CE-230: Hydraulics and Hydraulic Machinery

Chapter 4

Flow Measurement

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Rate of Flow or Discharge (Q)

- It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel.

- For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second.

- For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section. Thus,

  i. For liquids the units of Q are m^3/s or liters/s
  ii. For gases the units of Q is kgf/s or Newton/s

- Consider a liquid flowing through a pipe in which

  A = Cross-sectional area of pipe

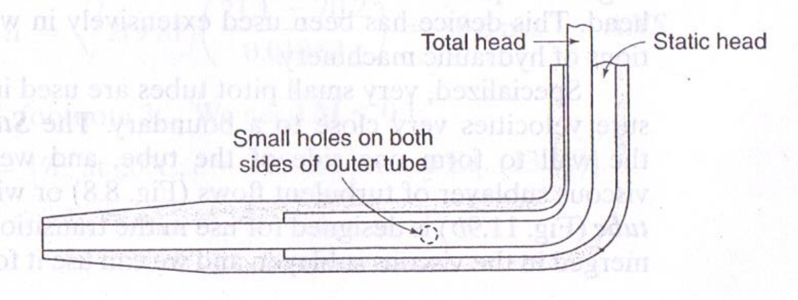
  V = Average velocity of fluid across the section

  Then discharge,

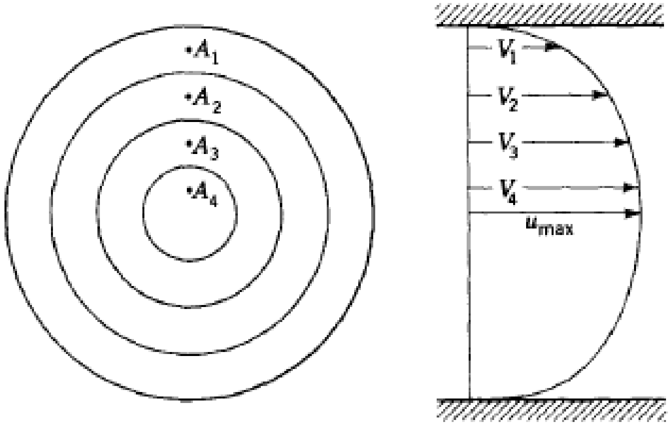
  $Q = A \times V$

Measurement of Discharge

- There are various ways of measuring discharge. In a pipe, for example, the velocity may be determined at various radii using a pitot-static tube (shown below) or a pitot tube in combination with a wall piezometer.



- The cross section of a pipe may then be considered as a series of concentric rings, each with a known velocity as shown below.

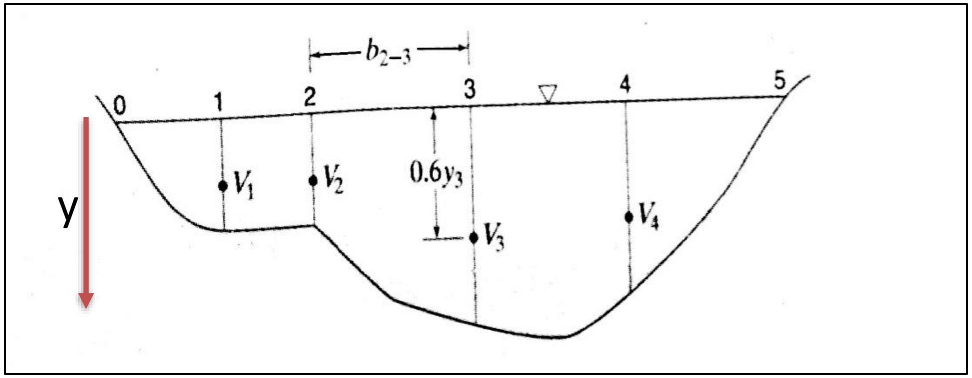


- The flow through these rings is summed up, as given below to determine the total flow rate (continuity equation).

$$Q = \sum A_i V_i = A_1 V_1 + A_2 V_2 + \cdots$$

Measurement of Discharge

- To determine the flow in a river or stream, the stream is divided into a number of convenient sections, and the average velocity in each section is measured, as shown below (area-velocity method).



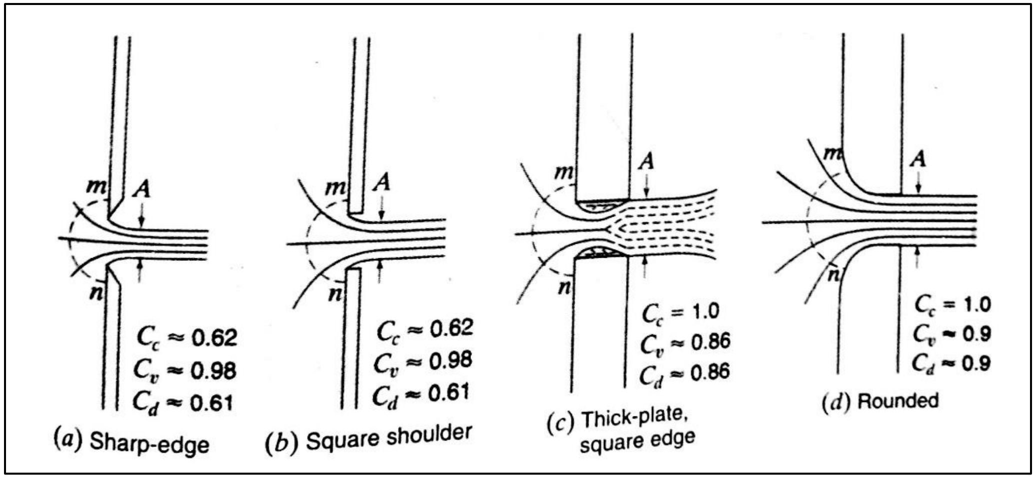
- A pitot tube could be used for such measurements, but a current meter is more commonly used.
- It has been found that the average velocity occurs at about 0.6 x depth, so the velocity is generally measured at that level.

$$Q_{2-3} = A \times V = \left[\left(\frac{y_2+y_3}{2}\right) b_{2-3}\right]\left(\frac{V_2+V_3}{2}\right)$$

- Another widely used method is to take the average of the velocities at 0.2 x depth and 0.8 x depth.
- A crude technique for estimating the average velocity of flow in a river or stream is to observe the velocity at which a float will travel down a stream [V = S/t].
- To get good results the reach of stream should be straight and uniform with a minimum of surface disturbances.
- The average velocity of flow **V** will generally be about (0.85 ± 0.05) times the float velocity.

Orifices

- Orifice is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side or at the bottom of a tank/reservoir, or in a plate normal to the axis of a pipe, the plate being either at the end of the pipe or in some intermediate location (orifice meter) through which a fluid is flowing.
- An orifice is characterized by the fact that the thickness of the wall is very small relative to the size of the opening.
- A **standard orifice** is the one with sharp edge, Fig (a), or an absolutely square shoulder, Fig (b), so that there is only a line contact with the fluid.



- Those shown in Figs (c) and (d) are not standard because the flow through them is affected by the thickness of the plate, the roughness of the surface, and for (d) the radius of curvature.
- Hence such orifices should be calibrated if high accuracy is desired.

Orifices

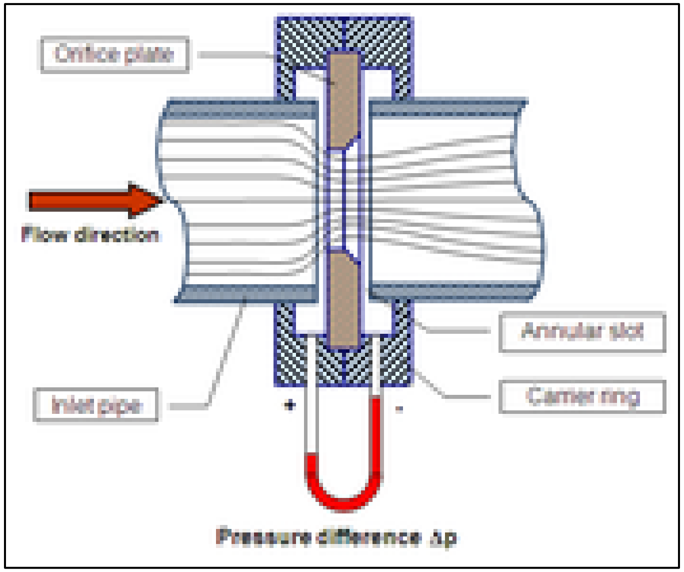
- The orifices are used for the measurement of discharge.
- The most important types of orifices are:

i. According to size:
   i. Small orifice
   ii. Large orifice

ii. According to shape:
   i. Circular orifice
   ii. Rectangular orifice
   iii. Square orifice
   iv. Triangular orifice

iii. According to shape of the upstream edge
   i. Sharp-edged orifice
   ii. Bell-mouthed

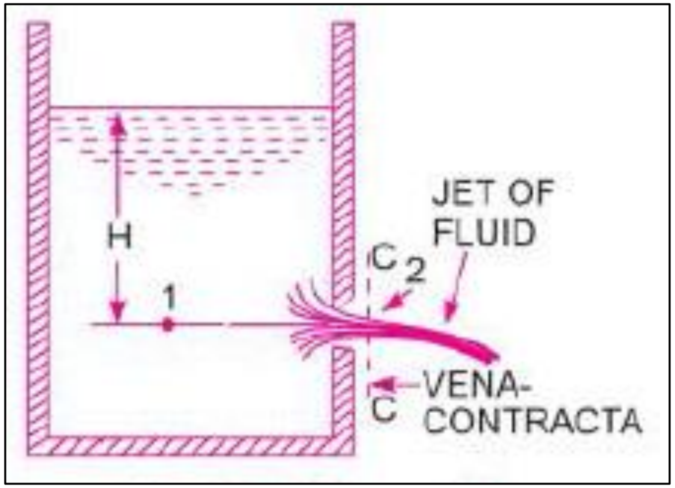
iv. According to nature of discharge
   i. Free discharging orifice
   ii. Drowned or submerged orifice
      a) Fully submerged
      b) Partially submerged



Important Terms Related to Orifices

1. Jet of Water:

- A jet is a stream issuing from an orifice, nozzle, or tube.
- It is not enclosed by solid boundary walls but is surrounded by a fluid whose velocity is less than its own.
- The two fluids may be different or they may be of the same kind.
- A **free jet** is a stream of liquid surrounded by a gas and is therefore directly under the influence of gravity.
- A **submerged jet** is a stream of any fluid surrounded by a fluid of the same type, that is, a gas jet discharging into a gas or a liquid jet discharging into a liquid.
- A submerged jet is buoyed up by the surrounding fluid and is not directly under the action of gravity.



Important Terms Related to Orifices

2. Vena Contracta:

- The jet of water, after leaving the orifice, gets contracted due to loss of energy during convergence towards orifice.
- The maximum contraction takes place at a section (C-C) slightly on the downstream side of the orifice, where the streamlines are straight and parallel to each other and perpendicular to the plane of orifice.
- Such a section of minimum area is known as vena contracta.
- Commonly this section is about $0.5d_2$ from the upstream edge of the opening, where d_2 is the diameter of the orifice.
- Beyond the vena contracta the jet commonly diverges because of frictional effects and is attracted in the downward direction by the gravity.

Consider two points 1 and 2:

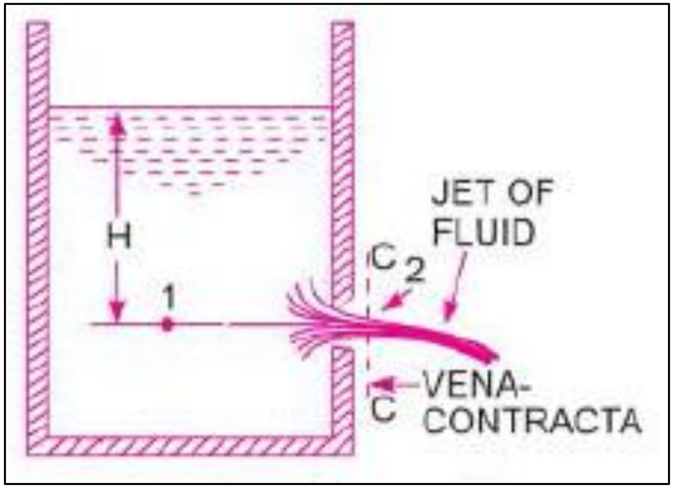
Applying Bernoulli's equation at 1 and 2:

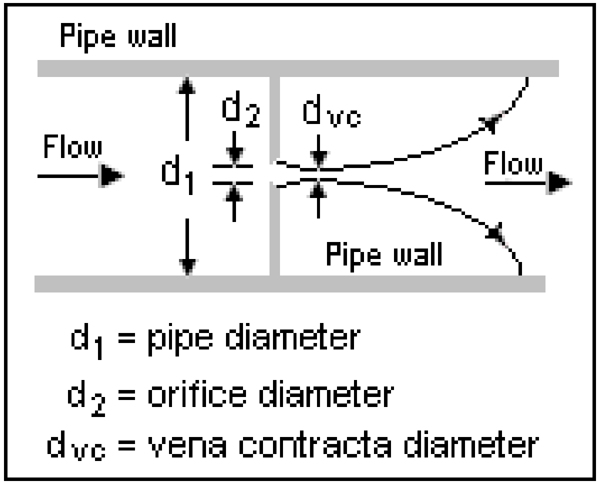
$$\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2$$

$$z_1=z_2 \text{ and } \frac{p_1}{\gamma}=H, \quad \frac{p_2}{\gamma}=0 \ (\text{P}_{atm})$$

$$H+0=0+\frac{v_2^2}{2g} \quad (\text{v}_1 \text{ is v.v.small w.r.to v}_2)$$

$$v_2=\sqrt{2gH} \quad \text{(theoretical velocity)}$$





Hydraulic Coefficients

1. Coefficient of Contraction (C_c):

- The ratio of area of the jet at vena contracta, to the area of the orifice is known as coefficient of contraction. Mathematically coefficient of contraction,

  C_c = Area of jet at vena contracta / Area of orifice

- The value of coefficient of contraction varies from 0.61 – 0.69 with the available head of the liquid, size and shape of the orifice. An average value of C_c is about 0.64.

2. Coefficient of Velocity (C_v):

- The ratio of actual velocity of the jet, at vena contracta, to the theoretical velocity is known as coefficient of velocity. Mathematically coefficient of velocity,

  C_v = Actual velocity of the jet at vena contracta / theoretical velocity of the jet

- The difference between the velocities is due to friction of the orifice.
- The value of coefficient of velocity varies slightly with the different shapes of the edges of the orifice. i.e., it is very small for sharp-edged orifices.
- An average value of C_v is about 0.98 (range: 0.95 – 0.99).
- The theoretical velocity of jet at vena contracta is given by $V = \sqrt{2gh}$ where h is the head of water at vena contracta.

Hydraulic Coefficients

3. Coefficient of Discharge (C_d):

- The ratio of actual discharge through an orifice to the theoretical discharge is known as coefficient of discharge. Mathematically coefficient of discharge,

  C_d = Actual discharge / Theoretical discharge

$$C_d = \frac{Q}{Q_{th}}$$

  C_d = (Actual area x Actual velocity) / (Theoretical area x theoretical velocity)

  $C_d = C_c \times C_v$

- An average of coefficient of discharge varies from 0.61 to 0.65.

4. Coefficient of Resistance (C_r):

- The ratio of loss of head in the orifice to the head of water available at the exit of the orifice is known as coefficient of resistance. Mathematically coefficient of resistance,

  C_r = Loss of the head in the orifice / Head of water

- The loss of head in the orifice takes place because the walls of orifice offer some resistance to the liquid as it comes out.
- It is generally neglected while solving numerical problems.

Discharge through a Small Rectangular Orifice

- An orifice is considered to be small if the head of water is more than 5 times the depth of the orifice.
- The actual discharge through such an orifice is given by

  $Q = C_d \, a \, V$

  $Q = C_d \, a \sqrt{2gh}$ as $h = V^2/2g$ (kinetic head) or $V = \sqrt{2gh}$

  $Q = C_d \, (L \times d) \sqrt{2gh}$ as $a = L \times d$

  where,

  C_d = Coefficient of discharge for the orifice,

  a = Cross-sectional area of the orifice,

  h = Height of the liquid above the center of the orifice,

  L = Length of the orifice, and

  d = Depth of the orifice

- In case of small orifice, the velocity in the entire cross-section of the jet is considered to be constant.

Discharge through a Large Rectangular Orifice

- In case of a large rectangular orifice the velocity of various liquid particles will not be constant, because there is considerable variation of head along the height of orifice.
- Consider a large rectangular orifice in one side of tank discharging freely into atmosphere under a constant head.
- Consider an elementary horizontal strip of depth 'dh' at a depth of 'h' below the free surface of the liquid in the tank.

Area of strip = L x dh

- Actual discharge through this strip is, dQ = C_d x Area of strip x Velocity

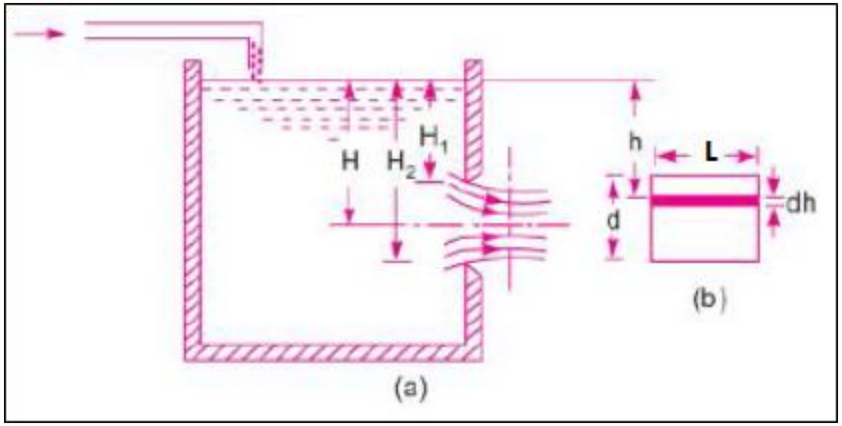
dQ = C_d x (L x dh) x $\sqrt{2gh}$ = C_d L x $\sqrt{2gh}$ dh

- By integrating the above equation between the limits H_1 and H_2, the total discharge through the whole orifice is obtained:

$$Q = \int_{H_1}^{H_2} C_d \times L \times \sqrt{2gh}\, dh = C_d \times L \times \sqrt{2g} \int_{H_1}^{H_2} \sqrt{h}dh$$

$$Q = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_{H_1}^{H_2}$$

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \left[H_2^{3/2} - H_1^{3/2} \right]$$



Example:

The head of water over an orifice of diameter 40 mm is 10 m. Find the actual discharge and actual velocity of the jet at vena contracta. Take C_d = 0.60 and C_v = 0.98.

Solution:

H = 10 m

d = 40 mm = 0.04 m

$$a = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.04)^2 = 0.001256\, m^2$$

C_d = 0.60

C_v = 0.98

$$C_d = \frac{Actual\ Discharge}{Theoretical\ Discharge} = \frac{Q}{Q_{th}}$$

$Q_{th} = a \times V_{th} = a \times \sqrt{2gH} = 0.001256 \times \sqrt{2 \times 9.81 \times 10} = 0.01758\, m^3/s$

$Q = C_d \times Q_{th}$ = 0.60 x 0.01758

Q = 0.01054 m³/s

$$C_v = \frac{Actual\, Velocity}{Theoretical\, Velcity} = \frac{V}{V_{th}}$$

$V = C_v \times V_{th}$ = 0.98 x 14

V = 13.72 m/s

Example:

The head of water over the center of an orifice of diameter 20 mm is 1 m. The actual discharge through the orifice is 0.85 lit/s. Find the coefficient of discharge.

Solution:

H = 1 m

Q = 0.85 lit/s = 0.00085 m^3/s

d = 20 mm = 0.02 m

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.02)^2 = 0.000314\, m^2$$

$$V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 1} = 4.429\, m/s$$

$$Q_{th} = a \times V_{th} = 0.000314 \times 4.429 = 0.00139\, m^3/s$$

$$C_d = \frac{Actual\ Discharge}{Theoretical\ Discharge} = \frac{Q}{Q_{th}} = \frac{0.00085}{0.00139}$$

C_d = 0.61

A jet discharges 5.19 L/s from a 35-mm-diameter orifice in a vertical plane under a head of 4 m. The jet center line passes through the point 4.28 m horizontally from the vena contracta and 1.2 m below the center of the orifice. Find the coefficients of (a) discharge, (b) velocity, and (c) contraction.

Eq. 5.45: $V = x\sqrt{g/2z} = 4.28\sqrt{9.81/(2 \times 1.2)} = 8.65$ m/s $= C_v\sqrt{2gh} = C_v\sqrt{2(9.81)4}$; $C_v = 0.977$ ◄

$A_o = (\pi/4)(0.035)^2 = 0.000962\ m^2$; Eq. 11.9: $0.00519 = C_d(0.000\,962)\sqrt{2(9.81)4}$; $C_d = 0.609$ ◄

From Eq. 11.11: $C_c = C_d/C_v = 0.609/0.977 = 0.623$ ◄

Alternatively: $A = Q/V = 0.005\,19/8.65 = 0.000\,600$; $C_c = A/A_o = 0.000\,600/0.000\,962 = 0.623$ ◄

Example:

Find the discharge through a rectangular orifice 2 m wide and 1.5 m deep fitted to a water tank. The water level in the tank is 3 m above the top edge of the orifice.

Take $C_d = 0.62$

Solution:

b = 2 m

d = 1.5 m

Height of water above top of the orifice = H_1 = 3 m

Height of water above top of the orifice = $H_2 = H_1 + d = 3 + 1.5 = 4.5$ m

$C_d = 0.62$

$$Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g}\left(H_2^{3/2} - H_1^{3/2}\right)$$

$$Q = \frac{2}{3} \times 0.63 \times 2 \times \sqrt{2 \times 9.81}\left(4.5^{3/2} - 3^{3/2}\right)$$

$$\boldsymbol{Q = 15.917\ m^3/s}$$

Notches

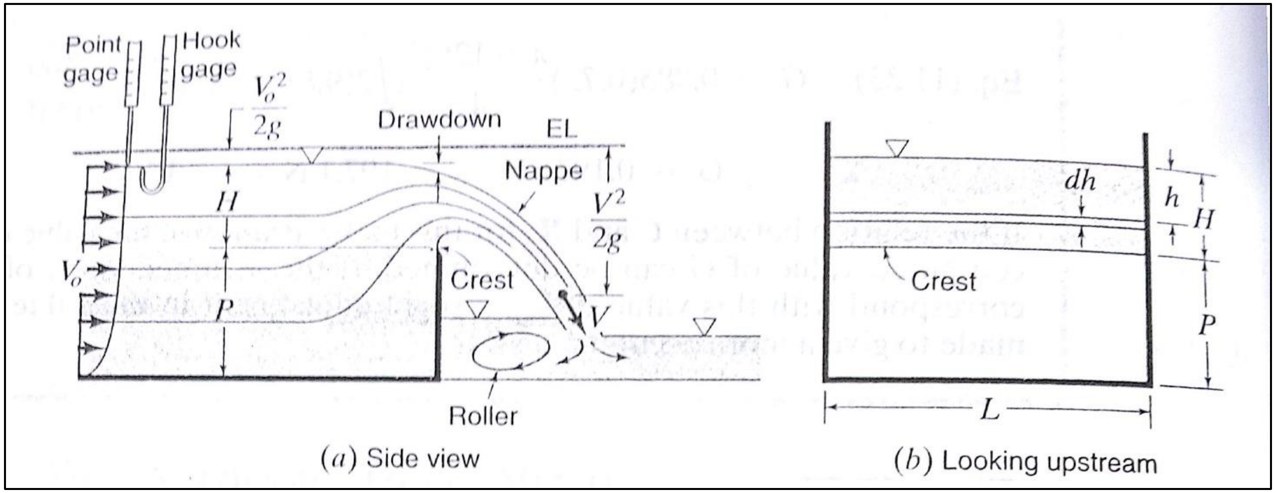
- A **notch** may be defined as an opening in one side of a tank or a reservoir, like a large orifice with the upstream liquid level below the top edge of the opening.
- Since the top edge of the notch is above the liquid level and serves no purpose, therefore a notch may have only the bottom edge and sides.
- The bottom edge over which the liquid flows is known as **sill** or **crest** of the notch and the sheet of liquid flowing over the notch is known as **nappe** or **vein**.
- A notch is usually made of a metallic plate and is used to measure the discharge of liquids.
- The notches are classified as:

1. According to the shape of the opening:
   a) Rectangular notch
   b) Triangular (V) notch
   c) Trapezoidal (Cipolletti) notch
   d) Stepped notch
2. According to the effect of the sides on the nappe:
   a) Notch with end contraction
   b) Notch without end contraction (suppressed notch)

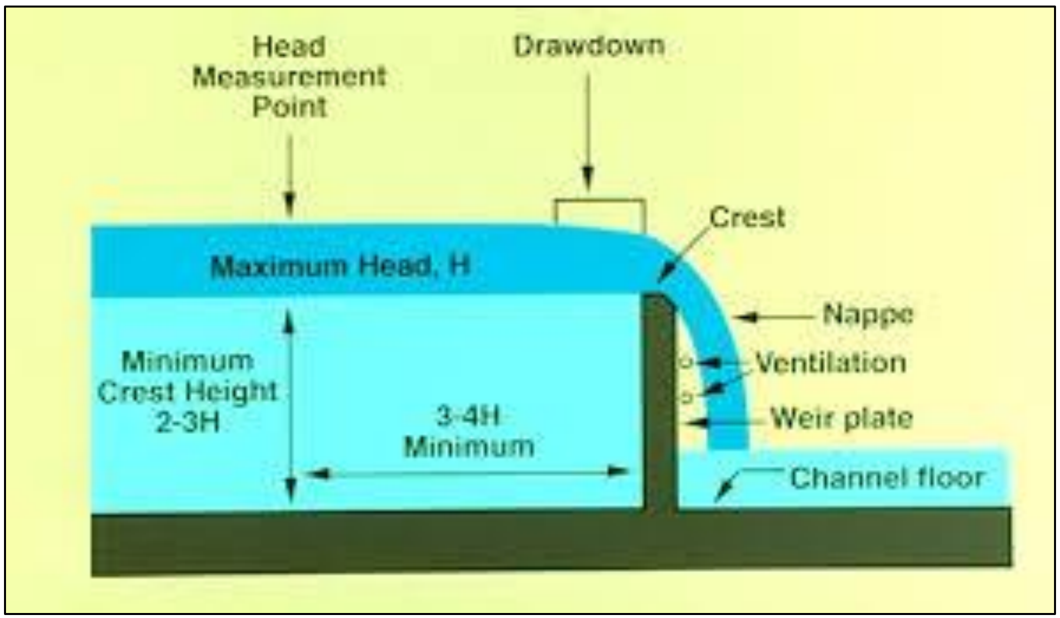
Fig : A Notch

Weirs

- A structure used to dam up a stream or river over which the water flows is called a weir.
- The weir has long been a standard device for the measurement of water in an open channel.
- The rate of flow is determined by measuring the height H (head), relative to the crest, at a distance upstream from the crest at least four times the maximum head (4H) corresponding to maximum flow, that is to be employed (as shown in next slide).
- The amount of drawdown at the crest is typically about 0.15H.
- The conditions of flow in the case of a weir are practically the same as those of a notch.
- That is why a notch is sometimes called as a weir and vice versa.
- The only difference between a notch and a weir is that the notch is of a small size and the weir is of a bigger one.
- Moreover, a notch is usually made in a plate, whereas a weir is usually made of masonry or concrete.
- The upstream face of the weir plate should be smooth, and the plate should be strictly vertical.



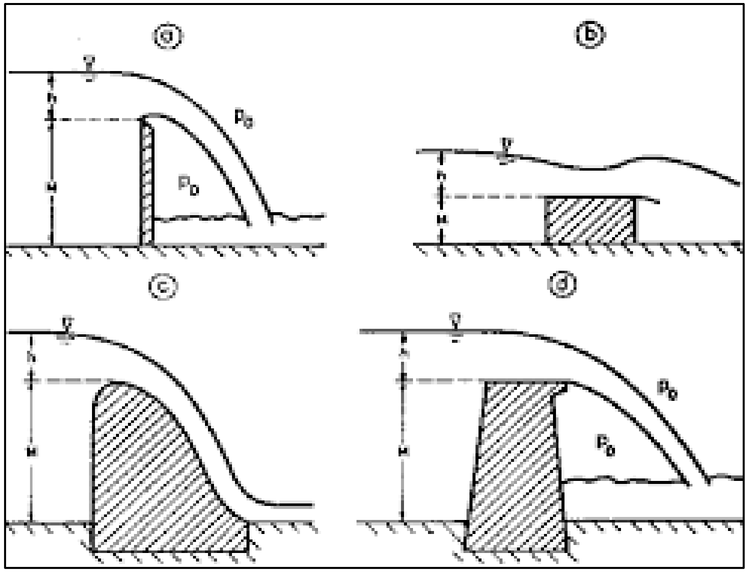
Flow over Sharp-crested Weir



Nomenclature of Weir

Weirs

- The crest should have a sharp, square upstream edge, and a bevel on the downstream side so that the nappe springs clear, making a line contact for all but the very lowest heads.
- If it does not spring clear, the flow cannot be considered as true weir flow and the experimentally determined coefficients do not apply.
- The velocity at any point in the nappe is related to the energy line as shown in previous slide.
- The approach channel should be long enough so that normal velocity distribution exists and the surface should be as free of waves as possible.
- The weirs are classified as:
  1. According to the shape:
     a) Rectangular weir
     b) Triangular weir
     c) Trapezoidal (Cippoletti) weir
  2. According to the width of crest:
     a) Narrow-crested weir
     b) Broad-crested weir
  3. According to the nature of crest:
     a) Sharp-crested weir
     b) Ogee weir

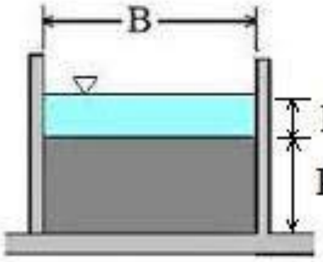


(a) Sharp-crested
(b) Broad-crested
(c) Ogee
(d) Broad-crested

Discharge Through Rectangular Notch/Weir (Suppressed)

- The rectangular notch/weir without end contractions is as wide as the channel and the width of the nappe or vein is the same as the length of the crest.
- As there are no contractions of the stream at the sides, we say that the end contractions are suppressed.
- it is essential that the sides of the channel upstream be smooth and regular.
- It is common to extend the sides of the channel downstream beyond the crest so that the nappe is confined laterally.
- The flowing water tends to entrain air from this enclosed space under the nappe, and unless this space is adequately ventilated there will be a partial vacuum and perhaps all the air may eventually be swept out.
- The water will then cling to the downstream face of the plate. and the discharge will be greater for a given head than when the space is vented.
- Therefore venting of a suppressed weir is necessary

  if the standard formulas are to be applied.



suppressed rectangular weir (L = B)

Discharge Through Rectangular Notch/Weir (Suppressed)

- Consider a rectangular notch or weir over which the water is flowing.
- Let, H = Height of the water above the crest

  L = Length of the crest

  C_d = Coefficient of discharge
- Let us consider a horizontal strip of water of thickness dh at a depth h from the water surface.

  Area of strip = L dh

  Theoretical velocity of water through the strip = $\sqrt{2gh}$

  Discharge through the strip = dq = C_d x Area of strip x Theoretical velocity

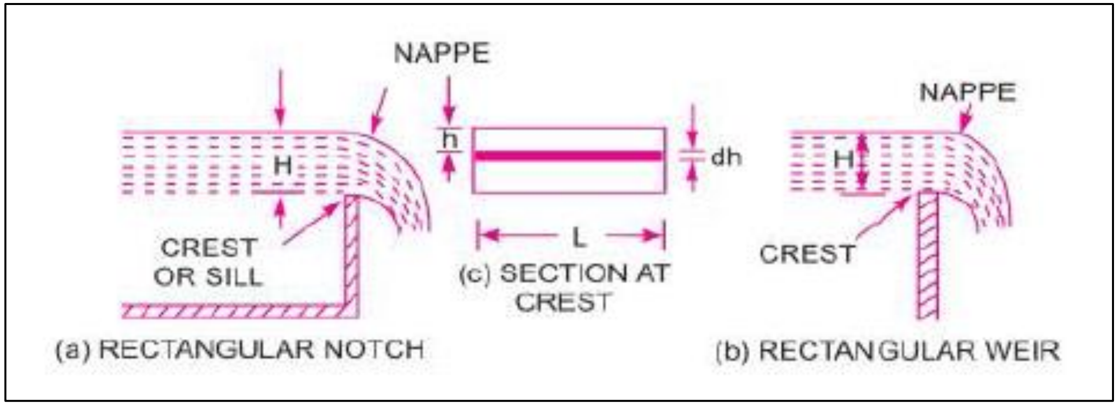
  = C_d L dh $\sqrt{2gh}$
- The total discharge is found out by integrating the above equation within the limits 0 and h.

$$Q = \int_0^H C_d L dh \sqrt{2gh}$$

$$Q = C_d L \sqrt{2g} \int_0^H h^{1/2}$$

$$Q = C_d L \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$Q = \frac{2}{3} C_d L \sqrt{2g} H^{3/2}$$

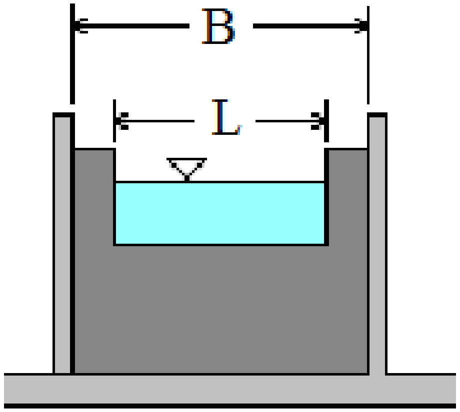


Rectangular Notch/Weir with End Contractions

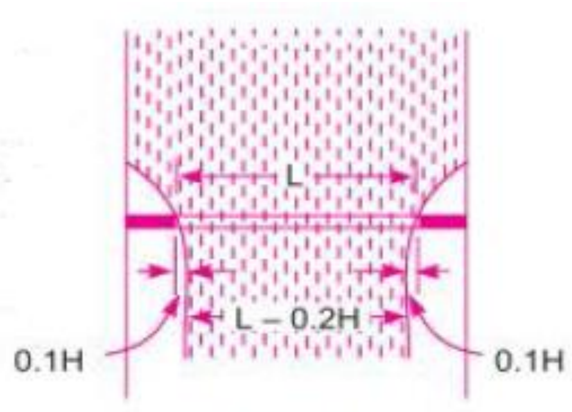
- When the length "**L**" of the crest of a rectangular weir is less than the width of the channel **"B"**, there will be a lateral contraction of the nappe so that its width is less than L.

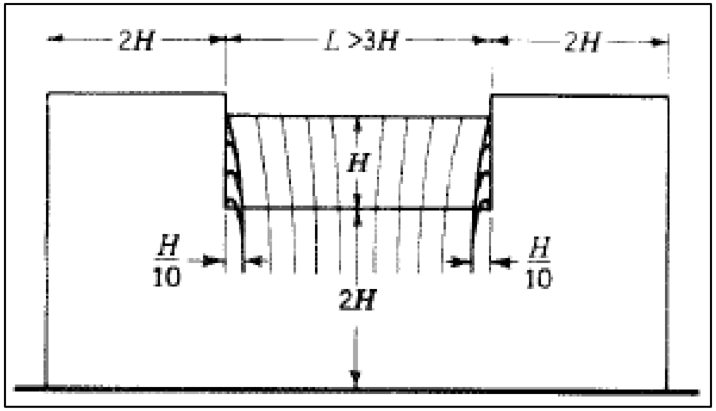
- Experiments by Francis indicated that under the conditions depicted as shown below, the effect of each side contraction is to reduce the effective width of the nappe by 0.1H.

- Hence for such a situation the flow rate may be computed by employing any of the three preceding equations and substituting (L – 0.lnH) for L, where n is the number of end contractions, normally 2 but sometimes 1.



contracted rectangular weir





Effect of End Contractions

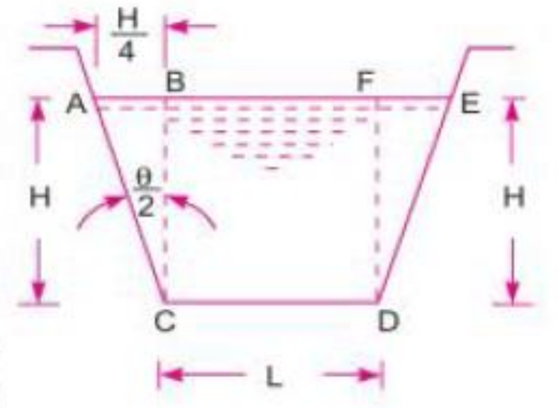
Trapezoidal (Cipolletti) Notch/Weir

- In order to avoid correcting for end contractions for rectangular weir, a Cipolletti weir is often used.
- It has a trapezoidal shape with side slopes of 1 horizontal to 4 vertical (1: ¼) as shown in the Figure.

$$tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{H/_4}{H} = \frac{1}{4}$$

$$\frac{\theta}{2} = tan^{-1}(\frac{1}{4}) = 14.04^o$$

- The widening of the weir as flow depth increases adds enough to the effective width of the stream to offset the lateral contraction.
- By giving this slope to the sides, an increase in discharge through the triangular portions ABC and DEF of the weir is obtained.
- If this slope is not provided the weir would be a rectangular one, and due to end contraction, the discharge would decrease.



- Thus, in case of cipolletti weir, the factor of end contraction is not required

Problem 11.11.2:

A 0.75 m high rectangular sharp-crested weir extends across a 2.2 m wide rectangular channel. When the head is 320 mm, determine the flow rate by neglecting the velocity of approach.

Solution:

$\mathrm{P} = 0.75\ \mathrm{m}$

$\mathrm{L} = 2.2\ \mathrm{m}$

$\mathrm{H} = 320\ \mathrm{mm} = 0.32\ \mathrm{m}$

$Q = 1.83LH^{3/2}$ for $C_d = 0.62$

$Q = 1.83 \times 2.2 \times (0.32)^{3/2}$

$Q = 0.729\ m^3/s$

Problem 11.11.3:

Suppose the rectangular weir of above Problem is contracted at both ends. (a) Find the flow rate for a head of 320 mm by the Francis formula. (b) What would be the maximum value of H for which the Francis formula could be used?

Solution:

For weir with end contractions,

$Q = 1.83(L - 0.1nH)H^{3/2}$

$Q = 1.83 \times [2.2 - (0.1 \times 2 \times 0.32)] \times (0.32)^{3/2}$

$Q = 0.708\ m^3/s$

For Francis formula, require L > 3H;

$H_{max} = {}^{L}/_{3} = {}^{2.2}/_{3} = 0.733\ m$

Discharge Through Triangular Notch/Weir

- For relatively small flows the rectangular weir must be very narrow and thus of limited maximum capacity, or else the value of *H* will be so small that the nappe will not spring clear but will cling to the plate.
- For such a case the triangular weir has the advantage that it can function for a very small flow and also measure reasonably large flows as well.
- The vertex angle is usually between 10° and 90° but rarely larger.
- Let, H = Height of the water above the apex of the notch, θ = Angle of the notch

  C_d = Coefficient of discharge, Width of the notch at the water surface = 2 H tan θ/2

  Area of the strip = 2 (H – h) tan θ/2 dh,

  Theoretical velocity of water through the strip = $\sqrt{2gh}$

  Discharge over the notch = dq = C_d x Area of strip x Theoretical velocity

  = C_d 2 (H – h) tan θ/2 dh $\sqrt{2gh}$

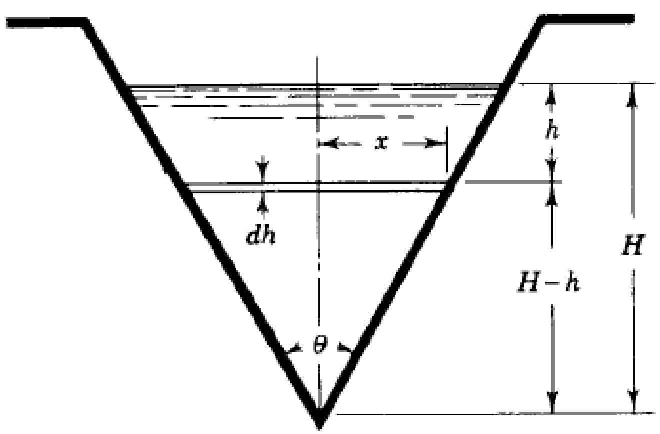
- The total discharge is found out by integrating the above equation within the limits 0 and h.

$$Q = \int_0^H C_d 2(H-h)\tan\frac{\theta}{2}\,dh\sqrt{2gh} = 2C_d\sqrt{2g}\tan\frac{\theta}{2}\int_0^H (H-h)\sqrt{h}\,dh$$

$$Q = 2C_d\sqrt{2g}\tan\frac{\theta}{2}\int_0^H (Hh^{1/2}-h^{3/2})dh = 2C_d\sqrt{2g}\tan\frac{\theta}{2}\left[\frac{Hh^{3/2}}{3/2}-\frac{h^{5/2}}{5/2}\right]_0^H$$

$$Q = \frac{8}{15}C_d\sqrt{2g}\tan\frac{\theta}{2}H^{5/2}$$

(If $\theta = 90^o$, $C_d = 0.6$ and g = 9.8 m/s^2, then Q = 1.417H$^{5/2}$)



Discharge Through Trapezoidal Notch/Weir

- A trapezoidal notch or weir is a combination of a rectangular and two triangular notches or weirs.
- Thus, the total discharge will be equal to the sum of discharge through a rectangular weir or notch and discharge through a triangular notch or weir.
- Consider a trapezoidal notch or weir ABCD, split it into a rectangular portion BCFE and two triangular portions ABE and DCF as shown, for analysis purpose.
- Let, H = Height of the water above the crest

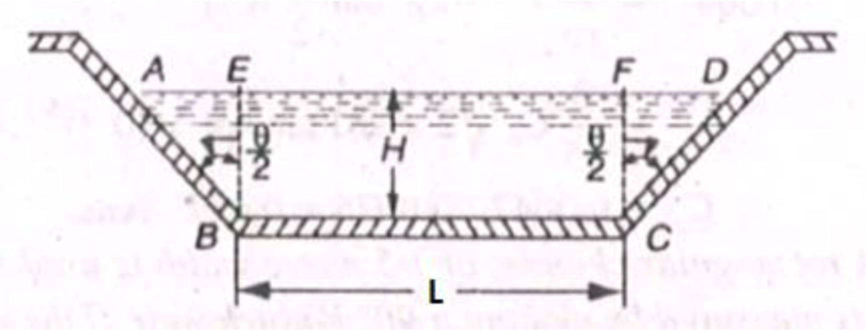
  L = Length of the crest

  C_{d_1} = Coefficient of discharge for rectangular portion

  C_{d_2} = Coefficient of discharge for triangular portion

  L = breadth of the rectangular portion

  θ/2 = angle, which the sides make with the vertical
- The purpose of slope on the sides is to obtain an increased discharge through the triangular portions, which, otherwise, would have been decreased due to end contractions in rectangular weirs.



$$Q = \frac{2}{3} C_{d_1} L \sqrt{2g} H^{3/2} + \frac{8}{15} C_{d_2} \sqrt{2g} \tan\frac{\theta}{2} H^{5/2}$$

Example:

Find the discharge of water flowing over a rectangular notch of 2 m length when the constant head over the notch is 300 mm. Take $C_d = 0.60$.

Solution:

Length of notch = L = 2 m

Head over notch = H = 300 mm = 0.3 m

$C_d = 0.6$

$Q = 2/3\ C_d\ L \sqrt{2g}\ H^{3/2}$

$Q = 2/3 \times 0.6 \times 2 \times \sqrt{(2 \times 9.81)} \times (0.3)^{3/2}$

$Q = 0.582\ m^3/s$

Problem 8.2 *Determine the height of a rectangular weir of length 6 m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8 m and discharge is 2000 litres/s. Take $C_d = 0.6$ and neglect end contractions.*

Solution. Given :

Length of weir, $L = 6$ m

Depth of water, $H_1 = 1.8$ m

Discharge, $Q = 2000$ lit/s $= 2$ m^3/s

$C_d = 0.6$

Let H is height of water above the crest of weir, and H_2 = height of weir (Fig. 8.2)

The discharge over the weir is given by the equation (8.1) as

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\ H^{3/2}$$

or
$$2.0 = \frac{2}{3} \times 0.6 \times 6.0 \times \sqrt{2 \times 9.81} \times H^{3/2}$$

$$= 10.623\ H^{3/2}$$

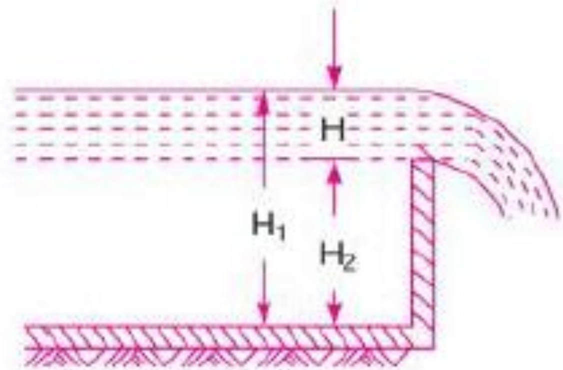


Fig. 8.2

$\therefore$
$$H^{3/2} = \frac{2.0}{10.623}$$

$\therefore$
$$H = \left(\frac{2.0}{10.623}\right)^{2/3} = 0.328 \text{ m}$$

$\therefore$ Height of weir,
$$H_2 = H_1 - H$$

= Depth of water on upstream side $- H$

$= 1.8 - .328 =$ **1.472 m. Ans.**

Problem 8.5 *Water flows over a rectangular weir 1 m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Taking C_d for the rectangular and triangular weir as 0.62 and 0.59 respectively, find the depth over the triangular weir.*

(Osmania University, 1990 ; A.M.I.E., Winter, 1975)

Solution. Given :

For rectangular weir, length, $L = 1$ m

Depth of water, $H = 150$ mm $= 0.15$ m

$C_d = 0.62$

For triangular weir, $\theta = 90°$

$C_d = 0.59$

Let depth over triangular weir $= H_1$

The discharge over the rectangular weir is given by equation (8.1) as

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 1.0 \times \sqrt{2 \times 9.81} \times (.15)^{3/2} \text{ m}^3\text{/s} = 0.10635 \text{ m}^3\text{/s}$$

The same discharge passes through the triangular right-angled weir. But discharge, Q, is given by equation (8.2) for a triangular weir as

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$\therefore \quad 0.10635 = \frac{8}{15} \times .59 \times \tan\frac{90}{2} \times \sqrt{2g} \times H_1^{5/2} \qquad \{\because \theta = 90° \text{ and } H = H_1\}$$

$$= \frac{8}{15} \times .59 \times 1 \times 4.429 \times H_1^{5/2} = 1.3936\, H_1^{5/2}$$

$$\therefore \quad H_1^{5/2} = \frac{0.10635}{1.3936} = 0.07631$$

$$\therefore \quad H_1 = (.07631)^{0.4} = 0.3572 \text{ m. Ans.}$$

Problem 8.7 *Find the discharge through a trapezoidal notch which is 1 m wide at the top and 0.40 m at the bottom and is 30 cm in height. The head of water on the notch is 20 cm. Assume* C_d *for rectangular portion = 0.62 while for triangular portion = 0.60.*

Solution. Given :

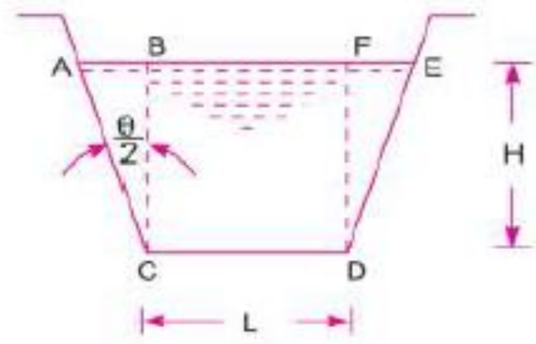


Fig. 8.5

Top width, $AE = 1$ m

Base width, $CD = L = 0.4$ m

Head of water, $H = 0.20$ m

For rectangular portion, $C_{d_1} = 0.62$

For triangular portion, $C_{d_2} = 0.60$

From ΔABC, we have

$$\tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{(AE - CD)/2}{H}$$

$$= \frac{(1.0 - 0.4)/2}{0.3} = \frac{0.6/2}{0.3} = \frac{0.3}{0.3} = 1$$

Discharge through trapezoidal notch is given by equation (8.4)

$$Q = \frac{2}{3} C_{d_1} \times L \times \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.4 \times \sqrt{2 \times 9.81} \times (0.2)^{3/2} + \frac{8}{15} \times .60 \times 1 \times \sqrt{2 \times 9.81} \times (0.2)^{5/2}$$

$= 0.06549 + 0.02535 = 0.09084$ m^3/s = **90.84 litres/s. Ans.**