CHAPTER 7
OSCILLATION

SIMPLE HARMONIC MOTION

$$F = -kx$$

F ↓ Applied Force; kx ↘ Elastic Restoring Force

$$a \propto -x$$
$$a = -\omega^2 x$$

$$\omega^2 = \frac{k}{m} \Rightarrow \text{constant}$$

k: constant → Depend on nature (physical shape and structure of spring)

* CHARACTERISTICS OF SHM

1. SHM always along a straight line
2. Acceleration depends upon displacement directed towards the mean position
   - At mean position : $a = 0$
   - At extreme position : a = maximum
3. Velocity :
   - At mean : Maximum
   - At extreme : Minimum

4. K.E

At mean : Maximum

At extreme : Minimum

5. P.E

At mean : Minimum

At extreme : Maximum

6. Total energy is conserved

MASS- SPRING SYSTEM

$$F = -kx$$

* When a body moves towards the mean position, $\vec{v}$ and $\vec{a}$ are parallel; body speeds up so Δv will be positive and $\vec{a}$ will be positive

* When a body moves away from the mean position, $\vec{v}$ and $\vec{a}$ are anti-parallel. Body will slow down Δv will be negative and there will be negative $\vec{a}$.

* $\vec{a}$ is always directed towards mean position while direction of velocity is in the direction of the bob or mass.

SPRING CONSTANT OR STIFFNESS CONSTANT (k)

k small for soft spring
k Large for hard spring

k depends on nature of the spring (physical shape and structure ; also depends inversely on length.

$$k \propto \frac{1}{l}$$

* A spring ~~consta~~ having a spring constant equal to k. What happens if the spring is cut into two equally sized pieces.
  Ans : The spring constant of each spring will be '2k'

* If length becomes double :

$$k' = \frac{k}{2}$$

* SERIES COMBINATION OF SPRINGS

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

or $\frac{1}{k} = \frac{k_1 + k_2}{k_1 k_2}$ or $k = \frac{k_1 k_2}{k_1 + k_2}$

* Two Identical Springs Connected in Series

$$k' = \frac{k}{2}$$

$k_{eq} = \frac{k}{n}$

PARALLEL COMBINATION OF SPRINGS

$$k = k_1 + k_2$$

For Identical Springs:

$$k' = nk$$

$k_{eq} = nk$

RATIO OF K_{series} TO $K_{parallel}$

$$\frac{K_s}{K_p} = \frac{1}{n^2}$$

* If spring is cut in two halves then double force is required to produce unit extension. Therefore spring constant becomes double

TIME PERIOD OF SHM

$$T = 2\pi\sqrt{\frac{m}{k}}$$

or $T = 2\pi\sqrt{\frac{l}{g}}$ (oscillating Vertically)

Timpe Period is independent of amplitude

TOTAL ENERGY OF SHM

The total energy of a particle executing SHM is directly proportional to the square of the amplitude

$$E \propto x_0^2$$

As $E_T = \frac{1}{2} k x_0^2$

* EFFECT ON TIME PERIOD WITH CHANGE IN SPRING CONSTANT (k)

If $k' = nk$

$$T' = \frac{T}{\sqrt{n}}$$

* RATIO OF T' to T

$$\frac{T'}{T} = \frac{1}{\sqrt{n}}$$

* Same weights producing different extension mean different values of spring constant (k)

MCQ: A mass-spring system oscillating vertically with a time period, $T = 2\pi\sqrt{x/g}$. If it is taken to moon, its time period will:

a. Increase
b. Decrease
✓c. Remain constant
d. None

Reason: As $T = 2\pi\sqrt{\frac{x}{g}}$, ~~wh~~ with decrease in g, applied force decrease ($W = mg$). With decrease in applied force x also decrease and hence x/g remains constant so T remains constant

* A mass-spring system oscillating horizontally has the same time period as the same mass-spring system oscillating vertically under similar conditions

* ANGULAR FREQUENCY (ω)

$$\omega = 2\pi f$$

* In SHM:

$$V_{max} = A\omega$$

$$a_{max} = A\omega^2$$

* TO CALCULATE % INCREASE IN TIME PERIO WHEN % INCREASE IN LENGTH IS GIVEN

→ If the length of a string is increased by 44%. what will the change in time period be in %age?

$$T = 2\pi\sqrt{l/g}$$

$$T' = 2\pi\sqrt{\frac{l'}{g}}$$

$$l' = l + 44\%\, l$$

$$= l + \frac{44}{100} l$$

$$= l + 0.44 l$$

$$l' = 1.44 l$$

$$T' = 2\pi\sqrt{\frac{1.44}{g}}$$

$$= 1.2\left[2\pi\sqrt{\frac{l}{g}}\right]$$

$$T' = 1.2T$$

$$\Delta T = T' - T$$

$$= 1.2T - T$$

$$\Delta T = 0.2T$$

$$\Delta T\% = \frac{\Delta T}{T} \times \frac{100}{100}$$

$$= \frac{0.2T}{T} \times \frac{100}{100}$$

$$\Delta T\% = 20\%$$

RELATION B·W 'K·E' and 'T' in SHM

$$K.E \propto \frac{1}{T^2}$$

KINETIC ENERGY FOR SHM

$$K.E = \frac{1}{2}k(x_0^2 - x^2)$$

POTENTIAL ENERGY FOR SHM

$$P.E = \frac{1}{2}kx^2$$

* In SHM angle between velocity and acceleration is 90°

RESONANCE

When an external periodic force acts on a vibrating system such that external periodic frequenc matches with the natural frequency of the body the system starts vibration with a large amplitude. This phenomenon is called resonance.

* At resonance, the energy transfer is maximum
* Amplitude increases bcz of absorption of maximum energy

MCQ: The natural frequency of a vibrating string is 5Hz.* Which frequency will resonance occur at:

i) 7 Hz
ii) 13 Hz
iii) 10 Hz ✓
iv) 14 Hz

Reason: Periodic force applied to a body such that the amplitude increase as an integral multiple

DAMPING

The decrease in amplitude with the passage of time (due to loss of energy)

* Resonance is the constructive interference of two waves with the same frequency

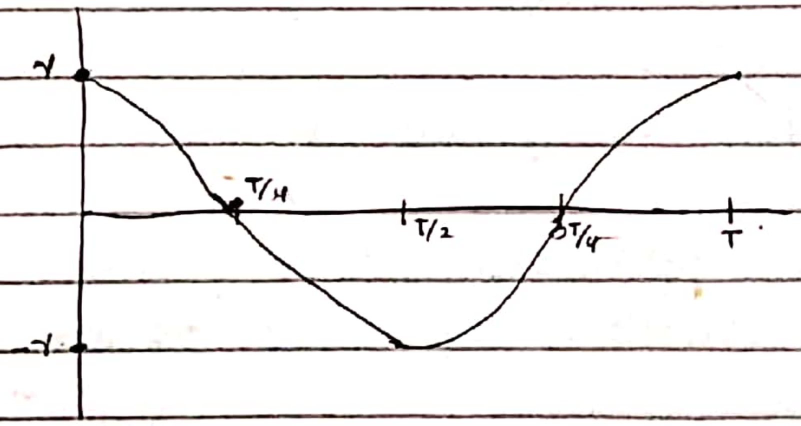
WAVEFORM OF SHM

The graph plotted b·w displacement and time for a wave is called waveform of SHM

CYANOCIDAL FUNCTION:

The function which alternates b·w maximum and minimum is known as cyanocidal.

$x = r\cos\left(\frac{2\pi}{T}\right)(t)$ $x = x_0 \cos\omega t$



t	0	T/4	T/2	3T/4	T
x	r	0	−r	0	r

Whether you choose $x = x_0 \cos\omega t$ or $x = x_0 \sin\omega t$ doesnot really matter too much. It just depends on whether you've chosen $x = 0$ or $x = x_0$ to be the displacement at $t=0$

* If $x = 0$ at $t = 0$ → i-e starting from mean position → then $x = x_o \sin \omega t$

* If $x = x_o$ at $t = 0$ i-e starting from extreme position then $x = x_o \cos \omega t$

PHASE

Phase is the quantity which shows the state of motion of an oscillator

$$x = x_o \cos(\omega t + \phi)$$

$\theta = \omega t + \phi$ is the phase angle

The quantity ϕ represents the phase difference b/w the states of motion of two oscillators

Phase Shift:

Phase shift is any change that occurr in the phase of one quantity, or in the phase difference between two or more quantities

Phase Difference: ϕ

Phase difference is the difference, expressed in degree or time, between two waves having the same frequency and referenced to the same point in time

* In Phase:

Two oscillators having the same frequency and no phase difference are said to be in phase.

* Out of Phase

Two oscillators that have the same frequency and different phases have a phase difference and the oscillators are said to be out of phase with each other.

* Path Difference:

Path difference is the difference b/w the distance travelled by two waves meeting at a point

* The acceleration of a body executing SHM leads the velocity by a phase $\pi/2$ radian.

* If the elevator is accelerated upward the net acceleration is the sum of 'a' and 'g'

If the elevator is accelerated downward the net acceleration is the difference of a and g

* Time Period during Upward Acceleration

$$T = 2\pi\sqrt{\frac{l}{g+a}}$$

* Time Period during Downward Acceleration

$$T = 2\pi\sqrt{\frac{l}{g-a}}$$

* SIMPLE PENDULUM

$T = 1 sec$

$f = 1 Hz$

$\ell = 0.248 cm$

* 2nd Pendulum:

$T = 2 sec$

$f = 0.5 Hz$

$\ell = 0.99 cm$

MASS-SPRING SYSTEM

* $\vec{a}$ is always directed towards mean position while direction of velocity is in the direction of the bob or mass
* When body moves towards the mean position, $\vec{v}$ and $\vec{a}$ are parallel; body speeds up so Δv will be +ve and $\vec{a}$ will be +ve
* When body moves away from the mean position, $\vec{v}$ and $\vec{a}$ are anti-parallel. Body will slow down Δv will be negative and there will be a negative acceleration

* Time taken by a body to complete one amplitude: $T/4$
* Time taken by vibrating body to complete displacement $A/2$ is : $T/8$

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* When Spring is cut in different parts:

$$T' = \frac{T}{\sqrt{n}}$$

where n: number of parts

* If spring is cut in two halves then double force is required to produce unit extension. Therefore spring constant becomes double.

* If spring constant of a spring is k. When the spring is cut into two halves the spring constant of one half is : 2k

* The tension in simple pendulum at extreme position is : $mg\sin\theta$

* Centripetal Acceleration:

$$a_c = \frac{v^2}{r} = r\omega^2$$

* Velocity of Projection : V_p

$$V_p = \omega\sqrt{r^2 - x^2}$$

* For Circular Motion

$$x = r\cos\theta$$

$$x = r\cos\omega t$$

$$x = r\cos\left(\frac{2\pi}{T}\right)(t)$$

SIMPLE PENDULUM

* Length of pendulum = Sum of length of string plus radius 'r' of metallic bob

* The only force acting on a bob is its weight
* The y-component of weight produces motion in the bob

$$a = -g \sin\theta$$

when θ is very small : $a = -g\theta$

$$a = -\left(\frac{g}{l}\right)x$$

$$\frac{g}{l} = \omega^2 = \text{constant}$$

$$a \propto -x$$

* $T = 2\pi\sqrt{\frac{l}{g}}$

  * At extreme Tension is minimum
  * At mean Tension is maximum

* The longer the pendulum the greater will be its time period
* The time period of the pendulum is independent of mass of bob