CHAPTER 4
WORK AND ENERGY

* All energy forms can be classified into potential or kinetic energy

* Potential Energy → stored energy
  include chemical, gravitational, electrostatic elastic and nuclear energy.

* GRAVITATIONAL FIELD STRENGTH: (g)
  The gravitational force per unit mass on a body is known as gravitational field strength.
  Its SI Unit is $N\,kg^{-1}$ or ms^{-2}

* Conservative Field:
  The field in which work done along a closed path is zero
  e.g Elastic Force
  Magnetic Force
  Elastic Spring Force
  Gravitational Force

Non-Conservative Forces

* Frictional Forces
* Air resistance
* Tension in the string
* Normal Force
* Propulsion Force of a motor
* Propulsion Force of a rocket

All resistive forces are non-conservative

Potential Energy:

Energy in a body due to its position (w.r.t earth's surface)

$$P.E = mgh$$

(only applicable when 'g' is constant)

* Gravitational Potential Energy is a negative quantity

* Absolute Potential Energy:

The absolute P.E of an object at a certain position is the work done against gravitational force in displacing the object from that position to infinity where the force of gravity becomes zero

$$\text{Absolute P.E} = \frac{-GM_e m}{R_e}$$

GRAVITATIONAL POTENTIAL:

$$\text{Gravitational Potential} = \frac{\text{Absolute P.E}}{m}$$

$$\text{Gravitational Potential} = \frac{-GM_e}{R_e}$$

WORK

$W = \vec{F} \cdot \vec{d}$

$W = F_x d_x + F_y d_y + F_z d_z$

IN CASE OF STRETCHED STRING:

$$W = \frac{1}{2} k x^2$$

For same extension 'x'

$$W \propto k$$

For same force:

$$W \propto \frac{1}{k}$$

MAXIMUM WORK : $\theta = 0°$

$W = Fd \cos 0$

$W = Fd$

K.E : Increases

MINIMUM WORK $\theta = 90°$, $d = 0$

$W = Fd \cos 90$

$W = 0$

Work done due to centripetal force is minimum

NEGATIVE WORK $\theta = 180°$

$W = -Fd$

Required Work → e.g work done against friction

SI UNIT:

Joule = Nm

IN CGS SYSTEM

erg = dyne cm

IN FPS SYSTEM

ft-Lb

* WORK DONE BY VARIABLE FORCE

Area under the Fx curve gives work done

POWER

$$P = \frac{W}{t} \quad \text{or} \quad P = \frac{F \cdot d}{t} \quad \text{or} \quad P = \vec{F} \cdot \vec{v}$$

SI Unit : Watt → Js^{-1}

MCQ: A car moving on a level road with a constant speed of $20ms^{-1}$. If force of friction on road is 1000N, then power of the engine will be:

Ans: 20,000 W

Explanation:

Here as speed is constant means applied force and frictional force are equal so

$P = 1000 \times 20$

$P = 20,000\,W$

HORSE POWER:

If a machine is doing 550 ft-Lb work in one second, the power is equal to 1 hp.

hp: Unit of power in FPS system

$1hp \cong 746$ Watt

* kilowatt hour
Commercial unit of energy

$1\,kwh = 3.6\,MJ$

$J = 10^7\,Erg$

$1\,kwh = 3.6 \times 10^{13}\,Erg$

K.E IN TERMS OF MOMENTUM

$$K.E = \frac{P^2}{2m}$$

If P = constant

$$K.E \propto \frac{1}{m}$$

* If momentum increases by 20%, then % increase in K.E will be:

$$K.E = \left(2\Delta P \pm \frac{(\Delta P)^2}{100}\right)$$

$$= 2 \times 20 \pm \frac{20^2}{100}$$

$$= 40 \pm \frac{400}{100}$$

$K.E = 40 \pm 4$

$40 + 4 = 44\%$

$40 - 4 = 36\%$

* K.E increases by 300% the % increase in momentum will be

$\left(1+\frac{K.E}{100}\right)^{1/2} = \sqrt{1+\frac{K.E}{100}}$

$$\% P = \left[\left(1+\frac{K.E}{100}\right)^{1/2} - 1\right] \times 100\%$$

$$= \left[\left(1+\frac{300}{100}\right)^{1/2} - 1\right] \times 100\%$$

$$= \left[(1+3)^{1/2} - 1\right] \times 100\%$$

$$= (2-1) \times 100\%$$

$$= 100\%$$

* FOR PERCENT DECREASE

$$\% P = \left[1 - \left(1-\frac{K.E}{100}\right)^{1/2}\right] \times 100\%$$

* $P = \sqrt{2mK.E}$

$$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2} \times \frac{K.E_1}{K.E_2}}$$

WORK ENERGY PRINCIPLE

Work done on a body is equal to change in energy of the body

$$W = \Delta K.E + \Delta P.E$$

* A constant force of 10 N acts on a body which changes the energy from 40 J to 80 J, the displacement of the body will be :

Ans : 4m

Solution :

$$W = E$$

$$Fd = E$$

$$d = \frac{E}{F}$$

$$= \frac{40}{10}$$

$$d = 4m$$

* A car start from rest and accelerate to speed 'v' If power of the engine is 'P' then time taken will be :

Ans : $\frac{mv^2}{2P}$

Sol: $P = \frac{W}{t} = \frac{\frac{1}{2}mv^2}{t}$ $\quad t = \frac{mv^2}{2P}$

* A man of mass 50 kg can climb 100 steps in 10 sec, if each step is 10 cm high then power of the man is

Ans : 500 Watt

Sol:

$$P = \frac{W}{t} = \frac{mgh}{t} = \frac{50 \times 10 \times 10}{10} = 500 \text{Watt}$$

LAW OF CONSERVATION OF ENERGY

$E = mc^2$

Mass is a compact form of energy

Ideal Case:

Loss of P.E = Gain in K.E

Loss in K.E = Gain in P.E

* A body of mass 1 kg is placed on a smooth inclined plane at a height of 5m. When releases then velocity at bottom of the plane will b

Ans: $10 ms^{-1}$

Sol:

$$mgh = \frac{1}{2} mv^2$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

$$= \sqrt{2 \times 10 \times 5}$$

$$= \sqrt{100}$$

$$v = 10 ms^{-1}$$

* Energy of a body at a height 'h' is 'mgh'. When releases the energy at the midpoint of the motion will be

✓a) mgh b) $\frac{mgh}{2}$ c) $4mgh$

The total energy remains constant though K.E and P.E are changing

ESCAPE VELOCITY

The initial velocity, which a projectile must have at the earth's surface in order to go out of the Earth's gravitational field, is known as escape velocity

If a projectile is given an initial K.E equal to $\frac{GM_em}{R_e}$, it will reach an infinite distance from earth.

ESCAPE VELOCITY FORMULA:

For Earth:

$$V_{esc} = \sqrt{\frac{2GM_e}{R_e}}$$

General Formula:

$$V_{esc} = \sqrt{2gR_e}$$

Escape Velocity is :

* Independent of mass
* Independent of angle of projection
* Dependent on mass of planet

VALUES :

$G = 6.67 \times 10^{-11} Nm^2 kg^{-2}$

$R_e = 6.4 \times 10^6 m$

$M_e = 6 \times 10^{24} kg$

On earth :

$V_{esc} = 11.2 \times 10^3 ms^{-1}$

VALUES ON MOON :

Radius of moon : $R_m = 1.6 \times 10^6 m$

g on moon $= 1.6 ms^{-2}$

$V_{esc} = 2.3 \times 10^3 ms^{-1}$

Escape velocity of moon is less than earth bcz gravitational pull on moon is less than earth

Moon's gravity is due to earth. Moon is revolving around earth and hence its gravity is due to earth

FORMULA FOR 'g'

$$g = \frac{GM_e}{R_e^2}$$

* Work done in a closed path is always zero

* The slope of F vs x graph give same dimention as that of spring constant

* G is constant so its value remains same e.g both at earth and moon i-e $6.67 \times 10^{-11} Nm^2kg^{-2}$

* Velocity of freely falling object

$$v = \sqrt{2gh}$$

* For constant K.E, work done will be zero

* For a body moving with uniform velocity,

Distance = Displacement

* A monkey of mass 20kg is holding a rope. The rope will break when a mass of 25 kg is suspended from it. What is max acceleration which the monkey can climb up along a rope?

$$T = mg + ma$$

$$ma = T - mg$$

$$a = \frac{T}{m} - g$$

$$= \frac{25 \times g}{20} - g$$

$$= 1.25g - g$$

$$= 0.25\ g$$

$$= 0.25 \times 10$$

$$a = 2.5\ ms^{-2}$$