Chapter 6

Water Turbines

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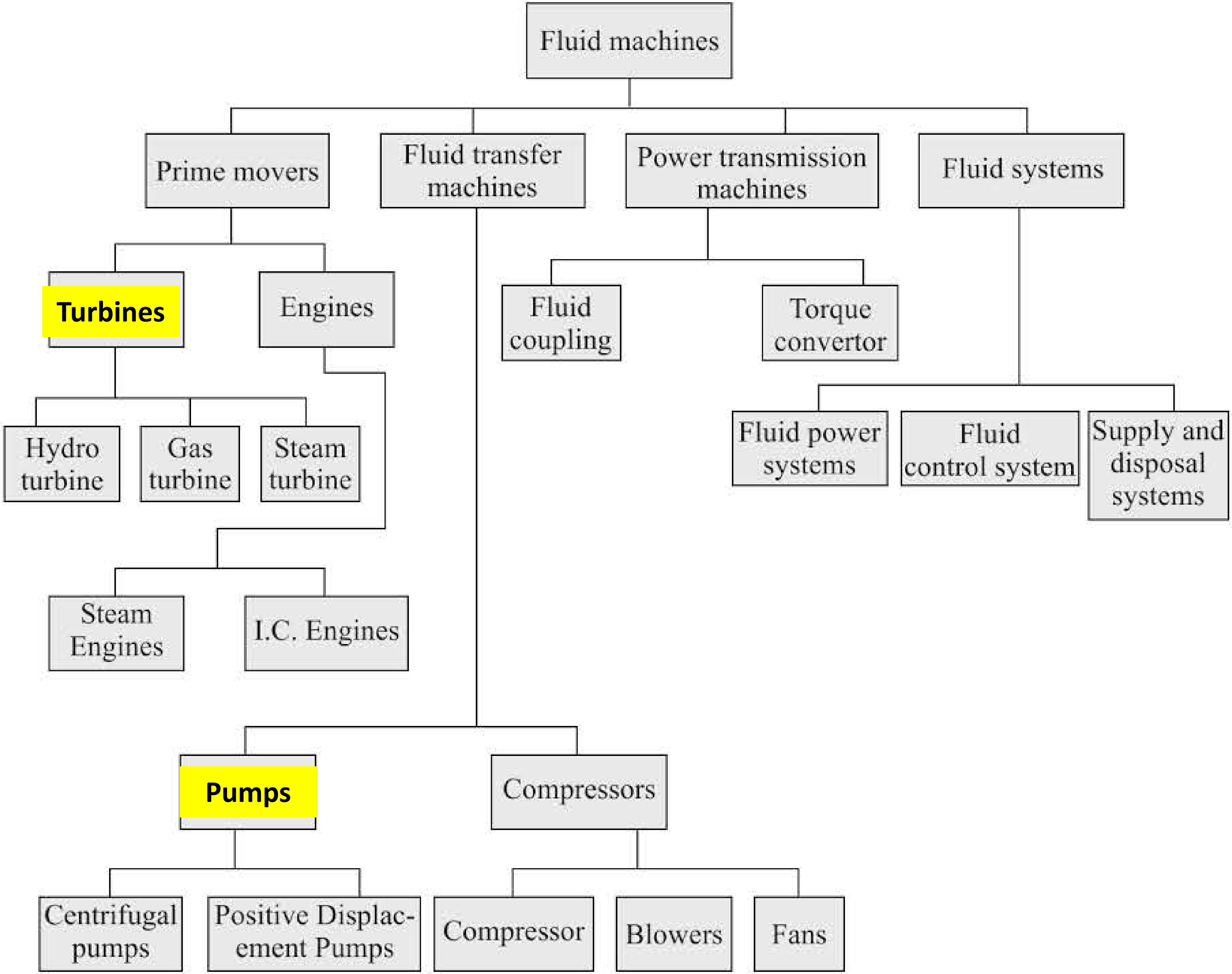
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Development of Water Turbines

- Electric power was formerly obtained from water by means of water wheels which were revolved by the action of water.
- The hydraulic energy was first converted into mechanical energy in India about 2200 years back, by passing the water through water wheels.
- Such type of water wheels were taken from India to Egypt and then to European countries, and finally to America.
- The major disadvantage of water wheels is that the hydro power was available mostly in rural and mountainous regions.
- As a result of this, the mills directly run by water wheels, had to be installed near the power stations and the prime movers had to run round the clock, even if some of the machines in the mills may remain idle.
- It was also observed that the slow moving water wheels were not suitable for all types of purposes.
- Lot of research was conducted by numerous scientists and engineers all over the world, to improve the working of water wheels in the 18^{th} and 19^{th} centuries.
- As a result of this research, water turbines were designed, which can operate under high head (highest head 1765 m in Austria) and can run at higher speeds.

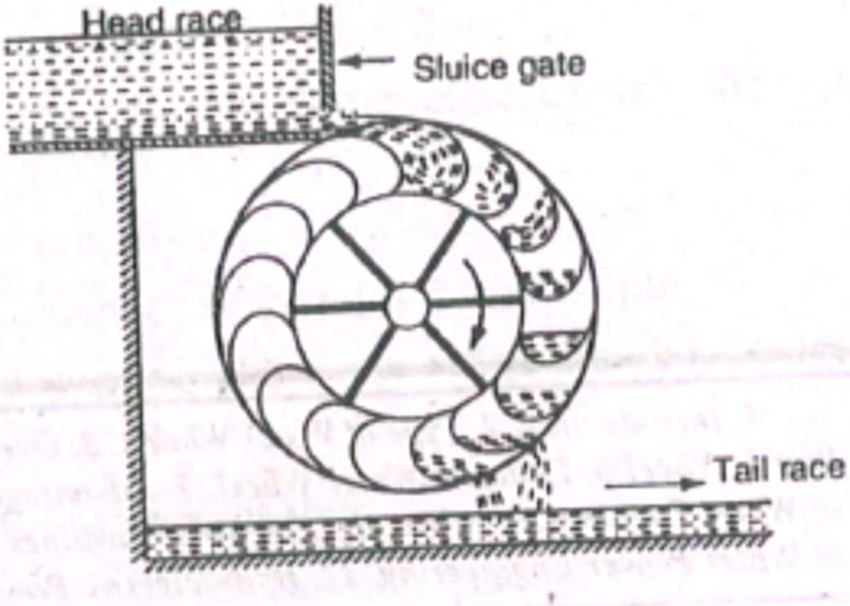


Fig. 31·1. Overshot water wheel.

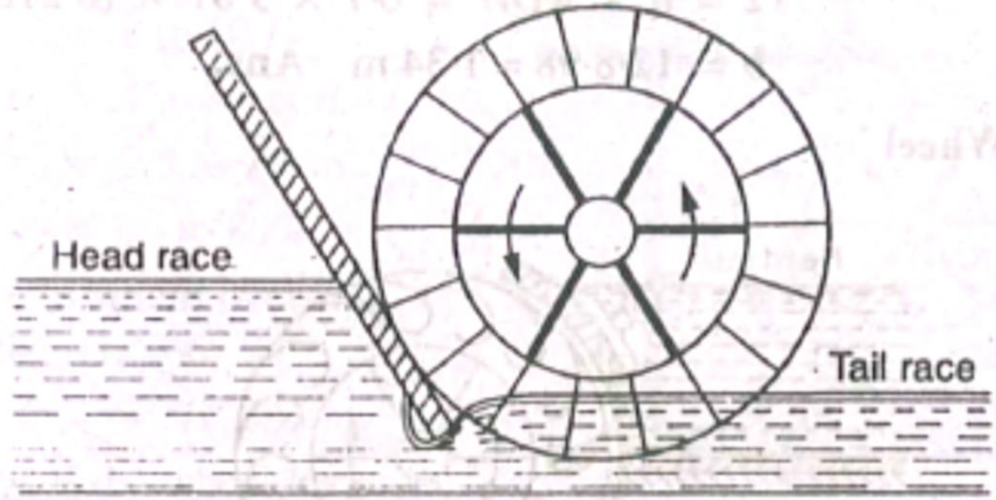


Fig. 31·3. Undershot water wheel.

Fig. 31·2. Breast water wheel

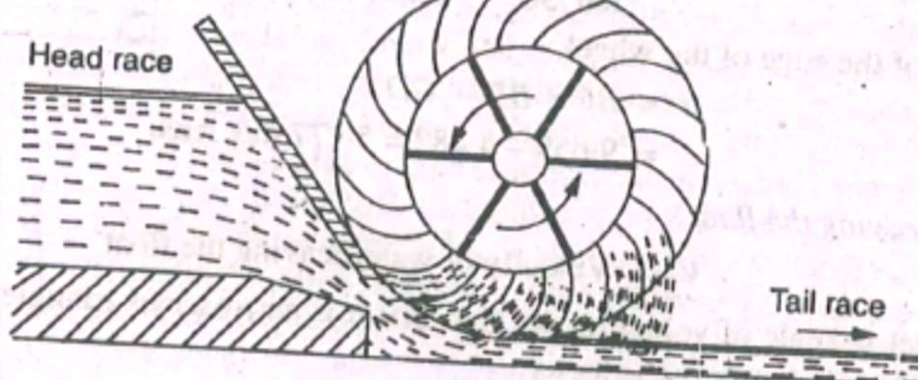


Fig. 31·4. Poncelet water wheel.

Advantages and Classification of Water Turbines

Advantages:

- The water wheels have been replaced by the water turbines.
- The water turbines have the following advantages over water wheels or any other type of prime movers:
  1. They have long life.
  2. They are efficient and can be easily controlled.
  3. They have outstanding ability to act as standby unit.
  4. They can be made automatic controlled.
  5. They can work under any head.

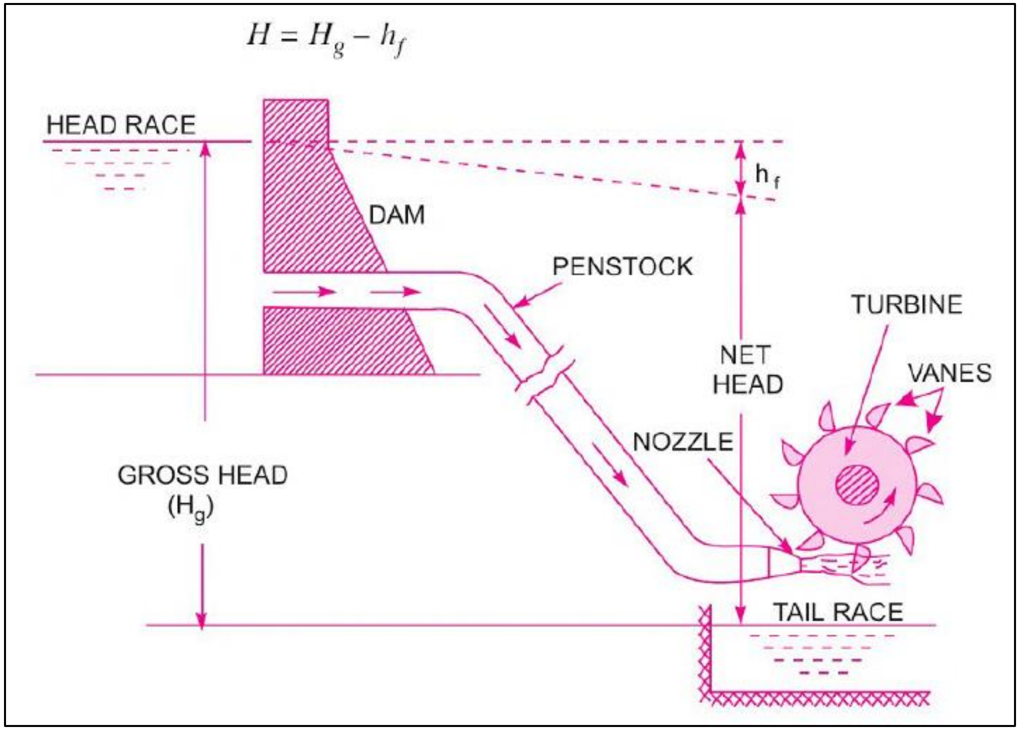
Classification:

- The water turbines may be broadly classified into the following two groups:
  1. Reaction or Pressure turbines
  2. Impulse or velocity turbines

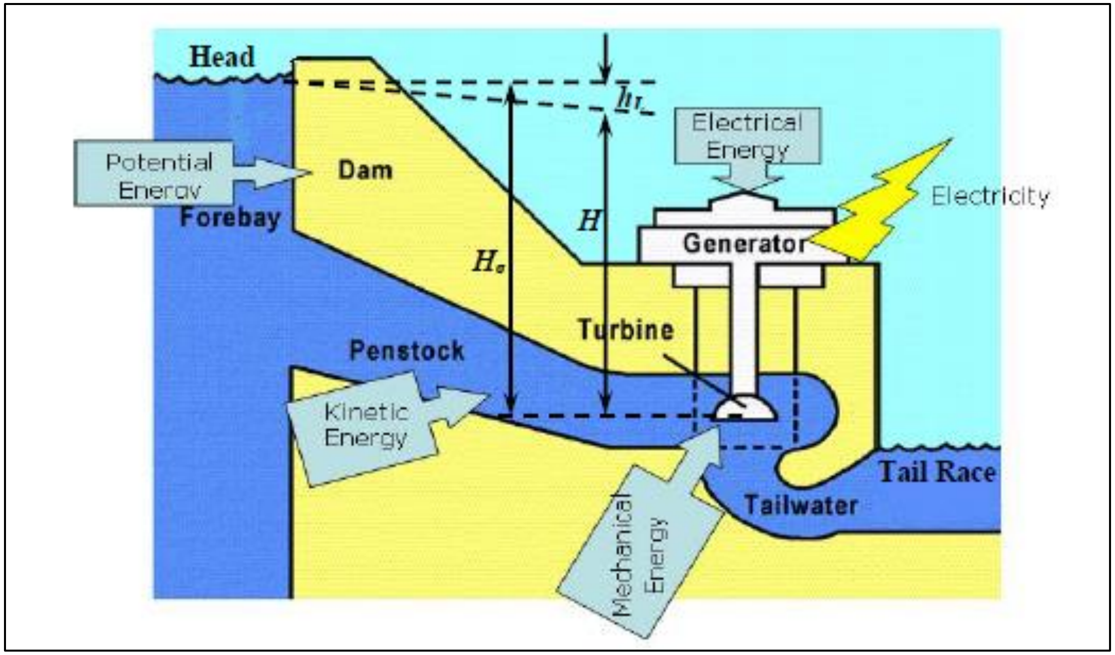
Water Turbines

- The device which converts hydraulic energy (of water) into mechanical energy or vice versa is known as Hydraulic Machines.
- The hydraulic machines which convert hydraulic energy (KE and/or PE) into mechanical energy (which is further converted into electrical energy by directly coupled generators) are known as hydraulic or water turbines.
- A hydroelectric plant consists of the following:

1. A **Dam** constructed across a river or a channel to store water. The reservoir is also known as **Headrace**.
2. Pipes of large diameter called **Penstocks** which carry water under pressure from storage reservoir to the turbines. These pipes are usually made of steel or reinforced concrete.
3. **Turbines** having different types of vanes or buckets or blades mounted on a wheel called runner.
4. **Tailrace** which is a channel carrying water away from the turbine after the water has worked on the turbines. The water surface in the tailrace is also referred to as tailrace.
5. The electric system components formed by the **synchronous generator** and its **control system**.



Pelton Wheel



Francis Turbine

Important Terms

Gross Head (H_g): It is the vertical difference between the headrace level and tail race level when there is no water flow. It is also known as total head of the turbine.

Net Head:(H): Net head or effective head is the actual head available at the inlet of the runner to work on the turbine. It is less than gross head by an amount equal to the friction losses occurring in the flow passage, when the water is flowing from the reservoir to the turbine inlet.

$$H = H_g - h_f = \left({}^{p}/_{\gamma} + {}^{V^2}/_{2g} \right) - h_f$$

where h_f is the total head loss during the transit of water from the headrace to tailrace which is mainly head loss due to friction, and is given by

$$h_f = {}^{4fLV^2}/_{2gD}$$

where **f** is the coefficient of friction of penstock depending on the type of material of penstock.

L = the total length of penstock

V = the mean flow velocity of water through the penstock

D = the diameter of penstock and

g = the acceleration due to gravity

Classes of Turbines

- Both reaction and impulse turbines are subdivided into classes based on the following criteria:

1. **According to the type of energy at inlet:**
   a. Impulse turbine – Pelton Wheel turbine
   b. Reaction turbine – Francis, Kaplan turbines
2. **According to the direction of flow through runner:**
   a. Tangential flow turbine – Pelton Wheel
   b. Radial flow turbine – Old Francis turbine
   c. Axial or Parallel flow turbine – Kaplan turbine, Propeller turbine
   d. Mixed flow turbine – Modern Francis turbine
3. **According to the head at the inlet of turbine:**
   a. High head turbine – Pelton Wheel turbine
   b. Medium head turbine – Francis Turbine
   c. Low head turbine – Kaplan turbine
4. **According to the specific speed of the turbine:**
   a. Low specific speed turbine – Pelton Wheel turbine
   b. Medium specific speed turbine – Francis turbine
   c. High specific speed turbine – Kaplan turbine
5. **According to the shaft of the turbine:**
   a. Horizontal shaft turbine – Pelton Wheel turbine
   b. Vertical shaft turbine – Francis turbine

Classes of Turbines

According to the direction of flow through runner:

a. **Tangential flow turbine:** If the water strikes the vanes along the tangent of the runner, the turbine is known as tangential flow turbine.

b. **Radial flow turbine:** If the flow of water on the vanes is perpendicular to the axis of rotation of the runner i.e. radial, the turbine is known as radial flow turbine and may be an **inward** or **outward flow** depending on whether the water enters at the outer circumference and flows inwards towards the center, or enters at the center and flows outward.

c. **Axial or Parallel flow turbine:** If the water flows parallel to the axis of rotation of the runner or turbine shaft, it is known as axial-flow turbine.

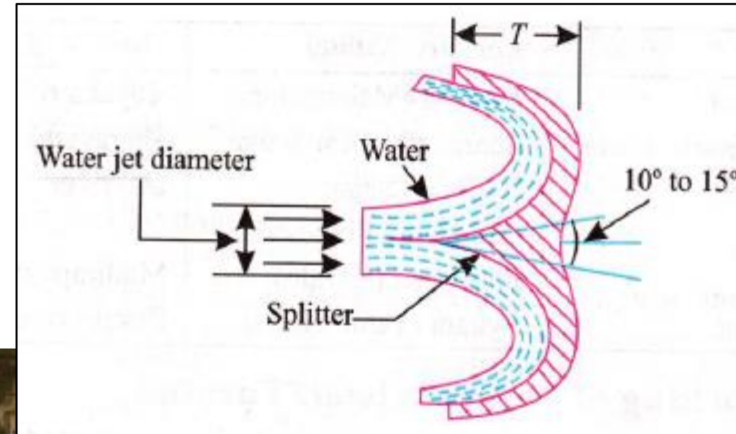
d. **Mixed flow turbine:** In some of the latest types of turbine the flow is partly radial (at inlet) and partly axial (at exit); such turbines are known as mixed flow turbines (e.g. modern Francis turbine).

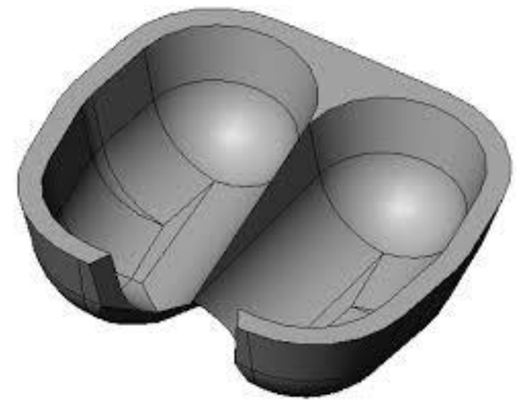
Types of Turbines on the Basis of H, Q and N_S

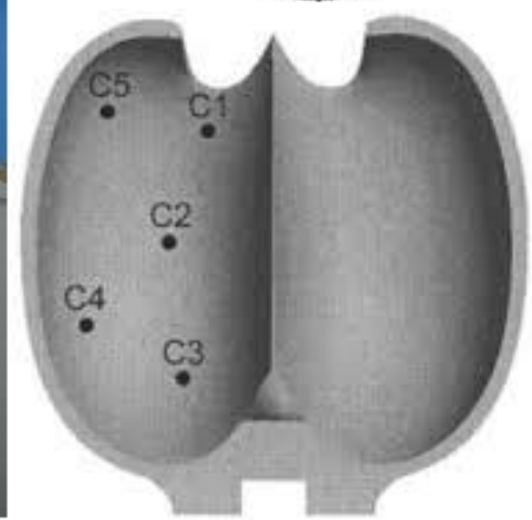
Turbine		Type of energy	Head	Discharge	Direction of flow	Specific Speed
Name	Type					
Pelton Wheel	Impulse	Kinetic	High Head > 250m to 1000m	Low	Tangential to runner	Low <35 Single jet 35 – 60 Multiple jet
Francis Turbine	Reaction Turbine	Kinetic + Pressure	Medium 60 m to 150 m	Medium	Radial flow Mixed Flow	Medium 60 to 300
Kaplan Turbine			Low < 30 m	High	Axial Flow	High 300 to 1000

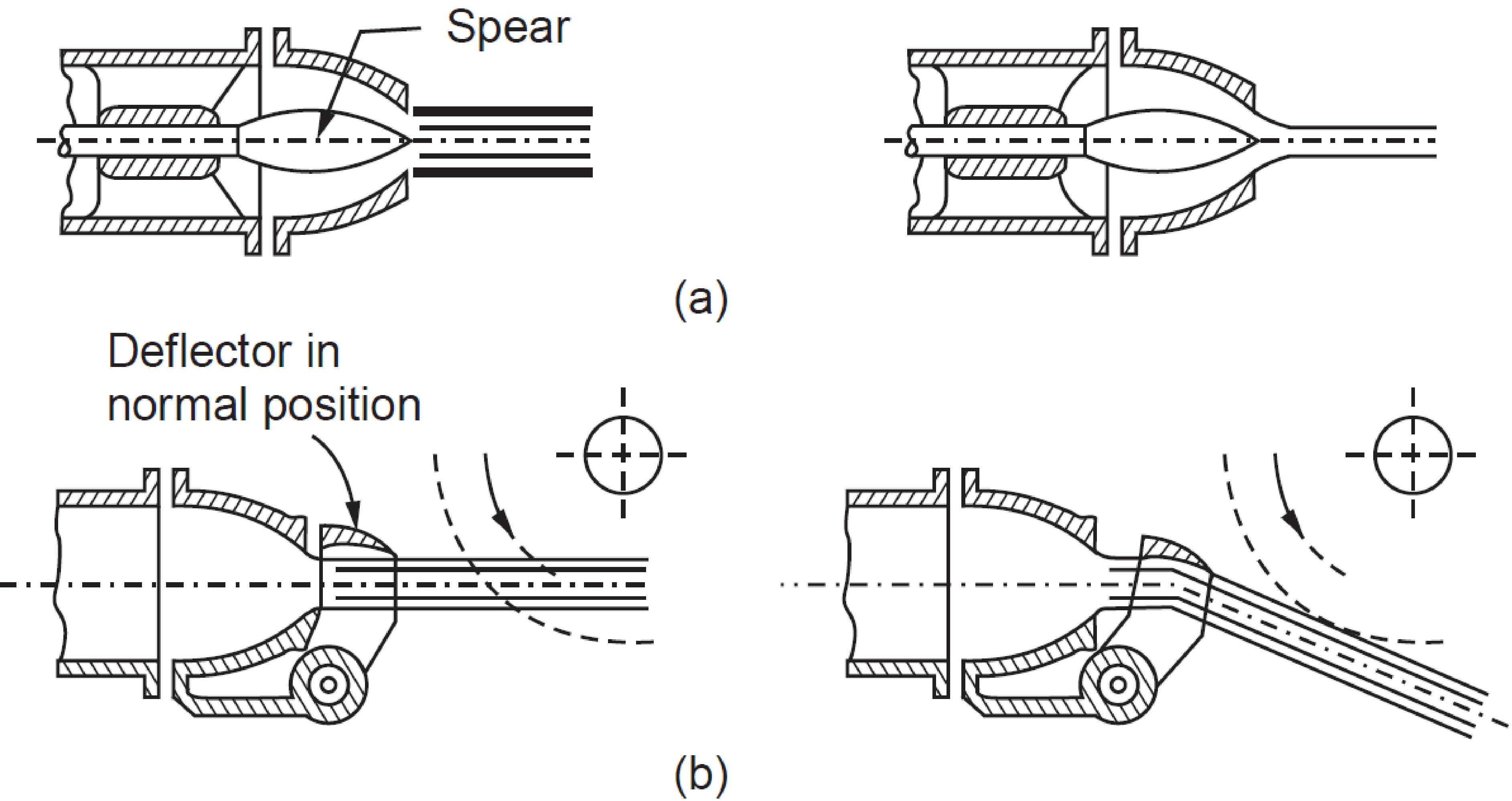
Impulse Turbines

- The impulse turbine is sometimes called an impulse wheel or Pelton wheel, so called in honor of American Engineer Lester Allan Pelton (1829-1908) in 1870s.
- In an impulse turbine the water from a dam is made to flow through a penstock, and then through guide mechanism and finally through the nozzle.
- All the available energy of flowing water is converted into kinetic energy or velocity head by passing it through a convergent nozzle at the end of penstock.
- The components of an impulse turbine are nozzle, runner and split hemispherical buckets, casing and breaking jet.
- When the jet strikes the dividing ridge of the bucket, it is split into two parts that discharges at both sides of the bucket.
- Each split bucket has a notch that enables the bucket to attain a position nearly tangent to the direction of the jet before the bucket lit intercepts the jet.
- Only one jet is used on small turbines, but two or more jets impinging at different points around the wheel are often used on large units.
- These jets are produced by a needle nozzle.
- Some jet velocities exceed 150 m/s.
- Water hammer effect is prevented by a bypass valve arrangement in the penstock that opens whenever the needle valve is closed quickly.


T
Water jet diameter
Water
10° to 15°
Splitter




C5
C1
C2
C4
C3


Spear
(a)
Deflector in normal position
(b)

Impulse Turbines

- The same effect could also be obtained with a jet deflector which deflects the jet away from the wheel when load drops.
- A governor is required to actuate the bypass or deflector units.
- The penstocks are usually made in large diameters to minimize energy losses in order to maximize energy delivered to turbine wheel.
- All the energy of the water is converted into velocity (kinetic energy) before entering the wheel by expanding through a nozzle or guide vanes.
- Impulse turbines are provided with housings to prevent splashing, but the air within the housing is at atmospheric pressure, hence the runner must not run full and it must be placed at the foot of the fall and above the tail race.
- The water may be admitted over part of the circumference only or over the whole circumference in the form of jets.

Energy absorbed by the runner per kg of water =

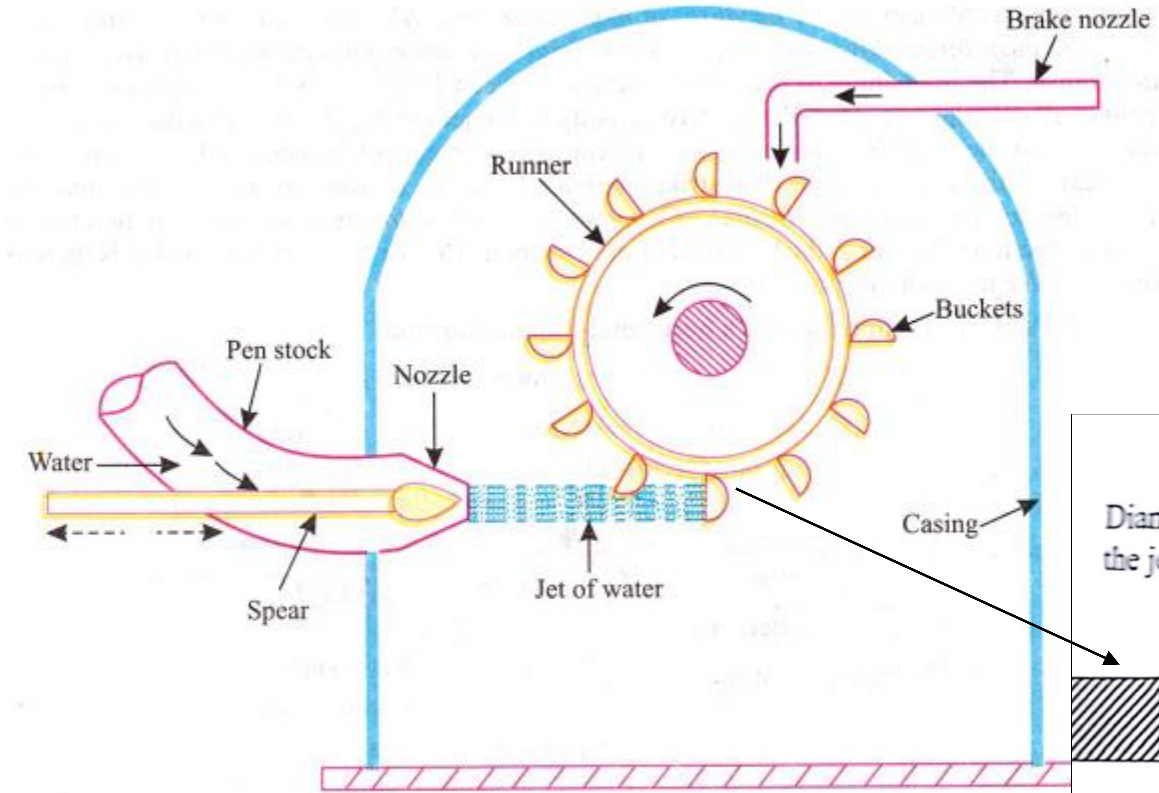
K.E. at penstock inlet – K.E. at nozzle outlet = $\frac{V_i^2}{2g} - \frac{V_o^2}{2g}$

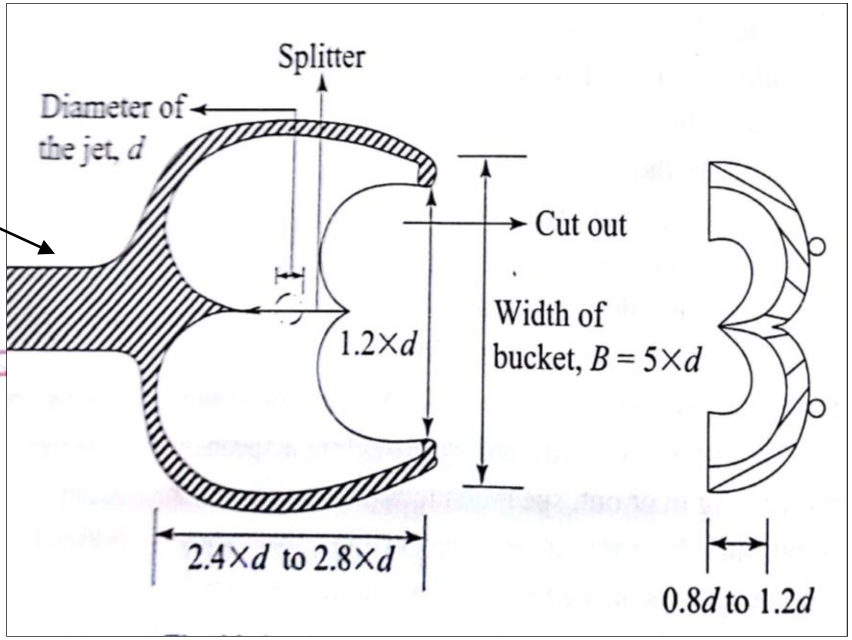
Also, Net Head = $\frac{p}{\gamma} + \frac{V^2}{2g}$

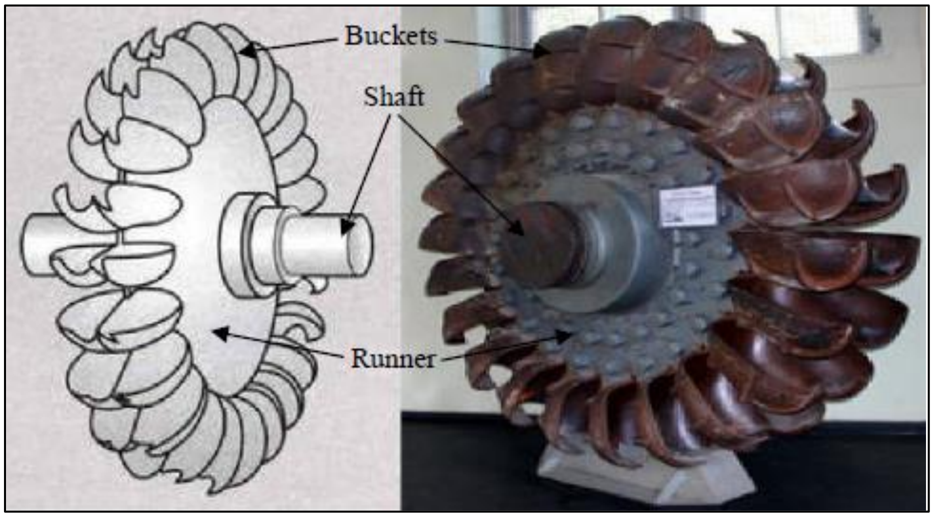
- The gravitational head "z" is not available as the runner is above the tail race.

Impulse Turbines

- For good efficiency the width of the bucket should be 3–4 times the jet diameter, and the wheel diameter is usually 15–20 times the jet diameter.
- The wheel diameter, also called pitch diameter is the diameter of the pitch circle, the circle to which the centerline of the jet is tangent.
- The diameters of impulse turbine range up to 5 m.
- The generator rotor is usually mounted on a horizontal shaft between two bearings with the runner installed on the projecting end of the shaft; this is known as single-overhung installation.
- Often runners are installed on both sides of the generator which is known as double-overhung construction, to equalize the bearings load.
- Some modern wheels are mounted on a vertical axis below the generator and driven by jets from several nozzles spaced uniformly around the periphery of the wheel.
- Such an arrangement simplifies the overall design.


Brake nozzle
Runner
Buckets
Pen stock
Nozzle
Water
Casing
Spear
Jet of water


Splitter
Diameter of the jet, d
Cut out
1.2×d
Width of bucket, B = 5×d
2.4×d to 2.8×d
0.8d to 1.2d


Buckets
Shaft
Runner

Efficiencies of a Turbine

- Depending on the considerations of input and output, the efficiencies can be classified as:

1. Hydraulic efficiency (η_h):

- It is the ratio of the power developed by the runner of a turbine to the power supplied at the inlet of a turbine.
- Since the power supplied is hydraulic, and the probable loss is between the striking jet and vane, it is rightly called hydraulic efficiency.

$$\eta_h = \frac{\text{Power developed by runner}}{\text{Power supplied at inlet}} = \frac{\text{R.P.}}{\text{W.P.}}$$

$$\text{Runner Power (R.P.)} = \frac{W}{g}\frac{[V_{w_1} \pm V_{w_2}]u}{1000}\ (\text{kW});\ \text{Pelton Turbine}$$

$$\text{Runner Power (R.P.)} = \frac{W}{g}\frac{[V_{w_1}u_1 \pm V_{w_2}u_2]}{1000}\ (\text{kW});\ \text{Radial flow Turbine}$$

$$\text{Water Power (W.P.)} = \frac{WH}{1000} = \frac{\rho g\, Q\, H}{1000} = g\, Q\, H\ (\text{kW})$$

Efficiencies of a Turbine

2. Mechanical efficiency (η_m):

- It is the ratio of the power available at the shaft to the power developed by the runner of a turbine.

$$\eta_m = \frac{\text{Power available at the shaft of turbine}}{\text{Power developed by turbine}} = \frac{\text{S.P.}}{\text{R.P.}}$$

- This depends on the slips and other mechanical problems that will create a loss of energy between the runner in the annular area between the nozzle and spear, the amount of water reduces as the spear is pushed forward and vice-versa.

3. Volumetric efficiency (η_v):

- It is the ratio of the volume of water actually striking the runner to the volume of water supplied to the runner.

$$\eta_v = \frac{\text{Volume of water striking the runner}}{\text{Volume of water supplied to runer}} = \frac{Q - \Delta Q}{Q}$$

- The volume of water striking the runner is slightly less than that of the supplied.
- Some of the volume of water is discharged (ΔQ) to the tail race without striking the runner.

Efficiencies of a Turbine

4. Overall efficiency (η_o):

- It is the ratio of the power available at the shaft to the power supplied at the inlet of a turbine.

- As this covers overall problems of losses in energy, it is known as overall efficiency.

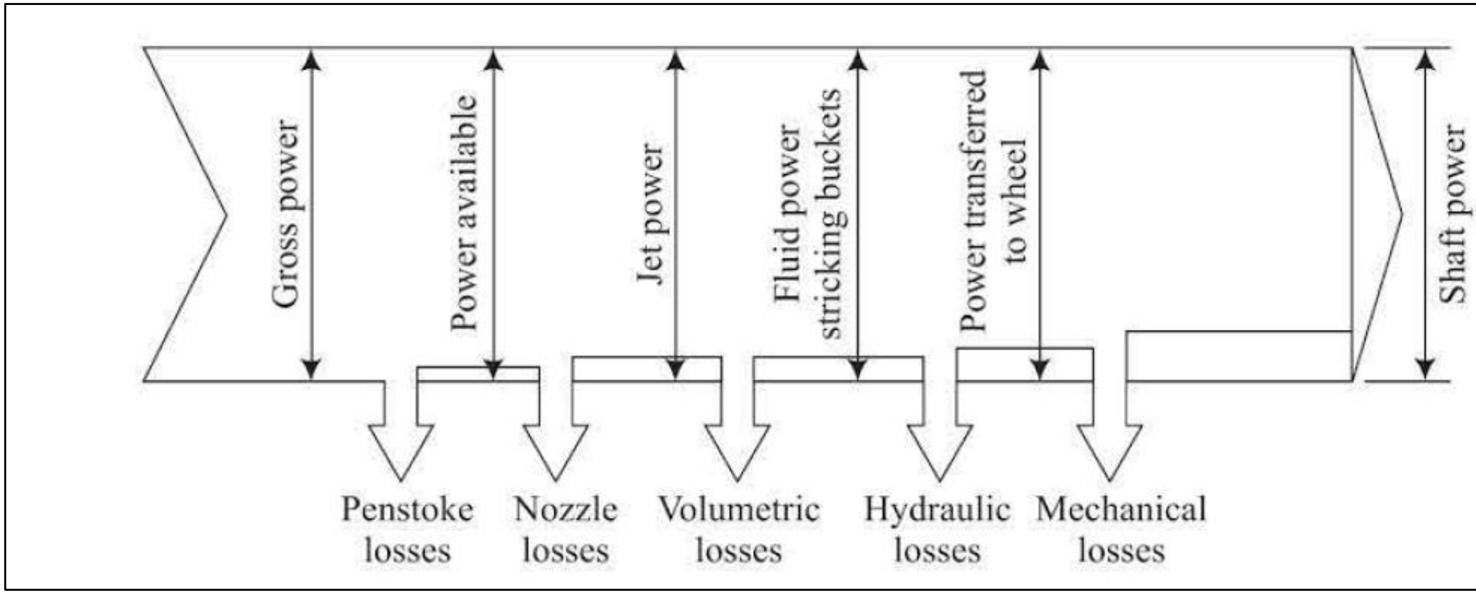
$$\eta_o = \frac{\text{Power available at the shaft}}{\text{Power supplied at inlet}} = \frac{\text{S.P.}}{\text{W.P.}} = \frac{T\,\omega}{\gamma\,Q\,H}$$

$$\eta_o = \frac{\text{S.P.}}{\text{R.P.}} \times \frac{\text{R.P.}}{\text{W.P.}}$$

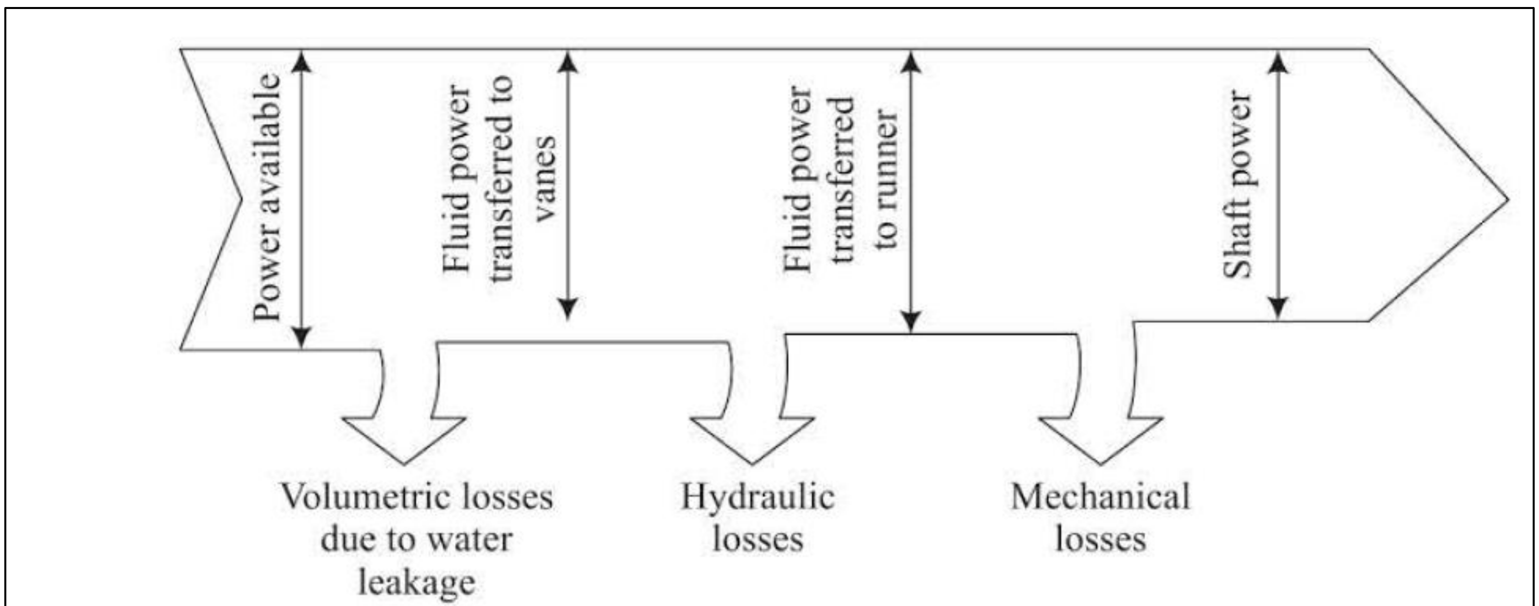
$$\eta_o = \eta_m \times \eta_h$$

- This depends on both the hydraulic losses and the slips and other mechanical problems that will create a loss of energy between the jet power supplied and the power generated at the shaft available for coupling of the generator.

Sankey Diagram (Energy Flow Diagram) for a Pelton Wheel

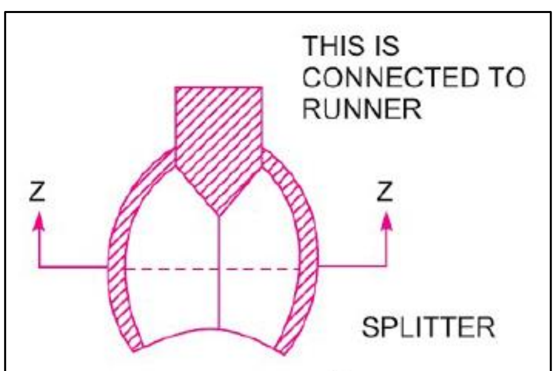


Sankey Diagram (Energy Flow Diagram) for a Francis Turbine

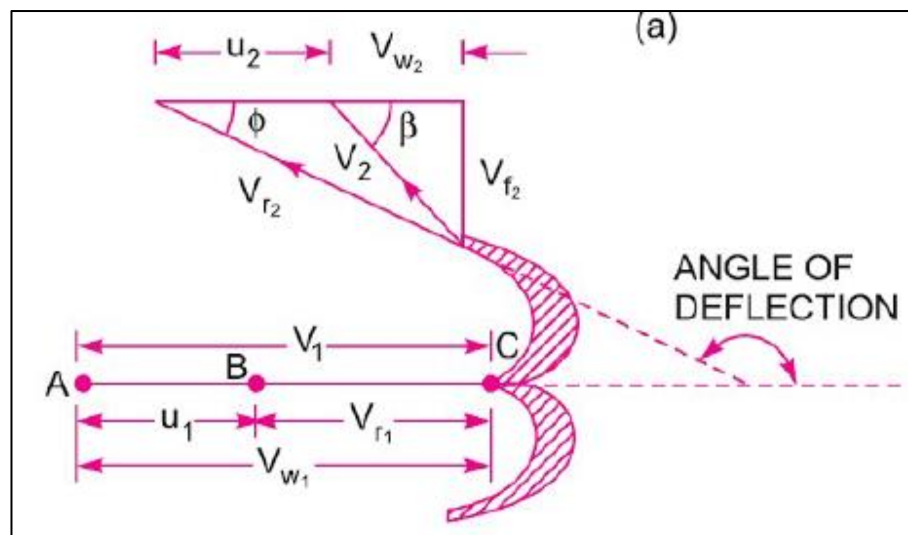


Work Done by an Impulse Turbine

- The jet of water issuing from the nozzle, strikes the bucket at its splitter. The splitter then splits up the jet into two parts.
- One part of the jet glides over the inside surface of one portion of the vane and leaves it at its extreme edge.
- The other part of the jet glides over the inside surface of the other portion of the vane and leaves it at its other extreme edge.
- A little consideration will show, that the mid-point of the bucket, where the jet strikes the splitter and gets divided forms the inlet tip, and the two extreme edges, where the divided jet leaves the bucket from the two outlet tips.
- The inlet velocity triangle is drawn at the splitter and the outlet velocity triangle is drawn at the outer edge of the bucket.



Section of Bucket at Z-Z



Work Done by an Impulse Turbine

- Let, H = net head acting on the Pelton Wheel = $H_g - h_f$

  $$h_f = {}^{4fLV^2}/_{2gD^*}$$

  D* = diameter of penstock

  N = speed of the wheel in rpm

  D = diameter of the wheel

  d = diameter of the jet

  $$V_1 = velocity\ of\ jet\ at\ inlet = \sqrt{2gH}$$

  u = velocity of vane

  $$u = u_1 = u_2 = \frac{\pi DN}{60}$$

- The velocity triangle at inlet will be a straight line where

  $$V_{r_1} = V_1 - u_1 = V_1 - u$$

  $$V_{w_1} = V_1 \cos\alpha = V_1$$

  $\alpha = 0$ and $\theta = 0$

- From the velocity triangle at outlet, we have

  $$V_{r_2} = V_{r_1} = V_1 - u_1 \text{ and } V_{w_2} = V_{r_2} cos\emptyset - u_2$$

$$cos\emptyset = \frac{V_{w_2} + u_2}{V_{r_2}}$$

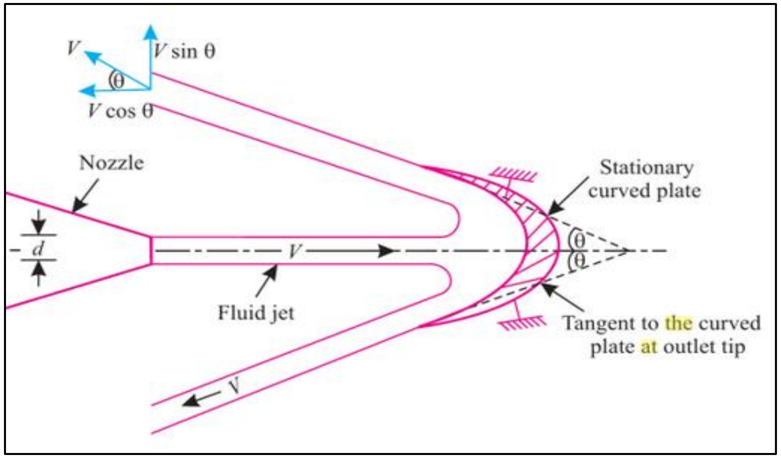
Work Done by an Impulse Turbine

- The force exerted by the jet of water in the direction of motion is given by

$$F_x = ma_x = \frac{\gamma a V_1}{g}[V_{w_1} + V_{w_2}] \qquad \text{where, } V_{w_1} = V_1 \text{ and } V_{w_2} = V_2 cos\beta$$

- For angle β < 90° + sign and β > 90° – sign should be taken.
- Now, work done by the jet on the runner per second

$$\text{WD/s} = F_x \times u = \frac{\gamma a V_1}{g}[V_{w_1} + V_{w_2}] \times u$$



- Power given by the jet to the runner

$$\text{P} = \frac{\frac{\gamma a V_1}{g}[V_{w_1}+V_{w_2}] \times u}{1000}$$

- Work done/s per unit weight of water striking/s

$$= \frac{\frac{\gamma a V_1}{g}[V_{w_1}+V_{w_2}] \times u}{\frac{\gamma a V_1}{g} \times g} = \frac{1}{g}[V_{w_1} + V_{w_2}] \times u$$

- Kinetic energy of jet per second = $\frac{1}{2} m V_1^2 = \frac{1}{2}\left(\frac{\gamma a V_1}{g}\right) V_1^2$

- Hydraulic efficiency (η_h) = $\frac{work\ done\ per\ second}{K.E. of\ jet\ per\ second} = \frac{\frac{\gamma a V_1}{g}[V_{w_1}+V_{w_2}] \times u}{\frac{1}{2}\left(\frac{\gamma a V_1}{g}\right) V_1^2} = \frac{2\,[V_{w_1}+V_{w_2}] \times u}{V_1^2}$

Work Done by an Impulse Turbine

$$\eta_h = \frac{2[V_1+(V_1-u)cos\emptyset-u]\times u}{V_1^2}$$

$$\eta_h = \frac{2(V_1-u)(1+cos\emptyset)\times u}{V_1^2}$$

From diagram,
$V_{w_1} = V_1$
$V_{w_1} = V_{r_2}cos\emptyset - u_2$
$V_{w_1} = V_{r_2}cos\emptyset - u$
$V_{r_2} = V_{r_1} = V_1 - u_1$

- The efficiency is maximum for a given value of V_1 when
- $\frac{d(\eta_h)}{du} = \frac{d}{du}\left(\frac{2(V_1-u)(1+cos\emptyset)\times u}{V_1^2}\right) = 0$
- Or by simplifying, $u = \frac{V_1}{2}$
- The expression for maximum efficiency of Pelton wheel is given by

$$\eta_h = \frac{1+cos\emptyset}{2}$$

Example:

A Pelton wheel having semi-circular buckets and working head of 140 m is running at 600 rpm. The discharge through nozzle is 500 lit/s and diameter of wheel is 600 mm. Find: (a) power available at the nozzle, and (b) hydraulic efficiency of the wheel, if coefficient of velocity is 0.98.

Solution:

$\phi = 180 - 180 = 0$	H = 140 m	N = 600 rpm
Q = 500 lit/s = 0.5 m^3/s	D = 600 mm = 0.6 m	$C_v = 0.98$

$$(a)\ Water\ Power = W.P. = \gamma QH = 9.81 \times 0.5 \times 140 = \mathbf{686.7\ kW}$$

$$(b)\ V = C_v\sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 140} = 51.36\ m/s$$

$$u = \frac{\pi DN}{60} = \frac{\pi \times 0.6 \times 600}{60} = 18.85\ m/s$$

$$\eta_h = \frac{2u(V - u)(1 + cos\emptyset)}{V^2} = \frac{2 \times 18.85 \times (51.36 - 18.85)(1 + cos0)}{51.36^2} = 0.929$$

$$\boldsymbol{Hydraulic\ Effciency = \eta_h = 92.9\%}$$

Design of Pelton Wheel Turbine - Summary

- Design of the Pelton wheel requires the following data to be determined:
  1. Diameter of the jet = d
  2. Diameter of wheel = D using $\frac{\pi D N}{60}$
  3. Width of bucket = 5 x d
  4. Depth of the bucket = 1.2 x d
  5. No of buckets on the wheel = $\frac{\text{total rate of flow through the turbine}}{\text{rate of flow of water through a single jet}} = \frac{D}{2d} + 15$

1. Jet Ratio (m) = Pitch Diameter of wheel / Dia. of Jet = D / d (11 – 16)
2. Speed Ratio (K_u) = Velocity of Wheel / Velocity of Jet = u / V = (0.43 – 0.48)
3. Velocity of Wheel $= \mathrm{u} = \mathrm{u}_1 = \mathrm{u}_2 = \frac{\pi D N}{60}$
4. Overall Efficiency $= \eta_o = \eta_m \times \eta_h$ Or $\eta_o = \frac{\text{S.P.}}{\text{W.P.}}$ (80 – 87%)
5. Water Power, W.P. = $\frac{1}{2} m V^2 = \rho g Q H = \gamma Q H$
6. Shaft Power, S.P. = $\frac{\gamma a V_1}{g}[V_{w_1} + V_{w_2}] \times u = \rho Q [V_{w_1} + V_{w_2}] \times u$
7. No. of Buckets = (0.5 x Jet Ratio) + 15
8. Angle of jet deflection = 165^o if not stated

Example:

Design a Pelton wheel for a head of 350 m at a speed of 300 rpm. Take overall efficiency of the wheel as 85% and ratio of jet to the wheel diameter as 1/10.

Solution:

H = 350 m N = 300 rpm η_o = 85% = 0.85 $\frac{D}{d} = \frac{1}{10}$

$$V = C_v\sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 350} = 81.2\ m/s$$

$$u = 0.46V = 0.46 \times 81.2 = 37.4\ m/s$$

$$\mathrm{u} = \frac{\pi DN}{60} \Rightarrow 37.4 = \frac{\pi \times D \times 300}{60} = 15.7D$$

$$(1)\ Diameter\ of\ Wheel = D = \mathbf{2.4\ m}$$

$$(2)\ Diameter\ of\ Jet = d = \frac{D}{10} = \frac{2.4}{10} = 0.24\ m = \mathbf{240\ mm}$$

$$(3)\ Width\ of\ Buckets = 5 \times d = 5 \times 0.24 = \mathbf{1.2\ m}$$

$$(4)\ Depth\ of\ Buckets = 1.2 \times d = 1.2 \times 0.24 = \mathbf{0.48\ m}$$

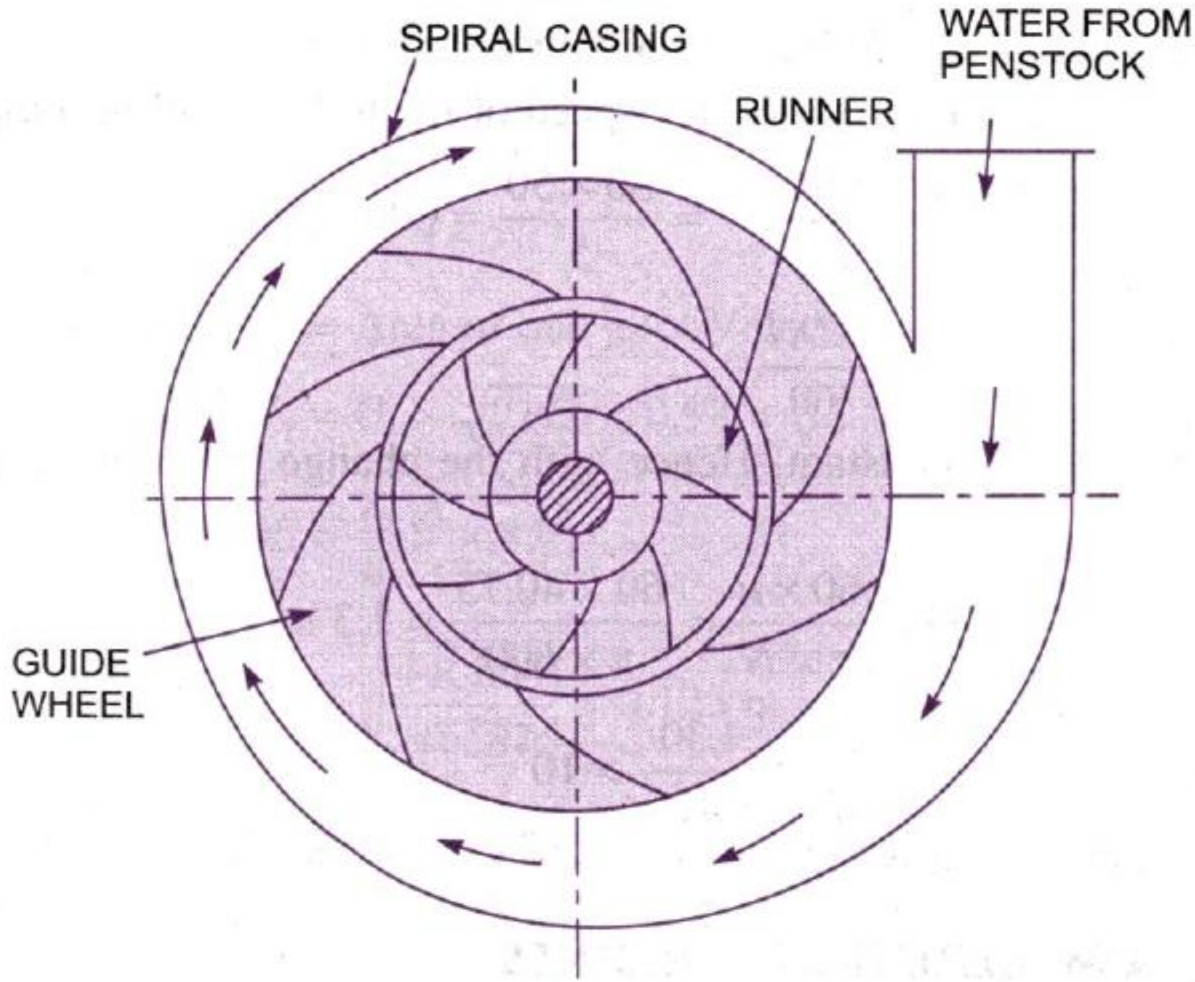
$$(5)\ No.of\ Buckets = \frac{D}{2d} + 15 = \frac{2.4}{2 \times 0.24} + 15 = \mathbf{20}$$

Reaction Turbines

- The first inward flow reaction turbine was built in 1849 by the eminent American hydraulic Engineer James B. Francis, who made an accurate test of this turbine.
- Only part of the available energy of water is converted into kinetic energy at the entrance to the runner and substantial amount remains as pressure energy.
- As the water flows through the vanes, the change from pressure energy to kinetic energy takes place gradually and finally reduces to atmospheric pressure.
- As the water is under pressure, the wheel must run full and may therefore be entirely submerged below the tail race.
- The difference of pressure between the inlet and outlet of the runner is called the reaction pressure, hence these turbines are called reaction turbines.
- The components of a reaction turbine are spiral casing, guide mechanism, turbine runner and draft tube.
- The water leaves the wheel with a large relative velocity but a small absolute velocity, practically the whole of its energy being given to the wheel.

  Let, H = total head of entering water

  V_2 = velocity of leaving water
- Then energy given to the wheel per kg of water = $\mathrm{H} - \frac{{V_2}^2}{2g} = \frac{1}{g}\left(V_{\omega_1}u_1 \pm V_{\omega_2}u_2\right)$
- The total head H consists partly of pressure head and partly of velocity head.
- The water must be admitted into a reaction turbine over the whole circumference of the wheel; the power is difficult to regulate without loss.

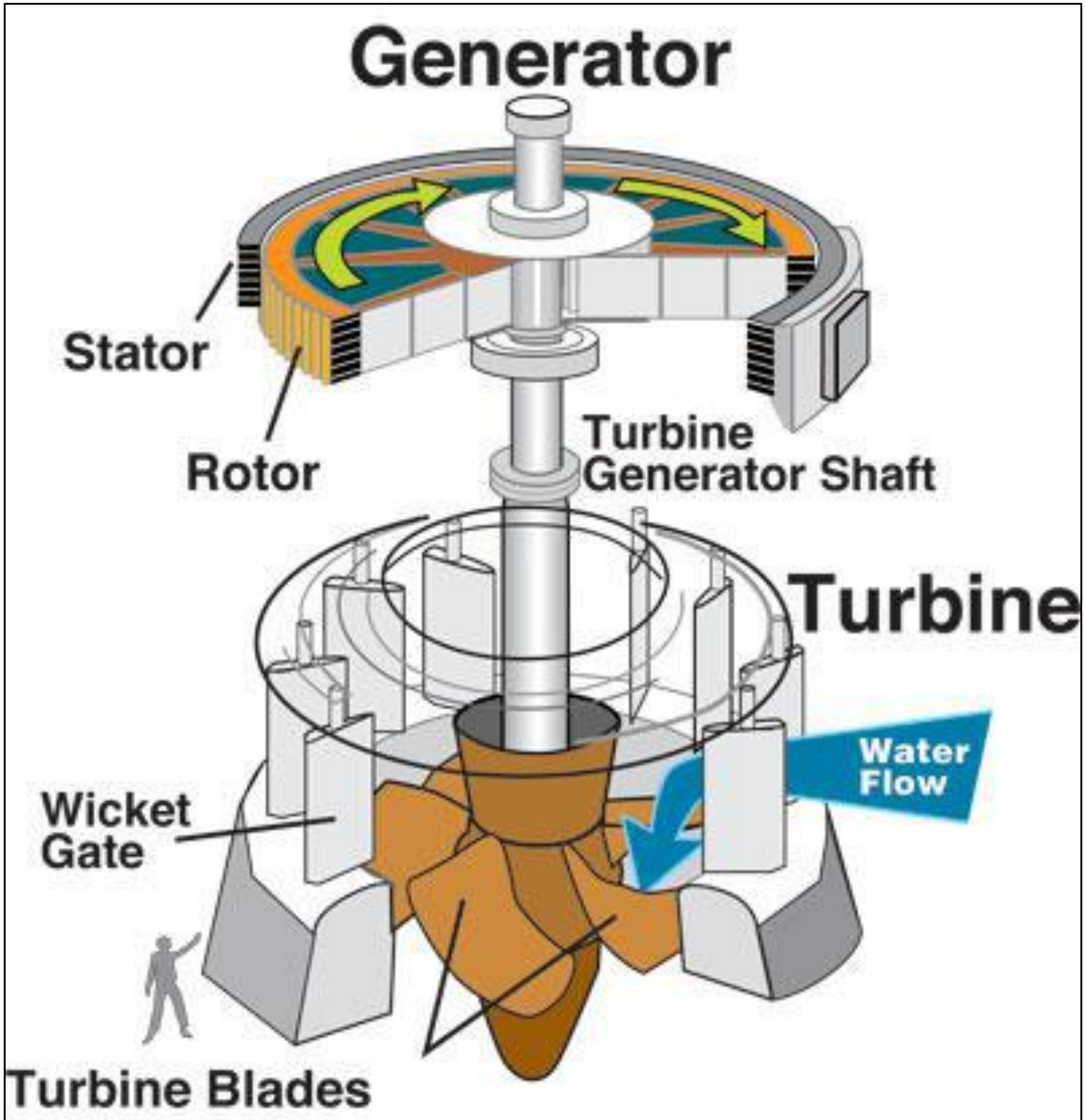


Parts of a Reaction Turbine (Inward Flow of Water)

Reaction Turbines

- In the usual **Francis turbine** water enters the scroll case and moves into the runner through a series of guide vanes with contracting passages that convert pressure head to velocity head (i.e. inward flow with radial discharge at outlet).
- These vanes are known as wicket gates, are adjustable so that the quantity and direction of flow can be controlled.
- Flow through usual Francis runner is at first inward in the radial direction, gradually changing to axial.

- The **propeller turbine** an axial-flow machine with its runner confined in a closed conduit, is commonly set on a vertical axis.
- The usual runner has four to eight blades (vanes) mounted on a hub and are not adjustable, with very little clearance between the blades and the conduit wall.
- The blades have free outer ends like a marine propeller.
- A **Kaplan turbine** (after Victor Kaplan, Austrian Engineer) is a propeller turbine with movable blades (3 – 8) whose pitch can be adjusted to suit existing operating conditions.
- Two other types of propeller turbine include the **Deriaz**; a reversible/diagonal flow turbine, and the **tube runner**; an inclined-axis type that is particularly well adapted to low-head installations.



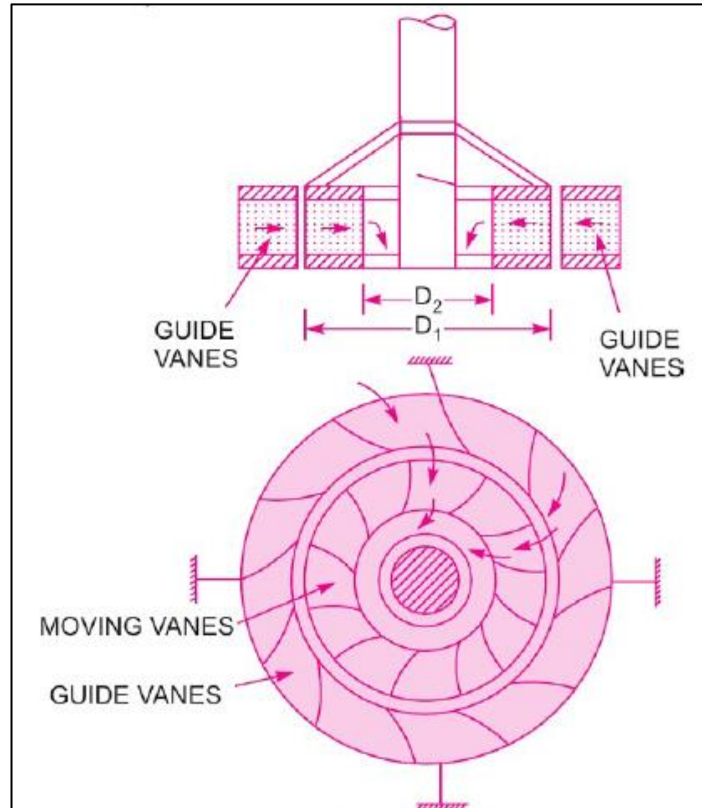


Fig. 18.11 *Inward radial flow turbine.*

Francis Turbine

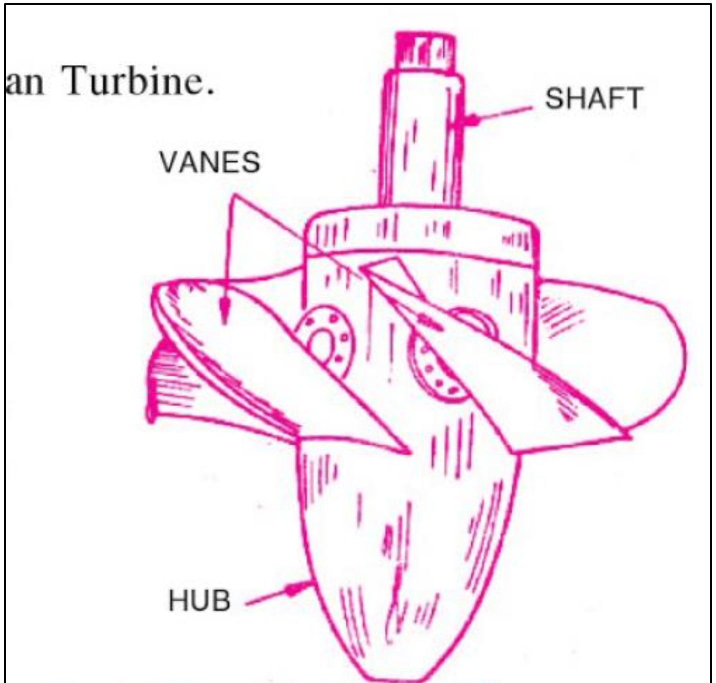


Fig. 18.25 *Kaplan turbine runner.*

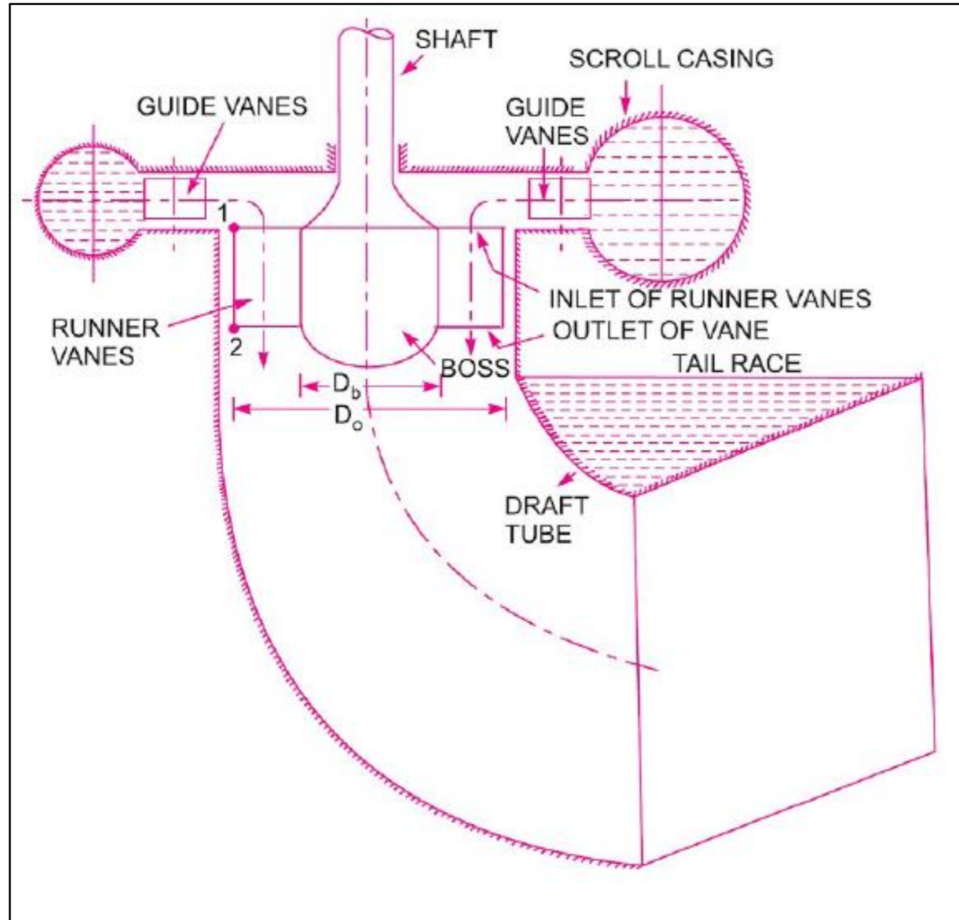
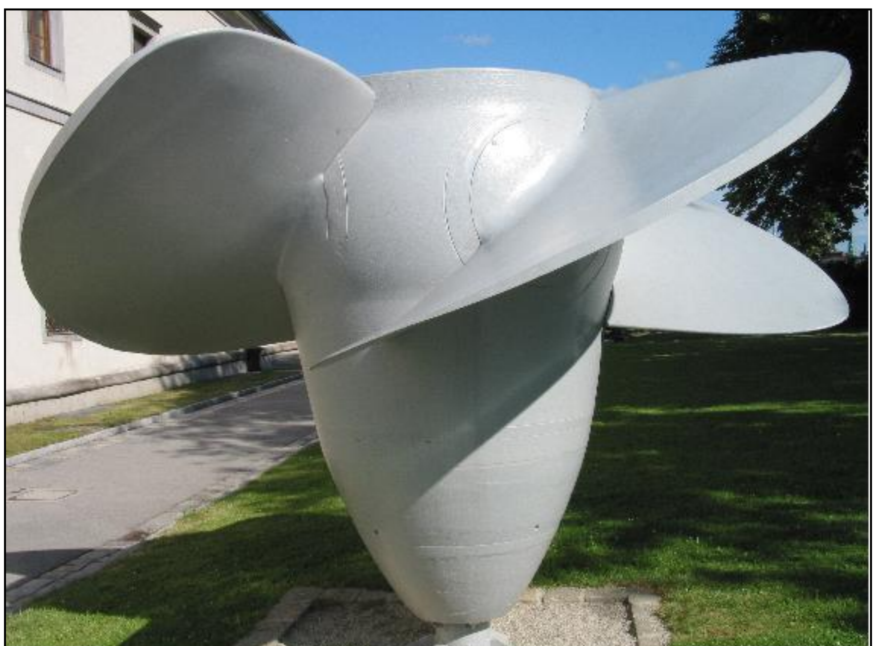


Fig. 18.26 *Main components of Kaplan turbine.*

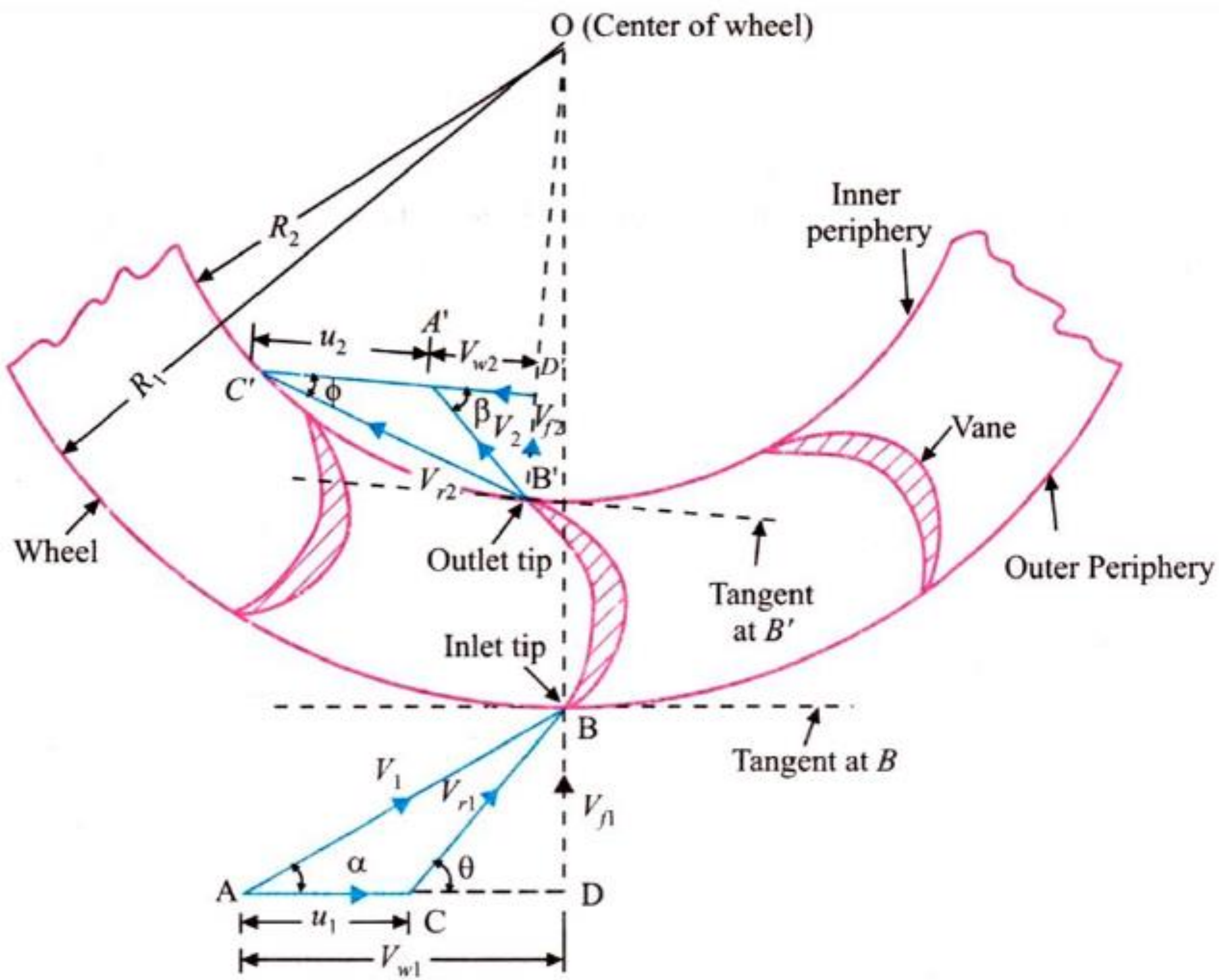


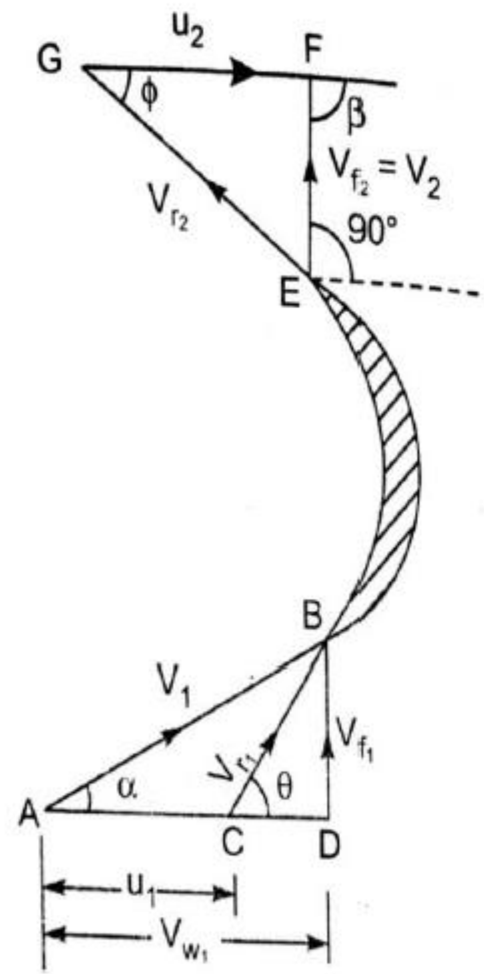
D_o = outer diameter of runner
D_b = diameter of hub or boss
Hub or boss acts as runner

Work Done by an Inward Flow Reaction Turbine

Read:

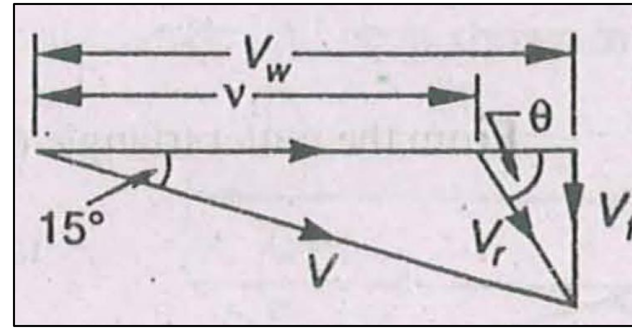
Topic: Impact of Jets on Radial Moving Vanes from Chapter 5.





Example:

An inward flow reaction turbine having an external diameter of 1·5 m runs at 400 r.p.m. The velocity of flow at inlet is 10 m/s. if the guide blade angle is 15°, find (a) absolute velocity of water, (b) velocity of whirl at inlet, (c) inlet vane angle of the runner, and (d) relative velocity at inlet.



Solution:

D = 1.5 m N = 400 rpm V_f = 10 m/s α = 15°

(a) Absolute Velocity of Water at Inlet (V),

$$V = \frac{V_f}{sin\alpha} = \frac{10}{sin15} = \mathbf{38.64\ m/s}$$

(b) Velocityof Whirl at Inlet (Vw),

$$V_w = V cos \propto = 38.64 \times cos15 = \mathbf{37.32\ m/s}$$

(c) Inlet Vane Angle of Runner (θ),

$$\text{Tangential Velocity of Vane at Inlet, u} = \frac{\pi DN}{60} = \frac{\pi \times 1.5 \times 400}{60} = 31.42\ m/s$$

$$tan\theta = \frac{V_f}{V_w - u} = \frac{10}{37.32 - 31.42} = 11.695\ \ or\ \ \boldsymbol{\theta = 59.5^o}$$

(d) Relative Velocity at Inlet (V_r),

$$V_r = \frac{V_f}{sin\theta} = \frac{10}{sin59.5} = \mathbf{11.61\ m/s}$$

Example:

Water is fed to an inward flow reaction turbine, running at 180 r.p.m. with a velocity of flow of 3 m/s. The diameter and width of the wheel at inlet are 1 m and 135 mm, respectively. Assuming the velocity of flow to be constant and radial discharge at inlet and outlet, find, (a) work done per kN of water, (b) power developed by the turbine, (c) head of water on the turbine, and (d) hydraulic efficiency of the turbine.

Solution:

N = 180 rpm $V_{f_1} = V_{f_2} = 3$ m/s D = 1 m b = 135 mm = 0.135 m

$(a)\ Work\ Done\ per\ kN\ of\ Water\ (WD),$

$$\text{Tangential Velocity of Vane at Inlet} = \mathrm{u}_1 = \frac{\pi DN}{60} = \frac{\pi \times 1 \times 180}{60} = 9.42\ m/s$$

$$Velocity\ of\ Whirl\ at\ Inlet, = V_{w_1} = u_1 = 9.42\ m/s \qquad \text{since discharge is radial at inlet}$$

$$\text{Inlet}\ Velocity\ of\ Whirl\ at\ Outlet = V_{w_2} = 0 \qquad \text{since discharge is radial at outlet}$$

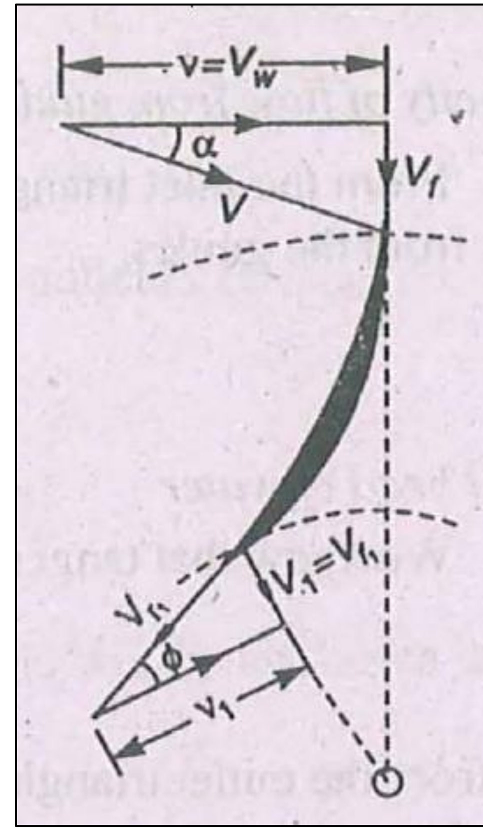
$$Work\ Done\ per\ kN = \frac{[V_{w_1} + V_{w_2}] \times u}{g} = \frac{[9.42 + 0] \times 9.42}{9.81} = \mathbf{9.05\ kN - ms}$$

Solution:

(b) *Power Developed by Turbine* (P),

$$Q = \pi D b V_f = \pi \times 1 \times 0.135 \times 3 = 1.272\ m^3/s$$

$$P = \gamma Q \frac{[V_{w_1} + V_{w_2}] \times u}{g} = 9.81 \times 9.05 = \mathbf{112.9\ kW}$$



(c) *Head of Water on Turbine* (H),

$$WD\ per\ kN\ of\ Water = H - \frac{V_1^2}{2g} = \frac{V_{f_1}^2}{2g}$$

$$9.05 = H - \frac{3^2}{2 \times 9.81}$$

$$H = 9.05 + 0.92 = \mathbf{9.97\ m}$$

(d) *Hydraulic Efficiency of Turbine* (η_h),

$$\eta_h = \frac{V_w u}{gH} = \frac{9.42 \times 9.42}{9.81 \times 9.97} = 0.907$$

$$\eta_h = \mathbf{90.7\%}$$

Design of Kaplan Turbine - Summary

1. Velocity of Wheel $= \mathrm{u}_1 = \mathrm{u}_2 = \frac{\pi D_m N}{60}$

   where, $Mean\ Diameter = D_m = \frac{D_t + D_h}{2}$

2. Velocities of Flow at Inlet and Outlet are Equal = $V_{f_1} = V_{f_2}$

3. Work Done per Sec = P = $\frac{\gamma a V_1}{g}[V_{w_1} + V_{w_2}] \times u$ = $\rho Q[V_{w_1} + V_{w_2}] \times u$

4. Discharge = $Q = \frac{\pi}{4}\left(D_t^2 - D_h^2\right) \times V_{f_1}$

5. Flow Ratio = $\frac{V_{f_1}}{\sqrt{2gH}}$

16.7.1 *A Kaplan turbine operating at 327.3 rpm develops a shaft power of 300 kW when the flow is 5.5 m^3/s. Find the torque transmitted from the flowing water to the shaft.*

SI

Eq. 16.10 (and 5.38): Shaft power $P = T\omega = 300$ kW = 300 000 N·m/s

$\omega = 327.3(2\pi/60) = 34.3$ rad/sec ; $T = 300\,000/\omega = 300\,000/34.3 = 8750$ N·m ◄

16.8.3 *A Kaplan turbine operating at 327.3 rpm develops a shaft power of 300 kW when the flow is 5.5 m^3/s. What is the net head on the runner if the turbine efficiency is 92%?*

SI

Eq. 16.8: Shaft power = $\gamma Q h \eta$; $300 = 9810(5.5)h(0.92)/1000$; $h = 6.04$ m ◄

Example:

A Kaplan turbine plant develops 3000 kW under a head of 10 m. While running at 60 rpm. The discharge is 350 m³/s. The tip diameter of the runner is 7.5 m and the hub to tip ratio is 0.43. Calculate the specific speed, turbine efficiency, the speed ratio

and flow ratio.

Solution:

P = 3000 kW H = 10 m N = 60 rpm Q = 350 m³/s

D_t = 7.5 m D_h/D_t = 0.43

Speed ratio is based on tip speed.

Hub diameter = D_h = 0.43 × 7.5 = 3.225 m

$$\text{Turbine efficiency} = \eta = \frac{P}{\gamma QH} = \frac{3000000}{09.81 \times 350 \times 10} = 0.8737 = 87.3\%$$

$$\text{Specific Speed} = N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{60 \times \sqrt{3000 \times 10^3}}{60 \times 10^{5/4}} = 308$$

$$\text{Runner Tip Speed} = u = \frac{\pi DN}{60} = \frac{\pi \times 7.5 \times 60}{60} = 23.56\ m/s$$

$$\text{Speed Ratio} = \frac{u}{\sqrt{2gH}} = \frac{23.56}{\sqrt{2 \times 9.81 \times 10}} = 1.68$$

$$\text{Flow Velocity} = V_{f_1} = \frac{Q}{\pi/_4 ({D_t}^2 - {D_h}^2)} = \frac{350 \times 4}{\pi (7.5^2 - 3.225^2)} = 9.72\ m/s$$

$$\text{Flow Ratio} = \frac{V_{f_1}}{\sqrt{2gH}} = \frac{9.72}{\sqrt{2 \times 9.81 \times 10}} = 0.69$$

Runner Dimensions

Runner diameter $= D = \frac{60\varphi}{\pi N}\sqrt{2gH} = \frac{60\times 2.2}{\pi\times 75}\times\sqrt{2\times 9.81\times 5} = 5.55\ m$

Hub diameter =

Water Flow

$\eta = \frac{P}{\gamma Q H}$ Assume, $\eta = 90\% = 0.90$

$$Q = \frac{P}{\eta\gamma H} = \frac{8000000}{0.90\times 9.81\times 5} = 181.22\ m^3/s$$

Comparison between Reaction and Impulse Turbines

SN	*Reaction turbine*	*Impulse turbine*
1	Only a fraction of the available hydraulic energy is converted into kinetic energy before the fluid enters the runner.	All the available hydraulic energy is converted into kinetic energy by a nozzle and it is the jet so produced which strikes the runner blades.
2.	Both pressure and velocity change as the fluid passes through the runner. Pressure at inlet is much higher than at the outlet.	It is the velocity of jet which changes, the pressure throughout remaining atmospheric.
3	The runner must be enclosed within a watertight casing (scroll casing).	Water-tight casing is not necessary. Casing has no hydraulic function to perform. It only serves to prevent splashing and guide water to the tail race
4.	Water is admitted over the entire circumference of the runner	Water is admitted only in the form of jets. . There may be one or more jets striking equal number of buckets simultaneously.
5.	Water completely fills at the passages between the blades and while flowing between inlet and outlet sections does work on the blades	The turbine does not run full and air has a free access to the buckets
6.	The turbine is connected to the tail race through a draft tube which is a gradually expanding passage. It may be installed above or below the tail race	The turbine is always installed above the tail race and there is no draft tube used
7.	The flow regulation is carried out by means of a guide-vane assembly. Other component parts are scroll casing, stay ring, runner and the draft tube	Flow regulation is done by means of a needle valve fitted into the nozzle.

Specific Speed of a Turbine

- After studying the behavior of a turbine, under unit conditions (i.e. unit power, unit speed, unit discharge), the next step is to know the characteristics of an imaginary turbine identical with the actual turbine, but reduced to such a size so as to develop a unit power under a unit head (i.e. 1 kW under a head of 1 meter).
- This imaginary turbine is called the specific turbine and its speed is known as specific speed.
- Thus the specific speed of a turbine may be defined as the speed of an imaginary turbine, identical in shape, geometrical dimensions, blade angles etc., with the given turbine, which will develop a unit power under a unit head.

  $$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

  N_s = specific speed of turbine

  N = operating speed of turbine (for highest possible efficiency)

  P = power produced by turbine

  H = net operating head of water
- For multi-jet impulse turbines the specific speed is based on the power produced per jet.

Specific Speed of a Turbine (Derivation)

$\eta_o = \frac{P}{\gamma Q H}$ $\quad$ $P = \eta_o \times \gamma Q H$ $\quad$ $\boldsymbol{P \propto Q.H}$

- The tangential speed of impeller is given by

$u = \frac{\pi D N}{60}$ $\quad$ $\boldsymbol{u \propto DN}$

As, $u = K_u\sqrt{2gH_m}$ $\quad$ $V_f = K_f\sqrt{2gH_m}$ $\quad$ $\boldsymbol{u \propto V_f \propto \sqrt{H_m}}$

$DN \propto \sqrt{H_m}$ $\quad$ $\boldsymbol{D \propto \frac{\sqrt{H_m}}{N}}$

- The discharge through impeller is given by, Q = Area x Velocity of flow

$Q = \pi D_1 B_1 V_{f_1} = \pi D_2 B_2 V_{f_2}$

$Q \propto D.B.V_f$ $\quad$ or $\quad$ $\boldsymbol{Q \propto D^2 V_f}$ $\quad$ as $B \propto D$

$P \propto \left(\frac{\sqrt{H_m}}{N}\right)^2 . \sqrt{H_m}. H_m$ $\quad$ $P \propto \frac{(H_m)^{\frac{5}{2}}}{N^2}$ $\quad$ $N^2 \propto \frac{(H_m)^{5/2}}{P}$

$N = K\frac{(H_m)^{5/4}}{\sqrt{P}}$ $\quad$ for H = 1, P = 1, $\quad$ $N_s = K$, $N = N_s\frac{(H_m)^{5/4}}{\sqrt{P}}$, $\quad$ $\boldsymbol{N_s = \frac{N\sqrt{P}}{H_m^{5/4}}}$

Governor for Turbines

- A Governor is a mechanism to regulate the speed of the shaft of a turbine.
- The turbine is directly coupled to the shaft of the electric generator, which is generating power/electricity.
- The power generated should have uniform rating of current and frequency which in turn depends on the speed of the shaft of the turbine.
- The constant speed of the generator is called synchronous speed, given by,

  $$N = \frac{60f}{p}$$

  f = frequency of the power generated (Hz) – usually (50 ± 0.15)

  p = number of pair of poles of generator
- Therefore, speed governors are provided on the turbine which control the rate of flow of water into the turbine corresponding to the load in order to keep the speed constant.
- The regulation system should be quick acting and accurate.
- On the other hand, it should be stable without too much sensitivity causing hunting.

Oil Pressure Governor for Turbines

- The main component parts of the oil pressure governor are:
  1. Actuator or Pendulum (Centrifugal governor)
  2. Distribution valve or control valve (Relay)
  3. Servomotor or Relay cylinder
  4. Oil Sump and Gear pump run by taking power from power shaft by belt drive
  5. Pipe system communicating with the control valve, servomotor and the sump
  6. Spear or needle

i. **Actuator or pendulum**

- The fly balls of centrifugal governor respond quickly to speed variation of turbine shaft and actuates the main lever.
- Modern turbines use electric tachogenerator (electronic means of frequency detection) which is extremely sensitive to frequency variation.

ii. **Relay valve**

- It is a piston type slide valve.
- The motion of the fly balls is transmitted to relay valve which distributes oil to either side of servomotor.

iii. **Servomotor**

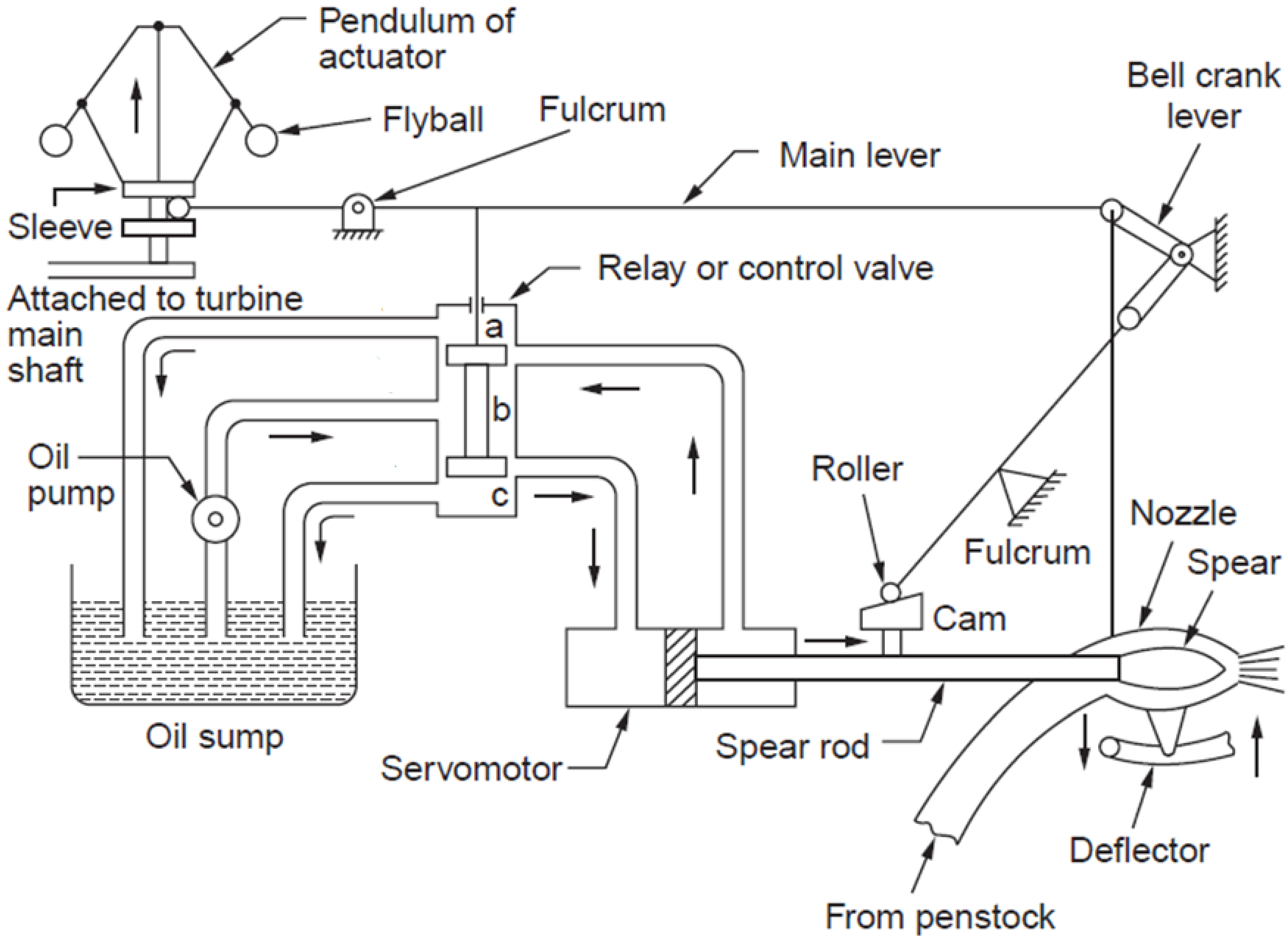
- Servomotor is a piston cylinder arrangement which transmits power to turbine guide mechanism.

Oil Pressure Governor for Turbines

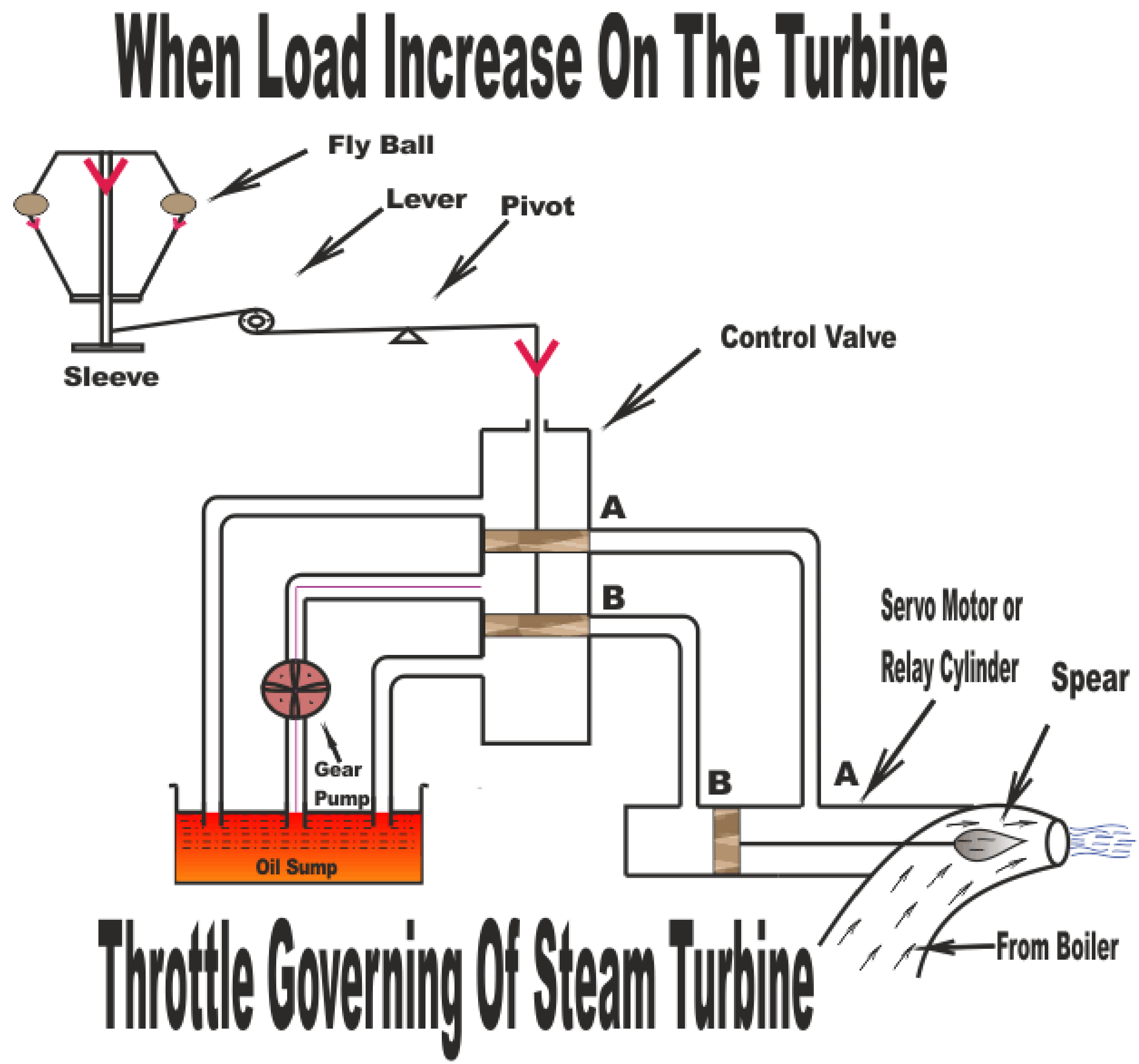
- In case of **Pelton turbine**, the guide mechanics is the spear rod (and deflector).
- In case of **Francis and Propeller turbines**, the guide mechanism is guide vanes.
- In case of **Kaplan turbine**, the power is transmitted to both guide vanes as well as runner vanes.

1. DOUBLE REGULATION SYSTEM OF PELTON TURBINE:

- The mechanical centrifugal governor is driven by the main shaft of turbine.
- The weights (fly balls) carry a sleeve which can move up and down the drive spindle.
- When the load decreases the turbine speeds up and the governor weights fly apart moving the sleeve up.
- The reverse happens when load increase on the turbine.
- The sleeve carries a lever which moves the control value in the relay cylinder.
- Oil under pressure is maintained at the central position of the relay cylinder.
- The top and bottom are connected on one side to the power cylinder and to the sump on the other side.
- Under steady load conditions the value rod closes both inlets to the power cylinder and the spear remains at a constant position.



Governing of Pelton Turbine


When Load Increase On The Turbine
Fly Ball
Lever
Pivot
Sleeve
Control Valve
A
B
Servo Motor or
Relay Cylinder
Spear
Gear
Pump
Oil Sump
B
A
From Boiler
Throttle Governing Of Steam Turbine

Oil Pressure Governor for Turbines

1. **DOUBLE REGULATION SYSTEM OF PELTON TURBINE:**

- When the turbine speeds up, the valve rod moves down connecting the oil supply to the left side of the power cylinder.
- The piston in the power cylinder mover to reduce the flow.
- At the some time the right side of the power cylinder is connected to the sump so that the oil in the right side can flow out.
- The opposite movement takes place when the turbine speed reduces.
- As sudden cut off is not desirable, a deflector is actuated by suitable mechanism to deflect the flow when sudden and rapid increase in speed takes place.
- When there is sudden fall of load, the spear moves rapidly to close the nozzle.
- There can be water hammer in the penstock due to sudden stoppage of water flow.
- In order to avoid sudden stoppage of water flow and water hammer, the spear is designed to allow some water flow during sudden load fall.
- The bell crank lever moves down and deflector moves up.
- The water is deflected and not allowed to strike the buckets.
- The deflected water goes as waste to the tail race.

Oil Pressure Governor for Turbines

2. GOVERNING OF REACTION TURBINE:

- The guide blades are pivoted and connected to regulating ring which in turn is connected to the piston rod of servomotor of oil pressure governor through regulating rods and regulating lever.
- When the servomotor moves to the right, the guide blades are rotated to decrease or close the water supply to the runner.
- The reverse happens when the servomotor moves to the right under decrease of generator load and increase of turbine speed.
- The guide blades are rotated to increase the water flow area to the turbine runner.
- In case of Kaplan turbine both guide vanes and runner blades are adjusted simultaneously in case of load fluctuation.
- There is an additional servomotor and a control valve interconnected to those of guide vanes so that for a given guide valve opening, there is a definite runner blade inclination.
- The runner blade operating mechanism is housed in the hub of the runner which is hollow.
- When the load suddenly falls on reaction turbine, a pressure relief valve is operated by speed governor.
- A portion of water from spiral casing is allowed to flow directly through relief valve to the tail race without striking the turbine runner.
- The pressure relief valve performs the function of protection system to avoid water hammer effects during sudden load drop.

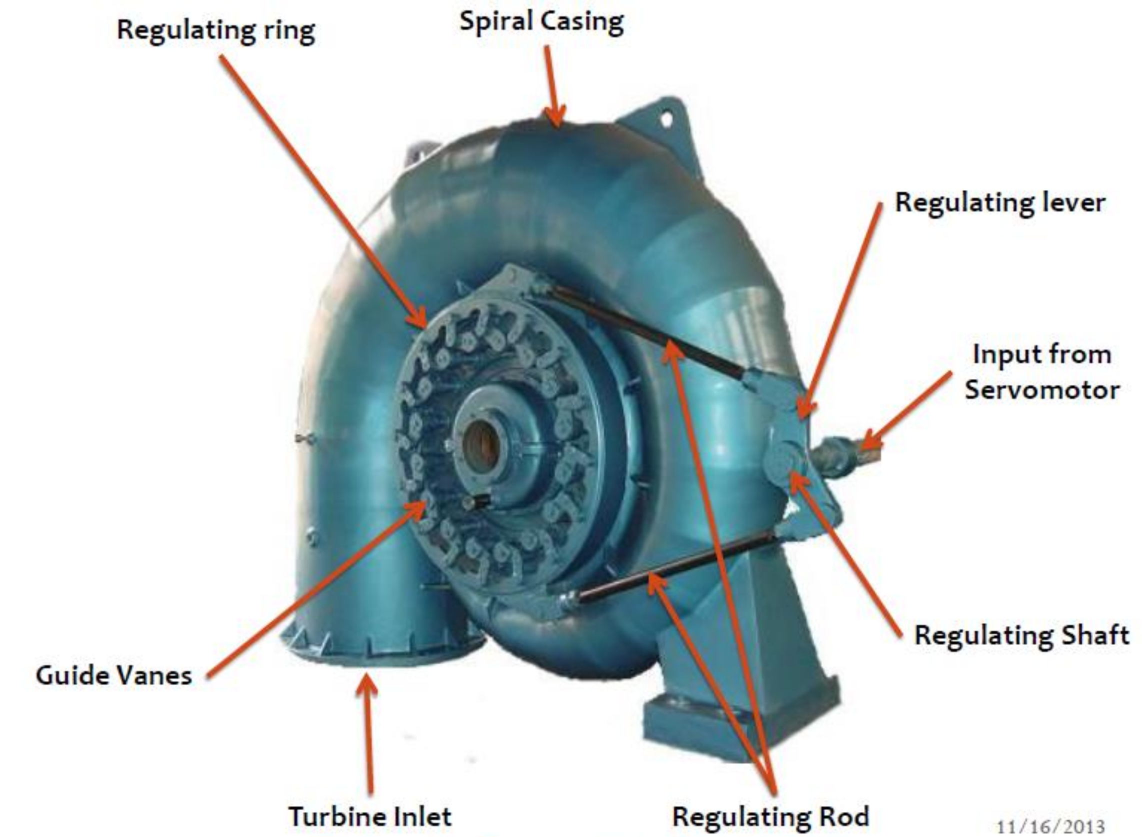


Fig.(2) Components of Governor mechanism of Francis Water Turbine

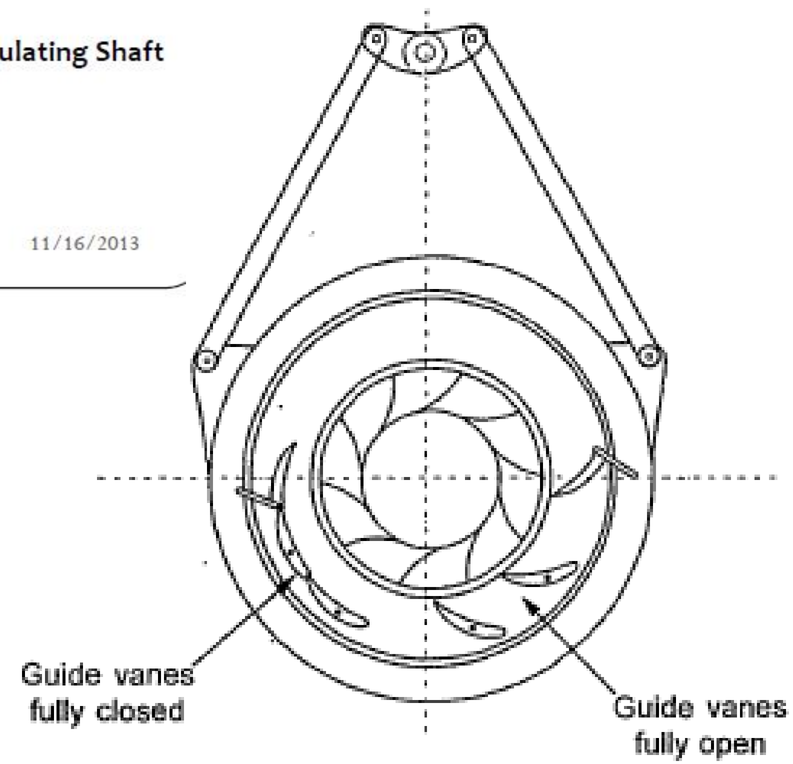
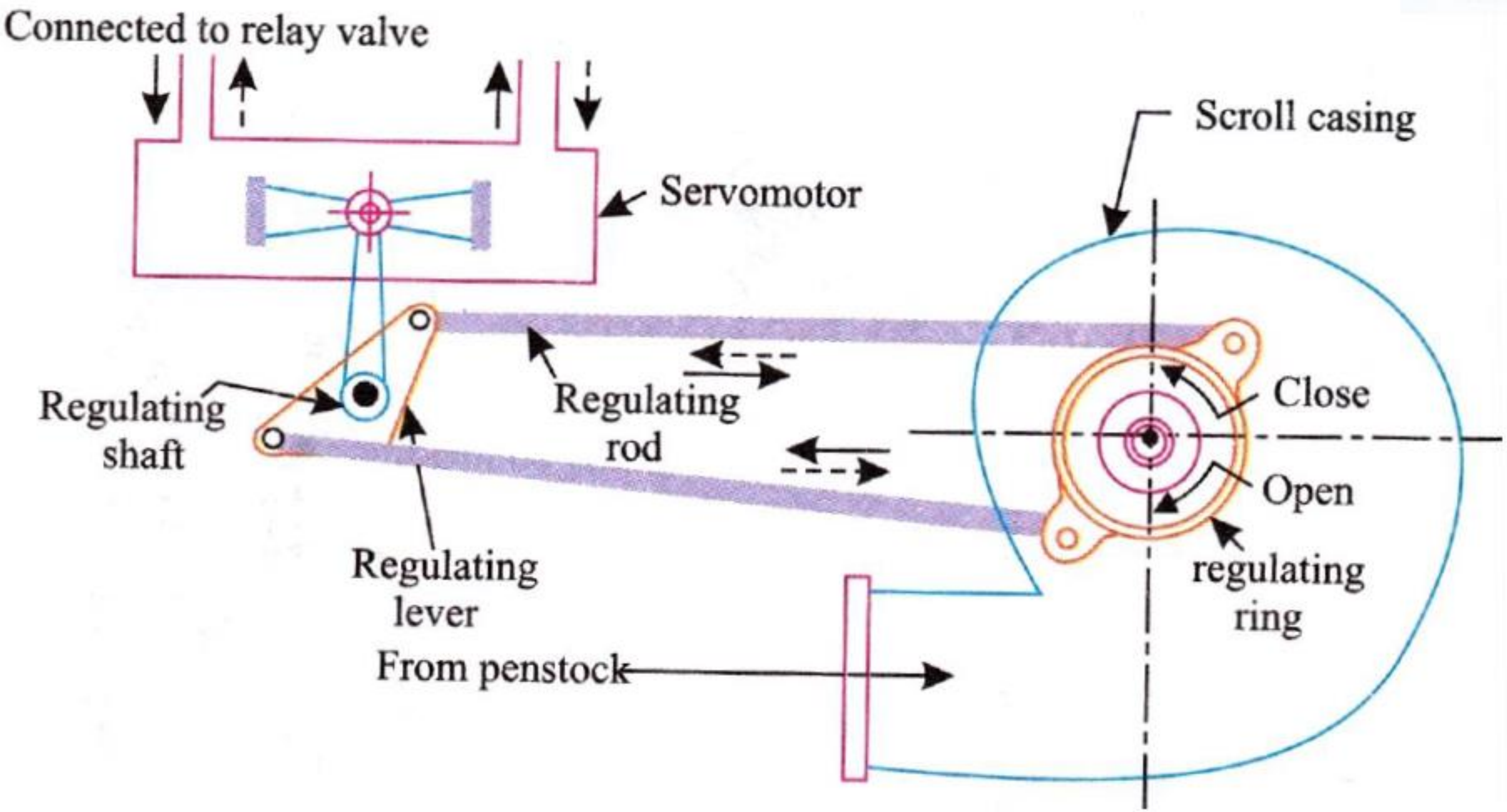


Fig. 6.10 Position of guide vanes during open and close position.



Governing of Francis Turbine

Water Hammer

- If water is flowing along a long pipe and is suddenly brought to rest by the closing of a valve, or by any similar cause, there will be a sudden rise in pressure due to the momentum of the water being destroyed.
- This will cause a wave of high pressure to be transmitted along the pipe with a velocity equal to the sound wave, which may setup noises known as Knocking.
- The magnitude of this pressure will depend on
  - i. The mean pipe flow velocity
  - ii. The length of the pipe
  - iii. The time taken to close the valve and
  - iv. The elastic properties of the pipe material and that of water.
- This sudden rise in pressure in the pipe due to the stoppage of the flow generating a high pressure wave, which will have a hammering effect on the walls of the pipe, is known as Water Hammer.
- The cases that can be studied under this are:
  1. Gradual closure of valve
  2. Sudden closure of valve
     a) Pipe is rigid
     b) Pipe is elastic

Water Hammer

❑ Critical Time:

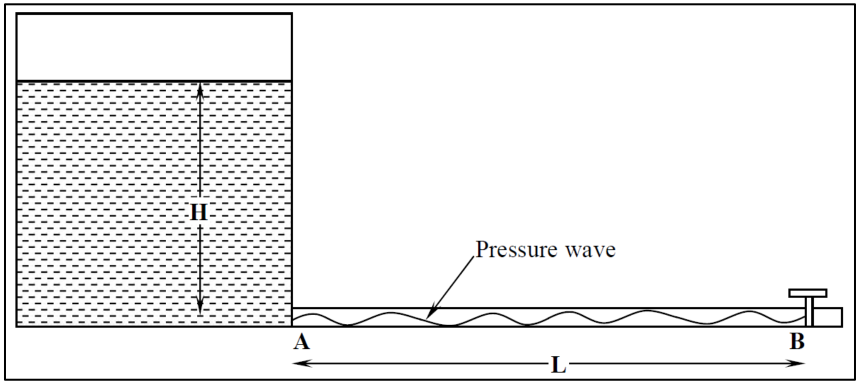
- It is defined as the time required for the pressure wave generated due to closure of valve to travel once from the point of origin to reservoir over the length of pipe and back to the point of origination.
- If **T** is the time required by the pressure wave to travel once up and down the pipe and **C** is the velocity of the pressure wave equal to the velocity of sound wave in water also called as Celerity, then from Newton's law, we have

Distance traveled = Average velocity x time

2L = C x T $\quad\quad T = \frac{2L}{C}$

- If t is the actual time of closure, T is the critical time, then

1. If, $t > T = \frac{2L}{C}$ then it is referred to as gradual closure, and
2. If, $t < T = \frac{2L}{C}$ then it is referred to as sudden closure



Water Hammer

1. Instantaneous Rise in Pressure in a Pipe Running Full due to Gradual Closure of Valve

- Consider a pipe AB of length **L** connected to a tank at A and a valve at B with water flowing in it as shown in the previous Figure.
- Let **V** be the mean flow velocity and **a** is the flow cross-sectional area, **p** the instantaneous rise in pressure due to gradual closure of valve and **t** be the actual time of closure of valve.
- From Newton's second law of motion, the retarding force generated against the flow direction is given by the rate of change momentum of the liquid along the direction of the force.

Retardation of water = Change in velocity / Time = $\frac{(V-0)}{t} = \frac{V}{t}$

Retarding force = Mass of water x Retardation = $\rho a L \frac{V}{t}$

The force generated due to pressure wave = Pressure intensity x area = p_i x a

Therefore, $p_i a = \rho a L \frac{V}{t}$ or $p_i = \rho L \frac{V}{t}$

Instantaneous rise in pressure head = $\mathrm{H} = \frac{p_i}{\rho g} = \frac{\rho L V}{\rho g t} = \frac{L V}{g t}$

- The above equation is valid only for rigid pipes with incompressible fluids flowing through it.

Water Hammer

2. *(a) Instantaneous Rise in Pressure in a Pipe Running Full due to Sudden Closure of Valve When the Pipe is Rigid*

- When the valve provided at the downstream end is closed suddenly and the pipe is rigid, then the converted pressure energy from the kinetic energy due to closure is to be absorbed by the fluid due to its compressibility only.

pressure energy converted from kinetic energy = pressure energy absorbed by water due to its compressibility

i.e. $E_k = E_w$

- Consider the pipe AB of length **L** and cross-sectional area a in which water of mass density **ρ**, weight density γ and bulk density **K** is flowing with a mean velocity **V** be suddenly stopped due to closure of valve provided at B.
- The kinetic energy of flowing water before closure of valve will be converted to strain energy, when the effect of friction and elasticity of pipe material are ignored.

Loss of kinetic energy, E_k = ½ x mass of water x V^2

Loss of kinetic energy, $E_k = \frac{1}{2}\rho aLV^2$

Gain in strain energy, $E_w = \frac{1}{2}\left(\frac{p_i^2}{K}\right) \times volume = \frac{1}{2}\left(\frac{p_i^2}{K}\right) \times aL$

therefore, $\frac{1}{2}\rho aLV^2 = \frac{1}{2}\left(\frac{p_i^2}{K}\right)aL$ $\quad \rho V^2 = \frac{p_i^2}{K}$ $\quad p_i^2 = \rho V^2 K$

$p_i = V\sqrt{\rho K}$ $\quad$ but, $C = \sqrt{\frac{K}{P}}$ $\quad$ therefore, $\boldsymbol{p_i = \rho V C}$

Water Hammer

2. *(b) Instantaneous Rise in Pressure in a Pipe Running Full due to Sudden Closure of Valve When the Pipe is Elastic*

- When the valve provided at the downstream end is closed suddenly and the pipe is elastic, then the converted pressure energy from the kinetic energy due to the valve closure is to be absorbed by both the fluid due to its compressibility and the elasticity of the pipe.

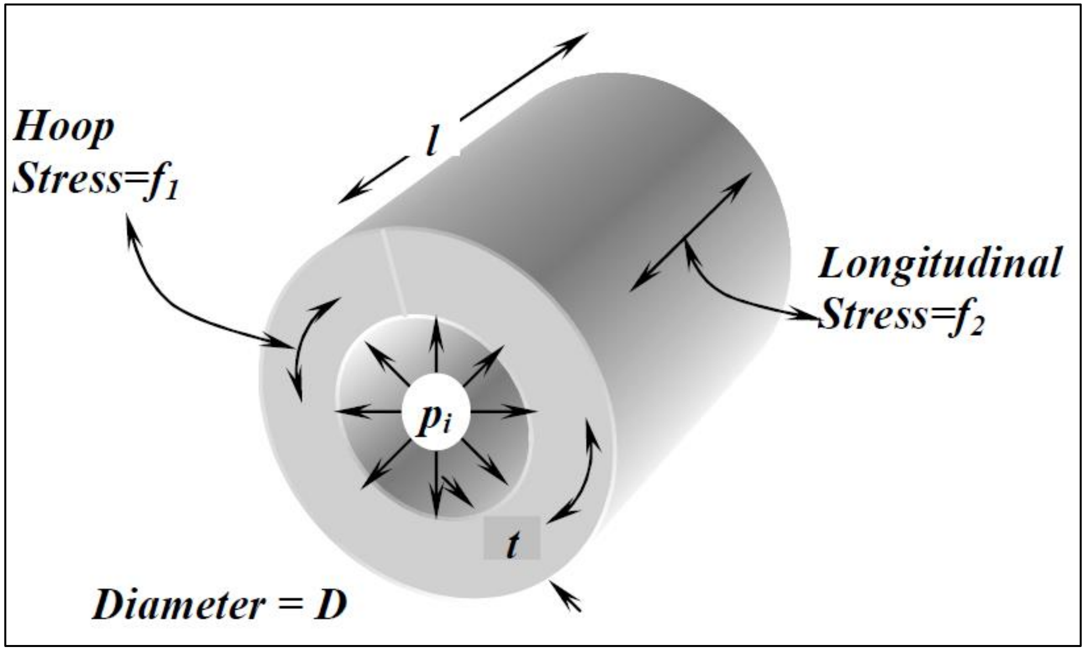
$$\left\{\begin{matrix}\text{pressuer energy converted}\\ \text{from kinetic energy}\end{matrix}\right\} = \left\{\begin{matrix}\text{pressure energy absorbed by}\\ \text{water due to its compressibility}\end{matrix}\right\} + \left\{\begin{matrix}\text{pressure energy absorbed by the}\\ \text{plastic pipe due to its expansion}\end{matrix}\right\}$$

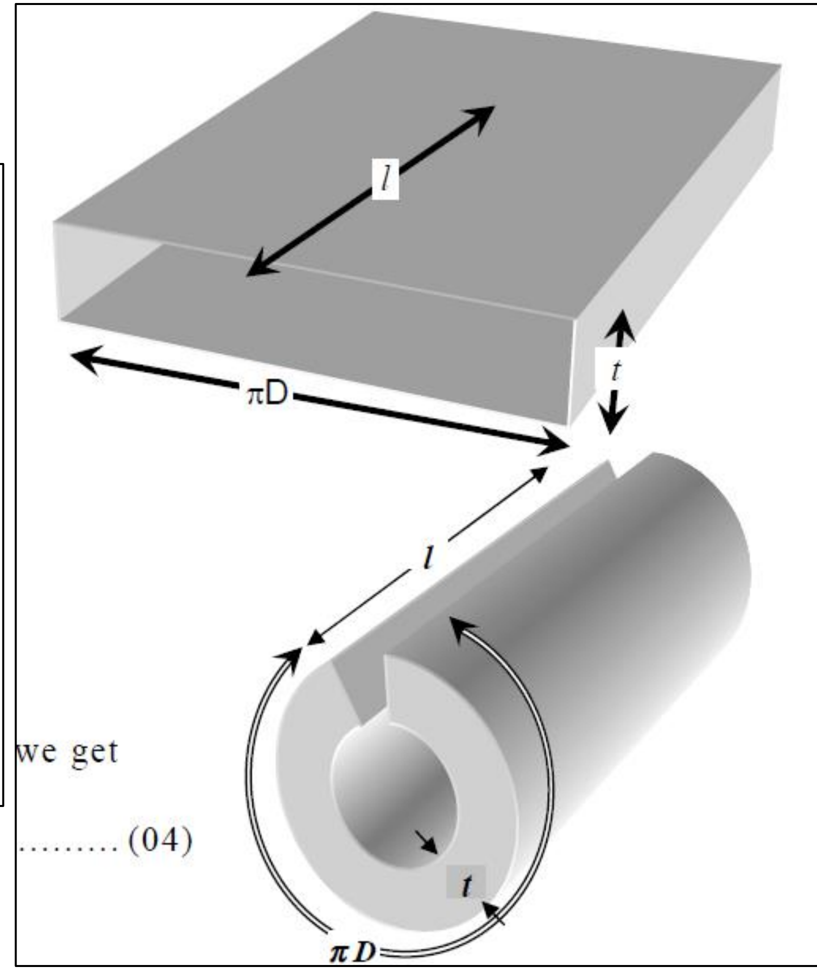
i.e. $E_k = E_w + E_p$

- Computation of $\mathbf{E_p}$ can be done by simulating the situation to the thick cylinder subjected to internal fluid pressure.
- Let **t** be the thickness of the elastic pipe wall and assume that it is small compared to its diameter **D**.
- Let $\mathbf{f_1}$ be the hoop or circumferential and stress $\mathbf{f_2}$ be the longitudinal stress as shown in Figure in next slide.
- Let the Young's modulus of the pipe material be **E** and Poison's ratio **1/m**.
- Let the instantaneous fluid pressure be $\mathbf{p_i}$.
- From the knowledge of strength of materials, we can write that

$$f_1 = \frac{p_i D}{2t} \quad \text{and} \quad f_2 = \frac{p_i D}{4t}$$

hence, $f_1 = 2f_2$


Hoop
Stress=f_1
l
Longitudinal
Stress=f_2
p_i
t
Diameter = D


l
πD
t
l
t
πD

we get

.... (04)

Water Hammer

- Further, the strain energy stored in pipe per unit volume is given by

$$\frac{E_p}{V_1} = \frac{1}{2E}\left[f_1^2 + f_2^2 - \frac{2f_1f_2}{m}\right]$$ since, $f_1 = 2f_2$

$$\frac{E_p}{V_1} = \frac{1}{2E}\left[4f_2^2 + f_2^2 - \frac{4f_2^2}{m}\right]$$

$$\frac{E_p}{V_1} = \frac{f_2^2}{2E}\left[5 - \frac{4}{m}\right]$$ substituting for f_2 and $V_1 = \pi Dtl$ (from diagram)

$$E_p = \frac{p_i^2 D^2}{16t^2} \times \frac{1}{2E}\left[5 - \frac{4}{m}\right]\pi Dtl$$

Since, $E_k = \frac{1}{2}\rho aLV^2$ and $E_w = \frac{1}{2}\left(\frac{p_i^2}{K}\right)aL$

$$\frac{1}{2}\rho aLV^2 = \frac{1}{2}\left(\frac{p_i^2}{K}\right)aL + \frac{p_i^2 D^2}{16t^2} \times \frac{1}{2E}\left[5 - \frac{4}{m}\right]\pi Dtl$$ as, $E_k = E_w + E_p$

$$\frac{1}{2}\rho aLV^2 = \frac{1}{2}L\left[\left(\frac{p_i^2}{K}\right)a + \frac{p_i^2 D}{4t} \times \frac{1}{E}\left(5 - \frac{4}{m}\right)\frac{\pi D^2}{4}\right]$$ as, $a = \frac{\pi D^2}{4}$

$$\rho V^2 = \left(\frac{p_i^2}{K}\right) + \frac{p_i^2 D}{4t} \times \frac{1}{E}\left[5 - \frac{4}{m}\right] = p_i^2\left[\frac{1}{K} + \frac{D}{4tE}\left(5 - \frac{4}{m}\right)\right]$$

$$p_i^2 = \frac{\rho V^2}{\left[\frac{1}{K} + \frac{D}{4tE}\left(5 - \frac{4}{m}\right)\right]} \qquad p_i = V\sqrt{\frac{\rho}{\left[\frac{1}{K} + \frac{D}{4tE}\left(5 - \frac{4}{m}\right)\right]}} = V\sqrt{\frac{\rho}{\left[\frac{1}{K} + \frac{D}{4tE}\left(\frac{5}{4} - \frac{4}{m}\right)\right]}}$$

- The above expression gives the instantaneous rise in pressure in an elastic pipe due to sudden closure of Valve.
- If the Poison's ratio is not given, it can be assumed as ¼, then above Eq. reduces to

$$p_i = V\sqrt{\frac{\rho}{\left[\frac{1}{K} + \frac{D}{4tE}\right]}}$$