CHAPTER 3
MOTION AND FORCE

VELOCITY

Average Velocity → Net Displacement per unit time

$$\langle v \rangle = \frac{\Delta x}{\Delta t}$$

Average Velocity can be either positive or negative
Negative → Reverse journey

INSTANTANEOUS VELOCITY:

To calculate instantaneous velocity, position vs. time graph of an object is sketched and tangent is drawn at the point at which the instantaneous velocity is required to be calculated.

The slope of the tangent gives the magnitude of instantaneous velocity

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

- Average velocity when $v_1 \neq v_2 \neq v_3$ and $t_1 \neq t_2 \neq t_3$

$$\langle v \rangle = \frac{v_1t_1 + v_2t_2 + v_3t_3}{t_1 + t_2 + t_3}$$

* AVERAGE VELOCITY

i) $\langle v \rangle = \dfrac{v_1 + v_2}{2}$

This is used when body is moving with uniform acceleration

ii) $\langle v \rangle = \dfrac{2v_1 v_2}{v_1 + v_2}$

This is used for non-uniform acceleration when $x_1 = x_2$ and v_1, v_2 are constant

ACCELERATION

Unit: ms^{-2}, cms^{-2}, fts^{-2}

Acceleration: change in velocity

Linear Acceleration: change in magnitude of velocity

Circular or Radial or Centripetal Acceleration: change in direction of velocity

AVERAGE ACCELERATION

$$\vec{a}_{av} = \frac{\Delta v}{\Delta t}$$

POSITIVE ACCELERATION

If velocity of a body increases with time, it is called positive acceleration

* Direction is parallel to velocity

NEGATIVE ACCELERATION

If velocity of a body decreases with time it is called negative acceleration or retardation or deceleration

* Direction is ~~per~~ antiparallel to velocity

* In circular acceleration, direction of acceleration is perpendicular to velocity

MCQ: The angle b/w velocity and acceleration is
✓ i) 0° ii) 90°
ii) 180° iii) None

If "All" is given then all will be selected. But here as None is given, it means this is referring to a special case. In general we take +ve acceleration as acceleration

MCQ: When velocity become parallel to acceleration then the body will:
✓i) Speed up
ii) Slow down
iii) will be constant
iv) All

* If force is applied at certain angle then both speed as well as direction changes and the body accelerates in a curved path

TIME OF FLIGHT

$$T = \sqrt{\frac{2h}{g}}$$

Relationship b·w height and Time of flight

$$h \propto T^2$$

$$T \propto \sqrt{h}$$

Relationship b·w speed and Time of flight

* $h \propto v^2$

$$v \propto \sqrt{h}$$

* A body is projected vertically upward with a velocity of 10m/s while another body is dropped at the same time from a height of 100m. At what time will they cross eachother.

Shortcut : $T = \frac{h}{V}$

$T = \frac{100}{10} = 10\,sec$

DISTANCE TRAVELLED IN n^{th} SECOND

When a = constant and time interval is equal i.e $t_1 = t_2 = t_3$

1. $S_n = (2n-1) S_1$

2. $S_n = \frac{a}{2}(2n-1)$

3. $S_n = 10t - 5$ → (For Free Fall)

where 'n' is a particular second

DISTANCE TRAVELLED IN n-seconds

1. $S = \frac{a}{2} n^2$

2. $S = 5t^2$ → (For Free Fall)

RATIO OF DISTANCES TRAVELLED IN n^{th} Seconds

$$\frac{S_{n_1}}{S_{n_2}} = \frac{2n_1 - 1}{2n_2 - 1}$$

RATIO OF DISTANCE TRAVELLED IN n^{th} Second TO DISTANCE TRAVELLED IN n-seconds

$$\frac{S_n}{S} = \frac{2n-1}{n^2}$$

UNIFORM ACCELERATION

If velocity of the body changes by equal amounts in equal interval of time, the body is said to have uniform acceleration

For a body moving with uniform acceleration, its average and instantaneous acceleration becomes equal

IMPULSE

$J = \Delta P$ (change of momentum)

* J is positive when $P_f > P_i$ or $v_f > v_i$
(Applied Force)

* J is -ve when $P_f < P_i$ or $v_f < v_i$
(Retarding Force)

LAW OF CONSERVATION OF MOMENTUM

This law states that for an isolated system, momentum before and after collision remains constant

OR

If there is no external force applied to a system, the linear momentum of that system remains constant

In most collisions between two objects, one object slows down and loses momentum while the other object speeds up and gains momentum

COLLISION

1. Elastic Collision : Momentum → Conserved
   K.E → Conserved

2. Inelastic Collision : Momentum → Conserved
   K.E → Not conserved

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$m_1(v_1 - v_1') = m_2(v_2' - v_2)$$

$$v_1 - v_2 = v_2' - v_1'$$

HEAD-ON COLLISION

CASE 1 : When $m_1 = m_2$
Then $v_1' = v_2$ and $v_2' = v_1$

CASE 2 : If A Massive Body Collides With A Light Stationary Target / Body i-e $m_1 >> m_2$
Initially $v_1 = v$, $v_2 = 0$
After collision $v_1' \cong v$, $v_2' \cong 2v$

CASE 3 : When A Light Body Collides With A Massive Body At Rest i-e $m_2 >> m_1$

Initially $v_1 = v$, $v_2 = 0$
After Collision $v_1' = -v$, $v_2' = 0$

GRAPH

Straight / curved line which show relationship b.w two interdependent physical quantities

SLOPE

$$\text{Slope} = \tan\theta = \frac{\Delta y}{\Delta x}$$

* Slope of distance-time graph show speed
* Slope of velocity-time graph show acceleration
* Slope of acceleration-time graph show change in velocity

FORMULAS

For uniform accelerated motion:

$$S = vt$$
$$v_f = v_i + at$$
$$S = v_i t + \frac{1}{2}at^2$$
$$2aS = v_f^2 - v_i^2$$

For non-uniform Acceleration:

Area under v-t curve gives us the distance travelled

NEWTON'S LAWS

First Law : $F = 0$, $a = 0$

Second Law: $F = ma$

Third Law = $F_{12} = -F_{21}$

Law of Inertia / First Law → Gives Qualitative Definition of force

Newton's Second Law → Gives Quantitative Definition of force

MCQ: The angle b/w force and -ve acceleration is: Zero

bcz at each and every instant angle b/w F and acceleration will ~~be~~ always be zero

$N = kg\,m\,s^{-2}$
$Dyne = gm\,cm\,s^{-2}$
$Lb = sluge\,ft\,sec^{-2}$

1 sluge = 14.6 kg
1 N = 10^5 dyne

RATE OF CHANGE OF MOMENTUM

Rate of change of momentum is equal to force acting on a body

$$F = \frac{\Delta P}{\Delta t}$$

* For free fall body rate of change of momentum is equal to its weight

* Law of Conservation of Momentum is in accordance with First Law of motion

* According to law of Conservation of Linear Momentum:

$$\Delta P = 0 \quad \text{or} \quad P = \text{constant}$$

PROJECTILE MOTION

Projectile Motion is a two-dimentional motion under the action of gravity

CHARACTERISTICS

1. The horizontal component of velocity remains constant throughout the flight

$$V_{ix} = V_{fx} = \text{constant} \quad , \text{so} \quad a_x = 0$$

2. The vertical component of velocity changes uniformly and is zero at highest point
   Reason: The vertical component of a projectile changes by $9.8\ ms^{-1}$ each second

3. Sign Convention:
   i) Whichever quantity is directed upward is considered positive
   ii) Whichever quantity is quantity is directed downward is considered negative

Therefore, acceleration in projectile motion is equal to $-g$ and that is along y-axis
Reason: Anything moving along the +ve direction of displacement has a positive velocity. Since acceleration due to gravity acts downward and our chosen direction for displacement is upward, the sign convention for 'g' is -ve

HEIGHT OF PROJECTILE

$$H = \frac{v_i^2 \sin^2\theta}{2g}$$

$H \propto v_i^2$ so when initial velocity is doubled, height is increased four times

$$\frac{H_1}{H_2} = \left(\frac{\sin\theta_1}{\sin\theta_2}\right)^2$$

TIME OF FLIGHT

It is the time during which a projectile completes its journey from the point of projection to the point of impact

$$T = \frac{2v_i \sin\theta}{g}$$

$$\frac{T_1}{T_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

TIME OF SUMMIT:

(Time to reach highest point)

$$T' = \frac{T}{2} = \frac{v_i \sin\theta}{g}$$

HORIZONTAL RANGE

$$R = \frac{V_i^2 \sin 2\theta}{g}$$

If $\theta = 45°$, $R = R_{max}$

$$R_{max} = \frac{V_i^2}{g}$$

For any specific velocity of projection the range of a projectile cannot exceed from a value equal to four times of the corresponding height.

$$R \leq 4H$$

$$R_{max} = 4H$$

COMPLEMENTARY ANGLES

Experiments have proven that for projectile motion, if speed of projection and 'g' remains constant then there are always two such angles for which the projectile travels same horizontal range. These angles are complementary angles of one another. Thus a projectile will travel same range for these angles:

(80°, 10°), (70°, 20°), (60°, 30°) etc

* The velocity of projectile during the motion is never zero
At maximum height vertical component of projectile velocity becomes zero. Therefore velocity has its minimum value at maximum height

* When projectile reaches to maximum height, velocity and acceleration becomes perpendicular

VELOCITY AT MAX HEIGHT

$$V_H = V\cos\theta$$

K.E AT MAX HEIGHT

$$K.E_H = \frac{1}{2} mv^2 \cos^2\theta$$

MOMENTUM AT MAX HEIGHT

$$P_H = mv\cos\theta$$

RATIO OF K.E at point OF PROJECTION TO K.E At Max HEIGHT

$$\frac{K.E_H}{K.E} = \cos^2\theta$$

RELATION BETWEEN HEIGHT AND TIME OF FLIGHT

$$H = \frac{T^2 g}{8} \quad \text{or} \quad T = \sqrt{\frac{8H}{g}}$$

RATIO OF H to T^2

$$\frac{H}{T^2} = \frac{g}{8}$$

So 5 : 4

θ At Which Range and Height Become ~~An~~ Equal

$$\theta = \tan^{-1}(4)$$

$$\theta = 76^\circ$$

FOR COMPLEMENTARY ANGLES

$$V_i = \sqrt{2g(H_1 + H_2)}$$

RELATION B.W MOMENTUM AND K.E

$$K.E = \frac{P^2}{2m}$$

* gravitational acceleration at moon = $\frac{1}{6}$ g of earth

* 'g' is the ratio Weight/Mass
As weight = mass x g
$$g = \frac{W}{m}$$

* A mass accelerates uniformly when the resultant force acting on it is → constant but not zero

* Newton's First law provides 1st condition of equilibrium

* A ball is thrown upward along a parabolic path. What is the ball's acceleration at its highest point?
g ($9.8\ ms^{-2}$), downward

* In the absence of air resistance, if light and heavy mass are dropped to the ground, both will reach the ground at the same time

* When a body falls freely it is weightless. As there is no vertical force to balance the weight of an object

* $$V_c = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$
V_c : combined velocity after collision