CHAPTER 2

VECTORS AND EQUILIBRIUM

Resultant:

$$R = \sqrt{R_x^2 + R_y^2}$$

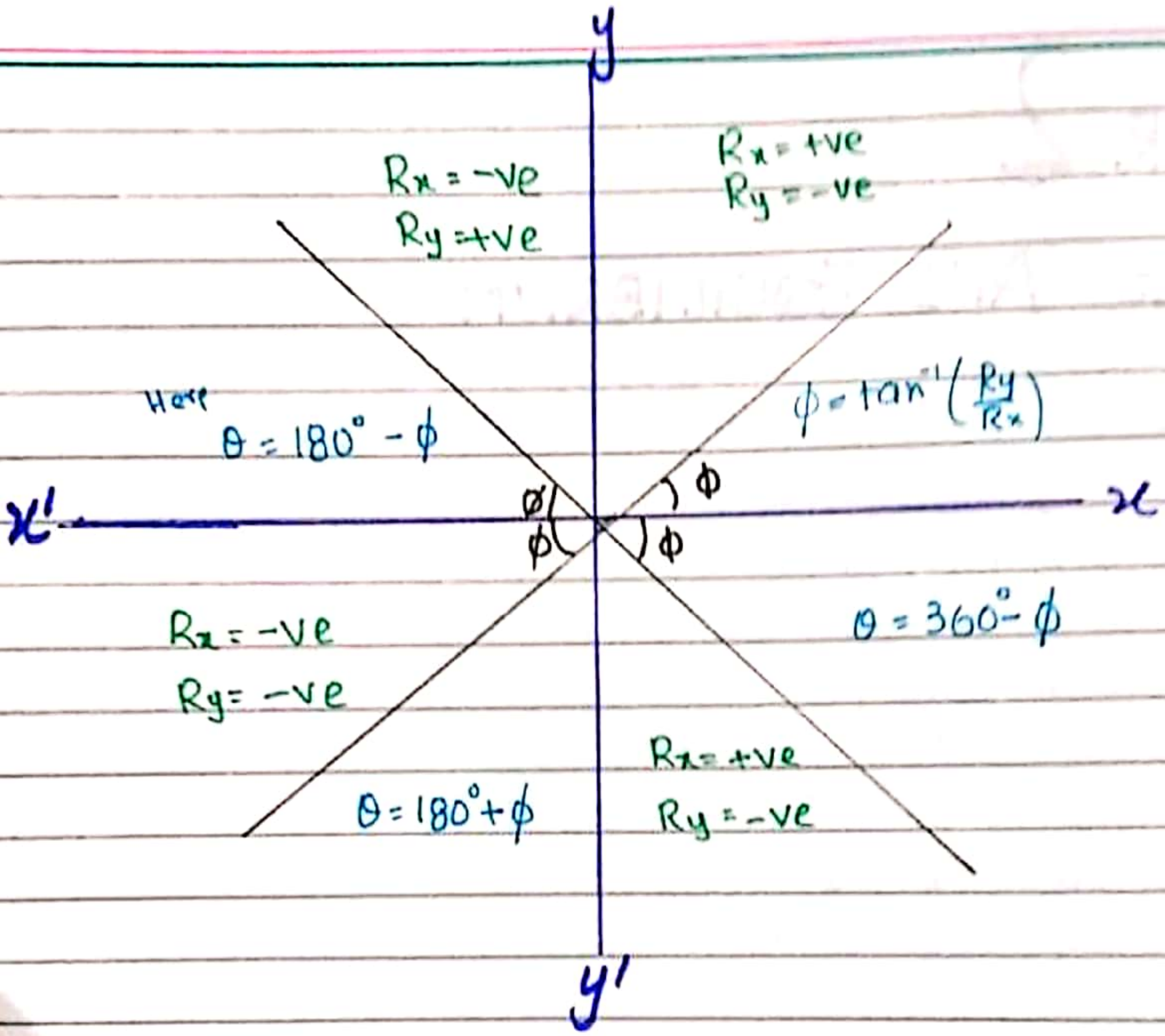
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

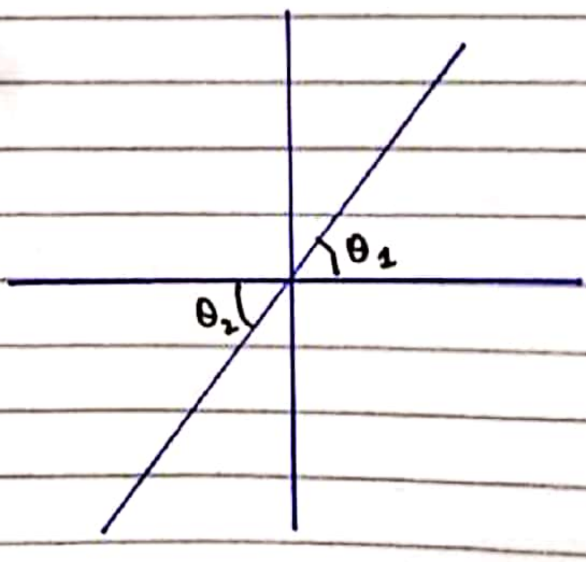
$$R_x = A_x + B_x + C_x \cdots$$
$$R_y = A_y + B_y + C_y \cdots$$

POSSIBLE RANGE OF RESULTANT

$$A - B \leq R \leq A + B$$

* Between ~~two~~ three Forces when $\Sigma F = 0$, the Resultant of any of the two forces is equal to the third force





θ_1 and θ_2 will be equal bcz these are vertical angles

UNIT VECTOR

- Its magnitude is equal to one.
- It represents direction only
- It is dimentionless

* $\hat{i}$ represent x-axis
* $\hat{j}$ represent y-axis
* $\hat{k}$ represent z-axis
* $\hat{n}$ represent direction of area
* $\hat{r}$ represent direction of position vector

DOT PRODUCT OF SIMILAR UNIT VECTORS

$i \cdot i = 1$ $\qquad$ $i \cdot i = i i \cos\theta = i^2 = 1$

↳Parallel vectors

$j \cdot j = 1$

$k \cdot k = 1$

DOT PRODUCT OF DIFFERENT UNIT VECTORS

$i \cdot j = 0$

$i \cdot k = 0$

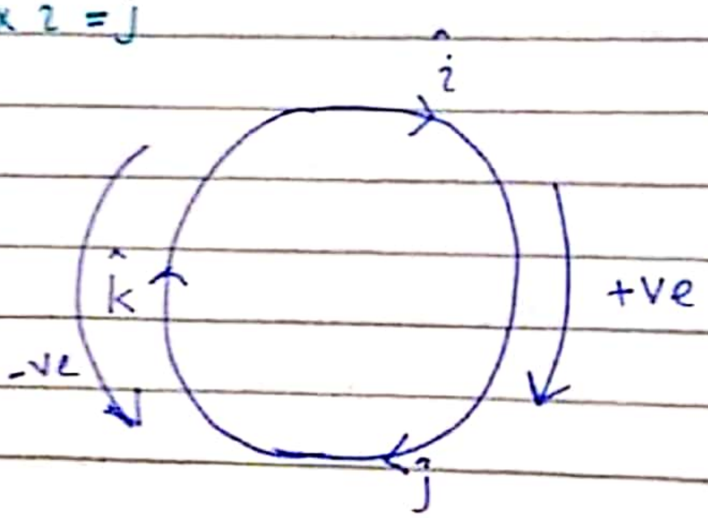
$j \cdot k = 0$

CROSS PRODUCT OF SIMILAR VECTORS

$i \times i = 0$
$j \times j = 0$
$k \times k = 0$

CROSS PRODUCT OF DIFFERENT VECTORS

$i \times j = k$
$j \times k = i$
$k \times i = j$



Similarly if we move in a.c.w direction

$k \times j = -i$
$j \times i = -k$
$i \times k = -j$

* Two vectors having equal magnitudes and having 120° between eachother then its resultant is also equal to one of theme.

i-e If $F_1 = F_2 = R$

$\theta = 120°$

* Three vectors having equal magnitudes and are acting on three sides of equilateral triangle results null vectors

If $A = B = C$ then $R = 0$

* Parallelogram Law Of Vector Addition:

If vectors represent adjacent sides of a parallelogram, the resultant is represented by diagonal

* Centre of gravity lies inside the object if it is uniform while it lies outside if it is non-uniform

* If $|A+B| = |A-B|$, then angle $= 90°$

* If vectors are parallel or anti parallel then they can be added or subtracted like scalar quantities and angle b·w them is 0° or 180°

* If $A \times B = A \cdot B$ then $\theta = 45°$

* $|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

* The minimum number of forces that keep the body in equilibrium are "Two"

* Angle b.w $A \times B$ and $B \times A$ $= 180°$

* The point at which an applied force produces linear motion but no rotatory motion is 'centre of gravity'

* Two equal, antiparallel and non concurrent forces that produce only angular acceleration are :
  Couple

* Two forces are acting on an object. The magnitude of their resultant is minimum when the angle between them is → 180°

* The dot and cross product give same result when the angle between them is 45°

* A torque applied to a rigid object always tends to produce : Rotational Acceleration

* Minimum number of unequal vectors which can give zero resultant are : Three

	0	30	45	60	90
	0	1	2	3	4
Sin	$\sqrt{0/4}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
(Divide each number by largest and take $\sqrt{}$)	0	$1/2$	$1/\sqrt{2}$ 0.707	$\sqrt{3}/2$ 0.866	1
Cos (For cos the table of sin reverses)	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan ($\tan\theta = \frac{\sin\theta}{\cos\theta}$)	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

* Angle B.w Vectors

$$\theta = \cos^{-1} \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{(A_x^2 + A_y^2 + A_z^2)(B_x^2 + B_y^2 + B_z^2)}}$$

* Finding Resultant Through Rectangular Components

$$R = \sqrt{F_x^2 + F_y^2} \quad \text{OR} \quad R = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$

* To Find Resultant Of Two Vectors

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

* To Find A Vector which is Perpendicular To A Given Vector:

i) If two perpendicular components are given exchange the co-efficients and change the sign of first component

e.g $3\hat{i} + 4\hat{j}$ is $\perp$ to $-4\hat{i} + 3\hat{j}$

ii) If two perpendicular unit vectors are given and in the options a third unit vector is given which is perpendicular to the two; then that option is selected:

e.g $3\hat{i} + 4\hat{j}$ is $\perp$ to $4\hat{k}$

* To Find If vectors are parallel

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$

If the ratio is the same then the vectors are parallel

* The resultant of cross product is always perpendicular to both vectors

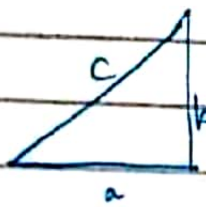
* Simple addition is only applicable when vector are parallel and the resultant will have its maximum value

* Simple subtraction is only applicable when vectors are antiparallel and the resultant will have its minimum value

* Equal vectors are always parallel.
  e.g $V_1 = nV_2$ $\theta = 0°$

* Division of vectors is not possible

* If two vectors A and B are such that $A + B = C$ and $A^2 + B^2 = C^2$ then angle b.w the vectors is → $90°$



$a + b = c$ → Head to Tail Rule
$a^2 + b^2 = c^2$ → Pythagorus Theorem

DETERMINING UNIT VECTOR

$$\hat{A} = \frac{\vec{A}}{|A|}$$

$\vec{A}$: Given

$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

e.g) What is the unit vector which has the same direction as $r = 2i - 5j$

$$|r| = \sqrt{2^2 + (-5)^2}$$
$$= \sqrt{29}$$

So

$$\hat{A} = \frac{2i - 5j}{\sqrt{29}}$$

$$\hat{A} = \frac{2i}{\sqrt{29}} - \frac{5j}{\sqrt{29}}$$

* To Find Angle b/w $A \times B$ and $A \cdot B$

$$\frac{A \times B}{A \cdot B} = \frac{\sin\theta}{\cos\theta}$$

$$\frac{A \times B}{A \cdot B} = \tan\theta$$

* The angle of vector $3i - 3j$ is
  i) 135°
  ii) 225°
  iii) 45°
  ✓ iv) 315°

Ans: As $A_x = +ve$ and $A_y = -ve$ so it lies in 4^{th} Quadrant. i-e Angle = $270^\circ - 360^\circ$

* 1^{st} Quadrant → $0 - 90^\circ$ $A_x = +ve$, $A_y = +ve$
  2^{nd} Quadrant → $90 - 180^\circ$ $A_x = -ve$, $A_y = +ve$
  3^{rd} Quadrant → $180^\circ - 270^\circ$ $A_x = -ve$, $A_y = -ve$
  4^{th} Quadrant → $270 - 360^\circ$ $A_x = +ve$, $A_y = -ve$

* Calculate angle for

$$A = -2i - 2j$$

$$\phi = \tan^{-1}\left(\frac{2}{2}\right)$$

$$\phi = \tan^{-1}(1)$$

$$\phi = 45^\circ$$

As $A_x = -ve$ and $A_y = -ve$ so θ lies in 3^{rd} Quadrant so we'll use:

$$\theta = 180^\circ + \phi$$

$$= 180^\circ + 45^\circ$$

$$\theta = 225^\circ$$

SCALAR PRODUCT

$A \cdot B = AB\cos\theta$

$A \cdot B = A(B\cos\theta)$

$B\cos\theta$: Scalar projection of B on A

$$\vec{A} \cdot \vec{B} = A(B\cos\theta)$$

$$\frac{\vec{A} \cdot \vec{B}}{A} = B\cos\theta$$

$$\frac{\vec{A}}{A} \cdot \vec{B} = B\cos\theta$$

$$\boxed{\hat{A} \cdot \vec{B} = B\cos\theta}$$

Similarly

$$\boxed{\vec{A} \cdot \hat{B} = A\cos\theta}$$

COMMUTATIVE PROPERTY

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

SCALAR PRODUCT IN COMPONENT FORM

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

MCQ:

$$\hat{i}\cdot(\hat{n}\cdot\hat{n}) = \hat{i}$$

$$\hat{j}\cdot(\hat{i}\cdot\hat{k}) = 0$$

VECTOR PRODUCT

* ANTI-COMMUTATIVE PROPERY

$$\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$$

Although : $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$

* SELF VECTOR PRODUCTS

$$\vec{A}\times\vec{A} = 0$$

MCQ:

$$j\cdot(i\times k) = j\cdot(-j)$$
$$= -(j\cdot j)$$
$$= -1$$

$j\cdot(j\times j)$ = zero

* DISTRIBUTIVE PROPERTY

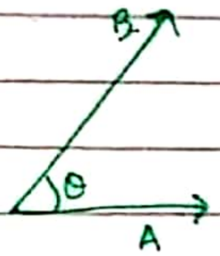
$$A\times(B+C) = (A\times B)+(A\times C)$$

FOR DIRECTION OF VECTOR PRODUCT

1. Place the vectors such that their tails are together
2. Rotate the vector which occur first in the product towards second through smaller angle

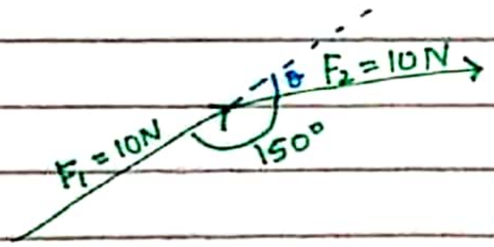
For vector product:

$$0^\circ < \theta < 180^\circ$$



$A \times B \rightarrow$ Direction outside paper

$B \times A \rightarrow$ Direction inside the paper



Direction : As tails are not together so we have to extend F_1

$\theta = 180^\circ - 150^\circ$

$\theta = 30^\circ$

Direction : Into the paper

FOR DIRECTION OF VECTOR PRODUCT

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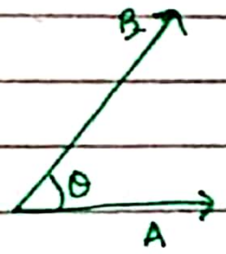

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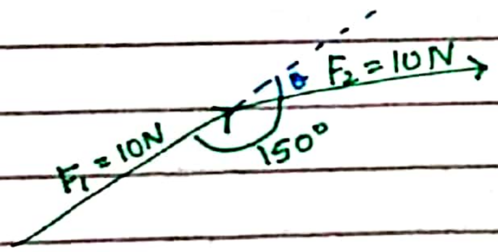

Direction : As tails are not together so we have to extend F_1

$$\theta = 180^\circ - 150^\circ$$
$$\theta = 30^\circ$$

Direction : Into the paper

EQUILIBRIUM

The state of a body in which acceleration is zero

1. TRANSLATIONAL EQUILIBRIUM

If linear acceleration, $a = 0$

a. STATIC TRANSLATIONAL EQUILIBRIUM

When body is at rest

b. DYNAMIC TRANSLATIONAL EQUILIBRIUM

When body is moving with uniform velocity

2. ROTATIONAL EQUILIBRIUM

When rotational acceleration, $\alpha = 0$

a. STATIC ROTATIONAL

Body is at rest

b. DYNAMIC ROTATIONAL

Body is rotating with constant speed

* In static translational, body is free to move
* In static rotational, body is fixed to a point

FIRST CONDITION OF EQUILIBRIUM

$\Sigma F = 0$

* First condition of equilibrium is only applicable for concurrent forces

* Concurrent Forces: Two or more than two forces whose line of action passes through a common point

* Concurrent Forces can be balanced by a single force

* If first condition of equilibrium is satisfied, body will be in translational equilibrium

SECOND CONDITION OF EQUILIBRIUM

$\Sigma \tau = 0$

$\Sigma \tau_{c.w} = \Sigma \tau_{a.c.w}$

$\tau_{net} = 0$

* Second Condition of Equilibrium is also known as principle of moment.

When clockwise torque becomes equal to anticlockwise torque then body is said to be in equilibrium

Torque acting on a uniformly rotating body is zero.

STATES OF EQUILIBRIUM

1. Stable

If centre of gravity is below axis of rotation

2. Neutral

Centre of Gravity lying on the axis of rotation. Equilibrium doesnot change even after applying force e.g ball

3. Unstable

Centre of gravity lying above the axis of rotation

TORQUE

Physical Quantity which produce rotational acceleration in the body

* Rotational Analogue of force
* Moment of force
* $\tau = \vec{r} \times \vec{F}$

* Torque acting on moon due to gravitational pull of earth is zero
  $\sin 180° = 0$

TORQUE DUE TO COUPLE

Couple : Two Forces having equal magnitude, opposite in direction, having different line of action

$\tau = l F \sin \theta$
l : couple Arm

Couple Arm : Perpendicular distance b/w the lines of action of couple forces