Lecture 5
Self GMD & Mutual GMD

9.7 Concept of Self-GMD and Mutual-GMD

The use of *self geometrical mean distance* (abbreviated as self-GMD) and *mutual geometrical mean distance* (mutual-GMD) simplifies the inductance calculations, particularly relating to multiconductor arrangements. The symbols used for these are respectively D_s and D_m. We shall briefly discuss these terms.

(*i*) **Self-GMD (D_s).** In order to have concept of self-GMD (also sometimes called Geometrical mean radius ; GMR), consider the expression for inductance per conductor per metre already derived in Art. 9.5

$$\text{Inductance/conductor/m} = 2 \times 10^{-7}\left(\frac{1}{4} + \log_e \frac{d}{r}\right)$$

$$= 2 \times 10^{-7} \times \frac{1}{4} + 2 \times 10^{-7} \log_e \frac{d}{r} \qquad \ldots(i)$$

In this expression, the term $2 \times 10^{-7} \times (1/4)$ is the inductance due to flux within the solid conductor. For many purposes, it is desirable to eliminate this term by the introduction of a concept called self-GMD or GMR. If we replace the original solid conductor by an equivalent hollow cylinder with extremely thin walls, the current is confined to the conductor surface and internal conductor flux linkage would be almost zero. Consequently, inductance due to internal flux would be zero and the term $2 \times 10^{-7} \times (1/4)$ shall be eliminated. The radius of this equivalent hollow cylinder must be sufficiently smaller than the physical radius of the conductor to allow room for enough additional flux to compensate for the absence of internal flux linkage. It can be proved mathematically that for a solid round conductor of radius r, the self-GMD or GMR = 0·7788 r. Using self-GMD, the eq. (*i*) becomes :

$$\text{Inductance/conductor/m} = 2 \times 10^{-7} \log_e d/D_s{}^*$$

$$\text{where} \quad D_s = \text{GMR or self-GMD} = 0{\cdot}7788\, r$$

It may be noted that self-GMD of a conductor depends upon the size and shape of the conductor and is independent of the spacing between the conductors.

(*ii*) **Mutual-GMD.** The mutual-GMD is the geometrical mean of the distances form one conductor to the other and, therefore, must be between the largest and smallest such distance. In fact, mutual-GMD simply represents the equivalent geometrical spacing.

(*a*) The mutual-GMD between two conductors (assuming that spacing between conductors is large compared to the diameter of each conductor) is equal to the distance between their centres *i.e.*

$$D_m = \text{spacing between conductors} = d$$

(*b*) For a single circuit 3-ϕ line, the mutual-GMD is equal to the equivalent equilateral spacing *i.e.*, $(d_1\, d_2\, d_3)^{1/3}$.

$$D_m = (d_1\, d_2\, d_3)^{1/3}$$

... double

(c) The principle of geometrical mean distances can be most profitably employed to 3-ϕ double circuit lines. Consider the conductor arrangement of the double circuit shown in Fig. 9·10. Suppose the radius of each conductor is r.

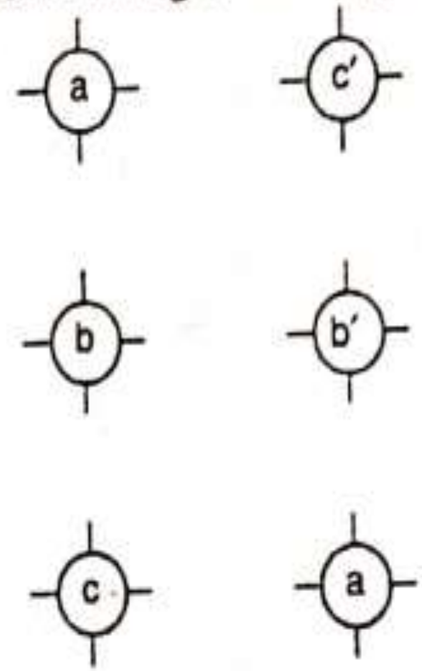

Fig. 9.10

Self-GMD of conductor = 0·7788 r

Self-GMD of combination aa' is

$$D_{s1} = (**D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a})^{1/4}$$

Self-GMD of combination bb' is

$$D_{s2} = (D_{bb} \times D_{bb'} \times D_{b'b'} \times D_{b'b})^{1/4}$$

Self-GMD of combination cc' is

$$D_{s3} = (D_{cc} \times D_{cc'} \times D_{c'c'} \times D_{c'c})^{1/4}$$

Equivalent self-GMD of one phase

$$D_s = (D_{s1} \times D_{s2} \times D_{s3})^{1/3}$$

The value of D_s is the same for all the phases as each conductor has the same radius.

Mutual-GMD between phases A and B is

$$D_{AB} = (D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'})^{1/4}$$

Mutual-GMD between phases B and C is

$$D_{BC} = (D_{bc} \times D_{bc'} \times D_{b'c} \times D_{b'c'})^{1/4}$$

Mutual-GMD between phases A and B is

$$D_{AB} = (D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'})^{1/4}$$

Mutual-GMD between phases B and C is

$$D_{BC} = (D_{bc} \times D_{bc'} \times D_{b'c} \times D_{b'c'})^{1/4}$$

Mutual-GMD between phases C and A is

$$D_{CA} = (D_{ca} \times D_{ca'} \times D_{c'a} \times D_{c'a'})^{1/4}$$

Equivalent mutual-GMD, $D_m = (D_{AB} \times D_{BC} \times D_{CA})^{1/3}$

It is worthwhile to note that mutual GMD depends only upon the spacing and is substantially independent of the exact size, shape and orientation of the conductor.

9.8 Inductance Formulas In Terms of GMD

The inductance formulas developed in the previous articles can be conveniently expressed in terms of geometrical mean distances.

* Basically, we have omitted the internal flux term while compensating for it by using an adjusted value for the radius of the conductor. Sometimes GMR is denoted by r'.

** D_{aa} or $D_{a'a'}$ means self-GMD of the conductor. $D_{aa'}$ means distance between a and a'.

(*i*) *Single phase line*

$$\text{Inductance/conductor/m} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

where $D_s = 0{\cdot}7788\, r$ and D_m = Spacing between conductors = d

(*ii*) *Single circuit 3-φ line*

$$\text{Inductance/phase/m} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

where $D_s = 0{\cdot}7788\, r$ and $D_m = (d_1\, d_2\, d_3)^{1/3}$

(*iii*) *Double circuit 3-φ line*

$$\text{Inductance/phase/m} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

where $D_s = (D_{s1}\, D_{s2}\, D_{s3})^{1/3}$ and $D_m = (D_{AB} \times D_{BC} \times D_{CA})^{1/3}$

9.12 BUNDLED CONDUCTORS

Stranded or composite conductors touch each other, however, bundled conductors are separated from each other by 30 cm or more and conductors of each phase are connected by conducting wires at particular length. Figure 9.14 shows the stranded and bundled conductors. For the voltage rating more than 230 kV, it is not possible to use the round conductors due to excessive corona loss. It is preferred to have the hollow conductor, normally in substations, and bundled conductors in transmission lines. The main advantages of using bundled conductors are:

- Reduced corona loss
- Reduced voltage gradient at the surface of the conductor
- Low reactance due to increase in the self-GMD
- Reduced radio interference
- Increase in capacitance
- Larger loading capability and
- Increase surge impedance loading.

The reactance of bundled conductors can be calculated with the help of GMD and GMR approach.

(a) Bundled conductors (b) Stranded conductors

Figure 9.14 Bundled and stranded conductors.

9.13 SKIN EFFECT

The alternating current distribution in a wire is not uniform. The current density near the surface is more than near to the centre. It is affected by the frequency of the current. If the frequency of current is more, the current distribution is more non-uniform. This effect is known as *skin effect*. Due to this effect the effective resistance (or ac resistance of the conductors) becomes more than the direct current (dc), where the current distribution is uniform, resistance (called the dc resistance). This can be understood by an example.

Consider a solid current carrying conductor of circular cross-section, as shown in Figure 9.15. This can be replaced by a large number of conductors bunched together with small radii. These conductors occupy the same cross-sectional area. If the current is same, the loss can be calculated in both the cases and thus the effective resistance. In n strands, each of resistance nR ohm carries a uniformly distributed current (of I/n ampere). The loss will be same as a single conductor (I^2R). Let us assume half of the conductors ($n/2$) carry currents of $[(I/n) + \Delta I]$ ampere and the other half ($n/2$) carry $[(I/n) - \Delta I]$ ampere. The total loss is

$$\frac{n}{2}\left(\frac{I}{n}+\Delta I\right)^2 nR + \frac{n}{2}\left(\frac{I}{n}-\Delta I\right)^2 nR = \frac{n^2R}{2}\left[2\left(\frac{I}{n}\right)^2 + 2\Delta I^2\right] = I^2R + n^2R\Delta I^2$$

which is greater than I^2R. This indicates that the effective resistance will be more than the dc resistance if current distribution is not uniform.

Figure 9.15 Skin effect.

Due to non-uniformity of current, skin effect, the flux linkages are reduced and thus skin effect reduces the effective internal reactance. The inner filaments carrying currents give rise to flux which links the inner filaments only whereas the flux due to the current carrying outer filaments enclose both inner and outer filaments.

9.14 PROXIMITY EFFECT

Like skin effect, the *proximity effect* also increases the resistance of the conductor. The alternating flux in a conductor caused by the current flowing in neighbouring conductors gives rise to circulating currents, which cause the non-uniformity of the current and thus increases resistance. Let us consider two wire system, as shown in Figure 9.16. When conductor *A* carries current, its flux links with the other conductor *B*. The flux linkages are

Figure 9.16 Proximity effect.

Figure 9.16 Proximity effect.

nearer to the conductor, as shown by shaded portion, than the opposite side. If the current in conductor B is opposite to the current in A, the current density will be more in the adjacent portion of the conductor. If the current direction is same, the current density will be more in the remote part of the conductor. Due to this non-uniformity, the effective resistance is more than the dc resistance.

This effect is more pronounced in cable where the phase conductors are nearer to each other. The proximity effect is negligible in overhead transmission line. Both skin effect and proximity effect depend on the conductor size, frequency of the supply, resistivity and permeability of the conductor material. For the circular conductors, the increase in effective resistance is proportional to $d^2 f\mu_r/\rho$, where d is the diameter of the conductor.