Chapter 2

Fluid Pressure

Engr. Khurram Sheraz

Assistant Professor

University of Engineering & Technology Peshawar

Contents

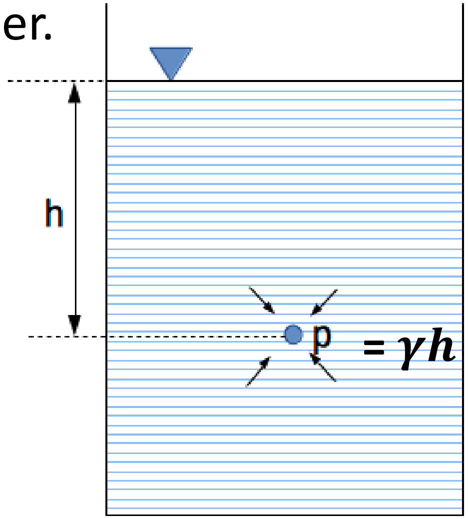
Introduction

- In **statics** (fluids at rest) there is no relative motion between adjacent fluid layers:
  - Shear stress is zero; only pressure can be acting on fluid surfaces
  - Gravity force acts on the fluid (body force)
- Whenever a liquid (such as water, oil etc.) is contained in a vessel, it exerts force at all points on the sides and bottom of the container.
- This force per unit area is called pressure.
- If F is the total normal force acting on some finite area A, then the average intensity of pressure is,

  $p = F / A$ (N/m^2 or Pascal, Pa or bar, (1 bar = 10^5 Pa))
- The direction of this pressure is always at right angles to the surface with which the fluid at rest comes in contact.

Pressure Head

- **Pressure head** (also static pressure head or static head) is the internal energy of a fluid due to the pressure exerted on its container.



- Consider a vessel containing some liquid with no pressure acting on its surface as shown in the Figure.
- Let, γ = specific weight of the liquid

  h = height of liquid in the vessel

  A= area of cylinder base
- A little consideration will show that there will be some pressure on the vessel base due to the weight of the liquid in it, therefore, pressure,

  p = weight (as force) of liquid in the vessel / area of the vessel base (W / A)

  $p = \gamma V / A = \gamma h A / A = \gamma h$ or $h = p / \gamma$ h (m) = (kN/m^2) / 9.81 = 0.1020 kPa
- This equation shows that the intensity of pressure at any point in a liquid is proportional to its depth (h) from the surface.
- Therefore the pressure can be expressed as:
  - *i. As a force per unit area (N/m^2) or*
  - *ii. As a height of the equivalent liquid column (m) i.e. pressure head*

Neglecting the pressure on the surface and the compressibility of water, what is the pressure in kPa at a depth of an instrument 4600 m below the surface of the ocean? The specific weight of ocean water under ordinary conditions is 10.05 kN/m³.

Eq. 3.4: $p = \gamma h = 10.05(4600) = 46\,700 \text{ kN/m}^2$ ◄

A pressure gage at elevation 4.8 m on the side of a storage tank containing oil reads 34.7 kPa. Another gage at elevation 2.2 m reads 57.5 kPa. Compute the specific weight, density, and specific gravity of the liquid.

From Eq. 3.3: $\gamma = \dfrac{\Delta p}{\Delta h} = \dfrac{(57.5 - 34.7 \text{ kN/m}^2)}{(4.8 - 2.2 \text{ m})} = 8.77 \text{ kN/m}^3$ ◄

$p - p_1 = -\gamma(z - z_1)$
- for elevations

Eq. 2.1: $\rho = \dfrac{\gamma}{g} = \dfrac{8770 \text{ N/m}^3}{9.81 \text{ m/s}^2} = \dfrac{8770 \text{ (kg·m/s}^2\text{)/m}^3}{9.81 \text{ m/s}^2} = 792 \text{ kg/m}^3$ ◄

Sec. 2.3: $s = \dfrac{\gamma}{\gamma_w} = \dfrac{8.77}{9.81} = 0.894$ ◄

If air had a constant specific weight of 12 N/m³ and were incompressible, what would be the height of air surrounding the earth to produce a pressure at the surface of 101.3 kPa abs?

Eq. 3.5: $h = p/\gamma = 101.3/12.00 = 8.44 \text{ km}$ ◄

An open tank contains water 1.40 m deep covered by a 2 m thick layer of oil (s = 0.855). What is the pressure head at the bottom of the tank, in terms of a water column?

$\gamma_{oil} = s\,\gamma_w = 0.855 \times 9.81 = 8.39\ kN/m^3$

Pressure of oil at interface = $p_{oil} = \gamma_{oil}\, h_{oil} = 8.39 \times 2 = 16.78$ kPa

Pressure head of oil in terms of water column = $h_{oil} = p_{oil}/\gamma_w = 16.78/9.81 = 1.71$ m of water

Pressure head at bottom = $h_b = h_{oil} + h_w = 1.71 + 1.4 = 3.11$ m of water

An open tank contains 5.0 m of water covered with 2 m of oil ($\gamma = 8.0\ kN/m^3$). Find the gage pressure (a) at the interface between the liquids and (b) at the bottom of the tank.

(*a*) Eq. 3.4: $p = \gamma h = (8\ kN/m^3)2\ m = 16\ kN/m^2 = 16$ kPa at interface ◀

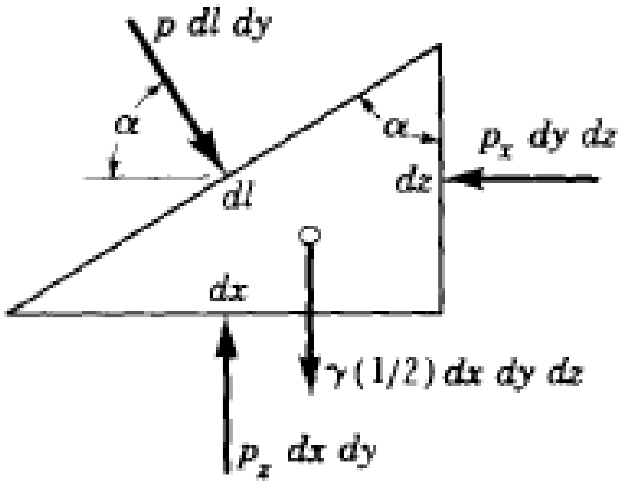
(*b*) $p_b = 16 + (9.81)5 = 65.1\ kN/m^2 = 65.1$ kPa at tank bottom ◀

Pascal's Law

- It states that *"the intensity of pressure at any point in a fluid at rest, is the same in all directions"*.
- Pressure is the normal force per unit area at a given point acting on a given plane within a fluid mass.
- Unlike in solids, in a fluid at rest, no tangential stresses can exist, and the only forces between adjacent surfaces are pressure forces normal to the surfaces.

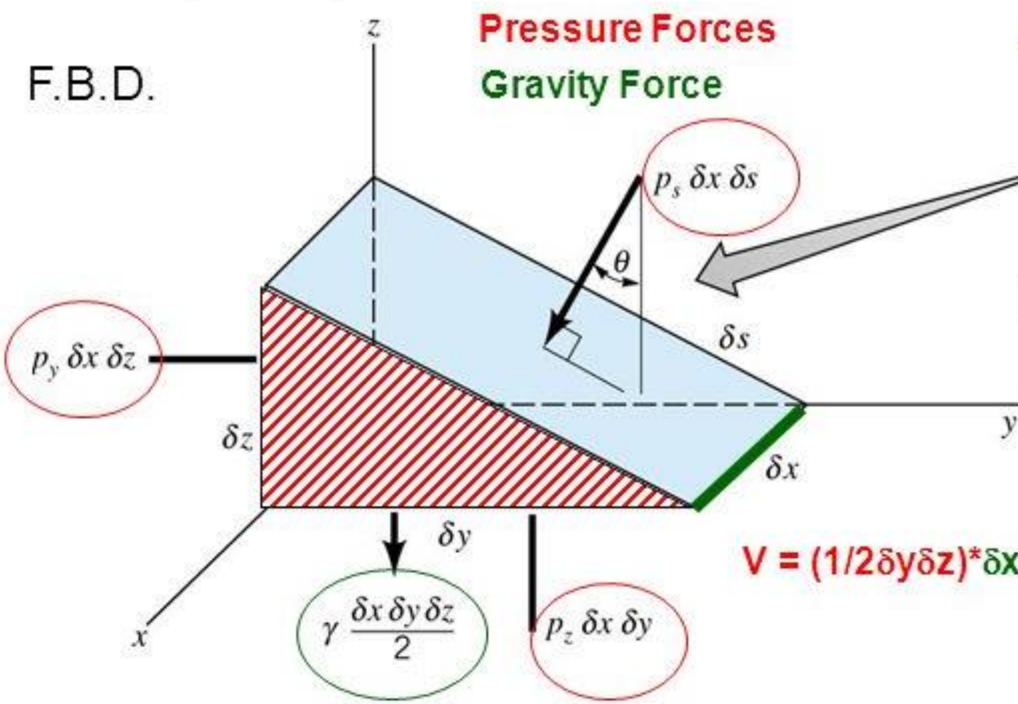
Blaise Pascal (1623-1662)

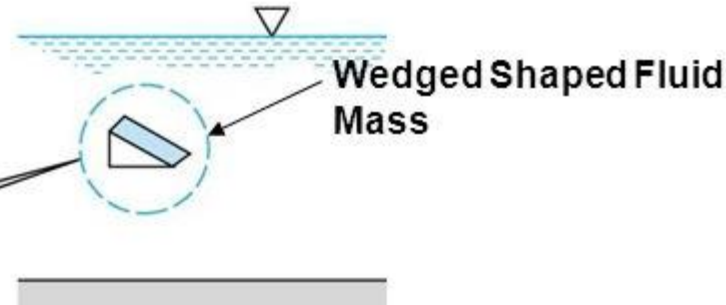
- Consider a very small wedge-shaped element of fluid at rest whose thickness perpendicular to the plane of the paper is equal to **dy**.
- Let **p** be the average pressure in any direction in the plane of the paper, $\mathbf{p_x}$ and $\mathbf{p_z}$ be the average pressures in the horizontal and vertical directions, respectively.



- For simplicity, the forces in **y** direction are not considered as they cancel.

F.B.D.





p is average pressure in the x, y, and z direction.

P_s is the average pressure on the surface

θ is the plane inclination

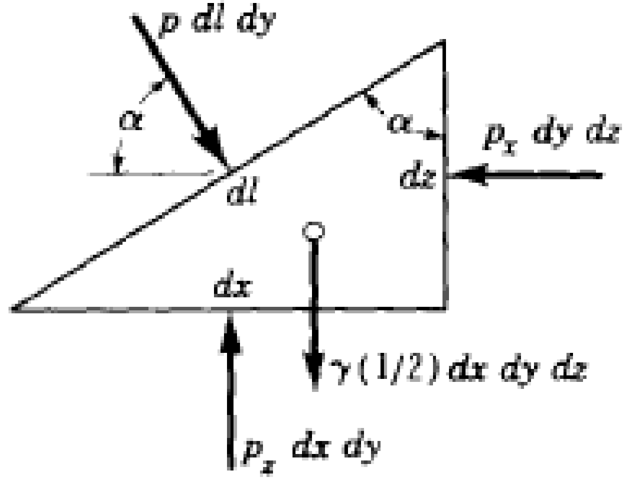
δ is the length is each coordinate direction, x, y, z

δs is the length of the plane

γ is the specific weight

Pascal's Law

- Applying the equations of static equilibrium to the fluid particle:



- $\sum Fx = p\ dl\ dy\ \cos\alpha - p_x\ dy\ dz = 0$

  $p\ dl\ dy\ \cos\alpha = p_x\ dy\ dz$

  Since, $\cos\alpha = dz / dl \Rightarrow dz = dl \cos\alpha$

  $p = p_x$

- $\sum Fz = p_z\ dx\ dy - p\ dl\ dy\ \sin\alpha - \gamma\ (dx\ dy\ dz)/2 = 0$

  neglecting 3rd term of higher power,

  $p\ dl\ dy\ \sin\alpha = p_z\ dx\ dy$

  Since, $\sin\alpha = dx / dl \Rightarrow dx = dl \sin\alpha$

  $p = p_z$

$W = \gamma V = \gamma A\ dy$
$W = \gamma\ (1/2\ dx\ dz)\ dy$
$W = \gamma\ (1/2)\ dx\ dy\ dz$

- In other words, ***"the pressure at a point in a fluid at rest, or in motion, is independent of the direction as long as there are no shearing stresses present"***.

- Pressure at any point is the same in all directions.
- In a fluid confined by solid boundaries, pressure acts perpendicular to the boundary.

TWO important principles about pressure

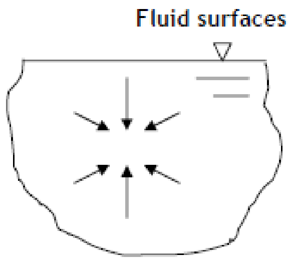


Figure Pressure acting uniformly in all directions

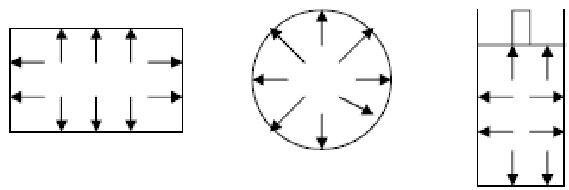
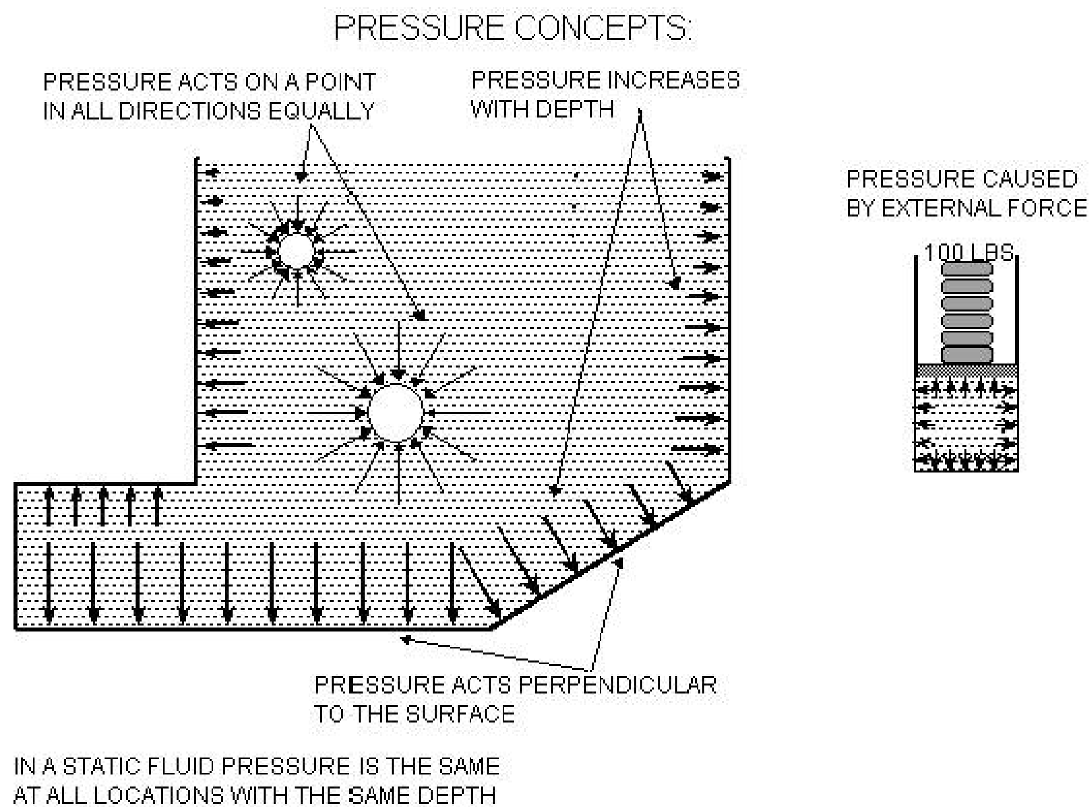


Figure Direction of fluid pressures on boundaries

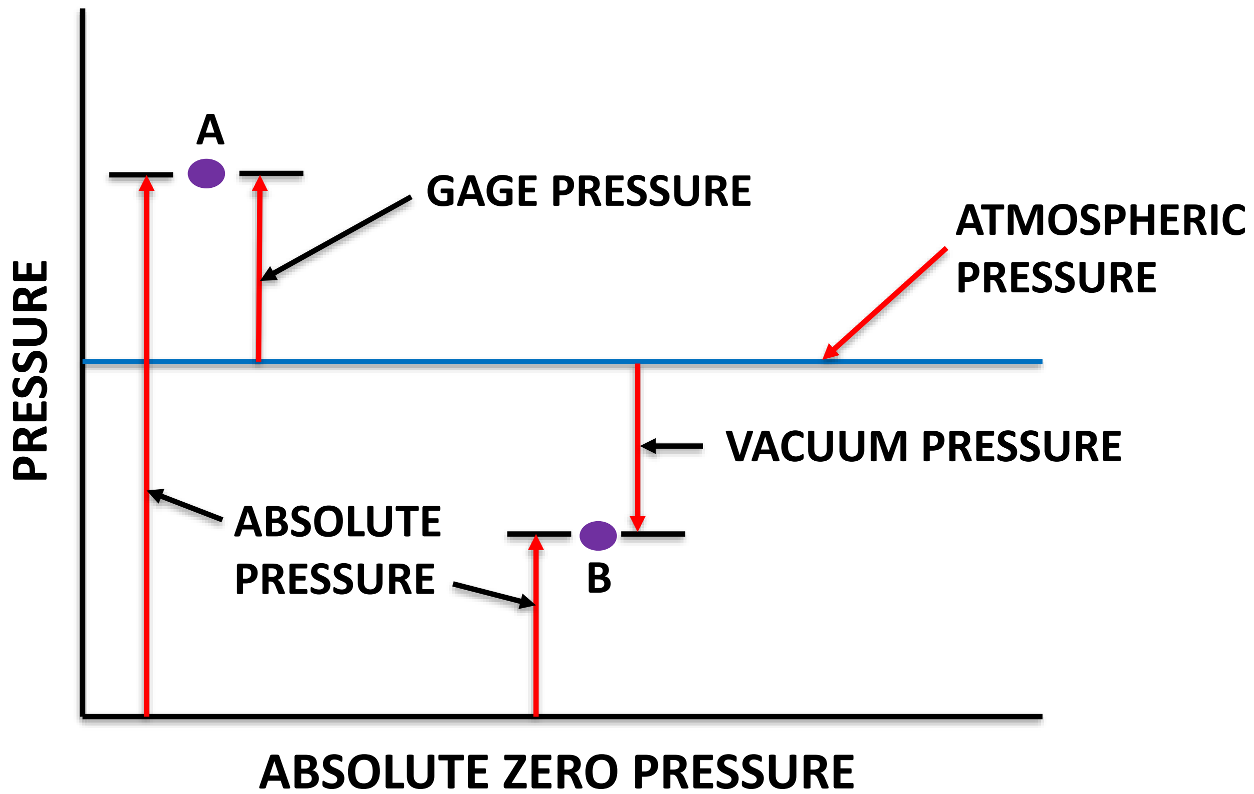


Absolute, Gage, and Atmospheric Pressures

- If pressure is measured relative to absolute zero, it is called **absolute pressure**.
- Absolute pressure is the actual pressure at a given point.
- All values of absolute pressure are positive, since a negative value would indicate tension, which is normally considered impossible in any fluid.

- When measured relative to atmospheric pressure as a base, it is called **gage pressure**.
- This is because practically all pressure gages register zero when open to the atmosphere and hence measure the difference between the pressure of the fluid to which they are connected and that of the surrounding air.
- The gage pressure is the pressure measured with the help of pressure measuring instrument, in which the atmospheric pressure is taken as datum.
- Gage pressures are positive if they are above that of the atmosphere and negative if they are vacuum

- If the pressure is less than atmospheric, it is called **vacuum pressure** and the gauge pressure value is negative.


A
GAGE PRESSURE
ATMOSPHERIC PRESSURE
PRESSURE
VACUUM PRESSURE
ABSOLUTE PRESSURE
B
ABSOLUTE ZERO PRESSURE

Absolute, Gage, and Atmospheric Pressures

- Absolute pressure is the algebraic sum of the gauge and atmospheric pressures:

$$p_{abs} = p_{gauge} + p_{atm}$$

- **Atmospheric pressure**, sometimes also called barometric pressure, is the pressure exerted by the weight of air in the atmosphere of Earth (or that of another planet).
- In most circumstances atmospheric pressure is closely approximated by the hydrostatic pressure caused by the weight of air above the measurement point.
- It varies with the altitude and also, at a given place it varies slightly from time to time because of changes in meteorological conditions.

p_{atm} at MSL at 15°C = 101.325 kPa = 1013.25 mbars = 10.34 m of water
= 760 mmHg = 14.696 psi = 33.91 ft of water = 29.92 inHg

The absolute pressure on a gas is 41 psia and the atmospheric pressure is 965 mb abs. Find the gage pressure in psi, kPa, and mb.

Inside front cover: $p_{abs} = 41 \text{ psia}(6.894\,76 \text{ kPa/psi}) = 282.7 \text{ kPa abs}$

Inside front cover: $p_{atm} = 965 \text{ mb abs} = (965 \text{ mb abs})(0.1 \text{ kPa/mb}) = 96.5 \text{ kPa abs}$

Eq. 3.7: Gage pressure $= p_{abs} - p_{atm} = 282.7 - 96.5 = 186.2 \text{ kPa}$ ◀

Gage pressure $= (186.2 \text{ kPa})(10 \text{ mb/kPa}) = 1862 \text{ mb}$ ◀

$$\text{Gage pressure} = \frac{186.2 \text{ kPa}}{6.894\,76 \text{ kPa/psi}} = 27.0 \text{ psi}$$ ◀

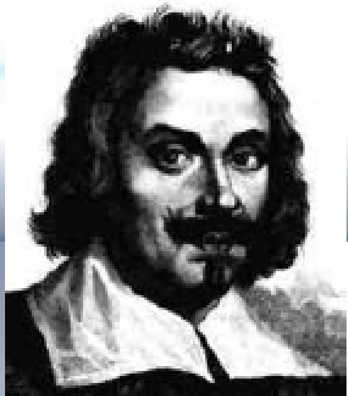
What will be the gauge pressure and absolute pressure of water at a depth 12 m below the surface? Take $\rho_{water} = 1000 \text{ kg/m}^3$ and $P_{atm} = 101 \text{ kN/m}^2$

Solution:

$P_{gage} = \rho g h = 1000 \times 9.81 \times 12$
$= 117.7 \text{ kN/m}^2$

$P_{abs} = P_{gage} + P_{atm}$
$= 117.7 + 101$
$= 218.7 \text{ kN/m}^2$

Barometer



Evangelista Torricelli (1608-1647)

The first mercury barometer was constructed in 1643-1644 by Torricelli. He showed that the height of mercury in a column was 1/14 that of a water barometer, due to the fact that mercury is 14 times more dense that water. He also noticed that level of mercury varied from day to day due to weather changes, and that at the top of the column there is a vacuum.

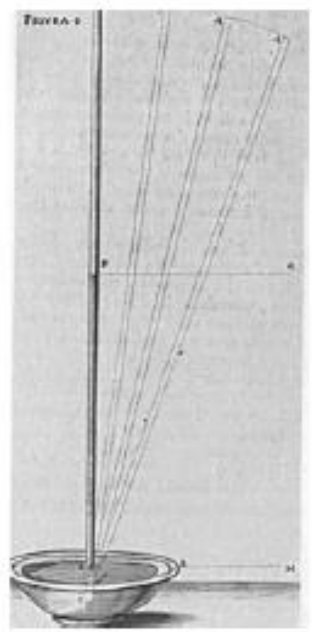
Torricelli studied a theory formulated by Aristotle (384-22 BC) stating that nature abhors a vacuum.

This theory is known as **"horror vacui"**.

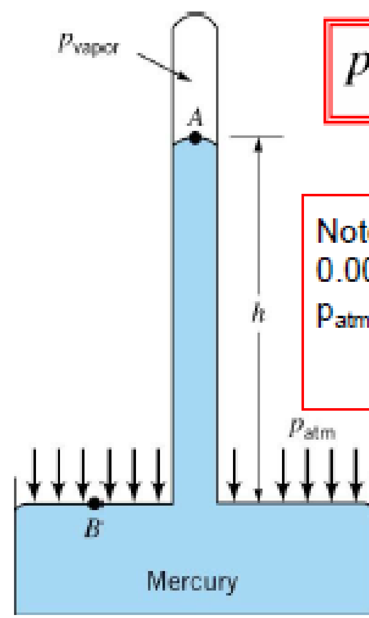
This suggests that nature does not favor absolute emptiness and therefore, draws in matter (gas or liquid) to fill the void.

Torricelli's study led to the discovery of the manometer.

Torricelli's Sketch



Schematic:



$$p_{\text{atm}} = \gamma h + p_{\text{vapor}}$$

Note, often p_{vapor} is very small, 0.0000231 psia at 68° F, and p_{atm} is 14.7 psi, thus:

$$p_{\text{atm}} \approx \gamma h$$

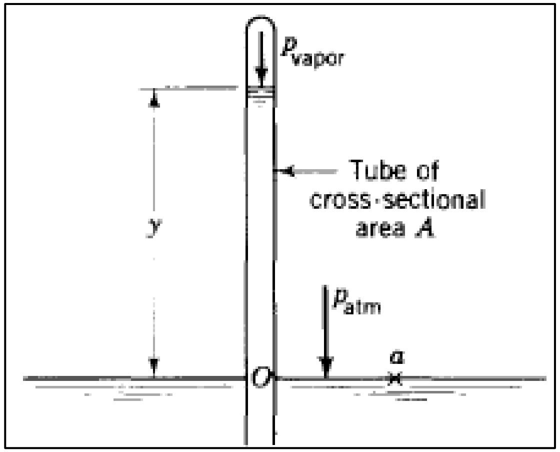
Barometer

- The absolute pressure of the atmosphere is measured by the barometer.
- If a tube has its lower open end immersed in a liquid which is exposed to the atmospheric pressure, and if air is exhausted from the tube, the liquid will rise in it as shown in the Figure below.
- If the air were completely exhausted, the only pressure on the surface of the liquid in the tube would then be that of its own vapor pressure and the liquid would have reached its maximum height.
- The pressure at 'O' within the tube and at 'a' at the surface of the liquid outside the tube must be the same; i.e., $p_o = p_a = p_{atm}$.
- From the conditions of static equilibrium of the liquid above 'O' in the tube of cross-sectional area A (i.e. $\Sigma F_y = 0$),

$$F_{atm} - F_{vapor} - W = 0$$

$$p_{atm}A - p_{vapor}A - \gamma\, Ay = 0 \qquad \text{as } W = \gamma\, V$$

$$p_{atm} = \gamma\, y + p_{vapor}$$



- If the vapor pressure on the surface of the liquid in the tube were negligible, then

$$p_{atm} = \gamma\, y \qquad \text{where } y = h$$

Barometer

- The liquid used in barometers is usually mercury, because its density is sufficiently great to enable a reasonably short tube to be used, and also because its vapor pressure is negligibly small at ordinary temperatures.

- If some other liquid were used, the tube necessarily would be so high as to be inconvenient and its vapor pressure at ordinary temperatures would be appreciable; hence a nearly perfect vacuum at the top of the column would not be attainable.

- The height attained by the liquid would consequently be less than the true barometric height and would necessitate applying a correction to the reading.

- When using a mercury barometer, to get as accurate a measurement of atmospheric pressure as possible, corrections for capillarity and vapor pressure should be applied to the reading.

Problem:

What would be the reading on a barometer containing carbon tetrachloride at 68°F at a time when the atmospheric pressure was equivalent to 30.26 inHg?

Solution:

p_{atm} = 30.26 (inHg) x 14.7 (psia)/29.92 (inHg)

p_{atm} = 14.86 psia

For carbon tetrachloride at 68°F (Table A.4):

ρ = 3.08 slugs/ft^3

p_{vapor} = 1.90 psia

$h = (p_{atm} - p_{vapor}) / \rho g$ since, $h = p / \gamma = p / \rho g$

h = (14.86 – 1.90) x 144 / (3.08 x 32.2)

h = 18.82 ft of carbontetrachloride

Devices for Measurement of Fluid Pressure

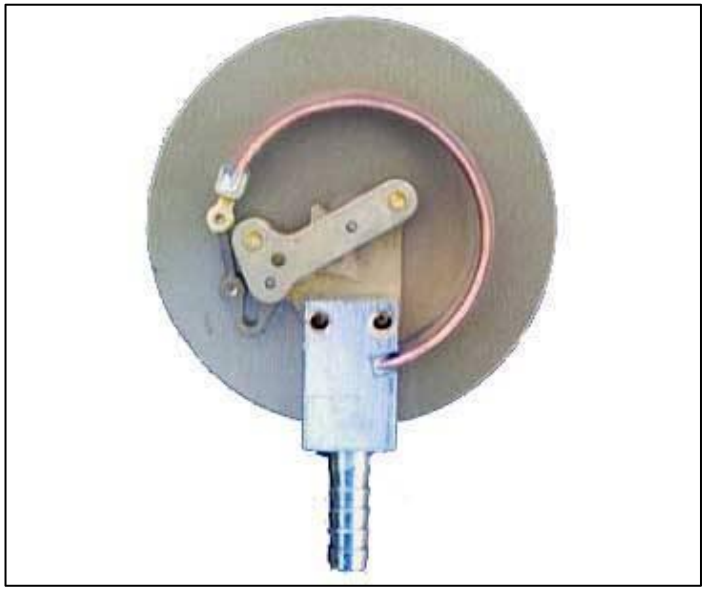
- The principles on which all the pressure measuring devices are based, are almost the same.
- However, for convenient, we may split up the same into the following two types:
  1. By balancing the liquid column (whose pressure is to be found out) by the same or another column; these are also called tube gages to measure the pressure.
  2. By balancing the liquid column (whose pressure is to be found out) by the spring or dead weight; these are also called mechanical gages to measure the pressure.
- The different devices used for measuring the fluid pressure are:
  1. Bourdon's gage
  2. Piezometer tube
  3. Manometer

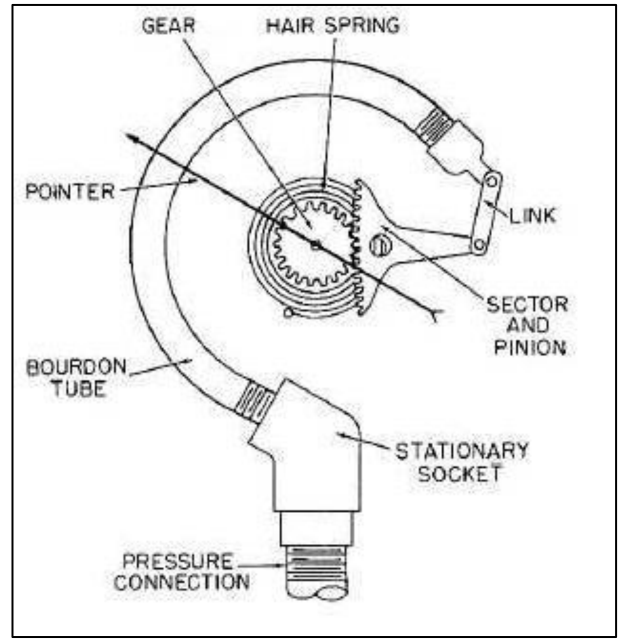
     (i) Simple manometer
     (ii) Differential manometer
     (iii) Micromanometer
     (iv) Inclined manometer

Bourdon’s Gage

- The pressure, above or below the atmospheric pressure may be easily measured with the help of a Bourdon’s gauge.
- In its simplest form, it consists of an elliptical tube, bent into an arc of circle.
- When the gage tube is connected to the fluid (whose pressure is required to be found out) the fluid under pressure flows into the tube and as a result of the increased pressure, the tube tends to straighten itself.
- Since the tube is encased in a circular cover, therefore it tends to become circular instead of straight.
- The moving end of the tube rotates a hand through a simple pinion and sector arrangement, i.e. the elastic deformation of the Bourdon's tube rotates the pointer.
- This pointer moves over a calibrated scale, which directly gives the pressure.
- A Bourdon’s gage is generally used for measuring high pressures.

Bourdon’s Gage





Construction and Working of Bourdon gauge

Bourdon's Gage

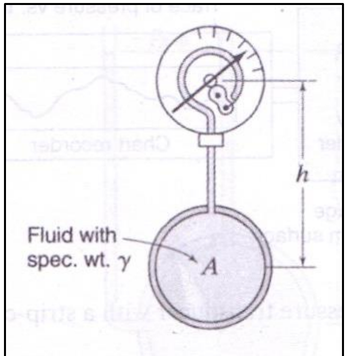


Figure 3.7
Bourdon gage.

- A pressure and vacuum gage combined into one is known as a compound gage.
- The pressure indicated by the gage is assumed to be that at its center.

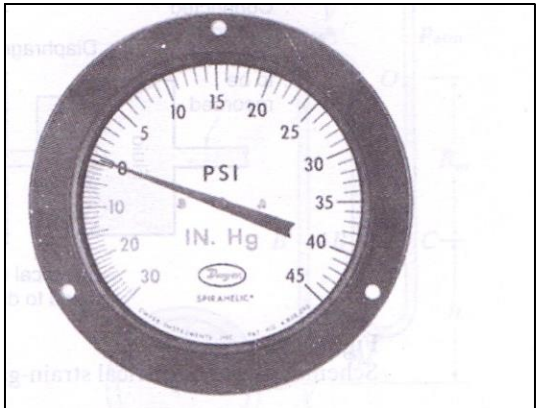


Figure 3.8
Compound pressure and vacuum gage. Pressures in pounds per square inch, vacuums in inches of mercury. (Courtesy of Dwyer Instruments, Inc.)

- If the connecting pipe is completely filled with fluid of the same density as that in A in Figure,
- In SI, p_A (kPa) = gage reading (kPa) + γ h
- In FPS, p_A (psi) = gage reading (psi) + γh (lb/ft^3)/144
- A vacuum gage, or the negative portion of a compound gage is normally graduated to read in 'mm' or 'inches' of mercury.

  h_A (vacuum in mmHg) = gage reading (mm) – h (mm)

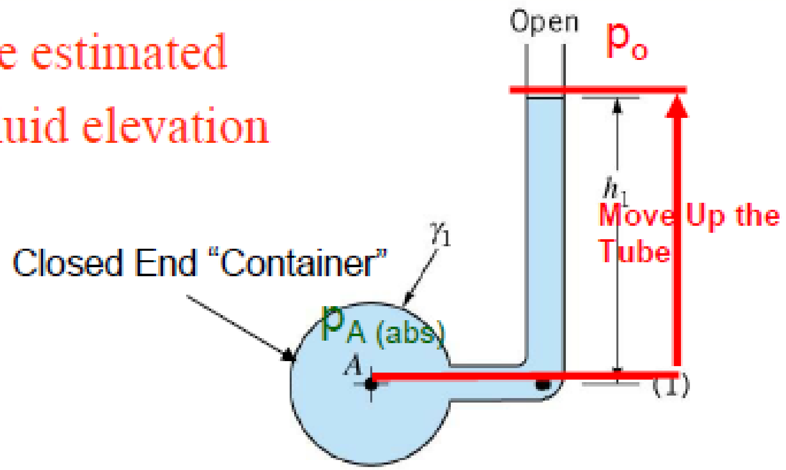
  h (inHg vacuum) = gage reading (inHg) - γh (lb/ ft^3)/144 x (29.92 inHg/14.7 psi)

Piezometer Tube

- A piezometer tube is the simplest form of instrument, used for measuring, moderate pressures.
- It consists of a tube, one end of which is connected to the pipeline in which the pressure is required to be found out.
- The other end is open to the atmosphere, in which the liquid can rise freely without overflow.
- The height, to which the liquid rises up in the tube, gives the pressure head directly ($p = \gamma h$).
- To reduce capillary error the tube diameter should be at least 12 mm.
- For measuring pressure of a flowing fluid, care should always be taken that the tube should not project inside the pipe beyond its surface.
- All burrs and roughness near the hole must be removed, and edge of the hole should be rounded off, and hole should be small (at most 3 mm).
- It may be noted that piezometer tube is meant for measuring gage pressure only as the surface of the liquid in the tube is exposed to the atmosphere.
- A piezometer tube is also not suitable for measuring negative pressure; as in such a case the air will enter in the pipe through the tube.

Piezometer Tube

•Pressure can be estimated by measuring fluid elevation



Disadvantages:
1)The pressure in the container has to be greater than atmospheric pressure.
2) Pressure must be relatively small to maintain a small column of fluid.
3) The measurement of pressure must be of a liquid.

Moving from left to right: $p_{A(abs)} - \gamma_1 h_1 = p_o$

Rearranging: $p_A - p_o = \gamma_1 h_1$

Gage Pressure

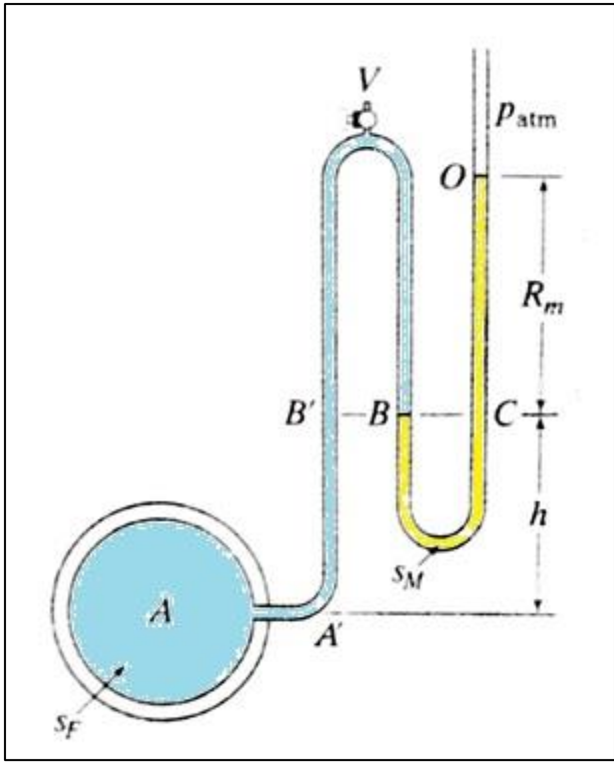
Then in terms of gage pressure, the equation for a Piezometer Tube:

$$p_A = \gamma_1 h_1$$

Note: $p_A = p_1$ because they are at the same level

Simple Manometer

- A manometer is an improved form of a piezometer tube with the help of which we can measure comparatively high pressures and negative pressures also, as the piezometer tube is too tall and cumbersome for high pressures.
- A simple manometer, in its simplest form, consists of a tube bent in U-shape, one end of which is attached to the gage point and the other is open to the atmosphere.
- The liquid used in the simple manometer is, generally, mercury which is 13.6 times heavier than water.
- To determine the pressure at A in terms of the liquid at A, one may write a gage equation based on the fundamental relations of hydrostatic pressures.
- Let s_M be the specific gravity of the fluid F whose pressure is being measured, and R_m be the manometer reading (height OC).
- If y' is the height of a column of measured fluid that would exert the same pressure at C as does the column of manometer Fluid OC (height R_m), then



Simple Manometer

$p_c = p_{B'}$

$p_c = \gamma_M R_m = \gamma_F y'$ since, $p = \gamma h$

$y' = (\gamma_M/\gamma_F) R_m = (\rho_M/\rho_F) R_m = (s_M/s_F) R_m$

$p_c = \gamma (s_M/s_F) R_m$

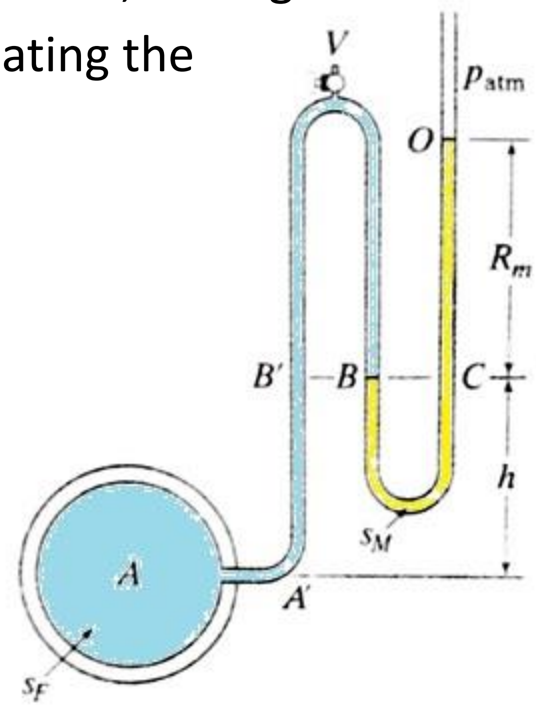
- This is also the pressure at B because the fluid is in balance (same level).
- The pressure at A is greater than this by γh.
- It is helpful to commence the equation at the open end of the manometer with the pressure head there, then proceed through the tube to A, adding terms when descending and subtracting when ascending, equating the result to the head at A, as

$$0 + \gamma (s_M/s_F) R_m + \gamma h = p_A$$

- Also, in terms of heads

$$0 + (s_M/s_F) R_m + h = p_A/\gamma$$

- If the absolute pressure at A is desired, then the zero of the first term will be replaced by the atmospheric pressure of the fluid whose pressure is to be measured.



Example – Simple Manometer

The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of specific gravity 0.9 is flowing. The center of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.

Specific gravity of fluid = $S_1 = 0.9$

Density of fluid = $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900\ kg/m^3$

Specific gravity of mercury = $S_2 = 13.6$

Density of mercury = $\rho_2 = S_2 \times 1000 = 13.6 \times 1000 = 13600\ kg/m^3$

Difference of mercury level = $h_2 = 20\ cm = 0.2\ m$

Height of fluid from A-A = $h_1 = 20 – 12 = 8\ cm = 0.08\ m$

Pressure of fluid in pipe = p

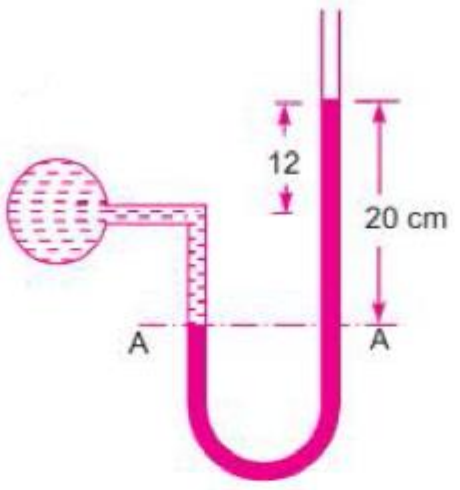
Equating the pressure in the left and right limbs above A-A:

$$p + \rho_1 g\, h_1 = \rho_2 g\, h_2$$

$$p + 900 \times 9.81 \times 0.08 = 13600 \times 9.81 \times 0.2$$

$$p = 26683 – 706 =$$

$$p = 25977\ N/m^2 = 25.977\ KPa$$



Differential Manometer

- It is a device used for measuring the difference of pressures, between two points in a pipe, or in two different pipes.
- A differential manometer, in its simplest form, consists of a U-tube containing a heavy liquid, whose two ends are connected to the points, whose difference of pressures is required to be found out.
- If the fluids in the two pipes are of the same density, then,

$$p_A - \gamma h_A + \gamma(s_M/s_F)\, R_m + \gamma h_B = p_B$$

$$p_A - p_B = \gamma h_A - \gamma h_B - \gamma(s_M/s_F)\, R_m$$

- And, in terms of heads,

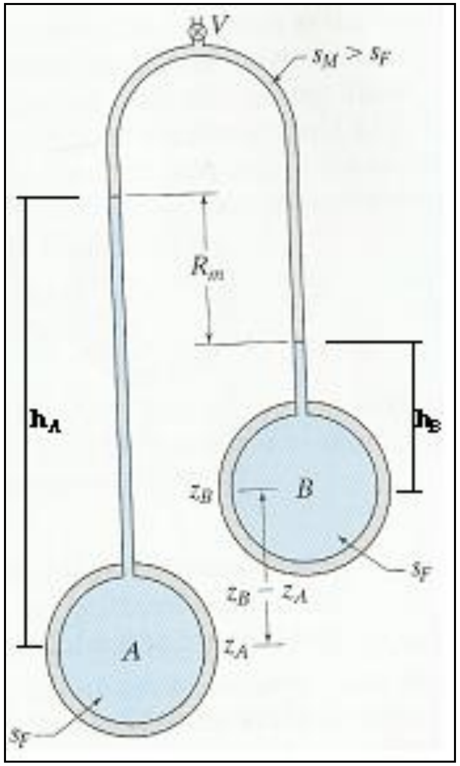
$$p_A/\gamma - p_B/\gamma = h_A - h_B - (s_M/s_F)\, R_m$$

- From the diagram,

$$h_A - h_B = z_B - z_A + R_m$$

$$p_A/\gamma - p_B/\gamma = z_B - z_A + R_m - (s_M/s_F)\, R_m$$

$$p_A/\gamma - p_B/\gamma = z_B - z_A + (1 - s_M/s_F)\, R_m$$

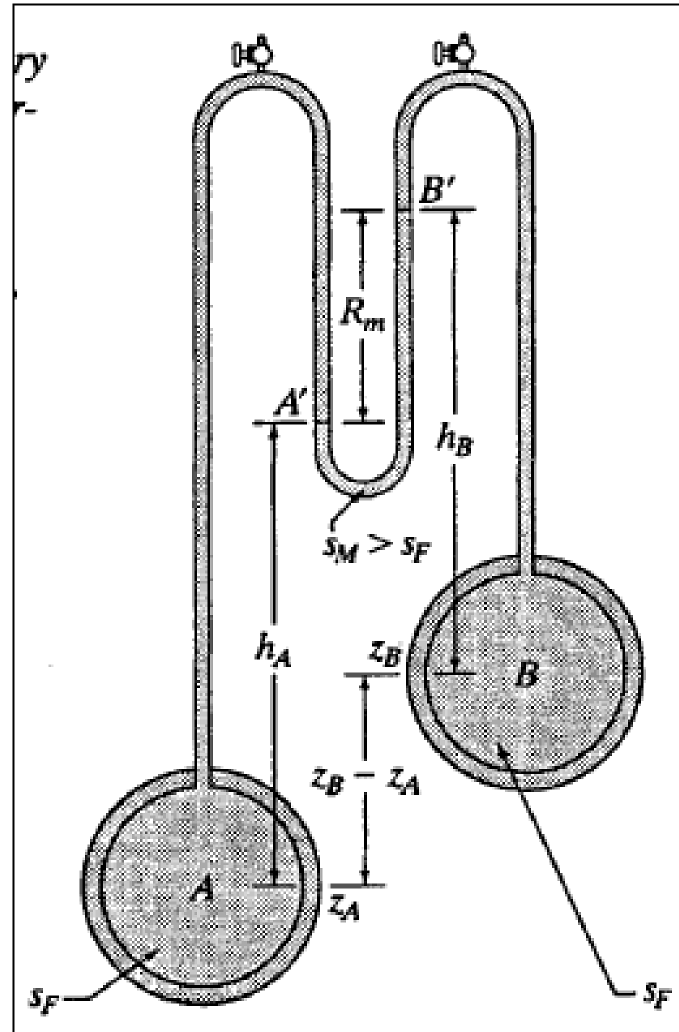


- The differential manometer, when used with a heavy fluid like mercury, is suitable for measuring large pressure differences.
- For a small pressure difference, light fluid, like oil or even air may be used.
- Naturally the fluid must be one which do not mix with the fluid in A or B.

Example – Differential Manometer

(a) Two vessels are connected to a differential manometer using mercury (s = 13.56), the connecting tubing being filled with water. The higher pressure vessel is 1.5 m lower in elevation than the other. Room temperature prevails. If the mercury reading is 100 mm, what is the pressure difference in m of water, and in kPa?

(b) If carbon tetrachloride (s = 1.59) were used instead of mercury, what would the manometer reading be for the same pressure difference?



Solution:

Given $z_B - z_A = 1.5$ m

(a)

$$\frac{p_A}{\gamma} - \frac{p_B}{\gamma} = z_B - z_A + \left(\frac{s_M}{s_w} - 1\right) R_m$$

$$\Delta p = 1.5 + \left(\frac{13.56}{1} - 1\right) 0.10$$

$$\Delta p = 2.76 \text{ m}$$

$$\Delta p = 2.76 \text{ x } 9.81 = 27 \text{ kPa}$$

(b)

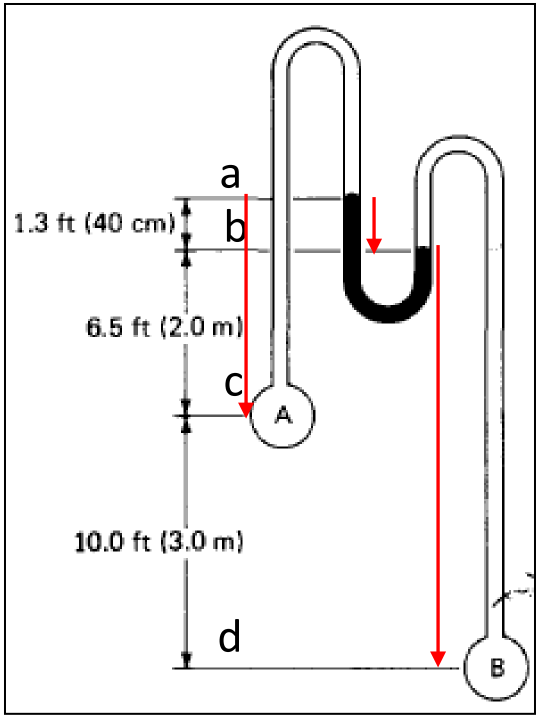
$$\Delta p = z_B - z_A + \left(\frac{s_C}{s_w} - 1\right) R_m$$

$$2.76 = 1.5 + \left(\frac{1.59}{1} - 1\right) R_m \rightarrow 2.76 - 1.5 = 0.59 R_m$$

$$R_m = 2.13\ m\ of\ CCl_4$$

Example – Differential Manometer

Liquid A weighs 8.4 kN/m^3, liquid B weighs 12.3 kN/m^3. Manometer liquid is mercury. If the pressure at B is 200 kN/m^2, find the pressure at A.



Solution:

Express all pressure heads in terms of the liquid in bulb B

$$\frac{p_A}{\gamma_B}-(z_a-z_c)\left(\frac{\gamma_A}{\gamma_B}\right)+(z_a-z_b)\left(\frac{\gamma_M}{\gamma_B}\right)+(z_b-z_d)\left(\frac{\gamma_B}{\gamma_B}\right)=\frac{p_B}{\gamma_B}$$

$$\frac{p_A}{\gamma_B}-2.4\left(\frac{\gamma_A}{\gamma_B}\right)+0.4\left(\frac{s_M\times\gamma_W}{\gamma_B}\right)+5.0=\frac{p_B}{\gamma_B}$$

$$\frac{p_A}{\gamma_B}-2.4\left(\frac{8.4}{12.3}\right)+0.4\left(\frac{13.55\times 9.81}{12.3}\right)+5.0=\frac{p_B}{\gamma_B}$$

$$\frac{p_A}{\gamma_B}-1.64+4.32+5.0=\frac{200}{12.3}$$

$$\frac{p_A}{\gamma_B}=8.6$$

$$p_A=8.6\times 12.3$$

$$p_A=106\ kN/m^2$$