Sag & Tension Calculation

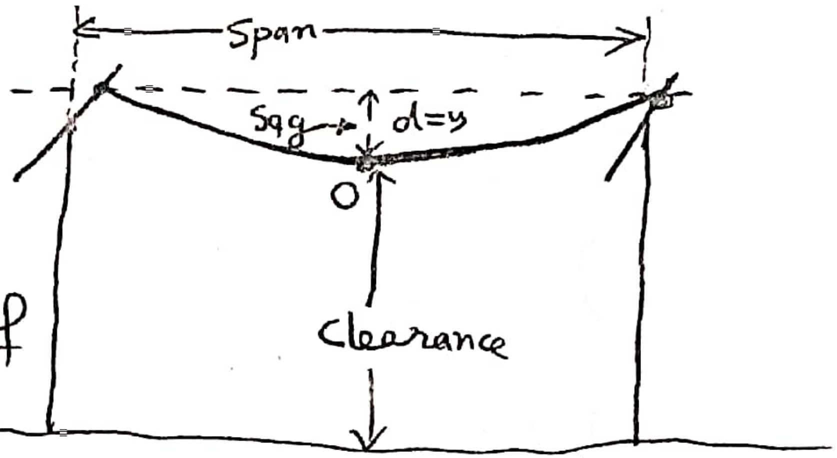
In order to specify the tension to be used in stringing the line cond., the values of Sag & Tension for different conditions must be known. The factors affecting the Sag of a cdt strung between supports are

1. Weight of cdt per unit length
2. Load due to Ice, Snow, wind etc
3. distance between the supports (span length)
4. Temperature
5. Conductor tension.

For Equal Level Support

SAG & SPAN TENSION Calculations.

an :- Part of an overhead line between the consecutive supports is called span



g:- The maximum vertical distance in the span of an overhead conductor line between a conductor and the straight line passing through the two top points of the support is called Sag. in figure d=y is Sag.

For Equal level Supports

A . L/2 L/2 Let Span l ...

Length of a Span cont

(i) With Wooden Poles 40 – 50 metres.

(ii) With steel tubular Poles 50 → 80 "

(iii) With R.C.C Poles 80–100 metres.

(iv) With Steel towers 100 – 300 metres

it should be remembered that

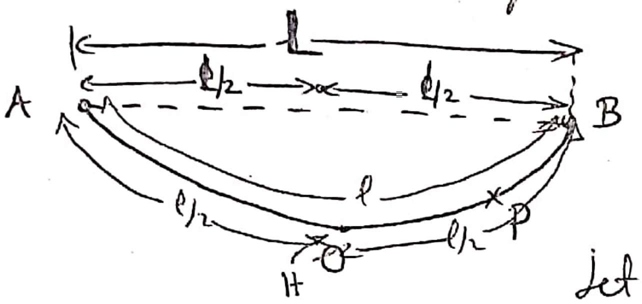
(i) the wires expand in the summer season and contract in the winter season, thereby decreasing and increasing the tension respectively

(ii) In order to keep the insulation cost min the span is kept ~~greater~~ Large with the increasing Voltages.

Sag Calculations

When the wire of Uniform X-Section is suspended between two points at the same level the wire sags down and assume the Shape of a "Catenary". Max stress will occur when the temperature is minimum and the line is covered with Ice, acted by a severe wind Pressure.

For Equal level Supports

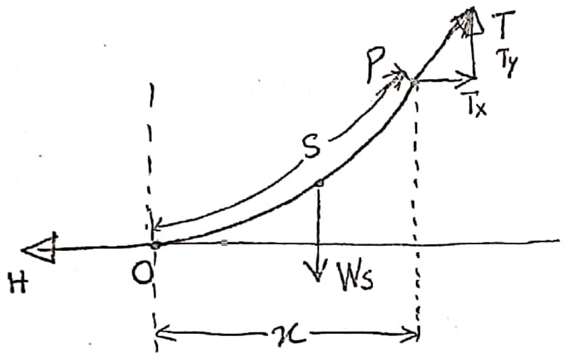


Let Span Length L

Weight Per Unit Length - W of cdt.

Let H – Tension at Point O

$T \rightarrow$ Tension at any Point P on conductor.



Let S – Length of cdt between O & P

Weight of cdt between O & P = $\cancel{Ws}$ = WS

As the cdt is in equilibrium so

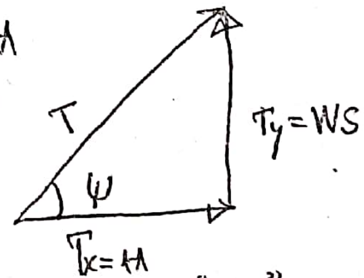
$$\sum F_y = 0$$

all forces acting in V. direction will be balanced = 0

$$-Ws + T_y = 0$$

or $T_y = WS$

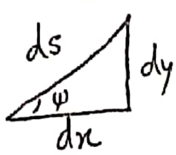
Similarly $\sum F_x = 0$ or $\cancel{Hx}$ $T_x = H$



Considered a small triangle

As $\tan \psi = T_y / T_x$

At Point "P" take small Length of cdt "ds" so "ds" Can be Considered as a straight line (as small section)



For straight line Equation

$$ds^2 = dx^2 + dy^2$$

Divid b. side by dx^2

$$ds^2/dx^2 = 1 + (dy/dx)^2$$

$$(ds/dx)^2 = 1 + \tan^2\psi \qquad \text{as} \qquad \tan\psi = T_Y/T_X = Ws/H$$

$$\therefore (ds/dx)^2 = 1 + (Ws/H)^2$$

$$ds/dx = \sqrt{1+(Ws/H)^2}$$

or

$$dx = \frac{ds}{\sqrt{1+(Ws/H)^2}}$$ take integration

$$\int dx = \int ds\Big/\sqrt{1+(Ws/H)^2} \Rightarrow x = \frac{H}{W}\sinh^{-1}\left(\frac{WS}{H}\right)+C_1 \quad (2)$$

Where C_1 is a Constant which can be determin. from initial conditions i,e $x=0$, $S=0$ put in Equation 2

$$0 = H/W \sinh^{-1}\left(\frac{W\times 0}{H}\right)+C_1$$

$C_1 = 0$ put in (2)

$$\frac{Wx}{H} = \sinh^{-1}\left(\frac{Ws}{H}\right)$$ or

$$S = \left(\frac{H}{W}\right)\sinh\left(\frac{Wx}{H}\right) \quad \text{------} (3)$$

As we know that $\sinh z = Z + Z^3/3! + Z^5/5! + \ldots\ldots$

So $S = H/W\left(\frac{Wx}{H} + \frac{W^3x^3}{H^3 3!} + \ldots\ldots\right) \ldots 4$

$$S = \frac{H}{W} \sinh \frac{Wx}{H} \quad \text{- - - - - - - cac length (3)}$$

At $x = L/2$ ($L \rightarrow$ span total Length) $S = l/2$ we obtain

$$l = \frac{2H}{W} \sinh \frac{WL}{2H}$$

Expanding $\sinh\left[\frac{WL}{2H}\right]$ and ignoring higher order terms

we get

$$l = \frac{2H}{W}\left[\frac{WL}{2H} + \frac{W^3L^3}{8 \cdot 3!\, H^3} + \dots\right] \qquad \sinh z = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \dots$$

$$\boxed{l = \left[L + \frac{W^2L^3}{24H^2}\right] = L\left[1 + \frac{W^2L^2}{24H^2}\right] \text{- - - - (3A)}}$$

Now Equation 3 can be rewritten as

$$\frac{WS}{H} = \sinh \frac{Wx}{H}$$

Since $\tan\varphi = \frac{WS}{H} = dy/dx$ we get

$$dy/dx = \sinh Wx/H \quad \text{or} \quad dy = \sinh \frac{Wx}{H}\, dx$$

Integrating both side, we get

$$y = \frac{H}{W} \cosh \frac{Wx}{H} + B$$

where B is the Integration Constant, and can be obtained with initial conditions at $x = 0$, $y = 0$ thus putting in the above Equation we get

$$B = -H/W \qquad \cosh 0 = 1$$

So we get

$$y = \frac{H}{W}\left[\cosh\left(\frac{Wx}{H}\right) - 1\right] \text{- - - - - 4}$$

it is equation of the Sag that is called "Catenary".

Expanding the Equation we get

$$y = \frac{H}{W}\left[1 + \frac{1}{2!}\left(\frac{Wx}{H}\right)^2 + \cdots - 1\right] \cdots$$

or

$$y \cong \frac{Wx^2}{2H}$$

This is a Parabola. For short span this equation is valid. Tension (T) at Point P will be

$$T^2 = T_x^2 + T_y^2 = H^2 + (WS)^2$$

substituting the value of S from Equation 3 and simplifying we have.

$$T^2 = H^2 + W^2\left[\frac{H^2}{W^2}\sinh^2\left(\frac{Wx}{H}\right)\right]$$

$$T^2 = H^2 + H^2\sinh^2\left(\frac{Wx}{H}\right) = H^2\left[1 + \sinh^2\frac{Wx}{H}\right]$$

$$T = H\left[1 + \sinh^2\frac{Wx}{H}\right]^{1/2} = H\cosh\frac{Wx}{H}$$

For short span & short sag, the difference between the max tension T and the horizontal tension H is small. The tension will be the highest at the conductor supports and will be minimum at the lowest point of catenary. Thus

$$T = \begin{cases} H & \text{at the lowest Point } x = 0 \\ H\cosh\frac{WL}{2H} & \text{at the supports.} \end{cases}$$

when $x = L/2$, y is equal to the sag or deflection "d"

Therefore, using Equation. 4

$$d = y = \frac{H}{W}\left[\cosh\left(\frac{WL}{2H}\right) - 1\right] \text{----}$$

Expanding & Ignoring high orders

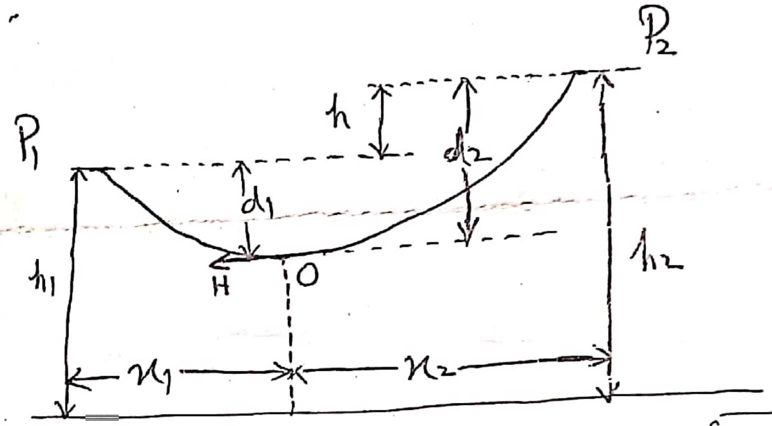
or $d \cong \frac{WL^2}{8H}$ ---- (5)

Using Equation 3A & 5, the length of wire (Perimeter of cdt) between span in terms of span length and sag can be calculated as

(6) $l = L\left[1 + \frac{8d^2}{3L^2}\right]$

Supports at different levels (unsymmetrical Span)

Let P_1 & P_2 are two Points at height h_1 & h_2 from the ground respectively, as shown in figure below. if the Point O is the Lowest Point, by using Equation (5), we can write sags in two sections as.



$$d_1 = \frac{Wx_1^2}{2H} \quad \& \quad d_2 = \frac{Wx_2^2}{2H}$$

As $2l = 2x = L$
$$d = \frac{W\,4x^2}{8H} = \frac{Wx^2}{2H}$$

Therefore,
$$d_2 - d_1 = h = \frac{W}{2H}(x_2^2 - x_1^2)$$

where h is difference between the elevation of two supports. if the span Length is L, $x_1 + x_2 = L$, Therefore

$$h = \frac{W}{2H}(x_1+x_2)(-x_1+x_2) = -\frac{W}{2H}(L)(x_1-x_2)$$

or
$$-x_1 + x_2 = \frac{2Hh}{WL} \quad -(6)$$

Using relation $x_1+x_2 = L$ & Equation (6)

$$x_1 = L/2 - \frac{Hh}{WL} \quad -(7)$$

$x_1 = L - x_2$ put in (6) we get

$$x_2 = x_1 + \frac{2Hh}{WL}$$

$$x_1 = L - x_1 - \frac{2Hh}{WL}$$

$$2x_1 = L - \frac{2Hh}{WL}$$

$$\boxed{x_1 = L/2 - \frac{Hh}{WL}}$$

&
$$x_2 = L/2 + \frac{Hh}{WL} \quad -(8)$$

from Equation 7

7)

x_1 Can be

$$x_1 > 0 \quad \text{mean} \quad L/2 > \frac{Hh}{wL}$$

$$x_1 < 0 \quad \text{mean} \quad L/2 < \frac{Hh}{wL}$$

$$x_1 = 0 \quad \text{mean} \quad L/2 = Hh/wL$$

if x_1 is negative, the lowest Point of the Curve lies outside the actual Span. it must be noted that the lowest Point is always be at a lower support Side. if x_1 is zero, the lowest Point of the Curve Will be at the lower - Support Point.

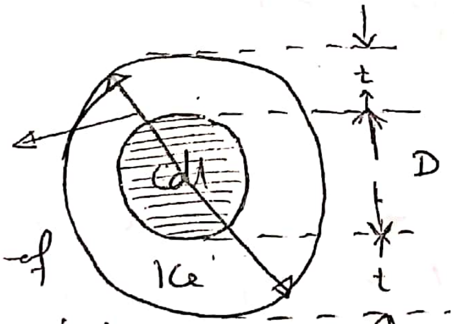
Effect of Ice & Wind

In snowy areas, Ice is deposited on cables. Sometime, thickness of Ice is more than the conductor's diameter and its accumulation on the conductor affects the design of the line:

(a) by increasing the weight per meter and

(b) by increasing the projected surface area subject to wind pressure.

Ice deposit may not be uniform but it can be assumed to be uniformly distributed around the conductor, as shown below. Due to Ice, the sag and tension of the conductor increases and therefore the line should be suitably designed to take care of this effect.



Let the diameter of conductor is D and the thickness of Ice is t, the cross-sectional area of Ice A_i will be the cross-section area of Ice along with the conductor minus cross-sectional area of the conductor. The cross-sectional area of Ice

$$A_i = \pi/4 \left[(D+2t)^2 - D^2 \right] = \pi (D+t)t$$

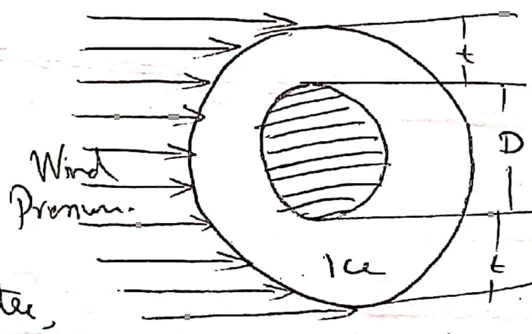
if the density of Ice is ρ_i (normally 915 Kg/m^3), the weight of Ice acting downward (assuming the Ice is spread throughout the span) will be

$\rho_i = \frac{W}{A_i}$ $\qquad W_i = \pi [(D+t)t] \rho_i$ Kg/m, $A_i * \rho_i$

where diameter and thickness are in metres. If the weight per unit length of the conductor is w and of ice is w_i, the total weight per unit length $W_T = w + w_i$.

Effect of Wind.

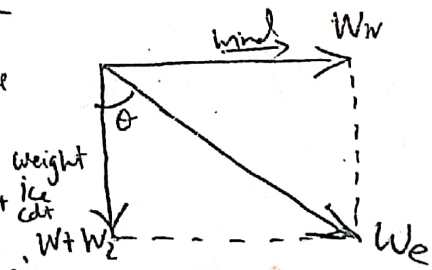
The effect of wind is taken horizontally across the projected area of the conductor covered with ice, if present, as shown below.



Therefore, it affects only to increase the transverse loading on the cdt. if D is dia of bare cdt in metre, t is the thickness of ice in metre, projected area for l-metre length will be $(D+2t)l$. if the wind pressure is p kg/m², the wind loading will be $W_w = (D+2t)p$ kg/m

The wind pressure depends on the shape of cdts. it is assumed to act on (2/3rd) of the projected area for cylindrical surfaces while it acts on full-face area in the square conductors. The wind pressure also depends on the square of the wind, which can be calculated by using $p = 0.006\,v^2$ kg/m², when v is the velocity of wind in km/hr. The effective load acting on the cdt will be

$$W_e = \sqrt{W_w^2 + (w+w_i)^2}$$



where, w & w_i are per unit weight of cdt & ice respectively. The loading is acting at an angle $\theta = \tan^{-1}\dfrac{W_w}{w+w_i}$ to the vertical plan, the total sag and tension must be calculated with an effective weight in case of ice & wind loaded lines.

Sag & Tension Calculation

Calculating area of cdt i.e ACSR. strand

A General formula for the total number of Strands "N" for n-layers (including central strand), of strands in cdt, if each strand is Uniform

$$N = 3n^2 - 3n + 1$$

layer

to calculate the outer diameter D of conductor, if dia of one strand is d,

$$D = (2n-1)d.$$

Example 12.3; An ACSR cdt has the following data normal Copper area = 120 mm², Size = (30+7)/6.30 mm; weight = 0.4 kg/m, tensile strength = 1250 kg Safety factor = 5, if span Length is 200 m, find

(a) Sag in still air,

(b) Sag if the cdt is covered with 0.5 cm thick ice (ice density of 915 kg/m³).

(c) Sag (total and Vertical) if the cdt is covered with ice of 0.5 cm thickness & a wind pressure of 10 kg/m² is acting on the projected area.

Solution The size of (30+7)/6.30 mm means the cdt has 30 Al strands & 7 steel strands of dia 6.30mm each. The total no of equal size strand will be 37 & the number of layers will be 4 (using $N = 3n^2-3n+1$) & the total dia of cdt will be (using

$D = (2n-1)d$,

$$D = (2\times4-1) * 6.30 = 44.1\,mm = 4.41\,cm$$

& Working stress

$$H = \frac{\text{Tensile strength}}{\text{Safety factor}} = \frac{1250}{5} = 250\,kg$$

(a) Sag in still air (d)

$$d = \frac{WL^2}{8H} = \frac{0.4 * 200^2}{8\times250} = \underline{8.0\,m}$$

(b) Wt of ice

$$W_i = \pi[(D+t)t]\,\rho_i$$

$$= \pi[(4.41+0.5)\times10^{-2} * 0.5\times10^{-2}]\times915$$

$$W_i = 0.706\ kg/m$$

Total weight $W_T = W+W_i = 0.4 + 0.706 = 1.106\ kg/m$

thus the sag 'd'

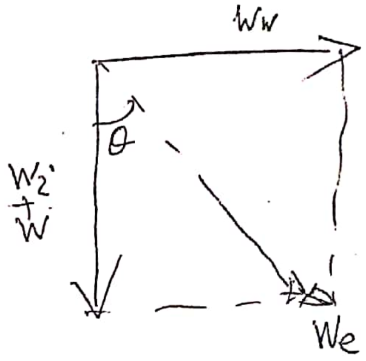
$$= \frac{WL^2}{8H} = \frac{1.106\times200^2}{8\times250} = \underline{22.11\,m}$$

(c) The wind loading $W_w = (D+2t)p = (4.41+1)\times10^{-2}\times10$

$t = 0.5$ $W_w = 0.541\ kg/m$

the effective Loading will be

$$W_e = \sqrt{W_w^2 + (W + W_i)^2}$$



$$W_e = \sqrt{0.541^2 + (0.4 + 0.706)^2} = 1.231 \text{ kg/m}$$

the total sag d

$$d = \frac{Wl^2}{8T} = \frac{1.231 \times 200^2}{8 \times 250} = 24.62 \text{ m}$$

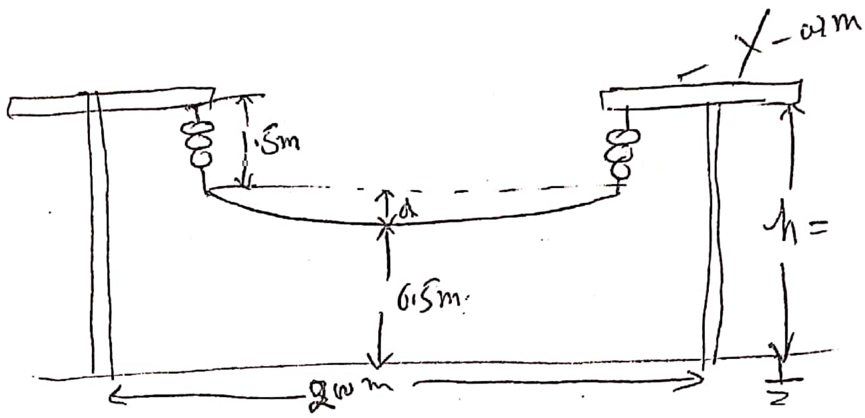
This sag will be acting at an angle

$$\theta = \tan^{-1}\frac{W_w}{W + W_i} = 26.07^\circ$$

to the vertical. Thus the vertical sag will be $24.62 \cos 26.07^\circ = 22.11$ m this is same as in (3) becoz the wind pressure is acting in transverse direction.

Example 4 A Tx ckt weigh equal to 2 kg/m is hung between two supports 200 m apart, the ice coating 1 kg/m, Wind Pressure is 1.5 kg/m. Calculate the total sag & height of the support to the lower cross-arm allowing 1.5 m for suspension gap. The breaking tension is 15000 kg & factor of safety is 2.5. Assume general clearance of 6.5 m; & the supports are at the same level.

Solution



$W = 2\,kg/m$

$W_i = 1\,kg/m$

$W_w = 1.5\,kg/m$

$W_e = \sqrt{W_w^2 + (W+W_i)^2}$

$W_e = \sqrt{1.5^2 + 3^2} = 3.354\,kg/m$

Breaking Tension 15000 kg

$S.f = 2.5$

H. T = Working strength

$= 15000/2.5$

$H = T = 600\,kg$

Sag

$$d \cong \frac{WL^2}{8H} = \frac{3.354 * 200^2}{8 * 600} = \underline{2.79m}$$

Total sag in vertical direction

$Sag = d \cos\theta$ $\qquad \theta = \tan^{-1}\frac{W_w}{W+W_i} = \tan^{-1}\frac{1.5}{3}$

$\theta = 26.5°$

$Sag = 2.79 * \cos 26.5$

$= 2.49$

Height of lower cross (section) arm will be $h = 6.5 + \underline{2.79 \cos 26.5} + 1.5 = \underline{10.5\ me}$