CHAPTER 8
WAVES

Waves:

Any moving disturbance is called wave

Three Types:

1. Mechanical Waves:
   Need medium for propagation

2. Electromagnetic Waves:
   Requires no medium

3. Particle Waves
   Waves associated with elementary particles

* Vibration is necessary for the formation of waves

* Waves transport energy without transporting matter. The energy transportation is carried by a disturbance, which spreads out from a source

* Whatever may be the nature of waves, the mechanism by which it transports energy is the same. A succession of oscillatory motions are always involved. The wave is generated by an oscillation in the vibrating body and propagation of wave through space is by means of oscillations

* PROGRESSIVE ~~EXAMPLES~~ WAVES
  → Propagates from particle to particle
  → Elasticity is necessary
  (The more elastic, the faster is the speed of propagation)

A wave which transfers energy by moving away from the source of disturbance, is called a progressive or travelling wave

There are two kinds of progressive waves:
1. Transverse Waves
2. Longitudenal Waves

1. Transverse Waves:
Particles propagate perpendicular to direction of propagation of waves

2. Longitudenal Waves:
Particles propagate parallel to direction of propagation of waves.

Both transverse and longitudenal waves can be set in solids. In fluids, however, transverse waves die out very quickly and usually cannot be produced at all. This is why sound waves are longitudenal in nature

In longitudenal waves, speed of compression as compared to rarefaction is same

STANDING WAVES:

Not progressive waves bcz the energy doesnot travel from one point to another

PERIODIC WAVES:

A wave that repeats itself after regular intervals of time.

Continuous, regular, rythmic disturbances in a medium result from periodic vibrations of a source which cause periodic waves in that medium.

A good example of a periodic vibrator is an oscillating mass-spring system

* Sound travels faster in solid than air. It is bcz solids are more elastic than gases

WAVELENGTH (λ)

Distance b.w two consecutive crest or trough is called wavelength

OR

Distance b.w two consecutive compressions or rarefactions

WAVE SPEED

→ Distance travelled by a wave per unit time.

$$v = f\lambda$$

* Speed of wave depends on:
  1. Type of wave
  2. Properties of medium

* Speed of Transverse Wave Pulse In An Elastic Stretched String or Spring

$$v = \sqrt{\frac{T}{m/l}}$$

where T : tension

m/l : mass per unit length

* Speed of transverse wave in a well stretched and thin string is greater as compared to a loose and thick one.

* Relation of Transverse Wave Speed And Area

$$v \propto \frac{1}{\sqrt{A}}$$

Derivation:

$$\rho = \frac{m}{V}$$

$$m = \rho V$$

$$m = \rho A l$$

Put in eq $v = \sqrt{\frac{Tl}{m}}$

$$v = \sqrt{\frac{Tl}{\rho A l}}$$

$$v = \sqrt{\frac{T}{\rho A}} \text{ --- } (*)$$

$$v \propto \frac{1}{\sqrt{A}}$$

* Relation of Transverse Wave Speed and 'r'

$$v = \frac{1}{r}\sqrt{\frac{T}{\rho \pi}}$$

Derivation:

$$A = \pi r^2$$

$$v = \sqrt{\frac{T}{\rho \pi r^2}}$$

$$v = \frac{1}{r}\sqrt{\frac{T}{\rho \pi}}$$

* WAVE SPEED OF COMPRESSIONAL OR LONGITUDENAL WAVES

$$v = \sqrt{\frac{E}{\rho}}$$

E : Elasticity
ρ : density

$\frac{E}{\rho}$ Solid > $\frac{E}{\rho}$ Liquid > $\frac{E}{\rho}$ Gas

* If Humidity is increased, speed of sound increases (bcz ρ is decreased)

FREQUENCY OF WAVES

The number of waves passing through a certain point in unit time is called frequency of the wave

* Frequency of the wave is equal to the frequency of the simple harmonic oscillating source

TIME PERIOD OF WAVE

* The time during which a wave passes through a certain point is called the time period of the wave

* The period of the wave is equal to the period of the simple harmonic oscillator

INTENSITY OF WAVE

The amount of energy transmitted per second per unit area placed perpendicular to the direction of propagation of waves is called intensity of the waves indicated by I.

$$I = \frac{E}{A \cdot t}$$

$$I = \frac{P}{A} \qquad \text{As } P = \frac{E}{t} \qquad (P: \text{Power})$$

* Relation of Intensity and Amplitude (x_0)

$$I \propto x_0^2$$

Derivation:

$$I = \frac{E}{A \times t}$$

$$= \frac{1}{2} k x_0^2$$

$$= \frac{\frac{1}{2} k x_0^2}{A \times t}$$

$$I \propto x_0^2$$

* MCQ Find the ratio of maximum to minimum intensity (as a result of superposition

$I_{max} \propto (x_2+x_1)^2$

$I_{min} \propto (x_2-x_1)^2$

$$\frac{I_{max}}{I_{min}} = \frac{(x_2+x_1)^2}{(x_2-x_1)^2}$$

SPEED OF SOUND

* The speed of mechanical wave in a medium depends upon two characteristics of medium:
  1. Density of the medium
  2. Elasticity of the medium

* The first unsuccessful attempt to determine the speed of Light was made by Galileo Galilee

* An expression for speed of sound in any medium was derived by Newton

$$V = \sqrt{\frac{E}{\rho}}$$

$$V = \sqrt{\frac{P}{\rho}}$$

$$V = \sqrt{\frac{\rho_m g h}{\rho}}$$ ρ_m : Density of mercury at STP

$\rho_m = 13.6\ gcm^{-3}$ 13546 kg/m^3

$g = 980\ cms^{-2}$

$h = 76\ cm$

Density of air = $\rho = 0.001293\ gcm^{-3}$

$v = 281\ ms^{-1}$

But the experimental value of speed of sound s 332 ms^{-1}. Thus the theoretical value is 16% less han the experimental value.

LAPLACE'S CORRECTION

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$\gamma P = E =$ Modulus of Elasticity for Adiabetic Process

$$v = \sqrt{\frac{\gamma \rho_m g h}{\rho}}$$

For air $\gamma = 1.42$

$v = 333\ ms^{-1}$

↳ Speed of sound at 0°C

A French mathematician Laplace explained the discrepancy in theoretical and experimental values of speed of sound in gas

Laplace argued that sound waves are longitudenal waves which consist of compressions and rarefactions

* At compression: The temp of air rises due to increase of pressure

* At rarefaction: Cooling effect is produced

So temp of gas doesnot remain constant hence Boyle's law is not applicable

* Laplace also said that air is a very poor conductor of heat and sound waves travel through it with a great speed ($330 ms^{-1}$). During compression air cannot lose heat and cannot gain heat during rarefaction. So the propagation of sound waves through air or gas is an adiabatic process. The rapid changes in the air pressure, volume and temperature takes place under adiabatic conditions.

Adiabetic Process:

A completely isolated process in which no heat transfer can take place

Laplace's Correction:

Laplace pointed out that the compressions and rarefactions occur so rapidly that heat of compressions remain confined to the regions which have undergone an expansion. Hence temp of the medium doesnot remain constant. In such case Boyle's law takes the form:

$$PV^{\gamma} = \text{constant}$$

Speed Of Soundin gas:-

$$v = \sqrt{\frac{\gamma RT}{m}}$$

* FACTORS AFFECTING SPEED OF SOUND IN AIR

1. DENSITY

$$v \propto \frac{1}{\sqrt{d}}$$

2. MOISTURE

→ Presence of moisture in air reduces resultant density

→ So speed of sound increases with humidity

3. PRESSURE

v is independent of pressure

✱ TEMPERATURE

→ For solids and liquids → change in speed of sound with temp is very small and can be neglected

→ For gases :

The increase in speed of sound with temp in gas is about $0.6 ms^{-1}$ for each $1°C$ rise in temp

$$v \propto \sqrt{T}$$

$$\frac{v}{v_0} = \sqrt{\frac{T}{T_0}}$$

$T_0 = 273 K$

$v_0 = 332 ms^{-1}$

At $0°C$, $v_0 = 332 ms^{-1}$

$$v = v_0 + \frac{332 t°C}{546}$$

$$v = v_0 + 0.61 t°C$$

$$v_0 = v - 0.61 t°C$$

* **WIND**
  → Speed of sound in direction of wind relative to ground

$$v + v_w$$

  → Speed of sound against the wind

$$v - v_w$$

v_w : speed of wind

* Speed of sound in water is ~~&~~ $1430\ ms^{-1}$

PRINCIPLE OF SUPERPOSITION OF WAVES

"When two or more waves are passing through the same region at the same time the total displacement at the point where they interact, is equal to the vector sum of the individual displacement due to each pulse at that point"

INTERFERENCE OF WAVES

"The effect produced by the superposition of waves from two coherent sources, passing through the same region is known as interference"

"Superposition of two waves having the same frequency and travelling in the same direction results in a phenomenon called interference"

* Coherent Sources:
  → having a constant phase difference
  → same frequency
  → same waveform

* Constant phase difference means having same frequency and amplitude

* CONSTRUCTIVE INTERFERENCE
  * Waves must be in-phase (0°)
  * Path difference must be either zero or integral multiple wavelength of λ.
    i.e $P.d = 0, 1\lambda, 2\lambda$
    or $P.d = m\lambda$
    where $m = 0, 1, 2, 3 \ldots$

* DESTRUCTIVE INTERFERENCE
  * Waves are out of phase (180°)
  * Path difference:
    $$d = \left(m + \frac{1}{2}\right)\lambda$$

BEATS

"The periodic vibration in the loudness of sound which is heard when two notes of nearly the same frequency are played simultaneously, is called beats'

"The periodic altermations of sound b/w maximum and minimum is known as beats"

* $N = f_1 - f_2$ → Beat Frequency

Number of beats per second is equal to the difference between the frequencies of the two forks

* PERIOD OF BEAT

$$f = \frac{1}{T}$$

f: beat frequency
$f = f_1 - f_2$

* For humans, if frequency $f > 10\,Hz$ then the sound can be distinguished.

* If $f < 10\,Hz$ then beats are observed

APPLICATIONS OF BEATS

1. FOR FINDING UNKNOWN FREQUENCIES

$$f_A = f_B \pm N$$

2. FOR TUNING THE MUSICAL INSTRUMENTS

Tuning is the process of adjusting the pitch of one or many tones from musical instruments until they form a desired arrangement

→ Tuning may be carried out by sounding two pitches and adjusting one of them to match or to relate the other

MCQ: NUMBER OF BEATS PRODUCED

$$n = f_A - f_B$$

MCQ: NUMBER OF BEATS OBSERVED

$$n = f_A - f_B$$

If n comes out to be greater than 10, then number of beats observed will be zero

MCQ: If $f_A = 300\,Hz$, $f_B = 250\,Hz$ Find

i) No. of beats observed = ?

ii) No. of beats produced = ?

$n = f_A - f_B$

$= 300 - 250$

$n = 50$

So no. of beats produced = 50 Hz

No. of beats observed = 0 Hz

RESULTS OF SUPERPOSITION OF WAVES

1. Interference
2. Beats
3. Standing Waves

1 INTERFERENCE:

Two waves having same frequency and travelling in the same direction.

2. BEATS:

Two waves of slightly different frequencie and travelling in same direction

3. STATIONARY WAVES

Two waves of equal frequency travelling in opposite direction

REFLECTION OF WAVES

* Regular Reflection

  Angule of incidence = Angle of refraction

  $\angle i = \angle r$

* Irregular Reflection

  $\angle i \neq \angle r$

* When reflection take place from a boundary of denser medium then phase change of 180° occur means shape changes i-e crest will convert to trough and vice versa

* When reflection takes place from the boundary of rare medium no phase change will occur

ECHO

The reflection of an original sound from a certain object is received at 0.1s later than the direct sound is called echo

$T = 0.1 sec$

$S = vt$

$= 332 \times 0.1$

$= 33.2$

$d = \frac{s}{2} = 17m$

The effective distance for echo is 17 m.

✤ REVERBERATION

When the reflecting surface is at a distance less than 17 m away from the source of sound, then the echo follows, so close upon the direct sound that they cannot be distinguished. This effect is known as reverberation

STATIONARY WAVES

→ When two plane waves having the same amplitude and frequency, travelling with the same speed in opposite direction along a line, are superposed, a wave obtained

→ Stationary waves can only be set up in mechanical waves

→ When reflected wave and original wave superpose with each other, they form stationary waves

Node: The end points of the cord which donot vibrate at all are called nodes, indicated by N.

Antinode: Midpoint b/w two successive nodes
→ Here amplitude of oscillation is maximum
→ Indicated by A

* Distance b/w two successive nodes or anti-nodes
$\lambda/2$

* Distance b/w adjacent node and antinode
$\lambda/4$

* At Node:
Energy → Minimum
Pressure → Max
Density → Max

* At Antinode
Energy → ~~Min~~ Max
Pressure → ~~Max~~ Min
Density → ~~Max~~ Min

Net Transfer of energy = Zero

* CASE 1: PLUCKED AT ITS MIDDLE i-e $L/2$

1 loop = $\lambda/2$

* Fundamental frequency or first harmonic (f_1)

$$f_1 = \frac{1}{2L}\sqrt{\frac{T \times L}{m}}$$

$$\boxed{f_1 = \frac{v}{2L}}$$

or $$f_1 = \frac{1}{2l}\sqrt{\frac{T}{\rho A}}$$

or $f_1 = \frac{1}{\gamma}\left[\frac{1}{2l}\sqrt{\frac{T}{\rho\pi}}\right]$

$\lambda_1 = 2L$

CASE 2:

STRING PLUCKED AT QUARTER LENGTH (L/4)

No. of loops formed = 2

$$f_2 = \frac{1}{L}\sqrt{\frac{T \times L}{m}}$$

This frequency is known as second harmonic or first overtone.

$$f_2 = 2f_1$$

CASE 3:

STRING PLUCKED AT ONE SIXTH OF LENGTH (L/6)

No. of loops formed = 3

$$f_3 = 3f_1$$

GENERAL CASE:
n loops

$$\lambda_n = \frac{2L}{n}$$

$f_n = n f_1$ $\qquad$ $f_n = n\left(\frac{v}{2L}\right)$

→ n frequency known as n^{th} harmonic
→ $(n-1)^{th}$ overtone

1^{st} Harmonic:
Nodes : 2
Antinodes : 1

2^{nd} Harmonic:
Nodes : 3
Antinodes : 2

3^{rd} Harmonic:
Nodes : 4
Antinodes : 3

n^{th} Harmonic:
Nodes : n+1
Antinodes : n

* When $f_1 = 50Hz$. If it is plucked at $L/10$.
What will be frequency

No. of loops $= \frac{10}{2} = 5$

$f = nf_1$

$= 5 \times 50$

$f = 250$

* $f_1 = 100Hz$. What will be frequency for 3rd overtone

$n = 3+1$

$= 4$

$f_4 = 4n_1$

$= 4 \times 100$

$= 400 Hz$

RESONANCE OF AIR COLUMN AND ORGAN PIPES

* Stationary waves can be set up in the air column.

* Resonating Column: The point where we hear loud sound is known as resonating column.

The resonance occurs when the frequency of the periodic force due to tuning fork becomes equal to the fundamental frequency of the air column.

* CLOSED ORGAN PIPE

Open from one side closed from the other.

* Node: Formed At Close End
* Antinode: Formed At Open End

FIRST HARMONIC

Nodes = 1

Antinodes = 1

$$f_1 = \frac{v}{4L}$$

n^{th} Harmonic:

Nodes = n

Antinodes = n

$$f_n = 2n-1\left(\frac{V}{4L}\right)$$

* OPEN ORGAN PIPE

* FUNDAMENTAL FREQUENCY

~~Number of Antinodes =~~

No. of nodes = 1

No. of Antinodes = 2

$$f_1 = \frac{V}{2L}$$

* n^{th} HARMONIC

No. of nodes = n

No. of antinodes = $n+1$

$$f_n = \frac{nV}{2L}$$

* RADAR WAVES:

Radio Audio Detection and Ranging waves

DOPPLER EFFECT

The apparent change in the frequency of sound, caused by the relative motion of either the source of sound or listener or both, is called Doppler effect

* Doppler effect inter relates the measured frequency of the wave to the relative velocity of the source of sound and receiver.

* Doppler effect can be observed for both sound and light

* WHEN BOTH SOURCE AND LISTENER ARE STATIONARY

$$v = f\lambda$$

$$f = \frac{v}{\lambda}$$

* WHEN SOURCE IS MOVING AND LISTENER IS AT REST

* SOURCE MOVING TOWARDS STATIONARY LISTENER

$$f' = \frac{v}{v-a} f$$

If the sounding source is approaching a stationary listener the frequency of sound increases. As a result pitch of sound increases.

* SOURCE MOVING AWAY FROM STATIONARY LISTENER

$$f' = \frac{v}{v+a} f$$

As $f' < f$, so pitch of sound decrease when the sounding source is moving away from stationary listener

* SOURCE IS AT REST AND LISTENER IS MOVING

Speed of sound = v
Speed of listener = b

* LISTENER MOVES AWAY FROM STATIONARY SOURCE

$$f' = \frac{v-b}{v} f$$

$f' < f$

* LISTENER MOVES TOWARDS STATIONARY SOURCE

$$f' = \frac{v+b}{v} f$$

So $f' > f$

* WHEN SOURCE AND LISTENER BOTH MOVES

v = speed of sound
a = speed of source
b = speed of listener

* SOURCE AND LISTENER MOVES TOWARDS EACHOTHER

$$f' = \frac{v+b}{v-a} f$$

$f' > f$

* SOURCE AND LISTENER MOVES AWAY FROM EACHOTHER

$$f' = \frac{v-b}{v+a} f$$

$f' < f$

FREQUENCY A HUMAN EAR CAN HEAR

20 Hz — 20,000 Hz

* **ULTRASONICS WAVES**

The sounds of frequencies higher than 20,000 Hz

* An ultrasonic wave is a pressure wave which has an extremely short wavelength bcz of its high frequency.

→ A wave is affected only by an object which is larger than its wavelength.

Read Pg 236 – 238

* There is no polarization property in sound waves

* A star moving away from the observer appears as red shift

* The earthquake is mechanical wave in nature

* A sound wave enters from air to water, its wavelength : Increases

* Transverse waves donot propagate through gases bcz gases have no : Bulk Modulus

* Electromagnetic waves are transverse in nature

* Relation b.w Phase Difference and Path Difference
  $\lambda = 360^\circ$

* When a stationary wave is formed, then its frequency is : → same as that of individual waves.

* At what temp speed of sound becomes double than at 0°C ?
  Ans. 1092 K

* For Transmitted Light constructive interference takes place at centre

* In the forced oscillation of a particle, the amplitude is max for a frequency ω_1 of the force while the energy is max for a frequency ω_2 of the force then:

$$\omega_1 = \omega_2$$

Reason : Both amplitude and energy get maximized when the frequency is equal to the natural frequency (Resonance)