CHAPTER # 5
ROTATIONAL AND CIRCULAR MOTION

* ANGULAR DISPLACEMENT

If $\Delta\theta$ is small → Angular Displacement is a vector quantity

If $\Delta\theta$ is large → Angular displacement is a scalar bcz then it doesnot follow laws of vector addition

* If rotation is in a.c.w direction → θ will be +ve
* If rotation is in c.w direction → θ will be -ve

ANGULAR ACCELERATION

The direction of angular acceleration is along the axis of rotation

Relation B.w Angular And Linear Displacement

$$S = r\theta$$

Relation B.w Angular And Linear Velocity

$$v = r\omega$$

$$(v = r\omega \sin\theta)$$

Relation B.w Linear And Angular Acceleration

$$a = r\alpha$$
$$(a = r\alpha \sin\theta)$$

* $S = r\theta$ is only applicable when r and θ ~~is~~ are perpendicular

ANGULAR VELOCITY

$$\omega = \frac{\theta}{t}$$

* Direction : Along x- axis of rotation and can be found by Right Hand Rule

* Direction of Angular Velocity doesnot change during the motion

$$\omega = \frac{2\pi}{T}$$

or $\omega = 2\pi f$

* Angular Velocity For Second Hand

$$\omega = \frac{2\pi}{T}$$
$$= \frac{2\pi}{60s}$$
$$\omega = \frac{\pi}{30}\ rad\ s^{-1}$$

* Angular Velocity For Minute Hand

$$\omega = \frac{2\pi}{60\,min}$$

$$\omega = \frac{\pi}{30}\ rad/min$$

$$\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$

* Relation b·w v, r, f

$$v = 2\pi r f$$

ANGULAR ACCELERATION

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Angular Velocity

↙ Magnitude

↘ Direction (No change)

Magnitude

↙ Speed Increase (+ve) ↓ Parallel to velocity

↘ Speed Decrease (−ve) ↓ Anti-parallel to velocity

TANGENTIAL ACCELERATION

In rotational motion, tangential acceleratio
is a measure of how quickly a tangential velocity changes. It always acts perpendicular to the centripetal acceleration of a rotating object.

$$a_T = \alpha \times r$$

CENTRIPETAL ACCELERATION

$$a_c = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r}$$

In Angular Form:

$$a_c = -\omega^2 r$$

A force always causes the centripetal acceleration:

* For a swing ball game, it is tension in the string
* For a satellite, it is the force of gravity
* For a car moving around a corner, it is frictional force b.w car and the road

CIRCULAR MOTION

When a body moves such that its distance from a fixed point remains constant.
In circular motion, direction of velocity chang at each point and hence acceleration is produce

In circular motion there are two types of acceleration:
1. Tangential acceleration which is responsible for change in magnitude
2. Centripetal Acceleration which is responsible for change in direction

UNIFORM CIRCULAR MOTION

* The magnitude of velocity is constant but the direction varies, which means angular velocity or speed is constant, hence the tangential acceleration is zero

$$\omega = \text{constant}$$
$$a_T = 0$$

* But the centripetal acceleration is always present which is responsible for the circular path as it is always directed towards the centre.

Uniform Circular Motion depends on the centripetal component which in turn depends upon velocity and radius which are constant, so centripetal

Finding Net Acceleration:

$$a = \sqrt{a_T^2 + a_R^2}$$

In uniform motion, only centripetal acceleration is present
In non-uniform motion, both centripetal as well as tangential acceleration is present so we take Resultant

Horizontal Circle

* In horizontal circle, tension is constant at all points and this tension provides necessary centripetal force.

$$F_{net} = F_c = T$$

Relation B.w F_c and Time Period

$$F_c = \frac{4\pi^2 mr}{T^2}$$

Derivation:

$$F_c = mr\omega^2$$

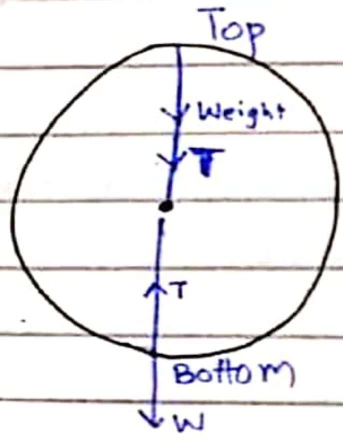
$$= mr\left(\frac{2\pi}{T}\right)^2$$

$$F_c = \frac{4\pi^2 mr}{T^2}$$

Relation B.w F_c and Frequency

$$F_c = 4\pi^2 m r f^2$$

VERTICAL CIRCLE



At Top:

$$F_c = W + T$$
$$T = F_c - W$$

At Bottom:

$$F_c = T - W$$

So At Top: Tension is minimum
At Bottom: Tension is maximum

* A stone tied to a string is rotated in a vertical circle at constant speed. The string is likely to break at the:

Bottom

Reason: Tension is max at bottom so apparant weight max at bottom

BANKING OF ROAD

The road is banked at a certain angle. The angle θ at which a bend should be banked is such that the centripetal force acting on the car arises entirely from a component of normal force 'F' on the road

Banking Angle:

$$\tan\theta = \frac{v^2}{gr}$$

$$\therefore \theta = \tan^{-1}\left(\frac{v^2}{gr}\right)$$

This shows that for a given radius of bend, the angle of banking is only correct for one speed

Maximum limit of 'v'

$$v = \sqrt{\mu R g}$$

$$\mu = f/F_n$$

For safe turning speed of car must be less than $\sqrt{\mu R g}$

$$\tan\theta = \mu$$

(co-efficient of friction)

Moment Of Inertia

$$I = mr^2$$

Torque

$$\tau = I\alpha$$

$$\alpha \propto \tau$$

$$\alpha \propto \frac{1}{I}$$

Rotational K.E

$$K.E_{rot} = \frac{1}{2} I\omega^2$$

Moment Of Inertia Of Different Objects:

1. Ring or thin walled cylinder : $I = mr^2$
2. Disc or solid cylinder : $I = \frac{1}{2} mr^2$
3. Disc ; $I = \frac{1}{2} m (r_2^2 + r_1^2)$
4. Solid sphere : $I = \frac{2}{5} mr^2$ Hollow sphere : $\frac{2}{3}$ m
5. Solid rod or meter stick : $I = \frac{1}{12} ml^2$
6. Rectangular plate : $I = \frac{1}{2} m (a^2 + b^2)$

Moment Of Inertia:

Reluctance of rotating rigid body to change its state of rest or uniform motion

It gives information and distribution of mass in a body.

KINETIC ENERGIES OF DIFFERENT SUBSTANCES

1. For A Ring $\quad I = mr^2$

$$K.E = \frac{1}{2} I\omega^2$$
$$= \frac{1}{2} (mr^2)\omega^2$$
$$K.E = \frac{1}{2} mv^2$$

$$K.E_{rotational} = K.E_{translational}$$
$$1 : 1$$

2. For a Disc $\quad (I = \frac{1}{2} mr^2)$

$$K.E_r = \frac{1}{2} I\omega^2$$
$$= \frac{1}{2} (\frac{1}{2} mr^2)\omega^2$$
$$= \frac{1}{2} (\frac{1}{2} mv^2)$$
$$K.E_r = \frac{1}{2} K.E_t$$

$$K.E_r = \frac{1}{2} K.E_T$$
$$\frac{K.E_r}{K.E_T} = \frac{1}{2}$$

$$1 : \frac{1}{2}$$

3. For A Solid Sphere $(I = 2/3\, mr^2)$

$$\frac{K.E_r}{K.E_T} = \frac{2}{3}$$

K.E OF A ROLLING OBJECT

$$K.E_{TOTAL} = K.E_T + K.E_r$$
$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$
$$K.E_{TOTAL} = mv^2$$

* $I_{RING} > I_{DISC} > I_{SHELL} > I_{SPHERE}$

ANGULAR DISPLACEMENT AFTER 'n' REVOLUTIONS

$$\theta = n(2\pi)$$

* A fan starts from rest moves with a constant acceleration. How many revolutions will it make in 10 seconds?

$$n = \frac{\theta}{2\pi}$$

To find θ we use

$$\theta = \cancel{\omega_i t}^{0} + \frac{1}{2}\alpha t^2$$
$$= \frac{1}{2}(2)(10)^2$$
$$\theta = 100$$

$$\boxed{n = \frac{100}{2\pi}}$$

ANGULAR MOMENTUM

$$L = rp$$

or $L = I\omega$

or $L = mr^2\omega$

Rate of change of angular momentum equals to torque

$$\frac{\Delta L}{\Delta t} = \tau$$

Relation Between L and K·E

$$L = \sqrt{2EI}$$

Relation Between L and K·E$_{rotational}$

$$K.E_r = \frac{1}{2} L\omega$$

LAW OF Conservation Of Angular Momentum

In the absence of any external torque, the angular momentum of a system remains constant

$$I_i\omega_i = I_f\omega_f$$

when $\tau = 0$

$$\frac{\Delta L}{\Delta t} = 0$$

$$\Delta L = 0$$

$$L_i = L_f = \text{constant}$$

Moment Of Inertia $\propto \frac{1}{\text{Angular Speed}}$

$$I \propto \frac{1}{\omega}$$

$$\omega \propto \frac{1}{r^2}$$

$$\frac{\omega_1}{\omega_2} = \left(\frac{r_2}{r_1}\right)^2$$

$$\frac{r_2}{r_1} = \sqrt{\frac{\omega_1}{\omega_2}}$$

ROLLING OF DISC AND HOOP DOWN THE INCLINED PLANE

Velocity of disc at bottom

$$v = \sqrt{\frac{4}{3}gh}$$

Velocity of hoop at bottom

$$v = \sqrt{gh}$$

The solid disk will move faster than hoop and will reach the bottom first.

$$F = \frac{G M_e m}{R_e^2}$$

so $W = \frac{G M_e m}{R_e^2}$ → Weight on surface of earth

* To Find Weight At Any Distance 'R''

$$W = \frac{G M_e m}{R'^2}$$

* To Find 'h' ~~when W is given~~

$$h = R' - R_e$$

Q: A satellite of ~~mass~~ weight $W_1 = 80N$ on surface of earth. In an orbit it has weight $W_2 = 20m$. Find height of orbit

$$W = \frac{G M_e m}{R_e^2}$$

$$80\,N = \frac{G M_e m}{R_e^2}$$

$$20 \times 4 = \frac{G M_e m}{R_e^2}$$

$$20 = \frac{G M_e m}{(2R_e)^2}$$

$$R' = 2R_e$$

$$R' = R_e + h$$

$$h = R' - R_e$$

$$h = 2R_e - R_e$$

$$\boxed{h = R_e}$$

* When $W' = W/4$ what will be height, h ?

$$W' = \frac{GM_e m}{4R_e^2}$$

$$= \frac{GM_e m}{(2R_e)^2}$$

$$h = R_e - 2R_e$$

$$h = R_e$$

* A satellite ~~was~~ having $W = 90N$ on surface of earth is in an orbit having radius 3 times radius of earth what will be its weight in orbit

$$W = \frac{GM_e m}{R_e^2}$$

$$W' = \frac{GM_e m}{R'^2}$$

$$W' = \frac{GM_e m}{(3R_e)^2}$$

$$W' = \frac{1}{9} W$$

$$W = \left(\frac{GM_e}{R_e^2}\right) m$$

constant $= 9.8\,ms^{-2} = g$

So $W = mg$

$$g = \frac{GM_e}{R_e^2}$$

"g" decrease as we move above the earth as well as when we move below the earth's surface. At the centre of earth 'g' becomes zero.

Value Of 'g' At Equator:

$$g' = g - R\omega\cos\theta$$

This factor is due to spin motion

g at equator < g at pole

If spin motion of earth is stopped 'g' at pole will be equal to 'g' at equator

Relation between Weight And Density

$$W = \rho G \tfrac{4}{3}\pi R_e m$$

$$W \propto \rho$$

APPARENT WEIGHT

The force which prevents the body from free-fall.

EXAMPLE : LIFT

1) At Rest

$$W = T$$
$$F_{net} = 0$$
$$a = 0$$

So real weight and apparent weight are equal when $a = 0$ i-e a body is either at rest or moving with uniform velocity

2) When Lift Accelerates Upward

$$T > W$$
$$T - W = F_{net}$$
$$T = W + F_{net}$$
$$T = mg + ma$$

3) When Lift Accelerates Downward

$$T < W$$
$$W - T = F_{net}$$
$$W - F_{net} = T$$
$$T = mg - ma$$

4) For Free Fall Motion

$$T = mg - mg$$
$$= 0$$

Apparent weight = 0 , situation known as weightlessness.

NEGATIVE WEIGHT

When a body is moving down with acceleration $a > g$ then it will have negative weight.

For downward motion:

$$T = mg - ma$$
$$= m(g-a)$$
$$T = -ve$$

SATELLITE

A smaller object revolving around a bigger object due to influence of that bigger object

To study satellite motion we need:

1. Newton's Laws of Motion
2. Newton's Law of Universal Gravitation
3. Keppler's Laws

A satellite experience no air resistance and no friction force ; ~~the~~ only 'g' acts on satellite which is perpendicular to motion of satellite so satellite starts circular motion.

ORBITAL SPEED

$$v = \sqrt{\frac{GM_e}{r}}$$

With increase in r , v decrease and so K.E decrease

CRITICAL SPEED

The speed at which we put a satellite in an orbit just above the surface of earth.

$$v = \sqrt{\frac{gR_e^2}{r}} \qquad v = \sqrt{gR}$$

RELATION BETWEEN ESCAPE VELOCITY AND ORBITAL SPEED

$$V_{esc} = \sqrt{2}\, V_o$$

GEOSTATIONARY SATELLITE

A satellite revolving around earth having time period , T = 24 hours i-e same as that of earth.

$$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \qquad \left(\Rightarrow \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3 \right)$$

ARTIFICIAL GRAVITY

Artificial Gravity is provided by spin motion which produce centripetal force that in turn produce centripetal acceleration

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{r}}$$

$$f \propto \frac{1}{\sqrt{r}}$$

For a satellite moving in an orbit of radius 'r'

$$v \propto \frac{1}{\sqrt{r}}$$

ROLLING OF DISC AND HOOP DOWN THE INCLINED PLANE

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Velocity of hoop at bottom:

$$v = \sqrt{gh}$$

The solid disc will move faster than hoop and will reach the bottom first

Read Book From Pg 132

* Orbital Radius For Geostationary Statellite

$$r_0 = 4.23 \times 10^4 \, km$$

* The whole surface of earth can be covered using 3 Geostationary satellites.

Each cover a longitude of $120°$

* Orbital Speed of moon

1 km/s

Formula For Angular Speed :-

$$\omega = \frac{2\pi}{T}$$

wher T : Time Period

* The angular speed of minute hand of a watch is

Ans: $\pi/1800$ m/s

Solution :

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3600} = \frac{\pi}{1800}$$

* When a mass is rotating in a plane about a fixed point, its angular momentum is directed along a line perpendicular to the plane of rotation.

* A boy rotates a stone in a vertical circle with the help of a string, the tension will be minimum at:

a. Lower end

✓ b. Upper end

c. Same at both ends

d. None of these

* The shortest distance of a satellite from earth is called: Perigee

* Torque on a uniformly rotating body is zero

* In circular motion of an object, the velocity and acceleration are perpendicular

* A satellite moving around the earth constitute non-inertial frame

* Values :

Critical Speed = $7.9 \ kms^{-1}$

Orbital Speed = $7.9 \ kms^{-1}$

* How much the velocity of moon should be increased so that it escapes away from Earth's gravitational field?

Ans. 41%

As $v_{esc} = \sqrt{2} \ V_{orbital}$

$V_{esc} = 1.4 \ V_{orb}$

Shortcut:

$1.41 = 1 + 0.41$

so 41% increase

(1+) means increase