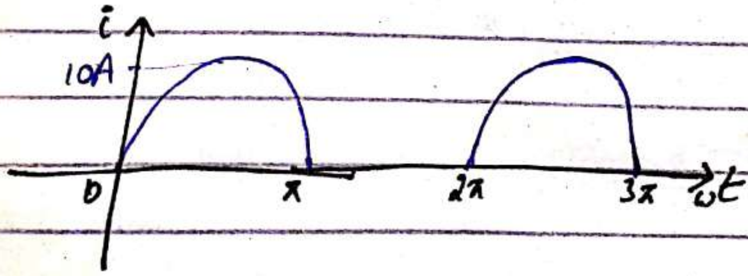
Example 2



calculate the rms $\quad I_m = 10A$

$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07\ A$

Example 3



$I_m = 10A$

$I_{avg} = ?$

$I_{rms} = ?$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$i = 10\sin\omega t \qquad 0 \le \omega t \le \pi$

$i = 0 \qquad \pi \le \omega t \le 2\pi$

$$I_{avg} = \frac{1}{2\pi}\int_0^{2\pi} i\, d\omega t$$

$$I_{avg} = \frac{1}{2\pi}\left[\int_0^{\pi} i\, d\omega t + \cancel{\int_{\pi}^{2\pi} i\, d\omega t}^{\,0}\right]$$

As $i = 0$ in this interval

$$I_{avg} = \frac{10}{2\pi}\int_0^{\pi} \sin\omega t\, d\omega t$$

$$I_{avg} = \frac{10}{2\pi}\left[-\cos\omega t\right]_0^{\pi} = \frac{10}{2\pi}\times 2$$

$$\boxed{I_{avg} = 3.18\,A}$$

koracademy.com koracademy.com koracademy.com

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\, d\omega t}$$

$$I_{rms} = \left[\frac{1}{2\pi}\left\{\int_0^{\pi} i^2\, d\omega t + \cancel{\int_{\pi}^{2\pi} i^2\, d\omega t}^{\,0}\right\}\right]^{1/2}$$

$$I_{rms} = \sqrt{\frac{100}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right)}$$

$$I_{rms} = \sqrt{\frac{100}{4\pi}\,\cancel{100}\,\left|\omega t\right|_0^{\pi}}$$

$$I_{rms} = \sqrt{25} = \underline{5A}$$

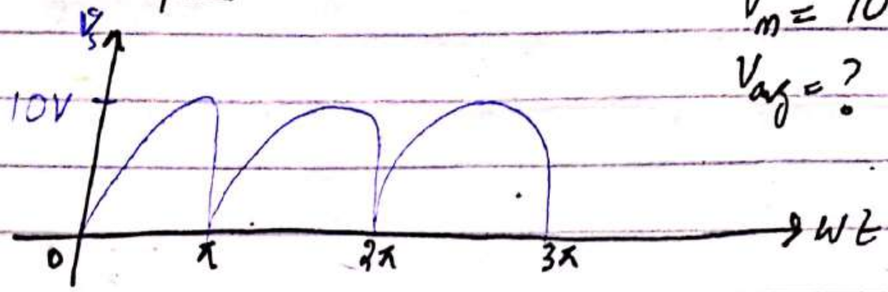
koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

DC voltmeter reads average value (or) constant value.
AC voltmeter reads rms value.
DC Ammeter reads the constant and average value.

Example 4



$V_m = 10\,V$

$V_{avg} = ?$ $\quad V_{rms} = ?$

$v_s = 10 \sin wt \qquad 0 \le wt \le \pi$

$v_s = 10 \sin(wt - \pi)$ or $v_s = -10 \sin wt \longrightarrow \pi \le wt \le 2\pi$

$$V_{s\,avg} = \frac{1}{\pi} \int_0^{\pi} v_s \, dwt = \frac{10}{\pi} \int_0^{\pi} \sin wt \, dwt$$

koracademy.com koracademy.com koracademy.com

$$= \frac{10}{\pi} \left[-\cos wt \right]_0^{\pi} = \frac{20}{\pi}$$

$$\boxed{V_{avg} = 6.36\,V}$$

$$V_{rms} = \sqrt{\frac{1}{\pi} \int_0^{\pi} v_s^2 \, dwt} = \sqrt{\frac{100}{\pi} \int_0^{\pi} \sin^2 wt \, dwt}$$

$$= \sqrt{\frac{100}{2\pi} \left[\int_0^{\pi} dwt - \int_0^{\pi} \cos 2wt \, dwt \right]}$$

$$= \sqrt{50} = 2\sqrt{2} = \boxed{7.07} \text{ volts.}$$

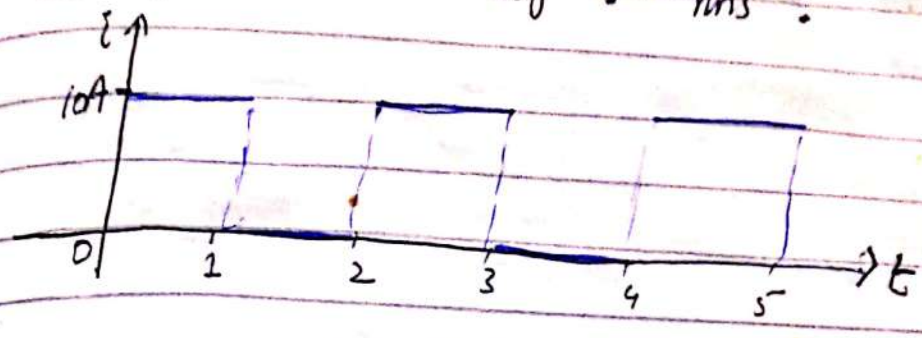
koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 5

$I_{avg} = ?\quad I_{rms} = ?$



This waveform is repeating itself after 2s so its time period is 2s.

$$T = 2s$$

$$i = 10\ A \qquad 0 \le t \le 1$$
$$i = 0\ A \qquad 1 \le t \le 2$$

$$I_{avg} = \frac{1}{T}\int_0^T i\,dt$$

koracademy.com koracademy.com koracademy.com

$$I_{avg} = \frac{1}{2}\left[\int_0^1 10\,dt + \int_1^2 0\,dt\right]$$

$$I_{avg} = \frac{10}{2}\left(t\Big|_0^1 + 0 = \boxed{5A}\right.$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

$$I_{rms} = \left[\frac{1}{2}\left\{\int_0^1 10^2\,dt + \int_0^1 0^2\,dt\right\}\right]^{1/2}$$

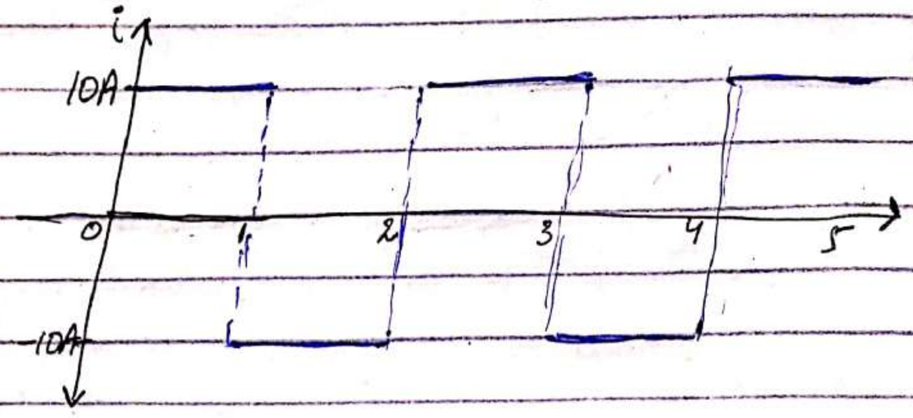
$$I_{rms} = \frac{100}{2}\left(t\Big|_0^1 + 0 = \sqrt{50}\right.$$

$$\boxed{I_{rms} = 7.07\,A}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 6: $I_{avg} = ?$ $I_{rms} = ?$ $T = 2\,sec$



$i = 10A \qquad 0 \le t \le 1$
$i = -10A \qquad 1 \le t \le 2$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{2}\left\{\int_0^1 10^2\,dt + \int_1^2 (-10)^2\,dt\right\}}$$

$$I_{rms} = \sqrt{\frac{100}{2}\left(t\right|_0^1 + \frac{100}{2}\left(t\right|_1^2} = \sqrt{50 + 100 - 50}$$

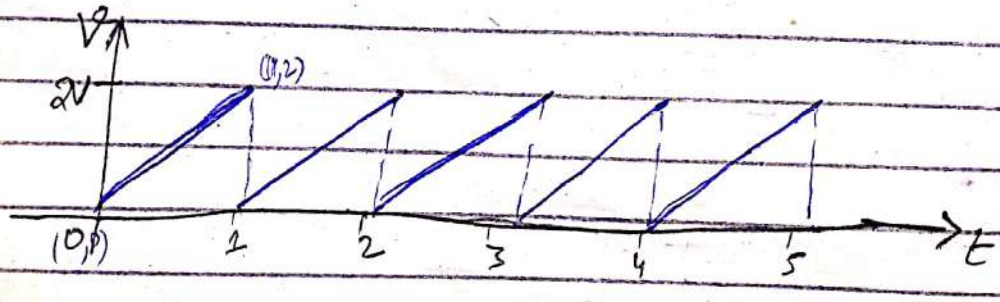
koracademy.com koracademy.com koracademy.com

$$\boxed{I_{rms} = 10A}$$

$$I_{avg} = \frac{1}{T}\int_0^T i\,dt = \frac{1}{2}\left\{\int_0^1 10\,dt + \int_1^2 -10\,dt\right\}$$

$$\boxed{I_{avg} = 0A}$$

Example 7:



$T = 1\,sec$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

The eq for a straight line passing through the origin $v = mt$

$$m = \frac{v_2 - v_1}{t_2 - t_1} = \frac{2-0}{1-0} = 2$$

$$v = 2t \qquad 0 \le t \le 1$$

Similarly
$$v = 2(t-1) \qquad 1 \le t \le 2$$
$$v = 2(t-2) \qquad 2 \le t \le 3$$
$$v = 2(t-3) \qquad 3 \le t \le 4$$
$$v = 2(t-4) \qquad 4 \le t \le 5$$

koracademy.com koracademy.com koracademy.com

$$V_{avg} = \frac{1}{T}\int_0^T v\,dt$$

$$V_{avg} = \frac{1}{1}\int_0^1 2t\,dt = 2\left(t^2/2\right|_0^1 = \boxed{1 \text{ volt}}$$

reading by DC voltmeter.

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

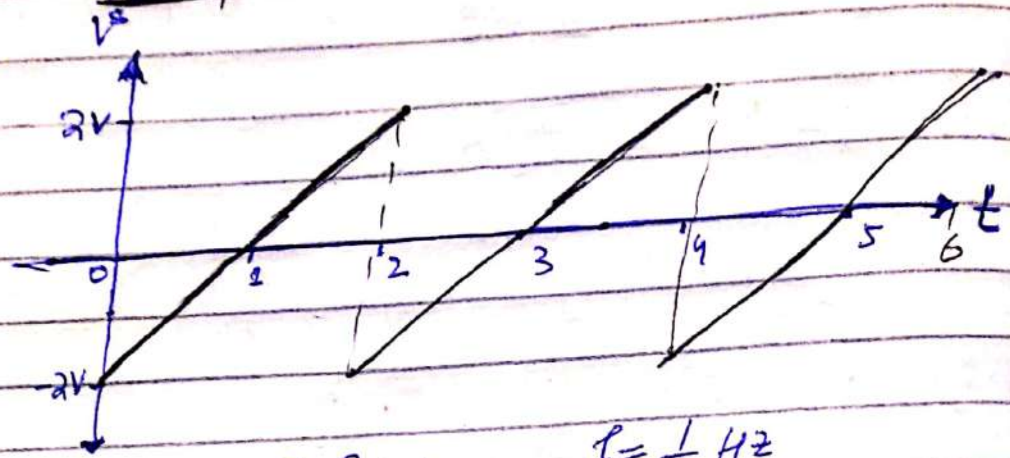
$$V_{rms} = \sqrt{\frac{1}{1}\int_0^1 (2t)^2\,dt} = \sqrt{4\left(t^3/3\right|_0^1} = \sqrt{\frac{4}{3}}$$

$$\boxed{V_{rms} = 1.154\,V}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 8



$T = 2\,sec$ $\quad f = \frac{1}{2}\,Hz$

$v = mt + C \quad (y = mx + c)$

$m = \frac{v_2 - v_1}{t_2 - t_1} = \frac{2-(-2)}{2} = 2$

$v = 2t + C$

Take any point (t, v) from graph to find c. let $(1, 0) \Rightarrow \boxed{C = -2}$

koracademy.com koracademy.com koracademy.com

$v = 2t - 2 \qquad 0 \le t \le 2$

$v = 2(t-2) - 2 \qquad 2 \le t \le 4$

$v = 2(t-4) - 2 \qquad 4 \le t \le 6$

$V_{avg} = \frac{1}{T}\int_0^T v\,dt = \frac{1}{2}\int_0^2 (2t-2)\,dt$

$V_{avg} = \frac{1}{2}\left[\int_0^2 2t\,dt - \int_0^2 2\,dt\right]$

$= \left(\frac{t^2}{2}\right|_0^2 - (t|_0^2 = \boxed{0\ volts}$

$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$= \sqrt{\frac{1}{2}\int_0^2 (2t-2)^2\,dt} = \sqrt{\frac{1}{2}\int_0^2 (4t^2+4-8t)\,dt}$$

$$= \left[\frac{1}{2}\left\{4\left(\frac{t^3}{3}\right|_0^2 + 4\left(t\right|_0^2 - 8\left(\frac{t^2}{2}\right|_0^2\right\}\right]^{1/2}$$

koracademy.com koracademy.com koracademy.com

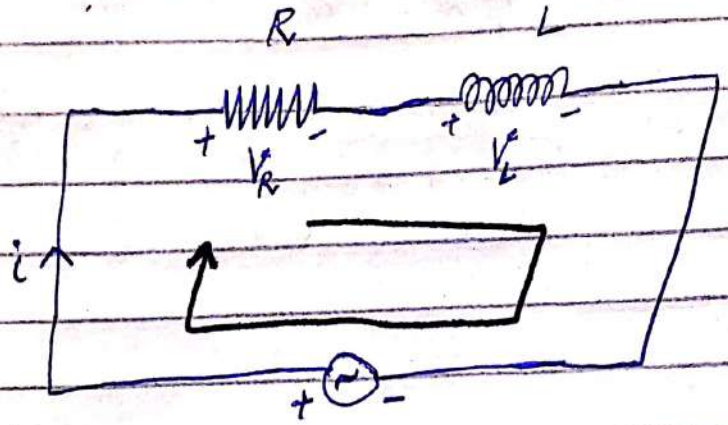
$$= \sqrt{2 \times \frac{8}{3} + 4 - 8} = 1.15 \text{ volts.}$$

koracademy.com koracademy.com koracademy.com

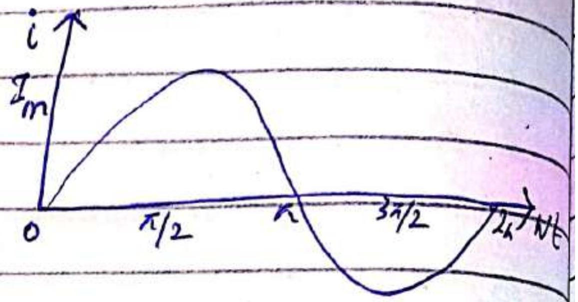
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

R-L Series Circuit



$$i = I_m \sin wt$$

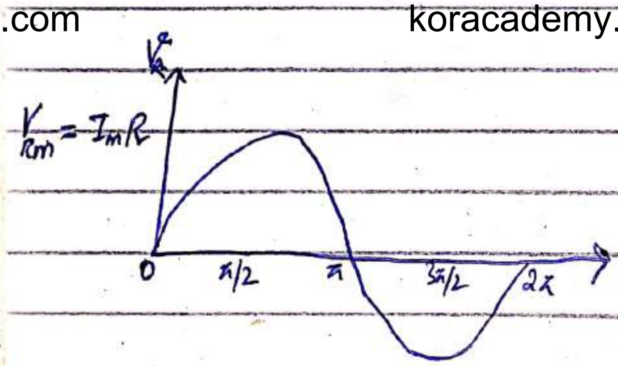


$$V_R = iR$$

$$\Rightarrow V_R = I_m R \sin wt$$

$$V_{Rm} = I_m R \quad \Rightarrow R = \frac{V_{Rm}}{I_m}$$

koracademy.com koracademy.com koracademy.com



The voltage across the resistor is always in phase with the current.

Also $V_L = L\frac{di}{dt} = L\frac{d}{dt}(I_m \sin wt)$

$$V_L = I_m w L \cos wt$$

$$V_{Lm} = I_m \times (wL)$$

This ratio gives the resistance of the inductor. $\longleftarrow$ $wL = \frac{V_{Lm}}{I_m}$

The resistance offered by inductor to AC is called inductive reactance.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

which is represented by X_L and

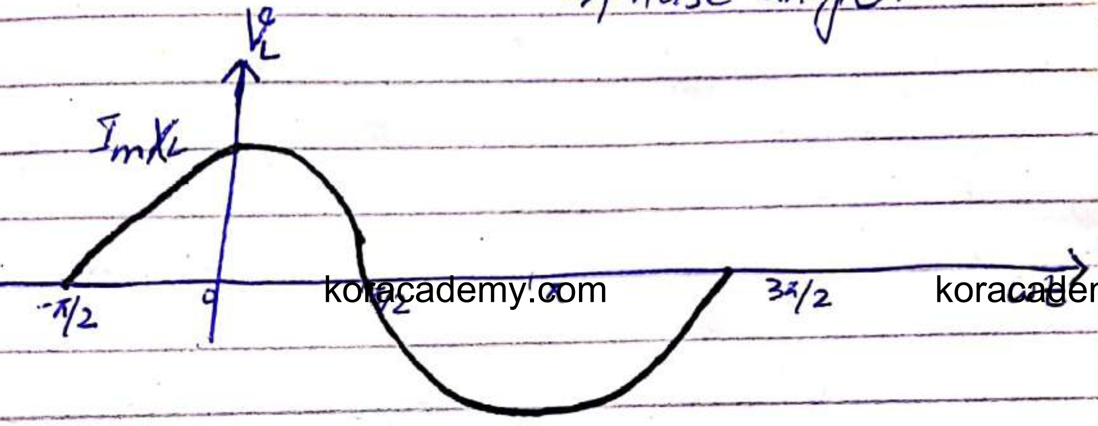
$$X_L = \omega L = 2\pi f L$$

Inductor does not offer any resistance to the flow of DC because its frequency is zero.

$$V_L = I_m X_L \cos\omega t$$

or $V_L = I_m X_L \sin(\omega t + 90^\circ)$

↳ phase angle.



koracademy.com koracademy.com koracademy.com

The voltage across the inductor leads the current by 90°.

(or) the current in the inductor lags behind the voltage across it by 90°.

There is a phase difference of 90°.

Apply KVL to the loop

$$V_s = V_R + V_L$$

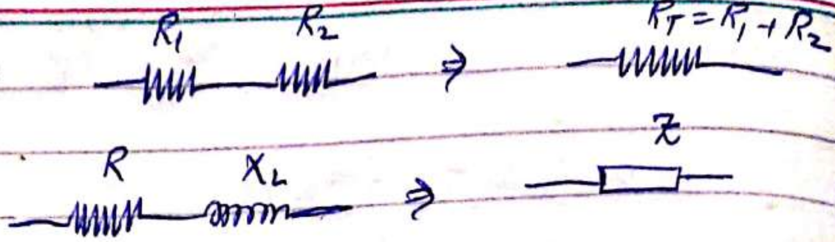
$$V_s = I_m R \sin\omega t + I_m X_L \cos\omega t \rightarrow (a)$$

Let $V_s = V_m \sin(\omega t + \theta)$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

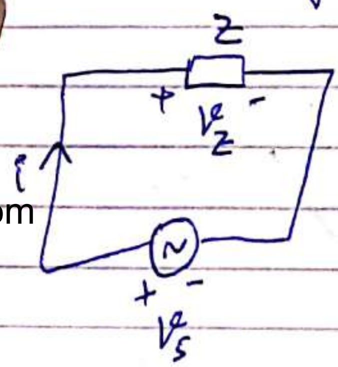
from 

↳ replace it theoretically.

total resistance $Z \rightarrow$ impedence

Apply the same source across this passive circuit element so the same current will flow through it.

The voltage, current and total resistance of circuit 1 is equal to c2 and apply KVL to it.



$\Rightarrow v_s^e = v_Z^e$

$V_m = V_{Zm} = I_m Z$

koracademy.com koracademy.com koracademy.com

$v_s = I_m Z \sin(\omega t + \theta)$

expand $\sin(\alpha + \beta)$

$v_s^e = I_m (Z \cos\theta) \sin\omega t + I_m (Z \sin\theta) \cos\omega t \rightarrow ⓑ$

Eqs ⓐ and ⓑ gives the same source voltage; these equations are equal if

$R = Z \cos\theta$

and $X_L = Z \sin\theta$

$R^2 = Z^2 \cos^2\theta \qquad X_L^2 = Z^2 \sin^2\theta$

Add

$R^2 + X_L^2 = Z^2 \cos^2\theta + Z^2 \sin^2\theta$

$Z^2 = R^2 + X_L^2$

$Z = \sqrt{R^2 + X_L^2}$ where $Z < R + X_L$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$\frac{X_L}{R} = \frac{Z\sin\theta}{Z\cos\theta} \Rightarrow \theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

The instantaneous power taken by a resistor

$$P_R = i^2 R$$

$$P_R = I_m^2 R \sin^2 wt$$

$$P_R = \frac{1}{\pi}\int_0^{\pi} I_m^2 R \sin^2 wt \; dwt;$$

$$P_R = \frac{I_m^2 R}{\pi}\int_0^{\pi} \sin^2 wt \; dwt$$

$$\sin^2 wt = \frac{1-\cos 2wt}{2}$$

koracademy.com koracademy.com koracademy.com

$$P_R = \frac{I_m^2 R}{2\pi}\int_0^{\pi} (1-\cos 2wt)\; dwt$$

$$P_R = \frac{I_m^2 R}{2\pi}\left\{\int_0^{\pi} dwt - \int_0^{\pi} \cos 2wt \; dwt\right\}$$

$$P_R = \frac{I_m^2 R}{2\pi}\left(wt\right|_0^{\pi}$$

$$P_R = \frac{I_m^2}{2} R = \left(\frac{I_m}{\sqrt{2}}\right)^2 R$$

$$\boxed{P_R = I_{rms}^2 R}$$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Power taken by inductor

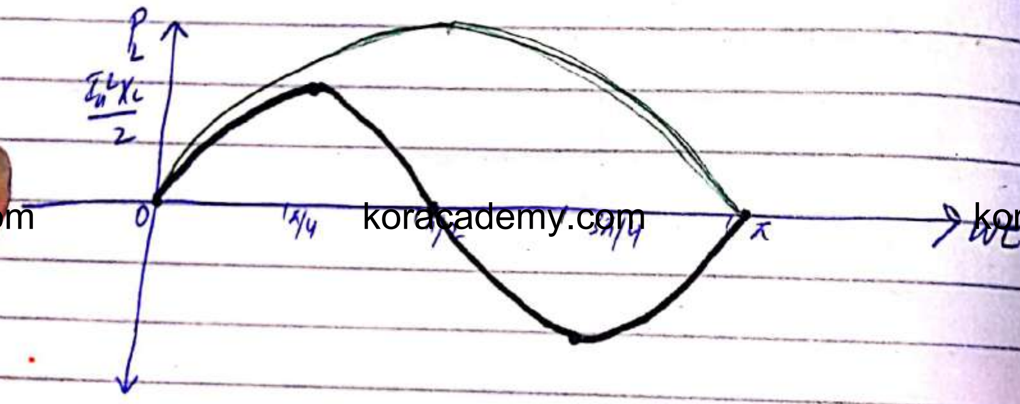
$$P_L = V_L \times i$$

$$P_L = (I_m X_L \cos wt)(I_m \sin wt)$$

$$P_L = \frac{I_m^2 X_L 2 \sin wt \cos wt}{2}$$

$$2 \sin wt \cos wt = \sin 2wt$$

$$P_L = \frac{I_m^2 X_L}{2} \sin 2wt$$



When current in inductor increases the inductor takes the power from voltage source. When the current decreases the same power is delivered back to the source.

The inductor does not consume this power so the average value of this power is zero which is calculated as,

$$P_L = \frac{1}{\pi} \int_0^{\pi} \frac{I_m^2 X_L}{2} \sin 2wt \, dwt$$

$$P_L = \frac{I_m^2 X_L}{2\pi} \int_0^{\pi} \sin 2wt \, dwt \qquad P_L = 0$$

koracademy.com koracademy.com koracademy.com
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Acurding to law of conservation of energy

$$P_S = P_R + P_L$$

$$\boxed{P_S = I_{rms}^2 R}$$

Example 1:

$R = 3\Omega \quad X_L = 4\Omega \quad i = 10\sqrt{2}\sin 100\pi t \; A$

$I_m = 10\sqrt{2} \; A \quad \omega = 100\pi \; rad/sec$

$v_R = I_m R \sin\omega t = 30\sqrt{2}\sin 100\pi t$ volts

$v_L = I_m X_L \sin(\omega t + 90^\circ) = 40\sqrt{2}\sin(100\pi t + 90)$

$v_S = I_m Z \sin(\omega t + \theta)$

koracademy.com koracademy.com koracademy.com

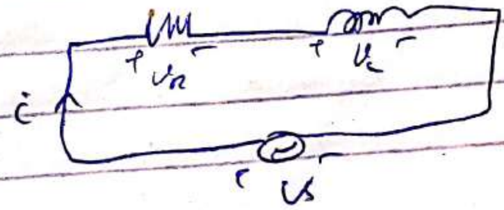
$Z = \sqrt{(R^2 + X_L^2)} = \sqrt{9+16} \Rightarrow Z = 5\Omega$

$\theta = Tan^{-1}\left(\frac{X_L}{R}\right) = Tan^{-1}\left(\frac{4}{3}\right) \Rightarrow \theta = 53.1^\circ$

$v_S = 50\sqrt{2}\sin(100\pi t + 53.1^\circ)$ volts.

$P_R = I_m^2 R \sin^2\omega t = 600\sin^2 100\pi t$ watts.

$P_L = \frac{I_m^2 X_L}{2}\sin 2\omega t = 400\sin 200\pi t$ watts.

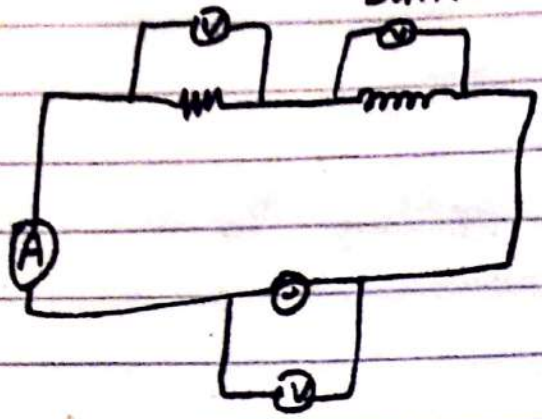


koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 2

Same data as example 1.



Readings = ?

Ammeter reads $I_{rms} = \frac{I_m}{\sqrt{2}} = 10A$

voltmeter across R $V_R = \frac{30\sqrt{2}}{\sqrt{2}} = 30$ volts.

koracademy.com voltmeter across koracademy.com L $V_{Lrms} = \frac{40\sqrt{2}}{\sqrt{2}} = 40$ volts. koracademy.com

$V_{s\,rms} = \frac{50\sqrt{2}}{\sqrt{2}} = 50$ volts.

Example 3

$R = 8\,\Omega \quad X_L = 6\,\Omega \quad i = 5\sqrt{2}\sin 100\pi t$

$I_{rms} = 5\,A \qquad V_R = I_m R \sin\omega t = 40\sqrt{2}\sin 100\pi t$

$V_{R\,rms} = 40\,V$

$V_L = I_m X_L \sin(\omega t + 90^\circ) = 30\sqrt{2}\sin(100\pi t + \pi/2)$

$V_{L\,rms} = 30$ volts.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

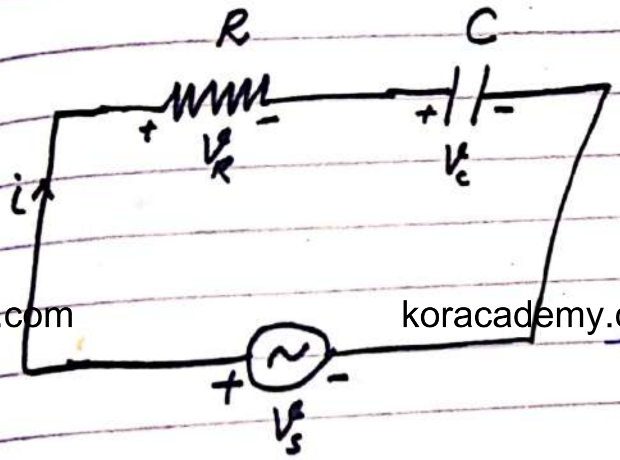
$$Z = \sqrt{64 + 36} = 10\,\Omega$$

$$\theta = \tan^{-1}\left(\frac{6}{8}\right) = 36.8^\circ$$

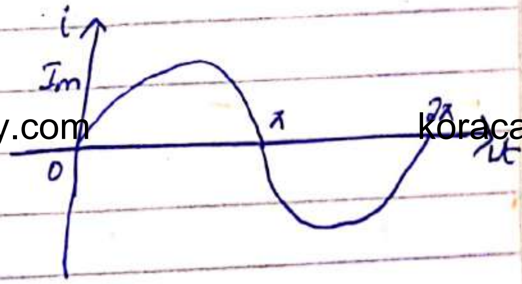
$$V_s = I_m Z \sin(\omega t + \theta)$$
$$= 50\sqrt{2} \sin(100\pi + 36.8^\circ)$$

$$V_{s\,rms} = 50 \text{ volts.}$$

R - C Series Circuit

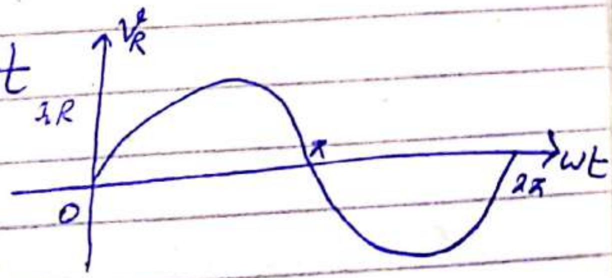


$$i = I_m \sin \omega t$$



koracademy.com koracademy.com koracademy.com

$$V_R = iR = I_m R \sin \omega t$$



The current and voltage through resistor are in phase with each other.

$$V_c = \frac{1}{C}\int i\,dt = \frac{1}{C}\int I_m \sin \omega t\,dt$$

$$V_c = -\frac{I_m}{\omega C}\cos \omega t = \frac{I_m}{\omega C}\sin(\omega t - 90^\circ)$$

$$V_{cm} = \frac{I_m}{\omega C}$$

koracademy.com koracademy.com
Scanned with CamScanner
koracademy.com

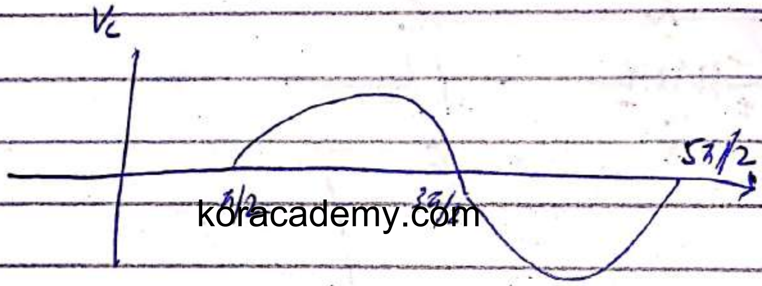
koracademy.com koracademy.com koracademy.com

$$\frac{1}{wC} = \frac{V_{cm}}{I_m} \quad \dots (\Omega)$$

This ratio gives the unit of resistance which is the resistance of the capacitor called as the capacitave reactance.

$$X_C = \frac{1}{wC} = \frac{1}{2\pi f C}$$

$$V_c = I_m X_c \sin(wt - 90^\circ)$$



koracademy.com koracademy.com koracademy.com

The voltage across capacitor laggs behind the current by 90° or the current leads leads voltage by 90°.

Apply KVL to this loop.

$$V_s = V_R + V_C$$

$$V_s = I_m R \sin wt - I_m X_c \cos wt \rightarrow (9)$$

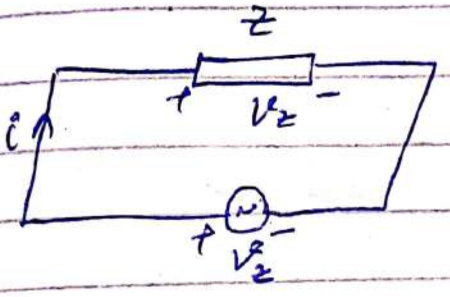
This is AC voltge which varys sinusoidally wrt to time. It will have proper maximum value and a proper phase angle which cannot be found out from this eqution. So we write generally.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$V_s^e = V_m \sin(\omega t - \theta)$$

Replace the combination of two circuit elements by a single circuit element. whose resistance is equal to the total resistance of circuit 1.



Apply KVL to it.

$$V_s^e = V_Z^e$$

$$V_m = V_{Zm} = I_m Z$$

$$V_s^e = I_m Z \sin(\omega t - \theta)$$

expand $\sin(\alpha - \beta)$

koracademy.com koracademy.com koracademy.com

(5) ← $V_s^e = I_m (Z\cos\theta)\sin\omega t - I_m (Z\sin\theta)\cos\omega t$

(4) = (5) as both represent both the source voltage.

These equations are equal if

$$R = Z\cos\theta \qquad X_C = Z\sin\theta$$

(or) $R^2 = Z^2\cos^2\theta \qquad X_C^2 = Z^2\sin^2\theta$

Add $R^2 + X_C^2 = Z^2(\cos^2\theta + \sin^2\theta)$

$$Z^2 = R^2 + X_C^2 \Rightarrow Z = \sqrt{R^2 + X_C^2}$$

$$\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Instantenous power by resistor

$$P_R = I_m^2 R \sin^2 \omega t = i^2 R$$

$$P_R = I_{rms}^2 R \rightarrow \text{average power}.$$

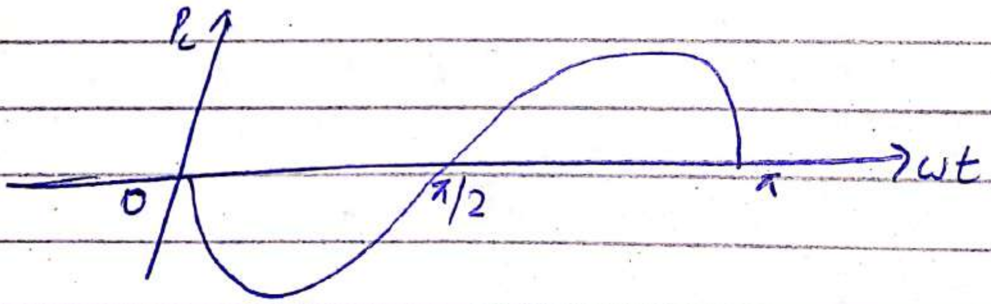
Instanous power by capacitor

$$P_c = V_c\, i = -\frac{I_m^2}{2} X_c\, 2 \sin\omega t \cos\omega t$$

$$2\sin\omega t \cos\omega t = \sin 2\omega t$$

koracademy.com koracademy.com koracademy.com

$$P_c = \frac{-I_m^2 X_c}{2} \sin 2\omega t$$



Average power.

$$P_c = \frac{1}{\pi} \int_0^{\pi} -\frac{I_m^2}{2} X_c \sin 2\omega t \, d\omega t$$

$$P_c = 0$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

According to law of conservation of energy

$$P_S = P_R + P_C$$

$$P_S = P_R$$

Example.

$R = 6\Omega \quad X_c = 8\Omega \quad i = 10\sqrt{2}\,\sin 100\pi t\ A$

$$Z = \sqrt{R^2 + X_c^2} = \sqrt{36+64} = \sqrt{100} \Rightarrow Z = 10\Omega$$

$$\theta = \tan^{-1}\frac{X_c}{R} = \tan^{-1}\frac{8}{6} \Rightarrow \theta = 53.1^\circ.$$

koracademy.com koracademy.com koracademy.com

$$V_R = I_m R \sin\omega t = 60\sqrt{2}\sin 100\pi t \text{ volts.}$$

$$V_c = I_m X_c \sin(\omega t - 90^\circ) = 80\sqrt{2}\sin(100\pi t - \pi/2).$$

$$V_s = I_m Z \sin(\omega t - 90^\circ) = 100\sqrt{2}\sin(100\pi t - 53.1^\circ)$$

$$P_R = I_m^2 R \sin^2\omega t = 1200 \sin^2 100\pi t \text{ watt.}$$

$$P_c = -\frac{I_m^2 X_c}{2}\sin 2\omega t = 800 \sin 200\pi t \text{ watts.}$$

Consider the data of above example and calculate the rms values of the current, voltage across resistor, and the capacitor and average power supplied by source to entire circuit.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} \Rightarrow I_{rms} = 10\ A$$

$$V_{R\,rms} = 60 \text{ volts.} = \frac{60\sqrt{2}}{\sqrt{2}}$$

$$V_{C\,rms} = \frac{80\sqrt{2}}{\sqrt{2}} = 80 \text{ volts.}$$

$$V_{s\,rms} = \frac{100\sqrt{2}}{\sqrt{2}} = \frac{V_{sm}}{\sqrt{2}} = 100 \text{ volts.}$$

$$P_s = P_R + P_C$$

$$P_C = 0 \qquad P_R = I_{rms}^2 R$$

$$P_s = I_{rms}^2 R = 600 \text{ watt.}$$

koracademy.com koracademy.com koracademy.com

AC ammeter reads rms value of crnt.

Example

$R = 4\,\Omega \qquad X_C = 3\,\Omega \qquad i = 5\sqrt{2}\sin 100\pi t\ A$

$$I_{rms} = 5\ A$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{16+9} = 5\,\Omega$$

$$\theta = \tan^{-1}\frac{X_C}{R} = \frac{3}{4} \Rightarrow \theta = 36.8^\circ$$

$$V_R = I_m R \sin\omega t = 20\sqrt{2}\sin 100\pi t \text{ volts.}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$V_C = I_m X_C \sin(\omega t - 90^\circ) = 15\sqrt{2}\sin(100\pi t - \pi/2)$$

$$V_{R\,rms} = 20 \text{ volts.} \qquad V_{C\,rms} = 15 \text{ volts.}$$

$$V_S = I_m Z \sin(\omega t - \theta) = 25\sqrt{2}\sin(100\pi t - 36.8^\circ) \text{ volts.}$$

$$P_R = I_m^2 R \sin^2 \omega t = 200 \sin^2 100\pi t \text{ watt.}$$

$$P_C = -\frac{I_m^2 X_C}{2} \sin 2\omega t = -75 \sin 2\omega t \text{ watts.}$$

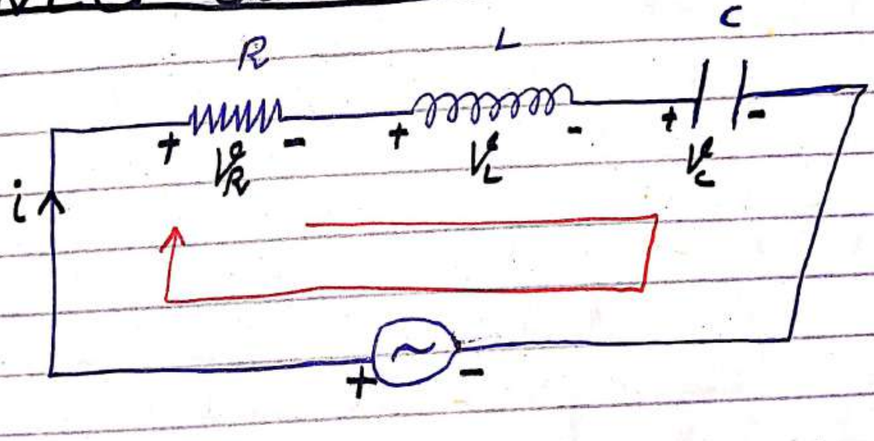
$$V_{S\,rms} = 25 \text{ volts.}$$

$$P_S = P_R + P_C \qquad P_C = 0 \quad P_R = I_{rms}^2 R$$

koracademy.com koracademy.com koracademy.com

$$P_S = 100 \text{ watts.}$$

(2) R L C Series Circuit



$$i = I_m \sin \omega t$$

$V_R = I_m R \sin \omega t$ → in phase with voltage

$V_L = I_m X_L (\sin(\omega t + 90^\circ))$ → leads current by 90°

$V_C = I_m X_C \sin(\omega t - 90^\circ)$ → laggs behind current by 90°.

waveforms of all 4.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Apply KVL to this loop

$$V_s = V_R + V_L + V_C$$

$$V_s = I_m R \sin\omega t + I_m X_L \cos\omega t - I_m X_C \cos\omega t$$

$$V_s = I_m R \sin\omega t + I_m (X_L - X_C) \cos\omega t \rightarrow (6)$$

X = net ~~reac~~ reactance $= (X_L - X_C)$

This voltage is AC voltage so it has a proper maximum value and a phase angle which cannot be easily determined from this equation although it is correct and gives the ~~value~~ instantaneous value.

Generally $V_s = V_m \sin(\omega t \pm \theta)$

koracademy.com koracademy.com koracademy.com

if this angle θ is 0 the source voltage is in phase with current and is a pure resistive circuit.

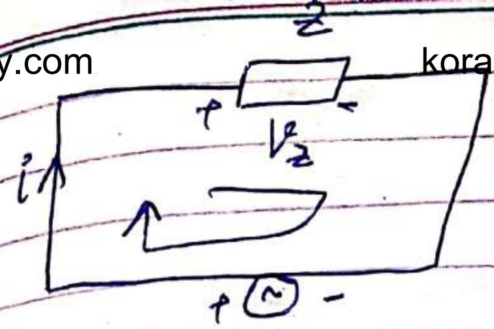
If is positive circuit will be inductive.
If is negative circuit will be capacitive.

Replace the series combination of these three circuit elements by a single passive circuit element theoretically.
It cannot be replaced practically due to the different nature of all the three elements

let Z is the total equivalent resistance of all 3 elements.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com



Apply KVL $\quad V_s = V_z$

$$\Rightarrow V_m = V_{mz} = I_{mz} Z$$

$$V_s = I_m Z \sin(\omega t \pm \theta)$$

Expand $\sin(\cancel{\omega t} \pm \beta)$.

$$V_s = I_m(Z\cos\theta)\sin\omega t \pm I_m(Z\sin\theta)\cos\omega t \rightarrow ⓑ$$

ⓐ = ⓑ

$$R = Z\cos\theta \quad \Rightarrow R^2 = Z^2\cos^2\theta$$

$$(X_L - X_C) = Z\sin\theta \quad \Rightarrow (X_L - X_C)^2 = Z^2\sin^2\theta \Big\} \text{Add}$$

koracademy.com koracademy.com koracademy.com

$$Z^2 = R^2 + (X_L - X_C)^2$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\theta = \frac{X_L - X_C}{R} \qquad \theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

(i) If $X_L = X_C \rightarrow$ entire circuit will behave like resistive circuit and this condition is known as resonance. Here the source voltage will be in phase with current.

(ii) If $X_L > X_C \Rightarrow \theta$ is positive and the entire circuit will behave like

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

inductive circuit.

(iii) If $X_L < X_C \Rightarrow \theta$ is negative and the circuit is capacitive.

Instententous power.

$$P_R = I_m^2 R \sin^2 \omega t$$

$$P_L = \frac{I_m^2 X_L}{2} \sin 2\omega t$$

$$P_C = -\frac{I_m^2 X_C}{2} \sin 2\omega t$$

Avege $P_R = I_{rms}^2 R \quad P_L = 0 \quad P_C = 0$

watmeter reads avege value of pow.

koracademy.com koracademy.com koracademy.com

law of conservation of energy

$$P_S = P_L + P_R + P_C$$

$$P_S = P_R = I_{rms}^2 R$$

Example 1

$R = 3\Omega \quad X_L = 4\Omega \quad X_C = 4\Omega$

$i = 10\sqrt{2} \sin 100\pi t$ A

$I_{rms} = 10$ A

$V_R = I_m R \sin \omega t = 30\sqrt{2} \sin 100\pi t$ volts.

$_R V_{rms} = 30$ volts

koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

$V_L = I_m X_L \sin(\omega t + \pi/2) = 40\sqrt{2} \sin(100\pi t + \pi/2)$

$V_{L\,rms} = 40$ volts.

$V_C = I_m X_C \sin(\omega t - \pi/2) = 40\sqrt{2} \sin(100\pi t - \pi/2)$

$V_{C\,rms} = 40$ volts.

$Z = \sqrt{R^2 + (X_L - X_C)^2} = R = 3\Omega$

$\theta = \tan^{-1} \frac{X_L - X_C}{R} = 0$

$V_S = I_m Z \sin(\omega t \pm \theta) = V_R = 30\sqrt{2} \sin 100\pi t.$

koracademy.com koracademy.com $V_{S\,rms} = 30$ volts. koracademy.com

under resonance condition,

→ Impedence $= R$

→ entire circuit offers minimum resistance and maximum current will flow.

→ $V_S = V_R$

$P_R = 600 \sin^2 \omega t$ $\quad P_L = 400 \sin 200\pi t$

$P_C = -400 \sin 2\omega t$ $\quad$ watts.

Example

$R = 8\Omega$ $\quad X_L = 16\Omega$ $\quad X_C = 10\Omega$

$i = 10\sqrt{2} \sin 100\pi t$ $\quad I_{rms} = 10A$

$V_R = I_m R \sin\omega t = 80\sqrt{2} \sin 100\pi t$

$V_{R\,rms} = 80$ volts.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$V_L = I_m X_L \sin(\omega t + \pi/2) = 160\sqrt{2}\sin(100\pi t + \pi/2)$$

$$V_{L\,rms} = 160 \text{ volts.}$$

$$V_C = I_m X_C \sin(\omega t - \pi/2) = 100\sqrt{2}\sin(100\pi t - \pi/2)$$

$$V_{C\,rms} = 100 \text{ volts.}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 10\Omega$$

$$\theta = \tan^{-1}\frac{X_L - X_C}{R} = 36.8^\circ$$

$$V_s = I_m Z \sin(\omega t \pm \theta) = 100\sqrt{2}\sin(100\pi t + 36.8^\circ) \text{ volts}$$

koracademy.com koracademy.com koracademy.com

$$V_{s\,rms} = 100 \text{ volts.}$$

$$P_R = 1600\sin^2 100\pi t \text{ watt}$$
$$P_L = 1600\sin 2\omega t$$
$$P_C = -100\sin 2\omega t$$

Example

$$R = 6\Omega \quad X_L = 10\Omega \quad X_C = 18\Omega$$

$$i = 10\sqrt{2}\sin 100\pi t \qquad I_{rms} = 10A$$

$$V_R = I_m R \sin\omega t = 60\sqrt{2}\sin 100\pi t \text{ volts.}$$
$$V_{R\,rms} = 60 \text{ volts.}$$

$$V_L = I_m X_L \sin(\omega t + \pi/2) = 100\sqrt{2}\sin(100\pi t + \pi/2)$$

$$V_{L\,rms} = 100 \text{ volts.}$$

koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

$$v_C = I_m X_C \sin(\omega t - \pi/2) = 180\sqrt{2}\sin(100\pi t - \pi/2)$$

$$V_{C\,rms} = 180\ V.$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 10\Omega$$

$$\theta = \tan^{-1}\frac{X_L - X_C}{R} = -53.2$$

$$v_s = I_m Z \sin(\omega t \pm \theta) = 100\sqrt{2}\sin(100\pi t - 53.2^\circ)$$

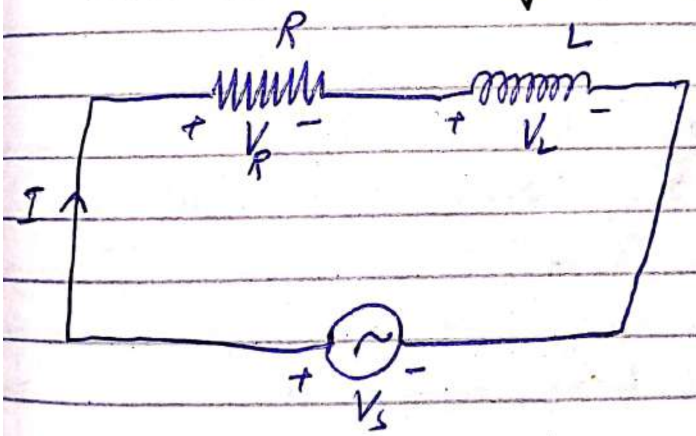
$$V_{s\,rms} = 100\ \text{volts}.$$

$$P_R = 1200\sin^2 100\pi t$$
$$P_C = -1800\sin 2\omega t$$
$$P_L = 1000\sin 2\omega t.$$

koracademy.com koracademy.com koracademy.com

Circuit

③ Phasor Analysis Of R-L Series



$$i = I_m \sin(\omega t + \theta)$$

Max Value $= I_m$

Rms values

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Phasor value $I = I_{rms}\angle 0^\circ = I_{rms}$

$$v_R = I_m R \sin(\omega t + 0^\circ)$$

Max value $= V_{Rm} = I_m R$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Rms value, $V_{rms} = \frac{I_m R}{\sqrt{2}}$

$$V_{rms} = I_{rms} R$$

Phasor value $V_R = I_{rms} R \angle 0^\circ = I_{rms} R$

As $\theta = 0^\circ$ $\sin 0 = 0$

$$V_L = I_m X_L \sin(\omega t + 90^\circ)$$

Max value $= I_m X_L$

Rms value, $V_{L_{rms}} = \frac{I_m X_L}{\sqrt{2}} = I_{rms} X_L$

koracademy.com koracademy.com koracademy.com

Phasor value $V_L = I_{rms} X_L \angle 90^\circ = \boxed{j I_{rms} X_L}$

as $\cos 90^\circ = 0$ imaginary phasor

Apply KVL;

The phasor sum of the voltage rises in a loop will be equal to the sum of all the voltage drops. phasor

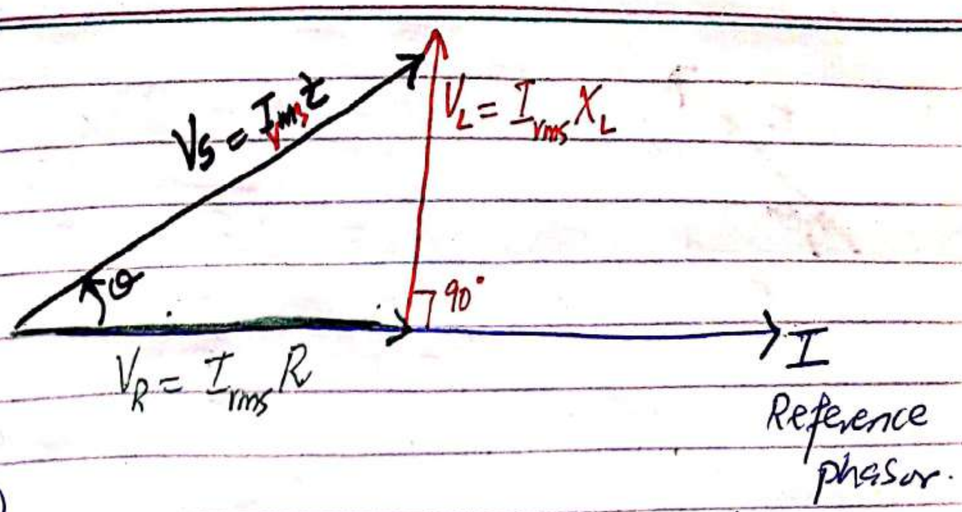
$$V_S = V_R + V_L$$

we add these two phasors by head to tail rule to get the phasor for the source voltage.

$$V_{S_{rms}} = \sqrt{V_R^2 + V_L^2}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com



OR Consider the instantaneous equation

$$V_S^t = I_m Z \sin(\omega t + \theta)$$

max Value $= I_m Z$ Rms $V_{s_{rms}} = \frac{I_m}{\sqrt{2}} Z$

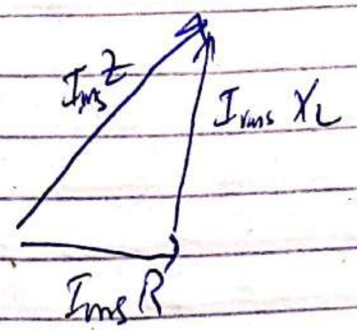
$$V_{s_{rms}} = I_{rms} Z$$

koracademy.com koracademy.com koracademy.com

$$I_{rms} = \frac{V_{s_{rms}}}{Z}$$

Phasor value $V_S = I_{rms} Z \angle \theta$

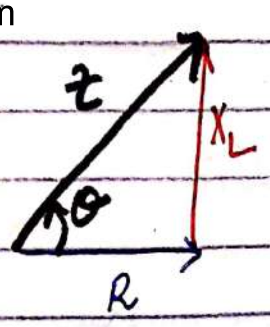
Consider the upper phasor diagram. let ignoring the current, now we dont call it a phasor diagram rather call it a voltage triangle.



As I_{rms} is common. Divide all the 3 sides of this ~~size~~ $\triangle$ by I_{rms} to get a new triangle

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com



$$Z = R + jX_L$$

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$\cos\theta = \frac{R}{Z}$$

Average values of power

$$P_S = P_R + \cancel{P_L}^{\,0} \qquad \boxed{P_S = I_{rms}^2 R}$$

$$P_S = I_{rms} \times I_{rms} \times R$$

$$\text{As } I_{rms} = \frac{V_{S\,rms}}{Z}$$

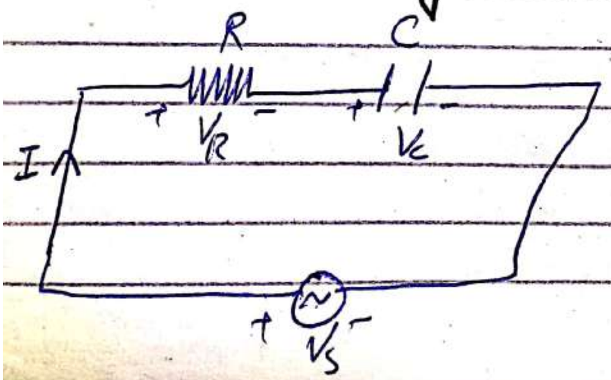
koracademy.com koracademy.com koracademy.com

$$P_S = V_{S\,rms} \times I_{rms} \times \frac{R}{Z}$$

$$\boxed{P_S = V_{S\,rms} I_{rms} \cos\theta}$$

$\cos\theta \rightarrow$ power factor
↳ lagging power factor.

Phasor Analysis of RC Series Circuit



$$i = I_m \sin(\omega t + \theta)$$

Max value $= I_m$

Rms value $= I_{rms}$

$$= \frac{I_m}{\sqrt{2}}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Phasor value $I = I_{rms} \angle 0^\circ = I_{rms}$

$v_R = I_m R \sin(\omega t + 0^\circ)$

max value $= V_{Rm} = I_m R$

Rms value $= V_{R\,rms} = \frac{I_m}{\sqrt{2}} R = I_{rms} R$

Phasor value $V_R = I_{rms} R \angle 0^\circ = I_{rms} R$

$v_C = I_m X_C \sin(\omega t - 90^\circ)$

max value $= I_m X_C$

Rms value ; $V_{C\,rms} = \frac{I_m}{\sqrt{2}} X_C = I_{rms} X_C$

Phasor value $V_C = I_{rms} X_C \angle -90^\circ = -j\, I_{rms} X_C$

koracademy.com koracademy.com koracademy.com

KVL $V_s = V_R + V_C$

$V_{s\,rms} = \sqrt{V_R^2 + V_C^2}$

$v_s = I_m Z \sin(\omega t - \theta)$

max value $= I_m Z$ $\quad$ Rms $= \frac{I_m}{\sqrt{2}} Z$

$V_{s\,rms} = I_{rms} Z$

$$\boxed{I_{rms} = \frac{V_{s\,rms}}{Z}}$$

Phasor value $V_s = I_{rms} Z \angle \theta$

Average power $P_s = P_R + \cancel{P_C} = I_{rm}{}^2 R$

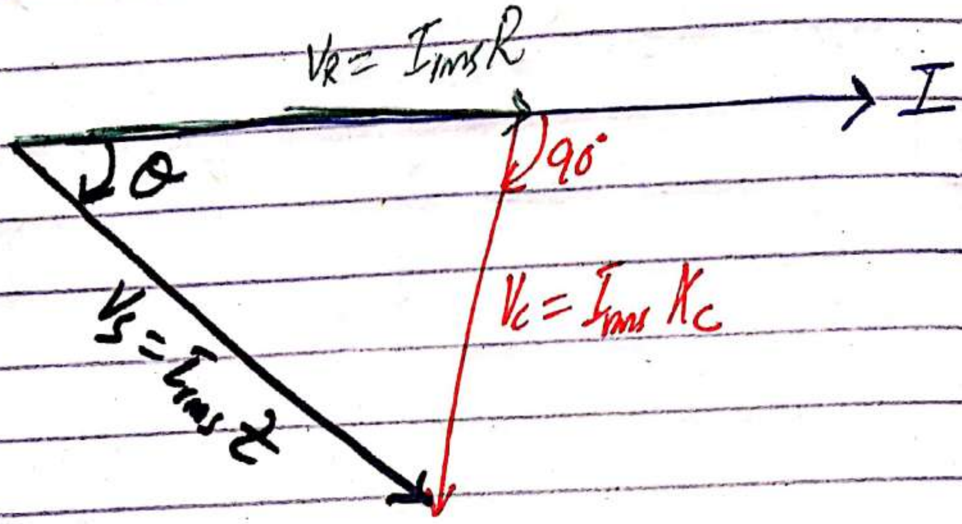
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

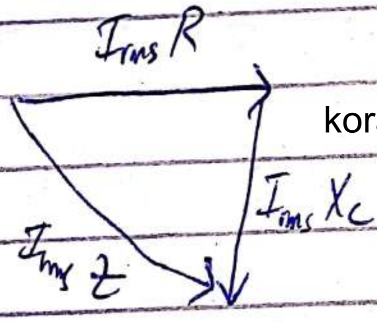
koracademy.com koracademy.com koracademy.com

$$\boxed{P_s = V_{s_{rms}} \times I_{rms} \times \cos\theta}$$

$\cos\theta \rightarrow$ power factor

Here leading power factor.

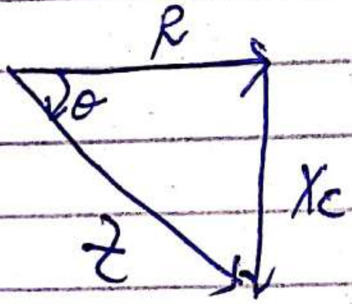




koracademy.com koracademy.com

voltage triangle.

koracademy.com



$$Z = R - jX_C$$

$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$$

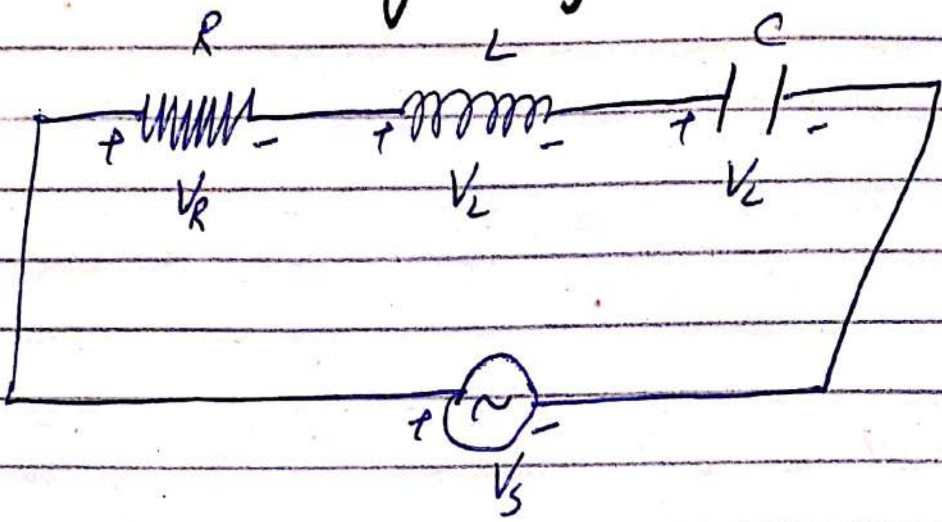
Power factor $\cos\theta = \frac{R}{Z}$ for this triangle.

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Phasor Analysis Of RLC Series Circuit



$$i = I_m \sin(\omega t + \theta)$$

Max value $= I_m$ Rms $I_{rms} = \frac{I_m}{\sqrt{2}}$

Phasor value $I = I_{rms} \angle 0^\circ = I_{rms}$

koracademy.com koracademy.com koracademy.com

$$V_R = I_m R \sin(\omega t + \theta)$$

Max value $V_{Rm} = I_m R$

Rms value $V_{R_{rms}} = \frac{I_m}{\sqrt{2}} R = I_{rms} R$

Phasor value of $V_R = I_{rms} R \angle 0^\circ = I_{rms} R$

$$V_L = I_m X_L \sin(\omega t + 90^\circ)$$

Max value $= I_m X_L$ Rms value $V_{rms} = \frac{I_m}{\sqrt{2}} X_L$

$$V_{Lrms} = I_{rms} X_L$$

Phasor value of $V_L = I_{rms} X_L \angle 90^\circ = j I_{rms} X_L$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$V_c = I_m X_C \sin(\omega t - 90^\circ)$$

Max value $= I_m X_C$

Rms value $\quad V_{C_{rms}} = \dfrac{I_m}{\sqrt{2}} X_C = I_{rms} X_C$

Phasor value of $V_C = I_{rms} X_C \angle 90^\circ = -j\, I_{rms} X_C$

Apply KVL

$$V_S = V_R + jV_L - jV_C$$

$$\boxed{V_S = V_R + j(V_L - V_C)}$$

koracademy.com koracademy.com koracademy.com

$$\boxed{V_{S_{rms}} = \sqrt{V_R^2 + (V_L - V_C)^2}} \rightarrow (1)$$

(i) Resistive Circuit

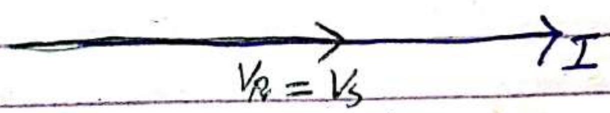
If $X_L = X_C$

$$I_{rms} X_L = I_{rms} X_C$$

$$V_{L_{rms}} = V_{C_{rms}}$$

$V_L - V_C = 0$

$$(1) \Rightarrow \quad V_S = V_R$$



koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

(ii) Inductive Circuit

if $X_L > X_C$

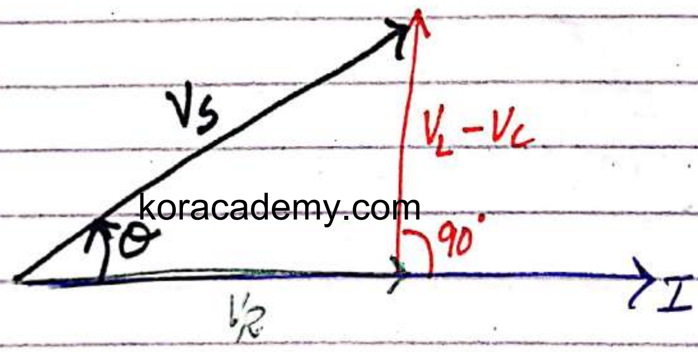
$I_{rms} X_L > I_{rms} X_C$

$V_{L_{rms}} > V_{C_{rms}}$

$V_L - V_C > 0 \Rightarrow j \rightarrow$ positive

The combination of inductor and capacitor behaves like an inductor.

voltage across inductor leads current by 90°.



koracademy.com koracademy.com koracademy.com

(iii) Capacitive Circuit

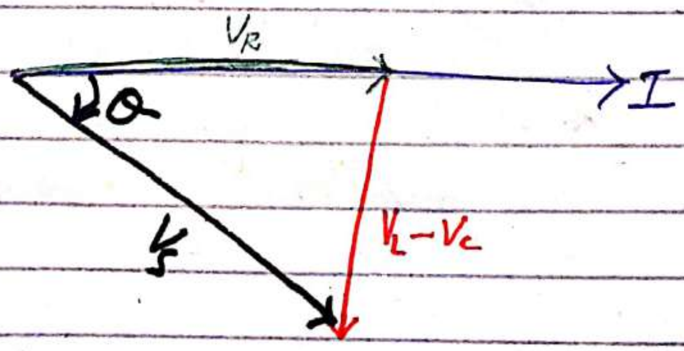
if $X_L < X_C$

$I_{rms} X_L < I_{rms} X_C$

$V_{L_{rms}} < V_{C_{rms}}$

$V_L - V_C = -ve$

$j = -ve$



koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

Example I:

R-L Circuit.

$R = 3\Omega \quad X_L = 4\Omega \quad V_{s\,rms} = 100$ volts

$Z = R + jX_L = 3 + j4\ \Omega$

$|Z| = \sqrt{9+16} = 5\ \Omega$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 51.3^\circ$

$I_{rms} = \frac{V_{s\,rms}}{Z} = \frac{100}{5} = 20A$

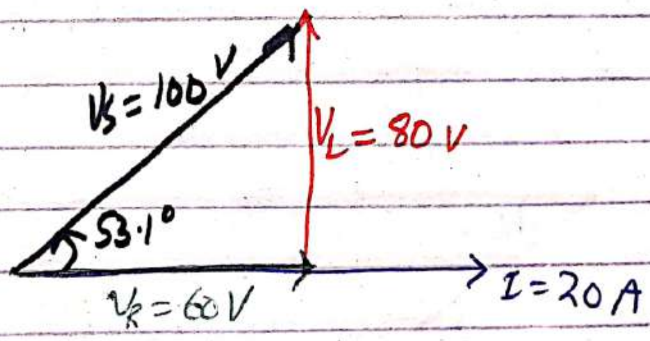
koracademy.com koracademy.com koracademy.com

$V_{R\,rms} = I_{rms} R = 60\ V$

$V_{L\,rms} = I_{rms} X_L = 80\ V$

Power factor $= \cos\theta = \frac{R}{Z} = \frac{3}{5} = 0.6$ lagging power factor.

$P_s = I^2_{rms} R = V_{s\,rms} \times I_{rms} \times \cos\theta = 100 \times 20 \times 0.6$
$= 1200$ watt.



koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 2

R–C circuit

$R = 3\Omega \quad X_C = 4\Omega \quad V_{s\,rms} = 100$ volts

$$Z = R - jX_C = 3 - j4\ \Omega$$

$$|Z| = \sqrt{9+16} = 5\Omega$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$$

$$I_{rms} = \frac{V_{s\,rms}}{Z} = \frac{100}{5} = 20\,A$$

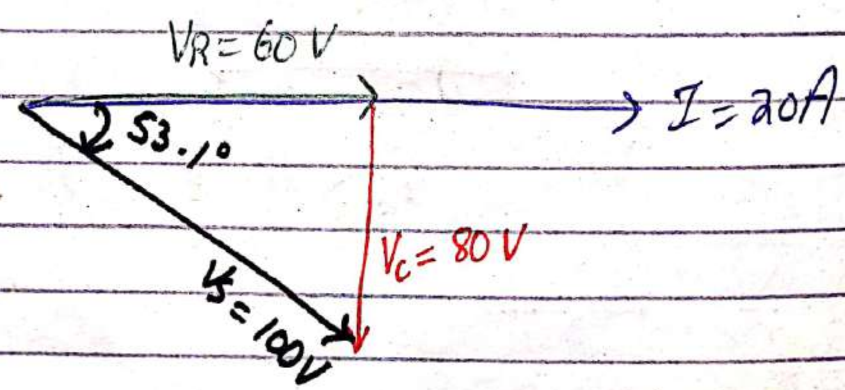
$$V_{R\,rms} = I_{rms} R = 60\,V$$

koracademy.com koracademy.com koracademy.com

$$V_{C\,rms} = I_{rms} X_C = 80\,V$$

$$\text{Power factor} = \frac{R}{Z} = \frac{3}{5} = 0.6 \rightarrow \text{leading P.f}$$

$$P_s = I_{rms}^2 R = V_{s\,rms} \times I_{s\,rms} \times \cos\theta = 1200\,W$$



koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 3:

RLC series Circuit

$X_L = 4\Omega \quad R = 3\Omega \quad X_C = 8\Omega \quad V_{s_{rms}} = 100\,V$

$$Z = R + j(X_L - X_C)$$

$$Z = R - jX = 3 - j4\,\Omega$$

$$|Z| = \sqrt{9+16} = 5\Omega \qquad \theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$$

$$I_{rms} = \frac{V_{s_{rms}}}{Z} = \frac{100}{5} = 20A$$

$$V_{R_{rms}} = I_{rms} R = 60\,V$$

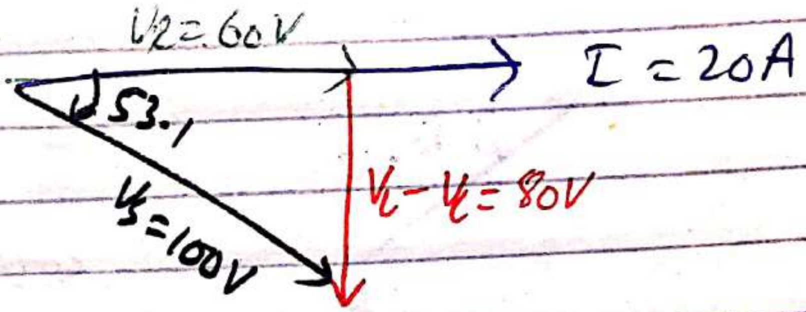
koracademy.com koracademy.com koracademy.com

$$V_{C_{rms}} = I_{rms} X_C = 160\,V$$

$$V_{L_{rms}} = I_{rms} X_L = 80\,V$$

$$\text{Power factor} = \frac{R}{Z} = 0.6 \rightarrow \text{leading power factor.}$$

$$P_s = I_{rms}^2 R = I_{rms} \times V_{s_{rms}} \times \cos\theta = 1200\,W$$



koracademy.com koracademy.com
Scanned with CamScanner
koracademy.com

koracademy.com koracademy.com koracademy.com

Example 4.

RLC Series Circuit

$X_C = 4\Omega \quad R = 3\Omega \quad X_L = 8\Omega \quad V_{s_{rms}} = 100\,V$

$Z = R + j(X_L - X_C)$

$Z = R + jX = 3 + j4\,\Omega$

$|Z| = \sqrt{9+16} = 5\Omega \qquad \theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$

$I_{rms} = \frac{V_{s_{rms}}}{Z} = \frac{100}{5} = 20\,A$

$V_{R_{rms}} = I_{rms} R = 60\,V$

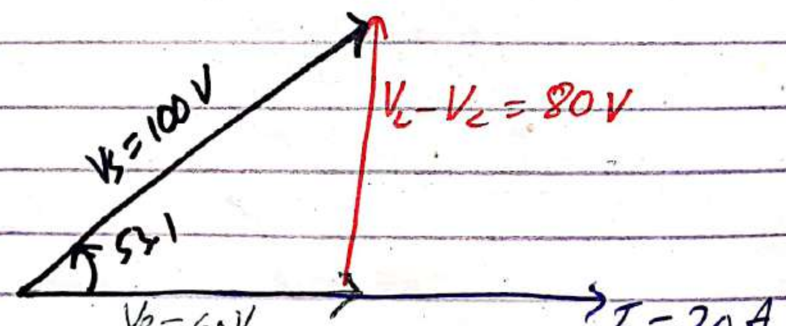
$V_{L_{rms}} = I_{rms} X_L = 160\,V$

koracademy.com koracademy.com koracademy.com

$V_{C_{rms}} = I_{rms} X_C = 80\,V$

Power factor $= \frac{R}{Z} = \frac{3}{5} = 0.6 \rightarrow$ ~~lead~~ lagging power factor.

$P_s = I_{rms}^2 R = I_{rms} \times V_{s_{rms}} \times \cos\theta = 1200$ watts.



purely inductive circuit.

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 5

$X_C = 8\,\Omega \quad R = 5\,\Omega \quad X_L = 8\,\Omega$

$V_{s_{rms}} = 100$ volts

$$Z = R + j(X_L - X_C)$$

$$Z = R = 5\,\Omega \qquad \theta = \tan^{-1}\left(\frac{0}{3}\right) = 0$$

$$I_{rms} = \frac{V_{s_{rms}}}{Z} = \frac{100}{5} = 20\,A$$

$$V_{s_{rms}} = V_{R_{rms}} = I_{rms} R = 100\,V.$$

$$V_{L_{rms}} = I_{rms} X_L = 160\,V$$

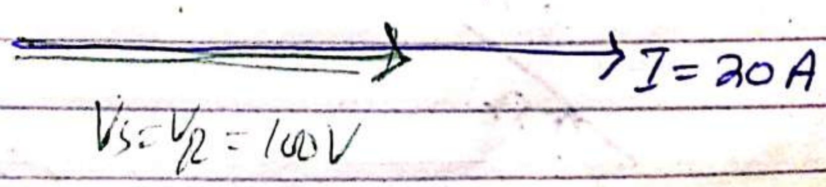
koracademy.com koracademy.com koracademy.com

$$V_{C_{rms}} = I_{rms} X_C = 160\,V.$$

Power factor $= \frac{R}{Z} = \frac{3}{3} = 1 \rightarrow$ unity power factor

$$P_s = I_{rms}^2 R = V_{s_{rms}} * I_{rms} \times \cos\theta = 2000 \text{ Watt}.$$

Circuit behaves purely resistive.



koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Example 6

$P_s = \cancel{1200}\ 600\,W \qquad P.f = \underbrace{0.6 \text{ lagging}}_{\text{RL series circuit}}$

$R = 6\,\Omega$

$P_s = P_R = I_{rms}^2 R$

$I_{rms}^2 = \frac{P_s}{R} = \frac{600}{6} = 100$

$I_{rms} = \sqrt{100} = 10\ A$

$\text{Power factor} = \frac{R}{Z} = 0.6 \Rightarrow Z = \frac{6}{0.6} = 10\,\Omega$

$Z^2 = R^2 + X_L^2 \Rightarrow X_L = \sqrt{-R^2 + Z^2}$

$X_L = \sqrt{100 - 36} = \sqrt{64} \Rightarrow X_L = 8\,\Omega$

koracademy.com koracademy.com koracademy.com

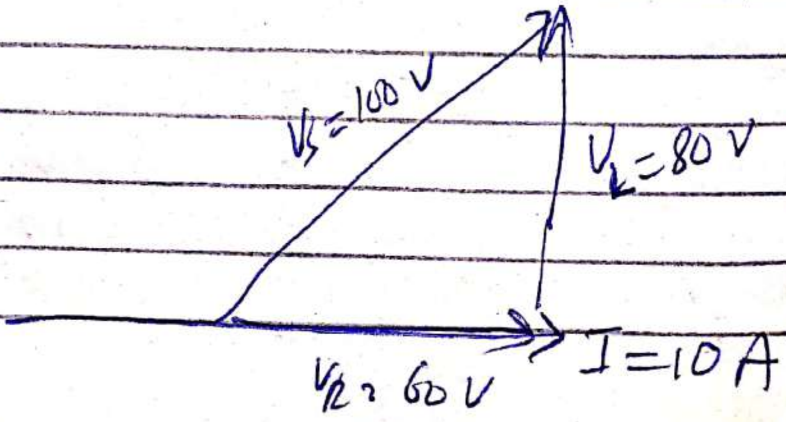
$\cos\theta = 0.6$

$\Rightarrow \theta = \cos^{-1}(0.6) = 53.1^\circ$

$V_{s\,rms} = I_{rms} Z = 10 \times 10 = 100\ V$

$V_{R\,rms} = I_{rms} R = 60\ V$

$V_{L\,rms} = I_{rms} X_L = 80\ V$



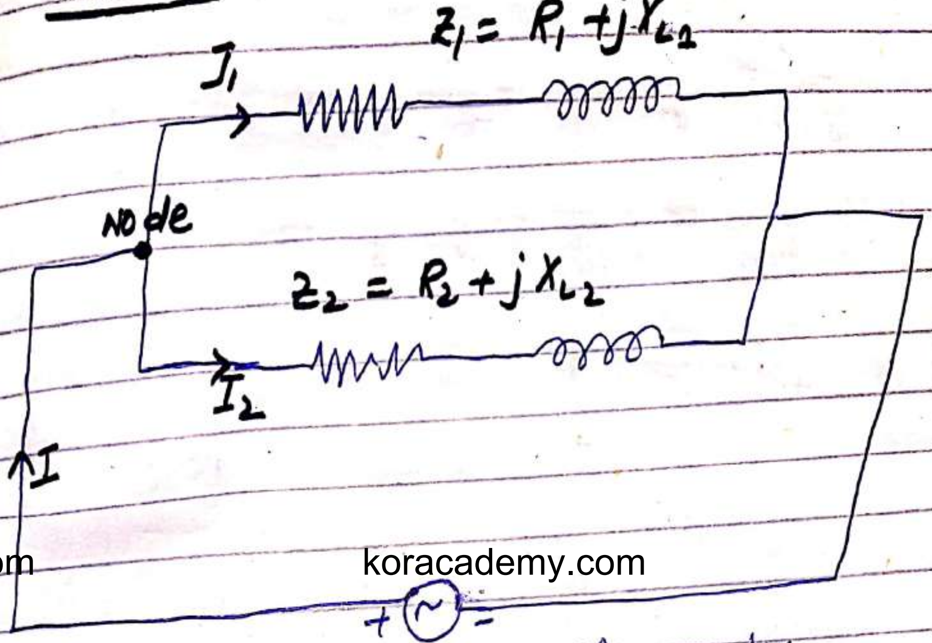
koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

(4) # Parallel Circuits.

Impedance method

Case 1



koracademy.com koracademy.com koracademy.com

$V_s = V_{rms} \angle 0^\circ$ → polar

$V_s = V_{rms}$ → rectangular

$Z_1 = |Z_1| \angle \theta_1$ $\qquad Z_2 = |Z_2| \angle \theta_2$

$$I_1 = \frac{V_s}{Z_1} = \frac{V_{rms} \angle 0^\circ}{|Z_1| \angle \theta_1^\circ}$$

$$I_1 = \frac{V_{rms}^{s}}{|Z_1|} \angle -\theta_1 = I_{1rms} \angle -\theta_1$$

In rectangula $I_1 = I_{a1} - jI_{b1}$

Similery $I_2 = \dfrac{V}{Z_2} = \dfrac{V_{rms} \angle 0^\circ}{|Z_2| \angle \theta_2}$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$I_2 = \frac{V_{s_{rms}}}{|Z_2|} \angle -\theta_2 = I_{2_{rms}} \angle -\theta_2$$

In rectangular form $\quad I_2 = I_{a2} - jI_{b2}$

Apply KCL to the only node of the circuit.

$$I = I_1 + I_2$$

$$I = I_{a1} - jI_{b1} + I_{a2} - jI_{b2}$$

$$I = (I_{a1} + I_{a2}) - j(I_{b1} + I_{b2})$$

$$I = I_{rms} \angle -\theta$$

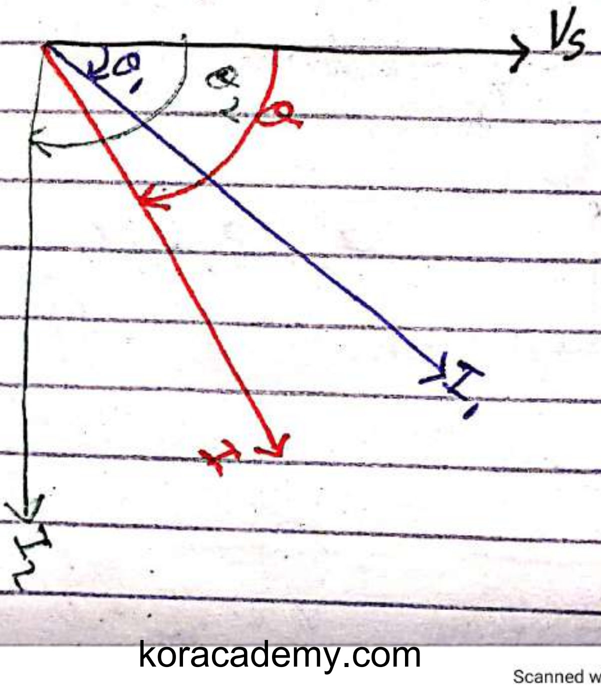
$$P.f_2 = \frac{R_1}{|Z_1|} = \cos\theta_1$$

koracademy.com koracademy.com koracademy.com

Lagging power factor.

$$P.f_2 = \frac{R_2}{|Z_2|} = \cos\theta_2$$

$$P.f = \cos\theta$$



koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

The power taken by the two branches:

$$P_1 = I_{1_{rms}}^2 R_1 = V_{s_{rms}} \times I_{1_{rms}} \times \cos\theta_1$$

$$P_2 = I_{2_{rms}}^2 R_2 = V_{s_{rms}} \times I_{2_{rms}} \times \cos\theta_2$$

$$P_s = P_t = P_1 + P_2$$

Total power delived by source

$$P_s = V_{s_{rms}} \times I_{rms} \times \cos\theta$$

Instantanous Expressions

$$v_s = \sqrt{2}\, V_{s_{rms}} \sin(wt + 0^\circ)$$

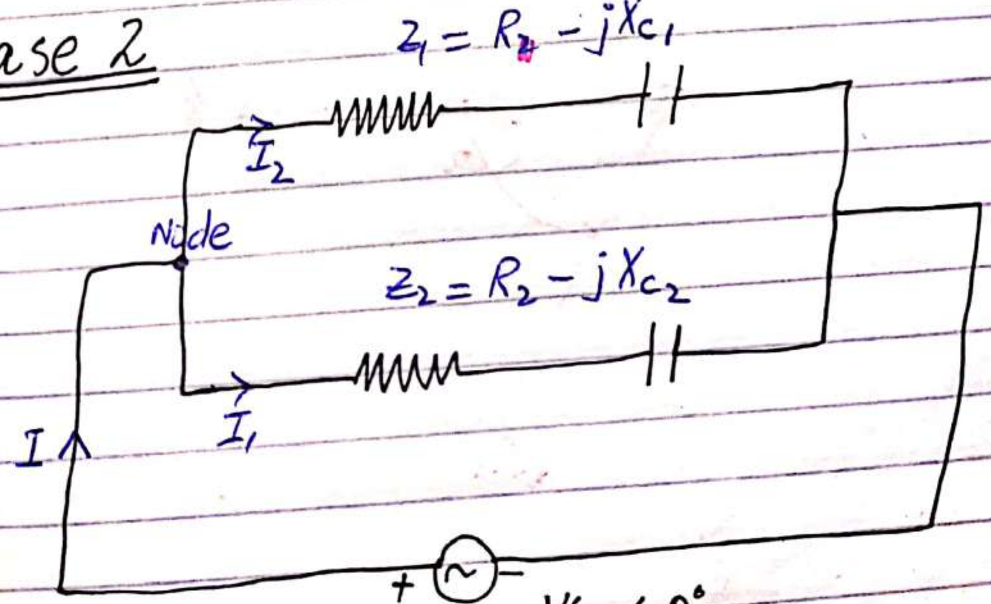
$$i_1 = \sqrt{2}\, I_{1_{rms}} \sin(wt - \theta_1)$$

koracademy.com koracademy.com koracademy.com

$$i_2 = \sqrt{2}\, I_{2_{rms}} \sin(wt - \theta_1)$$

$$i = \sqrt{2}\, I_{rms} \sin(wt - \theta).$$

Case 2



Polar $V_s = V_{s_{rms}} \angle 0^\circ$

Rectansular $V_s = V_{s_{rms}}$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$Z_1 = |Z_1| \angle -\theta_1 \quad Z_2 = |Z_2| \angle -\theta_2$

$$I_1 = \frac{V_s}{Z_1} = \frac{V_{s\,rms} \angle 0^\circ}{|Z_1| \angle -\theta_1}$$

$$I_1 = \frac{V_{s\,rms}}{|Z_1|} \angle +\theta_1 = I_{1\,rms} \angle +\theta_1$$

In rectangular form $I_1 = I_{a1} + j\, I_{b1}$

Similarly $I_2 = \frac{V_s}{Z_2} = \frac{V_{s\,rms} \angle 0^\circ}{|Z_2| \angle -\theta_2}$

$$I_2 = \frac{V_{s\,rms}}{|Z_2|} \angle +\theta_2 = I_{2\,rms} \angle +\theta_2$$

$$I_2 = I_{a2} + j\, I_{b2}$$

koracademy.com koracademy.com koracademy.com

Apply KCL to the only node.

$$I = I_1 + I_2$$

$$I = I_{a1} + j\, I_{b1} + I_{a2} + j\, I_{b2} = (I_{a1} + I_{a2}) + j\,(I_{b1} + I_{b2})$$

$$I = I_{rms} \angle +\theta$$

$P.f_1 = \frac{R_1}{|Z_1|} = \cos\theta_1 \qquad P.f_2 = \frac{R_2}{|Z_2|} = \cos\theta_2$

$P.f = \cos\theta$ leading.

$$P_1 = I_{1\,rms}^2 R_1 = V_{s\,rms} \times I_{1\,rms} \times \cos\theta_1$$

$$P_2 = I_{2\,rms}^2 R_2 = V_{s\,rms} \times I_{2\,rms} \times \cos\theta_2$$

koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

$$P_s = P_t = P_1 + P_2$$

$$P_s = V_{s_{rms}} \times I_{rms} \times \cos\theta .$$

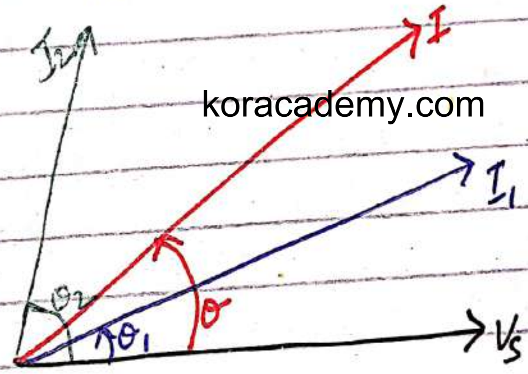
Instantaneous expressions

$$v_s = \sqrt{2}\, V_{s_{rms}} \sin(\omega t + 0^\circ)$$

$$i_1 = \sqrt{2}\, I_{1_{rms}} \sin(\omega t + \theta_1)$$

$$i_2 = \sqrt{2}\, I_{2_{rms}} \sin(\omega t + \theta_2) .$$

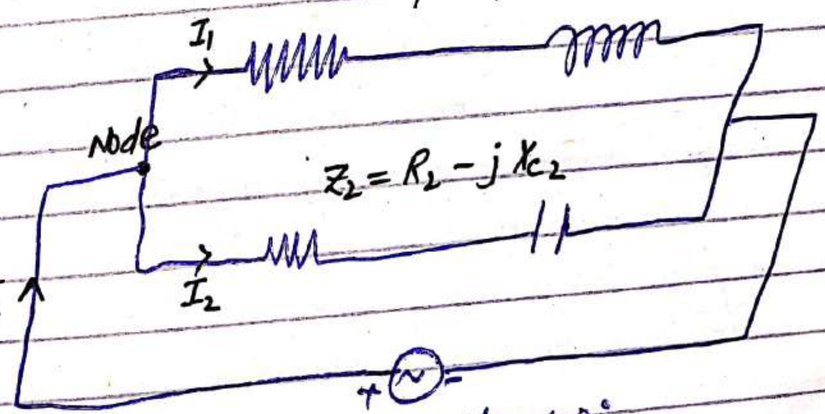
$$i = \sqrt{2}\, I_{rms} \sin(\omega t + \theta) .$$



koracademy.com koracademy.com koracademy.com

Case 3

$$Z_1 = R_1 + j X_{L_1}$$



$$V_s = V_{s_{rms}} \angle 0^\circ$$

$$V_s = V_{s_{rms}}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$Z_1 = |Z_1| \angle \theta_1 \qquad Z_2 = |Z_2| \angle -\theta_2$

$I_1 = \frac{V_s}{Z_1} = \frac{V_{s_{rms}} \angle 0^\circ}{|Z_1| \angle \theta_1} = \frac{V_{s_{rms}}}{|Z_1|} \angle -\theta_1 = I_{1_{rms}} \angle -\theta_1$

Rectangular $\quad I_1 = I_{a1} - j I_{b1}$

$I_2 = \frac{V_s}{Z_2} = \frac{V_{s_{rms}} \angle 0^\circ}{|Z_2| \angle -\theta_2} = \frac{V_{s_{rms}}}{|Z_2|} \angle +\theta_2 = I_{2_{rms}} \angle +\theta_2$

Rectangular $\quad I_2 = I_{a2} + j I_{b2}$

Apply KCL $\quad I = I_1 + I_2$

$I = I_{a1} - j I_{b1} + I_{a2} + j I_{b2}$

koracademy.com koracademy.com koracademy.com

$I = (I_{a1} + I_{a2}) \pm j(-I_{b1} + I_{b2})$

$I = I_{rms} \angle \pm \theta$

$P.f_1 = \frac{R_1}{|Z_1|} \cos\theta_1 \qquad P.f_2 = \frac{R_2}{|Z_2|} \cos\theta_2$

$P.f = \cos\theta$

$P_1 = I_{1_{rms}}^2 R_1 = V_{s_{rms}} \times I_{1_{rms}} \times \cos\theta_1$

$P_2 = I_{2_{rms}}^2 R_2 = V_{s_{rms}} \times I_{2_{rms}} \times \cos\theta_2$

$P_s = P_t = P_1 + P_2$

$P_s = V_{s_{rms}} \times I_{rms} \times \cos\theta$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Instantanous Expressions.

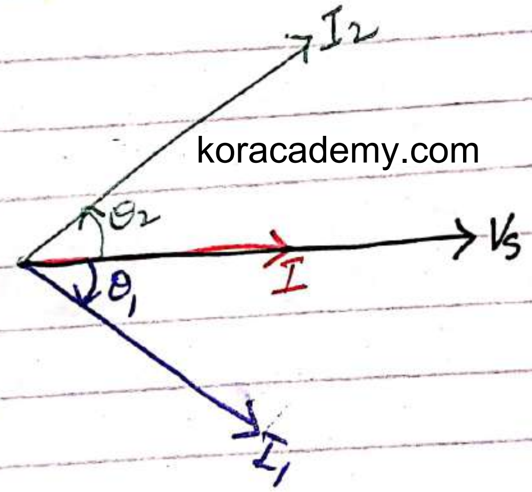
$$v_s = \sqrt{2}\, V_{s_{rms}} \sin(\omega t + 0^\circ)$$

$$i_1 = \sqrt{2}\, I_{1_{rms}} \sin(\omega t - \theta_1)$$

$$i_2 = \sqrt{2}\, I_{2_{rms}} \sin(\omega t + \theta_2)$$

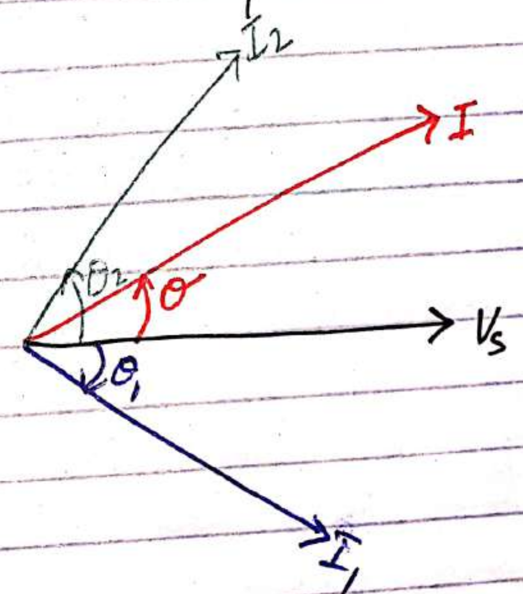
$$i = \sqrt{2}\, I_{rms} \sin(\omega t \pm \theta)$$

if $\theta = 0^\circ \Rightarrow$ Resistive circuit.



koracademy.com koracademy.com koracademy.com

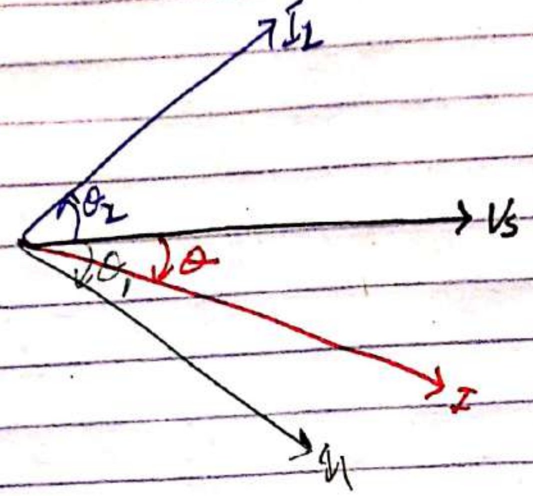
if $\theta = +ve \Rightarrow$ capacitive circuit



koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

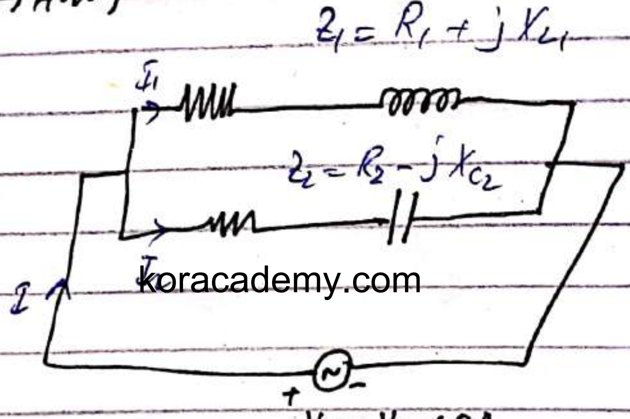
if $\theta = -ve \Rightarrow$ inductive circuit



Parallel Circuits By Admittance method.

(Case 1+2 → H.W)

Case 3



koracademy.com koracademy.com koracademy.com

$V_s = V_{s_{rms}} \angle 0°$

$V_s = V_{rms}$

$$I_1 = \frac{V_s}{Z_1} = V_s \times \frac{1}{Z_1}$$

$\frac{1}{Z_1} \rightarrow$ Admittance $= Y$ $\quad I_1 = V_s Y_1 \rightarrow (1)$

$$\frac{1}{Z_1} = \frac{1}{R + jX_L} \times \frac{R - jX_{L_1}}{R - jX_{L_1}}$$

$$\boxed{Y_1 = \frac{R_1}{R_1^2 + X_L^2} - \frac{jX_{L_1}}{R^2 + X_L^2}}$$

$$Y_1 = G_1 - jB_1$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$(1) \Rightarrow I_1 = V_{s_{rms}} (G_1 - j B_1).$

$I_1 = I_{G_1} - j I_{S_1} \qquad I_1 = I_{1_{rms}} \angle -\theta_1$

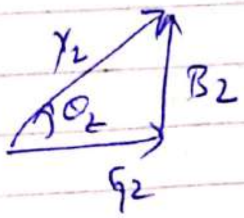
$I_2 = V_s \times \frac{1}{Z_2} = V_s Y_2 \rightarrow (2)$

$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_{C2}} \times \frac{R_2 + jX_{C2}}{R_2 + jX_{C2}}$

$Y_2 = \frac{R_2}{R^2 + X_{C2}^2} + j \frac{X_{C2}}{R_2^2 + X_{C2}^2}$

$Y_2 = \underbrace{G_2}_{\text{conductance}} + j \underbrace{B_2}_{\text{susceptance}}$

koracademy.com koracademy.com koracademy.com



$(2) \Rightarrow I_2 = V_{s_{rms}} (G_2 + jB_2)$

$I_2 = I_{G_2} + jI_{S_2}$

$I_2 = I_{2_{rms}} \angle \theta_2$

Apply KCL

$I = I_1 + I_2$

$I = I_{G_1} - jI_{S_1} + I_{G_2} + jI_{S_2}$

$I = (I_{G_1} + I_{G_2}) \pm j(-I_{S_1} + I_{S_2})$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$I = I_{rms} \angle \pm \theta$

$P.f_1 = \frac{Z_1}{|R_1|} \cos\theta_1 \quad P.f_2 = \frac{Z_2}{|R_2|} \cos\theta_2$

$P.f = \cos\theta$

$P_1 = I_{1rms}^2 R_1 = V_{s_{rms}} \times I_{1_{rms}} \times \cos\theta_1$

$P_2 = I_{2_{rms}}^2 R_2 = V_{s_{rms}} \times I_{2_{rms}} \times \cos\theta_2$

$P_s = P_t = P_1 + P_2$

$P_s = V_{s_{rms}} \times I_{rms} \times \cos\theta$

Instantaneous expressions

koracademy.com

$V_s = \sqrt{2}\, V_{s_{rms}} \sin(\omega t + 0^\circ)$

koracademy.com koracademy.com

$i_1 = \sqrt{2}\, I_{1_{rms}} \sin(\omega t - \theta_1)$

$i_2 = \sqrt{2}\, I_{2_{rms}} \sin(\omega t + \theta_2)$

$I = \sqrt{2}\, I_m \sin(\omega t \pm \theta)$

Example ex 3.

$Z_1 = 3 + j4\,\Omega \qquad Z_2 = 3 - j4\,\Omega$

$V_s = 100 \angle 0^\circ \quad V_s = 100\,V$

$Z_1 = 5 \angle 53.1^\circ\,\Omega \qquad Z_2 = 5 \angle -53.1^\circ\,\Omega$

$I_1 = \frac{V_s}{Z_1} = \frac{100}{5 \angle 53.1} \qquad I_1 = 20 \angle -53.1^\circ$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$I_1 = 12 - j16 \text{ A}$$

$$I_2 = \frac{V_s}{Z_2} = \frac{100}{5\angle -53.1^\circ} = 20\angle 53.1^\circ \text{ A}$$

$$I_2 = 12 + j16 \text{ A}$$

By Admittance

$$Y_1 = \frac{3}{25} - j\frac{4}{25} \ \mho$$

$$I_1 = V_s Y_1 = 100\left(\frac{3}{25} - j\frac{4}{25}\right)$$

$$I_1 = 12 - j16 \text{ A} = 20\angle -53.1^\circ \text{ A}$$

koracademy.com koracademy.com koracademy.com

$$Y_2 = \frac{3}{25} + j\frac{4}{25} \ \mho$$

$$I_2 = V_s Y_2 = 100\left(\frac{3}{25} + j\frac{4}{25}\right)$$

$$I_2 = 12 + j16 \text{ A} = 20\angle 53.1^\circ \text{ A}$$

$$I = I_1 + I_2 = 24 \text{ A}$$

$$P.f_1 = \cos\theta_1 = 3/5 = 0.6$$

$$P.f_2 = \cos\theta_2 = 3/5 = 0.6$$

$$P.f = \cos 0^\circ = 1.$$

$$P_1 = I_{1m}^2 R_1 = 1200 \text{ W}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$P_2 = I_{2rms}^2 R_2 = 1200\,W$

$P_s = P_1 + P_2 = 2400\,W$

Instantaneous expression

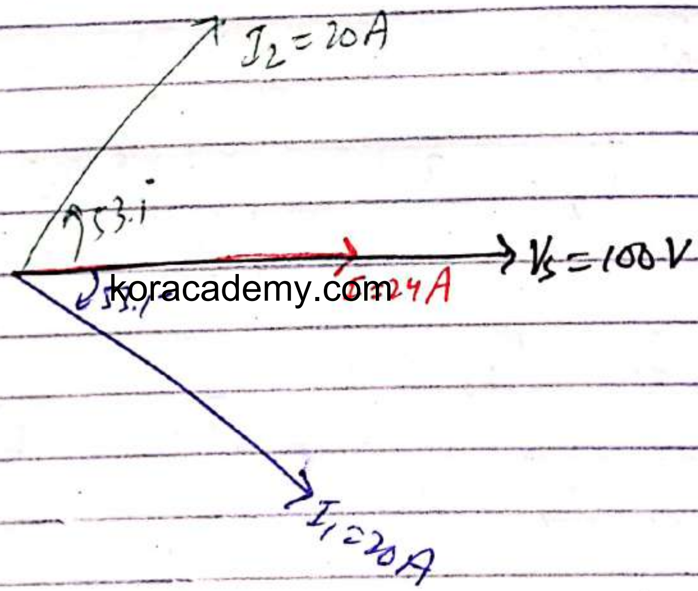
Let $f = 50\ Hz$

$V_s = 100\sqrt{2}\sin(100\pi t)$

$i_1 = 20\sqrt{2}\sin(100\pi t - 53.1^\circ)$

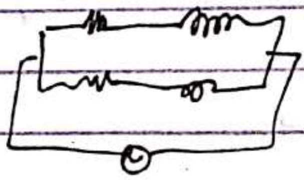
$i_2 = 20\sqrt{2}\sin(100\pi t + 53.1^\circ)$

$i = 24\sqrt{2}\sin(100\pi t + 0)$



koracademy.com koracademy.com koracademy.com

Example 2. Case a



$Z_1 = 3 + j4\,\Omega \qquad Z_2 = 3 + j4\,\Omega$

$V_s = 100\angle 0^\circ = 100\,V.$

$Z_1 = 5\angle 53.1^\circ\,\Omega \qquad Z_2 = 5\angle 53.1^\circ\,\Omega$

$I_1 = \frac{V_s}{Z_1} = \frac{100}{5\angle 53.1^\circ} = 20\angle -53.1^\circ\,A$

$I_1 = 12 - j16\,A$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$I_2 = \frac{V_s}{Z_2} = \frac{100}{5\angle 53.1^\circ} = 20\angle -53.1^\circ\ A$$

$$I_1 = 12 - j16\ A.$$

Admittance method

$$Y_1 = \frac{3}{25} - j\frac{4}{25}\ \mho$$

$$I_1 = V_s Y_1 = 100\left(\frac{3}{25} - j\frac{4}{25}\right)$$

$$I_1 = 12 - j16\ A = 20\angle -53.1^\circ\ A.$$

$$Y_2 = \frac{3}{25} - j\frac{4}{25}\ \mho$$

koracademy.com koracademy.com koracademy.com

$$I_2 = V_s Y_2 = 100\left(\frac{3}{25} - j\frac{4}{25}\right)$$

$$I_2 = 12 - j16\ A = 20\angle -53.1^\circ\ A$$

$$I = I_1 + I_2 = 24 - j32\ A = 40\angle -53.1\ A$$

$$P.f_1 = \cos\theta_1 = 3/5 = 0.6\ A = \cos\theta_2 = P.f_2$$

$$P.f = \cos 53.1 = 0.6.$$

$$P_1 = I_{1_{rms}}^2 R_1 = 1200\ W$$

$$P_2 = I_{2_{rms}}^2 R_2 = 1200\ W.$$

$$P_s = P_1 + P_2 = 2400\ W.$$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Instantaneous expressions

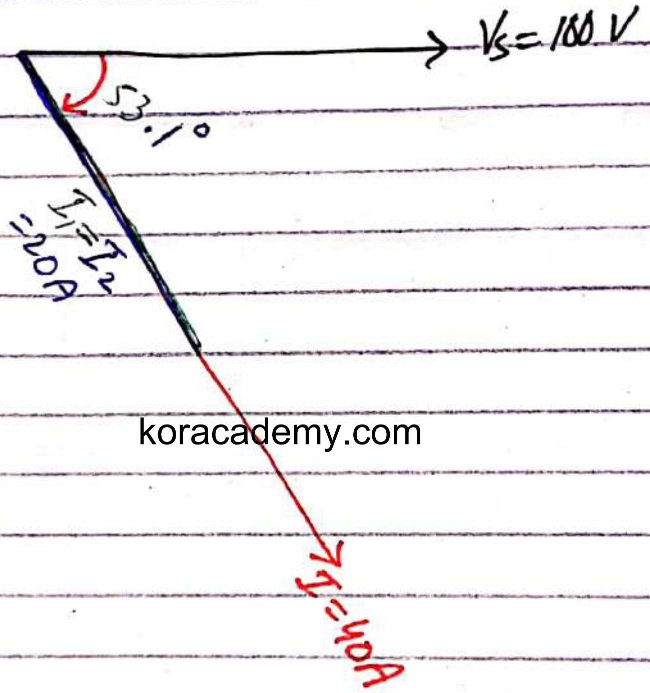
if $f = 50\ Hz$

$v_s = 100\sqrt{2}\sin(100\pi t)$

$i_1 = 20\sqrt{2}\sin(\omega t - 53.1^\circ)$

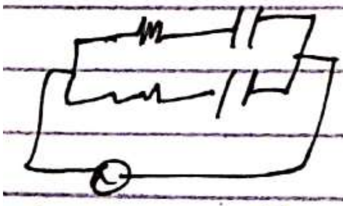
$i_2 = 20\sqrt{2}\sin(100\pi t - 53.1^\circ)$

$i = 24\sqrt{2}\sin(100\pi t \ 53.1^\circ)$



koracademy.com koracademy.com koracademy.com

Example 3 Case 2.



$Z_1 = 3 - j4\ \Omega \quad Z_2 = 3 - j4\ \Omega$

$V_s = 100\angle 0^\circ = 100\ V \quad f = 50\ Hz$

$Z_1 = 5\angle -53.1 \quad Z_2 = 5\angle -53.1$

$I_1 = \frac{V_s}{Z_1} = \frac{100}{5\angle -53.1} = 20\angle 53.1^\circ$

$I_1 = 12 + 16j\ A$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$I_2 = \frac{V_s}{Z_2} = \frac{100}{5\angle -53.1} = 20\angle 53.1^\circ$$

$$I_2 = 12 + j16\ A$$

Admittance

$$Y_1 = \frac{3}{25} + j\frac{4}{25}\ ℧$$

$$I_1 = V_s Y_1 = 100\left(\frac{3}{25} + j\frac{4}{25}\right) = 12 + j16\ A$$
$$= 20\angle 53.1^\circ\ A$$

$$Y_2 = \frac{3}{25} + j\frac{4}{25}\ ℧$$

$$I_2 = V_s Y_2 = 100\left(\frac{3}{25} + j\frac{4}{25}\right)$$

koracademy.com koracademy.com koracademy.com

$$I_2 = 12 + j16\ A = 20\angle 53.1^\circ\ A$$

$$I = I_1 + I_2 = 24 + j32 = 40\angle 53.1^\circ$$

$$P.f_1 = \cos\theta_1 = 3/5 = 0.6$$
$$P.f_2 = \cos\theta_2 = 3/5 = 0.6$$

$$P.f = \cos 53.1^\circ = 0.6$$

$$P_1 = I_{rms1}^2 R_1 = 1200\,W$$

$$P_2 = I_{rms2}^2 R_2 = 1200\,W$$

$$P_s = P_1 + P_2 = 2400\,W.$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

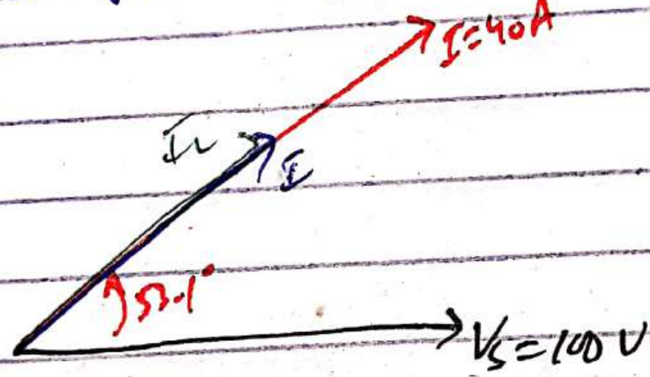
Instantaneous

$$V_s = 100\sqrt{2}\,\sin(100\pi t)$$

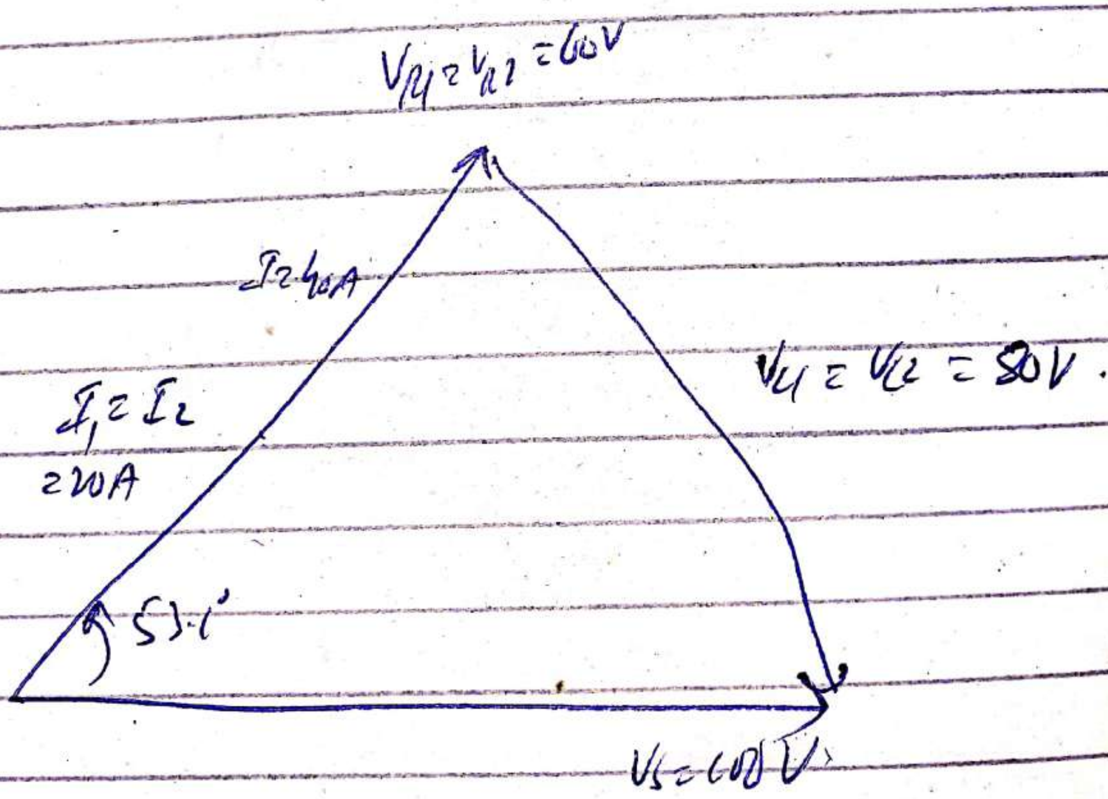
$$i_1 = 20\sqrt{2}\,\sin(100\pi t + 53.1^\circ)$$

$$i_2 = 20\sqrt{2}\,\sin(100\pi t + 53.1)$$

$$i = 24\sqrt{2}\,\sin(100\pi t + 53.1^\circ)$$



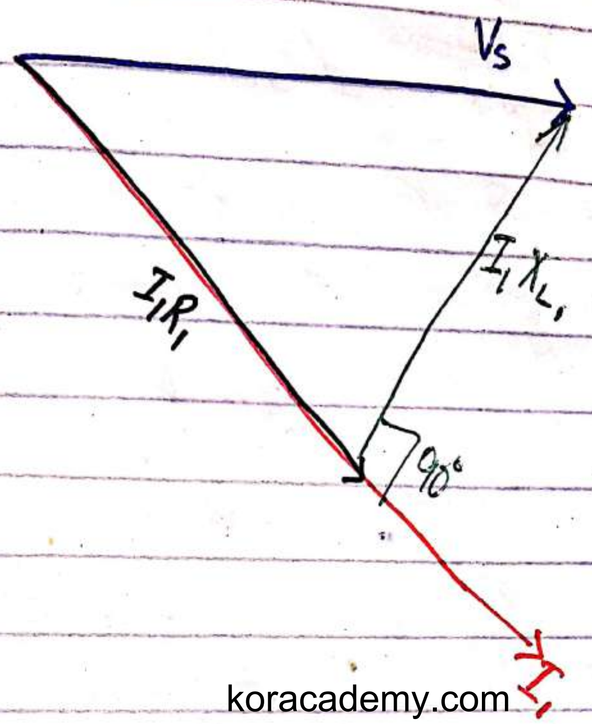
koracademy.com koracademy.com koracademy.com



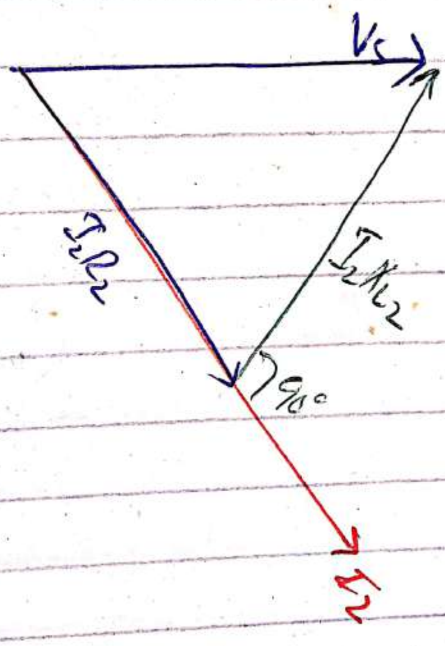
koracademy.com koracademy.com Scanned with CamScanner koracademy.com

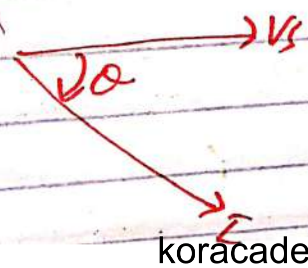
koracademy.com koracademy.com koracademy.com

For an inductive circuit



koracademy.com koracademy.com koracademy.com





koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Magnetic Circuits

Magnetic circuits are analogous to electrical circuits.

Magnetic flux ϕ is analogous to current.

Reluctance $\mathcal{R}$ is analogous to resistance R.
↳ the property of a magnetic material due to which it opposes the flow of magnetic flux.

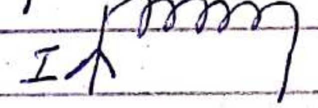
$$R = \frac{\rho l}{A} \qquad \mathcal{R} = \frac{l}{\mu A}$$

Permeability μ of a magnetic material is analogous to conductivity.

koracademy.com koracademy.com koracademy.com

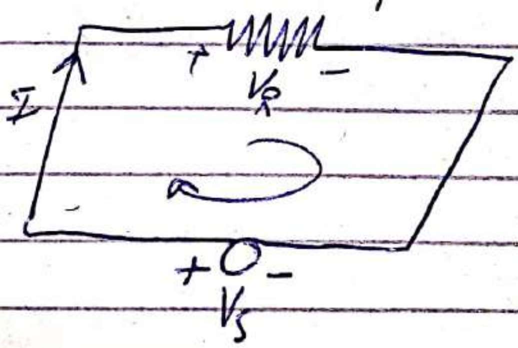
Magnetic motive force MMF is analogous to voltage V in electrical circuits.

For generation of magnetic flux, we need to have a current in a loop of N number of turns.



$$\boxed{MMF = F = NI} \quad (AT)$$

Review of An Electrical circuit



$$V_s = V_R$$

$$V_s = V_R = IR$$

$$I = \frac{V_s}{R}$$

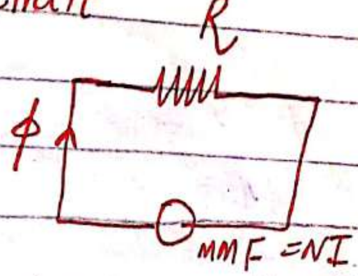
koracademy.com koracademy.com koracademy.com

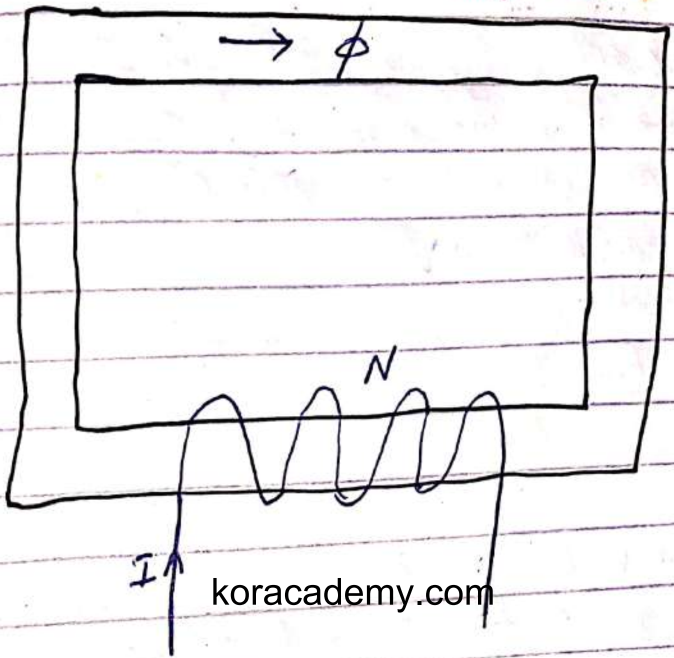
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Consider a magnetic material.

→ Equivalent ckt circuit





koracademy.com koracademy.com koracademy.com

Mean length = l Area = A Permeability = μ.

Strength of magnetic field H.

$$H \propto \frac{NI}{l}$$

$$H = \frac{NI}{l}$$

$$\Rightarrow \boxed{MMF = NI = Hl}$$

magnetic flux density is the flux per unit area → B

$$B = \mu H$$

$$\boxed{H = \frac{B}{\mu}}$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

As $B = \frac{\phi}{A}$ $\Rightarrow$ $\boxed{H = \frac{\phi}{\mu A}}$

$\Rightarrow$ $\boxed{MMF = NI = \phi \times \frac{l}{\mu A}}$

Reluctance $\boxed{\mathcal{R} = \frac{l}{\mu A}}$

$\Rightarrow$ $\boxed{MMF = \phi \mathcal{R}}$

$\boxed{\phi = \frac{MMF}{\mathcal{R}}}$

Example

$l = 1m$ $A = 0.5m^2$ $\mu_r = 100$ $N = 1000$

$\phi = 2 \times 10^{-3}$ Wb $I = ?$

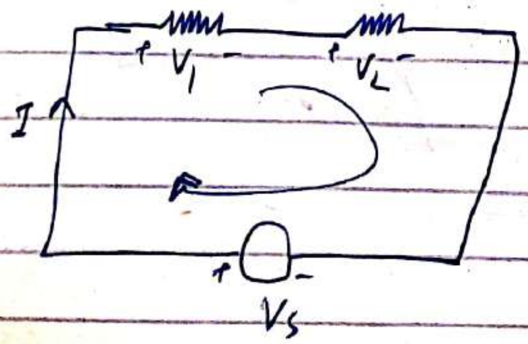
koracademy.com koracademy.com koracademy.com

$$\mathcal{R} = \frac{l}{\mu A} = \frac{1}{4 \times 10^{-7} \times 100 \times 0.5}$$

$$\Rightarrow \mathcal{R} = 1.591 \times 10^{3}\ AT/wb$$

$$I = \frac{\phi \mathcal{R}}{N} = \frac{2 \times 10^{-3} \times 1.591 \times 10^{3}}{1000} = 31.8\ mA.$$

Review Series Electrical Circuit



KVL $V_s = V_1 + V_2$

$V_1 = IR_1$ $V_2 = IR_2$

$V_s = I(R_1 + R_2)$

where

$R_T = R_1 + R_2$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

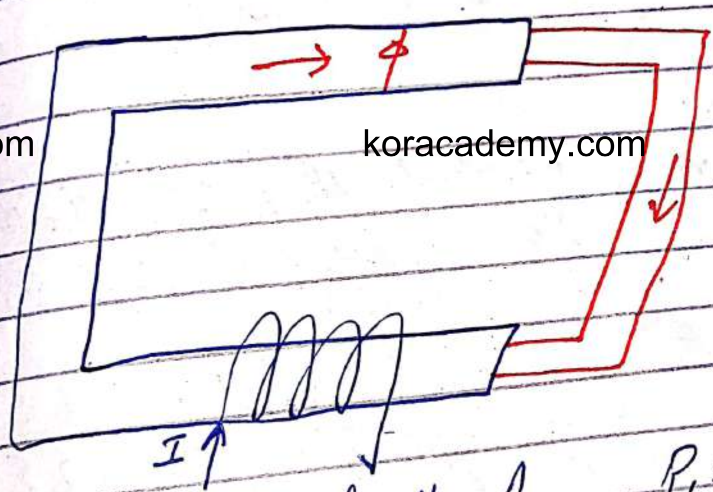
koracademy.com koracademy.com koracademy.com

Series Magnetic Circuit

Consider a circuit of two different materials. Put a coil of N turns on any material.

Similar direction of flux from more than one coil → +ve sign → add.
If in opposite direction → -ve sign → subtract.

In this case the same flux will flow in both the materials.

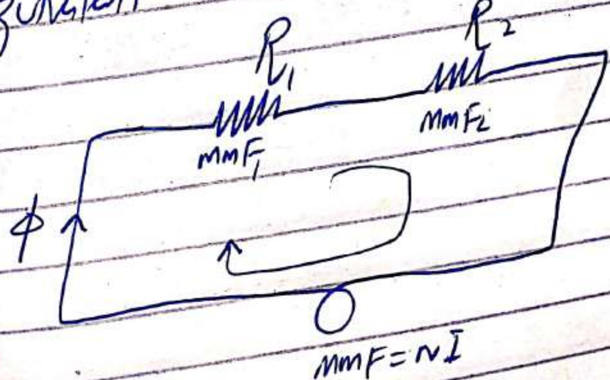


koracademy.com koracademy.com koracademy.com

Material 1 ⇒ l_1, μ_1, A_1 $\quad R_1 = \frac{l_1}{\mu_1 A_1}$

Material 2 ⇒ l_2, μ_2, A_2 $\quad R_2 = \frac{l_2}{\mu_2 A_2}$

equivalent circuit



koracademy.com koracademy.com Scanned with CamScanner koracademy.com

koracademy.com koracademy.com koracademy.com

A law that is analogous to KVL

$$MMF = MMF_1 + MMF_2$$

$$MMF_1 = \phi R_1$$
$$MMF_2 = \phi R_2$$

$$MMF = NI = \phi R_1 + \phi R_2 = \phi(R_1 + R_2)$$

where $R_T = R_1 + R_2$

<u>Example 4</u>

$l_1 = 1m$ $A_1 = 0.5m^2$ $\mu_{r1} = 100$

$R_1 = 1.591 \times 10^4$ AT/wb $l_2 = 1m$ $A_2 = 0.5m^2$

$\mu_{r2} = 1000$

$$R_2 = \frac{l_2}{\mu_2 A_2} = \frac{1}{4\pi \times 10^{-7} \times 1000 \times 0.5}$$

$$R_2 = 1.591 \times 10^3 \text{ AT/wb}$$

koracademy.com koracademy.com koracademy.com

$\phi = 3 \times 10^{-3}$ wb $I = 1A$ $N = ?$

As $MMF = NI = \phi(R_1 + R_2)$

$$\Rightarrow N = \frac{\phi(R_1 + R_2)}{I}$$

$$N = 3 \times 10^{-3}(1.59 \times 10^4 + 1.59 \times 10^3)$$

$$\underline{N = 53} \text{ turns.}$$

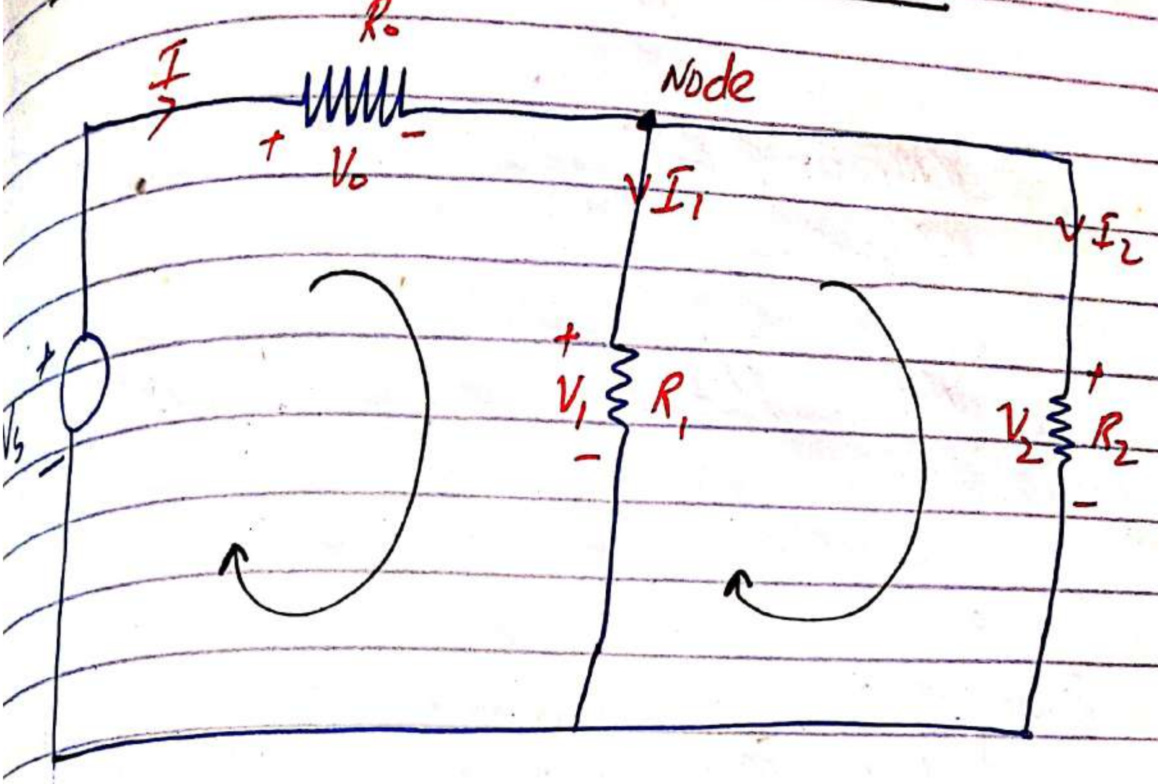
We dont have simple parallel magnetic circuits.

To understand series parallel magnetic circuits we first study series parallel electric circuits

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Series Parallel Electric Circuit



koracademy.com koracademy.com koracademy.com

Apply KVL to loop 1

$$V_s = V_0 + V_1$$

KVL to loop 2

$$V_1 = V_2$$

$$\Rightarrow V_s = V_0 + V_2$$

$V_0 = I R_0$ $\quad V_1 = I_1 R_1$ $\quad V_2 = I_2 R_2$

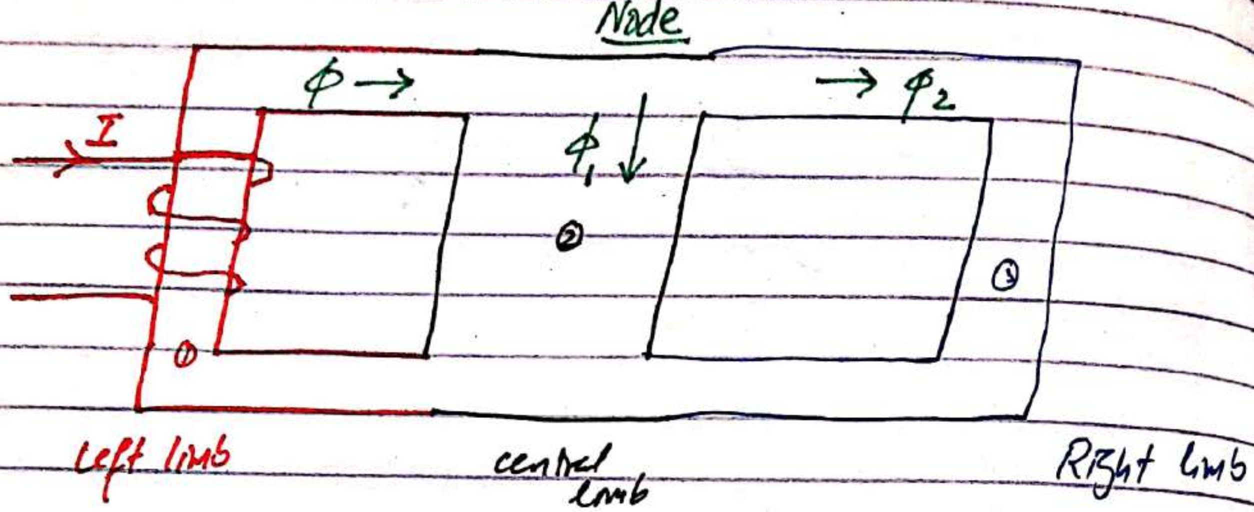
Apply KCL to the only node

$$I = I_1 + I_2$$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

Series Parallel Magnetic Circuit

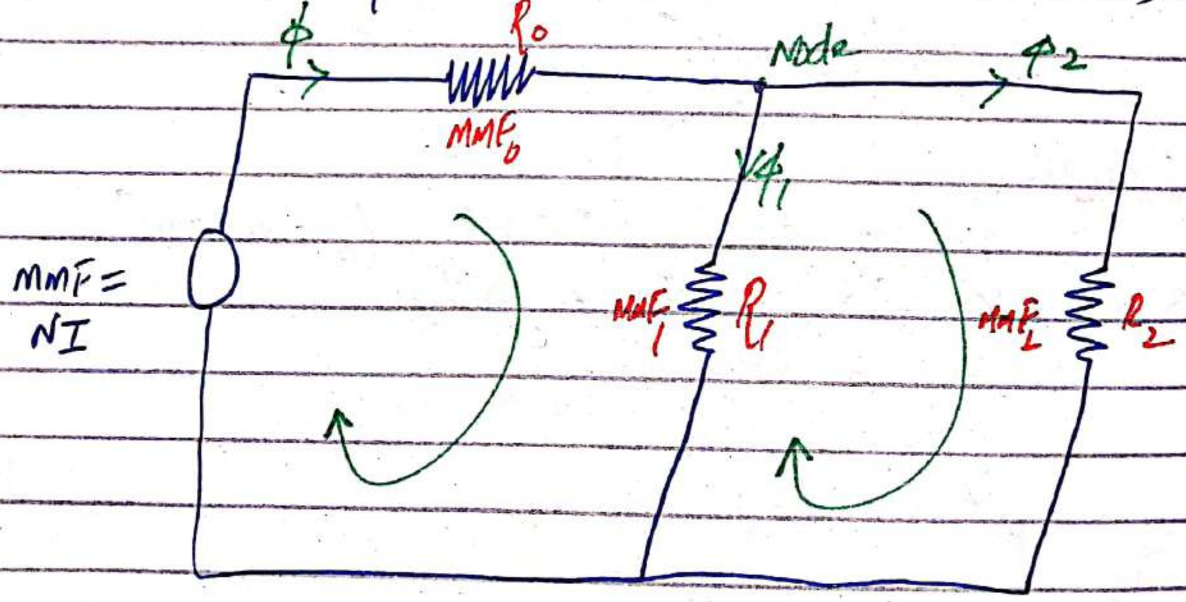


we divide this circuit in three sections.

Left limb $\Rightarrow$ μ_0, A_0, l_0 , $R_0 = l_0/\mu_0 A_0$
Right limb $\Rightarrow$ μ_2, A_2, l_2 $R_2 = l_2/\mu_2 A_2$
central limb $\Rightarrow$ μ_1, A_1, l_1 $R_1 = l_1/\mu_1 A_1$

koracademy.com koracademy.com koracademy.com

An equivalent circuit to the above is;



Apply a law equivalent to KVL; to loop 1.

$$MMF = MMF_0 + MMF_1$$

analogous to $V_s = V_0 + V_1$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

loop 2 $MMF_1 = MMF_2$

analgus to $V_1 = V_2$

$V_s = V_0 + V_2$

$MMF = MMF_0 + MMF_1$

$MMF_0 = \phi R_0$

algus to $V_0 = IR_0$

$MMF_1 = \phi_1 R_1$

analgus to $V_1 = I_1 R_1$

$MMF_2 = \phi_2 R_2$

analgus to $V_2 = I_2 R_2$

koracademy.com koracademy.com koracademy.com

Apply : law efevlet to KCL to the aly ckt

$\Rightarrow \phi = \phi_1 + \phi_2$

analg. to $I = I_1 + I_2$

Example

$N = 200$ turns $I = 2A$ $R_0 = 25 \times 10^3$

$\phi = 8 \times 10^{-3}$ wb $\phi_1 = 4 \times 10^{-3}$ wb

calculate R_1, R_2, ϕ_2

As $\phi = \phi_1 + \phi_2 \Rightarrow \phi_2 = \phi - \phi_1$

$\Rightarrow \phi_2 = 4 \times 10^{-3}$ wb

As $MMF = NI = MMF_0 + MMF_1$

$NI = \phi R_0 + \phi_1 R_1$

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$R_1 = \frac{NI - \phi R_0}{\phi_1} = \frac{200\times 2 - 8\times10^{-3}\times 25\times10^{3}}{4\times10^{-3}}$$

$$R_1 = 50\times10^{3} \ AT/Wb$$

As $MMF_2 = MMF_1$

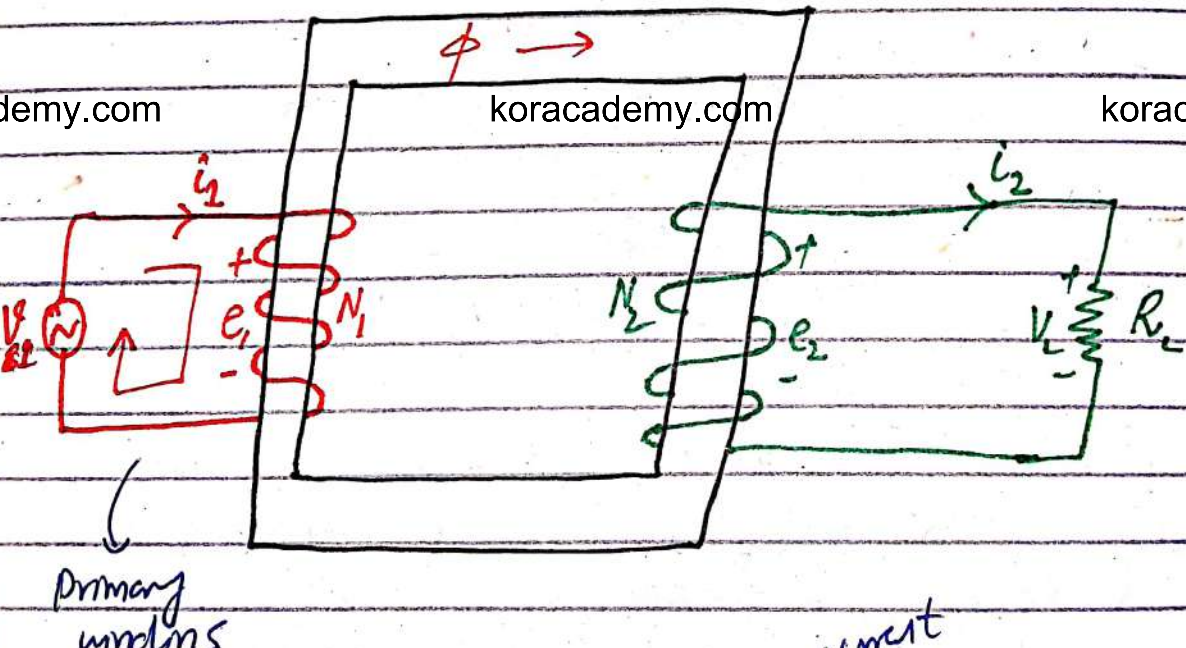
$$\phi_2 R_2 = \phi_1 R_1$$

$$R_2 = R_1 = 50\times10^{3} \ AT/Wb.$$

Single Phase AC Transformer

non magnetic material → whose relative permeability is 1. → because its reluctance is very high.

koracademy.com koracademy.com koracademy.com



primary winding

AC voltage → primary coil → current → magnetic flux → ac

The nature of flux depends on the nature of the current.

The current in the winding will induce a voltage through the winding.

koracademy.com koracademy.com koracademy.com
Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$i_1 = I_{m1} \sin \omega t$

$\phi = \phi_m \sin \omega t$

KVL $v_1 = e_1$

The induced voltage across the winding according to faraday's law is

$v_1 = e_1 = \dfrac{N_1 d\phi}{dt}$

$v_1 = \omega N_1 \phi_m \cos \omega t$

$v_1 = 2\pi f \phi_m N_1 \sin(\omega t + \pi/2)$

koracademy.com koracademy.com koracademy.com

$V_{m1} = 2\pi f \phi_m N_1$

$V_{1rms} = 0.707 \times 2\pi f \times N_1 \times \phi_m$

$V_{1rms} = 4.44 f N_1 \phi_m$

Secondary winding is placed in a time varying magnetic flux → so voltage is induced according to faraday's law.

if load is connected → v_2, i_2 ..

KVL $v_2 = e_2 = \dfrac{N_2 d\phi}{dt}$

$v_2 = \omega N_2 \phi_m \cos \omega t = 2\pi f N_2 \phi_m \sin(\omega t + \pi/2)$

koracademy.com koracademy.com koracademy.com

Scanned with CamScanner

koracademy.com koracademy.com koracademy.com

$$V_{2_{max}} = 2\pi f N_2 \phi_m$$

$$V_{2_{rms}} = 0.707 \times 2\pi f N_2 \phi_m = 4.44 f N_2 \phi_m$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \text{Turn ratio.}$$

Supposing this to be an ideal transformer.

$$P_1 = P_2$$

koracademy.com koracademy.com koracademy.com

$$V_{1_{rms}} \times I_{1_{rms}} \times \cos\phi = V_{2_{rms}} \times I_{2_{rms}} \times \cos\phi$$

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \text{Turn ratio.}$$

koracademy.com koracademy.com Scanned with CamScanner koracademy.com