Chapter 3

Hydrostatics, Kinematics, Hydrodynamics

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Hydrostatics

- The term hydrostatics means the study of pressure, exerted by a liquid at rest.
- It has been observed that the direction of such a pressure is always at right angles to the surface, on which it acts.
- The total pressure on a submerged surface, may be defined as the total pressure exerted by the liquid on it, mathematically total pressure (in terms of force):

  $F = p_1a_1 + p_2a_2 + p_3a_3$ ……………

  where

  p_1, p_2, p_3 are intensities of pressure on different strips of the surface, and

  a_1 , a_2, a_3 are areas of the corresponding strips.
- When a surface is submerged in a fluid, forces develop on the surface due to the weight of the fluid.
- The determination of these forces is important in the design of storage tanks, ships, dams, and other hydraulic structures.
- When a fluid is at rest, no tangential force can exist within the fluid and all forces are then normal to the surfaces in question.
- The position of an immersed surface may be:
  1. Horizontal
  2. Vertical
  3. Inclined

Hydrostatic Force on a Submerged Plane Horizontal Surface

- Consider a plane horizontal surface submerged in static water.
- As every point on the surface is at the same depth from the free surface of water, the pressure intensity will be equal on entire surface and equal to

$$\mathbf{p = \gamma\, h}$$

- If the pressure is *uniformly distributed* over an area, the force is equal to the pressure times the area, and the point of application of the force is at the centroid of the area.

$$\mathbf{F = p\, A = \gamma\, \bar{h}\, A}$$

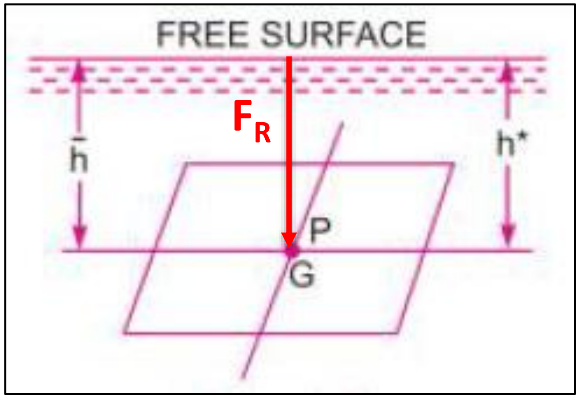
Note: $\bar{h} = h^* = h_p$ only for uniform forces

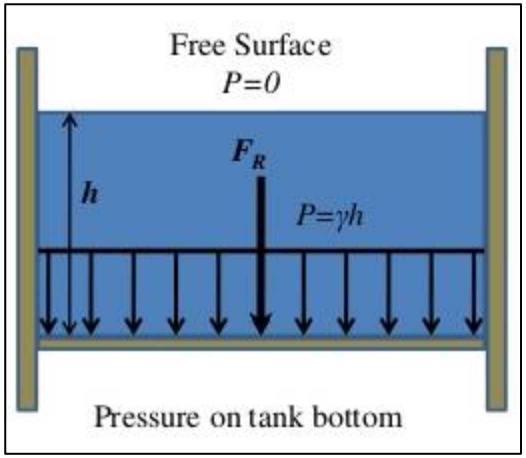
p is the uniform pressure on the submerged surface

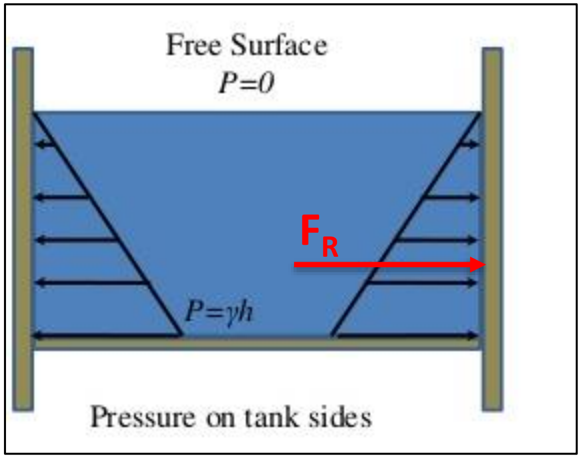
A is the total area of the submerged surface, and

$\bar{h} = h_c$ = depth of centroid (G) from free surface of water

$h^* = h_p$ = depth of center of pressure from free surface of water







Example:

A rectangular tank 4 m long 2 m wide contains water up to a depth of 2.5 m. Calculate the pressure at the base of the tank.

Solution:

l = 4 m b = 2 m $\bar{h}$ = 2.5 m

A = l x b = 4 x 2 = 8 m^2

F = $\gamma \bar{h}$ A = 9.81 x 8 x 2.5 = 196.2 kN

Example:

A tank 3 m x 4 m contains 1.2 m deep oil of specific gravity 0.8. Find (i) intensity of pressure at the base of the tank, and (ii) total pressure on the base of the tank.

Solution:

Size of tank (A) = 3 m x 4 m = 12 m^2, Depth of oil ($\bar{h}$) = 1.2 m, Specific gravity of oil = 0.8, specific weight of oil (γ) = 9.81 x 0.8 = 7.85 kN/m^3

i. Intensity of pressure at the base of the tank

   p = γ h = 7.85 x 1.2 = 9.42 kN/m^2= 9.42 kPa

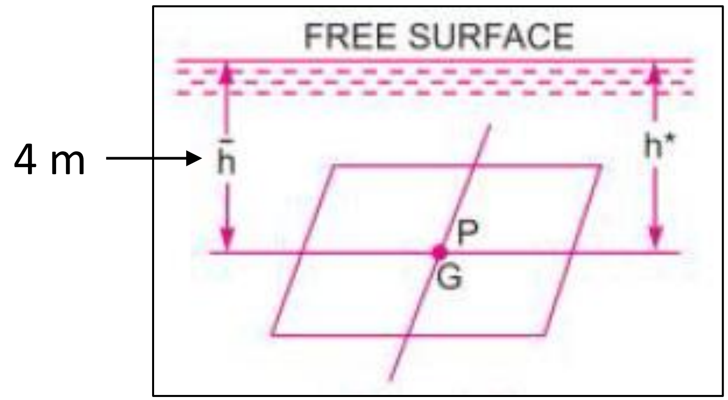
ii. Total pressure (pressure-force) on the base of the tank

   F = $\gamma \bar{h}$ A = 7.85 x 1.2 x 12 = 113.4 kN

Problem 3.7.15:

A rectangular area is 5 by 6 m, with the 5 m side horizontal. It is placed with its centroid 4 m below a water surface and rotated about a horizontal axis in the plane area and through its centroid. Find the magnitude of the force on one side.

Solution:



$F = \gamma\, \bar{h}\, A = \gamma\, h^*\, A$

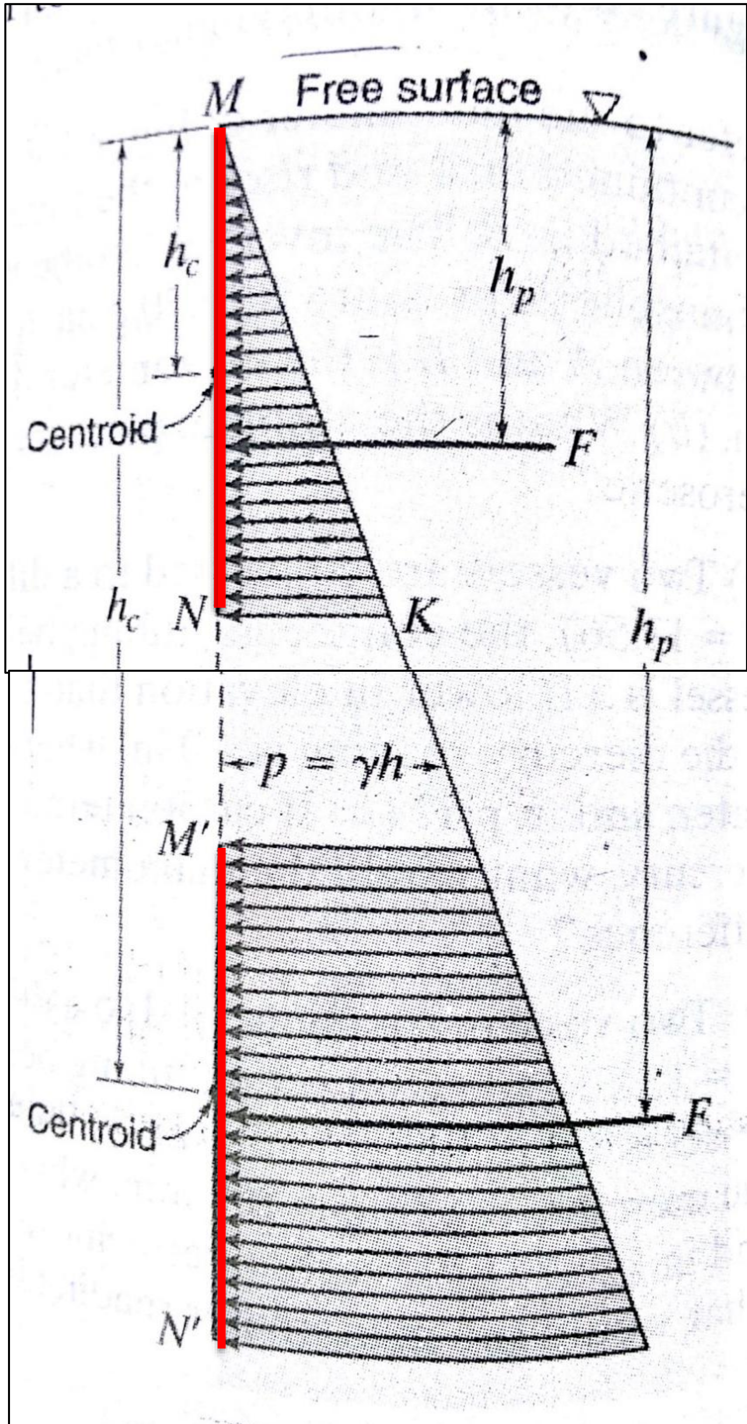
$F = 9.81 \times 4 \times (5 \times 6)$

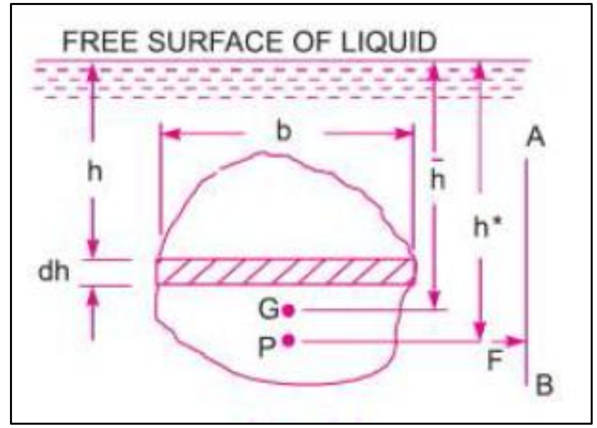
$F = 1177$ kN

for any angle because the rotation of the plane about G does not affect $\bar{h}$.

Hydrostatic Force on a Submerged Plane Vertical Surface

- In this case, the distribution of pressure is not uniform on the vertical side; hence further analysis is necessary.
- In the Figure shown on next slide, consider a vertical plane whose upper edge lies in the free surface of a liquid.
- Let this plane be perpendicular to the plane of the paper, so that MN is merely its trace.
- The pressure will vary from zero at M to NK at N.
- Thus, the total force F on one side is the sum of the products of the elementary areas and the pressure ($F = \sum p_i a_i$) upon them.
- It is apparent that the resultant of this system of parallel forces must be applied at a point (known as center of pressure) below the centroid of the area, because the centroid of an area is the point of application of the resultant of a system of uniform parallel forces.
- If the plane is lowered to M'N', the proportionate change of pressure from M' to N' is less than that from M to N.
- Hence, the center of pressure will be nearer to the centroid of the plane surface, and the deeper the plane is submerged, the more uniform the pressure becomes and the closer these two points will be together.





The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips.

The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

Consider a strip of thickness 'dh' and width 'b' at a depth of 'h' from free surface of liquid as shown in the above Figure.

Pressure intensity on the strip, $p = \gamma h$

Area of the strip, $dA = b\, dh$

Total pressure force on strip, $dF = p \times dA = \gamma h \times b\, dh$

Total pressure force on the whole surface,

$F = \int dF = \int \gamma h \times b\, dh = \gamma \int b\, h\, dh = \gamma \int h\, dA$

$F = \gamma A h_p$

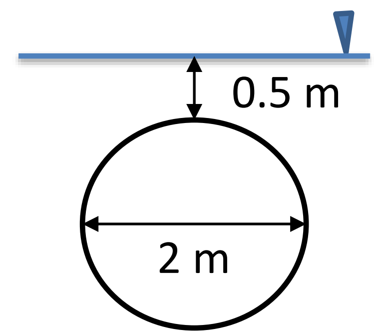
$\int h\, dA$ = Moment of surface area about (M) the free surface of liquid = Area of surface x Distance of C.G. from free surface

Pressure distributions on two vertical plane areas (viewed from edges)

Problem 3.7.6:

A plane surface is circular with a diameter of 2 m. If it is vertical and the top edge is 0.5 m below the water surface, find the magnitude of the force on one side.

Solution:



$h_c = 0.5 + d/2 = 0.5 + 2/2 = 1.50$ m

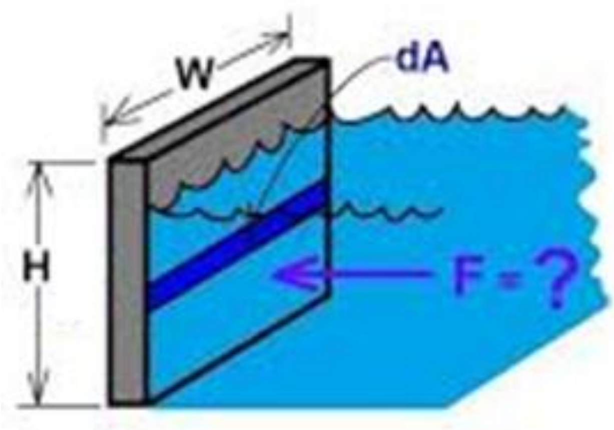
$F = \gamma\, h_c\, A = 9.81 \times 1.50 \times 3.1416 \times (1)^2$

$F = 46.2$ kN

Problem 3.7.12:

A rectangular plate submerged in water is 5 by 4 m, the 5-m side being horizontal and the 4-m side being vertical. Determine the magnitude of the force on one side of the plate if the top edge is (a) at the water surface; (b) 1 m below the water surface; (c) 100 m below the water surface.

Solution:



$F = \gamma\, h_c\, A = 9.81 \times h_c\, (4 \times 5 \text{ m}) = 196.2\, h_c$

(a) $h_c = 0 + 2 = 2$ m ; $F = 196.2 \times 2 = 392$ kN

(b) $h_c = 1 + 2 = 3$ m ; $F = 196.2 \times 3 = 589$ kN

(c) $h_c = 100 + 2 = 102$ m ; $F = 196.2 \times 102 = 20000$ kN

Hydrostatic Force on a Submerged Plane Inclined Surface

- In the Figure below, let MN be the trace of a plane area making an angle θ with the horizontal.
- To the right is the projection of this area upon a vertical plane.
- Let h be the variable depth to any point and y be the corresponding distance from OX, the intersection of the plane produced and the free surface.
- Consider an element of area so chosen that the pressure is uniform over it.
- Such an element is a horizontal strip.
- If x denotes the width of the area at any depth, then

$dA = x\,dy$ as $p = \gamma h$ and $h = y \sin\theta$

- the force dF on a horizontal strip is

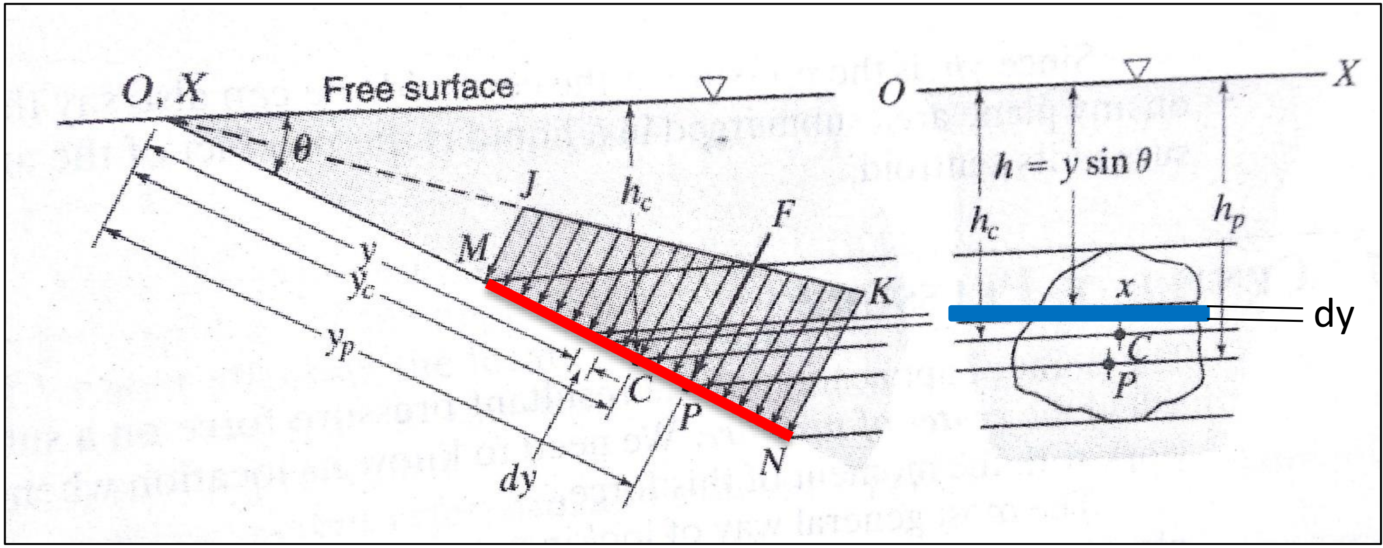
$dF = p\,dA = \gamma h\,dA$

$dF = \gamma y \sin\theta\,dA$

$F = \int dF$

$F = \gamma \sin\theta \int y\,dA$

$F = \gamma \sin\theta\, y_c A$



Hydrostatic Force on a Submerged Plane Inclined Surface

- where y_c is the distance to the centroid of the area A.
- If the vertical depth of the centroid is denoted by h_c, then

  $h_c = y_c \sin \theta$

  and

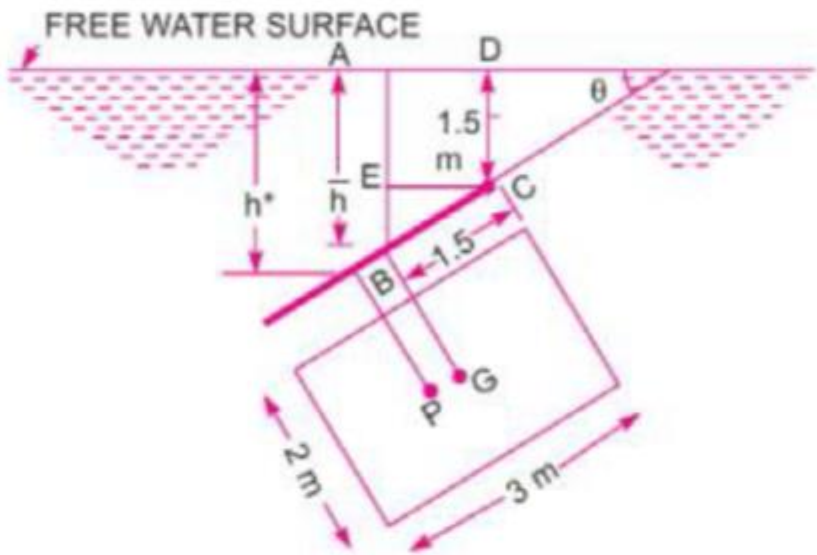
  $F = \gamma \, h_c \, A$
- Thus the total force on any plane area submerged in a liquid is found by multiplying the specific weight of the liquid by the product of the area and the depth of its centroid.
- The value of F is independent of the angle of inclination of the plane so long as the depth of its centroid is unchanged.
- Since $\gamma \, h_c$ is the pressure at the centroid, another statement is that the total force on any plane area submerged in a liquid is the product of the area and the pressure at its centroid.

- The depth to the center of pressure (h_P) can be determined using

$$h_P = {}^{I_C \sin^2\theta}/_{A\, h_C} + h_C$$

- For a plane, vertical surface the angle $\theta = 90^o$

A rectangular plane surface 2 m wide and 3 m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total pressure when the upper edge is 1.5 m below the free water surface.



Solution. Given :

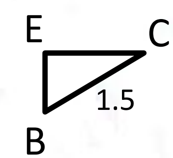
Width of plane surface, $b = 2$ m

Depth, $d = 3$ m

Angle, $\theta = 30°$

Distance of upper edge from free water surface = 1.5 m

(*i*) **Total pressure** force is given by equation (3.6) as



$\angle C = 30^o$

$sin30^o = \frac{EB}{BC}$

$$F = \rho g A \bar{h}$$

where $\rho = 1000$ kg/m^3

$A = b \times d = 3 \times 2 = 6$ m^2

$\{\because \quad \bar{h} = AE + EB = 1.5 + BC \sin 30° = 1.5 + 1.5 \sin 30°\}$

$\therefore \quad \bar{h}$ = Depth of C.G. from free water surface

$= 1.5 + 1.5 \sin 30°$

$\{\because \quad \bar{h} = AE + EB = 1.5 + BC \sin 30° = 1.5 + 1.5 \sin 30°\}$

$= 1.5 + 1.5 \times \frac{1}{2} = 2.25$ m

$\therefore \quad F = 1000 \times 9.81 \times 6 \times 2.25 =$ **132435 N. Ans.**

Summary:

- Pressure forces acting on a plane surface are distributed over every part of the surface.

- They are parallel and act in a direction normal to the surface.

- They can be replaced by a single resultant force **$F = \gamma\, h_c\, A$** acting normal to the surface and at the centroid of the plane.

- The point on the plane surface at which this resultant force acts is known as the ***center of pressure***.

- The **center of pressure** of any submerged plane surface is always below the centroid of the surface ($h_p > h_C$).

$$Y_P = \frac{\int_A y\, dF}{F} = \frac{\int_A y^2\, dA}{A\bar{y}} = \frac{I_x}{M_x} = \frac{I_o + A\bar{y}^2}{A\bar{y}} = \frac{I_o}{A\bar{y}} + \bar{y}$$

Types of Fluid Flow

1. Steady and Unsteady Flows:

- ***Steady flow*** is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc., at a point do not change with time. ${}^{dV}/_{dt} = 0$
- ***Unsteady flow*** is that type of flow, in which the velocity, pressure or density at a point changes with respect to time. ${}^{dV}/_{dt} \neq 0$

2. Uniform and Non-uniform Flows:

- ***Uniform flow*** is defined as that type of flow in which the velocity at any given time does not change with respect to space (i.e., length of direction of the flow).
- ***Non-uniform flow*** is that type of flow in which the velocity at any given time changes with respect to space.

3. Compressible and Incompressible Flows:

- ***Compressible flow*** is that type of flow in which the density of the fluid changes from point to point or in other words the density is not constant for the fluid.
- ***Incompressible flow*** is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases arc compressible.

4. Rotational and Irrotational Flows:

- ***Rotational flow*** is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis.
- ***Irrotational flow*** is that type of flow in which the fluid particles while flowing along stream-lines, do not rotate about their own axis.

Types of Fluid Flow

5. One-, Two- and Three-Dimensional Flows:

- ***One-dimensional flow*** is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x.
- For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinate only.
- The variation of velocities in other two mutually perpendicular directions is assumed negligible. $u = f(x), \quad v = 0, \quad w = 0$
- where u, v and w are velocity components in x, y and z directions, respectively.
- ***Two-dimensional flow*** is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates say x and y.
- For a steady two-dimensional flow the velocity is a function of two space co-ordinates only.
- The variation of velocity in the third direction is negligible.

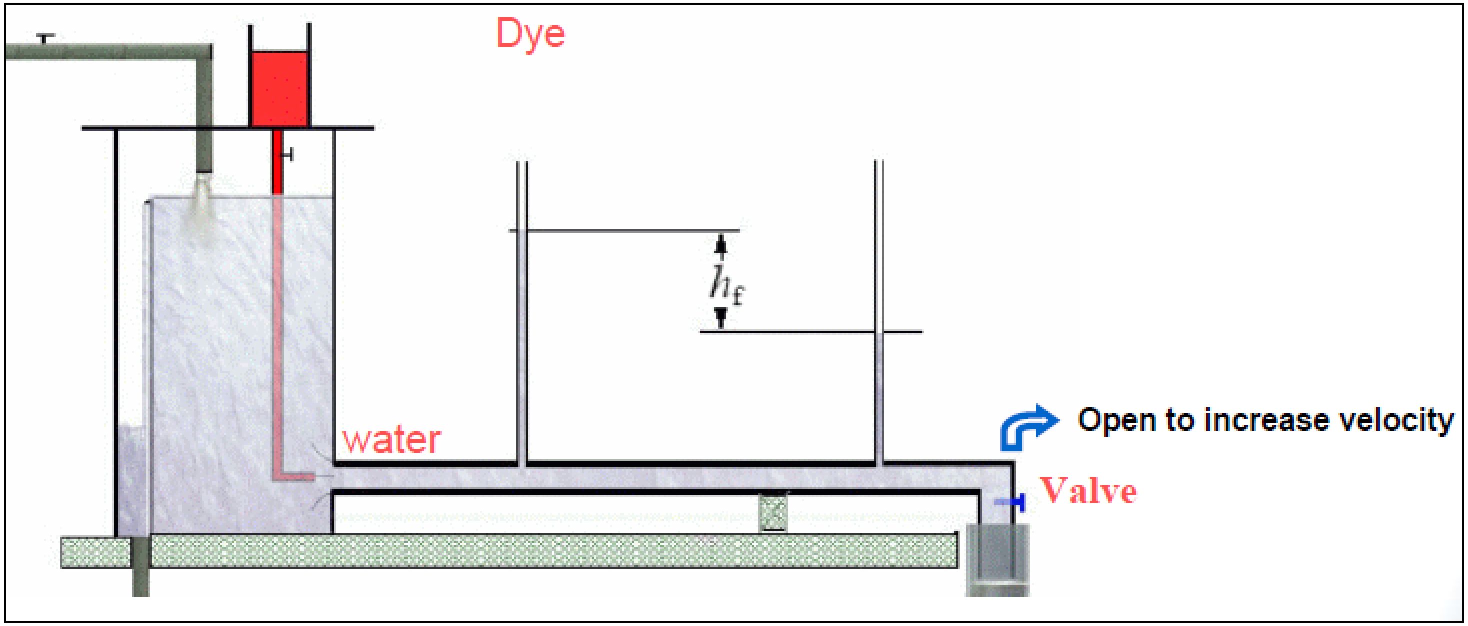
$$u = f_1(x, y), \qquad v = f_2(x, y), \qquad w = 0$$

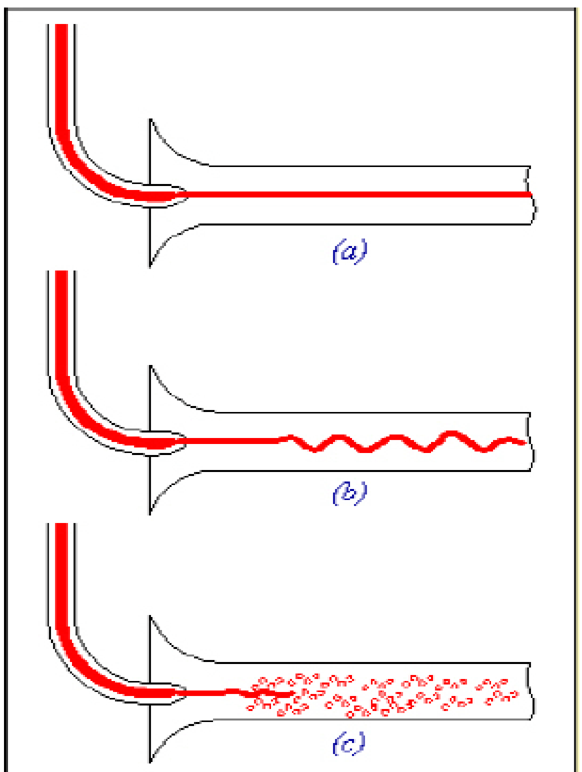
- ***Three-dimensional flow*** is that type of flow in which the velocity is a function of time and three mutually perpendicular directions.
- But for a steady three-dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only. U

$$u = f_1(x, y, z), \qquad v = f_2(x, y, z), \qquad w = f_3(x, y, z)$$

Laminar and Turbulent Flows

- There are two distinctly different types of fluid flow as demonstrated by Osborne Reynolds in 1883.
- He injected a fine, threadlike stream of colored liquid having the same density as water at the entrance to a large glass tube through which water was flowing from a tank, as shown in next slide.
- A valve at the discharge end permitted him to vary the flow.
- When the velocity in the tube was small, this colored liquid was visible as a straight line throughout the length of the tube, thus showing that the particles of water moved in parallel straight lines.
- As the velocity of the water was gradually increased by opening the valve further, there was a point at which the flow changed.
- The line would first become wavy, and then at a short distance from the entrance it would break into numerous vortices beyond which the color would be uniformly diffused so that no streamlines could be distinguished.
- In a series of experiments carried out at Owens College, Manchester (which has become the University of Manchester in 1880), Osborne Reynolds has shown the intermittent occurrence of turbulent flow within a pipe.





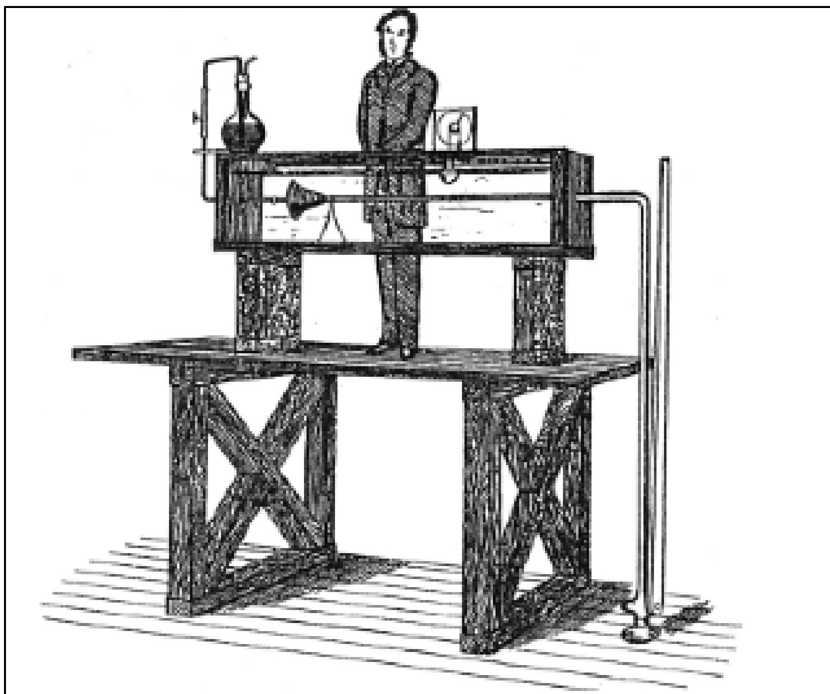
Laminar flow

- Dye streak stable
- Low velocity

Transition

Turbulent flow

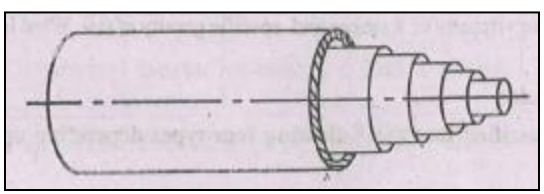
- Dye mixed up in the cross-section
- High velocity



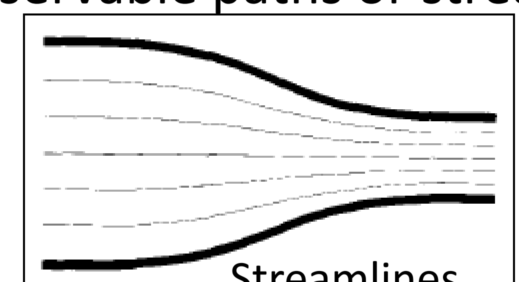
Reynolds' Turbulent flow experimental apparatus

Laminar Flow

- The first type of flow in the previous slide is known as **laminar**, **streamline**, or **viscous** flow.
- It is a flow, in which the viscosity of fluid is dominating over the inertia forces .
- It is more or less a theoretical flow, which rarely comes in contact with the engineers.
- A laminar flow can be best understood by the hypothesis that liquid moves in the form of concentric cylinders sliding one within the another.
- Or the fluid appears to move by the sliding of laminations of infinitesimal thickness relative to adjacent layers, with relative motion of fluid particles occurring at a molecular scale.
- The particles move in definite and observable paths or streamlines.



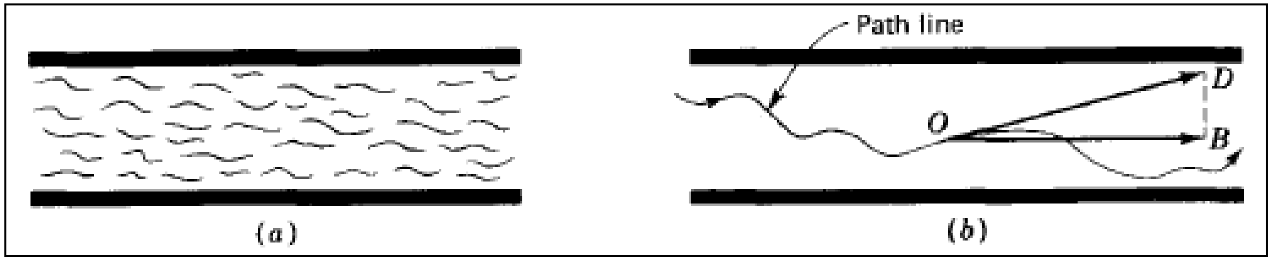
Concentric Cylinders



Streamlines

Turbulent Flow

- The second type is known as turbulent flow, as shown below, where (a) represents the irregular motion of a large number of particles during a very brief time interval, while (b) shows the erratic path followed by a single particle during a long time interval.



- It is a flow, in which the inertia force is dominating over the viscosity.
- A distinguishing characteristic of turbulence is its irregularity, there being no definite frequency, as in wave action, and no observable pattern, as in the case of eddies.
- Large eddies and swirls and irregular movements of large bodies of fluid, which can be traced to obvious sources of disturbance, do not constitute turbulence, but may be described as disturbed flow.
- By contrast, turbulence may be found in what appears to be a very smoothly flowing stream and one in which there is no apparent source of disturbance.

Turbulent Flow

- Turbulent flow is characterized by fluctuations in velocity at all points of the flow field.
- These fluctuations arise because the fluid as many small, discrete particles or packets called eddies, jostling each other around in a random manner.
- Although small, the smallest eddies are macroscopic in size, very much larger than the molecular sizes of the particles in laminar flow.
- They are the cause of the effective mixing action experienced with turbulent flow.
- They are often caused by rotation, particularly near boundaries, and often rotate themselves.
- They change shape and size with time as they move along with the flow.
- Each eddy dissipates its energy through viscous shear with its surroundings and eventually disappears.
- New eddies are continuously forming and the large eddies (large-scale turbulence) have smaller eddies within them giving rise to small-scale turbulence.
- The resulting fluctuations in velocity are rapid and irregular and can be detected by a fast-acting probe such as a hot-wire or hot-film anemometer.

Reynold’s Number

- Whether flow is laminar or turbulent depends on a dimensionless number.
- Osborne Reynold (1842-1912),English physicist and Professor found in 1882 that the value of critical velocity is governed by the relationship between the inertia force and viscous forces (i.e., viscosity).
- He derived a ratio of these two forces and found out a dimensionless number known as Reynold's number (Re) i.e.

$$\mathrm{Re} = \frac{Inertial\ Forces\left[F = ma = m\dfrac{V}{t}\right]}{Viscous\ Forces\left[\tau = \mu\dfrac{dV}{dy}A\right]} = \frac{(\rho L^3)\dfrac{V}{t}}{\mu\dfrac{V}{L}L^2} = \frac{\rho L^2\dfrac{1}{t}}{\mu} = \frac{\rho L\dfrac{L}{t}}{\mu} = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$$

- Reynold's number has much importance and gives us the information about the type of flow (i.e. laminar or turbulent).
- Reynold, after carrying out a series of experiments, found that if,

Re < 2000	the flow is a laminar
2000 < Re < 4000	the flow is transitional
Re > 4000	the flow is a turbulent

- It may be noted that the value of critical velocity corresponding with Re = 2000 is for a lower critical velocity and that corresponding with Re = 4000 is for a higher critical velocity, however, the value of the true critical Reynold’s number is 2000.

Illustrative Example 8.1. An oil ($s = 0.85$, $\nu = 1.8 \times 10^{-5}$ m²/s) flows in a 10-cm-diameter pipe at 0.50 ℓ/s. Is the flow laminar or turbulent?

$$V = \frac{Q}{A} = \frac{500 \text{ cm}^3/\text{s}}{\pi(10)^2 \text{ cm}^2/4} = 6.35 \text{ cm/s} = 0.0635 \text{ m/s}$$

$$N_R = \frac{DV}{\nu} = \frac{0.10 \text{ m}(0.0635 \text{ m/s})}{1.8 \times 10^{-5} \text{ m}^2/\text{s}} = 334$$

Since $N_R < 2{,}000$, the flow is laminar.

Oil with a kinematic viscosity of 0.185 St is flowing through a 150-mm-diameter pipe. Below what velocity will the flow be laminar?

Inside cover: $\nu = 0.142 \text{ St} = 0.142 \times 10^{-4} \text{ m}^2/\text{s}$

Eq. 8.1: $\mathbf{R} = 0.15V/(0.185 \times 10^{-4}) < 2000 = \mathbf{R}_{crit}$; so $V < 0.247$ m/s

Oil with a kinematic viscosity of 3 stokes flows through a 10 cm diameter pipe with a velocity of 5 m/s.

Is the flow laminar or turbulent?

υ = 33 St = 3×10^{-4} m²/s, D = 10 cm = 0.1 m, V = 5 m/s

$R = (5)(0.1)/3 \times 10^{-4} = 1667 < R_{crit} = 2000$

Therefore, the flow is laminar

Example 18.8. *An oil of specific gravity of 0·95 is flowing through a pipeline of 200 mm diameter at the rate of 50 litres/s. Find the type of flow, if viscosity for the oil is 1 poise.*

Solution. Given : Specific gravityof oil = 0·85; $d = 200$ mm = 0·2 m; $\theta = 50$ litres/s = 0·05 m^3/s and $\mu = 1\ P = 0{\cdot}1$ N-s/m^2.

We know that the density of oil,

$$\rho = 0{\cdot}95 \times 1000 = 950 \text{ kg/m}^2$$

and area of pipeline,

$$A = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0{\cdot}2)^2 = 0{\cdot}0314 \text{ m}^2$$

∴ Velocity of oil,

$$v = \frac{Q}{A} = \frac{0{\cdot}05}{0{\cdot}0314} = 1{\cdot}59 \text{ m/s}$$

We also know that the kinematic viscosity of the oil,

$$\nu = \frac{\mu}{\rho} = \frac{0{\cdot}1}{950} = 1{\cdot}05 \times 10^{-4} \text{ m}^2\text{/s}$$

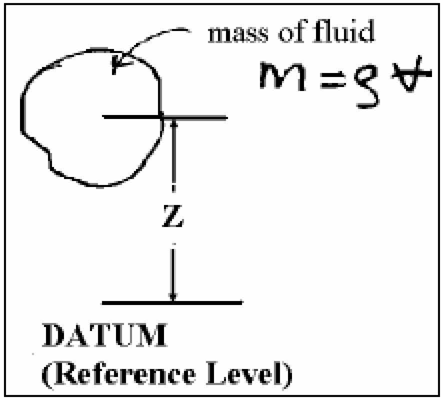
and Reynold's number,

$$R_N = \frac{vd}{\nu} = \frac{1{\cdot}59 \times 0{\cdot}2}{1{\cdot}05 \times 10^{-4}} = 3029$$

Energy of Flowing Water

- The energy, in general, may be defined as the capacity to do work.
- Though the energy exists in many forms, yet the following are important from the subject point of view:

1. Potential energy
2. Kinetic energy
3. Pressure energy
4. Internal energy



1. Potential Energy of a Liquid Particle in Motion

- It is energy possessed by a liquid particle by virtue of its elevation above an arbitrary datum plane.
- A liquid particle of weight **W** is **z** meters above a datum (arbitrarily chosen), the potential energy of the particle will be **Wz** (N-m/N = m) (i.e. P.E./W).
- The potential head of the liquid, at that point, will be **z** meters of the liquid.

$P.E = mg\,z = (\rho\forall)\,gz$

$P.E / W = \rho\forall g\,z / \rho g\forall = z$ $(W = mg = \rho\forall g)$

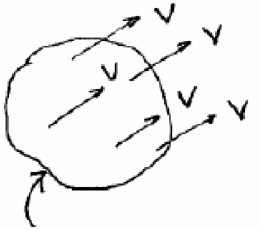
Energy of Flowing Water

2. Kinetic Energy of a Liquid Particle in Motion

- It is the energy, possessed by a liquid particle, by virtue of its motion or velocity.
- If a liquid is flowing with all particles moving at the same (or mean) velocity, then the kinetic energy of the liquid will be **$V^2/2g$** (N-m/N = m)
- Velocity head of the liquid, at that velocity, will be **$V^2/2g$** meters of the liquid.

$$K.E = \frac{1}{2} mV^2 = \frac{1}{2} (\rho\forall) V^2$$

$$K.E / W = \frac{1}{2} \rho\forall V^2 / \rho g\forall = V^2/2g$$



Mass of fluid with uniform velocity

3. Pressure Energy (Pressure Head) of a Liquid Particle in Motion

- It is the energy, possessed by a liquid particle, by virtue of its existing pressure.
- If a liquid particle is under a pressure of **p** kN/m^2 (i.e., kPa), then the pressure energy (although usually not referred to as pressure energy) of the particle will be **p/γ** (N-m/N = m), where γ is the specific weight of the liquid.
- Pressure head of the liquid under that pressure will be **p/γ** meters of the liquid.

$$p = \gamma h$$

$$h = p / \gamma \qquad \text{(pressure energy / weight)}$$

Energy of Flowing Water

4. Internal Energy

- Internal energy is stored energy that is associated with the molecular, or internal state of matter.
- It may be stored in many forms, including thermal, nuclear, chemical, and electrostatic.
- Thermal energy is energy due to the motion of molecules and forces of attraction between them ($i = g\ I$).

Total Energy of a Liquid Particle in Motion

- The total energy of a liquid in motion is the sum of its potential energy, kinetic energy and pressure energy, mathematically,

  $E = Z + V^2/2g + p/\gamma$

- The units of energy are in N-m (Joule) but according to the subject point of view, the units of energy are taken in terms of m of the liquid.

Total Head of a Liquid Particle in Motion

- The total head of a liquid in motion is the sum of its potential head, kinetic head and pressure head, mathematically,

  $E = Z + V^2/2g + p/\gamma$ m of liquid

Problem:

Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm^2 (gage) and with mean velocity of 2 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.

Solution:

Diameter of pipe = 5 cm = 0.05 m

Pressure = p = 29.43 N/cm^2 = 29.43×10^4 N/m^2

Velocity = V = 2 m/s

Datum head = Z = 5 m

Total head = pressure head + velocity head + datum head

Pressure head = $p/\gamma = 29.43 \times 10^4/(1000 \times 9.81) = 30$ m

Velocity head = $V^2/2g = (2)^2/(2 \times 9.81) = 0.204$ m

Total head = $p/\gamma + V^2/2g + Z = 30 + 0.204 + 5$

Total head = 35.204 m

Equations of Motion

- According to Newton's second law of motion, the net force F_x, acting on a fluid element in the direction of **x** is equal to mass **m** of the fluid element multiplied by the acceleration a_x in the x-direction.

$$F_x = m\, a_x$$

- In the fluid flow, the following forces are present:

i. F_g, gravity force.

ii. F_p, the pressure force.

iii. F_v, force clue to viscosity.

iv. F_t, force due to turbulence.

v. F_c, force due to compressibility.

- Thus in above equation, the net force:

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$$

1. If the force due to compressibility, F_c is negligible, the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

- and equation of motions are called **Reynold's** equations of motion.

2. For flow, where (F_t) is negligible, the resulting equations of motion are known as **Navier-Stokes** Equation.
3. If the flow is assumed to be ideal, viscous force (F_v) is zero and equation of motions are known as **Euler's** equation of motion.

Bernoulli’s Equation

- There are many basic assumptions involved in the derivation of this equation which are stated below:
  1) It assumes viscous (friction) effects are negligible.
  2) It assumes the flow is steady, but may vary with position.
  3) The equation applies along a streamline.
  4) It assumes the flow is irrotational.
  5) It assumes the flow to be incompressible, and
  6) It assumes no energy is added to or removed from the fluid along the streamline.

- It states “for a perfect incompressible liquid flowing in continuous stream the total energy of the particles remains the same while the particles moves from one point to another.

- The Bernoulli’s equation is a statement of the conservation of mechanical energy.

Bernoulli's Equation

- We shall consider the forces acting on a small cylindrical element of the fluid in the direction of the streamline and apply Newton's second law.
- The cross-sectional area of the element at right angles to the streamline may have any shape and varies from A to A+dA.
- The mass of the fluid element is,

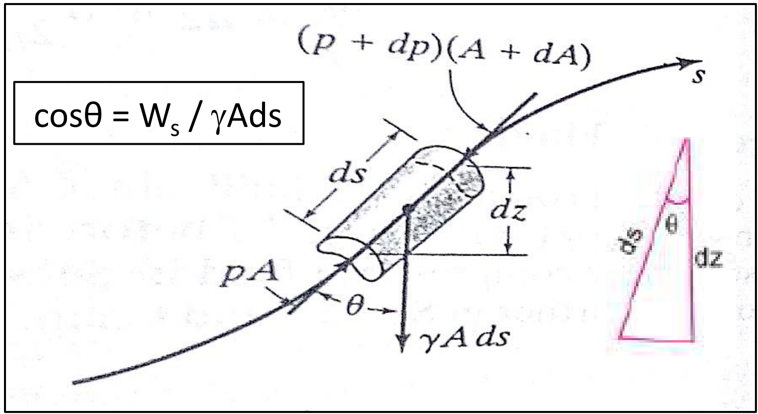
  $m = \rho V = \rho\,[\frac{A+(A+dA)}{2}]\,ds = \rho\,(A + \frac{dA}{2})\,ds = \boldsymbol{\rho A\, ds}$ neglecting second order term
- Let **θ** is the angle between the direction of flow and the line of action of the weight of element.
- The forces tending to accelerate or decelerate this mass along **s** are:

a) the pressure forces, $p\,A + (p + \frac{dp}{2})dA - (p + dp)\,(A + dA)$

$= -\,dp\,A - \frac{dp\,dA}{2} = \boldsymbol{-\,dp\,A}$ neglecting second term

b) the weight component (W_s) in the direction of motion which is

$-\,\gamma\,A\,ds\cos\theta = -\,\rho g\,ds\,A\,dz/ds = \boldsymbol{-\,\rho g\,A\,dz}$



- Applying $F = m\,a_s$ along the streamline, we get

  $-\,dp\,A - \rho g\,A\,dz = (\rho\,A\,ds)\,a_s$
- Dividing by the volume A ds

  $-dp/ds - \rho g\,dz/ds = \rho\,a_s$

Bernoulli's Equation

- This states that the pressure gradient along the streamline combined with the weight component in that direction causes the acceleration **a** of the element.
- Since, $a_s = \frac{dV}{dt} = \frac{dV}{ds}\frac{ds}{dt} + \frac{dV}{dt}$ but for steady flow, $\frac{dV}{dt} = 0$, and $\frac{ds}{dt} = V$
- Thus, $a_s = V\frac{dV}{ds}$

  $-dp/ds - \rho g\, dz/ds = \rho V\, dV/ds$

$$a_x = \left(\frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}\right) + \frac{\partial u}{\partial t}$$

$$a_x = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial t}$$

- Multiplying by ds/ρ and rearranging

  $dp/\rho + g\, dz + V\, dV = 0$

- This is the one dimensional Euler equation, because Leonhard Euler (1707-1783), a Swiss mathematician first derived it in about 1750.
- It applies to both compressible and incompressible flow, since the variation of **ρ** over the element length ds is small.
- Dividing through by g, we get

  $dp/\gamma + dz + d(V^2/2g) = 0$ where, $d(V^2/2g) = VdV$

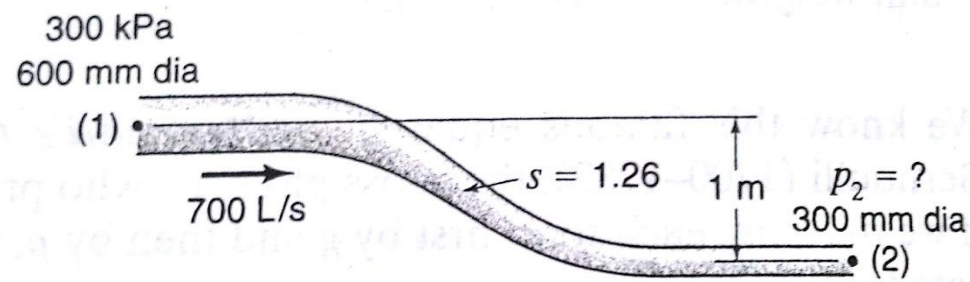
- For the case of incompressible fluid (γ = constant), we can integrate to give

  $p/\gamma + z + V^2/2g = \text{constant}$ (along a streamline)

- This equation is known as Bernoulli's theorem, in honor of Daniel Bernoulli (1700-1782), the Swiss physicist who presented this theorem in 1738.

Example 5.2:

Glycerin (specific gravity 1.26) in a processing plant flows in a pipe at a rate of 700 L/s. At a Point where the pipe diameter is 600 mm, the pressure is 300 kPa. Find the pressure at a second point where the pipe diameter is 300 mm if the second point is 1 m lower than the first point. Neglect head loss.



Solution:

Q_1 = 700 L/s = 0.7 m^3/s

r_1 = 600/2 = 300 mm = 0.3 m

r_2 = 300/2 = 150 mm = 0.15 m

Since, $Q = AV$

$V_1 = Q_1/A_1 = 700/3.1416(0.3)^2 = 2.48$ m/s **$A = \pi r^2$**

$V_2 = 4V_1 = 4 \times 2.48 = 9.90$ m/s

Since, $p_1/\gamma + z_1 + V_1^2/2g = p_2/\gamma + z_2 + V_2^2/2g$

$300/1.26(9.81) + 0 + (2.48)^2/2(9.81) = p_2/1.26(9.81) - 1 + (9.9)^2/2(9.81)$

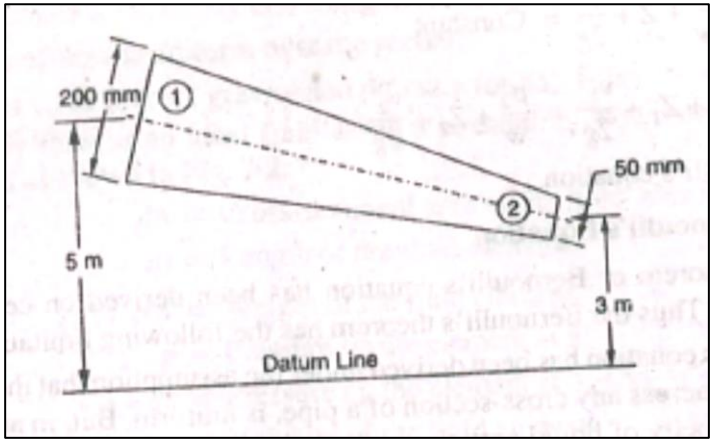
$p_2 = 254$ kN/m^2

Example:

The diameter of a pipe changes from 200 mm at a section 5 m above datum to 50 mm at a section 3 m above datum. The pressure of water at first section is 500 kPa. If the velocity of flow at the first section is 1 m/s, determine the intensity of pressure at the second section.

Solution:

d_1 = 200 mm = 0.2 m; Z_1 = 5 m; d_2 = 50 mm = 0.05 m; Z_2 = 3 m; p_1 = 500 kPa and V_1 = 1 m/s



$$a_1 = \frac{\pi}{4} d_1^{\ 2} = \frac{\pi}{4}(0.2)^2 = 0.03142 m^2$$

$$a_2 = \frac{\pi}{4} d_2^{\ 2} = \frac{\pi}{4}(0.05)^2 = 0.00196 m^2$$

Since, the discharge through the pipe is continuous

$$a_1 V_1 = a_2 V_2$$

$$V_2 = \frac{a_1 V_1}{a_2} = \frac{0.03142 \times 1}{0.00196} = 16 m/s$$

Applying Bernoulli's equation to both sections of the pipe

$$Z_1 + \frac{V_1^2}{2g} + \frac{p_1}{\gamma} = Z_2 + \frac{V_2^{\ 2}}{2g} + \frac{p_2}{\gamma}$$

$$5 + \frac{(1)^2}{2 \times 9.81} + \frac{500}{9.81} = 3 + \frac{(16)^2}{2 \times 9.81} + \frac{p_2}{9.81}$$

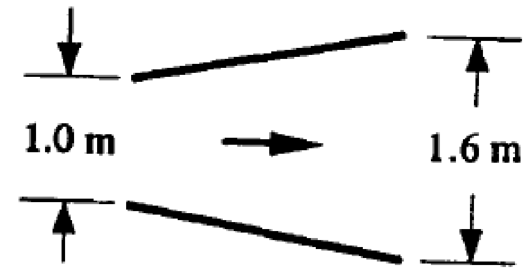
$$56.05 = 16.05 + \frac{p_2}{9.81}$$

$$p_2 = 392.4 kPa$$

Problem 5.3

Water flows through a long, horizontal, conical diffuser at the rate of 4.2 m^3/s. The diameter of the diffuser changes from 1.0 m to 1.6 m. The pressure at the smaller end is 9.5 kPa. Find the pressure at the downstream end of the diffuser, assuming frictionless flow. Assume also, that the angle of the cone is small enough that the flow does not separate from the walls of the diffuser.

Eq. 4.6: $V_1 = \dfrac{Q}{A_1} = \dfrac{4.2}{\pi(0.5)^2} = 5.35 \text{ m/s}$



$V_2 = \dfrac{Q}{A_2} = \dfrac{4.2}{\pi(0.8)^2} = 2.09 \text{ m/s}$

Eq. 5.7 with $z_1 = z_2$: $p/\gamma + V^2/2g = \text{constant}$

ie: $\dfrac{9.5 \text{ kN/m}^2}{9.81 \text{ kN/m}^3} + \dfrac{5.35^2}{2(9.81)} = \dfrac{p}{\gamma} + \dfrac{2.09^2}{2(9.81)}$; $0.968 \text{ m} + 1.458 \text{ m} = p/\gamma + 0.222$

$p/\gamma = 2.20 \text{ m}$; $p = 2.20(9.81) = 21.6 \text{ kN/m}^2$ ◄

Problem 5.2.3

Refer to Fig. X5.2.2. Assume $a = 1$ m, $b = 4$ m, and the flow to be frictionless in the siphon. Find the rate of discharge in m^3/s and the pressure head at B if the pipe has a uniform diameter of 150 mm.

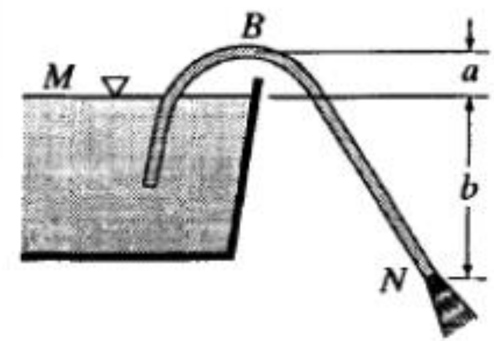


Figure X5.2.2

Eq. 5.7 from *M* to *N* (elevation datum at *N*):

$$0 + 4 + 0 = 0 + 0 + V_N^2/2g \;;\; V_N = V_B = 8.86 \text{ m/s}$$

$$Q = \pi(0.15/2)^2 8.86 = 0.1565 \text{ m}^3\text{/s} \quad \blacktriangleleft$$

Eq. 5.7 from *M* to *B*:

$$0 + 4 + 0 = p_B/\gamma + 5 + V_B^2/2g \;;\; p_B/\gamma = -5.00 \text{ m}$$