EE-287 Engineering Economics

Lecture Title:

Gradient Formulas

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Put on your headphones. Click Slideshow. Play the Speaker icon on each Slide and listen to the Lecture. Change Slide through Right/Left arrow keys or Page Down/Page Up keys

Gradient Cash Flows

Remember: The four (4) Uniform Series Formulas [They are used for Cash Flows of the same magnitude i.e. uniform magnitude (A) in each interest period]

But

Sometimes, Cash Flows occur in consecutive interest periods and are not of the same magnitude (A) i.e. they are not uniform. But such Cash Flows change in a predictable manner and are known as Gradient Cash Flows.

Keep the four (4) Uniform Series Formulas in mind for Gradient Cash Flow calculations. You'll need them!

Gradient Cash Flows are of two types:

1. Arithmetic Gradient Cash Flow
2. Geometric Gradient Cash Flow

&

Gradient Formulas are used for calculating the Present Worth (P) of such Gradient Cash Flows.

1. Arithmetic Gradient Cash Flow

Remember: Arithmetic Gradient Cash Flow changes (i.e. increases or decreases) by **same amount** in each period.

Example:

If Cash Flow in Period 1 is Rs.1000/- (**Base amount**)
& Cash Flow in Period 2 is Rs.2000/-
& Cash Flow in Period 3 is Rs.3000/-
& Cash Flow in Period 4 is Rs.4000/-
& So on
.
.

As you can see above that amounts are increasing by Rs.1000/- (**same amount**) in each subsequent interest period thus the unit **1000** is known as Arithmetic Gradient represented by capital **G**. So G = 1000.

Mathematically, $$P = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$$ ---------- (1)

Equation "**1**" calculates the present value of the Gradient only (i.e. G = 1000) not including the base amount of the money on which the gradient was built upon.

1. Arithmetic Gradient Cash Flow (Continued)

So, the Base Amount in the example on slide 3 (i.e. Rs.1000/-) in Period 1 must be accounted for separately as a Uniform Cash Flow Series.

So, generally, present worth of an Arithmetic Gradient Cash Flow Series is,

P = Present worth of Base Amount + Present worth of Gradient Amount -------- (2)

So, Equation (2) can be written as below

$$P = A\,(P/A, i\,\%, n) + G\,(P/G, i\,\%, n)$$

1st relation in Uniform Series Formulas

Equation 1 on slide 3

Important: If the Gradient Cash Flow decreases from one period to the next then minus (-ve) sign must be put above in Equation (2) in front of G

1. Arithmetic Gradient Cash Flow (Continued)

Example: Tarbela Dam: New Alternator has a useful life of 10 years.
Alternator Maintenance Cost: **Year 1**: Rs.50,000/-
Year 2: Rs.55,000/-
and to increase annually by Rs.5000/- through to **Year 10** at an interest rate of 10% per year.

Determine: Present Worth of 10 years of maintenance costs.
Solution: As we know, Gradient G = 5000
i = 10% per year
Base Amount = Rs.50,000/- starting in year 1
n = 10 (i.e. years/periods)

So from Equation 2,

$$P = A[(1+i)^n - 1/i(1+i)^n] + \frac{G}{i}\left[\frac{(1+i)^n-1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$$

$$P = 50{,}000[(1+0.1)^{10} - 1/0.1(1+0.1)^{10}] + \frac{5000}{0.1}\left[\frac{(1+0.1)^{10}-1}{0.1(1+0.1)^{10}} - \frac{10}{(1+0.1)^{10}}\right]$$

Calculate the answer yourself and get the required determination

2. Geometric Gradient Cash Flow

Remember: A Geometric Gradient Cash Flow is when the periodic cash flow (increases or decreases by a constant percentage and not by a constant amount as was the case in arithmetic gradient cash flows).

For Example:

If the first payment = Rs.100/- and the Geometric gradient "g" for successive payments is 10% (0.1).

Then, A_1 = Rs.100

A_2 = Rs.100(1 + g) = Rs.110/-

A_3 = Rs.100(1 + g)^2 = Rs.121/-

A_4 = Rs.100(1 + g)^3= Rs.133/

A_n = 100(1 + g)$^{n-1}$

–ve sign in front of "g" is used for decreasing series cash flow

To find the present worth "Pg" for a Geometric Gradient Cash Flow G

Mathematically, $P_g = A_1 \left[\frac{1 - \left(\frac{1+g}{1+i}\right)^n}{(i-g)} \right]$ ---------- (i) when g≠i

$P_g = A1 \left[\frac{n}{(1+i)} \right]$ ---------- (ii) when g=i

Also, for decreasing geometric gradient cash flow series, change the sign in front of both g_s in equation (i)

Thank You for listening